Inflationary cosmology as a probe of primordial quantum mechanics

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We show that inflationary cosmology may be used to test the statistical predictions of quantum theory at very short distances and at very early times. Hidden-variables theories, such as the pilot-wave theory of de Broglie and Bohm, allow the existence of vacuum states with nonstandard field fluctuations (''quantum nonequilibrium''). We show that inflationary expansion can transfer microscopic nonequilibrium to macroscopic scales, resulting in anomalous power spectra for the cosmic microwave background. The conclusions depend only weakly on the details of the de Broglie-Bohm dynamics. We discuss, in particular, the nonequilibrium breaking of scale invariance for the primordial (scalar) power spectrum. We also show how nonequilibrium can generate primordial perturbations with nonrandom phases and intermode correlations (primordial non-Gaussianity). We address the possibility of a low-power anomaly at large angular scales, and show how it might arise from a nonequilibrium suppression of quantum noise. Recent observations are used to set an approximate bound on violations of quantum theory in the early Universe.

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I. INTRODUCTION

According to inflationary cosmology [1], the early Universe underwent a period of exponential expansion, during which microscopic quantum fluctuations were stretched to macroscopic scales. The resulting (classical) primordial perturbations seem to be of the form required to explain the observed temperature anisotropy in the CMB, and are widely believed to have seeded the formation of large-scale structure generally. In this scenario, precision CMB measurements today can provide information about—and tests of—microscopic physics in the very early Universe. For this reason, many workers have turned to inflationary CMB predictions in the hope that these will provide a ''cosmic microscope'' with which to probe high-energy physics at very short distances and at very early times. However, if the primordial perturbations do indeed have a quantum origin, then inflationary CMB predictions will also be sensitive to the structure of quantum theory itself, as well as to that of high-energy physics. Therefore, inflationary cosmology and CMB measurements may equally be used to probe possible deformations of quantum theory at very short distances and at very early times.

In a typical inflationary scenario, at very early times the cosmological scale factor $a(t)$ undergoes a period of approximately exponential growth, $a \propto e^{Ht}$ with $H \approx$ const. During inflation, field perturbation modes have physical wavelengths $\lambda_{\rm phys} = a(t)\lambda \propto e^{Ht}$. (As usual, $\lambda = 2\pi/k$ is the wavelength today—the ''comoving wavelength''—and we set the scale factor today to be $a_0 = 1$.) A mode ''exits'' the Hubble radius H^{-1} when $\lambda_{\rm phys} \gtrsim H^{-1}$, at a time $t_{\rm exit} = t_{\rm exit}(k)$ [which can be defined by $2\pi a(t_{\rm exit})/k \sim H^{-1}$ or by $a(t_{\rm exit})/k \sim H^{-1}$]. Soon after $t_{\rm exit}(k)$, the perturbation ''freezes'' and becomes part of the primordial spectrum. After inflation ends, physical wavelengths $\lambda_{\rm phys} \propto a$ grow more slowly than the Hubble radius $H^{-1} \equiv a/\dot{a} \propto t$ (where $a \propto t^{1/2}$ or $t^{2/3}$, for radiation-dominated or matter-dominated expansion, respectively). Mode ''reentry'' occurs at a time $t_{\rm enter}(k)$ when $\lambda_{\rm phys} \lesssim H^{-1}$, after which the (formerly frozen) perturbations begin to grow, eventually giving rise to anisotropies in the CMB and to large-scale structure [2].

While there are many uncertainties surrounding the details of inflationary cosmology, there is a broad consensus that the formation of (frozen) primordial perturbations takes place when the corresponding physical wavelengths $\lambda_{\rm phys} \gtrsim H^{-1}$ are truly microscopic. Further, because of the huge expansion during the inflationary phase, the relevant modes will have had very short physical wavelengths, $\lambda_{\rm phys} \ll H^{-1}$, at the onset of inflation (where the shorter the wavelength, the later the time at which the mode exits the Hubble radius during the inflationary phase). Indeed, it appears that even modes with initial $\lambda_{\rm phys} \lesssim l_{\rm P}$, where $l_{\rm P} \approx 10^{-33}$ cm is the Planck length, may contribute to the primordial spectrum [3]. Clearly, if inflation did indeed occur, then precision measurements of the CMB (and of large-scale structure generally) can probe physics at very early times and at very short distances (possibly even at distances $\lesssim l_P$, to the extent that this might be meaningful).

A number of possible deformations of high-energy physics have been considered in an inflationary context. These include (a) modified dispersion relations (which may be introduced *ad hoc* [3,4], or which may be motivated by quantum-gravitational deformations of Lorentz invariance

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[5] or by quantum cosmology [6]), (b) an ultraviolet cutoff coming from a fundamental length associated with deformed uncertainty relations (possibly associated with quantum gravity or string theory) [7], and (c) short-distance noncommutative geometry [8]. Some authors consider that changes in physics at very high energies may have an effective description in terms of different choices of quantum vacuum [9] (for a review, see Ref. [10]). Excited, nonvacuum states have also been considered [11]. However, while it is generally agreed that inflationary primordial perturbations have a quantum origin, effects on the CMB arising from possible deformations of quantum theory itself are not usually considered. (By "quantum theory" we mean, essentially, the representation of physical states in Hilbert space, with unitary evolution, and with probabilities given by the Born rule.) An exception is Perez *et al.* [12], who discuss how predictions for the CMB could be affected by a hypothetical dynamical collapse of the wave function, a proposal that is motivated by the quantum measurement problem (which seems especially severe in a cosmological setting).

