$B_{s(d)}$-$\bar{B}_{s(d)}$ mixing constraints on flavor changing decays of t and b quarks

Amand Faessler,[1] Thomas Gutsche,[1] Sergey Kovalenko,[2] Valery E. Lyubovitskij,[1,*] and Ivan Schmidt[2]

[1]*Institut für Theoretische Physik, Universität Tübingen, Kepler Center for Astro and Particle Physics, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

[2]*Departamento de Física, Instituto de Estudios Avanzados en Ciencias e Ingeniería y Centro Científico Tecnológico de Valparaíso (CCTVal), Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile*

(Received 16 July 2010; published 20 October 2010)

We study those dimension 6 effective operators which generate flavor-changing quark-gluon transitions of the third generation quarks, with $t \to g + u(c)$ and $b \to g + d(s)$, and which could be of interest for LHC experiments. We analyze the contribution of these operators to $B_{s(d)}$-$\bar{B}_{s(d)}$ mixing and derive limits on the corresponding effective couplings from the existing experimental data. The standard model gauge invariance relates these couplings to the couplings controlling $t \to g + u(c)$. On this basis we derive upper limits for the branching ratios of these processes. We further show that forthcoming LHC experiments might be able to probe the studied operators and the physics beyond the standard model related to them.

DOI: 10.1103/PhysRevD.82.075012 PACS numbers: 12.10.Dm, 12.60.−i, 14.40.Nd, 14.65.Bt

The top quark is the least studied of the known quarks. Being the heaviest it may offer new ways of probing physics beyond the standard model (SM). The flavor-changing quark-gluon interactions leading to the decays $t \to g + u(c)$ and $b \to g + d(s)$ are examples for processes which are extremely suppressed in the SM, and therefore experimental observation of these decays would be a smoking gun for new physics. In the following we study these interactions within an effective Lagrangian approach. The most general effective operators of the lowest dimension 6 representing these interactions are [1,2]

$$O_{qG}^{ij} = i\bar{q}_{iL}\lambda^A\gamma^\mu D^\nu q_{jL}G^A_{\mu\nu} = i(\bar{u}_{iL}\lambda^A\gamma^\mu D^\nu u_{jL} + \bar{d}_{iL}\lambda^A\gamma^\mu D^\nu d_{jL})G^A_{\mu\nu}, \quad (1)$$

$$O_{qG\phi}^{ij} = \bar{q}_{iL}\lambda^A\sigma_{\mu\nu}d_{jR}\phi G^A_{\mu\nu} \to v\bar{d}_{iL}\lambda^A\sigma_{\mu\nu}d_{jR}G^A_{\mu\nu}, \quad (2)$$

$$O_{uG}^{ij} = i\bar{u}_{iR}\lambda^A\gamma^\mu D^\nu u_{jR}G^A_{\mu\nu}, \quad (3)$$

$$O_{uG\phi}^{ij} = \bar{q}_{iL}\lambda^A\sigma_{\mu\nu}u_{jR}\tilde{\phi}G^A_{\mu\nu} \to v\bar{u}_{iL}\lambda^A\sigma_{\mu\nu}u_{jR}G^A_{\mu\nu}. \quad (4)$$

Here $G^A_{\mu\nu}$ and ϕ are the gluon and Higgs fields, respectively; $\tilde{\phi}^j = \phi_i\epsilon^{ij}$, where ϵ^{ij} is the antisymmetric tensor; q_{iL} and d_{iR} are the notations for the left-handed doublet and the right-handed down quark of the ith generation. The form of the operators to the right of the arrows is taken after spontaneous symmetry breaking. For the vacuum expectation of the Higgs field ϕ we use $v = \langle\phi\rangle = 174$ GeV [1]. The operator (1) is the only one contributing both in the up and down quark sectors. It generates interactions in both sectors with the same coupling, as required by gauge invariance. Thus bounds on the flavor violating coupling of the b quark from low-energy B-meson phenomenology would lead to the same constraints on the corresponding couplings of the top quark. The latter contributes to the $t \to g + u(c)$ transition, which could be of interest for LHC experiments. The operators (3) and (4) also contribute to these decays. However, the low-energy constraints on their couplings could be deduced only at the loop level [3]. The second operator (2), despite that it is not related to the top decays, could also be interesting from the viewpoint of $B \to X_s g$ transitions at B factories [4].

