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$U_A(1)$ anomaly at high temperature: The scalar-pseudoscalar splitting in QCD

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We estimate the splitting between the spatial correlation lengths in the scalar and pseudoscalar channels in QCD at high temperature. The splitting is due to the contribution of the instanton/anti-instanton chains in the thermal ensemble, even though instanton contributions to thermodynamic quantities are suppressed. The splitting vanishes at asymptotically high temperatures as $\Delta M/M \propto (\Lambda_{\rm QCD}/T)^b$, where b is the beta function coefficient.

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There has been a lot of interest lately in the high temperature phase of quantum chromodynamics [1]. Although the main thrust of the recent activity, triggered by the Relativistic Heavy Ion Collider data, has been at temperatures close to the transition temperatures, there still are open questions about the high temperature deconfined phase of QCD. This regime may soon be experimentally accessible; the recently published Relativistic Heavy Ion Collider data [2] suggest a temperature in the range $300 \leq T \leq 600$ MeV, considerably above the expected transition temperature of $T_c \approx 170$ MeV.

At asymptotically high temperatures the basic QCD physics is perturbative; however some aspects of it cannot be understood without nonperturbative effects. The generation of magnetic mass is an example of such an effect [3]. Another, simpler, set of questions has to do with the contribution of instantons at high temperature [4]. Although the instanton contribution to thermodynamic quantities like pressure and energy is negligible, they must give the leading contribution to certain correlation functions. In particular, the question of whether and when the axial $U_A(1)$ symmetry of QCD is restored above the phase transition has been debated for some time. Clearly, since the anomaly is an operator equation, it remains true on the operator level also at nonzero temperature [5]. At low temperatures, the relation between the anomaly equation and π^0 decay has been elucidated in [6]. At very high temperatures, the effects of the anomaly are, however, generally believed to be very small because of the increasing irrelevance of instanton contributions at high temperature [4].

A convenient measure of the explicit breaking of the $U_A(1)$ symmetry in the theory with two massless flavors is the difference of the spatial correlation lengths in the scalar and pseudoscalar channels. This quantity has been calculated on the lattice and has been found to be small and probably nonvanishing [7–10], although the lattice studies are rather inconclusive because the signal is numerically delicate, especially in the presence of chiral zero modes. We are unaware of an analytic calculation of this splitting. The aim of this short paper is to provide such a calculation and to demonstrate that the instanton configurations contribute decisively to this quantity even though their contribution to pressure etc. may be negligible.

We stress that while the scalar-pseudoscalar mass splitting is well understood at low temperatures as being due to the spontaneous breakdown of chiral symmetry, this is clearly not the relevant mechanism in the high temperature regime where chiral symmetry is restored. In the chirally symmetric phase in the absence of the $U_A(1)$ anomaly, the scalar and pseudoscalar channels would be degenerate.

Our work is inspired by [11], where a very similar calculation was performed in the three-dimensional Georgi-Glashow model, which also exhibits a deconfinement phase transition [12]. Indeed there is a very close analogy between the symmetry structure of the 3D Georgi-Glashow model and QCD with massless quarks, as discussed in detail in [13].

We argue here that in QCD, chainlike configurations of instantons and anti-instantons are responsible for the scalar-pseudoscalar mass splitting, by studying their effect on correlators of fermion bilinears. In the following we consider only the high temperature regime with $T \gg \Lambda_{\rm QCD}$. The correlation lengths of interest in our calculation are of the order $1/T$. This regime is governed by the "standard" high temperature QCD, which does not require resummation of hard thermal loops, nor consideration of the extreme nonperturbative infrared physics associated with the scales of order of the inverse magnetic mass $1/\alpha_s T$. Thus even though there may be additional interesting nonperturbative effects associated with soft physics, we do not expect them to be relevant to the computation of the mass splittings discussed in this paper.