Despite the widespread reluctance to consider deformations of quantum theory itself, there is in fact no good scientific reason for believing that the structure of standard quantum theory is "final," or that the predictions of quantum theory will continue to hold under all conditions. The following arguments are often presented as evidence for the finality of quantum theory: that it provides a universal framework applicable to all systems independently of their composition (electrons, fields, atoms, etc.); that it is based on simple, elegant axioms; that it provides the basis for powerful new technologies; and, of course, that in all cases so far it agrees with experiment. However, arguments similar to these could have been made in the 18th and 19th centuries concerning the status of Newtonian mechanics: at that time, Newtonian mechanics seemed to provide a universal framework applicable to all systems independently of their composition (rocks, fluids, planets, etc.); it was based on simple, elegant axioms (Newton's three laws of motion); it provided the basis for powerful new technologies; and it agreed with all experiments performed to date. And yet, we now know that Newtonian mechanics is in fact merely approximate and emergent, arising from a classical and low-energy limit of relativistic quantum field theory. Of course, that Newtonian mechanics proved to be approximate and emergent does not imply that quantum theory will necessarily turn out likewise. However, the case of Newtonian mechanics does suggest that the above (frequently cited) arguments for the finality of a physical theory are not reliable.

The ultimate test of the domain of validity of a scientific theory is, of course, experiment. No matter how well a theory has been tested in the past, it will always be subject to possible modification in the future, in hitherto untested regimes. Therefore, in order to expand our knowledge of the domain of validity of any given theory, it is necessary to subject it to ever more stringent tests in ever more extreme conditions. To accomplish this, it is helpful to have a "foil" against which to test the theory in question—that is, to have a model reducing to the given theory only in some limit.

In the case of quantum theory, a number of alternatives or foils might be considered. Models with a nonlinear evolution or with a dynamical collapse of the wave function have, for example, been subjected to considerable experimental scrutiny. In this paper, we focus on a different possibility: that of nonequilibrium hidden variables [13–26].

A deterministic hidden-variables theory, such as the pilot-wave theory of de Broglie [27,28] and Bohm [29], agrees with quantum theory only in the limit in which the hidden parameters have a particular "quantum equilibrium" distribution [13–15,18,24,26]. A foil against which to test quantum theory may then be obtained from such a theory by allowing the hidden variables to have a nonstandard or "quantum nonequilibrium" distribution, resulting in statistical predictions that deviate from those of quantum theory [25].[1] Such possible corrections to quantum theory will be explored here, in the context of inflationary cosmology, where it will be shown how CMB observations may be used to set bounds on the presence of quantum nonequilibrium at very short distances and very early times.

If anomalies are observed in the CMB, one must of course ask if they are caused by corrections to quantum theory or by some other effect. (For example, the quantum state during the inflationary phase might differ significantly from the standard Bunch-Davies vacuum [11].) A similar issue arises for other proposed corrections to standard physics in the early Universe. Ideally, one would like to find a unique signature that could not be predicted by any quantum state compatible with inflation. In practice, one would at least require a quantitative prediction of a deviation from standard results. The present paper focuses on showing that early quantum nonequilibrium—for a given (standard) quantum state—could have observable consequences for the CMB. We also sketch two scenarios that would lead to a specific prediction: for example, deviations for wavelengths larger than a certain (predicted) infrared cutoff. But the full development of these scenarios, and the extraction of precise quantitative predictions from them, is left for future work.

In Sec. II, we review the notion of quantum nonequilibrium, in de Broglie-Bohm theory and in general (deterministic) hidden-variables theories, and we provide

[1]Note the clear distinction from the foils based on local hidden-variables models [30] or on a particular restricted class of nonlocal models [31]: such models disagree with the quantum predictions for *any* distribution (equilibrium or otherwise) of the hidden variables.

motivation for why quantum nonequilibrium might exist in the very early Universe. In Sec. III, we develop pilot-wave field theory on an expanding space, and we write down equations for the time evolution of arbitrary (nonequilibrium) distributions in an expanding universe. In Sec. IV, we discuss two scenarios whereby quantum nonequilibrium could exist during inflation: first, nonequilibrium for large-wavelength modes could survive from a preinflationary era, since under the right conditions relaxation can be suppressed at large wavelengths on an expanding space; second, nonequilibrium might be generated by novel gravitational processes at the Planck scale. In Sec. V, we review the standard theory of CMB temperature anisotropies, their explanation in terms of primordial curvature perturbations, and the production of the latter by inflaton fluctuations during inflation. In Sec. VI, we calculate the time evolution of quantum nonequilibrium in the Bunch-Davies vacuum on de Sitter space, and we show that the width $D_k(t)$ of the nonequilibrium distribution for each mode of wave number k remains in a fixed ratio $\sqrt{\xi(k)} \equiv D_k(t)/\Delta_k(t)$ with the equilibrium (quantum) width $\Delta_k(t)$. In Sec. VII, we show how the power spectrum for the primordial curvature perturbations is corrected by the factor $\xi(k)$. Some general remarks are made in Sec. VIII, concerning the transfer of microscopic nonequilibrium to cosmological scales, the effective quantum measurement of the inflaton field during the "quantum-to-classical" transition, and the weak dependence of our results on the details of pilot-wave dynamics. In Sec. IX, we use current CMB data to derive an approximate bound on quantum nonequilibrium during inflation; specifically, under certain assumptions, we show that the hidden-variable relative entropy $S_{\rm hv}(k)$ (which measures the difference between nonequilibrium and quantum probabilities for a mode of wave number k) satisfies the approximate bound $|S_{\rm hv}(k)| \lesssim 10^{-2}$ for values of k close to $k_0 = 0.002\ \mathrm{Mpc}^{-1}$. In Sec. X, we consider the possibility of a low-power anomaly at large angular scales, and we discuss how it might arise from a nonequilibrium suppression of quantum noise [$\xi(k) < 1$] in certain regions of k space. In Sec. XI, we show how nonequilibrium can generate primordial perturbations with nonrandom phases and intermode correlations. Our conclusions are given in Sec. XII.

II. QUANTUM EQUILIBRIUM AND QUANTUM NONEQUILIBRIUM

The notion of quantum nonequilibrium was first discussed in detail in terms of de Broglie-Bohm theory [13–15], and was later generalized to include all (deterministic) hidden-variables theories [18,19,21,24].

