

## Measurement of $CP$ violating asymmetries in $B^0 \rightarrow K^+ K^- K_S^0$ decays with a time-dependent Dalitz approach

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We report a measurement of  $CP$  violating asymmetries in  $B^0(\bar{B}^0) \rightarrow K^+K^-K_S^0$  decays with a time-dependent Dalitz approach. This analysis is based on a data sample of  $657 \times 10^6$   $B\bar{B}$  pairs accumulated at the  $Y(4S)$  resonance with the Belle detector at the KEKB asymmetric-energy  $e^+e^-$  collider. As the result of an unbinned maximum likelihood fit to the selected candidates, the mixing-induced and direct  $CP$  violation parameters  $\phi_1^{\text{eff}}$  and  $\mathcal{A}_{CP}$  are obtained for  $B^0 \rightarrow \phi(1020)K_S^0$ ,  $B^0 \rightarrow f_0(980)K_S^0$ , and other  $B^0 \rightarrow K^+K^-K_S^0$  decays. We find four solutions that describe the data. There are  $\phi_1^{\text{eff}}(B^0 \rightarrow \phi(1020)K_S^0) = (32.2 \pm 9.0 \pm 2.6 \pm 1.4)^\circ$ ;  $\phi_1^{\text{eff}}(B^0 \rightarrow \phi(1020)K_S^0) = (26.2 \pm 8.8 \pm 2.7 \pm 1.2)^\circ$ ;  $\phi_1^{\text{eff}}(B^0 \rightarrow \phi(1020)K_S^0) = (27.3 \pm 8.6 \pm 2.8 \pm 1.3)^\circ$ ; and  $\phi_1^{\text{eff}}(B^0 \rightarrow \phi(1020)K_S^0) = (24.3 \pm 8.0 \pm 2.9 \pm 5.2)^\circ$ . The values for the  $CP$  violating phase in  $B^0 \rightarrow \phi(1020)K_S^0$  are similar, but other properties of the Dalitz plot are quite different for the four solutions. These four solutions have consistent  $\phi_1^{\text{eff}}$  values for all three  $B$ -meson decay channels and none of them deviates significantly from the values measured in  $B \rightarrow (c\bar{c})K^0$  decays with the currently available statistics. In addition, we find no significant direct  $CP$  violation.

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$CP$  violation in the quark sector is described in the standard model (SM) by the Kobayashi-Maskawa (KM) theory [1]. In this theory, the existence of a single irreducible phase gives rise to  $CP$  violating asymmetries in the time-dependent rates of  $B^0$  and  $\bar{B}^0$  decays into a common  $CP$  eigenstate [2]. Specifically, for neutral  $B$ -meson decays dominated by  $b \rightarrow c\bar{c}s$  transitions such as  $B^0(\bar{B}^0) \rightarrow J/\psi K^0$ , we can measure  $CP$  violating quantities that determine the  $\phi_1$  [3] angle of the Unitarity Triangle. The measurements have been performed by Belle [4] and BABAR [5] Collaborations and provide a precise reference value for  $\phi_1$  because of the very small theoretical uncertainty.

Recently, measurements of time-dependent  $CP$  violation of  $b \rightarrow s$  penguin-mediated  $B$  decays have become interesting because these decay modes proceed via loop diagrams and are, therefore, expected to be sensitive probes of the physics beyond the SM. In these decay modes, searches for new physics effects are carried out by investigating deviations of  $CP$  violating parameters from those determined by  $b \rightarrow c\bar{c}s$  processes [6].

Among these  $B^0$  decays,  $B^0 \rightarrow K^+K^-K_S^0$  is one of the most promising modes because of its very small Cabibbo-suppressed tree diagram contribution. Previous Belle measurements of the  $CP$  violating asymmetries have been

performed separately in the  $K^+K^-$  mass region around the  $\phi(1020)$  mass [7] and at higher  $K^+K^-$  masses [8], while neglecting interference between intermediate states. It is, however, expected that the sensitivity to  $CP$  violating parameters would improve in a measurement using the time-dependent Dalitz plot distribution because of the correct treatment of interferences between various resonant and nonresonant  $B^0 \rightarrow K^+K^-K_S^0$  processes.

In the decay chain  $Y(4S) \rightarrow B^0\bar{B}^0 \rightarrow (K^+K^-K_S^0)f_{\text{tag}}$ , where one of the  $B$  mesons decays at time  $t_{\text{rec}}$  to the final state  $K^+K^-K_S^0$  and the other decays at time  $t_{\text{tag}}$  to a final state  $f_{\text{tag}}$  that distinguishes between  $B^0$  and  $\bar{B}^0$ , the decay rate has a time dependence given by

$$|A_{\text{sig}}(\Delta t, q)|^2 = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} [(|A|^2 + |\bar{A}|^2) - q(|A|^2 - |\bar{A}|^2)] \\ \times \cos(\Delta m_d \Delta t) + 2q \text{Im}(\bar{A}A^*) \\ \times \sin(\Delta m_d \Delta t)], \quad (1)$$

where  $\tau_{B^0}$  is the neutral  $B$  meson lifetime,  $\Delta m_d$  is the mass difference between the two neutral  $B$  mass eigenstates,  $\Delta t = t_{\text{rec}} - t_{\text{tag}}$ ,  $A(\bar{A})$  is the total amplitude of  $B^0(\bar{B}^0) \rightarrow K^+K^-K_S^0$ , and the  $b$ -flavor charge  $q = +1(-1)$  when the tagged  $B$  meson is a  $B^0(\bar{B}^0)$ . The Dalitz plot variables  $s_+$ ,  $s_-$ , and  $s_0$  are defined as  $s_{\pm} \equiv (p_{\pm} + p_0)^2$  and

$s_0 \equiv (p_+ + p_-)^2$ , where  $p_+$ ,  $p_-$ , and  $p_0$  are the four-momenta of the  $K^+$ ,  $K^-$ , and  $K_S^0$ , respectively. These variables satisfy  $s_+ + s_- + s_0 = m_{B^0}^2 + 2m_{K^+}^2 + m_{K_S^0}^2$  by energy-momentum conservation. In the isobar approximation [9], the total amplitude for  $B^0(\bar{B}^0) \rightarrow K^+K^-K_S^0$  is given by the sum of the decay channels with that final state:

$$\begin{aligned} A(s_+, s_-) &= \sum_i a'_i F_i(s_+, s_-), \\ \bar{A}(s_-, s_+) &= \sum_i \bar{a}'_i \bar{F}_i(s_-, s_+), \end{aligned} \quad (2)$$

where  $a'_i \equiv a_i e^{ib_i}$  is a complex coefficient describing the relative magnitude and phase for the  $i$  th decay channel, including the weak phase dependence. The Dalitz-dependent amplitudes  $F_i(s_+, s_-)$  contain only strong dynamics and, thus,  $F_i(s_+, s_-) = \bar{F}_i(s_-, s_+)$ . The amplitudes of the contributions considered in the  $B^0 \rightarrow K^+K^-K_S^0$  decay are summarized in Table I. We use the same formalism as Refs. [13,14]. We utilize Flatté [15] and relativistic Breit-Wigner (RBW) [10] line shapes to describe the resonances.

In the Dalitz-dependent amplitudes  $A$  [Eq. (2)], we choose a convention in which the  $B^0\bar{B}^0$  mixing phase ( $q/p$ ) is absorbed into the  $\bar{B}^0$  decay amplitude  $\bar{a}'_i$ . These complex coefficients  $a'_i$  and  $\bar{a}'_i$  can be redefined as

$$a'_i \equiv a_i(1 + c_i)e^{i(b_i+d_i)}, \quad \bar{a}'_i \equiv a_i(1 - c_i)e^{i(b_i-d_i)}, \quad (3)$$

in which case a resonance  $i$  has a direct  $CP$  violating asymmetry given by

$$\mathcal{A}_{CP}(i) \equiv \frac{|\bar{a}'_i|^2 - |a'_i|^2}{|\bar{a}'_i|^2 + |a'_i|^2} = \frac{-2c_i}{1 + c_i^2}, \quad (4)$$

where the  $c_i$ 's are restricted by definition to lie between  $-1$  and  $1$ .

