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D2-brane in the Penrose limits of $AdS_4 \times CP^3$

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We consider a D2-brane in the pp-wave backgrounds obtained from $AdS_4 \times CP^3$ when electric and magnetic fields have been turned on. Upon fixing the light-cone gauge, light-cone Hamiltonian and Bogomolni-Prasad-Sommerfield configurations are obtained. In particular we study Bogomolni-Prasad-Sommerfield configurations with an electric dipole on the two-sphere giant and a giant graviton rotating in transverse directions. Moreover we show that the gauge field living on the D2-brane is replaced by a scalar field in the light-cone Hamiltonian. We also present a matrix model by regularizing (quantizing) 2-brane theory.

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I. INTRODUCTION

AdS_5/CFT_4 correspondence identifies $\mathcal{N} = 4$ SU(N) superconformal gauge theory to type IIB superstring theory on the maximally supersymmetric $AdS_5 \times S^5$ background (where "S" stands for sphere). This correspondence is a weak/strong coupling corresponding and this makes it a powerful tool to compute the strong coupling region of either theory using the weak coupling of the other. Albeit helpful, this property makes it difficult to test AdS/CFT duality explicitly since neither type IIB superstring on the $AdS_5 \times S^5$ background nor strong coupling gauge theory are well understood.

Another maximal supersymmetric solution of type IIb supergravity is the pp-wave and it can be obtained by taking the Penrose limit of $AdS_5 \times S^5$. The superstring theory on this background was explicitly solved [1]. Therefore in the pp-wave background we know the string spectrum and can check that whether the same spectrum exists on the gauge theory side. Then we first need to understand how this specific limit translates to the gauge theory side. It was argued that the Penrose limit corresponds to considering a certain section of operators, namely, Berenstein-Maldacena-Nastase (BMN) operators [2]. A study of AdS/CFT correspondence in this specific limit opens a new way to test this conjecture more precisely.

Another example of AdS/CFT duality is AdS_4/CFT_3. $\mathcal{N} = 8$ CFT_3 was an open question for years and it was finally written in [3], the so-called Bagger-Lambert-Gustavsson (BLG) theory which is a $\mathcal{N} = 8$ three dimensional superconformal Chern-Simon theory. AdS_4/CFT_3 tells us that this theory is a suitable candidate to describe multiple M2-branes. But after a while it was shown that the BLG theory describes two coincident M2-branes [4]. Based on the BLG model, Aharony-Bergman-Jafferis-Maldacena (ABJM) theory has been nominated to describe the low energy of multiple M2-branes and to be dual to M-theory on $AdS_4 \times S^7/Z_k$ [5]. The ABJM model is a $\mathcal{N} = 6$ three dimensional superconformal U(N) × U(N) Chern-Simon theory of level k and $-k$. The duality between the ABJM model and type IIA string theory on $AdS_4 \times CP^3$ has been found when $N^{1/5} \ll k \ll N$ [5].

The pp-wave background has been also studied in the AdS_4/CFT_3 context. A Penrose limit of $AdS_4 \times CP^3$ with zero spacelike isometry was obtained in [6] and string spectrum and BMN-like operators were obtained. Also, pp-wave metrics with one flat direction and two spacelike isometries were found in [7,8]. In this paper we consider a D2-brane in a general pp-wave background [7] and will then find light-cone (LC) Hamiltonian and Bogomolni-Prasad-Sommerfield (BPS) configurations with electric field. This paper is organized as follows. In the next section we will review pp-wave backgrounds and in Sec. III the LC Hamiltonian for a D2-brane in pp-wave backgrounds will be obtained by using a LC gauge. Then we replace the gauge field on the D2-brane by a scalar field and find a matrix model by applying a suitable prescription. In Sec. IV, BPS configurations are given. The last section is devoted to discussion.

