Chern-Simons gravity with (curvature)^2 and (torsion)^2 terms and a basis of degree-of-freedom projection operators

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(Received 3 June 2010; published 8 September 2010)

The effects of (curvature)^2- and (torsion)^2-terms in the Einstein-Hilbert-Chern-Simons Lagrangian are investigated. The purposes are two-fold: (i) to show the efficacy of an orthogonal basis of degree-of-freedom projection operators recently proposed and to ascertain its adequacy for obtaining propagators of general parity-breaking gravity models in three dimensions; (ii) to analyze the role of the topological Chern-Simons term for the unitarity and the particle spectrum of the model squared-curvature terms in connection with dynamical torsion. Our conclusion is that the Chern-Simons term does not influence the unitarity conditions imposed on the parameters of the Lagrangian but significantly modifies the particle spectrum.

DOI: 10.1103/PhysRevD.82.064014 PACS numbers: 04.60.Kz, 04.90.+e, 11.10.Kk, 11.15.Bt

I. INTRODUCTION

In connection with the AdS/CFT correspondence in three dimensions, planar quantum gravity has been the object of renewed and raising interest [1]. As shown in Ref. [2], Einstein-Hilbert (E-H) gravity with a negative cosmological constant has black hole solutions and for this reason it is interestingly related to two-dimensional CFT theories on the AdS boundary. In the work of [3], Witten has reassessed other relevant aspects of three-dimensional gravity. In a subsequent paper, Li, Song, and Strominger [4] have proposed a chiral gravity model in three space-time dimensions and they focus their efforts on the study of topologically massive gravity.

In spite of E-H gravity in three dimensions having no propagating degrees of freedom, the introduction of (curvature)^2- terms allow new propagating modes. However, unitarity could be jeopardized due to the higher-derivative terms. Surprisingly, a recently proposed higher-derivative model, known as Bergshoeff-Hohm-Townsend model [5,6], was shown to be unitary and renormalizable [7,8]. This model is a specific combination of the E-H action with the wrong sign added with higher-derivative curvature terms, which is equivalent to the Pauli-Fierz Lagrangian at the linearized level.

In odd dimensional theories, it is tempting to consider a Chern-Simons term. In the case of vector fields, this term gives to the photon a mass in a gauge invariant way. For planar gravity, this term was first considered by Deser, Jackiw, and Templeton in Ref. [9]. In fact, the E-H Lagrangian with the wrong sign added to the Chern-Simons term propagates a parity-breaking massive spin-2 mode. In spite of the fact that the gravitational Chern-Simons term has three derivatives, it was shown that this model has neither ghosts nor acausalities. Furthermore, the presence of the three derivatives suggests that the ultraviolet divergences of the model could be stabilized rendering it power-counting renormalizable. Actually, this was explicitly shown in Refs. [10–12]. The issue of unitarity in extensions of these theories, such as the incorporation of quadratic terms, is not a trivial matter, as discussed in [13–16].

In four dimensions, massive gravity theories are motivated by the outstanding result that they could suitably modify Einstein's general relativity at very large distance scales (actually, cosmological scales), in such a way that the present accelerated expansion of our universe may be taken into account without invoking the idea of dark energy. Gravity models with dynamical torsion may naturally yield a way to generate mass for gravitons. So, we take the viewpoint that, a better understanding of both torsion propagation and massive gravitons in three-dimensional space may provide new insights into the way we comprehend massive gravitation in connection with dynamical torsion in four dimensions. More recently, in the work of Ref. [17], an interesting proposal is presented where the unitary of a Yang-Mills type formulation for massless and massive gravity with propagating torsion is investigated.

Once we are convinced of the relevance of investigating aspects of quantum gravity in three-dimensional space-time, we could go further and try to understand how the degrees of freedom associated with torsion may influence and affect properties of planar quantum gravity, previously contemplated in the absence of torsion. Our work sets out just to pursue an investigation of the possible effects torsion may induce on quantum-mechanical aspects of planar gravity. Of special interest for us is the emergence of massive gravitons, once torsion is allowed to be dynamical. For a deeper discussion on the role of torsion in

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four-dimensional quantum gravity, we address the reader to the works of Refs. [18–22].

In this paper, we shall be mainly interested in the Chern-Simons topological term, but with the modification that shall allow propagating torsion. This is appealing whenever we adopt the vielbein formalism for gravitation, since, as shown in [23,24], the spin connection and vielbein can be combined in such way that they form a connection for a truly gauge theory. In this way, the field-strength related to this connection carries both the Riemann and torsion tensor. Our main goal is to investigate the spectrum and unitary properties of the topological gauge gravity theory in the first-order formalism. By this, we mean the E-H Lagrangian with the Chern-Simons term considering the spin connection and vielbein as independent fields. Besides that, we also consider (curvature)^2- and (torsion)^2-terms, which renders our analysis more general.