TABLE I. Summary of the contributions in the signal model. Here  $L$  is the orbital angular momentum. All fixed parameters are taken from Ref. [10] except for those of the  $f_0(980)$  [11] and  $f_X$  [12].

Resonances	Fixed parameters (MeV)	Resonance shape	$L$
$f_0(980)$	$M = 965 \pm 10$ $g_\pi = 165 \pm 18$ $g_K = (4.21 \pm 0.33)g_\pi$	Flatté	0
$\phi(1020)$	$M = 1019.455 \pm 0.020$ $\Gamma = 4.26 \pm 0.04$	RBW	1
$f_X$	$M = 1524 \pm 14$ $\Gamma = 136 \pm 23$	RBW	0
$\chi_{c0}$	$M = 3414.75 \pm 0.31$ $\Gamma = 10.4 \pm 0.7$	RBW	0
$(K^+K^-)_{NR}$	no fixed parameters	$e^{-\alpha s_0}$	
$(K^+K_S^0)_{NR}$	no fixed parameters	$e^{-\alpha s_+}$	
$(K^-K_S^0)_{NR}$	no fixed parameters	$e^{-\alpha s_-}$	

For cases where the contribution  $i$  is a  $CP$  eigenstate, the mixing-induced  $CP$  violating parameter  $\phi_1^{\text{eff}}(i)$  equals the fitted parameter  $d_i$ ,

$$\phi_1^{\text{eff}}(i) \equiv \frac{\arg(a'_i \bar{a}'_i^*)}{2} = d_i \quad (5)$$

and is related to the mixing-induced  $CP$  violating asymmetry as

$$-\eta_i \mathcal{S}(i) \equiv \frac{-2 \text{Im}(\bar{a}'_i a'_i^*)}{|a'_i|^2 + |\bar{a}'_i|^2} = \frac{1 - c_i^2}{1 + c_i^2} \sin 2\phi_1^{\text{eff}}(i), \quad (6)$$

where  $\eta_i$  is the  $CP$  eigenvalue of the final state. Note that  $\mathcal{A}_{CP}(i)$  and  $\mathcal{S}(i)$  are restricted by these definitions to lie in the physical region.

This time-dependent Dalitz measurement of  $CP$  violating parameters in  $B^0 \rightarrow K^+K^-K_S^0$  decays is based on a large data sample that contains  $657 \times 10^6$   $B\bar{B}$  pairs collected with the Belle detector at the KEKB asymmetric-energy  $e^+e^-$  (3.5 on 8 GeV) collider [16] operating at the  $Y(4S)$  resonance. The  $Y(4S)$  is produced with a Lorentz boost factor of  $\beta\gamma = 0.425$  along the  $z$  axis, which is antiparallel to the positron beam direction. Since the  $B\bar{B}$  pairs are produced nearly at rest in the  $Y(4S)$  center-of-mass system (cms),  $\Delta t$  is determined from  $\Delta z$ , the distance between the two  $B$  meson decay vertices along the  $z$  direction;  $\Delta t \cong \Delta z/c\beta\gamma$ , where  $c$  is the speed of light.

The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprised of CsI(Tl) crystals (ECL) located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside the coil is instrumented to detect  $K_L^0$  mesons and to identify muons (KLM). The detector is described in detail elsewhere [17]. Two inner detector configurations were used. A 2.0 cm radius beam pipe and a 3-layer silicon vertex detector were used for the first sample of  $152 \times 10^6$   $B\bar{B}$  pairs, while a 1.5 cm radius beam pipe, a 4-layer silicon detector, and a small-cell inner drift chamber were used to record the remaining  $505 \times 10^6$   $B\bar{B}$  pairs [18].

We reconstruct  $B$  candidates from an oppositely-charged kaon pair and a  $K_S^0$  candidate. The charged kaons are selected from the charged tracks having their impact parameters consistent with coming from the interaction point (IP). To suppress background from particle misidentification, charged tracks that are positively identified as pions, protons, or electrons are excluded. The particle species are identified by using particle information (PID) from the CDC, ACC, TOF, and ECL systems. We reconstruct  $K_S^0$  candidates from pairs of oppositely-charged tracks having invariant mass within 12 MeV/ $c^2$  of the  $K_S^0$  mass. The direction of the  $K_S^0$  momentum is required to be consistent

with the direction of vertex displacement with respect to IP [19].

We combine the  $K^+K^-$  pair and  $K_S^0$  to form a neutral  $B$  meson. Signal candidates are identified by two kinematic variables defined in the cms: the beam-energy constrained mass  $M_{bc} \equiv \sqrt{E_{\text{beam}}^2 - (\vec{p}_B)^2}$  and the energy difference  $\Delta E \equiv E_B - E_{\text{beam}}$ , where  $E_{\text{beam}} = \sqrt{s}/2$  is the cms beam energy, and  $\vec{p}_B$  and  $E_B$  are the cms three momentum and energy of the reconstructed  $B$  meson candidate, respectively. We use candidates in a signal region defined as a  $3\sigma$  ellipse around the  $M_{bc}$  and  $\Delta E$  mean values;  $\frac{(M_{bc} - M_{B^0})^2}{(8 \text{ MeV}/c^2)^2} + \frac{(\Delta E)^2}{(45 \text{ MeV})^2} < 1$ , where  $M_{B^0}$  is the nominal neutral  $B$  meson mass [10]. A larger region in  $M_{bc}$  and  $\Delta E$ ,  $5.20 \text{ GeV}/c^2 < M_{bc}$  and  $-0.30 \text{ GeV} < \Delta E < 0.50 \text{ GeV}$ , is used to determine the signal and background fractions. The sideband regions used for the continuum background study are defined as  $5.20 \text{ GeV}/c^2 < M_{bc} < 5.26 \text{ GeV}/c^2$  and  $-0.10 \text{ GeV} < \Delta E < 0.50 \text{ GeV}$  for the  $\Delta t$  distribution,  $5.24 \text{ GeV}/c^2 < M_{bc} < 5.30 \text{ GeV}/c^2$  and  $-0.10 \text{ GeV} < \Delta E < 0.10 \text{ GeV}$ , excluding the rectangular region of  $5.268 \text{ GeV}/c^2 < M_{bc} < 5.30 \text{ GeV}/c^2$  and  $-0.05 \text{ GeV} < \Delta E < 0.05 \text{ GeV}$ , for the Dalitz distribution.

The dominant source of background is continuum  $e^+e^- \rightarrow q\bar{q}$  ( $q = u, d, s$ , and  $c$ ) production. To reduce it, we require that  $|\cos\theta_{\text{th}}| < 0.8$ , where  $\theta_{\text{th}}$  is the angle between the thrust axis of the  $B$  candidate and that of the rest of the event. This requirement retains 83% of the signal while 79% of the continuum events are removed. The  $B\bar{B}$  background is found to be mostly originating from  $b \rightarrow c$   $B$  decays, which peaks in the signal region with an estimated yield of 60 events:  $B^0 \rightarrow D^-[K_S^0K^-]K^+$  [20] and  $B^0 \rightarrow J/\psi K_S^0$  decays. There are also potential backgrounds from  $B^0 \rightarrow D_s^-[K_S^0K^-]K^+$  and  $B^0 \rightarrow \bar{D}^0[K^+K^-]K_S^0$ . Backgrounds due to  $K$ - $\pi$  misidentification are also found. All these peaking background decays are suppressed to a negligible level by applying  $\sim 1.5\sigma$  ( $J/\psi$ ) and  $\sim 2.5\sigma$  (other modes) vetoes on the invariant masses; these vetoes are summarized in Table II. For backgrounds that arise from misidentified particles, the invariant masses

TABLE II. Summary of the charm vetoes applied to  $B^0 \rightarrow K^+K^-K_S^0$  candidates. The subscript in the vetoed region indicates that an alternate mass hypothesis has been applied to the kaon candidates used to calculate the invariant mass term.