II. PP-WAVE BACKGROUNDS

In this section we will review three pp-wave backgrounds which are coming from $AdS_4 \times CP^3$ by taking the Penrose limit. One of the differences between them is concerned with the number of spacelike isometries. A general form of these metrics has been written in [7] which leads to three pp-wave backgrounds by choosing appropriate parameters. It is important to notice that the only meaningful pp-wave backgrounds in the AdS/CFT context are those which are derived from the Penrose limit of $AdS_4 \times CP^3$. The general form of pp-wave geometry is given by

$$ds^2 = -4dx^+dx^- + \sum_{\hat{i}=1}^{4}(du_{\hat{i}}^2 - u_{\hat{i}}^2(dx^+)^2) + \sum_{a=1}^{2}\bigg[dx_a^2 + dy_a^2 + \Big(\xi_a^2 - \frac{1}{4}\Big)(x_a^2 + y_a^2)(dx^+)^2 + 2((\xi_a - 2\zeta_a)x_a dy_a - (\xi_a + 2\zeta_a)y_a dx_a)dx^+\bigg], \quad (1)$$

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and by the following parameters we have:

$$\text{no flat direction} \leftrightarrow \xi_a = \zeta_a = 0, \tag{2a}$$

$$\text{one flat direction} \leftrightarrow \xi_1 = \tfrac{1}{2}, \qquad \xi_2 = b + \tfrac{1}{2}, \qquad \zeta_1 = \tfrac{1}{4}, \qquad \zeta_2 = 0, \tag{2b}$$

$$\text{two flat directions} \leftrightarrow \xi_a = -\tfrac{1}{2}, \qquad \zeta_a = \tfrac{1}{4}, \tag{2c}$$

where b is an arbitrary parameter. In addition, in the $\mathrm{AdS}_4 \times CP^3$ background there are two- and four-form Ramond-Ramond fields which after taking Penrose limit become

$$C_{+ij} = -\frac{1}{g_s}\epsilon_{ijk}u_k, \qquad C_+ = -\frac{1}{g_s}u_4, \tag{3}$$

where $i, j = 1, 2, 3$, and g_s is IIA string coupling constant. It is important to notice that these pp-wave backgrounds are not a deformation of the type IIA pp-wave background coming from the reduction of the maximally supersymmetric 11 dimensional pp-wave background [9].

$\mathrm{AdS}_4 \times \mathrm{S}^7$ is a maximally supersymmetric background. After taking the Z_k orbifolding of S^7 and reducing the M-theory background $\mathrm{AdS}_4 \times \mathrm{S}^7/Z_k$ to type IIA string background $\mathrm{AdS}_4 \times CP^3$, 24 out of 32 killing spinors remain [10]. It was shown that the case (2a) also preserves 24 supercharges [11]. More supersymmetric pp-waves in M-theory and their dimensional reduction to D0-brane or pp-waves in type IIA and T-dualization to solutions in type IIB theory are studied in [12]. Moreover, in each case of the above pp-wave backgrounds coming from $\mathrm{AdS}_4 \times CP^3$, the minimum bosonic symmetry is a SO(3) rotation acting on u^i as well as the translation symmetry in x^+ and x^- directions.

III. LIGHT-CONE HAMILTONIAN

The low energy effective action for a D2-brane in the general form of a pp-wave background is

$$S = \int d\tau d^2\sigma\sqrt{-\det N} + \int C^{(3)} + \int C^{(1)} \wedge F = \int d\tau d^2\sigma \mathcal{L}, \tag{4}$$

where

$$g_{\hat\mu\hat\nu} = -4\partial_{\hat\mu}x^-\partial_{\hat\nu}x^+ + \left[\left(\xi_a^2 - \frac{1}{4}\right)(x_a^2 + y_a^2) - u_{\hat i}^2\right] \times \partial_{\hat\mu}x^+\partial_{\hat\nu}x^+ + \partial_{\hat\mu}x^I\partial_{\hat\nu}x^I + 2(\xi_a - 2\zeta_a) \times x_a\partial_{\hat\mu}y_a\partial_{\hat\nu}x^+ - 2(\xi_a + 2\zeta_a)y_a\partial_{\hat\mu}x_a\partial_{\hat\nu}x^+, \tag{5a}$$

$$F_{\hat\mu\hat\nu} = \partial_{\hat\mu}A_{\hat\nu} - \partial_{\hat\nu}A_{\hat\mu}, \tag{5b}$$

$$N_{\hat\mu\hat\nu} = g_{\hat\mu\hat\nu} + F_{\hat\mu\hat\nu}. \tag{5c}$$

$g_{\hat\mu\hat\nu}$ ($\hat\mu$ and $\hat\nu$ denote world-volume indices) is an induced metric on the brane and $x^I = (u^{\hat i}, x^a, y^a)$. $C^{(3)}$, and $C^{(1)}$ are introduced in (3) and $F_{\hat\mu\hat\nu}$ is the field strength of the U(1) gauge field living on the D2-brane.

