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$X(1835)$ and the new resonances $X(2120)$ and $X(2370)$ observed by the BES collaboration

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We calculate the decay widths of both the second and the third radial excitations of η and η' within the framework of the $^3P_0$ model. After comparing the theoretical decay widths and decay patterns with the available experimental data of $\eta(1760)$, $X(1835)$, $X(2120)$, and $X(2370)$, we find that the interpretation of $\eta(1760)$ and $X(1835)$ as the second radial excitation of η and η' crucially depends on the measured mass and width of $\eta(1760)$, which is still controversial experimentally. We suggest that there may be sizable $p\bar{p}$ content in $X(1835)$. $X(2120)$ and $X(2370)$ cannot be understood as the third radial excitations of η and η'. $X(2370)$ is probably a mixture of $\eta'(4^1S_0)$ and glueball.

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I. INTRODUCTION

$X(1835)$ was first observed by BESII in the $\eta'\pi\pi$ invariant mass spectrum in the process $J/\psi \to \gamma\pi^+\pi^-\eta'$ with a statistical significance of 7.7σ. The fit with the Breit-Wigner function yields mass $M = 1833.7 \pm 6.1(\text{stat}) \pm 2.7(\text{syst})\ \text{MeV}/c^2$, width $\Gamma = 67.7 \pm 20.3(\text{stat}) \pm 7.7(\text{syst})\ \text{MeV}/c^2$, and the product branching fraction $\text{Br}(J/\psi \to \gamma X(1835))\text{Br}(X(1835) \to \pi^+\pi^-\eta') = (2.2 \pm 0.4(\text{stat}) \pm 0.4(\text{syst})) \times 10^{-4}$ [1]. Recently $X(1835)$ has been confirmed by BESIII collaboration in the same process with statistical significance larger than 25σ, and its mass and width are fitted to be $M = 1838.1 \pm 2.8$ MeV and $\Gamma = 179.5 \pm 9.1$ MeV. Moreover, two new resonances are reported, which are denoted as $X(2120)$ and $X(2370)$, respectively. Their masses and widths are determined to be $M_{X(2120)} = 2124.8 \pm 5.6$ MeV, $\Gamma_{X(2120)} = 101 \pm 14$ MeV, $M_{X(2370)} = 2371.0 \pm 6.4$ MeV, and $\Gamma_{X(2370)} = 108 \pm 15$ MeV [2,3].

The experimental observation of $X(1835)$ stimulated a number of theoretical speculations about its underlying structure. Some interpret $X(1835)$ as a $p\bar{p}$ bound state [4–8], a glueball candidate [9–12], or the radial excitation of η' [13,14], and some others interpret it as final state interaction or a rescattering effect [15–17]. Naïvely the observation of $X(2120)$ and $X(2370)$ seems to indicate that all the three resonances $X(1835)$, $X(2120)$, and $X(2370)$ are possibly the radial excitations of η or η'; they jump to the ground state η' through emitting two π [18]. Moreover, we note that before we consider the exotic structure hypothesis for some newly observed resonance, it is necessary to study whether the assignment of a conventional hadron is possible. Consequently, we shall investigate in this paper whether $X(1835)$, $X(2120)$, and $X(2370)$ could be canonical $q\bar{q}$ pseudoscalar mesons.

It is well known that there are nine pseudoscalar mesons π, K, η, and η', which form a good nonet in the limit of SU(3) flavor symmetry. From the Particle Data Group (PDG) [19], we see that the first radial excitations of these pseudoscalars have been well established; concretely, they are $\pi(1300)$, $K(1460)$, $\eta(1295)$, and $\eta(1475)$. As a result, if the three resonances $X(1835)$, $X(2120)$, and $X(2370)$ are canonical $q\bar{q}$ pseudoscalar mesons, the natural assignment would be $\eta(1760)$ and $X(1835)$ as the second radial excitation of η and η', $X(2120)$ and $X(2370)$ as the third radial excitation of η and η', respectively. In this work, we shall study the decays of these four resonances under the above assignment within the framework of the $^3P_0$ model. Our goal is to shed some light on the nature of these structures by comparing the predictions for the hadronic decay widths with the available experimental data.

This paper is organized as follows. First, we review the $^3P_0$ model briefly in Sec. II. The flavor mixing between the η and η' radial excitation and the allowed decay modes is presented in Sec. III. The Okubo-Zweig-Iizuka rule allowed strong decays of $\eta(1760)$, $X(1835)$, $X(2120)$, and $X(2370)$, which are studied in Sec. IV. Finally, we present our conclusions and some discussions in Sec. V.

II. REVIEW OF THE $^3P_0$ MODEL

The $^3P_0$ model for the decay of a $q\bar{q}$ meson A to mesons $B + C$ was proposed by Micu [20] and developed by Le Yaouanc *et al.* [21–23]. The $^3P_0$ model assumes that strong decay takes place via the creation of a pair of quark and antiquark with $J^{PC} = 0^{++}$ from the vacuum. The created quark pair together with the quark and antiquark in the initial meson recombine to final state mesons in two ways, as shown in Fig. 1. The decay amplitude is proportional to the overlap of wave functions (including spatial, spin, flavor, and color wave functions) between the initial state, the created quark pair, and the final state. The $^3P_0$ model has been widely applied to meson and baryon strong decays, with considerable success [21–28]. In this work, we shall use the diagrammatic technique developed in Ref. [23] to derive the amplitudes and the $^3P_0$ matrix elements. In this formalism, the $^3P_0$ model describes the strong decay process using a $q\bar{q}$ pair-production Hamiltonian, which is the nonrelativistic limit of

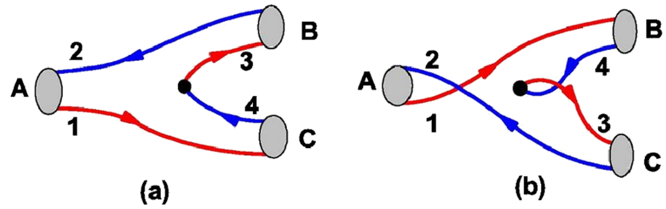

FIG. 1 (color online). Two possible diagrams contributing to the meson decay $A \to BC$ in the $^3P_0$ model.