In order to analyze the spectral properties of the model, the attainment of the propagator becomes a primary goal. There are various methods for the deriving propagators. The algebraic method based on the spin projection operators has been shown very efficient and has been widely used in the literature [25–27]. A three-dimensional analogous basis to the one proposed in [26] is not enough to handle the problem proposed in this paper. One possibility to circumvent this issue is to extend the basis by means of the decomposition of the transverse operator, θ, and the longitudinal operator, ω, into more fundamental ones. In three dimensions, the procedure is to decompose θ, which is a definite spin projector, into two degrees-of-freedom projectors, ρ and σ. In [28], the set of operators in terms of $\{\theta, \omega\}$ is rewritten in terms of ρ, σ, and ω. As it becomes clear in that work, additional mapping operators among the projectors are needed, since we are dealing with degree-of-freedom projectors rather than spin projectors. The physical appeal for this decomposition has to do with the different role played by parity in four and three dimensions. This extension of the basis enables one to handle terms with an explicit Levi-Civita tensor, which is necessary to describe parity-breaking and to take advantage of the dual aspects of the fields. The propagator and the conditions for the absence of ghosts and tachyons are, by this procedure, directly obtained.

Our results allow us to discuss the role played by the Chern-Simons term on the conditions for the unitarity and in the spectrum properties of the model. We also explicitly analyze the case where we discard the Chern-Simons term and compare them with the results obtained in [27] for $D = 3$, in order to verify the consistency of the proposed basis.

The outline of this paper is as follows: In Sec. II, we present the model and the conventions used in this work. In Sec. III, we work out the propagator of the model. Section IV tackles the issue of spectrum consistency by analyzing the conditions for absence of ghosts and tachyons in the massive and massless sectors. Finally, in Sec. V, we set up our concluding remarks. An appendix follows where we list the inverse of the spin matrices used to calculate the propagators of Sec. III.

II. DESCRIPTION OF THE MODEL

For the sake of generality, we investigate a general gravity Lagrangian which includes quadratic terms in the curvature and torsion added to a Chern-Simons (CS) term:

$$\begin{aligned}\mathcal{L} = e[&(u-s)R + \tfrac{1}{8}(3(\beta+\gamma) - \tfrac{1}{4}\xi)R^2 + \beta R_{\mu\nu}R^{\mu\nu} \\ &+ \gamma R_{\mu\nu}R^{\nu\mu} + \tfrac{1}{8}(u+r+2s)T_{abc}T^{abc} \\ &+ \tfrac{1}{4}(u+r-2s)T_{abc}T^{bca} \\ &+ \tfrac{1}{2}(u-2s-t)T_{ab}{}^{b}T^{ac}{}_{c}] + d\mathcal{L}_{\text{CS}}, \end{aligned} \tag{1}$$

where r, s, t, and u are arbitrary parameters with mass dimension equal to 1, whereas ξ, β, and γ are inverse mass parameters and d is dimensionless. Also, we should justify that this apparently unusual combination of parameters is a mere matter of convenience. Actually, in our analysis of the spectral conditions (subject of Sec. IV), the parameters associated to the terms of Lagrangian density above combine in such a way that the form we propose in (1) yields to considerable algebraic simplifications, without any loss of generality in our results. The term

$$\mathcal{L}_{CS} = \epsilon^{\alpha\beta\gamma}(R_{\alpha\beta ab}\omega_{\gamma}{}^{ab} + \tfrac{2}{3}\omega_{\alpha b}{}^{c}\omega_{\beta c}{}^{a}\omega_{\gamma a}{}^{b}) \tag{2}$$

is the well-known topological Chern-Simons term.

Some remarks are in order. First, the absence of terms of Riemann squared is due to the fact that, in three dimensions, the Riemann tensor can be written in terms of the Ricci tensor and the scalar curvature. Second, the absence of the cosmological constant and the translational Chern-Simons term, $\epsilon^{\mu\nu\lambda}T_{\mu\nu}{}^{a}e_{\lambda a}$ [29], are due to a more subtle reason. We shall adopt the position of expanding the graviton field around a Minkowski space. But, it is well known that free-matter solutions for theories with these two terms are nonflat. As an immediate consequence, the introduction of these terms would spoil the gauge symmetry that comes from the reparametrization symmetry of the nonlinear model. By expanding the action around the (flat) Minkowski background, we define our quantum theory by specifying this ground state and the symmetries of corresponding linearized action. By then computing the tree-level spectrum and concluding that tachyons and ghosts are absent (this shall be thoroughly done in the sequel), we make sure that we are expanding around a configuration stable against quantum corrections. Since the translational Chern-Simons and the cosmological constant terms would explicitly break the gauge symmetry of the linear theory, the possibility of inducing these two terms through radiative corrections is ruled out as long as we expand around Minkowski space.

