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Nonfactorizable contribution to $\overline{B_d^0} \to \pi^0 D^0$

Lars E. Leganger
Department of Physics, NTNU, Trondheim, Norway

Jan O. Eeg
Department of Physics, University of Oslo, Post Office Box 1048 Blindern, N-0316 Oslo, Norway
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The decay modes of the type $B \to \pi D$ are dynamically different. For the case $\overline{B_d^0} \to \pi^- D^+$ there is a substantial factorized contribution which dominates. In contrast, the decay mode $\overline{B_d^0} \to \pi^0 D^0$ has a small factorized contribution, being proportional to a very small Wilson coefficient combination. In this paper we calculate the relevant Wilson coefficients at one loop level in the heavy quark limits, both for the b quark and the c quark. We also emphasize that for the decay mode $\overline{B_d^0} \to \pi^0 D^0$ there is a sizeable nonfactorizable contribution due to long-distance interactions, which dominate the amplitude. We estimate the branching ratio for this decay mode within our framework, which uses the heavy quark limits, both for the b and the c quarks. In addition, we treat energetic light (u, d, s) quarks within a variant of large energy effective theory and combine this with a new extension of chiral quark models in order to estimate the effect of soft-gluon emission. For reasonable values of the model-dependent parameters of our model, we can account for at least $3/4$ of the amplitude needed to explain the experimental branching ratio $\simeq 2.6 \times 10^{-4}$.

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I. INTRODUCTION

There is presently great interest in decays of B mesons, due to numerous experimental results coming from *BABAR* and Belle. LHC will also provide data for such processes. Decays of B mesons like $B \to \pi\pi$ and $B \to K\pi$, where the energy release is big compared to the light meson masses, has been treated within QCD factorization [1] and soft collinear effective theory (SCET) [2]. In the high energy limit the amplitudes for such decay modes factorize into products of two matrix elements of weak currents, and some nonfactorizable corrections of order α_s which can be calculated perturbatively. The decays $B \to \pi\pi, K\pi$ are typical heavy to light decays. It was pointed out in previous papers [3] that for various decays of the type $\bar{B} \to D\bar{D}$, which are of heavy to heavy type, the methods of [1,2] are not expected to hold because the energy release is of order 1 GeV only. The so-called pQCD model [4] was also used for such decay modes [5].

The last two decades, b quarks, and some times also c quarks, were described within heavy quark effective field theory (HQEFT) [6]. Some transitions of heavy to heavy type have, in the heavy quark limits $(1/m_b) \to 0$ and $(1/m_c) \to 0$, been studied within heavy-light chiral perturbation theory (HLχPT) [7]. Typical cases are the Isgur-Wise function for $B \to D$ transition currents [8], and $B - \bar{B}$ mixing [9]. Also other $B \to D$ transitions, where the energy gap between the initial (B-meson) state and the final state (including a D meson) are substantial, have been analyzed within such a framework [8,10], even if it is not ideal. Especially in cases where the factorized amplitude is almost zero, calculations of nonfactorized amplitudes in terms of chiral loops or soft-gluon emission estimated within a chiral quark model might be fruitful [3,11,12], because they are expected to give results of reasonable order of magnitude.

The HQEFT covers processes where the heavy quarks carry the main part of the momentum in each hadron. To describe processes where energetic light quarks emerge from decays of heavy b quarks, large energy effective theory (LEET) was introduced [13] and used to study the current for $B \to \pi$ [14]. The idea was that LEET should do for energetic light quarks what HQEFT does for heavy quarks. In HQEFT one splits off the motion of the heavy quark from the heavy quark field, thus obtaining a reduced field depending on the velocity of the heavy quark. In LEET one splits off the large energy from the field of the light energetic quark, thus obtaining an effective theory for a reduced energetic light quark field depending on a light-like four vector. It was later shown that LEET in its initial formulation was incomplete and did not fully reproduce the physics of QCD [15]. Then LEET was further developed into an effective theory consistent with QCD, and became the SCET [2].

In this paper we consider decay modes of the type $B \to \pi D$. We restrict ourselves to processes where the b quark decays. This means the quark level processes $b \to c d \bar{u}$. Processes where the anti–b quark decays proceed analogously. The decay mode $\overline{B_d^0} \to \pi^- D^+$ has a substantial factorized amplitude, given by the Isgur-Wise function for $B \to D$ transition times the decay constant for π^-. The relevant Wilson coefficient is also the maximum possible, namely, of order 1 times the relevant Cabibbo-Kobayashi-Maskawa quark mixing factors and the Fermi coupling

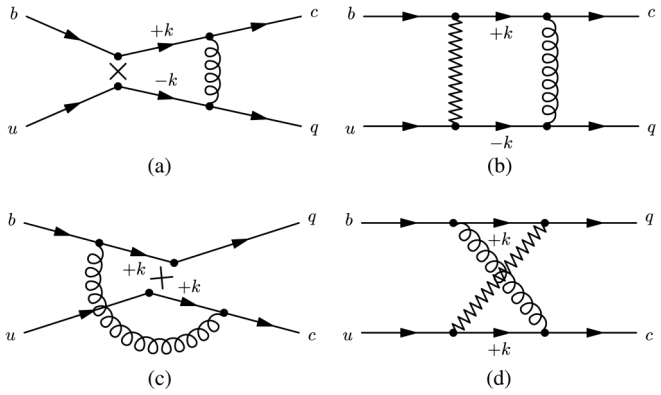

FIG. 1. QCD corrections for Q_A (top row) and Q_B (bottom row) when all quarks are considered light, i.e., for $\mu > m_b$. In the left column the weak interaction for an infinitely heavy W boson is marked by a cross. In the right column, the weak interaction is marked by a zigzag line. (In the lower right diagram, the zigzag line represent a fictitious "W^0" exchange.) In all cases the curly lines represent gluon exchanges.

constant. This is in contrast to the process $\overline{B_d^0} \to \pi^0 D^0$ where the factorized contribution to the decay mode is given by the $B \to \pi$ transition amplitude times the decay constant of the D^0 meson. This amplitude is almost zero because it is proportional to a very small Wilson coefficient combination. Thus, in the case $\overline{B_d^0} \to \pi^0 D^0$, the color suppressed amplitude is expected to dominate, as already pointed out in Ref. [16].

First, in Sec. III the Wilson coefficients will be calculated explicitly at one loop level within HQEFT for the b quark and the c quark, and scaled down to the scale $\mu \simeq 1$ GeV where perturbative QCD is matched to our long-distance framework. To our knowledge, this has not been down previously. It may be argued that to be completely consistent, we should also have treated the light energetic quark within a relevant large energy framework when calculating Wilson coefficients. But for this purpose LEET would be inappropriate because it is an incomplete theory as mentioned above. However, as we will see below, the uncertainty in our final amplitude for $\overline{B_d^0} \to \pi^0 D^0$ will be due to uncertainty in our model-dependent gluon condensate due to emission of soft gluons. Therefore the Wilson coefficients calculated in Sec. III here, as well as those within full QCD in [17,18], will be appropriate for our purpose.

Second, in the present paper we construct a modified version of the LEET used in [14] to study the $B \to \pi$ current, and in the next step construct a new model which we call large energy chiral quark model (LEχQM) [19]. It might be argued that we should have used the full SCET theory as the basis for our new model. However, the purpose of our paper is to estimate, in analogy with previous papers [9,11,12,20–24], the effects of soft-gluon emission in terms of gluon condensates. In this case transverse quark momenta and collinear gluons will not play a role. In any case we are making a model. Therefore, to construct our model, it suffices for our purpose to use the more simple formulation of LEET. We will combine LEET with chiral quark models (χQM) [25–28], containing only soft gluons making condensates. In our model an energetic quark is bound to a soft quark with an *a priori* unknown coupling, as proposed in [20]. The unknown coupling is determined by calculating the known $B \to \pi$ current matrix element within the model. This will fix the unknown coupling because the matrix element of this current is known [14]. Then, in the next step, we use this coupling to calculate the nonfactorized (color suppressed) amplitude contribution to $\overline{B_d^0} \to \pi^0 D^0$ in terms of the lowest dimension gluon condensate, as has been done for other nonleptonic decays [9,11,12,21,23]. After the quarks have been integrated out, we obtain an effective theory containing both soft light mesons as in HLχPT, and also fields describing energetic light mesons. A similar idea with a combination of SCET with HLχPT is considered in [29]. The LEχQM is constructed in analogy with the previous heavy-light chiral quark model (HLχQM) [21] and may be considered to be an extension of that model.