$$H_I = g\int d^3\mathbf{x}\,\bar{\psi}(x)\psi(x), \tag{1}$$

where ψ is a Dirac quark field and g is the coupling constant. The pair-production component of the $^3P_0$ Hamiltonian H_I can be written in terms of creation operators as

$$H_I = \sum_{s\bar{s}}\int d^3\mathbf{k}\,\frac{gm_q}{E_q}[\bar{u}_{\mathbf{k}s}v_{-\mathbf{k}\bar{s}}]b^\dagger_{\mathbf{k}s}d^\dagger_{-\mathbf{k}\bar{s}}, \tag{2}$$

where $b^\dagger_{\mathbf{k}s}$ creates a quark with momentum $\mathbf{k}$ and spin s, $d^\dagger_{-\mathbf{k}\bar{s}}$ creates an antiquark with momentum $-\mathbf{k}$, and spin $\bar{s}$, m_q is the mass of the created quark and antiquark. We note that each effective $3P_0$ quark pair-production vertex is associated with the factor $\frac{gm_q}{E_q}[\bar{u}_{\mathbf{k}s}v_{-\mathbf{k}\bar{s}}]$. We assume nonrelativistic $q\bar{q}$ wave function for the initial and final mesons,

$$|A\rangle = \int d^3\mathbf{p_1}\int d^3\mathbf{p_2}\Psi_{n_AL_AM_{L_A}}\times\left(\frac{m_2\mathbf{p_1}-m_1\mathbf{p_2}}{m_1+m_2}\right)\delta(\mathbf{P_A}-\mathbf{p_1}-\mathbf{p_2})|q_1(\mathbf{p_1})\bar{q}_2(\mathbf{p_2})>, \tag{3}$$

with explicit spin and flavor wave functions which are of the usual nonrelativistic quark model forms. n_A denotes the radial quantum number of meson A composed of quark q_1 and antiquark $\bar{q}_2$ with momentum $\mathbf{p_1}$ and $\mathbf{p_2}$ and mass m_1 and m_2, respectively, and $\mathbf{P_A}$ is the momentum of meson A. The wave functions of the final state mesons B and C can be written out directly in the same way. The spatial wave function Ψ is generally taken to be the simple harmonic oscillator (SHO) wave function. The SHO wave function enables analytical calculation of the decay amplitude, and it turned out to be a good approximation. Even if we use a more realistic wave function, the predictions would not be improved systematically due to the inherent uncertainties of the $^3P_0$ model. In momentum-space, the SHO wave function reads

$$\Psi_{nLM_L}(\mathbf{p}) = \frac{(-1)^n(-i)^L}{\beta^{3/2}}\sqrt{\frac{2n!}{\Gamma(n+L+3/2)}}\left(\frac{p}{\beta}\right)^L\times\exp\left(-\frac{p^2}{2\beta^2}\right)L_n^{L+1/2}\left(\frac{p^2}{\beta^2}\right)Y_{LM_L}(\Omega_p), \tag{4}$$

where β is the harmonic oscillator parameter, $Y_{LM_L}(\Omega_p)$ is the spherical harmonic function, and $L_n^{L+1/2}(\frac{p^2}{\beta^2})$ is the Laguerre polynomial.

One can now straightforwardly evaluate the Hamiltonian H_I matrix element for the decay $A \to B + C$ in terms of overlap integrals,

$$\begin{aligned}\langle BC|H_I|A\rangle_a &= I_{\rm signature}(a)I_{\rm flavor}(a)I_{\rm spin+space}(a)\delta\times(\mathbf{P_A}-\mathbf{P_B}-\mathbf{P_C})\\ \langle BC|H_I|A\rangle_b &= I_{\rm signature}(b)I_{\rm flavor}(b)I_{\rm spin+space}(b)\delta\times(\mathbf{P_A}-\mathbf{P_B}-\mathbf{P_C}),\end{aligned} \tag{5}$$

where the signature phase $I_{\rm signature}$ is equal to -1 for both diagrams (a) and (b) due to quark operator anticommutation. Starting from the flavor wave functions, we can directly obtain the flavor overlap factors $I_{\rm flavor}(a)$ and $I_{\rm flavor}(b)$ which result from contracting the explicit flavor states corresponding to Figs. 1(a) and 1(b), and listed in Table II for the decay modes concerned here. In the rest frame of meson A, the overlap integrals $I_{\rm spin+space}(a)$ and $I_{\rm spin+space}(b)$ are explicitly given by

$$\begin{aligned}I_{\rm spin+space}(a) &= \int d^3\mathbf{k}\,\Psi_{n_AL_AM_{L_A}}(\mathbf{k}-\mathbf{P_B})\Psi^*_{n_BL_BM_{L_B}}\times\left(\mathbf{k}-\frac{m_3}{m_2+m_3}\mathbf{P_B}\right)\Psi^*_{n_CL_CM_{L_C}}\\ &\quad\times\left(\mathbf{k}-\frac{m_3}{m_1+m_3}\mathbf{P_B}\right)g\frac{m_3}{E_3}[\bar{u}_{\mathbf{k}s_{q_3}}v_{-\mathbf{k}s_{\bar{q}_4}}],\\ I_{\rm spin+space}(b) &= \int d^3\mathbf{k}\,\Psi_{n_AL_AM_{L_A}}(\mathbf{k}+\mathbf{P_B})\Psi^*_{n_BL_BM_{L_B}}\times\left(\mathbf{k}+\frac{m_3}{m_1+m_3}\mathbf{P_B}\right)\Psi^*_{n_CL_CM_{L_C}}\\ &\quad\times\left(\mathbf{k}+\frac{m_3}{m_2+m_3}\mathbf{P_B}\right)g\frac{m_3}{E_3}[\bar{u}_{\mathbf{k}s_{q_3}}v_{-\mathbf{k}s_{\bar{q}_4}}],\end{aligned} \tag{6}$$

where the relevant spin factor has been omitted, and $E_3 = \sqrt{\mathbf{k}^2+m_3^2}$ is the energy of the created quark. We note that the spin factor and the labels s_{q_3} and $s_{\bar{q}_4}$ depend on the reaction considered; generally, the spin indexes s_{q_3} and $s_{\bar{q}_4}$ associated with Figs. 1(a) and 1(b) are different. As a result, the amplitude for the meson decay $A \to B + C$ is

$$\begin{aligned}\mathcal{M}(A\to B+C) &= I_{\rm signature}(a)I_{\rm flavor}(a)I_{\rm spin+space}(a)\\ &\quad + I_{\rm signature}(b)I_{\rm flavor}(b)I_{\rm spin+space}(b)\\ &\equiv h_{fi}.\end{aligned} \tag{7}$$

Taking into account the phase space, we get the differential decay rate

$$\frac{d\Gamma_{A\to BC}}{d\Omega} = 2\pi \frac{PE_BE_C}{M_A}|h_{fi}|^2, \quad (8)$$

where E_B and E_C are the energy of the mesons B and C, respectively, and P is the momentum of the final state mesons in the rest frame of meson A,

$$P = \sqrt{[M_A^2 - (M_B + M_C)^2][M_A^2 - (M_B - M_C)^2]}/(2M_A), \quad (9)$$

where M_A, M_B, and M_C are the masses of the mesons A, B, and C, respectively. To compare with experiments, we transform the amplitude h_{fi} into the partial wave amplitude $\mathcal{M}_{LS}$ by the recoupling calculation [29]. Then the decay width is

$$\Gamma(A \to B + C) = 2\pi \frac{PE_BE_C}{M_A}\sum_{LS}|\mathcal{M}_{LS}|^2. \quad (10)$$