Third, powers of curvature and torsion higher than two have not been considered in order to avoid higher derivatives that usually are hazardous to unitarity properties of the model. Finally, since we are considering the Chern-Simons term and thus a parity-broken model, there is the possibility of considering quadratic terms built from the dual of torsion and Ricci tensor, as, for example, $\epsilon_\mu{}^{\kappa\lambda}T_{\kappa\lambda}{}^a R^\mu{}_a$, $R\epsilon^{\mu\nu\kappa}T_{\mu\nu\kappa}$, $\epsilon_{\mu\nu}{}^\kappa T_{\kappa a}{}^a R^{\mu\nu}$. All these mixings couple the vielbein and spin-connection field strengths in a nontrivial way. It is interesting to notice that these terms may be regarded as originating from a Lorentz-symmetry violating gravity model in (1 + 3)-dimensions in the presence of a background vector, v^μ. The terms $\epsilon^{\mu\nu\kappa\lambda}v_\mu T_{\nu\kappa}{}^a R_{\lambda a}$, $\epsilon^{\mu\nu\kappa\lambda}v_\mu R T_{\nu\kappa\lambda}$, and $\epsilon^{\mu\nu\kappa\lambda}v_\mu T_{\nu a}{}^a R_{\kappa\lambda}$ yield, respectively, the $3D$-terms mentioned above for a spacelike background vector, $v^\mu = (0; 0, 0, \mu)$. In connection with a paper by Kostelecky [30] and the work of Ref. [31], an investigation of the possible origin and consequences of such $4D$-terms in the spectrum of gravity models with deviations from Lorentz symmetry and a full study of $3D$-gravity with the inclusion of the T-R-type terms above demands special attention and is the subject of an investigation we are pursuing.

In this paper, we shall work in the first-order formalism, where the vielbein (e^a_μ) and spin connection ($\omega_\mu{}^{ab}$) are taken as fundamental fields. We also set up the conventions for the Minkowski metric, $\eta_{\mu\nu} = (+1, -1, -1)$, the Levi-Civita symbol, $\epsilon_{012} = +1$, and the Riemann and torsion tensors,

$$R_{\mu\nu}{}^{ab} = \partial_\mu\omega_\nu{}^{ab} - \partial_\nu\omega_\mu{}^{ab} + \omega_\mu{}^a{}_c\omega_\nu{}^{cb} - \omega_\nu{}^a{}_c\omega_\mu{}^{cb}, \tag{3a}$$

$$T_{\mu\nu}{}^a = \partial_\mu e_\nu{}^a - \partial_\nu e_\mu{}^a + \omega_\mu{}^a{}_c e_\nu{}^c - \omega_\nu{}^a{}_c e_\mu{}^c, \tag{3b}$$

where the Greek indices refer to the world manifold and Latin ones for the frame indices. The contracted tensors are

$$R_\mu{}^a = e^\nu{}_b R_{\mu\nu}{}^{ab}, \tag{4a}$$

$$R = e^\mu{}_a e^\nu{}_b R_{\mu\nu}{}^{ab}. \tag{4b}$$

In order to settle down a quantum theory, we shall consider the following fluctuation around the Minkowski vacuum:

$$e_\mu{}^a = \delta_\mu{}^a + \tilde{e}_\mu{}^a \tag{5a}$$

$$\omega_\mu{}^{ab} = \tilde{\omega}_\mu{}^{ab} \tag{5b}$$

Henceforth, the distinction between Greek and Latin indices becomes unnecessary. It is also convenient to decompose the vielbein fluctuation, $\tilde{e}_{ab}$, into its symmetric, ϕ_{ab}, and antisymmetric, $\epsilon_{ab}{}^c\chi_c$, components. An analogous decomposition is done for the dual field of the spin-connection fluctuation, ψ_{ad} being the symmetric and $\epsilon_{ad}{}^e\lambda_e$ the antisymmetric components:

$$\tilde{e}_{ab} = \phi_{ab} + \epsilon_{ab}{}^c\chi_c, \tag{6a}$$

$$\tilde{\omega}_{abc} = \epsilon_{bc}{}^d(\psi_{ad} + \epsilon_{ad}{}^e\lambda_e). \tag{6b}$$

In the sequel, we shall consider ϕ, χ, ψ, and λ as the fundamental fields of the linearized model.

III. CALCULATION OF THE PROPAGATOR

It is our main goal to analyze the spectral consistency of the model (1). These aspects can be readily obtained by means of the propagator of the model. In order to accomplish this task, let us consider the Lagrangian up to second-order terms in the quantum fluctuations,

$$(\mathcal{L})_2 = \frac{1}{2}\sum_{\alpha,\beta}\Psi_\alpha\mathcal{O}_{\alpha\beta}\Psi_\beta, \tag{7}$$

where Ψ_α is a multiplet that carries the 18 components (ϕ_{ab}, ψ_{ab}, χ_a, λ_a), and $\mathcal{O}_{\alpha\beta}$ is the wave operator which contains η's, ϵ's, and at most two derivatives. The saturated propagator is written as

$$\Pi = i\sum_{\alpha,\beta}\mathcal{S}^*_\alpha\mathcal{O}^{-1}_{\alpha\beta}\mathcal{S}_\beta, \tag{8}$$

with $\mathcal{S}_\alpha$ being the sources for the fundamental fields.

The problem of the attainment of the propagator is reduced to the problem of inversion of the wave operator. Once a complete basis in which the wave operator can be expanded is found, the inversion of the operator becomes a lengthy but straightforward task.

A first step to treat the attainment of the propagator for Chern-Simons gravity in second-order formalism was carried out in [32,33], with the help of an extension of the Barnes-Rivers operators [34]. However, the consideration of Lagrangians with a larger number of free parameters, specially in first-order formalism, makes this task technically difficult [35,36], due to the nontrivial algebra that these operators satisfy.