Consider, for example, the very simple case of de Broglie-Bohm theory applied to a single nonrelativistic particle with mass m and no spin. The wave function $\psi = |\psi|e^{iS}$ (with units $\hbar = 1$) acts as a "pilot wave" that determines the velocity of the particle according to de Broglie's guidance equation $d\mathbf{x}/dt = (1/m)\nabla S$ [or $d\mathbf{x}/dt = (1/m)\mathrm{Im}(\nabla\psi/\psi)$]—an equation that determines the trajectory $\mathbf{x}(t)$ of the particle, given the initial position $\mathbf{x}(0)$ [assuming that $\psi = \psi(\mathbf{x}, t)$ is known for all $\mathbf{x}$ and t, by solving the Schrödinger equation with a given initial wave function $\psi(\mathbf{x}, 0)$]. Let ψ propagate in free space, then strike a screen with two slits, and finally strike a backstop where the particle is detected. The pilot wave undergoes interference upon traversing the screen. The location $\mathbf{x}(t)$ of the particle at any time t is determined (in principle) by the initial value $\mathbf{x}(0)$; in particular, where the particle lands on the backstop is determined by $\mathbf{x}(0)$. Because the velocity field $(1/m)\nabla S$ is equal to the usual quantum probability current $\mathbf{j}$ divided by the usual quantum probability density $|\psi|^2$, it follows trivially that an initial ensemble of particles guided by the same pilot wave ψ and with positions $\mathbf{x}(0)$ distributed according to the equilibrium rule $\rho(\mathbf{x}, 0) = |\psi(\mathbf{x}, 0)|^2$ will evolve into an equilibrium distribution $\rho(\mathbf{x}, t) = |\psi(\mathbf{x}, t)|^2$ at later times, resulting in the usual quantum distribution of particles at the backstop (showing the usual interference pattern). On the other hand, it is easy to see that, in general, an initial "nonequilibrium" ensemble with distribution $\rho(\mathbf{x}, 0) \neq |\psi(\mathbf{x}, 0)|^2$ results in a nonquantum distribution $\rho(\mathbf{x}, t) \neq |\psi(\mathbf{x}, t)|^2$ at the backstop. [For example, in the absence of a rapid divergence of neighboring trajectories, if $\rho(\mathbf{x}, 0)$ is concentrated around a single initial point $\mathbf{x}(0)$, then $\rho(\mathbf{x}, t)$ will be concentrated around a single trajectory $\mathbf{x}(t)$, and the usual interference pattern will be replaced by a single localized spot.]

The pilot-wave theory of a many-body system was first proposed by de Broglie at the 1927 Solvay conference [26–28]. For a system of n (nonrelativistic) particles with positions $\mathbf{x}_i(t)$ and masses m_i, de Broglie's law of motion takes the form

$$\frac{d\mathbf{x}_i}{dt} = \frac{1}{m_i}\,\mathrm{Im}\,\frac{\nabla_i\psi}{\psi} = \frac{\nabla_i S}{m_i}, \tag{1}$$

where $\psi = \psi(\mathbf{x}_1, \ldots, \mathbf{x}_n, t)$ is the many-body wave function. De Broglie regarded (1) as expressing a unification of the principles of Maupertuis and Fermat, resulting in a new form of dynamics based on velocities [28].

Writing the total configuration as $q = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)$, it is again readily shown that for an ensemble of systems guided by the same wave ψ and with configurations distributed according to $\rho(q, 0) = |\psi(q, 0)|^2$, the distribution of configurations at later times will be $\rho(q, t) = |\psi(q, t)|^2$.

As shown in detail by Bohm in 1952 [29], the above "de Broglian" dynamics may be applied to the process of quantum measurement itself, by treating the system being measured together with the measuring apparatus as a single many-body system of n particles. The total configuration $q = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)$ then defines the "pointer position" of the apparatus, as well as defining the configuration of the measured system. For each run of a quantum experiment, the evolution is deterministic: the initial conditions $q(0)$, $\psi(q, 0)$ determine the final conditions $q(t)$, $\psi(q, t)$. Over an ensemble of initial configurations $q(0)$ guided by the same wave function ψ, if we assume the initial quantum equilibrium condition $\rho(q, 0) = |\psi(q, 0)|^2$, then the statistical distribution of pointer positions at later times will agree with the predictions of quantum theory.

Schematically, during a standard quantum measurement, the initial packet $\psi(q, 0)$ on configuration space evolves into a superposition $\psi(q, t) = \sum_n c_n \psi_n(q, t)$ of terms $\psi_n(q, t)$ that separate with respect to the pointer degrees of freedom [that is, distinct $\psi_n(q, t)$ have negligible overlap with respect to the pointer degrees of freedom]. The final configuration $q(t)$ can then be in (the support of) only one "branch" of the superposition, say $\psi_i(q, t)$. For an initial equilibrium ensemble, it is readily shown that this occurs with probability $|c_i|^2$, in accordance with the Born rule. Further, inspection of de Broglie's velocity law (1) shows that the motion of $q(t)$ will then be affected by $\psi_i(q, t)$ alone, resulting in an effective "reduction" of the wave function.

As in the simple example of a single particle undergoing interference, for a general quantum measurement the distribution of outcomes depends crucially on the assumed initial distribution $\rho(q, 0)$ of initial configurations $q(0)$. For a nonequilibrium ensemble, $\rho(q, 0) \neq |\psi(q, 0)|^2$, the distribution of quantum measurement outcomes will generally disagree with the predictions of quantum theory (assuming that relaxation to equilibrium has not taken place in the meantime—see below).

De Broglie's dynamics may be readily applied to fields, where (say for a scalar field ϕ) the motion of the field configuration $q(t) = \phi(\mathbf{x}, t)$ is determined by the Schrödinger wave functional $\Psi[\phi(\mathbf{x}), t]$. Indeed, for any system with configuration q and Hamiltonian $\hat{H}$, as long as the Schrödinger equation $i\partial\psi/\partial t = \hat{H}\psi$ for $\psi(q, t)$ has an associated current $j = j[\psi] = j(q, t)$ in configuration space, obeying a continuity equation

$$\frac{\partial|\psi|^2}{\partial t} + \nabla \cdot j = 0$$

(with $\nabla \equiv \partial/\partial q$), one may define a de Broglian or pilot-wave dynamics for the system, by introducing the configuration-space velocity field

$$\frac{dq}{dt} = \frac{j}{|\psi|^2}. \tag{2}$$

Such a velocity field exists, in fact, whenever $\hat{H}$ is given by a differential operator [32]. (In this dynamics, ψ is viewed as a physical field or pilot wave in configuration space, guiding the motion of an individual system. Note that ψ has no *a priori* connection with probabilities. Furthermore, because ψ is not an ordinary field in spacetime, it does not itself carry an energy or momentum density.)