The pair production parameter g and the harmonic oscillator parameter β are fitted to the strong decay data, and they are found to be roughly flavor independent for decays involving production of $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ pairs. The typical values obtained from computation of light meson decays are $g = 0.334$ GeV and $\beta = 0.4$ GeV [23–25], assuming simple harmonic oscillator wave functions with a global scale, and they are frequently adopted by the literature. However, different quark models find different values of β (mostly in the range of $0.35 \sim 0.45$ GeV), so that there is the question of the sensitivity of our results to β; we will address this issue below. The masses of constituent quarks are chosen to be $m_u = m_d = 0.33$ GeV and $m_s = 0.55$ GeV, as usual. The masses used are the experimental values of well-established candidates, which are taken from the PDG [19]. Moreover, we have ignored the mass difference between the members of the same isospin multiplet. For the isoscalar, we assume ideal mixing $|\varphi_{\rm nonstrange}\rangle = 1/\sqrt{2}|u\bar{u} + d\bar{d}\rangle$, $|\varphi_{\rm strange}\rangle = |s\bar{s}\rangle$, where, except for the ground state pseudoscalar, we choose $|\eta\rangle = \cos\phi_p|u\bar{u} + d\bar{d}\rangle/\sqrt{2} - \sin\phi_p|s\bar{s}\rangle$ and $|\eta'\rangle = \sin\phi_p|u\bar{u} + d\bar{d}\rangle/\sqrt{2} + \cos\phi_p|s\bar{s}\rangle$ with the mixing angle $\phi_p = 39.2°$ [30]. The kaons and their excitations are not charge conjugation eigenstates, so mixing can occur among states with the same J^P that are forbidden for neutral states. For example, the $J^P = 1^+$ axial vector kaon mesons $K_1(1273)$ and $K_1(1402)$ are coherent superpositions of quark model ${}^3P_1$ and ${}^1P_1$ states [25],

$$|K_1(1273)\rangle = \sqrt{\frac{2}{3}}|{}^1P_1\rangle + \sqrt{\frac{1}{3}}|{}^3P_1\rangle|K_1(1402)\rangle$$
$$= -\sqrt{\frac{1}{3}}|{}^1P_1\rangle + \sqrt{\frac{2}{3}}|{}^3P_1\rangle. \quad (11)$$

III. MIXING BETWEEN THE η AND η' EXCITATIONS AND THE ALLOWED DECAY MODES

The radial excitation of η and η' are both isoscalar states with the same J^{PC} so there will be mixing between them. Consequently, the physical states are the mixture of the SU(3) flavor octet and singlet

$$\begin{aligned} |\eta(n^1S_0)\rangle &= \cos\theta|\eta_8(n^1S_0)\rangle - \sin\theta|\eta_0(n^1S_0)\rangle \\ |\eta'(n^1S_0)\rangle &= \sin\theta|\eta_8(n^1S_0)\rangle + \cos\theta|\eta_0(n^1S_0)\rangle, \end{aligned} \quad (12)$$

where n represents the radial quantum number and $|\eta_8(n^1S_0)\rangle$ and $|\eta_0(n^1S_0)\rangle$ are the octet and singlet states, respectively,

$$\begin{aligned} |\eta_8(n^1S_0)\rangle &\equiv \frac{1}{\sqrt{6}}|u\bar{u} + d\bar{d} - 2s\bar{s}\rangle \\ |\eta_0(n^1S_0)\rangle &\equiv \frac{1}{\sqrt{3}}|u\bar{u} + d\bar{d} + s\bar{s}\rangle. \end{aligned} \quad (13)$$

In order to explicitly exhibit the $u\bar{u} + d\bar{d}$ and $s\bar{s}$ components, we shall choose the so-called nonstrange-strange basis in this work:

$$\begin{aligned} |\eta(n^1S_0)\rangle &= \cos\phi|\eta_{\rm NS}(n^1S_0)\rangle - \sin\phi|\eta_{\rm S}(n^1S_0)\rangle \\ |\eta'(n^1S_0)\rangle &= \sin\phi|\eta_{\rm NS}(n^1S_0)\rangle + \cos\phi|\eta_{\rm S}(n^1S_0)\rangle, \end{aligned} \quad (14)$$

where $|\eta_{\rm NS}(n^1S_0)\rangle = |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$ and $|\eta_{\rm S}(n^1S_0)\rangle = |s\bar{s}\rangle$, and mixing angle ϕ is related to θ via $\phi = \theta + \arctan\sqrt{2} \simeq \theta + 54.7°$. We note that the mixing angle ϕ (or θ) is less constrained phenomenologically; its concrete value has to be determined experimentally. It is well known that $\eta - \eta'$ mixing has been measured by various means; however, there is still large uncertainty. As a result, we shall take the mixing angle ϕ as an undetermined parameter in the following analysis, and the dependence of the amplitudes and widths on ϕ will be considered.

We present the selection rules for the two-body decays of η and η' excitations in Table I. For the specific final states listed in Table I, all four states $\eta(1760)$, $X(1835)$, $X(2120)$, and $X(2370)$ could decay into them, if the process is not forbidden kinetically. We note that decays into two pseudoscalar or two scalar mesons are forbidden by parity and charge conjugation conservation. Moreover, the G parity forbids the decay processes $X \to \rho\pi$, $X \to \omega\eta$, $X \to \rho a_1(1260)$, $X \to \rho a_2(1320)$, $X \to \omega(\phi)f_1(1285)$, $X \to \omega(\phi)f_1(1420)$, $X \to \omega(\phi)f_2(1270)$, and $X \to \omega(\phi)f_2'(1525)$, where X denotes $\eta(1760)$, $X(1835)$, $X(2120)$, or $X(2370)$.

TABLE I. Allowed decay modes of η and η' radial excitations.

Decay modes	Final states
$X \to 1^1S_0 + 1^3S_1$	KK^*
$X \to 2^1S_0 + 1^3S_1$	$K(1460)K^*$
$X \to 1^1S_0 + 2^3S_1$	$KK^*(1410)$
$X \to 1^1S_0 + 1^3P_0$	$\pi a_0(1450)$, $KK_0^*(1430)$, $\eta f_0(1370)$, $\eta f_0(1710)$, $\eta' f_0(1370)$
$X \to 1^1S_0 + 1^3P_2$	$\pi a_2(1320)$, $KK_2^*(1430)$, $\eta f_2(1270)$, $\eta f_2'(1525)$, $\eta' f_2(1270)$
$X \to 1^1S_0 + 1^3D_1$	$KK^*(1680)$
$X \to 1^1S_0 + 1^3D_3$	$KK_3^*(1780)$
$X \to 1^3S_1 + 1^3S_1$	$\rho\rho$, K^*K^*, $\omega\omega$, $\phi\phi$
$X \to 1^1S_0 + 2^3S_1$	$\rho\rho(1450)$, $K^*K^*(1410)$, $\omega\omega(1420)$
$X \to 1^3S_1 + 1^3P_1$	$\rho b_1(1235)$, $K^*K_1(1273)$, $\omega h_1(1170)$, $\omega h_1(1380)$, $\phi h_1(1170)$, $\phi h_1(1380)$
$X \to 1^3S_1 + 1^3P_1$	$K^*K_1(1402)$
$X \to 1^3S_1 + 1^3P_2$	$K^*K_2^*(1430)$

IV. STRONG DECAYS OF $\eta(1760)$, $X(1835)$, $X(2120)$, AND $X(2370)$

Following the procedures presented in the previous sections, the total decay rate is given by the Hamilton matrix element squared, multiplied by the phase space, and summed over all final spin and charge states. Since we neglect mass splitting within the isospin multiplet, to sum over all channels, one should multiply the partial width into the specific charge channel by the flavor multiplicity factor $\mathcal{F}$ in Table II. This $\mathcal{F}$ factor also incorporates the statistical factor 1/2 if the final state mesons B and C are identical.

