Tetraquark state and multibody interaction

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The tetraquark states with diquark–anti-diquark configuration have been studied in the flux-tube model, in which the multibody confinement is used. In this model approach, the states $Y(2175)$, $f_0(600)$, $f_0(980)$, and $X(1576)$ can be assigned as tetraquark states. They are color confinement resonances with three-dimension structure. This study suggests that the multibody confinement should be employed in the quark model study of multiquark states instead of the additive two-body confinement.

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I. INTRODUCTION

Quantum chromodynamics (QCD) is widely accepted as the fundamental theory of strong interaction and has been verified in high momentum transfer processes. In the low energy region, such as hadron spectroscopy and hadron-hadron interaction study, the *ab initio* calculation directly from QCD becomes very difficult because of the complication of nonperturbative nature. Recently, lattice QCD and nonperturbative QCD method have made impressive progresses on hadron properties, even on hadron-hadron interactions [1,2]. However, QCD-inspired quark models are still an useful tool in obtaining physical insight for these complicated strong interaction systems. In the constituent quark model, hadrons are composed of $q\bar{q}$ (meson) and qqq (baryon). The states with dominant components beyond $q\bar{q}$ and q^3, such as multiquark $q^2\bar{q}^2$, $q^4\bar{q}$, q^6, quark-gluon hybrids $q\bar{q}g$, q^3g, glueballs gg, are called exotic hadrons. QCD does not deny the existence of exotic hadrons. The investigation of multiquark states included the two quarks and two antiquarks started by Jaffe [3] about 30 years ago. Since then, different models have been applied to the $qq\bar{q}\,\bar{q}$ studies [4–10]. Recently, the four-quark state studies have been revived [11–20] due to Belle, *BABAR*, and other experimental collaborations that have discovered a number of open and hidden charm meson states, which are difficult to fit into the conventional $q\bar{q}$ meson spectra [21].

The color confinement is the most prominent feature of QCD and should play an essential role in the low energy hadron physics. For $q\bar{q}$ meson and q^3 baryons, the color structures are unique. This makes the construction of quark models easier and effective. However, it also minimizes the effects of quark confinement. For multiquark systems and hadron-hadron interactions, the color structures are abundant (see Fig. 1). Is it reasonable to directly extend the two-body color confinement for usual hadrons to multiquark system by means of the Casimir scaling [22]? This is an open question. For example, the use of the Casimir scaling will lead to anticonfinement for some color structure in the multiquark system [23]. Lattice QCD calculations for mesons, baryons, tetraquarks, and pentaquarks reveal flux-tube or string-like structure [24,25] which links quark to form hadrons, as shown in Fig. 1, where a black dot represents a quark while a hollow dot represents an antiquark. The confinement of multiquark states is generally a multibody interaction. Lattice QCD calculations indicate that confinement potential energy is proportional to the minimum of the total length of strings which connect the quarks to form a multiquark state.

It is interesting to apply the multibody confinement to the multiquark calculations. This can be realized by employing the naive flux-tube or string model [26,27], which is based on lattice QCD picture by simplifying the multibody confinement with harmonic interaction approximation, i.e., the total length of strings is replaced by the sum of the square of the string lengths to simplify the calculation. A comparative study between linear and quadratic potential showed the inaccuracy of this replacement is quite small [26,27]. There are two reasons to expect this: One is that the spatial variations in separation of the quarks (lengths of the string) in different hadrons do not differ significantly, so the difference between the two functional forms is small and can be absorbed in the adjustable parameter, the stiffness. The second is that we are using a nonrelativistic description of the dynamics and, as was shown long ago [28], an interaction energy that varies linearly with separation between fermions in a relativistic, first order differential dynamics has a wide region in which a harmonic approximation is valid for the second order (Feynman-Gell-Mann) reduction of the equations of motion.

In our previous paper [27], a new string structure based on the flux-tube model, the quark benzene for six-quark system was proposed and its possible effect on NN scattering was discussed. In the present work, the same

