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# Radiation bounce from the Lee-Wick construction?

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It was recently realized that matter modeled by the scalar field sector of the Lee-Wick standard model yields, in the context of a homogeneous and isotropic cosmological background, a bouncing cosmology. However, bouncing cosmologies induced by pressureless matter are in general unstable to the addition of relativistic matter (i.e. radiation). Here we study the possibility of obtaining a bouncing cosmology if we add not only radiation, but also its Lee-Wick partner, to the matter sector. We find that, in general, no bounce occurs. The only way to obtain a bounce is to choose initial conditions with very special phases of the radiation field and its Lee-Wick partner.

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## I. INTRODUCTION

The inflationary scenario [1] is the current paradigm of early universe cosmology. It addresses some of the problems which the previous paradigm, the standard big bang model, could not address, and it gave rise to the first theory of cosmological structure formation based on fundamental physics [2] whose predictions were later confirmed by the precision observations of the cosmic microwave background. Inflationary models, however, are faced with serious conceptual problems (see e.g. [3]), among which the singularity problem and the ‘‘Trans-Planckian’’ problem for fluctuations. In the context of General Relativity as the theory of space-time, it has been shown [4] that inflationary models have a singularity in the past and therefore cannot yield a complete theory of the early universe. The ‘‘Trans-Planckian’’ problem for fluctuations [3,5] relates to the fact that in inflationary models, the wavelengths of perturbation modes which are observed today were smaller than the Planck scale in the early periods of inflation, and were thus in the ‘‘short wavelength zone of ignorance’’ in which we cannot trust the theory which is being used to track the fluctuations. In fact, in [5] it is shown that the predictions for observations are in fact rather sensitive to the physics assumed in this zone of ignorance. These conceptual problems of inflationary cosmology form one of the motivations for considering possible alternatives to inflation.

One of the alternative scenarios to inflation is the ‘‘matter bounce’’ paradigm (see e.g. [6,7] for introductory expositions). In this scenario it is assumed that the universe undergoes a nonsingular cosmological bounce. Time runs from $-\infty$ to $+\infty$. The time coordinate can always be adjusted such that the bounce point is at time $t = 0$. The Hubble radius $H(t)^{-1}$ is the scale which separates wavelengths on which microphysics dominates (sub-Hubble) from those where matter forces are frozen out (super-Hubble). If the contracting and expanding phases far away from the bounce point are described by General Relativity and we consider matter with pressure density $p > -\rho/3$ (where $\rho$ is the energy density), then it follows that scales which are currently observed exited the Hubble radius at some point during the contracting phase. As was realized in [8–10], if the curvature fluctuations start out early in the contracting phase on sub-Hubble scales in their vacuum state, then the growth of the perturbations on super-Hubble scales during the period of contraction leads to a scale-invariant spectrum of curvature fluctuations on super-Hubble scales before the bounce. Detailed analyses of the evolution of cosmological fluctuations through the nonsingular bounce performed in the context of specific bouncing models (see e.g. [11–13]) shows that the spectrum of curvature fluctuations is unchanged during the bounce on wavelengths which are large compared to the bounce time, a result which agrees with what is obtained by applying the Hwang-Vishniac matching conditions [14,15] to connect perturbations across a spacelike ‘‘matching’’ hypersurface between a contracting and an expanding Friedmann universe.

By construction, a bouncing cosmology is nonsingular. In such a model, the wavelength of fluctuations which are being probed in current observations always remains far larger than typical microphysical scales. If the energy density at the bounce point is set by the scale of particle physics grand unification, then the physical wavelength corresponding to the current Hubble radius is about 1 mm, to quote just one number. Hence, the fluctuations remain in the regime controlled by the infrared limit of the theory, far from the trans-Planckian zone of ignorance.

The challenge is to obtain a bouncing cosmology. One must either give up General Relativity as the theory of space-time, or else one must invoke a new form of matter which violates some of the ‘‘usual’’ energy conditions (see [16] for a discussion of the assumptions underlying the singularity theorems of General Relativity). For a recent review on how bouncing cosmologies can be obtained, see [17]. We here mention but a few recent attempts. Introducing higher derivative gravity terms can lead to

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nonsingular cosmologies, as in the ''nonsingular universe construction'' of [18]. Similarly, the ghost-free higher derivative action of [19] leads to a bouncing cosmological background. Horava-Lifshitz gravity also leads to a bouncing cosmology provided that the spatial curvature does not vanish [20]. Bouncing cosmologies may also arise from quantum gravity, as e.g. in loop quantum cosmology (see e.g. [21] for a recent review). If we maintain General Relativity as the theory of space and time, then one can obtain a bounce by introducing new forms of matter such as ''quintom'' matter [22]. In this case, in addition to the matter sector with regular sign kinetic action, there is a new sector (a ''ghost'' sector) which has an opposite sign kinetic action.

Several decades ago, Lee and Wick [23] introduced a field theory construction which involves degrees of freedom with opposite sign kinetic terms. The Lee-Wick (L-W) model aims at stabilizing the Higgs mass against quadratically divergent terms and is interesting to particle physicists since it can address the ''hierarchy problem.'' The Lee-Wick construction was recently resurrected and extended to yield a ''Lee-Wick standard model'' [24,25]. The Lee-Wick model can thus potentially provide a framework for obtaining a bouncing cosmology. In [26], the Higgs sector of the Lee-Wick standard model was analyzed and it was shown that, indeed, a bouncing cosmology emerges. However, the scalar field Lee-Wick bounce is unstable against the addition of regular radiation to the matter sector (as will be explained in Sect. II of this paper). Since we know that there is radiation in the universe, one may than worry whether the Lee-Wick bounce can be realized at all. However, to be consistent with the philosophy of the Lee-Wick construction, Lee-Wick radiation terms with opposite sign kinetic actions must be added. In this paper we address the question whether, in this context, a cosmological bounce can be achieved. We find that unless the phases of the two fields are chosen in a very special way, then no bounce will occur.