In Sec. II we present the weak four quark Lagrangian and its factorized and nonfactorizable matrix elements. In Sec. III we calculate the Wilson coefficients at one loop level in the heavy quark limits for both the b and the c quark. In Sec. IV we present our version of LEET, and in Sec. V we present the new model LEχQM to include energetic light quarks and mesons. In Sec. VI we calculate the nonfactorizable matrix elements due to soft gluons expressed through the (model-dependent) quark condensate. In Sec. VII we give the results and conclusion.

II. THE EFFECTIVE LAGRANGIAN AT QUARK LEVEL

We study $\overline{B_d^0}$ decays generated by the weak quark process $b \to c\bar{u}d$. The effective weak Lagrangian at quark level is [17,18]

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [c_A Q_A + c_B Q_B], \tag{1}$$

where the subscript L denotes the left-handed fields: $q_L \equiv Lq$, where $L \equiv (1 - \gamma_5)/2$ is the left-handed projector in Dirac space. The local operator products $Q_{A,B}$ are

$$Q_A = 4\bar{c}_L \gamma_\mu b_L \bar{d}_L \gamma^\mu u_L, \qquad Q_B = 4\bar{c}_L \gamma_\mu u_L \bar{d}_L \gamma^\mu b_L. \tag{2}$$

In these operators summation over color is implied. In (1), c_A and c_B are Wilson coefficients. At tree level $c_A = 1$ and $c_B = 0$. At one loop level, a contribution to c_B is also generated, and c_A is slightly increased. These effects are handled in terms of the renormalization group equations [17,18].

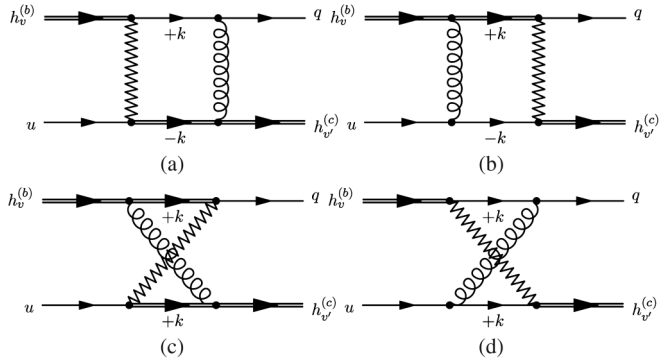

FIG. 2. QCD corrections for Q_A (top row) and Q_B (bottom row) in the case $\mu < m_c$, when both the b and the c quark are considered to be heavy. The heavy quarks are represented by double lines. The zigzag and curly lines have the same meaning as in Fig. 1.

Using the color matrix identity

$$2t^a_{in}t^a_{lj} = \delta_{ij}\delta_{ln} - \frac{1}{N_c}\delta_{in}\delta_{lj},$$

and Fierz rearrangement, the amplitudes for decays of $\overline{B^0_d}$ into $D\pi$ may be written as

$$\mathcal{M}_{D^+\pi^-} = 4\frac{G_F}{\sqrt{2}}V_{cb}V^*_{ud}\Big[\Big(c_A + \frac{1}{N_c}c_B\Big)\langle\pi^-|\bar{d}_L\gamma_\mu u_L|0\rangle \times \langle D^+|\bar{c}_L\gamma^\mu b_L|\overline{B^0_d}\rangle + 2c_B\langle D^+\pi^-|\bar{d}_L\gamma_\mu t^a u_L\bar{c}_L\gamma^\mu t^a b_L|\overline{B^0_d}\rangle\Big] \quad (3)$$

for charged mesons in the final state, and correspondingly for neutral mesons we have

$$\mathcal{M}_{D^0\pi^0} = 4\frac{G_F}{\sqrt{2}}V_{cb}V^*_{ud}\Big[\Big(c_B + \frac{1}{N_c}c_A\Big)\langle D^0|\bar{c}_L\gamma_\mu u_L|0\rangle \times \langle\pi^0|\bar{d}_L\gamma^\mu b_L|\overline{B^0_d}\rangle + 2c_A\langle D^0\pi^0|\bar{d}_L\gamma_\mu t^a b_L\bar{c}_L\gamma^\mu t^a u_L|\overline{B^0_d}\rangle\Big]. \quad (4)$$

Here the terms proportional to $2c_A$ and $2c_B$ with color matrices inside the matrix elements are the genuinely nonfactorizable contributions. These will be estimated in Sec. IV.

Since c_A is slightly bigger than one and c_B of order -0.4, we refer to the coefficients

$$c_f \equiv \Big(c_A + \frac{1}{N_c}c_B\Big) \simeq 1, \qquad c_{nf} \equiv \Big(c_B + \frac{1}{N_c}c_A\Big) \simeq 0, \quad (5)$$

as favorable (c_f) and nonfavorable (c_{nf}) coefficients, respectively. Thus, the decay mode $\overline{B^0_d} \to D^+\pi^-$ has a sizeable factorized amplitude proportional to c_f. In contrast, the decay mode $\overline{B^0_d} \to D^0\pi^0$ has a factorized amplitude proportional to the nonfavorable coefficient c_{nf} which is close to zero. In this case we expect the nonfactorizable term (involving color matrices) proportional to $2c_A$ to be dominant; i.e., the last line of Eq. (4) dominates. A substantial part of this paper is dedicated to the calculation of this nonfactorizable contribution to the $\overline{B^0_d} \to D^0\pi^0$ decay amplitude. Within the proposed LEχQM this amplitude will in Sec. VI be calculated in terms of emission of soft gluons assumed to make a gluon condensate.

III. PERTURBATIVE QCD CORRECTIONS TO ONE LOOP WITHIN HQEFT

Wilson coefficients for four quark operators for nonleptonic decays have been first calculated at the one loop level [17], and later at the two loop level [18]. In [3,11,12] the latter were used. Here we will calculate the Wilson coefficient completely within HQEFT at one loop level. Thus the heavy quarks will be described by the HQEFT Lagrangian [6]:

$$\mathcal{L}_{\text{HQEFT}} = \bar{Q}_v(iv\cdot D)Q_v + \mathcal{O}(1/m_Q), \quad (6)$$

where Q_v is the reduced heavy quark field (often named h_v in the literature), v its four velocity, and m_Q the mass of the heavy quark.

As usual, the renormalization of the four quark operators are performed in several steps: First, when the renormalization scale μ satisfies $m_b < \mu < M_W$, all the five quarks b, c, s, d, u are considered light. Then, for scales $m_c < \mu < m_b$, the b quark is considered heavy while the c quark is still considered light. Going further to the case $\mu < m_c$, the c quark is also considered heavy. Then the calculations are performed within strict HQEFT for both for the b and the c quark. By assumption the various chiral quark models work below the chiral symmetry breaking scale $\Lambda_\chi \simeq 1$ GeV. Also, HLχPT is applicable below the scale Λ_χ [3,11,23,24,26]. Therefore we will match the perturbative calculations with our model at $\mu = \Lambda_\chi$. For renormalization scales μ in the region $m_b < \mu < M_W$, where all the involved quarks are considered to be light (see Fig. 1), we obtain the well known result [17]

$$c^{(0)}_A(\mu) = \frac{1}{2}\Big[\Big(\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\Big)^{6/23} + \Big(\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\Big)^{-12/23}\Big], \quad (7)$$

$$c^{(0)}_B(\mu) = \frac{1}{2}\Big[\Big(\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\Big)^{6/23} - \Big(\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\Big)^{-12/23}\Big], \quad (8)$$

reflecting that the anomalous dimension matrix for the operator basis $Q_\pm = (Q_B \pm Q_A)$ is diagonal.