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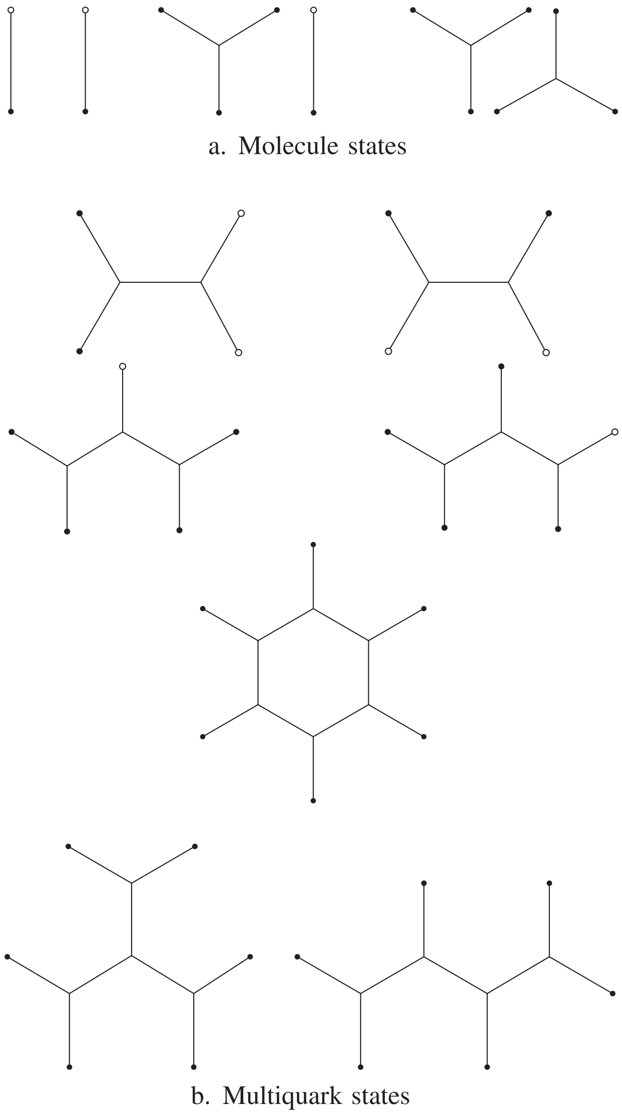

FIG. 1. Possible string structures of multiquark systems.

flux-tube model is extended to the study of tetraquark states by using Gaussian expansion method (GEM) [29], which is a powerful method for few-body system with high precision. The paper is organized as follows: in Sec. II the model Hamilton and parameters for describing mesons are given. In Sec. III we present the confinement potential of tetraquark state in the framework of flux-tube model. The brief introduction of GEM and the construction of the wave functions of tetraquark states are given in Sec. IV. Section V presents the numerical results and discussions. A summary is given in the last section.

II. MODEL AND PARAMETERS FOR $q\bar{q}$ MESON AND TETRAQUARK

The quark model inspired by QCD should include perturbative (effective one-gluon exchange) and nonperturbative (color confinement and the spontaneous breaking of chiral symmetry) properties. The origin of the constituent quark mass can be traced back to the spontaneous breaking of chiral symmetry and consequently constituent quarks should interact through the exchange of Goldstone bosons [30]. However, there are some controversies on the use of σ meson, and it is argued recently that the σ meson exchange used in the chiral quark model can be replaced by the quark delocalization and color screening mechanism [31]. Therefore, the σ meson is not included in the following Hamiltonian. The model Hamiltonian of the $q\bar{q}$ ($n = 2$) and tetraquark system ($n = 4$) is chosen as follows:

$$H = \sum_{i=1}^{n}\left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) - T_{\rm CM} + \sum_{i>j}^{n}(V_{ij}^C + V_{ij}^G + V_{ij}^\chi), \qquad (1)$$

$$\chi = \pi, K, \eta,$$

$$V_{ij}^\pi = \frac{g_{\rm ch}^2}{4\pi}\frac{m_\pi^2}{12m_im_j}\frac{\Lambda_\pi^2}{\Lambda_\pi^2 - m_\pi^2}m_\pi\sigma_i\cdot\sigma_j \times\left[Y(m_\pi r_{ij}) - \frac{\Lambda_\pi^3}{m_\pi^3}Y(\Lambda_\pi r_{ij})\right]\sum_{a=1}^{3}F_i^a\cdot F_j^a, \qquad (2)$$

$$V_{ij}^K = \frac{g_{\rm ch}^2}{4\pi}\frac{m_K^2}{12m_im_j}\frac{\Lambda_K^2}{\Lambda_K^2 - m_\pi^2}m_K\sigma_i\cdot\sigma_j \times\left[Y(m_K r_{ij}) - \frac{\Lambda_K^3}{m_K^3}Y(\Lambda_K r_{ij})\right]\sum_{a=4}^{7}F_i^a\cdot F_j^a, \qquad (3)$$