Vetoed mode	Vetoed region
$B^0 \rightarrow D^-[K_S^0K^-]K^+$	$ M(K_S^0K^-) - M_{D^-}  < 15 \text{ MeV}/c^2$
$B^0 \rightarrow J/\psi[K^+K^-]K_S^0$	$ M(K^+K^-) - M_{J/\psi}  < 15 \text{ MeV}/c^2$
$B^0 \rightarrow D_s^-[K_S^0K^-]K^+$	$ M(K_S^0K^-) - M_{D_s^-}  < 15 \text{ MeV}/c^2$
$B^0 \rightarrow \bar{D}^0[K^+K^-]K_S^0$	$ M(K^+K^-) - M_{\bar{D}^0}  < 15 \text{ MeV}/c^2$
$B^0 \rightarrow D^-[K_S^0\pi^-]K^+$	$ M(K_S^0K^-)_\pi - M_{D^-}  < 15 \text{ MeV}/c^2$
$B^0 \rightarrow \bar{D}^0[K^+\pi^-]K_S^0$	$ M(K^+K^-)_\pi - M_{\bar{D}^0}  < 15 \text{ MeV}/c^2$

are recalculated by assuming an alternate mass hypothesis for the charged kaon. The remaining contribution is included in the nominal fit as the  $B\bar{B}$  background component. Yields for signal, continuum, and  $B\bar{B}$  backgrounds as well as probability density functions (PDFs) for those are described in more detail later.

We identify the flavor of the accompanying  $B$  meson from inclusive properties of particles that are not associated with the reconstructed  $B^0 \rightarrow K^+K^-K_S^0$  candidate. The algorithm for flavor tagging is described in detail elsewhere [21]. To represent the tagging information we use two parameters,  $q$  defined in Eq. (1) and  $r$ . The parameter  $r$  is an event-by-event Monte Carlo (MC) determined flavor-tagging quality factor that ranges from  $r = 0$  for no flavor discrimination, to  $r = 1$  for unambiguous flavor assignment. It is used only for sorting data into seven intervals. The wrong tag fractions for the seven  $r$  intervals  $w_l$  ( $l = 0, 6$ ) and the difference in  $w$  between  $B^0$  and  $\bar{B}^0$  decays  $\Delta w_l$  are determined from data [21]. The vertex position for the  $K^+K^-K_S^0$  decay is reconstructed using the charged kaon pair and the transverse components of IP. The vertex position of  $f_{\text{tag}}$  is obtained using tracks that are not assigned to the  $K^+K^-K_S^0$  candidate and IP.

We find that 1.5% of the selected events have more than one  $B^0 \rightarrow K^+K^-K_S^0$  candidate. In these events, we choose the  $B^0$  candidate that is formed from the most kaonlike charged kaon candidate and the  $K_S^0$  candidate closest to the nominal  $K_S^0$  mass.

After all the selections are applied, we obtain 98982 candidates in the  $M_{bc}$ - $\Delta E$  fit region, of which 2333 are in the signal region. We extract the signal yield using a three-dimensional extended unbinned maximum likelihood fit to the distributions of  $\Delta E$ ,  $M_{bc}$  and the flavor-tag quality ( $r$ ) interval  $l$  for the selected  $B^0 \rightarrow K^+K^-K_S^0$  events. For the PDF of the signal component, we use a sum of two Gaussians (a single Gaussian) for the  $\Delta E$  ( $M_{bc}$ ) shape. All parameters of the PDFs are free in the fit, except the ratio of the area of the broader Gaussian component to that of the core Gaussian and the width of the broader Gaussian in  $\Delta E$ . These additional parameters are fixed from the results of a fit to a  $B^0 \rightarrow D^-[K_S^0\pi^-]\pi^+$  data control sample. For the continuum background component, the  $\Delta E$  ( $M_{bc}$ ) shape is modeled by a first-order polynomial (an ARGUS [22]) function, with shape parameters floated in the fit. The  $B\bar{B}$  background component is parameterized by two-dimensional binned histograms from MC. In the fit, the total signal, continuum, and  $B\bar{B}$  background yields are also free parameters. The fit yields  $1176 \pm 51$  signal events in the signal region. The projections of the  $\Delta E$ ,  $M_{bc}$ , and  $l$  distributions for the candidate events are shown in Fig. 1. The average signal, continuum, and  $B\bar{B}$  fractions in the signal ellipse are calculated to be  $\sim 50\%$ ,  $\sim 49\%$ , and  $\sim 1\%$ , respectively. The event-by-event signal probabilities as a function of  $\Delta E$ ,  $M_{bc}$ , and  $l$  obtained with this fit are used in the unbinned maximum likelihood fit with a

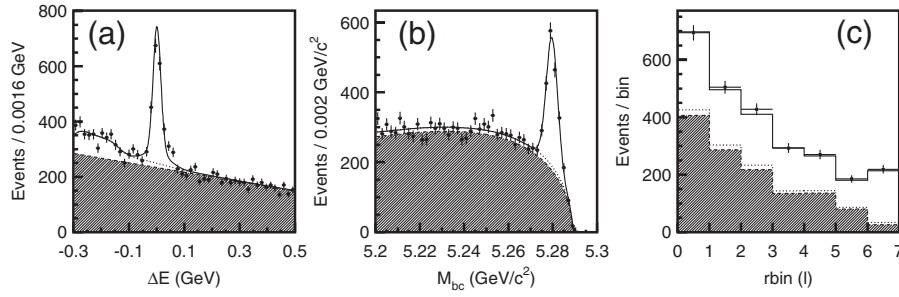


FIG. 1. Signal enhanced total projections of (a)  $\Delta E$  with  $5.272 \text{ GeV}/c^2 < M_{bc} < 5.288 \text{ GeV}/c^2$ , (b)  $M_{bc}$  with  $|\Delta E| < 0.045 \text{ GeV}$ , and (c)  $l$  in the  $(\Delta E, M_{bc})$  signal region for the  $B^0 \rightarrow K^+ K^- K_S^0$  candidate events. The solid curves show the fit projections, the hatched areas show the continuum background component, and the dotted curves show the total background contribution. The points with error bars are the data.

time-dependent Dalitz approach that is used to extract  $CP$  violation parameters.

In a Dalitz plot as a function of  $s_+$ ,  $s_-$ , signal and continuum events densely populate the kinematic boundaries with low  $s_0$ , which correspond to the  $\phi(1020)$  and  $f_0(980)$  resonances. Large variations in a small area of the Dalitz plane make it difficult to use histograms to describe the background. Therefore, we apply the transformation

$$ds_+ ds_- \rightarrow |\det J| dm' d\theta', \quad (7)$$

where  $J$  is the Jacobian of this transformation. The parameters  $m'$  and  $\theta'$  are given by the transformation

$$m' \equiv \frac{1}{\pi} \arccos\left(2 \frac{m_0 - m_0^{\min}}{m_0^{\max} - m_0^{\min}} - 1\right) \quad \text{and} \quad (8)$$

$$\theta' \equiv \frac{1}{\pi} \theta_0, \quad (9)$$

where  $m_0$  is  $K^+ K^-$  invariant mass,  $m_0^{\max}$  and  $m_0^{\min}$  are kinematic limits of  $m_0$ ,  $\theta_0$  is the helicity angle, defined as the angle between the  $K^-$  and the  $K_S^0$  in the  $K^+ K^-$  rest frame. With this transformation, the Dalitz plot turns into a ‘‘square Dalitz plot’’ with a smooth density variation.

Figure 2(a) and 2(b) shows the Dalitz distributions based on our signal model with the usual Dalitz parameterization  $s_+$  and  $s_-$ , the square Dalitz parameterization  $m'$  and  $\theta'$ , respectively. As can be seen, the highlighted region where most of the signal and background events are located is magnified in the square Dalitz parameterization. The square Dalitz plot is described in detail elsewhere [13,23].