In the LC gauge we fix a part of the area preserving diffeomorphism invariance which mixes world-volume time and spatial coordinates. In order to fix the LC gauge we separate the space and time indices on the brane world volume as $\sigma^{\hat\mu} = (\tau = \sigma^0, \sigma^r)$, $r = 1, 2$. The LC gauge is fixed by choosing

$$x^+ = \tau. \tag{6}$$

In order to ensure that the above condition is respected by dynamics we use the time-space mixing part of the area preserving diffeomorphism and set [13]

$$N^{0r} + N^{r0} \equiv G^{0r} = G_{0r} = (g - FgF)_{0r} = 0. \tag{7}$$

$G^{\hat\mu\hat\nu}$ is the symmetric part of $N^{\hat\mu\hat\nu}$ which has the interpretation of an open string metric [14] with $G_{\hat\mu\hat\nu}$ its inverse.

We noted that in the pp-wave background, x^+ and x^- are cyclic variables and their conjugate momenta are constants of motion. Then

$$p^+ = \frac{\partial\mathcal{L}}{\partial(\partial_\tau x^-)} = -\frac{2}{g_s}\sqrt{-\det N}N^{00}, \tag{8}$$

and the LC Hamiltonian is

$$\mathcal{H}_{\mathrm{LC}} = p^- = \frac{\partial\mathcal{L}}{\partial(\partial_\tau x^+)}. \tag{9}$$

From the above equation we have

$$\mathcal{H}_{\mathrm{LC}} = p^+\Big(\partial_\tau x^- - \frac{1}{2}\Big[\Big(\xi_a^2 - \frac{1}{4}\Big)(x_a^2 + y_a^2) - u_{\hat i}^2\Big] - \frac{1}{2}(\xi_a - 2\zeta_a)x_a\dot y_a + \frac{1}{2}(\xi_a + 2\zeta_a)y_a\dot x_a - \frac{1}{2p^+g_s}\epsilon^{ijk}u^i\{u^j, u^k\} - \frac{2Bu_4}{3p^+g_s}\Big), \tag{10}$$

where $B = F_{12}$ and $\{F, G\} = \epsilon^{rs}\partial_r F\partial_s G$. The last two terms of the above Hamiltonian are coming from the Chern-Simons term (last two terms) of the action (4).

Next we should eliminate $\partial_\tau x^-$. Using (5a), N_{00} is

$$N_{00} = -4\partial_\tau x^- + \Big(\xi_a^2 - \frac{1}{4}\Big)(x_a^2 + y_a^2) - (u^{\hat i})^2 + (\dot x^I)^2 + 2(\xi_a - 2\zeta_a)x_a\dot y_a - 2(\xi_a + 2\zeta_a)y_a\dot x_a. \tag{11}$$

Let us recall the definition of $\det N$ which is

$$\det N = \det(N_{rs})(N_{00} - N_{0r}N^{rs}N_{s0}), \tag{12}$$

where $N^{rp}N_{ps} = \delta^r_s$. It is important to note that $N^{00} \neq \frac{1}{N_{00}}$ because of off-diagonal electric-magnetic fields and hence

$$N^{00} = \frac{\det(N_{rs})}{\det N}. \tag{13}$$

The above two equations together with (8) lead to

$$N_{00} = -\left(\frac{2}{p^+ g_s}\right)^2 \det(N_{rs}) + N_{0r}N^{rs}N_{s0}. \tag{14}$$