A more efficient technique that greatly simplifies this issue is to decompose the wave operator of the linearized Lagrangian in an orthogonal basis of projector operators. In [25,26], an orthonormal basis of spin-parity operators in $4D$ is proposed which is suitable to study spectral properties of parity-preserving models containing a rank-2 tensor and a rank-3 tensor antisymmetric in two indices. The great advantage of an orthonormal basis of spin-parity operators over the standard ones is that it allows the decomposition of the wave operator into spin-parity sectors that can be inverted independently. Gravity Lagrangians in first-order formalism are fairly well accommodated in this treatment. Also, the gauge symmetries of the model are conveniently handled. Its generalization for arbitrary space-time dimensions [27] is straightforward and the particular three-dimensional case is the one relevant to this paper.

As is well known, the spin-parity operator basis obtained as a generalization from the one proposed in [26] cannot handle the Chern-Simons term in a straightforward way, since the wave operator contains the Levi-Civita symbol. In others words, this set of operators does not form a basis for parity-breaking Lagrangians, which is the sort of model in which the Chern-Simons term is encoded. In order to circumvent this problem, the authors of [28] propose a parity operator basis in 3-D that makes it possible to analyze parity-breaking gravity models on the same foot as those that do not violate parity. In [28], it is argued that each spin decomposition of the field does not have a definite parity and finding out degree-of-freedom operators is the convenient way to set up a basis operator in this case. In this vein, we perform in the sequel the wave operator decomposition of the linearized Lagrangian. (Further discussions and a list of these operators are given in [28].)

In the aforementioned basis, the wave operator is expanded as

$$\mathcal{O}_{\alpha\beta} = \sum_{J,ij} a_{ij}^{\varphi\vartheta}(J) P_{ij}^{\varphi\vartheta}(J^{PQ})_{\alpha\beta}. \tag{9}$$

Let us clarify the notation. The diagonal operators $P_{ii}^{\varphi\varphi}(J^{PP})$ are projectors in each of the degrees of freedom of the spin (J) and parity (P) sectors of the field φ, while the $P_{ij}^{\varphi\vartheta}(J^{PQ})$ (with $i \neq j$) are mappings between the projectors $P_{ii}^{\varphi\varphi}(J^{PP})$ and $P_{jj}^{\vartheta\vartheta}(J^{QQ})$. This can be read off in the following relations:

$$\sum_{\beta} P_{ij}^{\Sigma\Psi}(I^{PQ})_{\alpha\beta} P_{kl}^{\Lambda\Xi}(J^{RS})_{\beta\gamma} = \delta_{jk}\delta^{\Psi\Lambda}\delta^{IJ}\delta^{QR} P_{il}^{\Sigma\Xi}(I^{PS})_{\alpha\gamma}, \tag{10}$$

$$\sum_{i,J^{PP}} P_{ii}(J^{PP})_{\alpha\beta} = \delta_{\alpha\beta}. \tag{11}$$

The $a_{ij}^{\Sigma\Lambda}(J)$ are the coefficient in the wave operator expansion. These can be arranged in matrices representing the contribution to the spin (J). When these matrices are nonsingular, the saturated propagator (8) is given by

$$\Pi = i \sum_{\alpha,\beta,J^{PQ}} \mathcal{S}_{\alpha}^{*} a_{ij}^{-1\varphi\vartheta}(J) P_{ij}^{\varphi\vartheta}(J^{PQ})_{\alpha\beta} \mathcal{S}_{\beta}. \tag{12}$$

However, the considered Lagrangian (1) is invariant under local Lorentz and general coordinate transformations. This implies that the linearized Lagrangian is invariant under some local transformations of the fields. Gauge invariance makes the coefficient matrices become degenerate. In Ref. [25], it is shown that the correct gauge invariant propagator is obtained by taking the inverse any largest nondegenerate submatrix and then saturating it with sources.

For the model (1), the coefficients $a_{ij}^{\Sigma\Lambda}(J)$ form the 6×6 spin-0, 8×8 spin-1, and 4×4 spin-2 matrices. The spin-0 and spin-1 matrices are degenerate. We list below the largest nondegenerate submatrices obtained from them:

$$a(0) = \begin{pmatrix} 2u + 4r + 2(\beta - \gamma)p^2 & 2\sqrt{2}r & 0 & 8\sqrt{2}id\sqrt{p^2} \\ 2\sqrt{2}r & 2(u + r) & 0 & 0 \\ 0 & 0 & 2(u - t - s)p^2 & 2\sqrt{2}i\sqrt{p^2}t \\ -8i\sqrt{2}d\sqrt{p^2} & 0 & -2\sqrt{2}it\sqrt{p^2} & -4t + \xi p^2 \end{pmatrix}, \tag{13a}$$

$$a(1) = \begin{pmatrix} 2u + \beta p^2 & -4id\sqrt{p^2} & 0 & -iu\sqrt{p^2} \\ 4id\sqrt{p^2} & 2u + \beta p^2 & iu\sqrt{p^2} & 0 \\ 0 & -iu\sqrt{p^2} & \frac{1}{2}(u - t)p^2 & 0 \\ iu\sqrt{p^2} & 0 & 0 & \frac{1}{2}(u - t)p^2 \end{pmatrix}, \tag{13b}$$

$$a(2) = \begin{pmatrix} 2u + 2(\beta + \gamma)p^2 & 8id\sqrt{p^2} & 0 & 2iu\sqrt{p^2} \\ -8id\sqrt{p^2} & 2u + 2(\beta + \gamma)p^2 & -2iu\sqrt{p^2} & 0 \\ 0 & 2iu\sqrt{p^2} & 2sp^2 & 0 \\ -2iu\sqrt{p^2} & 0 & 0 & 2sp^2 \end{pmatrix}, \tag{13c}$$

where $p^2 = p_a p^a$, with p^a being the relativistic three-momentum. Their inverses, needed for the attainment of the propagators, are given in the Appendix.