We are interested in calculating the equilibrium (equal time) correlation functions

$$\langle S(x)S(y)\rangle, \qquad \langle P(x)P(y)\rangle, \tag{1}$$

where

$$\begin{aligned} S(x) &= \bar\psi_i\psi_i = \bar\psi_i^L\psi_i^R + \bar\psi_i^R\psi_i^L, \\ P(x) &= \bar\psi_i i\gamma_5\psi_i = i(\bar\psi_i^L\psi_i^R - \bar\psi_i^R\psi_i^L). \end{aligned} \tag{2}$$

The flavor index i takes values 1, 2. The action of the axial $U_A(1)$ symmetry, as usual, is

$$\psi_i^L \to e^{i\alpha}\psi_i^L, \qquad \psi_i^R \to e^{-i\alpha}\psi_i^R. \tag{3}$$

If the axial anomaly is absent, the diagonal correlators must vanish in the axially symmetric thermal ensemble

$$\langle \bar{\psi}_i^L(x)\psi_i^R(x)\bar{\psi}_j^L(y)\psi_j^R(y)\rangle = 0 = \langle \bar{\psi}_i^R(x)\psi_i^L(x)\bar{\psi}_j^R(y)\psi_j^L(y)\rangle, \tag{4}$$

and therefore

$$\langle S(x)S(y)\rangle = \langle P(x)P(y)\rangle. \tag{5}$$

Although the axial symmetry is anomalous, the effects of the anomaly are not felt on the level of perturbation theory, but rather are "transmitted" to the QCD spectrum via the nonperturbative instanton and anti-instanton contributions [14–17]. At zero temperature instantons are finite action solutions of Euclidean equations of motion. In the regular gauge the vector potential of an instanton in the $SU(2)$ gauge theory is

$$A_\mu^a(x) = \frac{2\eta_{a\mu\nu}x_\nu}{x^2+\rho^2}, \tag{6}$$

where the totally antisymmetric 't Hooft symbol is given by $\eta_{a\mu\nu} = \epsilon_{a\mu\nu}$; $\mu, \nu = 1, 2, 3$; $\eta_{a\mu 4} = \delta_{a\mu}$. The instanton in the $SU(N)$ theory is essentially the same configuration embedded into the $SU(N)$ group. All color orientations of the instanton have to be summed over in the path integral.

Because of conformal symmetry of the classical QCD action, the instanton solution is characterized by a size parameter ρ. The action of an instanton is independent of ρ classically,

$$S_{\rm I} = \frac{8\pi^2}{g^2}. \tag{7}$$

The most important quantum correction to this result stems from the fact that the coupling constant runs with scale, and so semiclassically

$$S_{\rm I} = \frac{8\pi^2}{g^2(\rho)}. \tag{8}$$

The measure of integration over the instanton size is $dn \propto \frac{d\rho}{\rho^5}(\rho\Lambda)^b$, where $b = (11N_c - 2N_f)/3$ is the coefficient of the one loop β function. The measure is peaked towards large size instantons, which makes instanton calculations at zero temperature uncontrollable. Therefore, while at such low temperatures we understand qualitatively that the mass splitting between the scalars and pseudoscalars is due to the spontaneous breaking of chiral symmetry, it is difficult to make accurate first-principles computations of the effects of instantonlike fluctuations in the vacuum. Nevertheless, the semiphenomenological instanton liquid model [18,19] works quite well.

At finite temperature the situation is similar, as long as the temperature does not significantly exceed the nonperturbative QCD scale $\Lambda_{\rm QCD}$ [17]. However, at high temperature the instanton calculations are much more under control since the temperature provides an effective infrared cutoff on the instanton size [20]. In the imaginary time formalism at finite temperature, instantons are periodic solutions of the classical equations of motion with period $\beta = 1/T$ in the imaginary time direction. Instantons with core size $\rho \ll \beta$ look essentially the same as at zero temperature, but large instantons with $\rho \gg \beta$ look very different. The core of size β is accompanied by a dyonlike field in the spatial dimensions, and so at large distances a large size instanton has a magnetic field which looks like that of a monopole [4]. Most importantly, the nonzero temperature T provides an external scale, and the instanton effective action depends on the instanton size, due to quantum fluctuations. As a result, large instantons with sizes $\rho > \beta$ are exponentially suppressed. Since the contribution to the path integral due to very small instantons is suppressed by the zero temperature measure, this means that the main contribution to physical observables comes from instantons with size $\rho \sim \beta$.