In the present paper we derive constraints on the operators (1) and (2) from the experimental data on the $B_{s(d)}$-$\bar{B}_{s(d)}$ mass differences [5–7]:

$$\begin{aligned} \Delta m_{B_d} &= 0.507 \pm 0.005 \text{ ps}^{-1}, \\ 17 \text{ ps}^{-1} &< \Delta m_{B_s} < 21 \text{ ps}^{-1}, \\ \Delta m_{B_s} &= 17.77^{+0.10}_{-0.10} \pm 0.07 \text{ ps}^{-1}. \end{aligned} \quad (5)$$

These data had a strong impact on the phenomenology for physics beyond the SM (for a review see, e.g., Refs. [8–10] and references therein).

The $B_{s(d)}$-$\bar{B}_{s(d)}$ meson mass difference is related to the matrix element of an effective Hamiltonian involving the $b \to \bar{b}$ transition [11,12]:

$$\Delta m_{B_q} = 2|\langle\bar{B}_q|\mathcal{H}_{\rm eff}|B_q\rangle|. \quad (6)$$

The operators (1) and (2) are constrained by these data since they contribute to $\mathcal{H}_{\rm eff}$. This contribution appears in second order of perturbation theory of the interaction Lagrangian

$$\mathcal{L} = \frac{1}{\Lambda^2}\sum_{i=1,2}[\alpha_{3i}O_{qG}^{3i} + \alpha_{i3}O_{qG}^{i3} + \beta_{3i}O_{dG\phi}^{3i} + \beta_{i3}O_{dG\phi}^{i3}] + \text{H.c.} \quad (7)$$

with dimensionless couplings α_{ij}, β_{ij} and the new physics scale Λ. The corresponding diagrams are shown in Fig. 1.

*On leave of absence from Department of Physics, Tomsk State University, 634050 Tomsk, Russia.

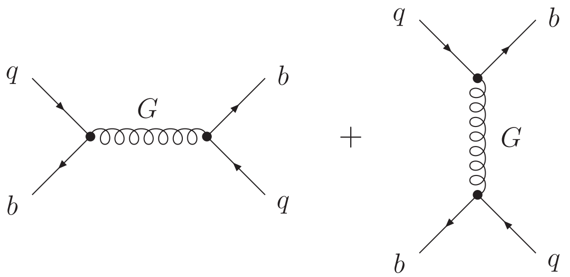

FIG. 1. Gluon-exchange diagrams contributing to the mixing of neutral B mesons.

Similar contributions to K-$\bar{K}$, D-$\bar{D}$, and B-$\bar{B}$ mixing were analyzed in the literature in relation to the SM dipenguin operators [13,14]. However, to the best of our knowledge this analysis has not yet been extended beyond the SM. Our approach is similar to Ref. [14]. It is also based on the direct analysis of the diagrams similar to Fig. 1 implying the perturbative QCD regime. This is justified by the fact that the scale of the momentum transfer through the gluon is set by the heavy quark mass $Q^2 \sim -m_b^2$, which is large in comparison with $\Lambda_{\rm QCD} \sim 200$ MeV.

With the Lagrangian (7) we obtain (see Appendix) the following Hamiltonian terms:

$$\begin{aligned}
\mathcal{H}_{\rm eff} &= \mathcal{H}^{(1)}_{\rm eff} + \mathcal{H}^{(2)}_{\rm eff},\\
\mathcal{H}^{(1)}_{\rm eff} &= c^{11}_{31} Q^{bd}_{11} + c^{12}_{31} Q^{bd}_{12} + c^{11}_{32} Q^{bs}_{11} + c^{12}_{32} Q^{bs}_{12} + \text{H.c.},\\
\mathcal{H}^{(2)}_{\rm eff} &= c^{21}_{31} Q^{bd}_{21} + c^{22}_{31} Q^{bd}_{22} + c^{23}_{31} Q^{bd}_{23} + c^{21}_{32} Q^{bs}_{21} + c^{22}_{32} Q^{bs}_{22}\\
&\quad + c^{23}_{32} Q^{bs}_{23} + \text{H.c.}
\end{aligned} \tag{8}$$

The effective couplings in Eq. (8) are expressed in terms of parameters of the underlying Lagrangian (7):

$$\begin{aligned}
c^{11}_{3i} &= c^{12}_{3i} = \frac{m_b^2}{\Lambda^4} \alpha^{*2}_{i3}, \qquad & c^{21}_{3i} &= -\frac{4v^2}{\Lambda^4} \beta^2_{3i},\\
c^{22}_{3i} &= -\frac{4v^2}{\Lambda^4} \beta^{*2}_{i3}, \qquad & c^{23}_{3i} &= -\frac{8v^2}{\Lambda^4} \beta_{3i} \beta^*_{i3}.
\end{aligned} \tag{9}$$