By means of (14) the LC Hamiltonian (10) becomes

$$\begin{aligned}\mathcal{H}_{\rm LC} = p^+\bigg(\left(\frac{1}{p^+ g_s}\right)^2 \det N_{rs} - \frac{1}{4}N_{0r}N^{rs}N_{s0} + \frac{1}{4}(\dot{x}^I)^2 \\ - \frac{1}{4}\left[\left(\xi_a^2 - \frac{1}{4}\right)(x_a^2 + y_a^2) - (u^{\hat{i}})^2\right] \\ - \frac{1}{2p^+ g_s}\epsilon^{ijk}u^i\{u^j, u^k\} - \frac{2Bu_4}{3p^+ g_s}\bigg),\end{aligned} \tag{15}$$

where Chern-Simon terms have been added. In the case of the D2-brane the first term in the Hamiltonian is

$$\det N_{rs} = \det g_{rs} + \det F_{rs} = \tfrac{1}{2}\{x^I, x^J\}^2 + B^2. \tag{16}$$

The second term of (15) can be simplified by using the momentum conjugate to the gauge field which is

$$p_E^r = \frac{\partial \mathcal{L}}{\partial F_{0r}} = \frac{1}{2g_s}\sqrt{-\det N}N^{0r}, \tag{17}$$

and one can then show

$$\begin{aligned}-p^+N_{0r}N^{rs}N_{s0} = \frac{16}{p^+}p_E^r g_{rs}p_E^s = \frac{16}{p^+}p_E^r\partial_r X^I p_E^s \partial_s X^I \\ = \frac{(4p_E^I)^2}{p^+}.\end{aligned} \tag{18}$$

Putting all these together we find the LC Hamiltonian to be

$$\begin{aligned}\mathcal{H}_{\rm LC} = \frac{(2p_E^I)^2}{p^+} + \frac{(p^{\hat{i}})^2}{p^+} + \frac{p^+}{4}\left(\frac{2p_x^a}{p^+} - (\xi_a + 2\zeta_a)y_a\right)^2 \\ + \frac{p^+}{4}\left(\frac{2p_y^a}{p^+} + (\xi_a - 2\zeta_a)x_a\right)^2 + \frac{1}{2p^+ g_s^2}\{x^I, x^J\}^2 \\ + \frac{B^2}{p^+ g_s^2} - \frac{p^+}{4}\left[\left(\xi_a^2 - \frac{1}{4}\right)(x_a^2 + y_a^2) - (u^{\hat{i}})^2\right] \\ - \frac{1}{2g_s}\epsilon^{ijk}u^i\{u^j, u^k\} - \frac{2Bu_4}{3g_s},\end{aligned} \tag{19}$$

where the third term in (15) was replaced by the following conjugate momenta:

$$\begin{aligned}p^{\hat{i}} &= \frac{\partial \mathcal{L}}{\partial(\partial_\tau u^{\hat{i}})} = -\frac{p^+}{2}\dot{u}_{\hat{i}}, \\ p_x^a &= \frac{\partial \mathcal{L}}{\partial(\partial_\tau x^a)} = -\frac{p^+}{2}[\dot{x}_a - (\xi_a + 2\zeta_a)y_a], \\ p_y^a &= \frac{\partial \mathcal{L}}{\partial(\partial_\tau y^a)} = -\frac{p^+}{2}[\dot{y}_a + (\xi_a - 2\zeta_a)x_a].\end{aligned} \tag{20}$$

Matrix model

The BMN (Banks-Fischler-Shenker-Susskind) matrix model is an interesting candidate for the discrete light cone quantization (DLCQ) of M-theory in terms of D0-branes in maximally supersymetric 11 dimensional pp-wave background (flat space) [2,15]. The Hamiltonian of this model is obtained as a regularized version of the M2-brane LC hamiltonian in 11 dimensional pp-wave background [16,17]. Moreover another matrix model describing the DLCQ of type IIB string theory on the maximally supersymetric ten dimensional pp-wave background has been introduced in [18], namely, the tiny graviton matrix model. By regularizing the spherical D3-brane in the ten dimensional pp-wave background, the Hamiltonian of the tiny graviton matrix model is obtained. In the following, the gauge field on the D2-brane is replaced by a scalar field and, by using the logic of [16,17], a matrix model is introduced.