In Ref. [27], one considers the same Lagrangian (1), except for the Chern-Simons term, in an arbitrary spacetime dimension. So, it is worthwhile to compare our results so far, whenever $d = 0$, with those in [27], for $2 + 1$ dimensions ($D = 3$), in order to verify the consistency of new basis of operators. At first glance, one notices that there are three more matrices than in our treatment. In fact, the spin-2^- and spin-0^-, which are contained in the spin-connection field decomposition, cannot appear here since the spin operators associated with these spins are identically zero in three dimensions. It also can be verified that the spin operators associated with the spin-1^+, in that work, are mapped into spin-0 operators when we use the duality relations for the fields. This is noticed in the spin-0 matrix above: for $d = 0$, it becomes block-diagonal with the blocks corresponding to the spin-0^+ and spin-1^+ that appear in [27]. The spin-2 and spin-1 matrices, compared with the spin-2^+ and spin-1^-, remain essentially the same. The differences are some rearrangements in the parameters of spin-1 and the duplication of the dimension of the matrices in this work, due to splitting into degrees of freedom instead of spins. It must be stressed that, by comparing the parameters in both works, one has to contemplate the fact that in three dimensions the Riemann tensor can be expressed in terms of the Ricci tensor and scalar curvature.

IV. SPECTRAL CONSISTENCY ANALYSIS

In this section, we analyze the spectral consistency of the model. With this study, we shall impose conditions on the parameters of Lagrangian (1) in such a way that it does not propagate unphysical particles, that is, ghosts and tachyons. For the sake of clarity, we split the discussions for the cases of massive and massless poles.

A. Massive poles

In terms of the inverse matrices (27)–(31), we can write the propagator as

$$\Pi(J^P) = i \sum_{ij,\alpha,\beta} A_{ij}^{\Sigma\Lambda}(J, m^2) S_\alpha^* P_{ij}^{\Sigma\Lambda}(J^{PQ})_{\alpha\beta} \mathcal{S}_\beta (p^2 - m^2)^{-1}, \tag{14}$$

where $A(J, m^2)$ is the 4×4 matrix which is degenerate at the pole $p^2 = m^2$.

The condition for absence of ghosts and tachyons are, respectively, given by

$$\Im \mathrm{Res}(\Pi|_{p^2=m^2}) > 0, \quad \text{and} \quad m^2 > 0. \tag{15}$$

The condition for absence of ghosts for each spin is directly related to the positivity of the matrices $(\sum A_{ij}(J, m^2) P_{ij})_{\alpha\beta}$.. However, it can be shown that these matrices have only one nonvanishing eigenvalue at the pole, which is equal to the trace of $A(J, m^2)$. Also, the operators P_{ij} themselves contribute only with a sign $(-1)^N$ when calculated at the pole, where N is the sum of the number of ρ's and σ's in each part of the projector. Therefore, the condition for absence of ghosts for each spin is reduced to

$$(-1)^N \mathrm{tr} A(J, m^2)|_{p^2=m^2} > 0. \tag{16}$$

Using the conditions (15) and (16) for the matrices (27)–(31), we have

$$\text{Spin} - \mathbf{2}\text{: } us(s-u) < 0; \quad (\beta + \gamma) > 0; \tag{17a}$$

$$\text{Spin} - \mathbf{1}\text{: } \beta < 0; \quad ut(u-t) < 0; \tag{17b}$$

$$\text{Spin} - \mathbf{0}\text{: } (s+t-u)(s-u)t > 0; \quad (r+u)u(u+3r) < 0; \quad \xi > 0; \quad (\beta+\gamma) > 0. \tag{17c}$$

It is remarkable that the conditions for absence of tachyons and ghosts are equivalent to the ones obtained in [27] in the three-dimensional case, even if the Chern-Simons term spoils the direct identification of the respective spin matrices.

The roots of the matrices denominators (28), (30), and (32), which are given in the Appendix, give us the masses of the propagating particles. A careful look at the parameter combination reveals that only the torsion terms are crucial for obtaining a massive spectrum (as discussed in [27]). This is a remarkable difference with the second-order formalism for gravity, where the Chern-Simons term brings up a massive graviton. However, this is due to the higher-derivative character of such a theory.