$$V_{ij}^\eta = \frac{g_{\rm ch}^2}{4\pi}\frac{m_\eta^2}{12m_im_j}\frac{\Lambda_\eta^2}{\Lambda_\eta^2 - m_\eta^2}m_\eta\sigma_i\cdot\sigma_j \times\left[Y(m_\eta r_{ij}) - \frac{\Lambda_\eta^3}{m_\eta^3}Y(\Lambda_\eta r_{ij})\right] \times[\cos\theta_P(F_i^8\cdot F_j^8) - \sin\theta_P], \qquad (4)$$

$$V_{ij}^G = \frac{1}{4}\alpha_s\lambda_i^c\cdot\lambda_j^c\left[\frac{1}{r_{ij}} - \frac{\pi}{2}\delta(\mathbf{r}_{ij})\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{3m_im_j}\sigma_i\cdot\sigma_j\right)\right], \qquad (5)$$

$$V_{ij}^C = k(r_{ij}^2 - \Delta), \quad \text{for } q\bar{q}, \qquad (6)$$

where $T_{\rm CM}$ is the center-of-mass kinetic energy, $Y(x)$ is the standard Yukawa function, and all symbols have their usual meaning. The delta function in the one-gluon exchange potential should be regularized [32]; the regularization is justified based on the finite size of the constituent quark and should, therefore, be flavor dependent [33,34],

$$\delta(\mathbf{r}_{ij}) = \frac{1}{4\pi}\frac{1}{r_{ij}r_0^2(\mu)}e^{-r_{ij}/r_0(\mu)}, \qquad (7)$$

where μ is the reduced mass of the $q\bar{q}$ system and $r_0(\mu) = \hat{r}_0/\mu$ and the $\hat{r}_0$ is a model parameter to be determined by ground state meson masses.

The model parameters are fixed as follows: the u, d-quark masses are taken as the same and are assumed to be exactly $\frac{1}{3}$ of the nucleon mass, namely, $m_u = m_d = 313$ MeV, the masses of π, K, η take the experimental values, the Λ_π, Λ_K, Λ_η, θ_p, Λ_0, and μ_0 are taken the same values as in Ref. [33], namely, $\Lambda_\pi = 4.2\ \mathrm{fm}^{-1}$, $\Lambda_K = \Lambda_\eta = 5.2\ \mathrm{fm}^{-1}$, $\theta_p = -15°$, $\Lambda_0 = 36.98$ MeV, and $\mu_0 = 0.113$ fm. The chiral coupling constant $g_{\rm ch}$ is determined from the πNN coupling constant through

$$\frac{g_{\rm ch}^2}{4\pi} = \left(\frac{3}{5}\right)^2 \frac{g_{\pi NN}^2}{4\pi} \frac{m_{u,d}^2}{m_N^2}, \tag{8}$$

and flavor $SU(3)$ symmetry is assumed, where $\frac{g_{\rm ch}^2}{4\pi} = 0.54$. The rest parameters m_s, k, Δ, α_0, and $\hat{r}_0$ are determined by fitting ground state meson spectra. An effective scale-dependent strong coupling constant [33] is given by

$$\alpha_s(\mu) = \frac{\alpha_0}{\ln[\frac{\mu^2+\mu_0^2}{\Lambda_0^2}]}. \tag{9}$$

The meson spectra are obtained by solving the following Schrödinger equation:

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V(\mathbf{r}) - E\right]\psi_{lm}(\mathbf{r})\psi_{\rm csf} = 0, \tag{10}$$

where ψ_{lm} is the relative orbital motion wave functions, $\psi_{\rm csf}$ is the color-spin-flavor wave function. $\psi_{lm}(\mathbf{r})$ can be expanded by means of Gaussian functions with different size [29],

$$\phi_{lm}(\mathbf{r}) = \sum_{n=1}^{n_{\max}} c_n N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}}). \tag{11}$$

Gaussian size parameters are taken as geometric progression

$$\nu_n = \frac{1}{r_n^2}, \quad r_n = r_1 a^{n-1}, \quad a = \left(\frac{r_{n_{\max}}}{r_1}\right)^{1/(n_{\max}-1)}. \tag{12}$$

With $r_1 = 0.1$ fm, $r_{n_{\max}} = 2.0$ fm, and $n_{\max} = 7$, we can obtain converged results. The model parameters and the obtained meson masses are shown in Tables I and II, respectively.

From Table II, one can see that the calculated meson masses are consistent with experimental values. The masses of mesons with orbital angular momentum $L = 1$ are also given in Table II.