The PDF expected for the signal distribution  $\mathcal{P}_{\text{sig}}$  is given by

$$\mathcal{P}_{\text{sig}}(m', \theta', \Delta t, q) = \epsilon(m', \theta') |A_{\text{sig}}(m', \theta', \Delta t, q)|^2 \otimes R_{\text{sig}}, \quad (10)$$

where

$$\begin{aligned} |A_{\text{sig}}(m', \theta', \Delta t, q)|^2 = & |\det J| \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} [(1 - q\Delta w_l) \\ & \times (|A|^2 + |\bar{A}|^2) - q(1 - 2w_l) \\ & \times (|A|^2 - |\bar{A}|^2) \cos(\Delta m_d \Delta t) \\ & + 2q(1 - 2w_l) \text{Im}(\bar{A}A^*) \\ & \times \sin(\Delta m_d \Delta t)], \end{aligned} \quad (11)$$

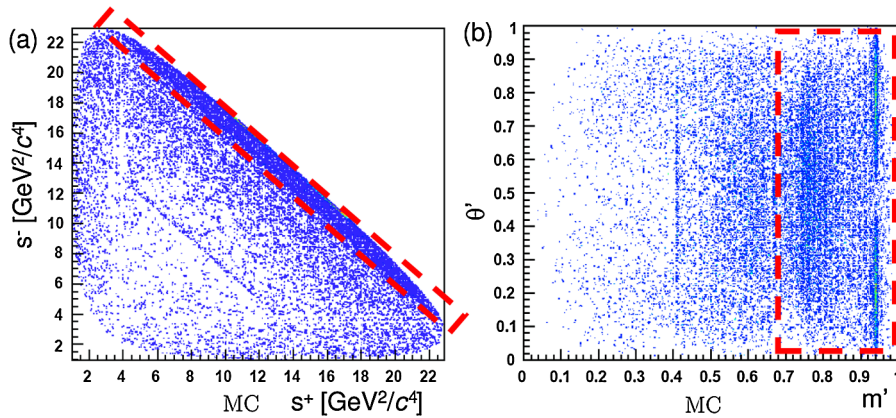


FIG. 2 (color online). The Dalitz distribution based on our signal model of GEANT-based signal MC [23] (a) with the normal Dalitz parameterization  $s_+$  and  $s_-$ , and (b) with the square Dalitz parameterization  $m'$  and  $\theta'$ . The dashed boxes indicate the regions where most of the signal components and the background are located.

which accounts for  $CP$  dilution from the incorrect flavor tagging. This function is convolved with the  $\Delta t$  resolution function  $R_{\text{sig}}$  [7]; the impact of detector resolution on the Dalitz plot is ignored because the intrinsic widths of the dominant resonances are larger than the mass resolution. We determine the variations of the signal detection efficiency across the Dalitz plane due to detector acceptance  $\epsilon(m', \theta')$  by using a large MC sample.

The PDF for continuum background is

$$\mathcal{P}_{q\bar{q}}(m', \theta', \Delta t, q) = \frac{1 + q\mathcal{A}_{q\bar{q}}(\theta')}{2} H_{q\bar{q}}(m', \theta') P_{q\bar{q}}(\Delta t), \quad (12)$$

where  $H_{q\bar{q}}$ ,  $\mathcal{A}_{q\bar{q}}$ , and  $P_{q\bar{q}}(\Delta t)$  are the Dalitz distribution PDF, the Dalitz-plot-dependent flavor asymmetry, and the  $\Delta t$  PDF, respectively. The function  $P_{q\bar{q}}$  is modeled as a sum of exponential and prompt components and is convolved with a double Gaussian that represents the resolution. All parameters of  $P_{q\bar{q}}$  are determined by a fit to the  $\Delta t$  distribution in the sideband region that is defined above. The Dalitz distribution PDF  $H_{q\bar{q}}$  is a two-dimensional binned histogram PDF. To determine the PDF, we use the sideband region around the signal region with a less restrictive requirement  $|\cos\theta_{\text{th}}| < 0.92$  to increase statistics. We have checked that the Dalitz distribution for this sideband region is similar to that for the signal region, using a MC sample. There is a flavor asymmetry  $\mathcal{A}_{q\bar{q}}$  due to the jetlike topology of continuum because a high momentum  $K^+(K^-)$  in  $f_{\text{rec}}$  is accompanied by a high momentum  $K^-(K^+)$  in  $f_{\text{tag}}$ ; to account for this, we extract the Dalitz plot asymmetry using almost the same region as the region used in  $H_{q\bar{q}}$  extraction; since we find no correlation between  $\theta'$  and  $M_{\text{bc}}$ , we enlarge the lower limit of the sideband region in this fit from 5.24 GeV/ $c^2$  to 5.2 GeV/ $c^2$  in  $M_{\text{bc}}$ .

Using high-statistics MC sample, we find no  $CP$  violating asymmetry in the background coming from charmless and charmed  $B$  decays. Therefore, the PDFs for  $B^+B^-$  and  $B^0\bar{B}^0$  backgrounds are given by

$$\mathcal{P}_{B^+B^-}(m', \theta', \Delta t) = H_{B^+B^-}(m', \theta') |A_{B^+B^-}(\Delta t)|^2 \otimes R_{B^+B^-}, \quad (13)$$

$$\mathcal{P}_{B^0\bar{B}^0}(m', \theta', \Delta t) = H_{B^0\bar{B}^0}(m', \theta') |A_{B^0\bar{B}^0}(\Delta t)|^2 \otimes R_{B^0\bar{B}^0}, \quad (14)$$

respectively. Dalitz distribution PDFs as well as  $H_{B^+B^-}$  ( $H_{B^0\bar{B}^0}$ ) are modeled with two-dimensional histograms from MC. The  $\Delta t$  PDF for both models  $|A_{B^+B^-}$  ( $|A_{B^0\bar{B}^0}|$ )<sup>2</sup> are described by exponential functions with effective lifetimes, while  $R_{B^+B^-}$  ( $R_{B^0\bar{B}^0}$ ) are the  $\Delta t$  resolution functions. The effective lifetimes are obtained from fits to the MC sample.

To account for a small fraction of events with large  $\Delta t$  values not yet described by either signal or background

PDFs, an outlier PDF is introduced  $\mathcal{P}_{ol} = H_{ol}G(\Delta t)$ , where  $G$  is a Gaussian and  $H_{ol}$  is the two-dimensional binned histogram PDF of the Dalitz plot of data itself.

For the  $j$ th event, the following likelihood function is evaluated:

$$P_j(m', \theta', \Delta t, q; \Delta E, M_{\text{bc}}, l) = (1 - f_{ol}) \left[ \sum_k f_k(\Delta E, M_{\text{bc}}, l) \mathcal{P}_k(m', \theta', \Delta t, q) \right] + f_{ol} \mathcal{P}_{ol}(m', \theta', \Delta t), \quad (15)$$

where  $k$  runs over a total of four components including signal and backgrounds. The probability of each component ( $f_j$ ) is calculated using the result of the  $\Delta E$ - $M_{\text{bc}}$ - $l$  fit on an event-by-event basis.

As there is only sensitivity to the relative amplitudes and phases between decay modes, we fix  $a_{(K^+K^-)_{\text{NR}}} = 60$  and  $b_{(K^+K^-)_{\text{NR}}} = 0^\circ$ . In addition,  $f_X$  and nonresonant contributions are combined and have a single common  $CP$  violating parameters. The combined component is referred to as ‘‘others’’ throughout this paper. The parameters  $d_{\chi_{c0}}$  and  $c_{\chi_{c0}}$  are fixed to the world average  $b \rightarrow c\bar{c}s$  values of  $21.5^\circ$  and  $0$ , respectively. We determine 19 parameters of the Dalitz plot and  $CP$  asymmetries by maximizing the likelihood function  $\mathcal{L} = \prod_j P_j$ , where the product is over all events.

We find four preferred solutions with consistent  $CP$  parameters but significantly different amplitudes for  $f_0(980)K_S^0$  and  $f_X K_S^0$ . The fitted results are summarized in Table III. These are obtained by performing a large number of fits with random input parameters. For each resonance  $i$  the relative fractions can be calculated as

$$f_i = \frac{(|a_i|^2 + |\bar{a}_i|^2) \int F_i(s_+, s_-) F_i^*(s_+, s_-) ds_+ ds_-}{\int (|A|^2 + |\bar{A}|^2) ds_+ ds_-}, \quad (16)$$

where the sum of fractions over all decay channels may not be 100% due to interference. Table IV summarizes the relative fractions for all solutions.