The mass spectrum,

$$\mathbf{2}\text{: } m_\pm^2 = \frac{8d^2}{(\beta+\gamma)^2} + \frac{u(u-s)}{s(\beta+\gamma)} \pm \sqrt{\left(\frac{8d^2}{(\beta+\gamma)^2}\right)^2 + 2\frac{u(u-s)}{s(\beta+\gamma)}\frac{8d^2}{(\beta+\gamma)^2}}, \tag{18a}$$

$$\mathbf{1}\text{: } m_\pm^2 = \frac{8d^2}{\beta^2} + \frac{2ut}{\beta(u-t)} \pm \sqrt{\left(\frac{8d^2}{\beta^2}\right)^2 + 2\frac{2ut}{\beta(u-t)}\frac{8d^2}{\beta^2}}, \tag{18b}$$

$$\mathbf{0}\text{: } m_\pm^2 = \left(\frac{32d^2}{\xi(\beta-\gamma)} + \frac{2t(s-u)}{\xi(s+t-u)} - \frac{u(3r+u)}{2(\beta-\gamma)(r+u)}\right) \pm \sqrt{\left(\frac{32d^2}{\xi(\beta-\gamma)} + \frac{2t(s-u)}{\xi(s+t-u)} - \frac{u(3r+u)}{2(\beta-\gamma)(r+u)}\right)^2 + 4\frac{tu(s-u)(3r+u)}{\xi(\beta-\gamma)(r+u)(s+t-u)}}, \tag{18c}$$

is significantly changed by the Chern-Simons term. In the spin-1 and spin-2 sectors the number of particles changes from one to two. The influence of the Chern-Simons term in the spin-0 sector is restricted to shifting the particle masses. All this happens due to the parity-breaking property of the Chern-Simons term. In three dimensions, every massive particle has one degree of freedom [37]. Since spin is represented by a pseudoscalar operator in 3-D [38], there must be a doublet of spins with the same absolute value for the mass, $|m|$, so that an irreducible representation of the Lorentz group extended by time-inversion and parity transformations be constituted. On the other hand, in a parity-breaking theory, this doublet structure is lost and each component spin acquires a different value of $|m|$. This becomes explicit when one analyzes the role of the Chern-Simons term for the particle masses (18a)–(18c).

B. Massless poles

For the calculation of the massless propagators, some subtleties require extra care. The wave operator, as well its inverse, are Lorentz covariant, thus they can be expressed in terms of the set of following structures:

$$\omega_{ab}=\frac{p_a p_b}{p^2};\qquad \theta_{ab}=\eta_{ab}-\omega_{ab};\qquad \epsilon_{abc};\qquad p_a. \tag{19}$$

As we discussed earlier, for the attainment of the propagator, it is extremely convenient to decompose θ as $\rho+\sigma$ to build an orthonormal set of parity operators. However, for the calculation of the residue on the massless pole, the explicit dependence on p_a complicates the identification of the spin projectors. At this stage, we rewrite the propagator in terms of the set (19).

Furthermore, since the model is gauge invariant, there are constraints that the sources satisfy. They consistently appear in order to inhibit the nonphysical modes from propagating. The explicit expressions for these constraints are given in terms of the left null-eigenvectors of the degenerate coefficient matrices:

$$\sum V_j^{(L,n)}(J)P_{kj}(J^{PQ})_{\alpha\beta}\mathcal{S}_\beta=0. \tag{20}$$

The equation (20) implies in the following constraints for the fundamental sources:

$$p^a(S_{ab}+S_{ba}+\epsilon_{bca}\Omega^c)=0, \tag{21a}$$
$$p^a(\Sigma_{ab}+\Sigma_{ba})=0, \tag{21b}$$

where S_{ab}, Ω_c, and Σ_{ab} are the sources for the fields ψ, λ, and ϕ, respectively. To compare with previous results, we express the final answer for the massless propagator in terms of the source to the spin-connection field. The relation among the fields given in (6a) and (6b) enables us to write the fundamental sources as

$$S_{ab}=\tfrac{1}{2}(\epsilon_{pqb}\tau_a{}^{pq}+\epsilon_{pqa}\tau_b{}^{pq}), \tag{22a}$$
$$\Omega_a=-2\eta_{cd}\tau^{cd}{}_a, \tag{22b}$$

with τ_{abc} being the source of ω_{abc}.

Using (21a), (21b), (22a), and (22b), one can show that

$$\begin{aligned}\Pi(p^2=0)=&\frac{1}{2p^2(s-u)}(\tau^{ab*}\Sigma^{ab*})\begin{pmatrix}4 & 2i\\ -2i & 1\end{pmatrix}\\ &\times\left[\frac{1}{2}(\eta_{ac}\eta_{bd}+\eta_{bc}\eta_{ad})-(\eta_{ab}\eta_{cd})\right]\\ &\times(\tau^{de}\Sigma^{de})-\frac{2id}{p^2(s-u)^2}\Sigma^{ab*}\\ &\times\left[\frac{1}{2}(\eta_{ac}\epsilon_{dbe}+\eta_{bd}\epsilon_{cae})p^e\right]\Sigma^{cd}\\ &+\text{terms that do not contribute to residue,}\end{aligned} \tag{23}$$

where $\tau_{ab}=p^c\tau_{abc}$. This result, for $d=0$, agrees with the one obtained in [27] in the three-dimensional case. It is shown there that this part of the propagator vanishes. To explicitly calculate the other term, let us expand the source Σ_{ab} as

$$\Sigma_{ab}=c_1p_ap_b+c_2(p_a\epsilon_b+p_b\epsilon_a)+c_3\epsilon_a\epsilon_b, \tag{24}$$

where

$$p_a=(p_0,\vec{p}), \tag{25a}$$
$$q_a=(p_0,-\vec{p}), \tag{25b}$$

with

$$p^2=q^2=0, \tag{26a}$$
$$p.q=(p_0)^2+(\vec{p})^2, \tag{26b}$$
$$p.\epsilon=q.\epsilon=0, \tag{26c}$$
$$\epsilon^2=-1. \tag{26d}$$