III. THE CONFINEMENT OF A TETRAQUARK STATE

The diquark picture is often used in the multiquark studies because of its attractive property. Maiani *et al.* obtained a rather good description of light scalar mesons with diquark–anti-diquark configuration [35,36]. Iwasaki and Fukutome discussed the possibility of the existence of tetraquark states made of four quarks based on diquark–-anti-diquark picture in the string (flux-tube) model [37]. Ding and Yan interpreted $X(1576)$ as the P-wave excitation of a diquark–anti-diquark bound state [38]. Here the diquark–anti-diquark configuration is used to study the tetraquark state in the flux-tube model. The configuration is shown in Fig. 2, where a solid dot represents a quark, while a hollow dot represents an antiquark, $\mathbf{r}_i$ is quark's position, and $\mathbf{y}_i$ represents a junction where three strings (flux tubes) meet. A thin line connecting a quark and a junction represents a fundamental representation, i.e. color triplet, and a thick line connecting two junctions is for a color sextet or other representations, namely, a compound string. The different types of string may have different stiffness [22,39]. Color couplings satisfying overall color singlet of tetraquark are $[[qq]_{\bar{\mathbf{3}}}[\bar{q}\,\bar{q}]_{\mathbf{3}}]_{\mathbf{1}}$ and $[[qq]_{\mathbf{6}}[\bar{q}\,\bar{q}]_{\bar{\mathbf{6}}}]_{\mathbf{1}}$, and subscripts represent the dimensions of color representations.

In the flux-tube model with quadratic confinement, the confinement potential of the tetraquark state has the following form:

$$V^C = k[(\mathbf{r}_1 - \mathbf{y}_1)^2 + (\mathbf{r}_2 - \mathbf{y}_1)^2 + (\mathbf{r}_3 - \mathbf{y}_2)^2 + (\mathbf{r}_4 - \mathbf{y}_2)^2 + \kappa_d(\mathbf{y}_1 - \mathbf{y}_2)^2], \tag{13}$$

where k is the stiffness of the string with the fundamental representation **3** which is determined by meson spectrum, and $k\kappa_d$ is the different compound string stiffness. The compound string stiffness parameter κ_d [22] depends on the color representation **d** of the string,

TABLE I. The model parameters.

m_s (MeV)	m_c (MeV)	k (MeV fm^{-2})	$\hat{r}_0$ (MeV fm)	Δ (fm^2)	α_0
520	1700	213.3	30.85	0.5	4.25

TABLE II. Meson spectra (MeV).

Meson:	π	K	ρ	K^*	ω	ϕ	D^0	D^*	D_s	D_s^*	η_c	J/ψ
Theor. ($L = 0$)	139	502	761	897	735	1023	1928	2001	2014	2112	2992	3144
Exp. ($L = 0$)	139	496	770	898	780	1020	1865	2007	1968	2112	2980	3097
Theor. ($L = 1$)	1054	1204	1102	1226	1098	1342	2304	2310	2401	2405	3419	3420

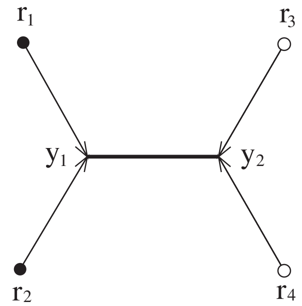

FIG. 2. Diquark–anti-diquark state.

$$\kappa_{\mathbf{d}} = \frac{C_{\mathbf{d}}}{C_{\mathbf{3}}}, \tag{14}$$

where $C_{\mathbf{d}}$ is the eigenvalue of the Casimir operator associated with the $SU(3)$ color representation $\mathbf{d}$ of the string, $C_{\mathbf{3}} = \frac{4}{3}$, $C_{\mathbf{6}} = \frac{10}{3}$, and $C_{\mathbf{8}} = 3$.

For given quark positions $\mathbf{r}_i$, we can fix the positions of the junctions $\mathbf{y}_i$ by minimizing the energy of the system. After fixing $\mathbf{y}_i$, a set of canonical coordinates $\mathbf{Q}_i$ is introduced to simplify expressions of the energies of the system. The orthogonal transformation between quark's coordinates $\mathbf{r}_j$ and $\mathbf{Q}_i$ can be written in the form of

$$\mathbf{Q}_i = \sum_{j=1}^{4} a_{ij}\mathbf{q}_j, \tag{15}$$

where $\mathbf{q}_j = \sqrt{m_j}\mathbf{r}_j$ and m_i is the mass of the quark q_i. Then the kinetic and potential energy of the system read