The gauge field living on a D2-brane has only one physical degree of freedom and it can be replaced by a scalar field in three dimensions. We are going to replace electric and magnetic fields in the LC Hamiltonian by derivative of scalar field. In the case of the D2-brane, it is easy to show that

$$\begin{aligned}\mathcal{L}_{\rm DBI} &= \sqrt{\det g\left(1 + \frac{1}{2}F_{\hat{\mu}\hat{\nu}}F^{\hat{\mu}\hat{\nu}}\right)} \\ &= -\frac{1}{2p}\det g + \frac{p}{2}\left(1 + \frac{1}{2}F_{\hat{\mu}\hat{\nu}}F^{\hat{\mu}\hat{\nu}}\right),\end{aligned} \tag{21}$$

where p is a Lagrangian multiplier (and DBI refers to Dirac-Born-Infeld). Let us define

$$F^{\hat{\mu}\hat{\nu}} = \beta\epsilon^{\hat{\mu}\hat{\nu}\hat{\alpha}}t_{\hat{\alpha}}, \tag{22}$$

where β and $t_{\hat{\alpha}}$ are arbitrary constant and vector, respectively. By using the equation of motion for the gauge field coming from (21) together with (22) we find

$$t_{\hat{\alpha}} = \partial_{\hat{\alpha}}\varphi, \qquad P_E^r = \beta\epsilon^{rs}\partial_s\varphi, \qquad B = \beta\dot{\varphi}. \tag{23}$$

Terms including electric and magnetic fields in the LC Hamiltonian are thus simplified as follows:

$$\begin{aligned}\frac{(2p_E^I)^2}{p^+} = \frac{1}{2p^+ g_s^2}\{x^I, \varphi\}^2, \\ \frac{B^2}{p^+ g_s^2} - \frac{2Bu_4}{3g_s} + \frac{p^+}{4}u_4^2 = p^+\left(\frac{p_\varphi}{p^+} - \frac{1}{3}u_4\right)^2 + \frac{5p^+}{36}u_4^2,\end{aligned} \tag{24}$$

where $\beta = \frac{1}{2\sqrt{2}g_s}$ and $p_\varphi = \frac{\dot{\varphi}}{\beta g_s}$. Since we are looking for a DLCQ description we need to compactify x^- on a circle of radius R_-

$$x^- \equiv x^- + 2\pi R_-. \tag{25}$$

This leads to the quantization of the LC momentum p^+

$$p^+ = \frac{J}{R_-}. \tag{26}$$

By following [17,18], we replace $x^{\hat{I}}$, $p^{\hat{I}}$ with $J \times J$ matrices, i.e.,

$$x^{\hat{I}} \leftrightarrow X^{\hat{I}}, \qquad p^{\hat{I}} \leftrightarrow JP^{\hat{I}}, \tag{27}$$

together with

$$p^+ \int d^2\sigma \leftrightarrow \frac{1}{R_-}\,\mathrm{Tr}, \qquad \{F, G\} \leftrightarrow J[F, G], \tag{28}$$

where $x^{\hat{I}} = (x^I, \varphi)$. Equation (19) then becomes

$$\begin{aligned} \mathrm{H} = R_-\mathrm{Tr}\bigg[&(P^i)^2 + \frac{1}{4}\bigg(2P_x^a - \frac{1}{R_-}(\xi_a + 2\zeta_a)Y_a\bigg)^2 \\ &+ \bigg(P_\varphi - \frac{1}{3R_-}U_4\bigg)^2 + \frac{1}{4}\bigg(2P_y^a + \frac{1}{R_-}(\xi_a - 2\zeta_a)X_a\bigg)^2 \\ &+ \frac{1}{2g_s^2}[X^{\hat{I}}, X^{\hat{J}}]^2 + \frac{5}{36R_-^2}U_4^2 - \frac{1}{4R_-^2}\bigg[\bigg(\xi_a^2 - \frac{1}{4}\bigg) \\ &\times (X_a^2 + Y_a^2) - (U^i)^2\bigg] - \frac{1}{2R_- g_s}\epsilon^{ijk}U^i[U^j, U^k]\bigg]. \end{aligned} \tag{29}$$

Inspired by [17,18], this matrix model describes the DLCQ of M-theory on the uplifted pp-wave backgrounds obtained from $AdS_4 \times CP^3$.