The outline of this paper is as follows: in the next section we briefly review the philosophy behind the Lee-Wick model and discuss why the scalar sector of the Lee-Wick model taken alone would yield a bouncing cosmology. In Sec. III we introduce the Lagrangian for Lee-Wick electromagnetism and derive the expression for the energy density. In order to study the cosmological implications of our action, we need to know how plane waves of the Lee-Wick partner of the radiation field evolves. This is the focus of Sec. IV. After understanding how regular and Lee-Wick radiation evolve, we can then study under which conditions a bouncing cosmology might result.

## II. REVIEW OF THE LEE-WICK MODEL AND THE SCALAR LEE-WICK BOUNCE

We will review the Lee-Wick model and the Lee-Wick bounce in the simple case of a single scalar field $\hat{\phi}$. The hypothesis of Lee and Wick [23] was to add an extra scalar degree of freedom designed to cancel the quadratic divergences in scattering matrix elements. Originally, the new degree of freedom was introduced by adding a higher derivative term of the form $(\partial^2\hat{\phi})^2$ to the action, yielding a higher order differential equation and hence a new degree of freedom. It is, however, simpler to isolate the new degree of freedom by introducing an auxiliary scalar field $\tilde{\phi}$ and redefining the ''physical'' field to be $\phi$ (see [24]). After doing this and after a field rotation, the Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{1}{2}M^2\tilde{\phi}^2 - \frac{1}{2}m^2(\phi - \tilde{\phi})^2 - V(\phi - \tilde{\phi}), \tag{1}$$

where $M \gg m$ is the mass scale of the new degree of freedom, and $V$ is the original potential which after the field redefinition depends on both fields.

The field $\tilde{\phi}$ is called the Lee-Wick partner of $\phi$. It has the opposite sign kinetic Lagrangian and the opposite sign of the mass square term. Hence, without any coupling to other fields or to gravity the evolution of $\tilde{\phi}$ would be stable and would consist of oscillations about $\tilde{\phi} = 0$. However, in the presence of any coupling of $\tilde{\phi}$ with other fields there are serious potential instability and unitarity problems [27–30]. Ways to make the theory consistent were discussed many years ago in [31] and more recently in [32] in the case of interest in the current paper, namely, Lee-Wick electromagnetism. In [32], a proposal for a ultraviolet (UV) complete theory of quantum electrodynamics via the Lee-Wick construction was made. It was argued that the presence of ghost poles in virtual state propagators and the loss of microcausality do not necessary mean that causality is violated at macroscopic scales. This would be the case if the Lee-Wick particles decayed fast enough [33].

The Lee-Wick model has been resurrected in [24] with the goal of studying signatures of this alternative model to supersymmetry in LHC experiments. For some projects to try to test experimentally the predictions of the Lee-Wick model see e.g. [34,35].

Let us now review [26] how a nonsingular bouncing cosmology can emerge from the scalar sector of a Lee-Wick model. In fact, for this to happen no coupling between these fields is required, and hence we will assume $V = 0$ in the following discussion. We take initial conditions at some initial time in which both the scalar field $\phi$ and its Lee-Wick partner $\tilde{\phi}$ are both oscillating about their ground states, and that the positive energy density of $\phi$ exceeds the absolute value of the negative energy density of $\tilde{\phi}$, i.e. we start in a phase dominated by regular matter. We assume that the universe is contracting with a Hubble rate dictated by the Friedmann equations.

Initially both fields are oscillating and their energy densities both scale as $a^{-3}(t)$, where $a(t)$ is the cosmic scale factor. Since $M \gg m$ while the energy density of $\tilde{\phi}$ is smaller than that of $\phi$, the amplitude $\tilde{\mathcal{A}}$ of $\tilde{\phi}$ must be much smaller than the amplitude $\mathcal{A}$ of $\phi$. During the initial period of contraction, both amplitudes increase at the same rate. At some point, however, $\tilde{A}$ becomes comparable to $m_{\rm pl}$, the four dimensional Planck mass. As we know from the dynamics of chaotic inflation [36], at super-Planckian field values $\phi$ will cease to oscillate—instead, it will enter a ''slow-climb'' regime, the time reverse of the inflationary slow-roll phase. During this phase, the energy density of $\phi$ increases only slightly. However, $\tilde{\phi}$ continues to oscillate and its energy density increases in amplitude exponentially (still proportional to $a^{-3}$). The energy in $\tilde{\phi}$ (i.e. its absolute value) will hence rapidly catch up with that of $\phi$. When this happens, $H$ will vanish. Since the kinetic energy of $\tilde{\phi}$ overwhelms that of $\phi$, $\dot{H} > 0$ and thus a nonsingular bounce will occur [26], and the Universe will begin to expand.

The matter bounce in the Lee-Wick scalar field model was analyzed in detail in [26]. In particular, it was verified explicitly that initial vacuum fluctuations on sub-Hubble scales in the contracting period develop into a scale-invariant spectrum of curvature fluctuations on super-Hubble scales after the bounce. A distinctive prediction of this scenario is the shape and amplitude of the three-point function, the ''bispectrum'' [37].