The operators Q^{bq}_{ij} can be expressed in terms of operators of the so-called supersymmetric basis [15–19]:

$$\begin{aligned}
O^{bq}_1 &= (\bar{b}^a_L \gamma^\mu q^a_L)(\bar{b}^b_L \gamma_\mu q^b_L), \qquad & \tilde{O}^{bq}_1 &= (\bar{b}^a_R \gamma^\mu q^a_R)(\bar{b}^b_R \gamma_\mu q^b_R),\\
O^{bq}_2 &= (\bar{b}^a_R q^a_L)(\bar{b}^b_R q^b_L), & \tilde{O}^{bq}_2 &= (\bar{b}^a_L q^a_R)(\bar{b}^b_L q^b_R),\\
O^{bq}_3 &= (\bar{b}^a_R q^b_L)(\bar{b}^b_R q^a_L), & \tilde{O}^{bq}_3 &= (\bar{b}^a_L q^b_R)(\bar{b}^b_L q^a_R),\\
O^{bq}_4 &= (\bar{b}^a_R q^a_L)(\bar{b}^b_L q^b_R), & &\\
O^{bq}_5 &= (\bar{b}^a_R q^b_L)(\bar{b}^b_L q^a_A), & &
\end{aligned} \tag{10}$$

as

$$\begin{aligned}
Q^{bq}_{11} &= \frac{4}{3} O^{bq}_1,\\
Q^{bq}_{12} &= -\frac{2}{3} O^{bq}_2 + 2 O^{bq}_3,\\
Q^{bq}_{21} &= \frac{4}{3} \tilde{O}^{bq}_1 - \frac{2}{3} \tilde{O}^{bq}_2 + 2 \tilde{O}^{bq}_3,\\
Q^{bq}_{22} &= \frac{4}{3} O^{bq}_1 - \frac{2}{3} O^{bq}_2 + 2 O^{bq}_3,\\
Q^{bq}_{23} &= -4 \tilde{O}^{bq}_2 + \frac{4}{3} \tilde{O}^{bq}_3 - \frac{2}{3} O^{bq}_4 + 2 O^{bq}_5,
\end{aligned} \tag{11}$$

where $q = d$ or s; a and b are color indices. The operators $\tilde{O}^{bq}_1$ in Eq. (10) are obtained from O^{bq}_1 by exchanging $L \leftrightarrow R$.

For the calculation of the matrix elements of the effective Hamiltonian $\mathcal{H}_{\rm eff}$ we use the relations [15–18]

$$\begin{aligned}
\langle \bar{B}_q | O^{bq}_1(\mu) | B_q \rangle &= \langle \bar{B}_q | \tilde{O}^{bq}_1(\mu) | B_q \rangle = \frac{1}{3} m_{B_q} f^2_{B_q} B^q_1(\mu),\\
\langle \bar{B}_q | O^{bq}_2(\mu) | B_q \rangle &= \langle \bar{B}_q | \tilde{O}^{bq}_2(\mu) | B_q \rangle\\
&= -\frac{5}{24} \xi_{B_q}(\mu) m_{B_q} f^2_{B_q} B^q_2(\mu),\\
\langle \bar{B}_q | O^{bq}_3(\mu) | B_q \rangle &= \langle \bar{B}_q | \tilde{O}^{bq}_4(\mu) | B_q \rangle\\
&= \frac{1}{24} \xi_{B_q}(\mu) m_{B_q} f^2_{B_q} B^q_3(\mu),\\
\langle \bar{B}_q | O^{bq}_4(\mu) | B_q \rangle &= \frac{1}{4} \xi_{B_q}(\mu) m_{B_q} f^2_{B_q} B^q_4(\mu),\\
\langle \bar{B}_q | O^{bq}_5(\mu) | B_q \rangle &= \frac{1}{12} \xi_{B_q}(\mu) m_{B_q} f^2_{B_q} B^q_5(\mu),\\
\xi_{B_q}(\mu) &= \left[\frac{m_{B_q}}{m_b(\mu) + m_q(\mu)} \right]^2,
\end{aligned} \tag{12}$$

where m_{B_q} and f_{B_q} are the mass and decay constant of the B_q meson. The $B_i(\mu)$ are the so-called "bag" parameters, which take into account the mismatch between the vacuum saturation approximation and the actual value for each of the matrix elements (see detailed discussion in Refs. [15–18]). All O^{bq}_i and $\tilde{O}^{bq}_i$ operators are renormalized at the same scale $\mu = m_b$. Because of parity conservation in strong interactions, the matrix elements of the operators $\tilde{O}^{bq}_i$ and O^{bq}_i coincide with each other [17].