Expansion (24) is the most general one for a symmetric rank-2 tensor that satisfies (21a). Using it in (23), one can show that the term due to Chern-Simons also identically vanishes. Therefore, even for $d\neq 0$, there is no massless particles propagating in the model and the relations (17a)–(17c) are the only ones that must be imposed to ensure the unitarity of the Lagrangian (1).

Though our efforts have been made to find the correct relations among the parameters to ensure absence of tachyons and ghosts, it must become clear that these are tree-level conditions, valid at linear level, by analyzing residues and poles of the free propagators. Interactions and loop corrections might give rise to unphysical modes that could be suppressed by reanalyzing the spectrum and finding new conditions among the loop-corrected parameters, so that ghosts do not show up. Therefore, at the nonlinear level, the spectrum might become plagued by ghosts, whose suppression has to be reassessed at each order in perturbation theory.

V. CONCLUDING REMARKS

We have considered a general gravity Lagrangian without higher derivatives and with a parity-breaking Chern-Simons term in the first-order formalism. It was our interest to investigate the possible unitary Lagrangians that one can obtain from it and determine the influence of the Chern-Simons term in the particle spectrum.

The proposal of applying the basis of spin projection operators developed in [28] was successfully implemented. Two striking features must be emphasized: first, its orthogonality properties makes the inversion of the wave operators easier and, second, the analysis of the symmetries of the model becomes a systematic procedure. Also, the results that appear in the literature for the same Lagrangian, but without the Chern-Simons term, are recovered, as was expected. For these reasons, it became clear that this basis of operators is not a mere algebraic convenience. The role of parity in three dimensions gives the operators a physical meaning. An analogous construction might be implemented for dealing with Lorentz-breaking theories in four dimensions, since parity-breaking terms in three dimensions are intrinsically related to Lorentz-symmetry-breaking terms in four dimensions.

From the analysis of the spectral consistency (Sec. IV), we see that the Chern-Simons term does not modify the unitary relations. Therefore, the possible unitary Lagrangians are the same as those obtained in [27] added up with the Chern-Simons term. The main contribution of this term, due to its parity-breaking character, is to raise the number of massive particles in the spin-1 and spin-2 sectors, as well as shifting their masses. Furthermore, only massive modes propagate. In the particular case of the Einstein-Hilbert-Chern-Simons Lagrangian in the first-order formalism, there are no massive particles and, consequently, we have no propagating modes. So, we conclude that the topological Chern-Simons term is compatible with the propagation of the torsion as long as unitarity is to be respected.

We understand that the Chern-Simons term does not alter the conditions for unitarity due to its lower derivative character and by virtue of its topological aspect. Actually, as we know, a Chern-Simons term alone does not yield local perturbations that we may identify as particles. The same should not be true for other parity-breaking terms (such as the ones listed in Sec. II), since they carry derivatives of a higher order. We have no concrete arguments in favor of this possibility, but, to our mind, they should be investigated. Also, we expect no modification in the particle content of the massive sector since, by analyzing the possible degrees of freedom that can be propagated, all massive modes have been consistently excited. The genuine massless spin-2 nonmassive mode (corresponding to the massless graviton) should not exist in three dimensions by a simple counting of the on-shell degrees of freedom. It remains to be elucidated if the remaining massless modes could propagate in a way compatible with unitarity.

ACKNOWLEDGMENTS

The authors are very grateful to Professor A. J. Accioly for a critical reading of their work. Thanks are also due to M. Plyushchay for a very pertinent comment on the massive representations of the 3-D Poincaré group. The authors express their gratitude to CNPq-Brazil and FAPERJ-Rio de Janeiro for financial support.

APPENDIX: INVERSE MATRICES

The inverse matrices that appear in the propagators are given by

$$a^{-1}(0) = \frac{1}{D_0}\begin{pmatrix} A_{11}^{(0)} & A_{12}^{(0)} & A_{13}^{(0)} & A_{14}^{(0)} \\ A_{12}^{(0)} & A_{22}^{(0)} & A_{23}^{(0)} & A_{24}^{(0)} \\ A_{13}^{(0)} & A_{23}^{(0)} & A_{33}^{(0)} & A_{34}^{(0)} \\ -A_{14}^{(0)} & -A_{24}^{(0)} & -A_{23}^{(0)} & A_{44}^{(0)} \end{pmatrix}, \tag{27}$$