However, the scalar field Lee-Wick bounce in unstable towards the addition of radiation before the bounce [38]: Since the energy density in radiation scales as $a^{-4}$ it becomes more important than that of $\tilde{\phi}$ as the universe decreases in size, and will hence destabilize the bounce. Can the addition of a Lee-Wick partner to regular radiation help restore the bounce? This is the question we ask in this work. We will follow the same type of reasoning as above, but for the case of radiation: we now introduce a Lee-Wick gauge field, the partner of the standard one, which will initially be dominant. We use the Lagrangian for a U(1) Lee-Wick gauge boson (see [24]) to which we add a coupling term between the normal and the Lee-Wick field in order to allow the energy to flow from one component to the other. Our goal is to see if we can get a bouncing universe using this setup.

## III. THE MODEL

We will consider the radiation sector of Lee-Wick quantum electrodynamics and will start with a higher derivative Lagrangian [24] for a $U(1)$ gauge field $A_\mu$ of the form

$$L_{hd} = -\frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} + \frac{1}{2M_A^2}\mathcal{D}^\mu\hat{F}_{\mu\nu}\mathcal{D}^\lambda\hat{F}^\nu_\lambda, \qquad (2)$$

where $F_{\mu\nu}$ is the field strength tensor associated with $A_\mu$ and $\mathcal{D}$ denotes the covariant derivative. Note the sign difference in the second term compared to [24]: This will prevent the appearance of a tachyonic massive L-W gauge boson. The mass $M_A$ corresponds to the mass of the new physics in the model. To solve the hierarchy problem of the standard model, this mass should be of the order of 1 TeV.

The higher derivative terms in the above Lagrangian lead to an extra propagating mode. We can isolate it using the usual Lee-Wick construction by introducing a new field $\tilde{A}$ ($\hat{A} = A + \tilde{A}$) called Lee-Wick partner, which depends on derivatives of the original field and adjusting the gauge fields such that the kinetic term of the Lagrangian becomes diagonal in $A_\mu$ and $\tilde{A}_\mu$. We find that the propagator for the $\tilde{A}_a$ field has pole at $p^2 = M_A^2$ and has an opposite sign compared to the normal one. Thus, it is a ghost field (with the associated problems of instability and nonunitarity mentioned in the previous section). The Lagrangian becomes

$$L = -\frac{1}{4}(F_{\mu\nu}F^{\mu\nu} - \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}) + cF_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{M_A^2}{2}\tilde{A}_a\tilde{A}^a. \qquad (3)$$

We have added a coupling term, with coupling constant $c$, in order to allow the energy density to be able to flow from the normal field to the Lee-Wick field. Since the Lee-Wick sector is not observed in experiments today, we choose the two fields to be weakly coupled. In the case when the coupling constant is equal to zero, $M_A$ is the mass of the L-W gauge field.

Note that the $U(1)$ gauge invariance of electromagnetism is broken by the addition of the Lee-Wick sector. In addition to the problem of ghosts, this is another serious potential problem for the model which we are currently investigating. Given that gauge invariance is violated, we need to justify our choice of the coupling between the two fields. We have used gauge invariance and power counting renormalizability to pick out the term we have added to the Lagrangian in order to describe the coupling. If the entire Lagrangian were gauge-invariant, this would clearly be the correct procedure. In the presence of a symmetry breaking term which is very small (for large values of $M_A$) we can use gauge invariance of the low energy terms in the action to justify neglecting small symmetry breaking coupling terms if we are interested in energy transfer between the two fields which should be operational already at low energies.

As our initial conditions in a contracting universe, we imagine that the usual radiation field dominates the energy-momentum tensor. This implies that we must set the initial amplitude of $\tilde{A}_\mu$ to be very small compared to that of the regular gauge field $A_\mu$. In this case, then if $M_A$ is large enough compared to the experimental energy scale, we would not expect to see the ghost radiation field in experiments.

The energy-momentum tensor following from (3) is

$$T_{\mu\nu} = -\frac{1}{4}g_{\mu\nu}(F_{\lambda\sigma}F^{\lambda\sigma} - \tilde{F}_{\lambda\sigma}\tilde{F}^{\lambda\sigma} - 4cF_{\lambda\sigma}\tilde{F}^{\lambda\sigma}) + F_{\mu}{}^{\lambda}F_{\nu\lambda} - \tilde{F}_{\mu}{}^{\lambda}\tilde{F}_{\nu\lambda} + \frac{1}{2}g_{\mu\nu}M_A{}^2\tilde{A}_a\tilde{A}^a - M_A^2\tilde{A}_\mu\tilde{A}_\nu - 4cF_{\mu\lambda}\tilde{F}_\nu^\lambda \tag{4}$$

and its trace is, contrary to the case of pure radiation, nonzero:

$$T_\mu^\mu = M_A^2\tilde{A}_a\tilde{A}^a. \tag{5}$$

Using the Friedmann metric given by (16), the energy density is equal to

$$T_{00} = \frac{1}{4}(F^2 - \tilde{F}^2) - cF_{\lambda\sigma}\tilde{F}^{\lambda\sigma} + F_0{}^\lambda F_{0\lambda} - \tilde{F}_0{}^\lambda\tilde{F}_{0\lambda} - M_A^2\left(\frac{\tilde{A}^2}{2} + \tilde{A}_0{}^2\right) - 4cF_{0\lambda}\tilde{F}_0^\lambda. \tag{6}$$