In the numerical calculations we use the following set of input parameters: a renormalization scale parameter $\mu = m_b = 4.6$ GeV, quark masses $m_d(\mu) = 5.4$ MeV, $m_s(\mu) = 150$ MeV, $m_b(\mu) = 4.6$ GeV, B-meson masses and decay constants $m_{B_d} = 5.279$ GeV, $m_{B_s} = 5.3675$ GeV, $f_{B_d} = 189$ MeV, $f_{B_s} = 230$ MeV. For the bag-parameters $B_i(\mu)$ we use the values computed in lattice QCD, with Wilson fermions and with the nonperturbative regularization independent momentum subtraction (RI/MOM) renormalization scheme [18]:

$$\begin{aligned} &B_1^d(\mu) = 0.87, && B_1^s(\mu) = 0.86,\\ &B_2^d(\mu) = 0.82, && B_2^s(\mu) = 0.83,\\ &B_3^d(\mu) = 1.02, && B_3^s(\mu) = 1.03,\\ &B_4^d(\mu) = 1.16, && B_4^s(\mu) = 1.17,\\ &B_5^d(\mu) = 1.91, && B_5^s(\mu) = 1.94. \end{aligned} \tag{13}$$

Now we calculate the contribution of the effective Hamiltonian $\mathcal{H}_{\rm eff}$ of Eq. (8) to the mass difference. The result is

$$\begin{aligned} \Delta m_{B_d} = &\frac{2|\alpha_{13}|^2}{9\Lambda^4} m_{B_d} m_b(\mu) f_{B_d}^2 B_{123}^d(\mu)\\ &+ \frac{16v^2}{9\Lambda^4} m_{B_d} f_{B_d}^2 [(|\beta_{13}|^2 + |\beta_{31}|^2) B_{123}^d(\mu)\\ &+ 2|\beta_{13}||\beta_{31}| B_{45}^d(\mu)], \end{aligned} \tag{14}$$

$$\begin{aligned} \Delta m_{B_s} = &\frac{2|\alpha_{23}|^2}{9\Lambda^4} m_{B_s} m_b(\mu) f_{B_s}^2 B_{123}^s(\mu)\\ &+ \frac{16v^2}{9\Lambda^4} m_{B_s} f_{B_d}^2 [(|\beta_{23}|^2 + |\beta_{32}|^2) B_{123}^s(\mu)\\ &+ 2|\beta_{23}||\beta_{32}| B_{45}^s(\mu)], \end{aligned} \tag{15}$$

where we introduced the following notations for the combinations of bag-parameters:

$$\begin{aligned} B_{123}^q(\mu) &= \left| 2B_1^q(\mu) + \xi_{B_q}(\mu)\left(\frac{5}{8}B_2^q(\mu) + \frac{3}{8}B_3^q(\mu)\right)\right|,\\ B_{45}^q(\mu) &= \xi_{B_q}(\mu)\left| -\frac{21}{4}B_4^q(\mu) + \frac{5}{4}B_5^q(\mu)\right|. \end{aligned} \tag{16}$$

From Eqs. (14) and (15) we derive the following upper limits for the parameters of the Lagrangian (7):

$$\begin{aligned} &\frac{|\alpha_{13}|^2}{\Lambda^4} < \frac{9\Delta m_{B_d}}{2m_b^2(\mu) f_{B_d}^2 m_{B_d} B_{123}^d(\mu)},\\ &\frac{|\alpha_{23}|^2}{\Lambda^4} < \frac{9\Delta m_{B_s}}{2m_b^2(\mu) f_{B_s}^2 m_{B_s} B_{123}^s(\mu)},\\ &\frac{|\beta_{13}|^2 v^2}{\Lambda^4} = \frac{|\beta_{31}|^2 v^2}{\Lambda^4} < \frac{9\Delta m_{B_d}}{16 f_{B_d}^2 m_{B_d} B_{123}^d(\mu)},\\ &\frac{|\beta_{23}|^2 v^2}{\Lambda^4} = \frac{|\beta_{32}|^2 v^2}{\Lambda^4} < \frac{9\Delta m_{B_s}}{16 f_{B_s}^2 m_{B_s} B_{123}^s(\mu)}. \end{aligned} \tag{17}$$