A. Decays of $\eta(1760)$ and $X(1835)$

The experimental evidence for $\eta(1760)$ is controversial: Its existence evidence was first reported by the Mark III collaboration in the J/ψ radiative decays to $\omega\omega$ [31] and $\rho\rho$ [32]. Then it was further studied by the DM2 and BES collaborations. The various experimental results associated with $\eta(1760)$ are summarized in Table III. Obviously, there

TABLE II. Relevant flavor weight factors for η and η' excitation decays, where $|X_0\rangle = |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$ and $|X_s\rangle = |s\bar{s}\rangle$, $(n\bar{s}) = (u\bar{s})$ or $(d\bar{s})$ for n being an up and down quark, respectively. $(n\bar{n}')_{I=1} = (u\bar{d})$, $[(u\bar{u}) - (d\bar{d})]/\sqrt{2}$ and $(d\bar{u})$, $(n\bar{n})_{I=0} = [(u\bar{u}) + (d\bar{d})]/\sqrt{2}$.

Generic Decay	Example	$I_{\rm flavor}(a)$	$I_{\rm flavor}(b)$	$\mathcal{F}$
$X_0 \to (n\bar{s})(s\bar{n})$	$X_0 \to K^+ + K^-$	0	$-1/\sqrt{2}$	2
	$X_0 \to K^{*+} + K^-$	0	$-1/\sqrt{2}$	4
$X_s \to (n\bar{s})(s\bar{n})$	$X_s \to K^+ + K^-$	-1	0	2
	$X_s \to K^{*+} + K^-$	-1	0	4
$X_0 \to (u\bar{d})(d\bar{u})$	$X_0 \to \pi^+ + a^-$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	3
	$X_0 \to \pi^+ + \pi^-$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	3/2
	$X_0 \to \rho^+ + \rho^-$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	3/2
	$X_0 \to \rho^+ + \rho^-(1450)$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	3
$X_s \to (u\bar{d})(d\bar{u})$	$X_s \to \pi^+ + a^-$	0	0	0
	$X_s \to \pi^+ + \pi^-$	0	0	0
	$X_s \to \rho^+ + \rho^-$	0	0	0
	$X_s \to \rho^+ + \rho^-(1450)$	0	0	0
$X_0 \to (\frac{u\bar{u}+d\bar{d}}{\sqrt{2}})(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}})$	$X_0 \to \eta_0 + f_0$	$1/\sqrt{2}$	$1/\sqrt{2}$	1
	$X_0 \to \omega + \omega$	$1/\sqrt{2}$	$1/\sqrt{2}$	1/2
$X_s \to (\frac{u\bar{u}+d\bar{d}}{\sqrt{2}})(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}})$	$X_s \to \eta_0 + f_0$	0	0	0
$X_0 \to (s\bar{s})(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}})$	$X_0 \to \eta_s + f_0$	0	0	0
$X_s \to (s\bar{s})(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}})$	$X_s \to \eta_s + f_0$	0	0	0
$X_0 \to (\frac{u\bar{u}+d\bar{d}}{\sqrt{2}})(s\bar{s})$	$X_0 \to \eta_0 + f_s$	0	0	0
$X_s \to (\frac{u\bar{u}+d\bar{d}}{\sqrt{2}})(s\bar{s})$	$X_s \to \eta_0 + f_s$	0	0	0
$X_0 \to (s\bar{s})(s\bar{s})$	$X_0 \to \eta_s + f_s$	0	0	0
$X_s \to (s\bar{s})(s\bar{s})$	$X_s \to \eta_s + f_s$	1	1	1
	$X_s \to \phi + \phi$	1	1	1/2

TABLE III. Summary of $\eta(1760)$ measurements.

Experiment	Mass (MeV)	Width (MeV)	Production
DM2 [33]	1760 ± 11	60 ± 16	$J/\psi \to \gamma\eta(1760)$, $\eta(1760) \to \rho\rho$
BES [34]	$1744 \pm 10 \pm 15$	$244^{+24}_{-21} \pm 25$	$J/\psi \to \gamma\eta(1760)$, $\eta(1760) \to \omega\omega$
PDG [19]	1756 ± 9	96 ± 70	

are big differences between the different measurements of $\eta(1760)$ width. In this work, both the mass and width are taken to be the world average listed in PDG. For $\eta(1760)$ and $X(1835)$ as the second radial excitation of η and η', the allowed decay channels, the corresponding decay amplitudes and partial widths are shown in Tables IV and V, respectively. Clearly, the decay amplitudes and widths depend strongly on the mixing angle ϕ, and measurements of any or several of the larger decay modes will provide constrained tests of the hypothesis and measurement of the mixing angle. We believe that the better way to determine the mixing angle is to compare the ratio between KK^* and $\rho\rho$ partial widths with experimental data, if both $\eta(1760)$ and $X(1835)$ are indeed conventional quark model states as assumed above. This is because the pair production parameter g cancels out in this ratio; consequently, there is less systematic uncertainty than in the decay rates. The partial widths of $\eta(1760)$ and $X(1835)$ as functions of the flavor mixing angle ϕ for fixed $\beta = 0.4$ GeV are shown in Fig. 2. Evidently large couplings of $\eta(1760)$ to $\rho\rho$ and $\omega\omega$ follow from moderate mixing, which could explain the observation of $\eta(1760)$ in the $\rho\rho$ and $\omega\omega$ final states by the DM2 and BES collaborations. Furthermore, we note that $\eta(1760)$ should have a sizable branching ratio into $\pi a_2(1320)$. Therefore, we urge experimentalists to search for $\eta(1760)$ in the process $J/\psi \to \gamma\eta(1760) \to \gamma\pi a_2(1320)$, which is an important test of our scenario. Obviously, the partial width of $X(1835) \to \eta f_2(1270)$ is particularly small. Taking into account the variation of the mixing angle ϕ, we find that $X(1835)$ may have a large branching ratio into $\rho\rho$, $\pi a_2(1320)$, and KK^* final states under the assignment of $\eta'(3^1S_0)$ $q\bar{q}$ meson. Experimental search for $X(1835)$ in these modes is suggested.