For scales μ satisfying $m_c < \mu < m_b$, the b quark is considered to be heavy, while the c quark is still light. In this range of μ we find that some of the diagrams which contributed for $m_b < \mu < M_W$ are now zero. As a consequence, in the (Q_A, Q_B) basis the anomalous dimension matrix is now [using the definition $\gamma \equiv (\alpha_s/2\pi)\hat{\gamma}$]

$$\hat{\gamma}(m_c < \mu < m_b) = \frac{1}{2}\begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix}, \tag{9}$$

which is half of what it is above $\mu = m_b$. The beta function to lowest order is proportional to $b_0^{(1)} = 11 - 2N_f/3$, where N_f is the number of effective flavors. With the bottom quark integrated out, $N_f = 4$, thus $b_0^{(1)} = 25/3$. Defining the quantity

$$D(\mu) \equiv \left(\frac{\alpha_s(m_b)}{\alpha_s(\mu)}\right)^{3/25}\left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right)^{6/23}, \tag{10}$$

we obtain the Wilson coefficients for the case $m_c < \mu < m_b$:

$$\begin{aligned} c_A^{(1)}(\mu) &= \frac{1}{2}[D(\mu) + (D(\mu))^{-2}], \\ c_B^{(1)}(\mu) &= \frac{1}{2}[D(\mu) - (D(\mu))^{-2}]. \end{aligned} \tag{11}$$

For the range $\Lambda_\chi < \mu < m_c$, where the b and the c quark are both considered as heavy (see Fig. 2), we obtain a more nonstandard anomalous dimension matrix

$$\gamma(\Lambda_\chi < \mu < m_c) = \frac{1}{2}\left(1 + \frac{2}{3}\omega r(\omega)\right)\begin{pmatrix} 0 & 0 \\ 3 & -1 \end{pmatrix}. \tag{12}$$

Then we finally get the result for $\Lambda_\chi < \mu < m_c$ the coefficients

$$\begin{aligned} c_A^{(2)}(\mu) &= c_A^{(1)}(m_c), \\ c_B^{(2)}(\mu) &= 3(1-\tau)c_A^{(1)}(m_c) + \tau c_B^{(1)}(m_c), \end{aligned} \tag{13}$$

where

$$\tau \equiv \left(\frac{\alpha_s(m_c)}{\alpha_s(\mu)}\right)^{\bar{\omega}}, \qquad \bar{\omega} \equiv -\frac{1}{18}\left(1 + \frac{2}{3}\omega r(\omega)\right), \tag{14}$$

the function $r(\omega)$ being the well known

$$r(\omega) \equiv \frac{1}{\sqrt{\omega^2 - 1}}(\omega + \sqrt{\omega^2 - 1}). \tag{15}$$

An analogous result for $b \to dc\bar{c}$ has been obtained in [30].

We observe that c_A is not further renormalized below $\mu = m_c$, while c_B and thereby c_{nf} get a small additional renormalization through the factor τ for $\Lambda_\chi = \mu < m_c$. Numerically, τ is close to 1. At $\mu = \Lambda_\chi \simeq 1$ GeV and the relevant value $\omega \simeq 1.6$, we have $c_A^{(2)} \simeq 1.2$ and $c_B^{(2)} \simeq -0.44$, giving $c_f \simeq 1.1$ and $c_{nf} \simeq -0.04$.

From the numerical point of view, the calculation performed in this section has not given us much new information. However, we think it is useful to have a calculation performed completely within HQEFT, and to our knowledge this is not presented anywhere else in the literature.

IV. AN ENERGETIC LIGHT QUARK EFFECTIVE DESCRIPTION (LEETδ)

An energetic light quark might, similarly to a heavy quark, carry practically all the energy E of the meson it is a part of (i.e., it has momentum fraction x close to 1). But the mass of the energetic quark is close to zero compared to m_Q and E, which are assumed to be of the same order of magnitude. Assuming that the energetic light quark is coming from the decay of a heavy quark Q with momentum $p_Q = m_Q v + k$, the momentum of the energetic quark q can be written (in the LEET approximation)

$$p_q^\mu = En^\mu + k^\mu, \qquad |k^\mu| \ll |En^\mu|, \qquad m_q \ll E, \tag{16}$$

where m_q is the light quark mass and n is the lightlike four vector which might be chosen to have the space part along the z axis, $n^\mu = (1; 0, 0, 1)$, in the frame of the heavy quark where $v = (1; 0, 0, 0)$. Then $(v \cdot n) = 1$ and $n^2 = 0$. Inserting this in the regular quark propagator, we obtain

$$S(p_q) = \frac{\gamma \cdot p_q + m_q}{p_q^2 - m_q^2} = \frac{E\gamma \cdot n + \gamma \cdot k + m_q}{2En \cdot k + k^2 - m_q^2}. \tag{17}$$

In the limit where the approximations in (16) are valid, we obtain the propagator

$$S(p_q) \to \frac{\gamma \cdot n}{2n \cdot k}. \tag{18}$$

This propagator is the starting point for the LEET constructed in Ref. [14].

Unfortunately, the combination of LEET with χQM will lead to infrared divergent loop integrals for $n^2 = 0$ (see Sec. V). Therefore, in the following we modify the formalism and instead of $n^2 = 0$, we use $n^2 = \delta^2$, with $\delta = \nu/E$ where $\nu \sim \Lambda_{\rm QCD}$, such that $\delta \ll 1$. In the following we derive a modified LEET [14] where we keep $\delta \neq 0$ with $\delta \ll 1$. We call this construction LEETδ and define the almost lightlike vectors

$$n = (1, 0, 0, +\eta), \qquad \tilde{n} = (1, 0, 0, -\eta), \tag{19}$$

where $\eta = \sqrt{1 - \delta^2}$. This means that

$$\begin{aligned} n^\mu + \tilde{n}^\mu &= 2v^\mu, \qquad n^2 = \tilde{n}^2 = \delta^2, \\ v \cdot n &= v \cdot \tilde{n} = 1, \qquad n \cdot \tilde{n} = 2 - \delta^2. \end{aligned} \tag{20}$$

Using the above equations, we choose the set of projection operators given by

$$\begin{aligned} \mathcal{P}_+ &= \frac{1}{N^2}\gamma \cdot n(\gamma \cdot \tilde{n} + \delta), \\ \mathcal{P}_- &= \frac{1}{N^2}(\gamma \cdot \tilde{n} - \delta)\gamma \cdot n, \end{aligned} \tag{21}$$

where $N = \sqrt{2n \cdot \tilde{n}} = 2 + \mathcal{O}(\delta^2)$. We factor out the main energy dependence, just as was analogously done in HQEFT, and define the projected reduced quark fields $q_\pm$ [14]:

$$q_\pm(x) = e^{iEn\cdot x}\mathcal{P}_\pm q(x),$$
$$q(x) = e^{-iEn\cdot x}[q_+(x) + q_-(x)]. \tag{22}$$

The adjoint fields are

$$\bar{q}_\pm = q_\pm^\dagger\gamma^0 = e^{-iEn\cdot x}\bar{q}\overline{\mathcal{P}}_\pm, \qquad \overline{\mathcal{P}}_\pm \equiv \gamma^0\mathcal{P}_\pm^\dagger\gamma^0. \tag{23}$$