We can split this into three different terms: the contribution of normal radiation,

$$\rho_A = \frac{1}{4}(F^2 + F_0{}^\lambda F_{0\lambda}), \tag{7}$$

the contribution from Lee-Wick radiation,

$$\rho_{\tilde{A}} = -\frac{1}{4}(\tilde{F}^2 + \tilde{F}_0{}^\lambda\tilde{F}_{0\lambda}) - M_A^2\left(\frac{\tilde{A}^2}{2} + \tilde{A}_0{}^2\right), \tag{8}$$

and the term coming from the mixing between the two fields,

$$\rho_{A-\tilde{A}} = -c(F_{\lambda\sigma}\tilde{F}^{\lambda\sigma} + 4F_{0\lambda}\tilde{F}_0^\lambda). \tag{9}$$

The equation of state is like that of radiation but with an additional term proportional to the mass of the Lee-Wick gauge field:

$$w \equiv \frac{p}{\rho} = \frac{\rho}{3\rho} + \frac{T_\mu^\mu}{3\rho} = \frac{1}{3} + \frac{M_A^2\tilde{A}_a\tilde{A}^a}{3T_{00}}. \tag{10}$$

We note that this expression is valid only when the total energy density is nonzero, and thus it would not be valid at the bouncing point if there were a bounce.

We can actually define three different equation of state parameters, one for each type of energy:

$$w_A = w_{A-\tilde{A}} = \frac{1}{3} \quad \text{and} \quad w_{\tilde{A}} = \frac{1}{3} + \frac{M_A^2\tilde{A}_a\tilde{A}^a}{\rho_{\tilde{A}}}, \tag{11}$$

the last of which is nonconstant in time. The equation of state parameter for the coupling term is the same as the one for normal radiation since the trace of the coupling energy-momentum tensor vanishes.

Our goal is to see under which conditions the above matter Lagrangian leads to a cosmological bounce. We will initially turn off the coupling between the two fields (i.e. set $c = 0$), derive the solutions of the equations of motion for both fields, and study what scaling with the cosmological scale factor $a(t)$ these solutions imply for the three contributions to the energy density discussed above. We find that—unlike what happens for the scalar field Lee-Wick model of [26]—there is no mechanism which leads to a faster increase in the energy density of the Lee-Wick partner field than that of the original radiation field. Thus, a bounce can only occur if there is a mechanism which drains energy from the original gauge field sector to the Lee-Wick partner field. It is for this reason that we have introduced a direct coupling term between the two fields in our Lagrangian. We will then study the effects of the coupling between the two fields, working in Fourier space and making use of the Green function method. We find that the sign of the energy transfer depends not only on the sign of the coupling coefficient $c$, but also on the phases of the oscillations of the two fields. Averaging over the phases, we find no net energy transfer, and hence there can be no cosmological bounce.

As initial conditions we choose a state in the contracting phase in which the regular radiation field is in thermal equilibrium at some initial time $t_i$. Since we want to start with a state which looks like the time reflection of the state we are currently in, we assume that the energy density in $\tilde{A}_\mu$ is initially subdominant. We, however, do assume that $\tilde{A}_\mu$ has excitations for modes with wave-number comparable to the initial temperature.

In the absence of coupling between the two fields, the distribution of $A_\mu$ would remain thermal, with a temperature $T$ which blueshifts as the universe contracts. The corresponding energy density would scale as $a^{-4}$. The presence of coupling will lead to a departure from thermal equilibrium. We will assume, however, that $a(t)$ continues to scale like $\sqrt{t}$, the scale factor of radiation. If there were a bounce, this approximation would fail at some point sufficiently close to the bounce time.

## IV. EQUATIONS OF MOTION

The equations of motion obtained from varying the Lagrangian with respect to $A_\mu$ and $\tilde{A}_\mu$ are

$$\partial_\mu(F^{\mu\nu} - 2c\tilde{F}^{\mu\nu}) + 3H(F^{0\nu} - 2c\tilde{F}^{0\nu}) = 0 \tag{12}$$

$$-M_A^2\tilde{A}^\nu + \partial_\mu(\tilde{F}^{\mu\nu} + 2cF^{\mu\nu}) + 3H(\tilde{F}^{0\nu} + 2cF^{0\nu}) = 0. \tag{13}$$

Combining them, we find that the L-W field will act as a source term for the normal field:

$$\partial_\mu F^{\mu\nu} + 3HF^{0\nu} = \frac{2cM_A^2}{1 + 4c^2}\tilde{A}^\nu \tag{14}$$

but that the L-W field is decoupled from the normal one and therefore only depends on the initial conditions:

$$\partial_\mu \tilde{F}^{\mu\nu} + 3H\tilde{F}^{0\nu} - \frac{M_A^2}{1+4c^2}\tilde{A}^\nu = 0. \quad (15)$$

From this last equation, we can also read off the new mass which the Lee-Wick partner field obtains in the presence of coupling: $M_A' = \frac{M_A}{\sqrt{1+4c^2}}$, which is about the same as $M_A$ at weak coupling. We can notice that at very strong coupling, the L-W gauge field becomes massless and therefore would evolve like a normal photon.

We will consider a homogeneous and isotropic universe with metric

$$ds^2 = -dt^2 + a^2(t)[dx^2 + dy^2 + dz^2], \quad (16)$$

where $t$ is physical time, $x$, $y$ and $z$ are the three spatial comoving coordinates, and we have for notational simplicity assumed that the universe is spatially flat.

Since the equations of motion are linear, we can work in Fourier space, i.e. with plane wave solutions. There will be no coupling between different plane waves. For simplicity, we focus on waves propagating along the $z$-axis with the same wave number, $k$, for the Lee-Wick and the normal gauge field. We work in the real basis of Fourier modes $\cos(kz)$ and $\sin(kz)$.