We note that the mixing angle appearing in the $\eta(1760)$ and $X(1835)$ flavor wave functions is the same, so that a large number of decays are correlated, as is demonstrated

TABLE IV. Partial widths of $\eta(1760)$ as the second radial excitation of η, where ϕ_1 is the flavor mixing angle, $s \equiv \sin\phi_1$, and $c \equiv \cos\phi_1$. Note that a factor of i has been suppressed in all odd partial wave amplitudes.

	$\eta(1760) = \cos\phi_1 \vert u\bar{u} + d\bar{d}\rangle/\sqrt{2} - \sin\phi_1 \vert s\bar{s}\rangle$	
Modes	Γ(MeV)	Amps. (GeV$^{-1/2}$)
KK^*	$30.85c^2 + 89.25cs + 64.56s^2$	$\mathcal{M}_{11} = 0.074c + 0.11s$
$\rho\rho$	$155.36c^2$	$\mathcal{M}_{11} = 0.30c$
$\omega\omega$	$49.50c^2$	$\mathcal{M}_{11} = -0.30c$
$\pi a_0(1450)$	$44.36c^2$	$\mathcal{M}_{00} = -0.22c$
$\pi a_2(1320)$	$60.93c^2$	$\mathcal{M}_{22} = -0.17c$
Total	$341.00c^2 + 89.25cs + 64.56s^2$	

TABLE V. Partial widths of $X(1835)$ as the second radial excitation of η', where ϕ_1 is the mixing angle, $s \equiv \sin\phi_1$, and $c \equiv \cos\phi_1$. Note that a factor of i has been suppressed in all odd partial wave amplitudes.

	$X(1835) = \sin\phi_1 \vert u\bar{u} + d\bar{d}\rangle/\sqrt{2} + \cos\phi_1 \vert s\bar{s}\rangle$	
Modes	Γ(MeV)	Amps. (GeV$^{-1/2}$)
KK^*	$43.18c^2 - 79.34cs + 36.45s^2$	$\mathcal{M}_{11} = -0.080c + 0.074s$
$\rho\rho$	$188.24s^2$	$\mathcal{M}_{11} = 0.30s$
K^*K^*	$29.84c^2 + 23.08cs + 4.46s^2$	$\mathcal{M}_{11} = 0.16c + 0.060s$
$\omega\omega$	$62.23s^2$	$\mathcal{M}_{11} = -0.30s$
$\pi a_0(1450)$	$47.85s^2$	$\mathcal{M}_{00} = -0.18s$
$\pi a_2(1320)$	$136.48s^2$	$\mathcal{M}_{22} = -0.22s$
$\eta f_2(1270)$	$0.051s^2$	$\mathcal{M}_{22} = 0.014s$
Total	$73.02c^2 - 56.26cs + 475.76s^2$	

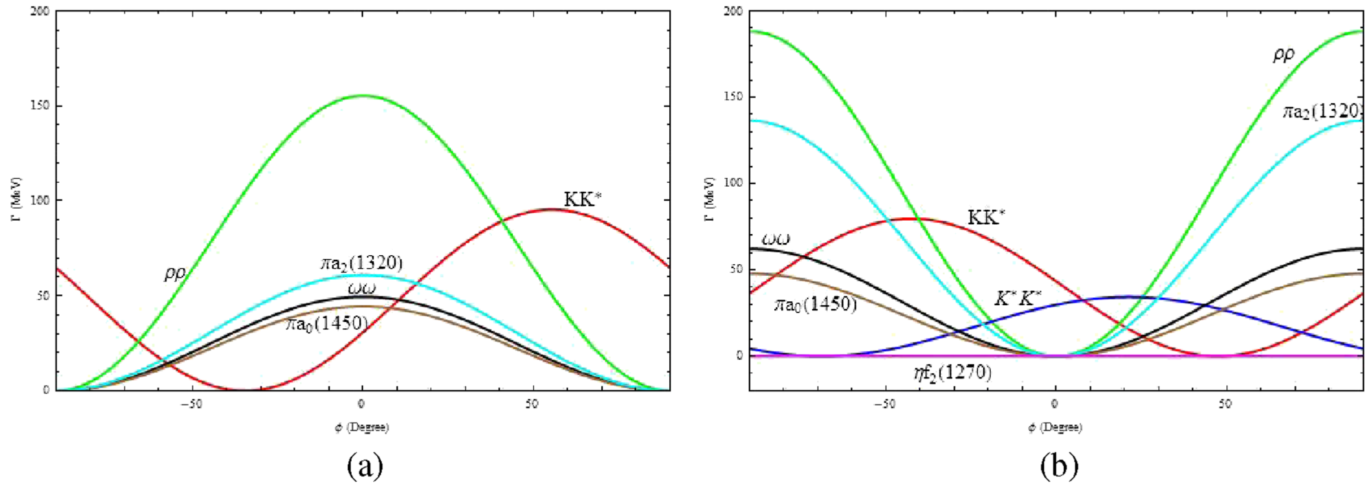

FIG. 2 (color online). Partial decay widths of $\eta(1760)$ and $X(1835)$ vs the flavor mixing angle ϕ. The left figure is for $\eta(1760)$, and the right figure is for $X(1835)$.

in Tables IV and V. It is essential to investigate whether there exists a certain region of mixing angle ϕ so that the predicted widths of $\eta(1760)$ and $X(1835)$ agree with the experimental observations within acceptable errors. Since the masses of $\eta(1760)$ and $X(1835)$ are measured precisely enough, their central values are used, and the harmonic oscillator parameter β is allowed to vary in the range of $0.35 \sim 0.45$ GeV. The total decay widths of $\eta(1760)$ and $X(1835)$ as functions of the mixing angle are shown in Fig. 3. Obviously, we see that there is not a value of ϕ such that the resulting widths of both $\eta(1760)$ and $X(1835)$ lie in the experimentally allowed range. The same conclusion is reached for the $\eta(1760)$ parameters measured by the DM2 collaboration, as is obvious from Fig. 4(a). It seems inappropriate to identify $\eta(1760)$ and $X(1835)$ as the second radial excitation of η and η', simultaneously. However, if