An instanton at zero and finite temperatures is accompanied by fermionic zero modes. There is one left-handed zero mode for ψ_i of each flavor, which is also a right-handed zero mode for $\bar{\psi}_i$ of each flavor. Because of the presence of these zero modes every tunneling event associated with the instanton is accompanied by the change of the axial $U_A(1)$ charge in the vacuum, giving a concrete manifestation of the anomaly.

The presence of the zero modes also modifies significantly the interaction between the instantons and anti-instantons. In pure Yang-Mills theory, an instanton (I) and an anti-instanton (A) interact weakly with the interaction "potential," decreasing as a power of the distance. Put differently, a field configuration corresponding to an instanton/anti-instanton pair separated by a distance R (assuming the sizes of the instantons are much smaller than the separation) has the action

$$S_{\rm IA}(R) \sim \frac{16\pi^2}{g^2}\left[1 + c\frac{\rho_{\rm I}^2\rho_{\rm A}^2}{R^4}\right]. \tag{9}$$

The actual formula is somewhat more complicated since the instantons also possess a color orientation, and thus the interaction depends on the relative color orientation of the instanton and an anti-instanton. Here we consider the high temperature molecular phase where it is known that the favored orientation is such that I and A are parallel in color space [17,19].

In the presence of the fermion zero modes the I-A interaction becomes longer range. At zero temperature the wave function of the zero mode decreases as $1/R^3$ away from the center of the instanton, and as a result the I-A interaction is logarithmic. At high temperatures on the other hand, the fermionic zero mode wave function has a characteristic exponential decay in space with the "mass" equal to the lowest Matsubara frequency πT. As a result the effective I-A interaction becomes linear. For an I-A pair separated by a large distance R in the spatial direction, the interaction at high temperature due to $2N_f$ zero modes depends linearly on R:

$$S_{\rm IA}(R) \approx S_{\rm I} + S_{\rm A} + 2N_f \pi T R. \qquad (10)$$

At these high temperatures the effective "confining" linear interaction between instantons binds them into pairs of net zero topological charge. It is believed that the transition to the "molecular" phase coincides with the chiral symmetry restoration and the deconfinement phase transition and happens around $T_c \approx 170$ Mev in the theory with two massless flavors [17]. We stress that this interaction between instantons is not an interaction two body potential *per se*, and is solely due to the fermion zero modes, being a reflection of the axial anomaly. Thus we should not analyze the effect of many instantons and anti-instantons by a statistical sum as would be appropriate for pseudoparticles interacting via a two body interaction. Rather, as is well known, the appropriate computation is that of the overlap matrix elements of the fermion zero modes [19] (see also below).

A simpler model which exhibits a very similar behavior and instanton/anti-instanton binding above the deconfinement phase transition is the Georgi-Glashow model in $2 + 1$ dimensions [11–13]. Although in this model the instantons are bound in pairs at high temperature, they nevertheless have a direct effect on some observables. As was shown in [11], in the $2 + 1$-dimensional Georgi-Glashow model they give the main contribution to the mass (spatial correlation length) splitting between the scalar and the pseudoscalar channels. We expect similar effects in QCD. Instanton effects should be present in a theory with any number of flavors, but they are simplest to see in the two flavor case, so in the following we therefore concentrate on $N_f = 2$.