As mentioned earlier, the background pp-waves considered here (1) are not a deformation of the dimensionally reduced 11 dimensional maximally supersymmetric pp-wave. Hence, as can also be seen from (29), the matrix model is not a deformation of the BMN matrix model. Nonetheless, the zero energy vacuum configurations of (29) which are given through $X_a = Y_a = U_4 = 0$ and

$$H = \frac{1}{4R_-}\,\mathrm{Tr}\bigg[\bigg(U^i - \frac{R_-}{g_s}\epsilon^{ijk}[U^j, U^k]\bigg)^2\bigg] \tag{30}$$

are the same as those of BMN matrix model [2,16]. These vacuum configurations are of the form concentric fuzzy sphere giant graviton.

IV. BPS CONFIGURATION

In this section we study BPS configurations involving electromagnetic fields. The case of our interest is the static electromagnetic fields. Our solutions include giant graviton and deformed giant graviton. Moreover a giant graviton rotating in transverse directions will be found as a BPS state.

A. Giant-like solution

We start with the case where $u^i \neq 0$ while other fields are set to be zero. In this case the LC Hamiltonian is

$$\begin{aligned} \mathcal{H}_{\mathrm{LC}} &= \frac{p^+}{4}\bigg(u_i^2 + \frac{2}{(p^+ g_s)^2}\{u^i, u^j\}^2 - \frac{2}{p^+ g_s}\epsilon_{ijk}u^i\{u^j, u^k\}\bigg) \\ &= \frac{p^+}{4}\bigg(u^i - \frac{1}{p^+ g_s}\epsilon^{ijk}\{u^j, u^k\}\bigg)^2. \end{aligned} \tag{31}$$

We consider the following ansatz:

$$u^i = \frac{\alpha}{2}p^+ g_s J^i, \tag{32}$$

where α is a constant and J^i's satisfy

$$\{J^i, J^j\} = \epsilon^{ijk}J^k, \tag{33}$$

which specifies a two-sphere whose radius is one. By substituting (32) in the LC Hamiltonian we then have

$$\mathcal{H}_{\mathrm{LC}} = \tfrac{1}{16}(p^+)^3 g_s^2 \alpha^2 (1-\alpha)^2. \tag{34}$$

The usual BPS argument tells us that $\mathcal{H}_{\mathrm{LC}}$ is minimized when

$$\alpha = 0 \quad \text{or} \quad \alpha = 1. \tag{35}$$

The above solutions (35) are graviton ($\alpha = 0$) and giant graviton[1] ($\alpha = 1$) where their radii are zero and $\frac{1}{2}p^+ g_s$, respectively. These are $\frac{1}{2}$ BPS [12 out of 24 in the case (2a)] configurations whose LC energy is zero and preserve SO(3) symmetry.

One can turn on a constant magnetic field on the spherical D2-brane, i.e., $B = F_{12} =$ constant. This magnetic field does not change the spherical shape and the radius of the giant graviton but moves its center of mass from $u_4 = 0$ to $u_4 = \frac{3B}{p^+ g_s}$.

B. BIGGons solution

For the pure electric field, (19) simplifies to

$$\begin{aligned} \mathcal{H}_{\mathrm{LC}} &= \frac{4}{(p^+)^2}((P_E^i)^2 + (\tilde{u}^i)^2) \\ &= \frac{4}{(p^+)^2}((\tilde{u}^i \pm R^{ij}P_E^j)^2 \mp 2\tilde{u}^i R^{ij} P_E^j), \end{aligned} \tag{36}$$

where $\tilde{u}^i = \frac{(p^+)^{3/2}}{4}(u^i - \frac{1}{p^+ g_s}\epsilon^{ijk}\{u^j, u^k\})$ and R^{ij} is a SO(3) rotation. Hence, the BPS equation is

$$\tilde{u}^i = R^{ij}P_E^j. \tag{37}$$

This BPS equation was discussed in Sec. 3.1 of [13] for the case of a three sphere giant graviton where the electric field is turned on. There, the shape deformation induced by the electric field sourced by two equally and opposite point charges placed on the North and South Poles of the three spherical brane was obtained. The findings of [13] generalize BIons [20] to spherical D3-brane BIGGons.

[1]The giant graviton in the $AdS_4 \times CP^3$ background is discussed in [19].