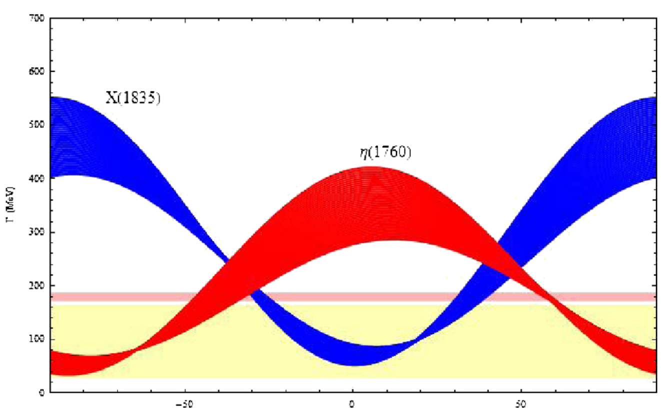

FIG. 3 (color online). Total decay widths of $\eta(1760)$ and $X(1835)$ as functions of the mixing angle, where the harmonic oscillator parameter β varies from 0.35 to 0.45 GeV. The horizontal (yellow and pink) bands denote the experimental errors of $\eta(1760)$ and $X(1835)$ widths, where the mass and width of $\eta(1760)$ is taken to be the world average.

we take the $\eta(1760)$ mass and width to be the BES measurement [the corresponding decay widths are shown in Fig. 4(b)], we find that the theoretical widths of $\eta(1760)$ and $X(1835)$ could be consistent with experimental data for the mixing angle ϕ in the range $-31° \sim -24°$ or $30° \sim 40°$. Therefore, experimentally resolving the inconsistency between the DM2 and the BES collaboration results for $\eta(1760)$ is important for understanding $X(1835)$. Remembering that $X(1835)$ is close to the threshold of proton and antiproton (i.e., $p\bar{p}$), "dressing" of the $q\bar{q}$ singlet meson $\eta'(3^1S_0)$ with two $q\bar{q}$ pairs can create nucleon-antinucleon, and final state interactions enhance the probability of this transition. In this way, the $\eta'(3^1S_0)$ meson can mix with the $p\bar{p}$ final state, and its wave function develops a sizable $p\bar{p}$ component. As a result, $X(1835)$ could be a mixture of $\eta'(3^1S_0)$ and a $p\bar{p}$ molecule; then all experimental facts related to $X(1835)$ can be understood qualitatively. To shed light on the nature of $X(1835)$, a coupled channel analysis necessary, but this topic is beyond the scope of the present work.

B. Decays of $X(2120)$ and $X(2370)$

Under the assignment of $\eta(4^1S_0)$ and $\eta'(4^1S_0)$ $q\bar{q}$ mesons, the decay amplitudes and partial widths of $X(2120)$ and $X(2370)$ in terms of the general mixing angles are shown in Tables VI and VII, respectively. Since $X(2120)$ and $X(2370)$ have larger masses, many strong decay modes are allowable. $X(2120)$ has large partial widths to $\pi a_2(1320)$ and $KK^*(1410)$, and the main decay modes of $X(2370)$ are $\rho\rho(1450)$, $\rho b_1(1235)$, $\omega\omega(1420)$, $\pi a_2(1320)$, $K^*K^*(1410)$ and $KK_2^*(1430)$; the corresponding partial widths as functions of the flavor mixing angle ϕ are shown in Fig. 5. It is obvious that the modes $\pi a_2(1320)$ and $KK^*(1410)$ are important to the search for $X(2120)$ because, if the signal of $X(2120)$ is accidently suppressed in one mode, it should be evident in the other. The same is

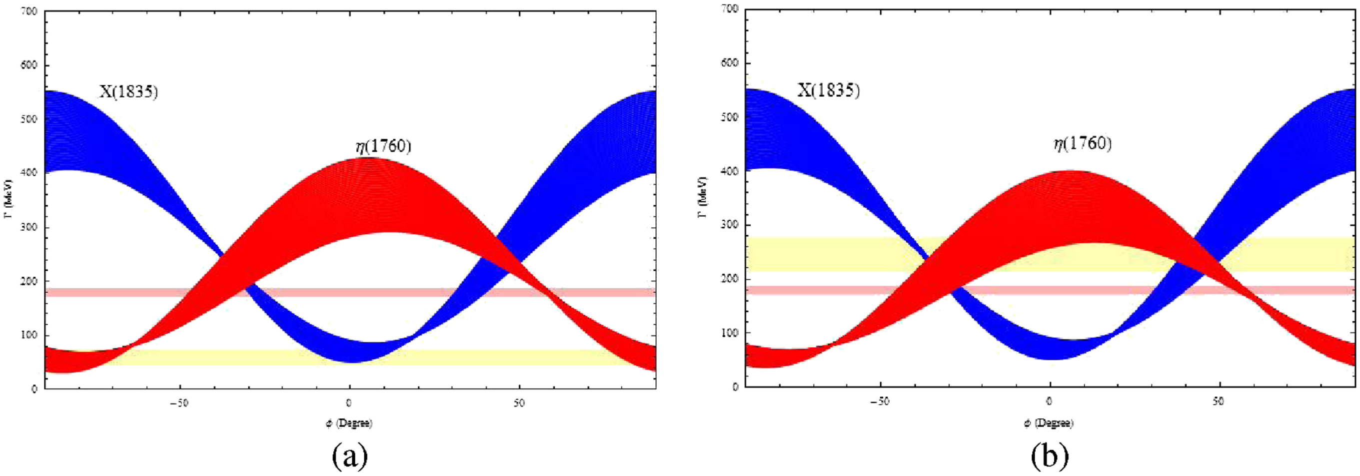

FIG. 4 (color online). The same details as in Fig. 3. Here, the parameters of $\eta(1760)$ are chosen to be the measurements of the DM2 (left) and BES (right) collaborations, respectively.

true for the $X(2370)$ decay modes $\rho\rho(1450)$ and $K^*K^*(1410)$. We note that the branching ratios of the KK^* and $\rho\rho$ modes in both $X(2120)$ and $X(2370)$ decays are predicted to be smaller, despite their larger phase space, as they are accidentally near the node in the $^3P_0$ decay amplitude for the physical masses and $\beta = 0.4$ GeV. The $X(2120)$ decay modes $\rho b_1(1235)$ and $\omega h_1(1170)$ are interesting because the two subamplitudes $^1S_0$ and $^5D_0$ are comparable and individually proportional to $\cos\phi$; thus the D/S amplitude ratio is independent of the mixing angle ϕ. The measurement of the $\rho b_1(1235)$ and $\omega h_1(1170)$ subamplitudes directly accesses $\cos\phi$, although these modes may be too weak to allow this measurement. Similarly, $X(2370)$ can decay into $\rho b_1(1235)$, $\omega h_1(1170)$, $K^*K_1(1273)$, and $K^*K_1(1402)$ in both S-wave and D-wave, and the D/S ratio for the latter two modes strongly depends on the flavor mixing angle.

For the harmonic oscillator parameter β in the range of $0.35 \sim 0.45$ GeV, the total widths of $X(2120)$ and $X(2370)$ against the flavor mixing angle ϕ are displayed in Fig. 6. Since $X(2370)$ has many decay modes, its width is predicted to be larger than 300 MeV. Even if the width is overestimated by a factor of 2, it is still larger than the measured value. Obviously, there does not exist an appropriate value of the mixing angle such that the theoretically predicted widths of $X(2120)$ and $X(2370)$ lie in

TABLE VI. Partial widths of $X(2120)$ as the third radial excitation of η, where $s \equiv \sin\phi_2$ and $c \equiv \cos\phi_2$, ϕ_2 is the mixing angle between $X(2120)$ and $X(2370)$, and the factor of i has been suppressed in all odd partial wave amplitudes.