Following the procedure of [14], we eliminate q_- and obtain for $q_+ \equiv q_n$ the effective Lagrangian:

$$\mathcal{L}_{\mathrm{LEET}\delta} = \bar{q}_n\left(\frac{\gamma\cdot\tilde{n}+\delta}{N}\right)(in\cdot D)q_n + \frac{1}{E}\bar{q}_n X q_n + \mathcal{O}(E^{-2}), \tag{24}$$

which (for $\delta = 0$) is the first part of the SCET Lagrangian. Equation (24) yields the LEETδ quark propagator

$$S_n(k) = \mathcal{P}_+\left[\frac{\gamma\cdot\tilde{n}+\delta}{N}(n\cdot k)\right]^{-1} = \frac{\gamma\cdot n}{N(n\cdot k)}, \tag{25}$$

which reduces to (18) in the limit $\delta \to 0$. In addition, the SCET propagator [2] for light energetic quarks will for transverse quark momentum $p_\perp \to 0$ be reduced to this LEET form in (25). Our $\mathcal{O}(E^{-1})$ term is given by

$$X = -\frac{1}{2}(i\gamma\cdot D)\gamma\cdot v\left[(i\gamma\cdot D) - \frac{(\gamma\cdot\tilde{n}+\delta)}{N}(in\cdot D)\right] - \frac{1}{2}\left[\gamma\cdot(i\overleftarrow{D}) - \frac{(\gamma\cdot\tilde{n}+\delta)}{N}(in\cdot\overleftarrow{D})\right]\gamma\cdot v(i\gamma\cdot D). \tag{26}$$

It was pointed out in [14] that in the formal limits $M_H \to \infty$ and $E \to \infty$, a heavy H (e.g., B or maybe also D) meson decaying by a vector weak current V^μ to a light pseudoscalar meson P has a matrix element $\langle P|V^\mu|H\rangle$ of the form

$$\langle P|V^\mu|H\rangle = 2E[\zeta^{(v)}(M_H, E)n^\mu + \zeta_1^{(v)}(M_H, E)v^\mu], \tag{27}$$

where

$$\zeta^{(v)} = C\frac{\sqrt{M_H}}{E^2}, \qquad C \sim (\Lambda_{\mathrm{QCD}})^{3/2}, \qquad \frac{\zeta_1^{(v)}}{\zeta^{(v)}} \sim \frac{1}{E}. \tag{28}$$

This behavior is consistent with the energetic quark having x close to 1, where x is the quark momentum fraction of the outgoing pion [14].

V. EXTENDED CHIRAL QUARK MODEL FOR HEAVY AND ENERGETIC LIGHT QUARKS (LEχQM)

The χQM [25,26] and the HLχQM [21] include meson-quark couplings and thereby allow us to calculate amplitudes and chiral Lagrangians for processes involving heavy quarks and low energy light quarks. In this section we will extend these models to include also hard, energetic light quarks.

For the pure light sector the χQM Lagrangian can be written as [23,25]

$$\mathcal{L}_{\chi\mathrm{QM}} = \bar{\chi}[\gamma\cdot(iD + \mathcal{V}) + \gamma\cdot\mathcal{A}\gamma_5 - m]\chi, \tag{29}$$

where m is the constituent mass term being due to chiral symmetry breaking. (The small current mass term is neglected here.) Here we have introduced the flavor rotated fields $\chi_{L,R}$:

$$\chi_L = \xi^\dagger q_L, \qquad \chi_R = \xi q_R, \tag{30}$$

where q is the light quark flavor triplet and

$$\xi = \exp\{i\Pi/f\},$$
$$\Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \tag{31}$$

Further, $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ are vector and axial vector fields, given by

$$\mathcal{V}_\mu \equiv \frac{i}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger),$$
$$\mathcal{A}_\mu \equiv -\frac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger). \tag{32}$$

To couple the heavy quarks to mesons there are additional meson-quark couplings within HLχQM [21]:

$$\mathcal{L}_{\mathrm{int}} = -G_H[\bar{\chi}_a\bar{H}_v^a Q_v + \bar{Q}_v H_v^a\chi_a], \tag{33}$$

where a is a SU(3) flavor index, Q_v is the reduced heavy quark field in (6), and H_v is the corresponding heavy $(0^-, 1^-)$ meson field,

$$H_v = \mathcal{P}_+(v)(\gamma\cdot P^* - i\gamma_5 P_5), \tag{34}$$

where $\mathcal{P}_+(v) = (1 + \gamma\cdot v)/2$ is a projection operator. Further, P_μ^* is the 1^- field and P_5 the 0^- field. These mesonic fields enter the Lagrangian of HLχPT:

$$\mathcal{L}_{\mathrm{HL}\chi\mathrm{PT}} = -\mathrm{Tr}(\bar{H}_v iv_\mu\partial^\mu H_v) + \mathrm{Tr}(\bar{H}_v{}^a H_v^b v_\mu\mathcal{V}_{ba}^\mu) - g_{\mathcal{A}}\,\mathrm{Tr}(\bar{H}_v{}^a H_v^b\gamma_\mu\gamma_5\mathcal{A}_{ba}^\mu), \tag{35}$$

where a, b are SU(3) flavor indices. The quark-meson coupling G_H is determined within the HLχQM to be [21] given by

$$G_H^2 = \frac{2m}{f_\pi^2}\rho, \tag{36}$$

where ρ is a hadronic quantity of order 1.

For hard light quarks and chiral quarks coupling to a hard light meson multiplet field M, we extend the ideas of χQM and HLχQM, and assume that the energetic light mesons couple to light quarks with a derivative coupling to an axial current:

$$\mathcal{L}_{\mathrm{int}q} \sim \bar{q}\gamma_\mu\gamma_5(i\partial^\mu M)q. \tag{37}$$

We combine LEETδ with the χQM and assume that the ingoing light quark and the outgoing meson are energetic,

and we pull out a factor $\exp(\pm iEn\cdot x)$ as in (22). To describe (outgoing) light energetic mesons, we use an octet 3×3 matrix field $M=\exp(+iEn\cdot x)M_n$, where M_n has the same form as Π in (31):

$$M_n=\begin{pmatrix}\frac{\pi_n^0}{\sqrt{2}}+\frac{\eta_n}{\sqrt{6}} & \pi_n^+ & K_n^+\\ \pi_n^- & -\frac{\pi_n^0}{\sqrt{2}}+\frac{\eta_n}{\sqrt{6}} & K_n^0\\ K_n^- & \bar{K}_n^0 & -\frac{2\eta_n}{\sqrt{6}}\end{pmatrix}. \tag{38}$$

Here π_n^0, π_n^+, K_n^+, etc., are the (reduced) meson fields corresponding to energetic light mesons with momentum $\sim En^\mu$.

Combining (37) with the use of the rotated soft quark fields in (30) and using $\partial^\mu\to iEn^\mu$, we arrive at the following ansatz for the LEχQM interaction Lagrangian:

$$\mathcal{L}_{\text{int}q\delta}=G_AE\bar{\chi}(\gamma\cdot n)Zq_n+\text{H.c.}, \tag{39}$$

where q_n is the reduced field corresponding to an energetic light quark having momentum fraction close to 1 [see (24)], and χ represents a soft quark [see Eq. (30)]. Further, G_A is an unknown coupling to be determined later by physical requirements. Further,

$$Z=\xi M_RR-\xi^\dagger M_LL. \tag{40}$$

Here M_L and M_R are both equal to M_n, but they have formally different transformation properties. This is analogous to the use of quark mass matrices $\mathcal{M}_q$ and $\mathcal{M}_q^\dagger$ in standard chiral perturbation theory (χPT). They are in practice equal, but have formally different transformation properties.

The axial vector coupling introduces a factor $\gamma\cdot n$ to the vertex [see (39)], which simplifies the Dirac algebra within the loop integrals. In order to calculate the nonfactorizable contribution, we must first find a value for the coupling G_A in (39) assumed to bind a large energy light quark and a soft (anti)quark to an energetic light meson. This will be done by requiring that our model be constistent with Eqs. (27) and (28). Applying the Feynman rules of LEχQM we obtain the following bosonized current (before soft-gluon emission forming the gluon condensate is taken into account):

$$J_0^\mu(H_v\to M_n)=-N_c\int đk\,\text{Tr}\{\gamma^\mu LiS_v(k)\times[-iG_HH_v]iS_\chi(k)[iEG_A\gamma\cdot nZ]iS_n(k)\}, \tag{41}$$

where $đk\equiv d^dk/(2\pi)^d$ (d being the dimension of space-time), and

$$S_v(k)=\frac{P_+(v)}{v\cdot k},\qquad S_\chi(k)=\frac{(\gamma\cdot k+m)}{k^2-m^2},\qquad S_n(k)=\frac{\gamma\cdot n}{Nn\cdot k} \tag{42}$$

are the propagators for the reduced heavy quark fields [Q_v in Eq. (6)], light constituent quarks [χ in Eq. (29)], and reduced light energetic quark fields [q_n in (24)], respectively. (Below we will use the leading order value $N=2$.)