Using data (5) we finally deduce the following bounds on the coupling constants α_{ij} and β_{ij}:

$$\begin{aligned} &\frac{|\alpha_{13}|}{\Lambda^2} < 3.6\times 10^{-7}\ \mathrm{GeV}^{-2},\\ &\frac{|\alpha_{23}|}{\Lambda^2} < 1.7\times 10^{-6}\ \mathrm{GeV}^{-2},\\ &\frac{|\beta_{13}|v}{\Lambda^2} = \frac{|\beta_{31}|v}{\Lambda^2} < 5.6\times 10^{-7}\ \mathrm{GeV}^{-2},\\ &\frac{|\beta_{23}|v}{\Lambda^2} = \frac{|\beta_{32}|v}{\Lambda^2} < 2.8\times 10^{-6}\ \mathrm{GeV}^{-2}. \end{aligned} \tag{18}$$

The operator of Eq. (1) contains both b and t quark terms with the same couplings to the gluon field, as dictated by the SM gauge invariance. Therefore, we can apply the above limits to the derivation of the decay rate involving the top quark flavor-changing neutral current. The corresponding formula for the decay rate is

$$\Gamma(t\to u_i g) = \frac{m_t^5}{12\pi}\frac{|\alpha_{3i} + \alpha_{i3}^*|^2}{\Lambda^4}. \tag{19}$$

Using the limits of Eq. (18) we get for the branching ratios

$$\frac{\Gamma(t\to ug)}{\Gamma_t} \le 1.6\times 10^{-3}, \qquad \frac{\Gamma(t\to cg)}{\Gamma_t} \le 3.6\times 10^{-2}, \tag{20}$$

where Γ_t is the top quark total decay width which can be approximated by the dominant mode [20]:

$$\Gamma_t \approx \Gamma(t\to bW) = 1.42|V_{tb}|^2. \tag{21}$$

These limits are to be compared to the existing CDF [21] limit derived in [22]:

$$\frac{\Gamma(t\to cg)}{\Gamma_t} \le 0.45. \tag{22}$$

For the LHC experiments preliminary estimations give [23,24]

$$\frac{\Gamma(t\to ug)}{\Gamma_t} \le 0.1, \tag{23}$$

which corresponds to a 10% precision measurement of Γ_t. As can be seen, this is not too far away from the limits of Eqs. (20), and with an improved precision on Γ_t these flavor-changing neutral current transitions could be probed by the LHC experiments.

Note that the above bounds (20) are obtained with an *ad hoc* assumption about the vanishing contribution of the operators (3) and (4) to the decay rate (19). As we mentioned in the introduction they are not constrained by low-energy processes at tree level. Therefore, taking them into account may significantly relax the constraints (20) essentially improving the prospects for searches involving the $t\to ug$ transition.

In conclusion, we analyzed a subset of effective dim$=6$ operators describing flavor-changing interactions of the 3rd generation quarks with gluons, representing one of the manifestations of physics beyond the SM. We derived their contribution to the $B_{s(d)}$-$\bar{B}_{s(d)}$ mass difference and extracted upper limits on the parameters of these operators from the experimental data. With these limits we evaluated constraints on the branching ratios of the top quark decays $t \to g + u(c)$, and found that the LHC experiments have good prospects to probe the studied operators and the new physics related to them.

This work was supported by the DFG under Contract No. FA67/31-2 and No. GRK683, by FONDECYT Projects No. 1100582 and No. 110287, and by Centro-Científico-Tecnológico de Valparaíso PBCT ACT-028. This research is also part of the European Community-Research Infrastructure Integrating Activity "Study of Strongly Interacting Matter" (HadronPhysics2, Grant Agreement No. 227431), Russian President Grant "Scientific Schools" No. 3400.2010.2, Russian Science and Innovations Federal Agency Contract No. 02.740.11.0238.

APPENDIX: DERIVATION OF THE SECOND-ORDER EFFECTIVE HAMILTIONIAN $\mathcal{H}_{\text{eff}}$

With the effective operators (1) and (2) we can generate at second order of perturbation theory matrix elements describing the s- and u-channel quark transition $q\bar{b} \to b\bar{q}$ (see diagrams in Fig. 1):

$$M^{(1)} = M_s^{(1)} + M_u^{(1)}, \qquad M^{(2)} = M_s^{(2)} + M_u^{(2)}, \tag{A1}$$

where the first and the second matrix elements correspond to the O_{qG} and $O_{dG\phi}$ operators, respectively. Here the subscripts s and u denote the s- and u-channel contributions. Below we show the results for the s-channel contributions $M_s^{(1)}$ and $M_s^{(2)}$ [the crossing u-channel results are obtained via the replacement $q_i \leftrightarrow -q_f$, $u_q(q_i) \leftrightarrow v_q(q_f)$]:

$$\begin{aligned} M_s^{(1)} &= -\frac{\Gamma^{(1)}_{\mu\alpha}}{\Lambda^4 s}\bar{u}_b(p_f)\lambda^A\gamma^\mu P_L v_q(q_f)\bar{v}_b(p_i)\lambda^A\gamma^\alpha P_L u_q(q_i), \\ \Gamma^{(1)}_{\mu\alpha} &= 4(\alpha_{3i}q_f^\nu - \alpha^*_{i3}p_f^\nu)(\alpha_{3i}q_i^\beta - \alpha^*_{i3}p_i^\beta)(g_{\nu\beta}p_\mu p_\alpha \\ &\quad + g_{\mu\alpha}p_\nu p_\beta - g_{\nu\alpha}p_\mu p_\beta - g_{\mu\beta}p_\nu p_\alpha) \end{aligned} \tag{A2}$$

and

$$\begin{aligned} M_s^{(2)} &= M^{(2)}_{s,LL} + M^{(2)}_{s,RR} + M^{(2)}_{s,LR} + M^{(2)}_{s,RL}, \\ M^{(2)}_{s,LL} &= \frac{v^2\beta^{*2}_{i3}}{\Lambda^4 s}\Gamma^{(2)}_{\mu\nu;\alpha\beta}\bar{u}_b(p_f)\lambda^A\sigma^{\mu\nu}P_L v_q(q_f)\bar{v}_b(p_i)\lambda^A\sigma^{\alpha\beta}P_L u_q(q_i), \\ M^{(2)}_{s,RR} &= \frac{v^2\beta^{2}_{3i}}{\Lambda^4 s}\Gamma^{(2)}_{\mu\nu;\alpha\beta}\bar{u}_b(p_f)\lambda^A\sigma^{\mu\nu}P_R v_q(q_f)\bar{v}_b(p_i)\lambda^A\sigma^{\alpha\beta}P_R u_q(q_i), \\ M^{(2)}_{s,LR} &= \frac{v^2\beta_{3i}\beta^{*}_{i3}}{\Lambda^4 s}\Gamma^{(2)}_{\mu\nu;\alpha\beta}\bar{u}_b(p_f)\lambda^A\sigma^{\mu\nu}P_L v_q(q_f)\bar{v}_b(p_i)\lambda^A\sigma^{\alpha\beta}P_R u_q(q_i), \\ M^{(2)}_{s,RL} &= \frac{v^2\beta_{3i}\beta^{*}_{i3}}{\Lambda^4 s}\Gamma^{(2)}_{\mu\nu;\alpha\beta}\bar{u}_b(p_f)\lambda^A\sigma^{\mu\nu}P_R v_q(q_f)\bar{v}_b(p_i)\lambda^A\sigma^{\alpha\beta}P_L u_q(q_i), \\ \Gamma^{(2)}_{\mu\nu;\alpha\beta} &= 4(g_{\nu\beta}p_\mu p_\alpha + g_{\mu\alpha}p_\nu p_\beta - g_{\nu\alpha}p_\mu p_\beta - g_{\mu\beta}p_\nu p_\alpha). \end{aligned} \tag{A3}$$

Here $P_L = (1-\gamma_5)/2$, $P_R = (1+\gamma_5)/2$; $p_i(p_f)$ and $q_i(q_f)$ are the momenta of the bottom and the light quark in the initial (final) state, respectively; p is the intermediate gluon momentum.

For the derivation of the effective operators contributing to the $B_{s(d)}$-$\bar{B}_{s(d)}$ mass difference, we consider static limit for the b quarks (their 3-momenta are equal to zero $\vec{p}_i = \vec{p}_f = 0$), which is well justified in the heavy quark limit $m_b \to \infty$. The momenta of quarks read as $p_i = (m_b, \vec{0})$, $p_f = (m_b, \vec{0})$, $q_i = (E_i, \vec{q}_i)$, and $q_f = (E_i, \vec{q}_f)$, where the energies and 3-momenta of the light quarks are of order of the constituent quark mass and are counted as $\mathcal{O}(1)$ in the heavy quark mass expansion. Then for the Mandelstam variables we get

$$\begin{aligned} &s = (p_i + q_i)^2 = (p_f + q_f)^2 = m_b^2 + m_q^2 + 2m_b E_{q_i}, \\ &t = (p_i - p_f)^2 = (q_i - q_f)^2 = 0, \\ &u = (p_i - q_f)^2 = (p_f - q_i)^2 = m_b^2 + m_q^2 - 2m_b E_{q_i}, \\ &s + t + u = 2(m_b^2 + m_q^2). \end{aligned} \tag{A4}$$