$X(2120) = \cos\phi_2\lvert u\bar{u} + d\bar{d}\rangle/\sqrt{2} - \sin\phi_2\lvert s\bar{s}\rangle$		
Mode	Γ(MeV)	Amps. (GeV$^{-1/2}$)
KK^*	$2.39c^2 - 9.26cs + 8.98s^2$	$\mathcal{M}_{11} = -0.015c + 0.029s$
$KK^*(1410)$	$31.90c^2 + 106.14cs + 88.29s^2$	$\mathcal{M}_{11} = 0.082c + 0.14s$
$\pi a_0(1450)$	$0.013c^2$	$\mathcal{M}_{00} = -0.0018c$
$KK_0^*(1430)$	$2.98c^2 - 3.70cs + 1.15s^2$	$\mathcal{M}_{00} = 0.025c - 0.016s$
$\eta f_0(1370)$	$1.61c^2$	$\mathcal{M}_{00} = -0.036c$
$\pi a_2(1320)$	$149.68c^2$	$\mathcal{M}_{22} = 0.16c$
$KK_2^*(1430)$	$3.72c^2 - 23.98cs + 38.63s^2$	$\mathcal{M}_{22} = 0.028c - 0.092s$
$\eta f_2(1270)$	$21.55c^2$	$\mathcal{M}_{22} = -0.12c$
$\eta f_2'(1525)$	$0.25s^2$	$\mathcal{M}_{22} = -0.021s$
$\rho\rho$	$1.09c^2$	$\mathcal{M}_{11} = -0.017c$
K^*K^*	$5.60c^2 - 6.20cs + 1.71s^2$	$\mathcal{M}_{11} = -0.038c + 0.021s$
$\phi\phi$	$4.67s^2$	$\mathcal{M}_{11} = -0.097s$
$\omega\omega$	$0.52c^2$	$\mathcal{M}_{11} = 0.021c$
$\rho b_1(1235)$	$50.80c^2$	$\mathcal{M}_{00} = 0.082c$
		$\mathcal{M}_{22} = 0.093c$
$\omega h_1(1170)$)	$22.75c^2$	$\mathcal{M}_{00} = -0.051c$
		$\mathcal{M}_{22} = -0.12c$
Total	$294.60c^2 + 63.00cs + 143.69s^2$	

TABLE VII. Partial widths of $X(2370)$ as the third radial excitation of η', where $s \equiv \sin\phi_2$ and $c \equiv \cos\phi_2$, ϕ_2 is the mixing angle between $X(2120)$ and $X(2370)$, and the factor of i has been suppressed in all odd partial wave amplitudes.

$X(2370) = \sin\phi_2\lvert u\bar{u} + d\bar{d}\rangle/\sqrt{2} + \cos\phi_2\lvert s\bar{s}\rangle$		
Modes	Γ(MeV)	Amps. (GeV$^{-1/2}$)
KK^*	$14.33c^2 + 1.94cs + 0.066s^2$	$\mathcal{M}_{11} = -0.032c - 0.0022s$
$K(1460)K^*$	$17.65c^2 - 13.33cs + 2.52s^2$	$\mathcal{M}_{11} = -0.10c + 0.036s$
$KK^*(1410)$	$2.64c^2 - 22.59cs + 48.37s^2$	$\mathcal{M}_{11} = -0.017c + 0.075s$
$\pi a_0(1450)$	$14.83s^2$	$\mathcal{M}_{00} = -0.047s$
$KK_0^*(1430)$	$9.41c^2 + 0.15cs + 0.00064s^2$	$\mathcal{M}_{00} = -0.033c - 0.00027s$
$\eta f_0(1370)$	$1.74s^2$	$\mathcal{M}_{00} = 0.028s$
$\eta' f_0(1370)$	$5.61s^2$	$\mathcal{M}_{00} = -0.084s$
$\eta f_0(1710)$	$2.61c^2$	$\mathcal{M}_{00} = 0.055c$
$\pi a_2(1320)$	$127.35s^2$	$\mathcal{M}_{22} = 0.12s$
$KK_2^*(1430)$	$66.79c^2 + 78.46cs + 23.04s^2$	$\mathcal{M}_{22} = 0.089c + 0.052s$
$\eta f_2(1270)$	$36.25s^2$	$\mathcal{M}_{22} = -0.12s$
$\eta' f_2(1270)$	$7.50s^2$	$\mathcal{M}_{22} = -0.072s$
$\eta f_2'(1525)$	$11.60c^2$	$\mathcal{M}_{22} = 0.083c$
$KK^*(1680)$	$9.30c^2 - 6.24cs + 1.05s^2$	$\mathcal{M}_{11} = -0.047c + 0.016s$
$KK_3^*(1780)$	$2.12c^2 - 0.72cs + 0.061s^2$	$\mathcal{M}_{33} = -0.026c + 0.0044s$
$\rho\rho$	$12.56s^2$	$\mathcal{M}_{11} = 0.050s$
K^*K^*	$9.15c^2 - 10.41cs + 2.96s^2$	$\mathcal{M}_{11} = 0.040c - 0.023s$
$\phi\phi$	$3.88c^2$	$\mathcal{M}_{11} = 0.059c$
$\omega\omega$	$3.84s^2$	$\mathcal{M}_{11} = -0.048s$
$\rho\rho(1450)$	$435.60s^2$	$\mathcal{M}_{11} = 0.34s$
$K^*K^*(1410)$	$161.05c^2 + 138.87cs + 29.94s^2$	$\mathcal{M}_{11} = 0.21c + 0.089s$
$\omega\omega(1420)$	$165.10s^2$	$\mathcal{M}_{11} = -0.33s$
$\rho b_1(1235)$	$189.78s^2$	$\mathcal{M}_{00} = -0.028s$ $\mathcal{M}_{22} = 0.17s$
$K^*K_1(1273)$	$12.76c^2 + 26.21cs + 14.04s^2$	$\mathcal{M}_{00} = -0.0087c$ $\mathcal{M}_{22} = 0.042c + 0.045s$
$\omega h_1(1170)$	$68.36s^2$	$\mathcal{M}_{00} = 0.038s$ $\mathcal{M}_{22} = -0.16s$
$K^*K_1(1402)$	$24.78c^2 - 18.75cs + 17.85s^2$	$\mathcal{M}_{00} = 0.034c - 0.066s$ $\mathcal{M}_{22} = -0.070c$
$K^*K_2^*(1430)$	$9.01c^2 - 4.51cs + 0.56s^2$	$\mathcal{M}_{22} = -0.052c + 0.013s$
Total	$357.08c^2 + 169.10cs + 1208.97s^2$	