Consider the equal time correlation function

$$G_{LL}(x-y) = \langle \bar\psi_i^R(x)\psi_i^L(x)\bar\psi_j^R(y)\psi_j^L(y)\rangle. \qquad (11)$$

As mentioned above, in the absence of the $U_A(1)$ anomaly this correlation function vanishes, since it has a net axial $U_A(1)$ charge. Thus the perturbative calculation of this correlator at high temperature gives a vanishing result. It is, however, easy to see that this correlator does not vanish on a single-instanton configuration. The instanton has four zero modes—exactly the right number to be saturated by the fermionic operators in the observables. Thus, as long as the points x and y are very far away from each other, $|\vec x - \vec y| \gg 1/T$, an instanton centered at a point $\vec a$ gives a contribution

$$\langle \bar\psi_i^L(x)\psi_i^R(x)\bar\psi_j^L(y)\psi_j^R(y)\rangle_{\rm I} \approx aT^6 e^{-S_{\rm I}} e^{-2\pi T\{|\vec x-\vec a|+|\vec y-\vec a|\}}, \qquad (12)$$

where a is a constant of order one. We do not estimate numerical prefactors here, as we concentrate on the dominant parametric dependence on the temperature.

We argue below that the maximal contribution comes from the instantons and anti-instantons located along the straight line connecting points x and y. Any deviation from this chainlike configuration will involve additional exponential factors, making these contributions classically highly suppressed. The fluctuation effects away from the straight line are important in determining the subleading nonexponential prefactor, similarly to how the Lüscher term follows from fluctuations of a confining string. In this paper, however, we do not concern ourselves with these subleading corrections. Within this framework, it is straightforward to compute the effect of the instanton/anti-instanton chain, using standard arguments [21,22] familiar from computing energy splittings due to instantons. The instanton contribution does not depend on the position of the instanton along the line. Integrating over the position of the instanton with the weight Tdz, we obtain

$$\langle \bar\psi_i^R(x)\psi_i^L(x)\bar\psi_j^R(y)\psi_j^L(y)\rangle_{\rm I} \approx (aT^6)(cT)|\vec x - \vec y| e^{-S_{\rm I}(\beta)} e^{-2\pi T|\vec x-\vec y|}, \qquad (13)$$

where we have set the instanton size to be equal to the inverse temperature and have allowed for a dimensionless constant c, of order one, which must appear due to the integration over the instanton sizes. It should be possible to estimate c using the dependence of the instanton action on ρ [4].

The next contribution comes from a configuration of two instantons and one anti-instanton, alternating along the straight line connecting $\vec x$ and $\vec y$. The zero modes are again completely saturated and so this contribution does not vanish. The main contribution comes from the configurations where the two instantons are located closer to the endpoints x and y, with the anti-instanton between them. Two of the zero modes of each instanton are then saturated by the zero modes of the anti-instanton. The typical distances between the instantons clearly is greater than their size, and thus we can neglect all interactions between them except for that induced by the fermionic zero modes. The contribution of this configuration is therefore

$$\langle \bar\psi_i^R(x)\psi_i^L(x)\bar\psi_j^R(y)\psi_j^L(y)\rangle_{\rm IAI} \approx (aT^6)\tfrac{1}{2}c^3T^3|\vec x - \vec y|^3 e^{-3S_{\rm I}(\beta)} e^{-2\pi T|\vec x-\vec y|}. \qquad (14)$$

If the instantons and anti-instantons are not aligned along the straight line, the exponential factor instead is $e^{-2\pi T\Sigma_{i=1}^4 L_i}$, where L_i are the length of the straight segment of the line connecting the points x, y and the positions of the instantons and anti-instantons. For any arrangement of instantons different from the straight line between x and y, this introduces a large suppression factor. Thus the dominant contribution comes from the straight line arrangement with alternating instantons and anti-instantons.