$$T = \frac{1}{2}\sum_{i=1}^{4}\dot{\mathbf{Q}}_i^2, \qquad V^C = \frac{1}{2}\sum_{i=1}^{4}\omega_i^2\mathbf{Q}_i^2, \tag{16}$$

respectively. If all quarks have same mass, the set of canonical coordinates are simplified as

$$\begin{aligned} \mathbf{Q}_1 &= \sqrt{\frac{m}{2}}(\mathbf{r}_1 - \mathbf{r}_2), \qquad \mathbf{Q}_2 = \sqrt{\frac{m}{2}}(\mathbf{r}_3 - \mathbf{r}_4), \\ \mathbf{Q}_3 &= \sqrt{\frac{m}{4}}(\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_3 - \mathbf{r}_4), \\ \mathbf{Q}_4 &= \sqrt{\frac{m}{4}}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4). \end{aligned} \tag{17}$$

In the center-of-mass system of four quarks, the confinement potential V^C can be written as

$$V^C = \frac{k}{m}\left(\mathbf{Q}_1^2 + \mathbf{Q}_2^2 + \frac{\kappa_d}{1+\kappa_d}\mathbf{Q}_3^2\right). \tag{18}$$

Taking into account potential energy shift [see Eq. (6) above], the confinement potential V^C used in the present calculation has the following form:

$$\begin{aligned} V^C = k\bigg[&\left(\frac{(\mathbf{r}_1 - \mathbf{r}_2)^2}{2} - \Delta\right) + \left(\frac{(\mathbf{r}_3 - \mathbf{r}_4)^2}{2} - \Delta\right) + \frac{\kappa_d}{1+\kappa_d} \\ &\times\left(\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} - \frac{\mathbf{r}_3 + \mathbf{r}_4}{2}\right)^2 - \Delta\right)\bigg]. \end{aligned} \tag{19}$$

The confinement potential is a multibody interaction. In this way, our model is different from the string model of Ref. [37], where the four-body problem is simplified to two-body one by treating diquark as a antiquark and anti-diquark as a quark.

IV. $Y(2175)$ AND WAVE FUNCTIONS

Recently, *BABAR* Collaboration observed a resonance $Y(2175)$ near the threshold in the process $e^+e^- \to \phi f_0(980)$ via initial-state radiation [40–42]. The Breit-Wigner mass is $M = 2.175 \pm 0.010 \pm 0.015$ GeV, and the width is narrow $\Gamma = 0.058 \pm 0.016 \pm 0.020$ GeV. It is claimed as an isospin singlet and its spin-parity is determined to be $J^{PC} = 1^{--}$. It was also confirmed by the BES collaboration in the process $J/\psi \to \eta\phi f_0(980)$ [43]. Various theoretical interpretations have been proposed to explain this resonance. Ding and Yan interpreted it as a strangeonium hybrid and studied its decay properties in the flux-tube model and the constituent gluon model [44,45]. Wang studied $Y(2175)$ as a tetraquark state $ss\bar{s}\bar{s}$ by using QCD sum rule and suggested that there might be tetraquark components in the state $Y(2175)$ [46]. Zhu reviewed $Y(2175)$ and indicated that the possibility of $Y(2175)$ arising from S-wave threshold effects cannot be excluded [47]. Napsuciale *et al.* studied the reaction $e^+e^- \to \phi\pi\pi$ for pions in an isoscalar S-wave channel. By selecting the $\phi f_0(980)$ contribution as a function of the $e + e$ energy, they reproduced the experimental data except for the narrow peak, yielding support to the existence of a 1^{--} resonance [48]. Bystritskiy *et al.* calculated the total probability and the differential cross section of the process $e^+e^- \to \phi f_0(980)$ by using the local Nambu-Jona-Lasinio model [49]. Torres *et al.* performed a Faddeev calculation for the three meson system, ϕKK, taking the interaction between two pseudoscalar mesons and between a vector and a pseudoscalar meson from the chiral unitary approach which generates dynamically the low lying scalar mesons and the low lying axial vector mesons. They obtained a neat resonance peak around a total mass of 2150 MeV [50]. Chen *et al.* studied the mass of the state $Y(2175)$ in the QCD sum rule and obtained a mass around 2.2–2.4 GeV [51]. They also discussed possible decay properties of $Y(2175)$ if it is a tetraquark state.