It should be emphasized that for the loop diagram for $B\to\pi$ in Fig. 3 (lower part of the diagram), we have the following picture: The large energy ($M_B\simeq m_b$) of the heavy b quark and the large energy ($E\simeq M_B/2$) of the hard d quark are floating through the (lower part of the) loop diagram. The loop momenta of the reduced quark fields for the heavy quark, energetic light quark are then carrying the same soft loop momentum k (with $|k|<\Lambda_\chi\simeq 1$ GeV) as the soft light (anti)quark ($\bar{d}$), which justifies the use of our model.

The presence of the left projection operator L in Z ensures that we only get contributions from the left-handed part, that is, $Z\to-\xi^\dagger M_LL$. The momentum integrals have the form

$$K_{rst}=\int\frac{đk}{(v\cdot k)^r(k\cdot n)^s(k^2-m^2)^t}, \tag{43}$$

$$K_{rst}^\mu=\int\frac{đkk^\mu}{(v\cdot k)^r(k\cdot n)^s(k^2-m^2)^t}=K_{rst}^{(v)}v^\mu+K_{rst}^{(n)}n^\mu, \tag{44}$$

where r, s, t are integer numbers. These integrals have the important property that $K_{rst}^{(n)}$ dominates over $K_{rst}^{(v)}$ and K_{rst} with one power of $1/\delta$. In the present model, we choose $\nu=m$ which is of order Λ_{QCD}. Thus $\delta=m/E$ in the following.

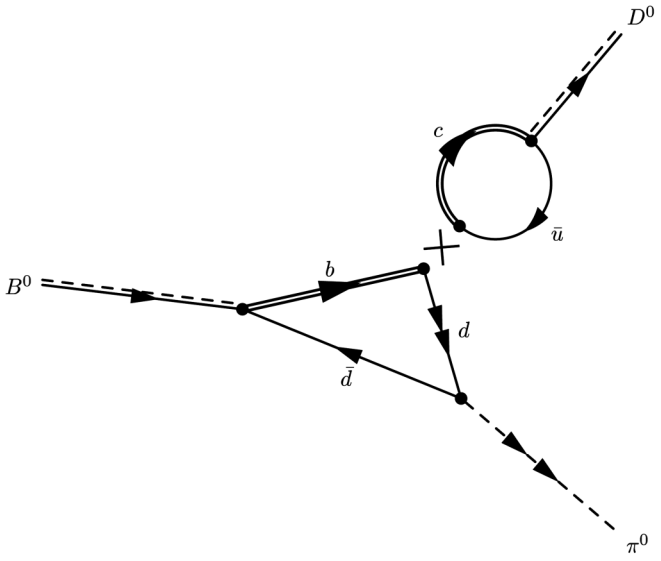

FIG. 3. The factorized contribution to the $B^0\to D^0\pi^0$ decay, as described in combined HLχQM and LEχQM. Double lines, single lines, and the single line with two arrows represent heavy quarks, light soft quarks, and light energetic quarks, respectively. Heavy mesons are represented by a single line combined with a parallel dashed line, and the light energetic pion is represented by a dashed line with double arrow.

The contribution in (41), corresponding to the $B \to \pi$ part of Fig. 3, with no gluon condensate contribution included, contains K_{111} and K^{μ}_{111} and turns out to be proportional to the formally linearly divergent integral previously called $I_{3/2}$ [21]. There are also other contributions with two emitted soft gluons making a condensate [21,26]. To calculate emission of soft gluons we have used the framework of Novikov *et al.* [31]. In this framework the ordinary vertex containing the gluon field A^a_μ will be replaced by the soft-gluon version containing the soft-gluon field tensor $G^a_{\mu\nu}$:

$$ig_s t^a \Gamma^\mu A^a_\mu \to -\frac{1}{2} g_s t^a \Gamma^\mu G^a_{\mu\nu} \frac{\partial}{\partial p_\nu} \ldots |_{p=0}, \tag{45}$$

where p is the momentum of the soft gluon. (Using this framework one has to be careful with the momentum routing because the gauge where $x^\mu A^a_\mu = 0$ has been used.) Here $\Gamma^\mu = \gamma^\mu$, v^μ, or $n^\mu(\gamma \cdot \tilde{n} + \delta)/N$ for a light soft quark, heavy quark, or light energetic quark, respectively. Our loop integrals are *a priori* depending on the gluon momenta $p_{1,2}$ which are sitting in some propagators. These gluon momenta disappear after having used the procedure in (45). It is understood that the derivatives in (45) have to be taken with respect to all propagators in the loop integral.

There is a contribution to the $H_v \to M_n$ current where two soft gluons are emitted from the light quark line. This contribution contains K_{114} and K^μ_{114} and is finite. Emission from the heavy quark or light energetic quark are expected to be suppressed. This will be realized in most cases because the gluon tensor is antisymmetric, and therefore such contributions are proportional to

$$G^a_{\mu\nu} v^\mu v^\nu = 0, \quad \text{or} \quad G^a_{\mu\nu} n^\mu n^\nu = 0. \tag{46}$$

However, there are also contributions proportional to

$$G^a_{\mu\nu} v^\mu n^\nu \neq 0, \tag{47}$$

analogous to what happens in some diagrams for the Isgur-Wise diagram where there are two different velocities v_b and v_c [32]. In that case the corresponding contributions are proportional to $(v_b \cdot v_c - 1)$ which is zero for $v_c \to v_b$. Such contributions (proportional to K_{331} and K^μ_{331}) appear within our calculation when two soft gluons are emitted from the heavy quark line. (This statement is however gauge dependent. With another momentum routing such a contribution would come from another diagram. But summing all diagrams, gauge invariance is fulfilled.)

Using the prescription [21,23,26,31]

$$g_s^2 G^a_{\mu\nu} G^a_{\rho\lambda} \to 4\pi^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{1}{12} (g_{\mu\rho} g_{\nu\lambda} - g_{\mu\lambda} g_{\nu\rho}) \tag{48}$$

for the gluon condensate, one obtains a total bosonized current of the form

$$J^\mu_{\text{tot}}(H_v \to M_n) = -i \frac{G_H}{2} (EG_A) \delta^2 \,\text{Tr} \times \{\gamma^\mu L H_v [R^{(v)} + R^{(n)} \gamma \cdot n] \xi^\dagger M_L\}, \tag{49}$$

where the relevant quantity needed is (to leading order in δ)

$$R^{(n)} = \frac{m}{\delta} F,$$
$$F \equiv \frac{1}{m} \left(-iN_c I_{3/2} + \frac{\pi}{8 \cdot 16 m^3} \left[\frac{2}{3} - 1 \right] \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \right). \tag{50}$$

Here the contribution $\sim 2/3$ within the parenthesis is coming from the diagram where two gluons are emitted from the heavy quark line. This contribution is due to (47). Note that F is dimensionless.