For an ensemble of systems, each with the same wave function $\psi(q, t)$, we may consider an arbitrary initial distribution $\rho(q, 0) \neq |\psi(q, 0)|^2$, whose time evolution $\rho(q, t)$ is determined by the de Broglian velocity field $\dot{q}$ in accordance with the continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla \cdot (\rho\dot{q}) = 0.$$

Because $|\psi|^2$ obeys the same equation, an initial distribution $\rho(q, 0) = |\psi(q, 0)|^2$ evolves into $\rho(q, t) = |\psi(q, t)|^2$. This is the state of quantum equilibrium, but the theory clearly allows one (in principle) to consider any initial distribution—just as classical mechanics allows one to consider any initial distribution departing from thermal equilibrium.

It is worth emphasizing that pilot-wave theory is a radically new form of dynamics, very different from classical (Newtonian or Hamiltonian) mechanics. This was in fact de Broglie's original point of view, but it was unfortunately obscured by Bohm's pseudo-Newtonian reformulation of the theory in terms of a law for acceleration (involving a "quantum potential") [28].

Pilot-wave dynamics is grounded in configuration space, where ψ propagates. While the dynamics is local in configuration space, it is highly nonlocal when projected down to 3-space (as required by Bell's theorem). For example, if a particle with position $\mathbf{x}_1$ is entangled with a particle with position $\mathbf{x}_2$, then the velocity $\dot{\mathbf{x}}_1$ depends instantaneously on $\mathbf{x}_2$ (no matter how remote $\mathbf{x}_2$ may be from $\mathbf{x}_1$), and changing the local Hamiltonian at $\mathbf{x}_2$ is found to have an instantaneous effect on the distant velocity $\dot{\mathbf{x}}_1$. Such nonlocal effects are erased upon averaging over an equilibrium ensemble $\rho = |\psi|^2$; but in nonequilibrium, $\rho \neq |\psi|^2$, there are (in general) nonlocal signals at the statistical level [14,15], suggesting the existence of an underlying preferred foliation of spacetime [33].

Pilot-wave dynamics—as originally formulated by de Broglie—is also first order in time in configuration space (rather than in phase space): the fundamental law of motion determines velocities, not accelerations. This last feature has important implications for the associated kinematics: for particles, the natural state of motion is rest (instead of uniform motion in a straight line), and there is indeed a natural preferred foliation of spacetime with a fundamental time parameter t (consistent with the fundamental nonlocality of the theory) [34].

Quantum nonequilibrium may be considered, not only in pilot-wave theory, but also in any deterministic hidden-variables theory [18,19,21,24]. For any such theory, given macroscopic experimental settings M, there is a mapping $\omega = \omega(M, \lambda)$ from initial hidden variables λ to final outcomes ω of quantum measurements. There is also a quantum equilibrium probability measure $\rho_{\mathrm{QT}}(\lambda)$, defined on the set of hidden variables, that yields quantum probabilities $P_{\mathrm{QT}}(\omega)$ for the outcomes. [In the case of pilot-wave theory, $\rho_{\rm QT}(\lambda)$ is given by $\rho = |\psi|^2$.] Once such a theory has been constructed, one may consider arbitrary "nonequilibrium" probability measures $\rho(\lambda) \neq \rho_{\rm QT}(\lambda)$, resulting in outcome probabilities $P(\omega) \neq P_{\rm QT}(\omega)$ that depart from the predictions of quantum theory.

In this paper we shall be studying quantum nonequilibrium in the context of inflationary cosmology, using the pilot-wave theory of fields as a concrete example. However, we emphasize that similar studies could be made in any deterministic hidden-variables theory, simply by making the replacement $\rho_{\rm QT}(\lambda) \rightarrow \rho(\lambda)$.

At present, pilot-wave theory is the only deterministic hidden-variables theory of broad scope that we possess, though some attempts have been made to construct alternative theories. For example, in the 1980s, Smolin attempted to construct a deterministic hidden-variables theory of an N-body system, based on the classical Hamiltonian dynamics of a certain $N \times N$ matrix $M_{ij}(t)$, whose eigenvalues correspond to particle positions and whose off-diagonal elements correspond to nonlocal hidden variables associated with pairs of particles [35]. Adopting a classical action principle for the (deterministic) dynamics of the matrix, Smolin made a number of assumptions, including a statistical assumption to the effect that the coarse-grained evolution of the off-diagonal terms amounts to a Brownian motion. In the limit of a large number N of particles with masses m_i, it was shown from these assumptions that the particle positions also undergo a Brownian motion, that the ith particle current velocity $\mathbf{v}_i$ (the average of the mean forward and backward velocities) is given by a gradient, $\mathbf{v}_i = \boldsymbol{\nabla}_i S/m_i$, where S is a function on configuration space, and that the complex function $\psi \equiv \sqrt{\rho} e^{iS}$ (where ρ is the particle probability distribution on configuration space) satisfies the Schrödinger equation for a many-body nonrelativistic system. Smolin's strategy was to show that his assumptions led, in the limit of large N, to the basic postulates of Nelson's stochastic mechanics [36]. As was already known, in Nelson's theory—which is based on a form of Brownian motion subject to special conditions, including the condition that $\mathbf{v}_i = \boldsymbol{\nabla}_i S/m_i$ for some function S—the derived quantity $\psi \equiv \sqrt{\rho} e^{iS}$ indeed satisfies the Schrödinger equation.

More recently, a model similar to the above (though based on the bosonic part of the classical matrix models used in string and M theory) was again investigated by Smolin, with similar assumptions and results [37]. In Ref. [35], it had also been suggested that one might consider a model in which the off-diagonal matrix elements of $M_{ij}(t)$ are constant, with fluctuations in a local system arising from the nonlocal transmission of fluctuations from other particles in remote regions of space. This last model has recently been recast in terms of the dynamics of a graph with N nodes [38]: assuming that the edges of the graph do not evolve in time, the corresponding adjacency matrix is constant, and is taken to be the off-diagonal part of matrices $M_{ij}(t)$. Again, as in Smolin's original model, assumptions are made so as to arrive at Nelson's stochastic mechanics in some approximation.