Remarkably, (37) and its solutions are the same to those found in [13]. In other words we have found BIGGons solutions for spherical D2-branes. Physically this family of solutions describes open strings ending on a two-sphere giant graviton.

C. Rotating giant graviton solution

Another family of solutions that we consider are rotating giant gravitons. We turn on $u^i(\tau, \sigma^r)$, $x^a(\tau)$, and $y^a(\tau)$ fields and the LC Hamiltonian thus becomes

$$\begin{aligned}\mathcal{H}_{\rm LC} = \frac{p^+}{4}\Big(u^i - \frac{1}{p^+ g_s}\epsilon^{ijk}\{u^j, u^k\}\Big)^2 &+ \Big(p_x^a \pm \frac{1}{2}p^+\alpha_+ y_a\Big)^2 + \Big(p_y^a \mp \frac{1}{2}p^+\alpha_- x_a\Big)^2 \\ &+ (\mp\alpha_+ - \xi_a - 2\zeta_a)p_x^a y_a + (\xi_a - 2\zeta_a \pm \alpha_-)p_y^a x_a,\end{aligned} \tag{38}$$

where

$$\alpha_\pm^2 = (\xi_a \pm 2\zeta_a)^2 - \Big(\xi_a^2 - \frac{1}{4}\Big). \tag{39}$$

If the coefficients of the last two terms in (38) are equal they will show an angular momentum. Let us consider case (2a). In this case $\alpha_\pm = \frac{1}{2}$ and hence[2]

$$\begin{aligned}\mathcal{H}_{\rm LC} = \frac{p^+}{4}\Big(u^i - \frac{1}{p^+ g_s}\epsilon^{ijk}\{u^j, u^k\}\Big)^2 + \Big(p_x^a \pm \frac{1}{4}p^+ y_a\Big)^2 \\ + \Big(p_y^a \mp \frac{1}{4}p^+ x_a\Big)^2 \mp \frac{1}{2}(p_x^a y_a - p_y^a x_a).\end{aligned} \tag{40}$$

The BPS equations are given by

$$\begin{aligned}u^i = \frac{1}{p^+ g_s}\epsilon^{ijk}\{u^j, u^k\}, \qquad p_x^a = \pm\frac{1}{4}p^+ y_a, \\ p_y^a = \mp\frac{1}{4}p^+ x_a,\end{aligned} \tag{41}$$

and the LC Hamiltonian is

$$\mathcal{H}_{\rm LC} = \tfrac{1}{16}p^+(x_a^2 + y_a^2) = L_{x_a y_a}. \tag{42}$$

The above solution (41) describes a giant graviton rotating in the $x^a - y^a$ plane whose angular momentum is $\frac{1}{16}p^+(x_a^2 + y_a^2)$. This configuration is $\frac{1}{4}$ BPS and preserves $\mathrm{SO}(3)\times \mathrm{U}(1)\times \mathrm{U}(1)$. Obviously one can also consider a giant graviton rotating in the $x_1 - x_2$ or $y_1 - y_2$ plane.

[2] A similar solution exists for the case $\alpha_\pm = -\frac{1}{2}$.

V. CONCLUSION

There are three different pp-wave backgrounds coming from $\mathrm{AdS}_4 \times CP^3$ where they have a different number of spacelike isometry. We consider a D2-brane in these pp-wave backgrounds and the LC Hamiltonian of this system is found by applying LC gauge fixing. There is a contribution coming from the gauge field living on the D2-brane in the LC Hamiltonian considered as a electric and magnetic fields. We show that in three dimensions these fields are replaced by the derivative of a scalar field. Using the idea of a matrix model [16,17], we propose a matrix theory describing M-theory on the uplifted pp-wave backgrounds.

We then find BPS configurations. Half-BPS solutions are graviton and giant graviton with $SO(3)$ symmetry. For a pure electric field, we reproduce BIGGons configurations describing open strings ending on a giant graviton. These are $\frac{1}{4}$ BPS configurations.

A giant graviton rotating in transverse directions is another $\frac{1}{4}$ BPS configuration. Our solution has $\mathrm{SO}(3)\times \mathrm{U}(1)\times \mathrm{U}(1)$ symmetry and rotates in the $x^a - y^a$ plane. Rotation can be easily extended to other planes in transverse directions.

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