Note that the u variable on general kinematical grounds can vanish at $E_q = (m_b^2 + m_q^2)/2m_b$, leading to the pole in the u-channel diagram and introducing an uncontrollable long-distance contribution. However, it is well known that the heavy quark carries nearly the whole part of the heavy-light meson momentum, so that the meson distribution amplitude, depending on the heavy quark momentum fraction x, is strongly peaked at $x \sim 1$ [14]. The light quark momentum depends on the confinement potential and its typical average values lie around 0.5–0.6 GeV or are even smaller [25]. Therefore, the relative contribution of the kinematical configuration leading to the u pole is strongly suppressed and in a reasonable approximation we may neglect in Eqs. (A4) both the light quark mass m_q and its energy E_q. A more accurate approach, based on pQCD, using model distribution amplitudes, was applied in Ref. [14] for the evaluation of the standard model dipenguin diagrams similar to those analyzed in the present paper. For our rough estimations we simply take $s \simeq u \simeq m_b^2$.

Next, we simplify the matrix elements (A2) and (A3) using the equations of motion for quark u and antiquark v spinors, applying the heavy quark limit $m_q/m_b \ll 1$ and using the Fierz identities for the spinor and color matrices:

$$\begin{aligned}
(\gamma^\mu)_{\alpha\beta}(\gamma_\mu)_{\rho\sigma} &= \delta_{\alpha\sigma}\delta_{\rho\beta} - (\gamma_5)_{\alpha\sigma}(\gamma_5)_{\rho\beta} - \frac{1}{2}(\gamma^\mu)_{\alpha\sigma} \\
&\quad \times (\gamma_\mu)_{\rho\beta} - \frac{1}{2}(\gamma^\mu\gamma_5)_{\alpha\sigma}(\gamma_\mu\gamma_5)_{\rho\beta}, \\
(\gamma^\mu P_{R/L})_{\alpha\beta}(\gamma_\mu P_{R/L})_{\rho\sigma} &= -(\gamma^\mu P_{R/L})_{\alpha\sigma}(\gamma_\mu P_{R/L})_{\rho\beta}, \\
(\gamma^\mu P_{R/L})_{\alpha\beta}(\gamma_\mu P_{L/R})_{\rho\sigma} &= 2(P_{L/R})_{\alpha\sigma}(P_{R/L})_{\rho\beta}
\end{aligned} \tag{A5}$$

and

$$\begin{aligned}
\delta_{ab}\delta_{cd} &= \frac{1}{3}\delta_{ad}\delta_{cb} + \frac{1}{2}\lambda^A_{ad}\lambda^A_{cb}, \\
\lambda^A_{ab}\lambda^A_{cd} &= \frac{16}{9}\delta_{ad}\delta_{cb} - \frac{1}{3}\lambda^A_{ad}\lambda^A_{cb}.
\end{aligned} \tag{A6}$$

With these identities we derive relations between the different four-quark operators under investigation and express them in terms of the supersymmetric basis:

$$\begin{aligned}
\bar{b}^a_L\gamma^\mu\lambda^A_{ab}q^b_L\bar{b}^c_L\gamma_\mu\lambda^A_{cd}q^d_L &= \frac{4}{3}\bar{b}^a_L\gamma^\mu q^a_L\bar{b}^b_L\gamma_\mu q^b_L = \frac{4}{3}O^{bq}_1, \\
\bar{b}^a_R\gamma^\mu\lambda^A_{ab}q^b_R\bar{b}^c_R\gamma_\mu\lambda^A_{cd}q^d_R &= \frac{4}{3}\bar{b}^a_R\gamma^\mu q^b_R\bar{b}^b_R\gamma_\mu q^a_R = \frac{4}{3}\tilde{O}^{bq}_1, \\
\bar{b}^a_R\gamma^\mu\lambda^A_{ab}q^b_R\bar{b}^c_L\gamma_\mu\lambda^A_{cd}q^d_L &= -2\bar{b}^a_R\lambda^A_{ab}q^d_L\bar{b}^c_L\lambda^A_{cd}q^b_R \\
&= \frac{4}{3}O^{bq}_5 - 4O^{bq}_4, \\
\bar{b}^a_R\lambda^A_{ab}q^b_L\bar{b}^c_R\lambda^A_{cd}q^d_L &= -\frac{2}{3}O^{bq}_2 + 2O^{bq}_3, \\
\bar{b}^a_L\lambda^A_{ab}q^b_R\bar{b}^c_L\lambda^A_{cd}q^d_R &= -\frac{2}{3}\tilde{O}^{bq}_2 + 2\tilde{O}^{bq}_3, \\
\bar{b}^a_R\lambda^A_{ab}q^b_L\bar{b}^c_L\lambda^A_{cd}q^d_R &= -\frac{2}{3}O^{bq}_4 + 2O^{bq}_5.
\end{aligned} \tag{A7}$$