Here we also treat the $Y(2175)$ as a tetraquark state with two s and two $\bar{s}$ quarks. Since the $Y(2175)$ has quantum numbers $J^{PC} = 1^{--}$, the combined effects of the negative parity and the total angular momentum $J = 1$ require a unit of orbital angular momentum excitation. The Jacobi coordinates of the tetraquark are defined as (see Fig. 2)

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \qquad \mathbf{R} = \mathbf{r}_3 - \mathbf{r}_4, \tag{20}$$

$$\mathbf{X} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2} - \frac{m_3\mathbf{r}_3 + m_4\mathbf{r}_4}{m_3 + m_4}, \tag{21}$$

where particles 1 and 2 are the s quarks, and particle 3 and 4 are the $\bar{s}$ quarks. We assume that the $Y(2175)$ is the first orbital angular momentum excitation between a diquark and anti-diquark, $([ss]_{s\text{-wave}}[\bar{s}\,\bar{s}]_{s\text{-wave}})_{P\text{-wave}}$ and with total spin $S = 0$. The total wave function for $Y(2175)$ can be written as a sum of the following direct products of color, isospin, spin, and configuration terms,

$$\Phi_{IJ_TM_T} = [[[\phi^G_{l_1m_1}(\mathbf{r})\Psi_{s_1m_{s_1}}]_{J_1M_1} \times [\psi^G_{l_2m_2}(\mathbf{R})\Psi_{s_2m_{s_2}}]_{J_2M_2}]_{J_{12}M_{12}}\chi^G_{LM}(\mathbf{X})]_{J_TM_T} \times [\Psi_{c_1}\Psi_{c_2}]_c[\Psi_{I_1}\Psi_{I_2}]_I, \tag{22}$$

where I_i, s_i, and c_i, $i = 1, 2$ represent isospin, spin, and color of diquark and anti-diquark, respectively. The four-quark state is an overall color singlet with well defined parity $P = (-1)^{l_1+l_2+L}$, isospin I, and total angular momentum J_T. For $Y(2175)$, $I = 0$, $J_T = 1$, $l_1 + l_2 + L =$ odd. In this calculation, $l_1 = 0$, $l_2 = 0$, and $L = 1$ are assumed. Because orbital and flavor wave functions are symmetric under exchange of two identical particles, the combinations of color, and spin wave functions should be antisymmetric, which leads to two possibilities: channel I color $[[ss]_{\mathbf{6}}[\bar{s}\,\bar{s}]_{\bar{\mathbf{6}}}]_{\mathbf{1}}$ and spin $[[ss]_0[\bar{s}\,\bar{s}]_0]_0$, channel II color $[[ss]_{\bar{\mathbf{3}}}[\bar{s}\,\bar{s}]_{\mathbf{3}}]_{\mathbf{1}}$, and spin $[[ss]_1[\bar{s}\,\bar{s}]_1]_0$. The diquarks in channel I and channel II are not "good" diquarks [52] because of the orbital and flavor wave functions are symmetric.

To obtain a reliable solution of few-body problem, a high precision method is indispensable. In this work, the GEM [29], which has been proven to be rather powerful to solve few-body problem, is used to study four-quark systems in the flux-tube model. In GEM, three relative motion wave functions are written as

$$\phi^G_{l_1m_1}(\mathbf{r}) = \sum_{n_1=1}^{n_{1\max}} c_{n_1}N_{n_1l_1}r^{l_1}e^{-\nu_{n_1}r^2}Y_{l_1m_1}(\hat{\mathbf{r}}), \tag{23}$$

$$\psi^G_{l_2m_2}(\mathbf{R}) = \sum_{n_2=1}^{n_{2\max}} c_{n_2}N_{n_2l_2}R^{l_2}e^{-\nu_{n_2}R^2}Y_{l_2m_2}(\hat{\mathbf{R}}), \tag{24}$$

$$\chi^G_{LM}(\mathbf{X}) = \sum_{n_3=1}^{n_{3\max}} c_{n_3}N_{LM}X^{L}e^{-\nu_{n_3}X^2}Y_{LM}(\hat{\mathbf{X}}). \tag{25}$$

The choice of Gaussian size is the same as that of mesons.

V. NUMERICAL RESULTS AND DISCUSSIONS

The energy of $Y(2175)$ is obtained by solving the four-body Schrödinger equation

$$(H - E)\Phi_{IJ_TM_T} = 0 \tag{26}$$

with Rayleigh-Ritz variational principle. In GEM the calculated results are converged with $n_{1\max} = 10$, $n_{2\max} = 10$, and $n_{3\max} = 10$. Minimum and maximum ranges of the bases are 0.1 and 3.0 fm for coordinates $\mathbf{r}$, $\mathbf{R}$, and $\mathbf{X}$, respectively. Eventually, the model space is constructed by 2000 basis functions.