Without loss of generality we can restrict attention to one polarization mode which we take to be the electric field in the $x$ direction and the magnetic field in the $y$ direction. In this case, the only nonzero components of the field strength tensors are $F^{01}$, $F^{13}$, $\tilde{F}^{01}$, and $\tilde{F}^{13}$. Using the temporal gauge where $\tilde{A}_0 = A_0 = 0$, we find that only the first component of the gauge fields are nonzero, and we can make the ansatz

$$A_1(k,t) = f(t)\cos(kz) \quad \text{and} \quad \tilde{A}_1(k,t) = g(t)\cos(kz) \quad (17)$$

or equivalently

$$A^1(k,t) = a(t)^{-2} f(t)\cos(kz) \quad \text{and} \quad \tilde{A}^1(k,t) = a(t)^{-2} g(t)\cos(kz). \quad (18)$$

From (14) and (15), we obtain two linear second order differential equations with a damping term for the coefficient functions $f(t)$ and $g(t)$:

$$\ddot{f}(t) + H\dot{f}(t) + \left[\frac{k}{a(t)}\right]^2 f(t) = -\frac{2c}{1+4c^2}M_A^2 g(t) \quad (19)$$

$$\ddot{g}(t) + H\dot{g}(t) + \left[\left[\frac{k}{a(t)}\right]^2 + \frac{M_A^2}{1+4c^2}\right]g(t) = 0. \quad (20)$$

For $\frac{k}{a} \ll \frac{M_A}{\sqrt{1+4c^2}}$, the L-W field behaves as a harmonic oscillator with angular frequency $\frac{M_A}{\sqrt{1+4c^2}}$. As a consequence of the cosmological dynamics the oscillator undergoes damping (in an expanding universe) or antidamping (in the case of interest to us, that of a contracting universe). The regular radiation field satisfies the equation of a driven oscillator, again subject to cosmological damping or antidamping. Notice that (19) has a particular solution $f_p(t) = 2cg(t)$. The driving term can lead to energy transfer between the regular radiation field and its L-W partner. In the following we wish to study if the energy transfer is able to drain enough energy from the regular radiation field to enable a bounce to occur.

To solve these equations for any H(t), it is easier to use the conformal time $\eta = \int \frac{dt}{a}$, and to make things clearer, we introduce new functions $u$ and $v$ such that $u(\eta) = f(\eta)$ and $v(\eta) = g(\eta)$. Equations (19) and (20) can thus be rewritten as

$$u''(\eta) + k^2 u(\eta) = -a(t)^2\frac{2c}{1+4c^2}M_A^2 v(\eta) \quad (21)$$

$$v''(\eta) + \left[k^2 + a(t)^2\frac{M_A^2}{1+4c^2}\right]v(\eta) = 0, \quad (22)$$

where $'$ denotes the derivative with respect to $\eta$. For a radiation-dominated universe, we have $a(\eta) = \eta$.

From (21) we see that in the absence of coupling, we get simple oscillations in conformal time with frequency $k$ for the normal gauge field. For the L-W field we get oscillations in conformal time, with a time dependant frequency $\tilde{k} = \sqrt{k^2 + a(t)^2\frac{M_A^2}{1+4c^2}}$. In physical time these correspond to

$$f(t) = C\cos(2\sqrt{t}k + \phi) \quad (23)$$

$$g(t) = \frac{\alpha}{t^{1/4}} WhM\left(\frac{-ik^2\sqrt{1+4c^2}}{2M_A}, \frac{1}{4}, \frac{2iM_A t}{\sqrt{1+4c^2}}\right) + \frac{\beta}{t^{1/4}} WhW\left(\frac{-ik^2\sqrt{1+4c^2}}{2M_A}, \frac{1}{4}, \frac{2iM_A t}{\sqrt{1+4c^2}}\right) \quad (24)$$

where $WhM$ and $WhW$ are the Whittaker functions (see e.g. [39]), $\alpha$ and $\beta$ are constants characterizing the phase of $g(t)$ and $\phi$ is the phase of $f(t)$.

Before discussing the solutions of these equations, we must specify our initial conditions. We consider a contracting phase dominated by regular radiation. Since we have in mind an initial state which looks like the time reverse of a state in the early radiation phase of our expanding cosmology, we will start at some time $t_i$ in thermal equilibrium with a temperature much smaller than the Planck scale. The occupation numbers of the Fourier modes of the regular radiation field are hence given by the thermal distribution, with the peak wave number being set by the temperature and hence much larger than the Hubble rate. We are thus considering modes inside the Hubble radius. Since we are interested in studying the possibility of obtaining a bounce, we will work at temperature higher than the mass $M_A$.

We assume that the energy density of the L-W radiation field is subdominant at the initial time $t_i$. The most conservative assumption is that the distribution of wave numbers is also peaked at the initial temperature. These assumptions will allow us to pick out the limiting cases of the solutions of the above equations (to be discussed in the following section) which are relevant for us.

## V. SOLUTIONS

### A. Solutions for the Lee-Wick field

Depending on whether the physical wave-number is larger or smaller than the mass of the L-W gauge field, $M'_A = \frac{M_A}{\sqrt{1+4c^2}}$, we get different behaviors for the solution $g$. Since we are interested in exploring the solutions at high densities, close to the hypothetical bounce point, we will assume that the temperature is larger than the mass L-W field. We will focus on wave numbers close to the peak of the thermal distribution function, and hence $k/a > M'_A$. In this limit, the solutions for the L-W gauge field will simply be oscillating in conformal time with frequency $k$:

$$g(t) = \tilde{C}\cos(\eta k) = \tilde{C}\cos(2\sqrt{t}k), \tag{25}$$

where we have used the scaling of $a(t)$ of a radiation-dominated universe to express the conformal time $\eta$ in terms of physical time $t$, and where $\tilde{C}$ is a constant amplitude.