The leading contribution from a configuration with an alternating chain of n instantons and $n-1$ anti-instantons is

$$\langle \bar{\psi}_i^R(x)\psi_i^L(x)\bar{\psi}_j^R(y)\psi_j^L(y)\rangle_{\mathrm{I}^n\mathrm{A}^{n-1}} \approx (aT^6)(cTe^{-S_1(\beta)})^{2n-1}e^{-2\pi T|\vec{x}-\vec{y}|} \times \int_x^y dy_n \int_x^{y_n} dx_{n-1} \int_x^{x_{n-1}} dy_{n-1} \ldots \int_x^{x_1} dy_1. \quad (15)$$

Summing all the contributions we obtain

$$\langle \bar{\psi}_i^R(x)\psi_i^L(x)\bar{\psi}_j^R(y)\psi_j^L(y)\rangle \approx (aT^6)e^{-2\pi T|\vec{x}-\vec{y}|}\sinh(cTe^{-S_1(\beta)}|\vec{x}-\vec{y}|). \quad (16)$$

In this calculation, we have assumed that all the instantons and anti-instantons have the same orientation in color space. At high T the integration over the orientations is indeed dominated by the configuration where instantons and anti-instantons are parallel [17]. One can, however, integrate over the color orientations independently. The integration over orientations results in the same functional form but with a different value for the preexponential factor c (see discussion below).

Turning our attention to the axially symmetric correlator, we note that perturbatively

$$\langle \bar{\psi}_i^L(x)\psi_i^R(x)\bar{\psi}_j^R(y)\psi_j^L(y)\rangle_{\mathrm{perturbative}} \approx (aT^6)e^{-2\pi T|\vec{x}-\vec{y}|}. \quad (17)$$

The origin of the exponential factor is now not the zero modes, but rather just the contribution of the free lowest Matsubara frequency contribution. The constant factor aT^6 is the same as in Eqs. (12)–(17), since it originates from the disconnected piece associated with points x and y separately. Clearly, instanton/anti-instanton chains give a contribution to this correlator in exactly the same way as before. The only difference is that now the number of instantons and anti-instantons in the chain must be equal in order to saturate all the fermionic zero modes. The result is then

$$\langle \bar{\psi}_i^L(x)\psi_i^R(x)\bar{\psi}_j^R(y)\psi_j^L(y)\rangle \approx (aT^6)e^{-2\pi T|\vec{x}-\vec{y}|}\cosh(cTe^{-S_1}|\vec{x}-\vec{y}|). \quad (18)$$

We now present an alternative, somewhat more rigorous derivation of the results (16) and (18), using a path integral calculation of the correlators. Consider the calculation of the correlation function G_{LL} on a configuration of widely separated n instantons and $n-1$ anti-instantons. As we are interested in long-distance behavior, we can neglect the contribution of nonzero modes and restrict ourselves to the subspace of the fermionic Hilbert space spanned by the zero modes of the instantons and the anti-instantons [19]. We expand the quark fields as

$$\psi_i^L(x) = \Sigma_{a=1}^n \phi_a(x)\psi_i^{La}, \qquad \psi_i^R(x) = \Sigma_{b=1}^{n-1}\phi_b(x)\psi_i^{Rb}, \quad (19)$$

and similarly for $\bar{\psi}$. Here $\phi_a(x)$ is the wave function of the zero mode of the ath instanton, and ψ_i^{La} is the Fermi operator associated with this zero mode, and similarly for the anti-instanton modes $\phi_b(x)$, ψ_i^{Ra}. Since the zero modes are not exactly orthogonal to each other, the operators ψ_i^{Ra} and ψ_i^{La} are not strictly independent fermionic operators. However, the overlap between the zero modes is exponentially small, since we are considering far separated instantons. With good accuracy we can then consider ψ_i^{Ra} and ψ_i^{La} to have standard anticommutation relations and, in the path integral formalism, to be independent Grassman fields. The calculation of the correlator on this particular instanton-anti-instanton configuration then amounts to calculating the path integral

$$G_{LL}(x,y) = \int [\Pi_{iab} d\psi_i^{La} d\psi_i^{\dagger Ra} d\psi_i^{Rb} d\psi_i^{\dagger Lb}] \times (\Sigma_{b=1}^n \phi_b^*(x)\psi_i^{\dagger Rb})(\Sigma_{a=1}^n \phi_a(x)\psi_i^{La}) \times (\Sigma_{b=1}^n \phi_b^*(y)\psi_j^{\dagger Rb})(\Sigma_{a=1}^n \phi_a(y)\psi_j^{La})e^{-S[\psi]}. \quad (20)$$