However, while it is often claimed that Nelson's theory is empirically equivalent to quantum theory, unfortunately, as shown by Wallstrom [39], the two theories are in fact not equivalent, because Nelson's function S does not have the specific multivalued structure required for the phase of a single-valued (and continuous) complex field ψ. The Schrödinger equation is indeed derived, but only for the exceptional set of wave functions with no nodes, for which the circulation of $\boldsymbol{\nabla}_i S$ around all closed curves vanishes. Since almost all wave functions have nodal points (where $\psi = 0$), quantum theory cannot be derived from Nelson's theory, or from any model that leads to Nelson's theory. (Note that there is no such problem in pilot-wave theory, where ψ is regarded as a basic entity.)

Thus, as they stand, the deterministic models of Refs. [35,37,38] seem to yield derivations of Nelsonian mechanics, but not of quantum mechanics. Some basic element is missing. One must somehow ensure that the circulation of $\boldsymbol{\nabla}_i S$ around nodes of ρ can be nonzero but always restricted to integer multiples of 2π. (And if one wishes to derive the wave function, then of course one cannot simply assume at the outset that S is the phase of a complex-valued field.) Still, if some way were found to solve Wallstrom's phase problem, then such derivations of Nelsonian mechanics as an average over a certain statistical state could again be generalized to arbitrary statistical states, yielding nonequilibrium departures from quantum theory in the sense considered here.[2]

As another example, Adler [42] has constructed what appears to be a deterministic hidden-variables theory, in which the parameters λ are matrices with Grassmann (even and odd) valued elements, obeying a generalized form of classical Hamiltonian dynamics. The state of thermal equilibrium, defined in the usual way on phase space, is argued to lead (after some approximations) to a quantum-like phenomenology with a dynamical wave function collapse. The precise nature of Adler's theory seems to require further elucidation; but if it is indeed a hidden-variables theory in the sense meant here, then thermal nonequilibrium in Adler's theory should again correspond to quantum nonequilibrium.

[2]Smolin [40] has attempted to solve Wallstrom's phase problem by allowing discontinuous wave functions. However (even leaving aside the resulting divergences for expectation values of quantum observables such as kinetic energy), Smolin applies his prescription only to the case of a particle moving on a circle, which is too simple to capture the nature of the problem raised by Wallstrom. In higher dimensions—for example, even in two dimensions, and with just one node—allowing discontinuous wave functions results in an ill-defined (one-to-many) mapping from Nelsonian states to quantum states. For a full discussion, see Ref. [41].

In the author's view, because Hamiltonian dynamics is of second order in configuration space, it is not a natural framework for nonlocal theories with a preferred state of rest or preferred slicing of spacetime—unlike pilot-wave dynamics, which is first order in configuration space, and which therefore (as we have mentioned) provides a natural setting for such theories [34]. But even so, the above alternative theories based on Hamiltonian dynamics do illustrate that the idea of quantum nonequilibrium is a general one.

For the purposes of this paper, it suffices that there exists at least one model of quantum nonequilibrium, based on pilot-wave dynamics, that may serve as a foil against which to test quantum theory. To be able to provide quantitative bounds on violations of quantum theory in the early Universe is motivation enough to consider models with quantum nonequilibrium. Even so, before proceeding, let us briefly provide some further motivation for why quantum nonequilibrium might exist at very early times.

First, it has been shown that in pilot-wave theory the equilibrium state $\rho = |\psi|^2$ may be understood as arising from a process of relaxation that is analogous to classical thermal relaxation, where the former is defined on configuration space rather than on phase space. The difference between ρ and $|\psi|^2$ may be quantified by the H-function

$$H = \int dq \rho \ln(\rho/|\psi|^2) \tag{3}$$

(equal to minus the relative entropy of ρ with respect to $|\psi|^2$), which obeys a coarse-graining H-theorem analogous to the classical one, and where the minimum $H = 0$ corresponds to $\rho = |\psi|^2$ [13,15,17]. Further, numerical simulations for simple two-dimensional systems [23,43] show a remarkably efficient approach to equilibrium, with an approximately exponential decay of the coarse-grained H-function, $\bar{H}(t) \to 0$, and a corresponding coarse-grained relaxation $\bar{\rho} \to \overline{|\psi|^2}$ (assuming appropriate initial conditions for ρ and ψ).[3] Because all the systems we have access to (such as hydrogen atoms in the laboratory) have a long and violent astrophysical history, we would then *expect* to see quantum equilibrium in these systems. While it is logically possible, of course, that the Universe was simply born in a state of quantum equilibrium, it seems more natural to consider that the equilibrium we see today arose from relaxation processes in the remote past [16,17], in which case the very early Universe is the natural place to look for nonequilibrium phenomena.

Second, an appealing feature of this picture concerns the status of locality in physics. It may be shown that quantum nonequilibrium for entangled systems leads to nonlocal signals at the statistical level, in pilot-wave theory (as already mentioned) and indeed in any deterministic hidden-variables theory; while in equilibrium, the underlying nonlocal effects cancel out at the statistical level [14,15,18,19,26]. Locality is therefore a contingency (or emergent feature) of the equilibrium state. Similarly, standard uncertainty-principle limitations on measurements are also contingencies of equilibrium [14,15,20,24]. These results provide an explanation for the otherwise mysterious "conspiracy" in the foundations of current physics, according to which (roughly speaking) quantum noise and the uncertainty principle prevent us from using quantum nonlocality for practical nonlocal signaling. From the above perspective, this "conspiracy" is not part of the laws of physics, but merely a contingent feature of the equilibrium state (much as the inability to convert heat into work, in a state of global thermal equilibrium, is not a law of physics but a contingency of the state). On this view, quantum physics is merely the effective description of a particular state—just as, for example, the standard model of particle physics is merely the effective description of (perturbations around) a particular vacuum state (arising from spontaneous symmetry breaking). If one takes this view seriously, it suggests that nonequilibrium phenomena should exist somewhere (or some time) in our Universe. And again, the early Universe seems the natural place to look.