The normal gauge field satisfies a harmonic oscillator equation with a driving term with which the L-W field acts on it. The strength of the driving term is proportional to the coupling constant $c$ in the Lagrangian. The general solution of the inhomogeneous equation for $u$ is the general solution of the homogeneous equation plus a particular solution of the inhomogeneous equation whose amplitude is proportional to $c$ and which can be determined using the Green function method (see later). The homogeneous solution for $u$ is oscillating with frequency $k$.

For large wavelength, i.e. $\frac{k}{\eta} \ll M'_A$, the solutions for $g$ behave like a combination of Bessel functions:

$$g(t) = \alpha t^{1/4} J\left(\frac{1}{4}, \frac{M_A t}{\sqrt{1+4c^2}}\right) + \beta t^{1/4} Y\left(\frac{1}{4}, \frac{M_A t}{\sqrt{1+4c^2}}\right), \tag{26}$$

where $\alpha$ and $\beta$ are constants that can be determined using the initial conditions and $J$ and $Y$ are, respectively, the Bessel functions of order $\frac{1}{4}$ of the first and the second kind.

A more physical way of understanding the behavior is to rewrite the solutions in the asymptotic limits. For large values of $t$ and for $M'_A t \gg |\frac{1}{16} - 1|$, the L-W gauge field oscillates with a frequency corresponding to the mass of the L-W gauge field, $M'_A$. Indeed, in this case,

$$J\left(\frac{1}{4}, M'_A t\right) \approx \sqrt{\frac{2}{\pi M'_A t}} \cos\left(M'_A t - \frac{3\pi}{8}\right) \tag{27}$$

$$Y\left(\frac{1}{4}, M'_A t\right) \approx \sqrt{\frac{2}{\pi M'_A t}} \sin\left(M'_A t - \frac{3\pi}{8}\right). \tag{28}$$

Therefore, in this limit the L-W gauge field scales like $g(t) \propto t^{-1/4} \sim a(t)^{-1/2}$ when we are in a radiation-dominated period, which we are during a certain time since the initial state is dominated by regular radiation.

To better understand the behavior of the solutions in the small $k$ limit and at large times, we can rewrite the solution using powers of the scale factor. The two independent solutions are

$$g(t) \approx a(t)^{-(1/2)} e^{\pm \int 1/2 \sqrt{H(t)^2 - \frac{4M_A^2}{1+4c^2}} dt}, \tag{29}$$

though this expression is valid only when the square root term in the exponential is approximately constant. Choosing the initial time $t_i$ such that $\frac{M_A^2}{1+4c^2} \gg H(t_i)^2$, we see that this inequality stays valid only a finite period of time since $H(t_i)$ increases with time in a radiation phase of a contracting universe. We immediately get $g(t) \propto a(t)^{-1/2} \cos(\frac{M_A}{\sqrt{1+4c^2}} t)$ which is in agreement with the behavior we found using asymptotic values of the Bessel functions.

In the opposite case, when $t$ is close to 0 (and we are still considering large wave numbers), the asymptotic forms of the Bessel functions of first and second kind scale as a power of $t$:

$$t^{1/4} J\left(\frac{1}{4}, \frac{M_A t}{\sqrt{1+4c^2}}\right) = \frac{2^{5/4}(\frac{M_A}{\sqrt{1+4c^2}})^{1/4}\Gamma(\frac{3}{4})\sqrt{t}}{\pi} + o(t^2) \tag{30}$$

$$\begin{aligned} t^{1/4} Y\left(\frac{1}{4}, \frac{M_A t}{\sqrt{1+4c^2}}\right) = & \frac{-2^{3/4}}{(\frac{M_A}{\sqrt{1+4c^2}})^{1/4}\Gamma(\frac{3}{4})} \\ & + \frac{2^{3/4}(\frac{M_A}{\sqrt{1+4c^2}})^{1/4}\Gamma(\frac{3}{4})\sqrt{t}}{\pi} \\ & + \frac{1}{3}\frac{2^{3/4}M_A^2 t^2}{(\frac{M_A}{\sqrt{1+4c^2}})^{1/4}(1+4c^2)\Gamma(\frac{3}{4})} \\ & + \mathcal{O}(t^2). \end{aligned} \tag{31}$$

If we choose the amplitude of the two Bessel functions to be equal and opposite in (26), we get a cancellation of the square root term in $g(t)$ and thus the L-W gauge field scales as $g(t) \approx C_3 - C_4 t^2 + o(t^2)$. In the general case we get $g(t) \approx C_3 + C_5\sqrt{t}$ where $C_3$ and $C_5$ are constants.

Note that the closer we get to $t = 0$, less and less modes will satisfy the condition $k \ll |\eta| \frac{M_A}{\sqrt{1+4c^2}}$. Instead, they will evolve into the large wave-number regime discussed at the beginning of this subsection. They will oscillate and behave exactly as normal radiation.

We note that since g(t) is just oscillating, its effect on the normal field will decrease with time in a contracting phase as the source will scale as $a(t)^2 \sim t$ in a radiation-dominated era and time runs from $-\infty$ to 0 in the contracting phase.