By translating the fit results using Eqs. (4) and (5), we determine the time-dependent  $CP$  violating parameters of  $B^0 \rightarrow f_0(980)K_S^0$  and  $B^0 \rightarrow \phi(1020)K_S^0$  decays and other  $B^0$  decays with the  $K^+K^-K_S^0$  final state. Table V summarizes the  $CP$  violating parameters for all solutions.

In Table IV,  $f_{\phi(1020)K_S^0}$  is similar for all four solutions but  $f_{f_0(980)K_S^0}$  and  $f_{f_X K_S^0}$  are significantly different. These four solutions are due to interference between the  $f_0(980)$  and nonresonant component and interference between the  $f_X$  and nonresonant component and are characterized by different relative fractions for  $f_0(980)$  and  $f_X$ . In order to distinguish these solutions with the current statistics, we use external information from  $B^0 \rightarrow \pi^+ \pi^- K_S^0$  and the property that  $f_0(980)$  decays to either  $\pi^+ \pi^-$  or  $K^+ K^-$ .

TABLE III. Time-dependent Dalitz plot fit results for the four solutions with statistical errors. The phases,  $b_i$  and  $d_i$ , and  $\alpha$  are given in degrees and  $\text{GeV}^{-2}c^4$ , respectively.

Parameter	Solution 1	Solution 2	Solution 3	Solution 4
$a_{f_0(980)}$	$29.3^{+2.6}_{-2.7}$	$53.0^{+7.3}_{-19.9}$	$31.8^{+3.0}_{-3.5}$	$64.1^{+7.0}_{-5.8}$
$a_{\phi(1020)}$	$0.53^{+0.07}_{-0.06}$	$0.67^{+0.10}_{-0.23}$	$0.56^{+0.06}_{-0.06}$	$0.71^{+0.13}_{-0.10}$
$a_{f_X}$	$5.2 \pm 0.8$	$7.0^{+1.2}_{-2.5}$	$15.6^{+1.5}_{-1.4}$	$23.9^{+3.9}_{-3.1}$
$a_{\chi_{c0}}$	$2.03^{+0.31}_{-0.28}$	$2.53^{+0.46}_{-0.89}$	$2.16^{+0.29}_{-0.28}$	$2.89^{+0.56}_{-0.45}$
$a(K_S^0 K^+)_{\text{NR}}$	$6.5^{+8.2}_{-6.7}$	$20.8^{+9.0}_{-7.6}$	$10.3^{+7.5}_{-6.2}$	$24.3^{+6.8}_{-6.2}$
$a(K_S^0 K^-)_{\text{NR}}$	$25.9^{+4.9}_{-4.3}$	$40.2^{+7.1}_{-6.4}$	$29.7^{+6.4}_{-5.4}$	$21.7^{+5.9}_{-6.0}$
$b_{f_0(980)}$	$-16.0^{+10.0}_{-13.2}$	$83.4^{+8.8}_{-7.3}$	$-1.8^{+10.0}_{-13.5}$	$109.3^{+10.5}_{-8.2}$
$b_{\phi(1020)}$	$-34.5^{+14.0}_{-14.5}$	$108.7^{+15.8}_{-15.5}$	$-7.0^{+14.1}_{-14.5}$	$149.2^{+17.5}_{-16.6}$
$b_{f_X}$	$-32.6^{+8.3}_{-8.9}$	$-106.0^{+12.9}_{-13.8}$	$92.1^{+8.7}_{-8.2}$	$35.7^{+6.9}_{-5.9}$
$b_{\chi_{c0}}$	$-28.0^{+22.5}_{-26.6}$	$-36.2^{+25.0}_{-28.3}$	$-35.7^{+25.3}_{-27.8}$	$45.96^{+22.4}_{-30.2}$
$b(K_S^0 K^+)_{\text{NR}}$	$126.5^{+39.7}_{-82.7}$	$113.6^{+13.6}_{-16.2}$	$113.5^{+19.0}_{-31.4}$	$98.8^{+23.0}_{-21.8}$
$b(K_S^0 K^-)_{\text{NR}}$	$-123.5^{+13.4}_{-11.4}$	$-143.5^{+8.3}_{-7.6}$	$-141.2^{+9.8}_{-8.8}$	$23.7^{+22.2}_{-24.6}$
$c_{f_0(980)}$	$0.16^{+0.16}_{-0.15}$	$0.10^{+0.08}_{-0.07}$	$-0.01 \pm 0.11$	$0.09 \pm 0.07$
$c_{\phi(1020)}$	$-0.02 \pm 0.10$	$-0.04 \pm 0.09$	$0.01 \pm 0.10$	$-0.10 \pm 0.09$
$c_{\text{others}}$	$0.07 \pm 0.06$	$0.03 \pm 0.08$	$0.01^{+0.04}_{-0.05}$	$-0.02^{+0.05}_{-0.06}$
$d_{f_0(980)}$	$31.3^{+9.0}_{-8.5}$	$26.1^{+7.0}_{-6.6}$	$25.6^{+7.6}_{-7.2}$	$26.3^{+5.7}_{-5.4}$
$d_{\phi(1020)}$	$32.2^{+9.0}_{-8.4}$	$26.2^{+8.8}_{-8.4}$	$27.3^{+8.6}_{-8.0}$	$24.3^{+8.0}_{-7.7}$
$d_{\text{others}}$	$24.9^{+6.4}_{-6.0}$	$29.8^{+6.6}_{-6.4}$	$26.2^{+5.9}_{-5.4}$	$23.8^{+5.5}_{-5.1}$
$\alpha$	$0.12^{+0.03}_{-0.04}$	$0.06 \pm 0.04$	$0.10 \pm 0.04$	$0.18 \pm 0.03$
$-2 \log \mathcal{L}$	10201.7	10198.6	10204.5	10208.9

TABLE IV. Summary of the relative fractions (%), the errors are statistical only.

Parameter	Solution 1	Solution 2	Solution 3	Solution 4
$f_{f_0(980)K_S^0}$	$26.0 \pm 7.4$	$54.0 \pm 9.6$	$26.4 \pm 7.8$	$68.1 \pm 12.3$
$f_{\phi(1020)K_S^0}$	$14.2 \pm 1.2$	$14.5 \pm 1.2$	$14.2 \pm 1.2$	$14.4 \pm 1.2$
$f_{f_X K_S^0}$	$5.10 \pm 1.39$	$5.89 \pm 1.86$	$39.6 \pm 2.6$	$59.0 \pm 3.0$
$f_{\chi_{c0} K_S^0}$	$3.73 \pm 0.74$	$3.71 \pm 0.73$	$3.68 \pm 0.73$	$4.15 \pm 0.79$
$f_{(K^+ K^-)_{\text{NR}} K_S^0}$	$138.4 \pm 44.8$	$175.0 \pm 52.6$	$157.4 \pm 29.5$	$48.1 \pm 11.7$
$f_{(K_S^0 K^+)_{\text{NR}} K^-}$	$1.65 \pm 4.17$	$21.0 \pm 17.3$	$4.63 \pm 6.76$	$7.87 \pm 4.78$
$f_{(K_S^0 K^-)_{\text{NR}} K^+}$	$26.0 \pm 12.9$	$78.0 \pm 36.2$	$38.6 \pm 18.1$	$6.27 \pm 3.81$
$F_{\text{tot}}$	$215.2 \pm 47.5$	$352.0 \pm 66.8$	$284.5 \pm 36.3$	$207.9 \pm 18.4$

We calculate the branching fraction  $\mathcal{B}(B^0 \rightarrow f_0(980) \times [\pi^+ \pi^-] K_S^0)$  based on the branching fraction of  $B^0 \rightarrow \pi^+ \pi^- K_S^0$  and the relative fraction of  $f_0(980) K_S^0$  in the  $B^0 \rightarrow \pi^+ \pi^- K_S^0$  decay [24]. Similarly, we can calculate the branching fraction  $\mathcal{B}(B^0 \rightarrow f_0(980) [K^+ K^-] K_S^0)$  from Table IV. The fraction  $f_{f_0(980) \rightarrow \pi\pi} =$

$\frac{\mathcal{B}(f_0(980) \rightarrow \pi\pi)}{\mathcal{B}(f_0(980) \rightarrow \pi\pi) + \mathcal{B}(f_0(980) \rightarrow KK)}$  is calculated to be  $0.47 \pm 0.10$  for Solution 1 and  $0.30 \pm 0.07$  for Solution 2. The value of  $f_{f_0(980) \rightarrow \pi\pi}$  is also determined by the BES Collaboration, which uses the same parameterization for the  $f_0(980)$  to be  $0.75 \pm 0.12$  [11]. Therefore, the solutions with a low  $f_0(980) K_S^0$  fraction (Solutions 1 and 3) are preferred. It is

TABLE V. Time-dependent  $CP$  violating parameters for the four solutions, where the first error is statistical, the second is systematic, and the third is the Dalitz plot model uncertainty.