Quantum nonequilibrium at very early times may also be motivated by the cosmological horizon problem, which may be avoided by the explicit nonlocality associated with nonequilibrium [14–16,19]—see Sec. IVA.

Finally, if one takes de Broglie-Bohm theory seriously, one should take the possibility of nonequilibrium seriously as well, since it is only in nonequilibrium that the underlying details of the theory become visible (via measurements more accurate than those allowed by quantum theory [20,24]). If instead the Universe is always and everywhere in quantum equilibrium, the details of de Broglie-Bohm trajectories will be forever shielded from experimental tests, and de Broglie-Bohm theory itself would be unacceptable as a scientific theory.

For the above reasons, then, we are led to consider the hypothesis of quantum nonequilibrium at or close to the big bang [13–17,25,26]. It is the purpose of this paper to show that inflationary cosmology provides a means of testing this hypothesis, through precision measurements of the cosmic microwave background.

III. PILOT-WAVE FIELD THEORY ON EXPANDING SPACE

For simplicity we restrict ourselves to a flat metric,

$$d\tau^2 = dt^2 - a^2 d\mathbf{x}^2, \tag{4}$$

where again $a(t)$ is the scale factor, with Hubble parameter $H \equiv \dot{a}/a$. As is customary, we take $a_0 = 1$ today (at time

[3]The understanding of relaxation in pilot-wave theory is subject to the usual caveats—familiar from classical statistical mechanics—associated with initial conditions and time reversal. For detailed discussions of this point, see Refs. [15–17,23].

t_0), so that $|d\mathbf{x}|$ is a comoving distance (or proper distance today).

A free (minimally coupled) massless scalar field ϕ has a Lagrangian density $\mathcal{L} = \frac{1}{2} g^{1/2} \partial_\alpha \phi \partial^\alpha \phi$ or

$$\mathcal{L} = \tfrac{1}{2} a^3 \dot{\phi}^2 - \tfrac{1}{2} a (\nabla \phi)^2, \tag{5}$$

with an action $\int dt \int d^3\mathbf{x} \mathcal{L}$ (where $\mathbf{x}$ are comoving coordinates). This implies a canonical momentum density $\pi = \partial \mathcal{L} / \partial \dot{\phi} = a^3 \dot{\phi}$ and a Hamiltonian density

$$\mathcal{H} = \frac{1}{2} \frac{\pi^2}{a^3} + \frac{1}{2} a (\nabla \phi)^2. \tag{6}$$

The equations of motion $\dot{\phi} = \delta H / \delta \pi$, $\dot{\pi} = -\delta H / \delta \phi$ (with $H = \int d^3\mathbf{x} \mathcal{H}$) lead to the classical wave equation

$$\ddot{\phi} + \frac{3\dot{a}}{a} \dot{\phi} - \frac{1}{a^2} \nabla^2 \phi = 0. \tag{7}$$

Pilot-wave field theory is defined in terms of the functional Schrödinger picture, with a preferred foliation of spacetime [15,16,29,44–49]. For an expanding universe with metric (4), containing a scalar field ϕ with Hamiltonian density (6), a general wave functional $\Psi[\phi, t] = \langle \phi(\mathbf{x}) | \Psi(t) \rangle$ [where $|\phi(\mathbf{x})\rangle$ is a field eigenstate] satisfies the functional Schrödinger equation[4]

$$i \frac{\partial \Psi}{\partial t} = \int d^3\mathbf{x} \left(-\frac{1}{2a^3} \frac{\delta^2}{\delta \phi^2} + \frac{1}{2} a (\nabla \phi)^2 \right) \Psi \tag{8}$$

(with the usual realizations $\hat{\phi} \to \phi$, $\hat{\pi} \to -i\delta / \delta \phi$). This implies the continuity equation

$$\frac{\partial |\Psi|^2}{\partial t} + \int d^3x \frac{\delta}{\delta \phi} \left(|\Psi|^2 \frac{1}{a^3} \frac{\delta S}{\delta \phi} \right) = 0 \tag{9}$$

(where $\Psi = |\Psi| e^{iS}$), from which one may identify the de Broglie velocity

$$\frac{\partial \phi}{\partial t} = \frac{1}{a^3} \frac{\delta S}{\delta \phi} \tag{10}$$

for an individual field configuration. Here, again, Ψ is interpreted as a physical field in configuration space, guiding the evolution of an individual field $\phi(\mathbf{x}, t)$ in 3-space. [Note that S is defined only locally, as $S = \mathrm{Im} \ln \Psi$. One may equally write (10) as $\frac{\partial \phi}{\partial t} = \frac{1}{a^3} \mathrm{Im} \frac{1}{\Psi} \frac{\delta \Psi}{\delta \phi}$, without mentioning S.)

A similar construction may be given in any globally hyperbolic spacetime, by choosing a preferred foliation [22]. Thus there is no need for spatial homogeneity.

Over an ensemble of field configurations guided by the same pilot wave Ψ, there will be some (in principle, arbitrary) initial distribution $P[\phi, t_i]$, whose time evolution $P[\phi, t]$ will be determined by

$$\frac{\partial P}{\partial t} + \int d^3x \frac{\delta}{\delta \phi} \left(P \frac{1}{a^3} \frac{\delta S}{\delta \phi} \right) = 0. \tag{11}$$

If $P[\phi, t_i] = |\Psi[\phi, t_i]|^2$, then $P[\phi, t] = |\Psi[\phi, t]|^2$ for all t, and empirical agreement is obtained with standard quantum field theory [29,45–49]. On the other hand, for an initial nonequilibrium distribution $P[\phi, t_i] \neq |\Psi[\phi, t_i]|^2$, for as long as P remains in nonequilibrium, the predicted statistics will generally differ from those of quantum field theory. In any case, whatever form P may take (equilibrium or nonequilibrium), its time evolution will be given by (11).