### B. Scaling of the energy densities

The energy densities for each type of radiation can be rewritten in terms of $f$, $g$ and their derivatives for each mode $k$ by averaging $<\cos(kz)^2>$ over the $z$-direction:

$$\rho_A(t,k) = \frac{1}{4a^2}\left[\left(\frac{k}{a}\right)^2 f(t)^2 + \dot{f}(t)^2\right] \tag{32}$$

$$\rho_{\tilde{A}}(t,k) = -\frac{1}{4a^2}\left[\left(\left(\frac{k}{a}\right)^2 + \frac{M_{\tilde{A}}^2}{2}\right)g(t)^2 + \dot{g}(t)^2\right] \tag{33}$$

$$\rho_{A-\tilde{A}}(t,k) = -\frac{c}{a^2}\left[\left(\frac{k}{a}\right)^2 f(t)g(t) + \dot{f}(t)\dot{g}(t)\right]. \tag{34}$$

Rewriting this in term of conformal time, $\eta$, we get

$$\rho_A(\eta,k) = \frac{1}{4a(\eta)^4}[u'(\eta)^2 + k^2u(\eta)^2] \tag{35}$$

$$\rho_{\tilde{A}}(\eta,k) = \frac{-1}{4a(\eta)^4}\left[v'(\eta)^2 + \left[k^2 + \frac{M_{\tilde{A}}^2}{2}a(\eta)^2\right]v(\eta)^2\right] \tag{36}$$

$$\rho_{A-\tilde{A}}(\eta,k) = \frac{-c}{a(\eta)^4}[u'(\eta)v'(\eta) + k^2u(\eta)v(\eta)]. \tag{37}$$

In the absence of coupling between the two fields the solutions for $u$ correspond to undamped oscillations. Hence, the energy density of the regular radiation field scales as $a^{-4}$ as we know it must. The contribution of all short wavelength modes to the L-W energy density also scales as $a^{-4}$ since for these modes $v$ is oscillating with constant amplitude. The coefficient is negative as expected for a ghost field. The third energy density, that due to interactions, also scales as $a^{-4}$ for short wavelengths.

The contribution of long wavelength modes to the energy density of the L-W field and to the interaction energy density scale as $a^{-p}$ with a power $p$ which is smaller than 4. For large times, the power $p$ is 3 in the energy density for the L-W field, i.e. a scaling like that of nonrelativistic matter. Close to $t = 0$ the power changes to $p = 2$. This can be seen most clearly from (33) and from the scalings of $g(t)$ derived earlier.

Hence, we conclude that in the absence of coupling between the two fields (i.e. for $c = 0$), the energy density in the regular radiation field will dominate throughout the contracting phase if it initially dominates, and hence no cosmological bounce will occur. In fact, for temperatures $T < M'_A$, modes of $v$ with values of $k$ close to the peak of the thermal distribution scale as matter. Hence, the ratio of the energy density in the L-W field to the energy density in the regular radiation field decreases which renders it even more difficult to obtain a bounce. Once $T > M'_A$, the energy densities in both fields scale as radiation.

### C. Solution for the regular radiation field

We now consider the evolution of the regular radiation field in the presence of a nonvanishing coupling with the L-W radiation field. Our starting point is the set of equations of motion (21) and (22). From (22) it follows that the ghost field $v$ evolves independently. In turn, it influences the evolution of the regular radiation field $u$ as a source term. We expect the coupling constant $c$ to be small.

First, we show that the correction to the energy density in the presence of nonvanishing coupling is very small, namely, of order $c^2$. We observe that if we turn on the coupling, the following is a solution of (21):

$$u(\eta)_{c\neq 0} = u(\eta)_{c=0} + 2cv(\eta). \tag{38}$$

Inserting this into $\rho_A(k,\eta)$ [see (35)] yields

$$\rho_{Ac\neq 0} = \rho_{Ac=0} - 4c^2\left(\rho_{\tilde{A}} + \frac{M_{\tilde{A}}^2}{4}a(\eta)^{-2}v(\eta)^2\right) + \frac{c}{a(\eta)^4}[u'(\eta)v'(\eta) + k^2u(\eta)v(\eta)]. \tag{39}$$

Note that $\rho_{\tilde{A}}$ and $v$ stay the same when we turn the coupling on. We also have a change in the expression for the coupling term in the energy density since it also depends on u:

$$\rho_{A-\tilde{A}c\neq 0} = -\frac{c}{a(\eta)^4}[u'(\eta)v'(\eta) + k^2u(\eta)v(\eta)] - \frac{2c^2}{a(\eta)^4}[v'(\eta)^2 + k^2v(\eta)^2]. \tag{40}$$

The total energy density when the coupling is turned on is

$$\begin{aligned}\rho_{\mathrm{tot}\,c\neq 0} &= \rho_{A\,c\neq 0} + \rho_{\tilde{A}} + \rho_{A-\tilde{A}} \\ &= \rho_{A\,c=0} + (1+4c^2)\rho_{\tilde{A}} - c^2M_{\tilde{A}}^2a(\eta)^{-2}v(\eta)^2\end{aligned} \tag{41}$$

This looks very much like the total energy we had before adding any coupling ($\rho_{\mathrm{tot}\,c=0} = \rho_{A\,c=0} + \rho_{A-\tilde{A}}$) but with two correction terms of order $c^2$. Both correction terms appear to decrease the total energy density (recall that $\rho_{\tilde{A}}$ is negative). The second correction term [the last term in (41)], however, increases less fast in a contracting background than the other terms, and the first correction term corresponds to a small time-independent renormalization of the energy density in the L-W field. Thus, it appears that if the energy density of the regular radiation field dominates initially, then it will forever and no bounce will occur. In the following we will confirm this conclusion by means of an analysis which compares solutions with and without coupling with the same initial conditions.