	Solution 1	Solution 2	Solution 3	Solution 4
$\mathcal{A}_{CP}(f_0(980)K_S^0)$	$-0.30 \pm 0.29 \pm 0.11 \pm 0.09$	$-0.20 \pm 0.15 \pm 0.08 \pm 0.05$	$+0.02 \pm 0.21 \pm 0.09 \pm 0.09$	$-0.18 \pm 0.14 \pm 0.08 \pm 0.06$
$\phi_1^{\text{eff}}(f_0(980)K_S^0)$	$(31.3 \pm 9.0 \pm 3.4 \pm 4.0)^\circ$	$(26.1 \pm 7.0 \pm 2.4 \pm 2.5)^\circ$	$(25.6 \pm 7.6 \pm 2.9 \pm 0.8)^\circ$	$(26.3 \pm 5.7 \pm 2.4 \pm 5.8)^\circ$
$\mathcal{A}_{CP}(\phi(1020)K_S^0)$	$+0.04 \pm 0.20 \pm 0.10 \pm 0.02$	$+0.08 \pm 0.18 \pm 0.10 \pm 0.03$	$-0.01 \pm 0.20 \pm 0.11 \pm 0.02$	$+0.21 \pm 0.18 \pm 0.11 \pm 0.05$
$\phi_1^{\text{eff}}(\phi(1020)K_S^0)$	$(32.2 \pm 9.0 \pm 2.6 \pm 1.4)^\circ$	$(26.2 \pm 8.8 \pm 2.7 \pm 1.2)^\circ$	$(27.3 \pm 8.6 \pm 2.8 \pm 1.3)^\circ$	$(24.3 \pm 8.0 \pm 2.9 \pm 5.2)^\circ$
$\mathcal{A}_{CP}(\text{others})$	$-0.14 \pm 0.11 \pm 0.08 \pm 0.03$	$-0.06 \pm 0.15 \pm 0.08 \pm 0.04$	$-0.03 \pm 0.09 \pm 0.08 \pm 0.03$	$+0.04 \pm 0.11 \pm 0.08 \pm 0.02$
$\phi_1^{\text{eff}}(\text{others})$	$(24.9 \pm 6.4 \pm 2.1 \pm 2.5)^\circ$	$(29.8 \pm 6.6 \pm 2.1 \pm 1.1)^\circ$	$(26.2 \pm 5.9 \pm 2.3 \pm 1.5)^\circ$	$(23.8 \pm 5.5 \pm 1.9 \pm 6.4)^\circ$

TABLE VI. Comparison of external information with each of the four solutions.

	$f_{f_0(980)\rightarrow\pi\pi}$	$\frac{\mathcal{B}(f_0(1500)\rightarrow\pi\pi)}{\mathcal{B}(f_0(1500)\rightarrow KK)}$
Solution 1	$0.47 \pm 0.10$	$1.0 \pm 0.7$
Solution 2	$0.30 \pm 0.07$	$0.91 \pm 0.64$
Solution 3	$0.46 \pm 0.10$	$0.13 \pm 0.09$
Solution 4	$0.25 \pm 0.06$	$0.09 \pm 0.06$
External information	$0.75 \pm 0.12$ [11]	$4.1 \pm 2.5$ [10]

likely that the  $f_X$ , described in this analysis and the  $B^0 \rightarrow \pi^+ \pi^- K_S^0$  analysis [24], is the same state  $f_0(1500)$ . If this is the case, the ratio  $\frac{\mathcal{B}(f_0(1500)\rightarrow\pi\pi)}{\mathcal{B}(f_0(1500)\rightarrow KK)}$  is calculated to be  $1.0 \pm 0.7$  for Solution 1 and  $0.13 \pm 0.09$  for Solution 3. As the world average of this ratio is  $4.1 \pm 2.5$  [10], the solutions with a low  $f_X K_S^0$  fraction (Solutions 1 and 2) are preferred. Table VI summarizes these values for each solution. Altogether, we conclude that Solution 1 is preferred from all currently available external measurements. The mass projections onto (a)  $M(K_S^0 K^+)$ , (b)  $M(K_S^0 K^-)$ , and (c)  $M(K^+ K^-)$  distributions for Solution 1 are shown in Fig. 3, and (a)  $\Delta t$  distribution and (b) raw asymmetry in the

$\phi(1020)K_S^0$  region are shown in Fig. 4. The full correlation matrix is given in Tables VII and VIII. Likelihood scans of  $\phi_1^{\text{eff}}$  for all four solutions are obtained by fixing  $\phi_1^{\text{eff}}$  and redoing the fit. We also perform scans that include the systematic and model errors by convolving the likelihood with a Gaussian with the width set to the quadratic sum of the systematic and mode uncertainties. These are shown in Figs. 5 and 6.

The sources of systematic uncertainties and their contributions are summarized in Table IX. The systematic errors in the vertex reconstruction include uncertainties in the IP constraint, charged track selection based on track helix errors, vertex reconstruction quality,  $\Delta t$  requirement, tracking error corrections,  $\Delta z$  bias, and imperfect SVD alignment. The parameters for flavor tagging and resolution function, physics parameters, background  $\Delta t$  shape, and signal probability are varied by  $\pm 1\sigma$ . For each histogram, systematic errors are estimated using 100 sets of pseudoexperiments generated by statistically fluctuating samples to create the histogram. Samples of pseudoexperiments showed some fitting bias for  $CP$  parameters due to low statistics in each sample. We take this bias as a systematic uncertainty. The effect of misreconstruction is