Note that the contributions of the s- and u-channel diagrams are equal to each other within the approximations used in our analysis. Finally, we derive the expressions for the effective Hamiltonians $\mathcal{H}^{(1)}_{\rm eff}$ and $\mathcal{H}^{(2)}_{\rm eff}$ corresponding to the matrix elements $M^{(1)}$ and $M^{(2)}$. The result is shown in Eqs. (8)–(11).

[1] W. Buchmuller and D. Wyler, Nucl. Phys. **B268**, 621 (1986).
[2] J. A. Aguilar-Saavedra, Nucl. Phys. **B812**, 181 (2009).
[3] A. Faessler, T. Gutsche, J. C. Helo, S. Kovalenko, V. E. Lyubovitskij, and I. Schmidt (unpublished).
[4] J. Drobnak, S. Fajfer, and J. F. Kamenik, Phys. Rev. Lett. **104**, 252001 (2010).
[5] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[6] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **97**, 021802 (2006).
[7] A. Abulencia *et al.* (CDF-Run II Collaboration), Phys. Rev. Lett. **97**, 062003 (2006); M. Jones (CDF-Run II Collaboration), in *Intersections of Particle and Nuclear Physics: 9th Conference on the Intersections of Particle and Nuclear Physics*, AIP Conf. Proc. No. 870 (AIP, New York, 2006), p. 116.
[8] A. Lenz and U. Nierste, J. High Energy Phys. 06 (2007) 072
[9] P. Ko and J. H. Park, Phys. Rev. D **80**, 035019 (2009).
[10] M. Blanke, A. J. Buras, B. Duling, S. Gori, and A. Weiler, J. High Energy Phys. 03 (2009) 001.
[11] L. B. Okun, *Leptons and Quarks* (North-Holland, Amsterdam, Netherlands, 1982), p. 361.
[12] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
[13] J. O. Eeg and I. Picek, Z. Phys. C **39**, 521 (1988); Nucl. Phys. **B292**, 745 (1987); Phys. Lett. **160B**, 154 (1985).
[14] A. A. Petrov, Phys. Rev. D **56**, 1685 (1997).
[15] F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, Nucl. Phys. **B477**, 321 (1996).
[16] C. R. Allton *et al.*, Phys. Lett. B **453**, 30 (1999).
[17] M. Ciuchini *et al.*, J. High Energy Phys. 10 (1998) 008.
[18] D. Becirevic, V. Gimenez, G. Martinelli, M. Papinutto, and J. Reyes, J. High Energy Phys. 04 (2002) 025.
[19] W. Altmannshofer, A. J. Buras, S. Gori, P. Paradisi, and D. M. Straub, Nucl. Phys. **B830**, 17 (2010).
[20] D. Chakraborty, J. Konigsberg, and D. L. Rainwater, Annu. Rev. Nucl. Part. Sci. **53**, 301 (2003); W. Wagner,

Rep. Prog. Phys. **68**, 2409 (2005); A. Denner and T. Sack, Nucl. Phys. **B358**, 46 (1991); G. Eilam, R. R. Mendel, R. Migneron, and A. Soni, Phys. Rev. Lett. **66**, 3105 (1991); A. Czarnecki and K. Melnikov, Nucl. Phys. **B544**, 520 (1999); K. G. Chetyrkin, R. Harlander, T. Seidensticker, and M. Steinhauser, Phys. Rev. D **60**, 114015 (1999).

[21] J. Incandela (CDF Collaboration), Nuovo Cimento Soc. Ital. Fis. **109A**, 741 (1996).

[22] T. Han, K. Whisnant, B. L. Young, and X. Zhang, Phys. Rev. D **55**, 7241 (1997).

[23] P. M. Ferreira, O. Oliveira, and R. Santos, Phys. Rev. D **73**, 034011 (2006).

[24] Y. K. Kim (CDF and D0 Collaborations), Int. J. Mod. Phys. A **17**, 3099 (2002).

[25] M. B. Voloshin, Surv. High Energy Phys. **8**, 27 (1995); D. S. Hwang, C. S. Kim, and W. Namgung, Phys. Rev. D **54**, 5620 (1996); F. De Fazio, Mod. Phys. Lett. A **11**, 2693 (1996).