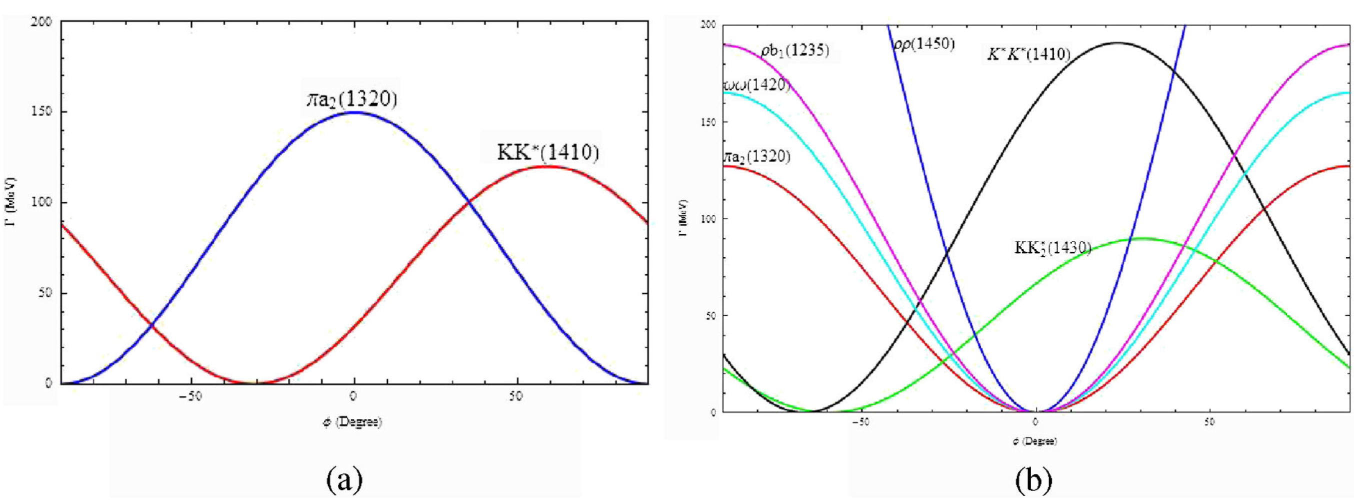

FIG. 5 (color online). Partial decay widths of the leading decay modes of $X(2120)$ and $X(2370)$ vs the flavor mixing angle ϕ. The left figure shows $X(2120)$, and the right figure shows $X(2370)$.

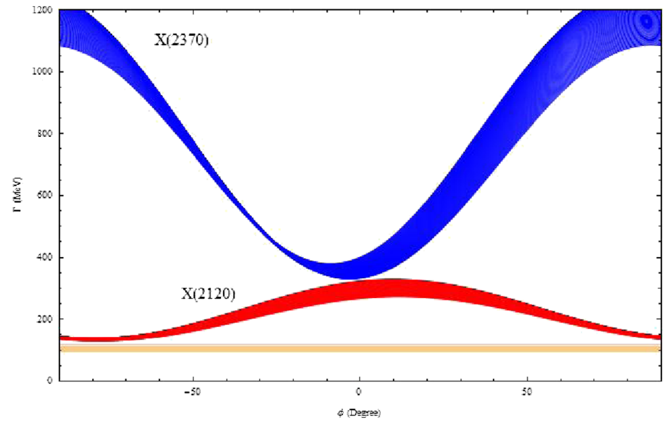

FIG. 6 (color online). Total decay widths of $X(2120)$ and $X(2370)$ vs the flavor mixing angle ϕ, where the harmonic oscillator parameter β varies from 0.35 to 0.45 GeV. The horizontal (yellow and pink) bands represent the widths of $X(2120)$ and $X(2370)$, respectively, as fitted by the BES collaboration; they are close to each other.

the experimentally allowed range. Therefore, it seems unlikely that $X(2120)$ and $X(2370)$ can be understood as the third radial excitation of η and η' simultaneously. The lattice QCD simulations predict that the 0^{-+} glueball is about $2.3 \sim 2.6$ GeV [35], so it would mix with the nearby pseudoscalar isoscalar mesons. Consequently, $X(2370)$ may be a mixture of $\eta'(4^1S_0)$ and glueball, if its quantum numbers turn out in future to be $J^{PC} = 0^{-+}$. To understand the nature of $X(2370)$, partial wave analysis is important.

V. SUMMARY AND DISCUSSIONS

In this work, we investigate whether the resonances $X(1835)$, $X(2120)$, and $X(2370)$ newly observed by the BES collaboration could be conventional $q\bar{q}$ mesons. If they are indeed canonical pseudoscalar mesons, the natural assignments are $\eta(1760)$ and $X(1835)$ as the second radial excitation of η and η', respectively, and $X(2120)$ and $X(2370)$ as the third radial excitation of η and η'. To do so, we calculate all kinematically allowed two-body strong decays of $\eta(3^1S_0)$, $\eta'(3^1S_0)$, $\eta(4^1S_0)$, and $\eta'(4^1S_0)$ states within the framework of the $^3P_0$ model.

The decay amplitudes and widths turn out to be strongly dependent on the flavor mixing angle. If the mass and width of $\eta(1760)$ are chosen to be the world average listed in PDG or the DM2 measurement, we cannot find a proper value of the mixing angle so that both the theoretically predicted widths of $\eta(1760)$ and $X(1835)$ lie in the experimentally allowed range. However, if the BES results for $\eta(1760)$ are taken to be true, the theoretical predictions could be consistent with the experimental data within error for the flavor mixing angle ϕ in the range of $-31° \sim -24°$ or $30° \sim 40°$. Further experimental study of $\eta(1760)$ is important to understand the nature of $X(1835)$. Since the $\eta'(3^1S_0)$ $q\bar{q}$ meson would mix with $p\bar{p}$ due to the dressing effect and final state interaction, we suggest $X(1835)$ is the mixture of $\eta'(3^1S_0)$ and the $p\bar{p}$ molecule. Then we can naturally understand all the observations associated with $X(1835)$.

Under the assignment of $X(2120)$ and $X(2370)$ as $\eta(4^1S_0)$ and $\eta'(4^1S_0)$ $q\bar{q}$ mesons, $X(2120)$ dominantly decays into $\pi a_2(1320)$ and $KK^*(1410)$; the modes KK^* and $\rho\rho$ are suppressed by the decay amplitude node. $X(2370)$ is predicted to be rather broad (i.e., its width should be larger than 300 MeV), so it is unlikely that $X(2120)$ and $X(2370)$ can be understood as the third radial excitation of η and η' simultaneously. Since $X(2370)$ is close to the 0^{-+} glueball $2.3 \sim 2.6$ GeV predicted by lattice QCD, we suggest it may be a mixture of the $\eta'(4^1S_0)$ meson and glueball, if its quantum numbers are determined by future experiments to be $J^{PC} = 0^{-+}$.

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