TABLE III. Numerical results for different models (MeV).

Models	Channel I	Channel II	Channel coupling
Native	2559	2442	2422
Chiral	2538	2413	2387
Flux-tube	2290	2188	2176

Numerical results of different models are shown in Table III. In the flux-tube model, the energy of channel I is 2290 MeV, about 110 MeV higher than the experimental value. The energy of channel II is 2188 MeV which is very close to experimental value. The difference between channel I and channel II mainly comes from the one-gluon exchange. Channel coupling reduce the energy to 2176 MeV, which is consistent with experimental value. For comparison, the results of naive [53] and chiral quark models, where the confinement of multiquark systems is the sum of two-body interactions, are also shown in Table III, which are higher than experimental value. The chiral quark gives similar results as the naive quark model, which means the Nambu-Goldstone-boson exchange does not pull down the energy of the system much. Generally, the Goldstone-boson exchange is important for the splitting within a multiplet of hadrons but not important for the average energy of the multiplet and the splitting due to Goldstone boson exchange in tetraquark systems are several MeV to several tens of MeV. To find the spatial structure of the tetraquark, the distances between any two quarks are calculated and shown in the following:

$$\langle \mathbf{r}^2_{12}\rangle = \langle \mathbf{r}^2_{34}\rangle = 1.0\ \text{fm}^2, \tag{27}$$

$$\langle \mathbf{r}^2_{13}\rangle = \langle \mathbf{r}^2_{14}\rangle = \langle \mathbf{r}^2_{23}\rangle = \langle \mathbf{r}^2_{24}\rangle = 1.5\ \text{fm}^2. \tag{28}$$

Obviously, the $Y(2175)$ can not be planar, as shown in the Fig. 3.

To check the universality of the multibody confinement potential for the multiquark states in the flux-tube model, the calculation is extended to other tetraquark states with the same configuration and parameters. For nonstrange

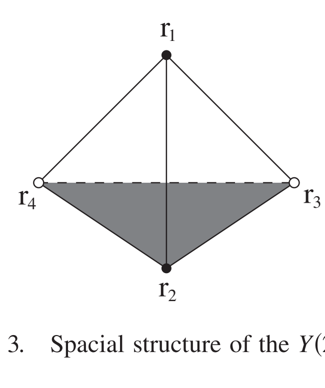

FIG. 3. Spacial structure of the $Y(2175)$.

TABLE IV. Energies for some tetraquark states (MeV), $n = u, d$.

States	$nn\bar{n}\bar{n}$	$nn\bar{n}\bar{n}$	$ns\bar{n}\bar{s}$	$ns\bar{n}\bar{s}$	$ss\bar{s}\bar{s}$
I^GJ^{PC}	0^+0^{++}	0^+0^{++}	1^-0^{++}	1^-1^{--}	0^+0^{++}
Energy	587	1019	1306	1715	1925
Candidate	$f_0(600)$	$f_0(980)$	$a_0(1450)$	$X(1576)$	$f_0(2020)$
States	$ss\bar{s}\bar{s}$	$cn\bar{c}\bar{n}$	$cn\bar{c}\bar{n}$	$cn\bar{c}\bar{n}$	$cn\bar{c}\bar{n}$
Energy	2176	3776	4020	3644	3978
Candidate	$Y(2175)$		$X(3872)$		$Y(4008)$
Stages	$cs\bar{c}\bar{s}$	$cs\bar{c}\bar{s}$			
I^GJ^{PC}	0^+0^{++}	0^+1^{--}			
Energy	4038	4292			
Candidate		$Y(4260)$			

tetraquark, the ground state and the first radial excited state with quantum numbers $I^GJ^{PC} = 0^+0^{++}$ are investigated. The masses 587 and 1019 MeV are obtained (see Table IV), which are consistent with the experimental values of $f_0(600)$ and $f_0(980)$. So the scalar mesons $f_0(600)$ and $f_0(980)$ are identified as tetraquark states in the present calculation, other than the $q\bar{q}$ P-wave excited states (see Table II). Chen *et al.* have also studied the lowest-lying scalar mesons in the QCD sum rule; the same conclusion is obtained [54].