In order to obtain the HLχPT Lagrangian terms in (35), one calculates quark loops with attached heavy meson fields and vector and axial vector fields $\mathcal{V}^\mu$ or $\mathcal{A}^\mu$. Then, as explained in previous papers [21,23,25,26], logarithmic and linearly divergent integrals I_2 and $I_{3/2}$ (as well as quadratic diverget integtral I_1) will appear. These might be regularized, say, with ultraviolet cutoffs of order Λ_χ [27,28]. The explicit expressions of the divergent integrals in terms of the cutoffs will depend on the details of the regularization procedure. We will however not go into these details, but simply identify the divergent integrals by appropriate quantities regarded as physical within our model. That is, we the use identification [25,26]

$$-iN_c I_2 = \frac{1}{4m^2} \left(f_\pi^2 - \frac{1}{24m^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \right) \equiv \frac{f_\pi^2}{4m^2} \lambda, \tag{51}$$

for the logarithmically divergent integral, and [21]

$$-iN_c I_{3/2} = \frac{3 f_\pi^2}{8m\rho} (1 - g_A) + \frac{N_c m}{16\pi} - \frac{(8 - 3\pi)}{256 m^3} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle, \tag{52}$$

for the linearly divergent integral. The parameter λ defined in (51) is of order 10^{-1} and rather sensitive to small variations in the parameters m and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$. Using (51) and (52) it can be shown that the parameter ρ in (36) is given by [21,32]

$$\rho = \frac{(1 + 3g_A)}{4(1 + \frac{m^2 N_c}{8\pi f_\pi^2} - \frac{\eta_H}{2m^2 f_\pi^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle)}, \tag{53}$$

where $\eta_H = (8 - \pi)/64$. Then we obtain for the quantity F

$$F = \frac{N_c}{16\pi} + \frac{3 f_\pi^2}{8 m^2 \rho} (1 - g_A) - \frac{(24 - 7\pi)}{768 m^4} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle. \tag{54}$$

Numerically, $F \simeq 0.08$.

In order to fix G_A in (39), we compare (27) with (49). In our case where no extra soft pions are going out, we put $\xi \to 1$, and $M_L \to k_M \sqrt{E}$, with the isospin factor

$k_M = 1/\sqrt{2}$ for π^0 (while $k_M = 1$ for charged pions). Moreover for the B meson with spin-parity 0^- we have $H_v \to P_+(v)(-i\gamma_5)\sqrt{M_H}$. Using this, we obtain the traces

$$\mathrm{Tr}\{\gamma^\mu L H_v \xi^\dagger M_L\} \to -i\sqrt{M_H}(k_M\sqrt{E})v^\mu, \tag{55}$$

$$\mathrm{Tr}\{\gamma^\mu L H_v^{(+)} \gamma^\sigma \xi^\dagger M_L\} \to +i\sqrt{M_H}(k_M\sqrt{E})g^{\mu\sigma}. \tag{56}$$

Then we obtain the following matrix element of the current,

$$J^\mu_{\mathrm{tot}}(H_v \to M_n) = \frac{G_H}{2}(EG_A)\sqrt{M_H}(k_M\sqrt{E})\delta^2 \times [-R^{(v)}v^\mu + R^{(n)}n^\mu], \tag{57}$$

where $R^{(v)}/R^{(n)} \sim \delta$; i. e., we obtain $R^{(v)}/R^{(n)} \to 0$ as $E \to \infty$, as we should according to (27) and (28). Using Eqs. (27), (54), and (57), we obtain

$$G_A = \frac{4\zeta^{(v)}}{m^2 G_H F}\sqrt{\frac{E}{M_H}}, \tag{58}$$

where $\zeta^{(v)}$ is numerically known [33] to be $\simeq 0.3$. Within our model, the analogue of Λ_{QCD} is the constituent light quark mass m. To see the behavior of G_A in terms of the energy E we therefore write C in (28) as $C \equiv \hat{c}m^{3/2}$, and obtain

$$G_A = \left(\frac{4\hat{c}f_\pi}{mF\sqrt{2\rho}}\right)\frac{1}{E^{3/2}}, \tag{59}$$

which explicitly displays the behavior $G_A \sim E^{-3/2}$. In terms of the number N_c of colors, $f_\pi \sim \sqrt{N_c}$ and $F \sim N_c$ which gives the behavior $G_A \sim 1/\sqrt{N_c}$, i.e., the same behavior as for the coupling G_H in (33).

VI. NONFACTORIZABLE PROCESSES IN LEχQM

In this section we calculate the nonfactorizable contribution to $\overline{B^0_d} \to \pi^0 D^0$ in the second line of Eq. (4). This contribution will be expressed as a quasifactorized product of two colored currents, as illustrated in Fig. 4. Then the nonfactorized aspects enter through color correlation between the two parts, using Eq. (48). Such a calculation within HLχQM was done previously [3] for $\overline{B^0_{s,d}} \to D^0\overline{D^0}$, where the relevant colored current for decay of a D meson was calculated. What we will calculate here is the colored current for $B \to \pi$ with soft one gluon emission, within the LEχQM presented in the preceding section. One relevant diagram is given in Fig. 4. Using the values for G_A and G_H from the preceding section, we find an expression for the nonfactorizable $\overline{B^0_d} \to D^0\pi^0$ decay amplitude, which may be compared with experiment.

For a low energy quark interacting with one soft gluon, one might in simple cases use the effective propagator [23,34]

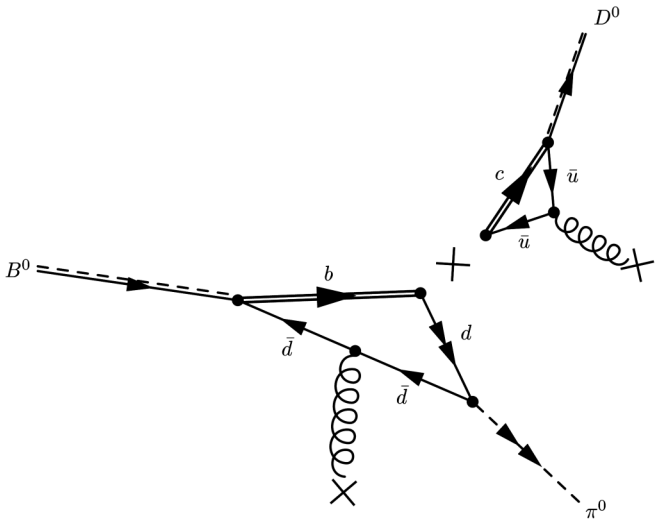

FIG. 4. The part of the nonfactorizable contribution containing large energy light fermions and mesons. The curly lines represent soft gluons, and the cross in the end of these symbolizes the gluon condensate. Otherwise the various symbols have the same meaning as in Fig. 3.

$$S_1^G(k) = \frac{g_s}{4}t^a G^a_{\mu\nu}\frac{(2m\sigma^{\mu\nu} + \{\sigma^{\mu\nu}, \gamma\cdot k\})}{(k^2 - m^2)^2}, \tag{60}$$

where $\{a, b\} \equiv ab + ba$ denotes the anticommutator. This expression is consistent with the prescription in (45), and can be used for the diagram in Fig. 4. Then we obtain the following contribution to the bosonized colored $B \to \pi$ current corresponding to this diagram:

$$J^\mu_{1G}(H_v \to M_n)^a_4 = -\int đk\,\mathrm{Tr}\{\gamma^\mu L t^a i S_v(k) \times [-iG_H H_v] i S_1^G(k)[iEG_A \gamma\cdot nZ] i S_n(k)\}. \tag{61}$$

Once more, we deal with the momentum integrals of the types in (43) and (44). Taking the color trace, we obtain a contribution of the form

$$J^\mu_{1G}(H_v \to M_n)^a = g_s G^a_{\alpha\beta} T^{\mu;\alpha\beta}(H_v \to M_n), \tag{62}$$

where the contribution from Fig. 4 alone is to leading order in δ

$$T^{\mu;\alpha\beta}(H_v \to M_n)_4 = \frac{G_H G_A}{128\pi}\epsilon^{\sigma\alpha\beta\lambda}n_\sigma\,\mathrm{Tr}(\gamma^\mu L H_v \gamma_\lambda \xi^\dagger M_L), \tag{63}$$

where $E\cdot\delta = m$ has been explicitly used.

There is also a diagram not shown where the soft gluon is emitted from the energetic quark. This diagram is zero due to (46). Furthermore, there is a diagram not shown where the gluon is emitted from the heavy quark which contains a nonzero part due to (47). In this case we have to

stick to the general rule in (45). This gives the additional contribution

$$T^{\mu;\alpha\beta}(H_v \to M_n)_6 = i\frac{G_H G_A}{64\pi} v^\alpha n^\beta \mathrm{Tr}(\gamma^\mu L H_v \gamma\cdot n \xi^\dagger M_L). \quad (64)$$

The total contribution to (62) is given by the right-hand sides of (63) and (64). Below we will use all the expressions for the various $J^\mu_{1G}(H_v \to M_n)^a$ for a decaying B meson; i.e. we have $v = v_b$, where v_b is the velocity of the b quark.