It will prove convenient to rewrite the dynamics in Fourier space. Expressing $\phi(\mathbf{x})$ in terms of its Fourier components

$$\phi_{\mathbf{k}} = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{x} \phi(\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}},$$

and writing

$$\phi_{\mathbf{k}} = \frac{\sqrt{V}}{(2\pi)^{3/2}} (q_{\mathbf{k}1} + i q_{\mathbf{k}2})$$

for real $q_{\mathbf{k}r}$ ($r = 1, 2$), where V is a box normalization volume, the Lagrangian $L = \int d^3\mathbf{x} \mathcal{L}$ becomes

$$L = \sum_{\mathbf{k}r} \frac{1}{2} (a^3 \dot{q}_{\mathbf{k}r}^2 - a k^2 q_{\mathbf{k}r}^2).$$

[For $V \to \infty$, $\frac{1}{V} \sum_{\mathbf{k}} \to \frac{1}{(2\pi)^3} \int d^3\mathbf{k}$ and $V \delta_{\mathbf{k}\mathbf{k}'} \to (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}')$. The reality of ϕ requires $\phi_{\mathbf{k}}^* = \phi_{-\mathbf{k}}$ or $q_{\mathbf{k}1} = q_{-\mathbf{k}1}$, $q_{\mathbf{k}2} = -q_{-\mathbf{k}2}$, so that a sum over physical degrees of freedom should be restricted to half the values of $\mathbf{k}$.] Introducing the canonical momenta

$$\pi_{\mathbf{k}r} \equiv \frac{\partial L}{\partial \dot{q}_{\mathbf{k}r}} = a^3 \dot{q}_{\mathbf{k}r},$$

the Hamiltonian becomes

$$H = \sum_{\mathbf{k}r} \left(\frac{1}{2a^3} \pi_{\mathbf{k}r}^2 + \frac{1}{2} a k^2 q_{\mathbf{k}r}^2 \right).$$

The Schrödinger equation for $\Psi = \Psi[q_{\mathbf{k}r}, t]$ is then

$$i \frac{\partial \Psi}{\partial t} = \sum_{\mathbf{k}r} \left(-\frac{1}{2a^3} \frac{\partial^2}{\partial q_{\mathbf{k}r}^2} + \frac{1}{2} a k^2 q_{\mathbf{k}r}^2 \right) \Psi, \tag{12}$$

which implies the continuity equation

$$\frac{\partial |\Psi|^2}{\partial t} + \sum_{\mathbf{k}r} \frac{\partial}{\partial q_{\mathbf{k}r}} \left(|\Psi|^2 \frac{1}{a^3} \frac{\partial S}{\partial q_{\mathbf{k}r}} \right) = 0 \tag{13}$$

and the de Broglie velocities

$$\frac{dq_{\mathbf{k}r}}{dt} = \frac{1}{a^3} \frac{\partial S}{\partial q_{\mathbf{k}r}} \tag{14}$$

(again with $\Psi = |\Psi| e^{iS}$). The time evolution of an arbitrary distribution $P[q_{\mathbf{k}r}, t]$ will then be given by

[4] As usual in this context, some sort of regularization is implicitly assumed.

$$\frac{\partial P}{\partial t} + \sum_{\mathbf{k}r} \frac{\partial}{\partial q_{\mathbf{k}r}} \left(P \frac{1}{a^3} \frac{\partial S}{\partial q_{\mathbf{k}r}} \right) = 0. \tag{15}$$

For product states

$$\Psi[q_{\mathbf{k}r}, t] = \prod_{\mathbf{k}r} \psi_{\mathbf{k}r}(q_{\mathbf{k}r}, t) \tag{16}$$

(such as the Bunch-Davies vacuum during inflation), the wave function $\psi_{\mathbf{k}r}$ for a single mode $\mathbf{k}r$ satisfies

$$i\frac{\partial \psi_{\mathbf{k}r}}{\partial t} = \left(-\frac{1}{2a^3} \frac{\partial^2}{\partial q_{\mathbf{k}r}^2} + \frac{1}{2} a k^2 q_{\mathbf{k}r}^2 \right) \psi_{\mathbf{k}r}. \tag{17}$$

Writing $\psi_{\mathbf{k}r} = |\psi_{\mathbf{k}r}| e^{i s_{\mathbf{k}r}}$ (where $S = \sum_{\mathbf{k}r} s_{\mathbf{k}r}$), the de Broglie velocity for $q_{\mathbf{k}r}$ is then

$$\frac{dq_{\mathbf{k}r}}{dt} = \frac{1}{a^3} \frac{\partial s_{\mathbf{k}r}}{\partial q_{\mathbf{k}r}}. \tag{18}$$

If the initial distribution $P[q_{\mathbf{k}r}, t_i]$ also takes the product form

$$P[q_{\mathbf{k}r}, t_i] = \prod_{\mathbf{k}r} \rho_{\mathbf{k}r}(q_{\mathbf{k}r}, t_i), \tag{19}$$

then the time evolution of $\rho_{\mathbf{k}r}(q_{\mathbf{k}r}, t)$ will be given by

$$\frac{\partial \rho_{\mathbf{k}r}}{\partial t} + \frac{\partial}{\partial q_{\mathbf{k}r}} \left(\rho_{\mathbf{k}r} \frac{1}{a^3} \frac{\partial s_{\mathbf{k}r}}{\partial q_{\mathbf{k}r}} \right) = 0. \tag{20}$$

Note that the factorizability condition (19) for the probability distribution P is logically independent of the factorizability condition (16) for the pilot wave Ψ. Thus, even for a vacuum state, in nonequilibrium it is still possible to have intermode correlations. For simplicity, in Sec. VI, we shall restrict ourselves to the case of uncorrelated nonequilibrium modes. The correlated case is discussed in Sec. XI.

IV. QUANTUM NONEQUILIBRIUM IN THE VERY EARLY UNIVERSE

In this paper, the focus is on setting experimental bounds on possible violations of quantum theory during inflation. Before proceeding with this, however, let us indicate how one might (in future work) be able to predict details of such violations. The scenarios sketched in this section also serve to give a preliminary idea of the kinds of violations one might expect to find.