The evolution of $u$ in the presence of the coupling with $v$ can be determined using the Green function method. The general solution $u(\eta)$ of (21) is the sum of the solution $u_0(\eta)$ of the homogeneous equation which solves the same initial conditions as $u$ and the particular solution $\delta u(\eta)$

with vanishing initial conditions. The particular solution is given by

$$\delta u(\eta) = u_1(\eta)\int_{\eta_I}^{\eta} d\eta' \epsilon(\eta')u_2(\eta')s(\eta') - u_2(\eta)\int_{\eta_I}^{\eta} d\eta' \epsilon(\eta')u_1(\eta')s(\eta'), \quad (42)$$

where $u_1$ and $u_2$ are two independent solutions of the homogeneous equation, $\eta_I$ is the initial conformal time, $\epsilon(\eta)$ is the Wronskian

$$\epsilon(\eta) = (u_1' u_2 - u_2' u_1)^{-1}, \quad (43)$$

and $s(\eta)$ is the source inhomogeneity

$$s(\eta) = -a^2 \frac{2c}{1+4c^2} M_A^2 v(\eta). \quad (44)$$

In our case, the solutions of the homogeneous equation are $u_1(\eta) = \cos(k\eta)$ and $u_2(\eta) = \sin(k\eta)$ and the Wronskian is $\epsilon(\eta) = -1/k$.

Since it is less hard to imagine a bounce once the energy densities in both fields scale as radiation, and since to study the possibility of a bounce it is important to investigate the dynamics at very high temperatures when the bulk of the Fourier modes of both fields scale as radiation, we will consider in the following Fourier modes for which $v$ is oscillating.

We will now show that the sign of the energy transfer between the two fields depends on the relative phase between the oscillations of $u_0(\eta)$ and $v(\eta)$. We are interested in conformal time scales long compared to the oscillation time $k^{-1}$ but short compared to the cosmological time. Hence, we can approximate the scale factor in (42) by a constant. A simple calculation then shows that if we choose phases for which $v(\eta) = v_0 \sin(k\eta)$ and $u_0 = \mathcal{A}\cos(k\eta)$ then

$$u(\eta) \simeq \left(\mathcal{A} - \frac{cv_0}{1+4c^2}\frac{M_A^2}{4k}(\eta - \eta_I)\right)\cos(k\eta). \quad (45)$$

For a coupling constant $c > 0$ this choice of phase hence leads to draining of energy density from the regular radiation field. On the other hand, the phase choice $v(\eta) = v_0 \cos(k\eta)$ and $u_0(\eta) = \mathcal{A}\sin(k\eta)$ leads to

$$u(\eta) \simeq \left(\mathcal{A} + \frac{cv_0}{1+4c^2}\frac{M_A^2}{4k}(\eta - \eta_I)\right)\sin(k\eta) \quad (46)$$

and hence to a relative increase in the energy density of the regular radiation field.

We need to consider the full phase space of Fourier modes. Even if we only consider modes with fixed value of $k$ given by the peak of the thermal distribution, we must sum over the different angles. Since there is no reason why the phases for different Fourier modes should be the same, we must take the expectation value of the energy transfer averaged over all possible choices of phases. This average obviously vanishes. Hence, we conclude that without unnatural fine tuning of phases it is not possible to obtain the required draining of the energy density from $u$ to $v$.

## VI. CONCLUSIONS AND DISCUSSION

If the scalar field sector of the Lee-Wick standard model is coupled to Einstein gravity, then—in the absence of anisotropic stress—it is known that a bouncing cosmology can be realized. Since the energy density in radiation increases at a faster rate in a contracting universe compared to that of nonrelativistic matter, the cosmological bounce is unstable to the addition of radiation to the initial conditions early in the contracting phase. However, one may entertain the hope that the presence of the ghost radiation which is present in the Lee-Wick model might allow a bounce to occur in analogy to how the presence of ghost scalar field matter is responsible for the bounce in the scalar field Lee-Wick model.

For a Lee-Wick radiation bounce to occur, either the energy density of the ghost radiation would have to increase faster intrinsically than that of regular radiation, or there would have to be a mechanism which drains energy density from the regular radiation sector to the ghost sector. We have shown that neither happens, unless the initial phases of regular and ghost radiation are tuned in a very special way. Thus, we have shown that in the Lee-Wick standard model, the presence of radiation prevents a cosmological bounce from occurring.

The methods we have used in this paper could be applied to other proposals to obtain a bouncing cosmology by modifying the matter sector. Rather generically, one needs to worry whether any given proposal is robust towards the addition of radiative matter. The stability can be studied using the methods we have developed. Whether a channel to effectively drain energy density from radiation to ghost matter will exist may depend rather sensitively on the specific model. Here, we have shown that in the Lee-Wick standard model this does not happen. The same Green function method could be used to study the energy transfer in other models.

Cosmologies in which the bounce is induced by extra terms in the gravitational sector such as in the ''nonsingular universe construction'' [18], the model of [19] or the Horava-Lifshitz bounce [20] are more likely to be robust against the addition of matter. Specifically, the constructions of [18,19] are based on theories which are asymptotically free in the sense that at high curvatures the coupling of any kind of matter to gravity goes to zero. This means that a bounce will not be affected by adding radiative matter. In Horava-Lifshitz gravity, there are higher spatial derivative gravitational terms which act as ghost matter scaling as $a^{-4}$ and $a^{-6}$. The latter are present if we go beyond the ''detailed balance case'' and we allow for spatial curvature. In this case, once again radiative matter can be added without preventing a cosmological bounce.

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