The colored left-handed current for the outgoing D^0 was found in [3] to be

$$(\overline{Q^{(+)}_{v_c}} t^a \gamma^\alpha q_L)_{1G} \to J^\mu_{1G}(\overline{H_{v_c}})^a = g_s G^a_{\alpha\beta} T^{\mu;\alpha\beta}(\overline{H_{v_c}}), \quad (65)$$

where

$$T^{\mu;\alpha\beta}(\overline{H_{v_c}}) = -\frac{G_H}{64\pi}\mathrm{Tr}\left[\xi\gamma^\mu L\left(\sigma^{\alpha\beta} - \frac{2\pi f_\pi^2}{m^2 N_c}\lambda\{\sigma^{\alpha\beta}, \gamma\cdot v_c\}\right)\overline{H_{v_c}}\right], \quad (66)$$

where λ is defined in (51) and v_c is the velocity of the c quark.

Now we use (48) and also include the Fermi coupling the Cabibbo-Kobayashi-Maskawa matrix elements, and the coefficient $2c_A$ for the nonfactorizable contributions to the amplitude, where c_A is the Wilson coefficient for the $\mathcal{O}_A$ local operator. Then we find an effective Lagrangian at a mesonic level relevant for the nonfactorizable contribution to $\overline{B^0_d} \to D^0\pi^0$:

$$\mathcal{L}^{\mathrm{LE}\chi\mathrm{QM}}_{\mathrm{Nonfact}} = \frac{4\pi^2 c_A}{3}\left(4\frac{G_F}{\sqrt{2}}V_{cb}V^*_{ud}\right) \times \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle S(H_{v_b} \to M_n H_{v_c}), \quad (67)$$

where $S(H_{v_b} \to M_n H_{v_c})$ is the tensor product

$$S(H_{v_b} \to M_n H_{v_c}) \equiv T^{\mu;\alpha\beta}(H_{v_b} \to M_n) T_{\mu;\alpha\beta}(\overline{H_{v_c}}). \quad (68)$$

The four vector products $(v_b\cdot v_c)$, $(v_b\cdot n)$, and $(v_c\cdot n)$ can be related to physical parameters by the equations for momentum and energy conversation. From

$$M_B v_b^\mu = M_D v_c^\mu + E n^\mu, \qquad E = \frac{M_B^2 - M_D^2}{2M_B}, \quad (69)$$

we obtain [up to $\mathcal{O}(\delta^2)$]

$$(v_b\cdot v_c) = \frac{M_B^2 + M_D^2}{2M_B M_D}, \qquad (v_b\cdot n) = 1, \qquad (v_c\cdot n) = \frac{M_B}{M_D}. \quad (70)$$

Using $\delta = m/E$ and (70), we find an explicit expression for $S(H_{v_b} \to M_n H_{v_c})$ in the case $\overline{B^0} \to D\pi^0$:

$$S(\overline{B^0} \to \pi^0 D^0) = \frac{G_H^2 G_A}{32\cdot 64\pi^2}\sqrt{M_B M_D E}\left(\frac{M_B}{M_D}\right) \times \left(1 + \frac{6\pi f_\pi^2}{N_c m^2}\lambda\right)\frac{1}{\sqrt{2}}. \quad (71)$$

Inserting the expressions for G_H in (36) and G_A in (58), we obtain

$$S(\overline{B^0} \to \pi^0 D^0) = \frac{\sqrt{2\rho}\zeta^{(v)}}{8\cdot 64\pi^2 F f_\pi}\sqrt{\frac{m}{M_D}}\left(\frac{E M_B}{m^2}\right) \times \left(1 + \frac{6\pi f_\pi^2}{N_c m^2}\lambda\right)\frac{1}{\sqrt{2}}. \quad (72)$$

We will now compare this nonfactorizable amplitude for $\overline{B^0} \to D^0\pi^0$ with the factorized amplitude which dominates $\overline{B^0} \to D^+\pi^-$:

$$\mathcal{M}_{D^+\pi^-} = 4\frac{G_F}{\sqrt{2}}V_{cb}V^*_{ud}\left(c_A + \frac{1}{N_c}c_B\right)\cdot\left(\frac{1}{2}f_\pi E n_\mu\right) \cdot\left(\frac{1}{2}\sqrt{M_B M_D}(v_b + v_c)^\mu \xi(\omega)\right), \quad (73)$$

where $\xi(\omega)$ is the Isgur-Wise function.

The ratio between the nonfactorized and factorized amplitudes is now

$$r = \frac{\mathcal{M}(\overline{B^0} \to \pi^0 D^0)_{\mathrm{Nonfact}}}{\mathcal{M}(\overline{B^0} \to \pi^- D^+)_{\mathrm{Fact}}} = \frac{c_A}{c_f}\frac{h}{(1 + \frac{M_D}{M_B})}\frac{\zeta^{(v)}}{\xi(\omega)}\sqrt{\frac{m}{M_B}}, \quad (74)$$

where h is our model-dependent hadronic factor

$$h = \frac{\sqrt{\rho}\langle\frac{\alpha_s}{\pi}G^2\rangle}{96\cdot F f_\pi^2 m^2}\left(1 + \frac{6\pi f_\pi^2}{N_c m^2}\lambda\right), \quad (75)$$

which behaves as $h \sim 1/N_c$ with respect to color.

It will be interesting how the ratio r scales in the limit $M_B^2 \gg M_D^2 \gg m^2$. Then we use the scaling of $\zeta^{(v)}$ given in (28) with $C = \hat{c}m^{3/2}$ as in (59). The scaling of $\xi(\omega)$ for $M_B^2 \gg M_D^2$ is not so well established. Under certain assumptions [35] it is found that the Isgur-Wise function $\xi(\omega)$ has the form

$$\xi(\omega) = \left(\frac{2}{1+\omega}\right)^\gamma, \quad (76)$$

where $\omega = v_b\cdot v_c$. In the so-called Bogomol'nyi-Prasad-Sommerfeld limit [36] one obtains [35] $\gamma = 3/2$. The Isgur-Wise function calculated within a bag model [37] has the same form as in (76). Within chiral HLχQM calculations, the Isgur-Wise function will have terms of the type in (76) for $\gamma = 1$, and some terms which for $\omega \gg 1$ scale as $\ln\omega/\omega$ [21,27,32,37], where $\omega \sim (M_B/2M_D)$.

Using the simple form (76) and (70), we find for r for $M_B^2 \gg M_D^2 \gg m^2$

$$r \simeq \frac{c_A}{c_f} \frac{h\hat{c}}{4^{(\gamma-1)}} \frac{m^2}{(M_D)^{\gamma}(M_B)^{(2-\gamma)}}. \quad (77)$$

Anyway, our calculations show that the ratio r of the amplitudes is suppressed by $1/N_c$ and by inverse powers of heavy meson masses, as expected.

Concerning numerical predictions from our model, we have to stick to Eq. (74). The measured branching ratios for $\overline{B^0_d} \to \pi^- D^+$ and $\overline{B^0_d} \to \pi^0 D^0$ are $\simeq (2.68 \pm 0.13) \times 10^{-3}$ and $(2.62 \pm 0.24) \times 10^{-4}$, respectively [38]. In order to predict the experimental value solely with the mechanism considered in this section, we should have $r \simeq 1/3$. For typical values $m \sim 200$ to 220 MeV and $\langle \frac{\alpha_s}{\pi} G^2 \rangle \sim 300$ to 320 MeV, we find that $h \sim 3$. Further, numerically $\zeta^{(v)} \simeq 1/3$ [33], and $\xi(\omega) \sim 2/3$, and $(1 + M_D/M_B) \simeq 4/3$. Thus we obtain a ratio r of order $1/4$; i.e., our model can account for roughly $3/4$ of the amplitude needed to explain the experimental branching ratio.