The fermionic action is given by the original action reduced to the zero mode subspace

$$S[\psi] = (2n-1)S_{\mathrm{I}} + \Sigma_{ab}[\bar{\psi}_i^{Ra}T_{ab}\psi_i^{Rb} + \bar{\psi}_i^{Lb}T_{ba}^*\psi_i^{La}], \quad (21)$$

with the overlaps

$$T_{ab} = \int dx \phi_a^*(x) \not{D} \phi_b(x). \quad (22)$$

Note that the zero mode wave functions and the overlap matrix elements are both exponential,

$$\phi_{a(b)}(x) \sim e^{-\pi T|x-x_{a(b)}|}, \qquad T_{ab} \sim e^{-\pi T|x_a-x_b|}, \quad (23)$$

where x_a (x_b) is the location of the instanton (anti-instanton).

The calculation of the path integral in Eq. (20) is now straightforward. Expanding the exponential of the action and integrating over the Grassman zero mode fields, we generate a sum of terms where the two zero modes at point x are contracted into two (possibly different) instantons, likewise the two modes at the point y, while the other zero modes are all contracted between instantons and anti-instantons in all possible ways. Each such term is proportional to the product of appropriate exponential factors. Clearly, the largest such contribution is when the zero modes at x are both contracted into the same, closest instanton, as are the modes at y, while the rest of the instantons and anti-instantons alternate in a chainlike configuration. Such a configuration is proportional to $e^{-2\pi T|x-y|}$, while any other arrangement of contractions between the zero modes is suppressed by a larger exponential factor. This leading contribution is precisely the one given in Eq. (15). The overall proportionality coefficient depends on the color orientation of the instantons. When all (anti-)instantons have the same orientation, it is simply a power of some constant factor as in Eq. (15). In fact, one should integrate over the color orientations of the (anti-) instantons independently. Since the integration over the orientations and over the coordinates in the calculation of the overlaps T_{ab} commute, this leads to the result of the same form with a slightly different value of the constant c. In fact, the calculation then is completely equivalent to forgetting about explicit instantons, and instead including the effective 't Hooft vertex in the fermion part of the action [19]. Thus the value of the coefficient c is calculable, although it is not of interest to us in the present work. The integration over the instanton coordinates and summation over their number reproduces Eqs. (16) and (18).

We note that in addition to the instantons and anti-instantons which are members of the chain, there are also vacuum fluctuations, which in the high temperature phase are linearly bound instanton/anti-instanton pairs. Those fluctuations in the leading approximation are disconnected and therefore do not affect the correlation function. A more careful account of these extra pairs would be necessary to calculate the preexponential factor, as they presumably have a similar effect to that of instanton chains of non-minimal length.

From Eqs. (16) and (18) we find

$$\langle S(x)S(y)\rangle \propto e^{-M_S|x-y|}, \qquad \langle P(x)P(y)\rangle \propto e^{-M_P|x-y|},$$
$$\langle S(x)P(y)\rangle = 0, \tag{24}$$

with

$$M_S = 2\pi T - cTe^{-S_1(\beta)}, \quad M_P = 2\pi T + cTe^{-S_1(\beta)}. \tag{25}$$

Thus we find that the splitting between the scalar and pseudoscalar correlation lengths is given by

$$\frac{\Delta M}{M} \propto e^{-S_1(\beta)} \propto \left(\frac{\Lambda_{\rm QCD}}{T}\right)^b, \tag{26}$$

which is the advertised result. In the end, this is essentially the expected semiclassical quantum mechanical splitting determined by exponentiation of the single-instanton action [21,22]. The key observation is that at high T the problem reduces to one of a *linear* chain of alternating instantons and anti-instantons, with the total number being either different by 1 (antisymmetric correlator) or equal (symmetric correlator). This is very similar to the behavior found in the Georgi-Glashow model [11], as we expected.