where

$$\begin{aligned} D_0 = 8p^2[(u+r)((u+2r+(\beta-\gamma)p^2) \\ \times (((u-t-s)\xi p^2 - 4t(u-s))) \\ - 64d^2p^2(u-t-s)), \\ - 2r^2((u-t-s)\xi p^2 - 4t(u-s))], \end{aligned} \tag{28}$$

$$A_{11}^{(0)} = 4(u+r)(-4t(u-s) + (u-t-s)\xi p^2))p^2, \qquad A_{12}^{(0)} = -2\sqrt{2}r(2(u-t-s)(-4t+\xi p^2) - 8t^2)p^2,$$

$$A_{22}^{(0)} = ((2u+4r+2(\beta-\gamma)p^2)(2(u-t-s)(-4t+\xi p^2) - 8t^2) - 256d^2p^2(u-t-s))p^2,$$

$$A_{13}^{(0)} = 64dt(r+u)p^2, \qquad A_{14}^{(0)} = -32\sqrt{2}id(u-t-s)(r+u)\sqrt{p^2}p^2,$$

$$A_{22}^{(0)} = 4((u+2r+(\beta-\gamma)p^2)(-4(u-s)t + (u-t-s)\xi p^2) - 64d^2p^2(u-t-s))p^2, \qquad A_{23}^{(0)} = -64\sqrt{2}drtp^2,$$

$$A_{24}^{(0)} = 64idr(u-t-s)\sqrt{p^2}p^2, \qquad A_{33}^{(0)} = 4(-4t+\xi p^2)(u(3r+u) + (u+r)(\beta-\gamma)p^2) - 256p^2d^2(u+r),$$

$$A_{34}^{(0)} = -8\sqrt{2}i\sqrt{p^2}t(u(3r+u) + (\beta-\gamma)(u+r)p^2), \qquad A_{44}^{(0)} = 8p^2(u-t-s)(u(3r+u) + (\beta-\gamma)(u+r)p^2).$$

$$a^{-1}(1) = \frac{1}{D_1}\begin{pmatrix} A_{11}^{(1)} & A_{12}^{(1)} & A_{13}^{(1)} & A_{14}^{(1)} \\ -A_{12}^{(1)} & A_{11}^{(1)} & -A_{14}^{(1)} & A_{13}^{(1)} \\ A_{13}^{(1)} & A_{14}^{(1)} & A_{33}^{(1)} & A_{34}^{(1)} \\ -A_{14}^{(1)} & A_{13}^{(1)} & -A_{34}^{(1)} & A_{33}^{(1)} \end{pmatrix}, \tag{29}$$

where

$$D_1 = \tfrac{1}{2}p^2(4d^2(u-t)^2p^2 - (\tfrac{1}{2}p^2(u-t)\beta - ut)^2), \tag{30}$$

$$A_{11}^{(1)} = -\frac{1}{4}(u-t)\Big(\frac{1}{2}(u-t)\beta p^2 - ut\Big)p^2,$$
$$A_{12}^{(1)} = -\frac{i}{2}(u-t)^2 d\sqrt{p^2}p^2,$$
$$A_{13}^{(1)} = -du(u-t)p^2,$$
$$A_{14}^{(1)} = -\frac{i}{2}\sqrt{p^2}u\Big(\frac{1}{2}(u-t)\beta p^2 - ut\Big),$$
$$A_{33}^{(1)} = -\bigg(\Big(\frac{1}{2}p^2\beta - t\Big)(u-t)^2 + \Big(\frac{1}{4}p^4\beta^2 - 2t^2\Big)(u-t) - t^2\Big(\frac{1}{2}\beta p^2 + t\Big) - 4(u-t)d^2p^2\bigg),$$
$$A_{34}^{(1)} = -2idu^2\sqrt{p^2},$$
$$a^{-1}(2) = \frac{1}{D_2}\begin{pmatrix} A_{11}^{(2)} & A_{12}^{(2)} & A_{13}^{(2)} & A_{14}^{(2)} \\ -A_{12}^{(2)} & A_{11}^{(2)} & -A_{14}^{(2)} & A_{13}^{(2)} \\ A_{13}^{(2)} & A_{14}^{(2)} & A_{33}^{(2)} & A_{34}^{(2)} \\ -A_{14}^{(2)} & A_{13}^{(2)} & -A_{34}^{(2)} & A_{33}^{(2)} \end{pmatrix}, \tag{31}$$

and where

$$D_2 = 2p^2(16d^2s^2p^2 - (u(s-u) + p^2s(\beta+\gamma))^2),$$
$$A_{11}^{(2)} = -s(su + s(\beta+\gamma)p^2 - u^2)p^2,$$
$$A_{12}^{(2)} = 4ids^2\sqrt{p^2}p^2,$$
$$A_{13}^{(2)} = -4dusp^2,$$
$$A_{14}^{(2)} = iu\sqrt{p^2}(u(s-u) + s(\beta+\gamma)p^2),$$
$$A_{33}^{(2)} = -\frac{1}{s}((u(s-u) + s(\beta+\gamma)p^2)^2 - 16s^2d^2p^2 + u^2(u(s-u) + s(\beta+\gamma)p^2)),$$
$$A_{34}^{(2)} = -4idu^2\sqrt{p^2}. \tag{32}$$

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