VII. CONCLUSION

We have presented perturbative QCD corrections for the quark process $b \to c\bar{u}q$ calculated completely within HQEFT at one loop level, and scaled with the renormalization group equation down to $\mu = \Lambda_\chi \simeq 1$ GeV. We have shown that the factorized amplitude for process $\overline{B^0_d} \to \pi^0 D^0$ is proportional to a Wilson coefficient combination close to zero. Thus the nonfactorizable contribution dominates the amplitude for this decay mode. To handle the nonfactorizable contributions we have extended previous chiral quark models for the pure light quark case [25] used in [23,24,26], and the heavy-light case [21] used in [3,9,11,12,20,22], to include also energetic light quarks. Thus, within our framework, the heavy and the energetic light quarks are represented by reduced fields corresponding to the redundant soft(er) interactions obtained when we split off the hard momenta, being of order m_b or m_c for heavy quarks and $E \simeq m_b/2$ for the light energetic quark. *A priori* we should have built our model on the SCET formalism using for light energetic quarks the SCET propagator depending also on transverse quark momenta [2,15]. But, as explained in the introduction, for our purpose this is an unnecessary complication which in our model has no significant numerical effects. Therefore the simplified LEET formalism (but keeping $\delta \neq 0$) is used within our model named LEχQM.

We have found that within our model we can account for $3/4$ of the amplitude needed to explain the experimental branching ratio [38]. In addition to our contributions one might think of mesonic loops like for processes of the type $B \to D\bar{D}$ [3] and $B \to \gamma D$ [12], but for such mesonic loops one has to insert *ad hoc* farm factors, or they should be handled within dispersion relation techniques. In both cases such calculations are beyond the scope of this paper. Anyway, final state interactions should be present [39] and give additional contributions.

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[1] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999).
[2] C. W. Bauer, S. Fleming, and M. Luke, Phys. Rev. D **63**, 014006 (2000); C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, Phys. Rev. D **63**, 114020 (2001); C. W. Bauer, D. Pirjol, I. Z. Rothstein, and I. W. Stewart, Phys. Rev. D **70**, 054015 (2004).
[3] J. O. Eeg, S. Fajfer, and A. Hiorth, Phys. Lett. B **570**, 46 (2003); J. O. Eeg, S. Fajfer, and A. Prapotnik, Eur. Phys. J. C **42**, 29 (2005). See also J. O. Eeg, S. Fajfer, and J. Zupan, Phys. Rev. D **64**, 034010 (2001).
[4] Y. Y. Keum, H-n. Li, and A. I. Sanda, Phys. Lett. B **504**, 6 (2001).
[5] Ying Li, Ciai-Diam Lü, and Zhen-Jun Xiao, J. Phys. G **31**, 273 (2005).
[6] See, for instance, M. Neubert, Phys. Rep. **245**, 259 (1994); A. V. Manohar and M. B. Wise, *Heavy Quark Physics* (Cambridge University Press, Cambridge, England, 2000).
[7] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. **281**, 145 (1997).
[8] C. G. Boyd and B. Grinstein, Nucl. Phys. **B451**, 177 (1995).
[9] A. Hiorth and J. O. Eeg, Eur. Phys. J. direct C **30**, 006 (2003), and references therein; republished as **32**, s69 (2004).
[10] B. Grinstein and R. F. Lebed, Phys. Rev. D **60**, 031302(R) (1999).
[11] J. O. Eeg, A. Hiorth, and A. D. Polosa, Phys. Rev. D **65**, 054030 (2002).
[12] J. A. Macdonald Sørensen and J. O. Eeg, Phys. Rev. D **75**, 034015 (2007).
[13] M. J. Dougan and B. Grinstein, Phys. Lett. B **255**, 583 (1991).
[14] J. Charles, A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Phys. Rev. D **60**, 014001 (1999).
[15] C. Balzereit, T. Mannel, and W. Kilian, Phys. Rev. D **58**, 114029 (1998); U. Aglietti and G. Corbò, Int. J. Mod. Phys. A **15**, 363 (2000).
[16] M. Neubert and A. Petrov, Phys. Lett. B **519**, 50 (2001).

[17] M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. **33**, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. B **52**, 351 (1974); A. I. Vainstein, V. I. Zakharov, and M. A. Shifman, Zh. Eksp. Teor. Fiz. **72**, 1275 (1977) [Sov. Phys. JETP **45**, 670 (1977)].
[18] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996), and references therein.M. Ciuchini, E. Franco, G. Martinelli, and L. Reina, Nucl. Phys. **B415**, 403 (1994).
[19] Lars E. Leganger, Master thesis, Univ. of Oslo, 2007.
[20] J. O. Eeg and A. Hiorth, Fiz. B **14**, 41 (2005).
[21] A. Hiorth and J. O. Eeg, Phys. Rev. D **66**, 074001 (2002).
[22] A. Hiorth and J. O. Eeg, Eur. Phys. J. direct C **39 S1**, 27 (2005).
[23] S. Bertolini, J. O. Eeg, and M. Fabbrichesi, Nucl. Phys. **B449**, 197 (1995); V. Antonelli, S. Bertolini, J. O. Eeg, M. Fabbrichesi, and E. I. Lashin, Nucl. Phys. **B469**, 143 (1996); S. Bertolini, J. O. Eeg, M. Fabbrichesi, and E. I. Lashin, Nucl. Phys. **B514**, 63 (1998); **B514**, 93 (1998).
[24] J. O. Eeg and I. Picek, Phys. Lett. B **301**, 423 (1993); **323**, 193 (1994); A. E. Bergan and J. O. Eeg, Phys. Lett. B **390**, 420 (1997); J. O. Eeg, K. Kumerički, and I. Picek, Phys. Lett. B **669**, 150 (2008).
[25] See, for example, A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984); D. Espriu, E. de Rafael, and J. Taron, Nucl. Phys. **B345**, 22 (1990); and references therein.
[26] A. Pich and E. de Rafael, Nucl. Phys. **B358**, 311 (1991).
[27] W. A. Bardeen and C. T. Hill, Phys. Rev. D **49**, 409 (1994); A. Deandrea, N. Di Bartolomeo, R. Gatto, G. Nardulli, and A. D. Polosa, Phys. Rev. D **58**, 034004 (1998); A. Polosa, Riv. Nuovo Cimento Soc. Ital. Fis. **23 N11**, 1 (2000); D. Ebert, T. Feldmann, R. Friedrich, and H. Reinhardt, Nucl. Phys. **B434**, 619 (1995).
[28] A. A. Andrianov and V. A. Andrianov, Z. Phys. C **55**, 435 (1992); J. Bijnens, E. de Rafael, and H. Zheng, Z. Phys. C **62**, 437 (1994).
[29] B. Grinstein and D. Pirjol, Phys. Lett. B **615**, 213 (2005).
[30] B. Grinstein, W. Kilian, T. Mannel, and M. B. Wise, Nucl. Phys. **B363**, 19 (1991); R. Fleischer, Nucl. Phys. **B412**, 201 (1994).
[31] V. Novikov, A. I. Vainstein, V. I. Zakharov, and M. A. Shifman, Fortschr. Phys. **32**, 585 (1984).
[32] K. Kumerički and J. O. Eeg, Phys. Rev. D **81**, 074015 (2010); arXiv:0910.5428.
[33] See, for example, P. Ball, J. High Energy Phys. 09 (1998) 005; F. De Fazio, T. Feldman, and T. Hurth, J. High Energy Phys. 02 (2008) 031.
[34] L. J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rep. **127**, 1 (1985).
[35] F. Jugeau, A. Le Yaouanc, L. Oliver, and J.-C. Raynal, Phys. Rev. D **74**, 094012 (2006).
[36] N. Uraltsev, Phys. Lett. B **585**, 253 (2004).
[37] H. Hogaasen and M. Sadzikowski, Z. Phys. C **64**, 427 (1994).
[38] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[39] A. B. Kaidalov and M. I. Vysotsky, Phys. At. Nucl. **70**, 712 (2007).