$X(1576)$ was considered as the tetraquark state $qs\bar{q}\bar{s}$ with quantum numbers $J^{PC} = 1^{--}$ by Karliner and Lipkin [55]. In our calculation, the mass 1715 MeV is obtained, which is a little higher than experimental value but it is still in the region of error bar, $1576^{+49}_{-55}(\text{stat})^{+98}_{-91}(\text{syst})$ [56]. The ground state of $qs\bar{q}\bar{s}$ with quantum numbers $I^GJ^{PC} = 1^-0^{++}$ in the flux-tube model has energy 1306 MeV, which is lower than the mass of the possible candidate $a_0(1450)$. Channel coupling with $q\bar{q}$ may be needed to push the energy up to make this assignment possible.

In Table IV, the energy of the tetraquark state $ss\bar{s}\bar{s}$ with quantum numbers $I^GJ^{PC} = 0^+0^{++}$ is also given. The energy 1925 MeV is about 100 MeV lower than the mass of scalar meson $f_0(2020)$.

The calculation is also extended to charmonium states. The energies of states $cq\bar{c}\bar{q}$, $q = u, d, s$ with $J^P = 0^{++}$, 1^{--} and 1^{+-} are given in Table IV. From the mass, the states $Y(4008)$ and $Y(4260)$ can be assigned to be tetraquark states $cq\bar{c}\bar{q}$, $q = u, d$ (mass 3978 MeV) and $cs\bar{c}\bar{s}$ (mass 4292 MeV) with $J^{PC} = 1^{--}$. Assuming $X(3872)$ as a tetraquark state $cq\bar{c}\bar{q}$ with $J^P = 1^{+-}$, the mass is about 100 MeV lower than experimental values. The tetraquark state $cs\bar{c}\bar{s}$ with $J^{PC} = 0^{++}$ was taken as the candidate of $Y(4140)$ by the Ding [57], In the present work the state has energy 3953 MeV, which is about 100 MeV lower than the experimental value.

From Table IV, we can see an interesting phenomenon. Taking as the possible candidates of scaler meson observed experimentally, the ground states that contain s, c quarks in our model have energies about 100 MeV lower than experiment values, while the excited states are consistent with experiment values. Further study is needed to clarify the situation.

For tetraquark systems, lattice QCD calculations show that a three-dimensional tetrahedral structure is rather stable against the transition into two mesons [58]. The decay of the tetraquark state $Y(2175)$ and others into color singlet hadrons requires the breakup of the nonplanar flux-tube structure into conventional color singlet hadrons, which involves flux-tube structure rearrangement which is similar to the structure transformation as in isomer in chemistry. This might be the reason for the narrow width of the $Y(2175)$ but certainly needs further quantitative calculation.

VI. SUMMARY

By employing multibody confinement, which is consistent with the lattice QCD calculations, the flux-tube model is used to investigate the tetraquark states with diquark–-anti-diquark configuration. The model parameters are determined by meson spectra. So the masses of tetraquark states are all theoretical prediction. In the present calculation, the scalar meson $f_0(600)$, $f_0(980)$ can be identified as nonstrange tetraquark states with quantum numbers $I^GJ^{PC} = 0^+0^{++}$. $X(1576)$ is tetraquark $qs\bar{q}\bar{s}$ with quantum numbers $I^GJ^{PC} = 1^-1^{--}$. $Y(2175)$ is hidden strange tetraquark $ss\bar{s}\bar{s}$ with $I^GJ^{PC} = 0^+1^{--}$. $Y(4008)$ and $Y(4260)$ can be assigned to be tetraquark states $cq\bar{c}\bar{q}$, $q = u, d$, and $cs\bar{c}\bar{s}$ with $J^{PC} = 1^{--}$. The multibody interaction plays an important role in reducing the energy of the system. Of course, the identification should be checked by decay calculation.

Because all the tetraquark states studied here can have $q\bar{q}$ as their component, the calculation with two-quark and four-quark mixing is needed. Even limiting to a four-quark system, the mixing with other configurations, e.g., dimeson, should also be considered. This is still a difficult task at present, because no reliable information about the transition interaction between different color structures available. Lattice QCD may be helpful here.

Multibody flux-tube confinement connects the four quarks into a tetraquark state. The states cannot decay into two colorful hadrons directly due to color confinement. They must transform into color singlet mesons before decaying. The situation is similar to compound nucleus formation and therefore should induce a resonance. Those are ‘‘color confined, multiquark resonance’’ states [27,59] in our model, which is different from all of those microscopic resonances discussed by Weinberg [60]. Its decay width is sensitively dependent on the transition interaction between color singlet mesons and genuine multiquark hidden color configuration. Both of these call for the transition interaction between different color structures.

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