We note that although we have discussed isosinglet correlators, the results apply also to isovectors since the chiral $SU(2)\otimes SU(2)$ symmetry is restored in the high temperature phase. Thus, for example, we also have

$$\frac{M_{a_0} - M_\pi}{M_\pi} \propto \left(\frac{\Lambda_{\rm QCD}}{T}\right)^b. \tag{27}$$

Although this ratio is not parametrically suppressed at $T \sim 2T_c$, the large power $b \sim 10$ may explain the fact that at these temperatures the difference in the correlation lengths is difficult to detect in lattice calculations [10]. It would be very interesting if the high T regime could be probed further with high statistics lattice computations.

A good consistency check on our calculation is to add a θ term to the QCD Lagrangian. In the chiral limit the spectrum of masses and correlation lengths should not depend on the value of θ, as it can be eliminated by the anomalous $U(1)$ rotation. This means that with $\theta \neq 0$ we should recover the same correlation lengths, but in the correlators of the axially rotated operators. We can include the effect of the QCD θ angle in our calculation by noting that the correlators with one extra instanton or anti-instanton in the chain will acquire a phase, while the correlators with equal numbers of instantons and anti-instantons will be unchanged. Thus, G_{LL} acquires an extra phase $e^{i\theta}$, G_{RR} acquires an extra phase $e^{-i\theta}$, while G_{LR} and G_{RL} are unchanged. Then we find

$$\begin{aligned}\langle S(x)S(y)\rangle &\propto 2e^{-M|x-y|}[\cosh(\Delta M|x-y|) \\ &\quad - \cos\theta \sinh(\Delta M|x-y|)], \\ \langle P(x)P(y)\rangle &\propto 2e^{-M|x-y|}[\cosh(\Delta M|x-y|) \\ &\quad + \cos\theta \sinh(\Delta M|x-y|)], \\ \langle P(x)S(y)\rangle &= \langle S(x)P(y)\rangle \\ &\propto 2e^{-M|x-y|}\sin\theta \sinh(\Delta M|x-y|),\end{aligned} \tag{28}$$

with $M = \frac{1}{2}(M_S + M_P)$, $\Delta M = \frac{1}{2}(M_P - M_S)$. The eigenvalues of the correlator matrix are the same as before, while the eigenvectors are rotated precisely by the axial rotation, as expected.

We conclude with two brief comments. First, for a theory with more than two massless flavors, the I-A chains do not contribute to the correlation function of two fermionic bilinears, since the number of zero modes on an instanton does not match the number of fermionic operators. Instead they contribute to the Green's functions with N_f bilinears. In this case we expect that the main contribution will not come from linear chains but from a more complicated geometrical arrangement of instantons. Second, at high temperature in QCD the instantons can be thought of as Skyrmions of the field A_0^a in the dimensionally reduced theory [23]. This is very similar to the situation in the 3D Georgi-Glashow model, where instanton-monopoles become vortices of the Abelian part of A_0. The interesting difference is that it is easy to understand the linear interaction between vortices [11,24] as a direct result of the nonexistence of a continuous $U(1)$ symmetry. On the other hand, it is more difficult to understand a linear potential between Skyrmions, since the Skyrmion field decays fast at infinity. Since this linear interaction is the consequence of the fermionic zero modes, it would be interesting to understand how this interaction arises in the effective Lagrangian of the dimensionally reduced theory due to the integration of the quark fields.

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