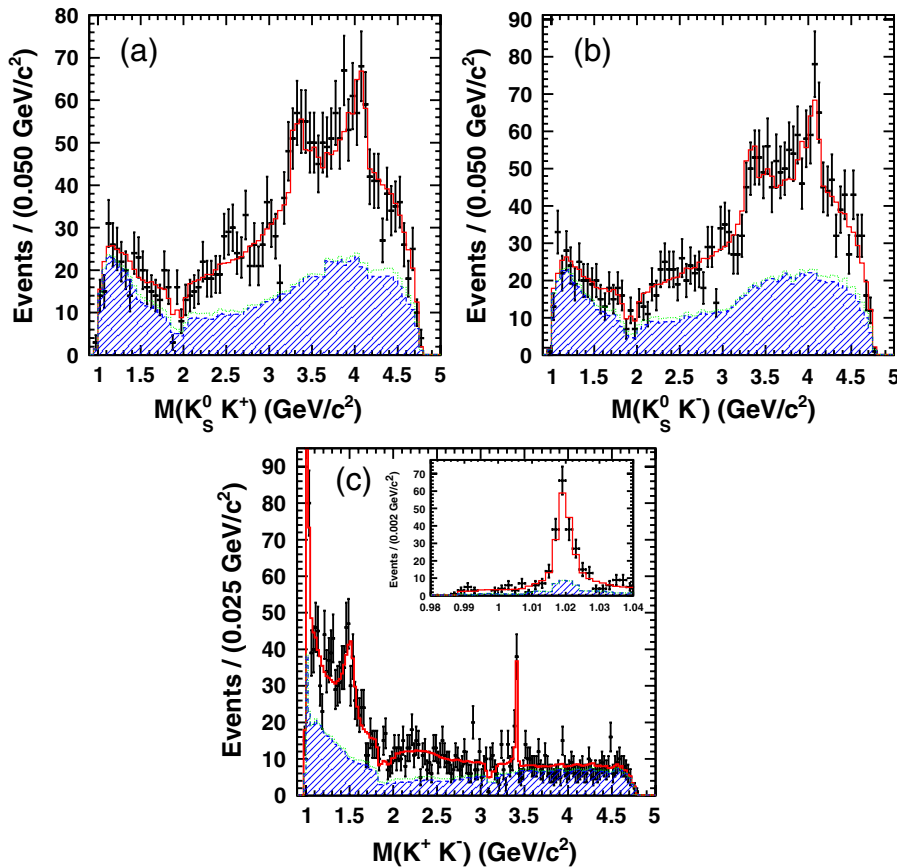


FIG. 3 (color online). The mass projections onto (a)  $M(K_S^0 K^+)$ , (b)  $M(K_S^0 K^-)$ , and (c)  $M(K^+ K^-)$  distributions (the inset shows the projection near the  $\phi(1020)$  resonance) for the  $B^0 \rightarrow K^+ K^- K_S^0$  candidate events in the signal region using Solution 1. In (a–c), the solid curves show the fit projections while the hatched areas and the dashed curves show the  $q\bar{q}$  and total background components, respectively. The points with error bars are the data.



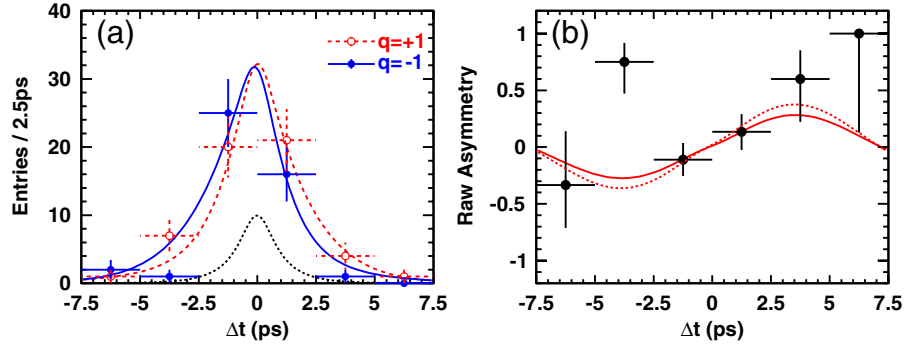


FIG. 4 (color online). (a)  $\Delta t$  distribution and (b) raw asymmetry for the  $B^0 \rightarrow K^+ K^- K_S^0$  candidates in the  $\phi(1020)K_S^0$  region  $|M_{K^+K^-} - M_{\phi(1020)}| < 0.01$  GeV/ $c^2$ , with good tags  $r > 0.5$ , using Solution 1. In (a), the solid and dashed curves show the fitted results with  $B^0$  and  $\bar{B}^0$  tags, respectively. The dotted curve shows the background component with  $B^0$  and  $\bar{B}^0$  tags. In (b), the solid curve shows the fit projection and the dashed curve shows the standard model expectation from the time-dependent  $CP$  asymmetry measurement in  $b \rightarrow c\bar{c}s$  decays.

TABLE VII. Statistical correlation matrix for Solution 1.

	$a_{f_0(980)}$	$a_{\phi(1020)K_S^0}$	$a_{f_X K_S^0}$	$a_{\chi_{c0} K_S^0}$	$a_{(K_S^0 K^+)_{NR}}$	$a_{(K_S^0 K^-)_{NR}}$	$b_{f_0(980)}$	$b_{\phi(1020)K_S^0}$	$b_{f_X K_S^0}$	$b_{\chi_{c0} K_S^0}$
$a_{f_0(980)}$	1.00	0.13	0.42	0.13	0.29	0.17	-0.04	0.22	-0.18	0.14
$a_{\phi(1020)K_S^0}$		1.00	0.57	0.71	0.54	0.71	-0.88	-0.35	-0.38	0.15
$a_{f_X K_S^0}$			1.00	0.45	0.50	0.51	-0.44	-0.09	-0.09	0.12
$a_{\chi_{c0} K_S^0}$				1.00	0.41	0.52	-0.71	-0.28	-0.31	0.37
$a_{(K_S^0 K^+)_{NR}}$					1.00	0.67	-0.51	-0.05	-0.12	0.17
$a_{(K_S^0 K^-)_{NR}}$						1.00	-0.73	-0.14	-0.29	-0.01
$b_{f_0(980)}$							1.00	0.38	0.40	-0.09
$b_{\phi(1020)K_S^0}$								1.00	0.17	-0.01
$b_{f_X K_S^0}$									1.00	-0.01
$b_{\chi_{c0} K_S^0}$										1.00

TABLE VIII. Statistical correlation matrix for Solution 1.

	$b_{(K_S^0 K^+)_{NR}}$	$b_{(K_S^0 K^-)_{NR}}$	$d_{f_0(980)K_S^0}$	$d_{\phi(1020)K_S^0}$	$d_{\text{others}}$	$c_{f_0(980)K_S^0}$	$c_{\phi(1020)K_S^0}$	$c_{\text{others}}$	$\alpha$
$a_{f_0(980)}$	0.02	-0.09	0.11	0.01	0.14	-0.20	-0.05	-0.01	-0.35
$a_{\phi(1020)K_S^0}$	0.15	-0.24	0.22	0.07	0.07	0.02	0.01	0.07	-0.72
$a_{f_X K_S^0}$	0.11	-0.28	0.15	0.08	0.06	0.01	0.01	0.05	-0.65
$a_{\chi_{c0} K_S^0}$	0.17	-0.12	0.19	0.06	0.08	0.00	0.02	0.05	-0.54
$a_{(K_S^0 K^+)_{NR}}$	0.26	-0.25	0.20	0.08	0.26	-0.14	-0.01	-0.05	-0.81
$a_{(K_S^0 K^-)_{NR}}$	-0.16	-0.50	0.22	0.07	0.19	-0.09	-0.01	-0.05	-0.77
$b_{f_0(980)}$	-0.21	0.29	-0.30	-0.11	-0.03	-0.09	-0.03	-0.16	0.72
$b_{\phi(1020)K_S^0}$	-0.20	-0.01	-0.00	-0.09	0.11	-0.20	-0.03	-0.09	0.15
$b_{f_X K_S^0}$	-0.09	0.01	-0.10	-0.04	-0.02	-0.00	-0.01	-0.03	0.21
$b_{\chi_{c0} K_S^0}$	0.00	0.18	0.11	0.05	0.17	-0.07	-0.01	0.01	-0.07
$b_{(K_S^0 K^+)_{NR}}$	1.00	0.03	0.01	-0.01	-0.05	0.02	0.02	0.05	-0.30
$b_{(K_S^0 K^-)_{NR}}$		1.00	-0.08	-0.02	-0.00	0.01	0.02	0.02	0.63
$d_{f_0(980)K_S^0}$			1.00	0.60	0.59	0.14	-0.02	0.47	-0.20
$d_{\phi(1020)K_S^0}$				1.00	0.35	0.45	0.08	0.37	-0.07
$d_{\text{others}}$					1.00	-0.39	-0.09	-0.13	-0.14
$c_{f_0(980)K_S^0}$						1.00	0.24	0.60	0.07
$c_{\phi(1020)K_S^0}$							1.00	0.14	0.00
$c_{\text{others}}$								1.00	0.00
$\alpha$									1.00

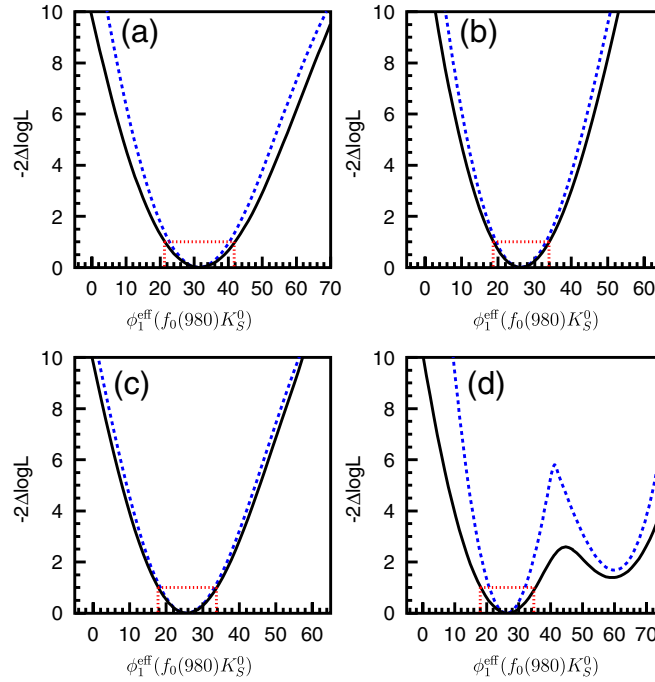


FIG. 5 (color online). Likelihood scans of  $\phi_1^{\text{eff}}(f_0(980)K_S^0)$  for (a) Solution 1, (b) Solution 2, (c) Solution 3, and (d) Solution 4. The solid (dashed) curve contains the total (statistical) error and the dotted box indicates the parameter range corresponding to  $\pm 1\sigma$ .

accounted for by comparing the fitted results of signal MC samples with and without misreconstructed events. The efficiency histogram also includes systematic uncertainties in the Dalitz-dependent correction factors for  $K_S^0$ , PID, and tracking efficiency. For tagside interference [25], pseudoexperiments are generated with and without tagside

interference, and the difference is taken as a systematic error. The fixed masses and widths of the resonance form factors in the signal model are varied by their errors; the changes in the results are taken into account in the systematic errors. To take into account a systematic uncertainty due to mass resolution in the  $\phi(1020)$  mass region, the

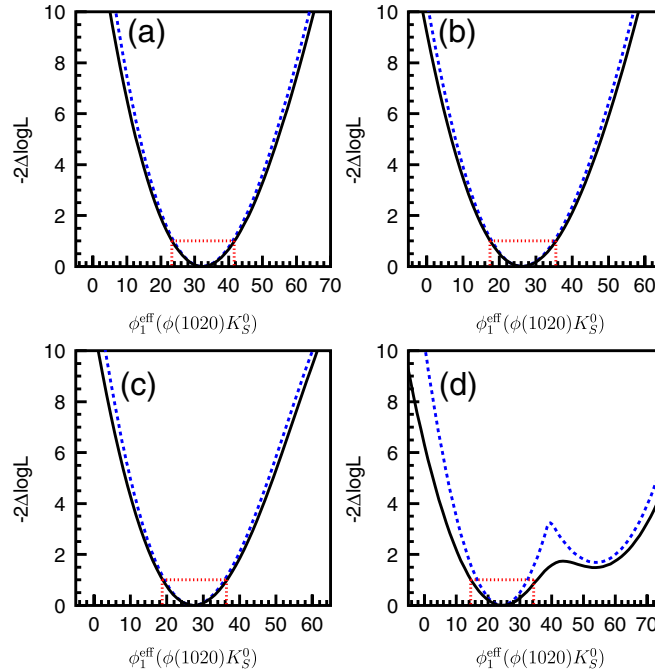


FIG. 6 (color online). Likelihood scans of  $\phi_1^{\text{eff}}(\phi(1020)K_S^0)$  for (a) Solution 1, (b) Solution 2, (c) Solution 3, and (d) Solution 4. The solid (dashed) curve contains the total (statistical) error and the dotted box indicates the parameter range corresponding to  $\pm 1\sigma$ .

TABLE IX. Summary of systematic uncertainties for Solution 1.

Category	$f_0(980)K_S^0$	$\phi(1020)K_S^0$ $\delta\phi_1^{\text{eff}} (^\circ)$	others	$f_0(980)K_S^0$	$\phi(1020)K_S^0$ $\delta\mathcal{A}_{CP}$	others
Vertex reconstruction	1.3	1.2	1.1	0.046	0.080	0.024
Wrong tag fraction	0.2	0.2	0.2	0.004	0.006	0.003
$\Delta t$ resolution function	1.9	1.9	1.5	0.018	0.011	0.010
Possible fit bias	2.2	0.9	0.4	0.067	0.008	0.026
Physics parameters	0.1	0.0	0.1	0.002	0.001	0.001
Background PDF	1.0	0.8	0.8	0.037	0.012	0.016
Signal fraction	0.2	0.4	0.3	0.013	0.006	0.004
Misreconstruction	0.1	0.0	0.0	0.000	0.000	0.001
Efficiency	0.2	0.2	0.1	0.011	0.004	0.005
Signal model	0.7	0.4	0.4	0.040	0.017	0.006
Tag-side interference	0.0	0.0	0.0	0.043	0.054	0.066
Total w/o Dalitz model	3.4	2.6	2.1	0.110	0.100	0.078
Dalitz model	4.0	1.4	2.5	0.089	0.019	0.032

width of the  $\phi(1020)$  is varied from  $4.26 \text{ MeV}/c^2$  to  $5.4 \text{ MeV}/c^2$ , and the difference in the fitted result is taken. The systematic uncertainty due to Blatt-Weisskopf barrier factors [26] in RBW is determined by taking the difference in the fitted results with and without these factors. The shape of the nonresonant component is empirically chosen, so different parameterizations are possible. This includes modeling the nonresonant part with the tail of a Breit-Wigner  $R_{\text{NR}}(s; \alpha) = i\alpha/(s + i\alpha)$  and a power law whose exponent is a fitted parameter  $R_{\text{NR}}(s; \alpha) = s^{-\alpha}$ . A possible variation in the model of the  $f_0(980)$  Flatté function is also considered using a different parameterization [27]. We also include a possible contribution from the spin 2  $f_2(1270)$  resonance in the signal model. The differences in the fit results from these alternate Dalitz plot parameterizations were summed in quadrature. The total systematic uncertainty is obtained by summing all of the above contributions in quadrature.

In summary, for the first time in Belle we perform a measurement of the  $CP$  violating asymmetries in  $B^0(\bar{B}^0) \rightarrow K^+K^-K_S^0$  decays with the time-dependent Dalitz approach. There are four solutions that describe the data well. These give similar values for the  $CP$  violating phase in the  $b \rightarrow s$  penguin mode  $B^0 \rightarrow \phi(1020)K_S^0$ :

$$\phi_1^{\text{eff}}(B^0 \rightarrow \phi(1020)K_S^0) = (32.2 \pm 9.0 \pm 2.6 \pm 1.4)^\circ;$$

$$\phi_1^{\text{eff}}(B^0 \rightarrow \phi(1020)K_S^0) = (26.2 \pm 8.8 \pm 2.7 \pm 1.2)^\circ;$$

$$\phi_1^{\text{eff}}(B^0 \rightarrow \phi(1020)K_S^0) = (27.3 \pm 8.6 \pm 2.8 \pm 1.3)^\circ;$$

and

$$\phi_1^{\text{eff}}(B^0 \rightarrow \phi(1020)K_S^0) = (24.3 \pm 8.0 \pm 2.9 \pm 5.2)^\circ.$$

These  $CP$  violating parameters are consistent at the current level of precision with the measurement of the  $CP$  violating phase in  $b \rightarrow c\bar{c}s$  processes such as  $B^0 \rightarrow J/\psi K^0$ , which is  $(22 \pm 1)^\circ$ . Previous measurements used  $K^+K^-$  selections around the  $\phi(1020)$  mass and in the higher

$K^+K^-$  mass region. Here we establish a superior analysis procedure for obtaining  $CP$  violating asymmetries without the uncertainty from interference among different resonant contributions, and therefore this represents an important step toward measurements with higher statistics such as in Super  $B$ -factory experiments.

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