A. Relic nonequilibrium from a preinflationary era

One reason to expect early nonequilibrium to exist is that, as sketched in Sec. II, according to de Broglie-Bohm theory ordinary matter corresponds to a "quantum equilibrium phase," and it is natural to suppose that this equilibrium state emerged from the violence of the big bang.

Another reason is that nonequilibrium at very early times would unleash the nonlocality inherent in all hidden-variables theories, thereby evading the horizon problem associated with an early Friedmann expansion (if there was one). For $a \propto t^{1/2}$ the horizon distance is (with $c = 1$)

$$l_{\mathrm{h}}(t) = a(t) \int_0^t \frac{dt'}{a(t')} = 2t,$$

and for any two comoving points separated by a coordinate distance $|\Delta \mathbf{x}|$, we have $l_{\mathrm{h}}(t) \ll a(t)|\Delta \mathbf{x}|$ for sufficiently small t. On this basis it has been widely argued that early homogeneity—over seemingly causally disconnected domains—is unnatural and puzzling.[5] As we have mentioned, the hypothesis of quantum nonequilibrium at the big bang was originally introduced partly to solve this problem [14–16,19]. For the above scalar field, for example, a generic wave functional Ψ will be entangled across space, so that the field velocity

$$\frac{\partial \phi(\mathbf{x}, t)}{\partial t} = \frac{1}{a^3} \frac{\delta S[\phi, t]}{\delta \phi(\mathbf{x})}$$

at a point $\mathbf{x}$ will depend on instantaneous values of the field at remote points $\mathbf{x}' \neq \mathbf{x}$, and in nonequilibrium this nonlocal dependence will not be hidden by statistical noise (as it is in quantum theory). Of course the horizon problem was also one of the historical motivations for introducing inflation: the period of exponential expansion ensures that our observable region originates from within a single causal patch [50]. However, even in an inflationary context, it appears that some models require homogeneity as an initial condition in order for inflation to begin [51]. Therefore, it is possible that consideration of a preinflationary era will revive the horizon problem, and that some form of early nonlocality may provide a resolution. The nonlocality could be generated by quantum nonequilibrium, or perhaps by some other means (other proposals include topological fluctuations [52] and an increased speed of light at early times [53]).

If we then assume—for whatever reason—that the preinflationary Universe was in a state of quantum nonequilibrium, the question is how the nonequilibrium will evolve in time, and, in particular, whether any of it will survive until entry into the inflationary era. To address this question, let us first summarize what is known so far about relaxation in pilot-wave theory.

As already mentioned, numerical simulations for simple two-dimensional systems show an efficient relaxation, with an approximately exponential decay of the coarse-grained H-function $\bar{H}(t)$ [23,43]. Specifically, for an ensemble of nonrelativistic particles in a two-dimensional box (on a static spacetime background), with a wave function consisting of a superposition of the first 16 modes, it was found that $\bar{H}(t) \approx \bar{H}_0 e^{-t/t_c}$ where, as discussed in Ref. [23], the time scale t_c coincides approximately with a

[5] Note, however, that the existence of the puzzle depends on assuming a classical Friedmann expansion $a \propto t^{1/2}$ all the way back to $t = 0$.

theoretical relaxation time scale τ defined by $1/\tau^2 \equiv -(1/\bar{H})d^2\bar{H}/dt^2$ [15], which under certain conditions may be roughly estimated as [17,23,54]

$$\tau \sim \Delta t \equiv 1/\Delta E,$$

where ΔE is the quantum energy spread and Δt is the usual quantum time scale over which the wave function evolves.

Similar results have been obtained for nonrelativistic particles in a two-dimensional harmonic oscillator potential [43], a case that has immediate implications for the field theory of a single decoupled mode $\mathbf{k}$.

Writing $\Psi = \psi_{\mathbf{k}}(q_{\mathbf{k}1}, q_{\mathbf{k}2}, t)\varkappa$, where $\varkappa$ depends only on degrees of freedom for modes $\mathbf{k}' \neq \mathbf{k}$, Eqs. (12) and (14) imply that the wave function $\psi_{\mathbf{k}}$ satisfies

$$i\frac{\partial \psi_{\mathbf{k}}}{\partial t} = -\frac{1}{2a^3}\left(\frac{\partial^2}{\partial q_{\mathbf{k}1}^2} + \frac{\partial^2}{\partial q_{\mathbf{k}2}^2}\right)\psi_{\mathbf{k}} + \frac{1}{2}ak^2(q_{\mathbf{k}1}^2 + q_{\mathbf{k}2}^2)\psi_{\mathbf{k}}, \tag{21}$$

while the de Broglie velocities for $(q_{\mathbf{k}1}, q_{\mathbf{k}2})$ are

$$\dot{q}_{\mathbf{k}1} = \frac{1}{a^3}\frac{\partial s_{\mathbf{k}}}{\partial q_{\mathbf{k}1}}, \qquad \dot{q}_{\mathbf{k}2} = \frac{1}{a^3}\frac{\partial s_{\mathbf{k}}}{\partial q_{\mathbf{k}2}} \tag{22}$$

(with $\psi_{\mathbf{k}} = |\psi_{\mathbf{k}}|e^{is_{\mathbf{k}}}$). The marginal distribution $\rho_{\mathbf{k}}(q_{\mathbf{k}1}, q_{\mathbf{k}2}, t)$ will evolve according to

$$\frac{\partial \rho_{\mathbf{k}}}{\partial t} + \sum_{r=1,2}\frac{\partial}{\partial q_{\mathbf{k}r}}\left(\rho_{\mathbf{k}}\frac{1}{a^3}\frac{\partial s_{\mathbf{k}}}{\partial q_{\mathbf{k}r}}\right) = 0. \tag{23}$$

As discussed elsewhere [25,54], these are identical to the equations of pilot-wave dynamics for an ensemble of nonrelativistic particles with time-dependent "mass" $m = a^3$, moving in the two-dimensional $q_{\mathbf{k}1} - q_{\mathbf{k}2}$ plane, in a harmonic oscillator potential of time-dependent angular frequency $\omega = k/a$. In the short-wavelength limit, $\lambda_{\mathrm{phys}} \ll \Delta n_{\mathbf{k}} \cdot H^{-1}$ (where $n_{\mathbf{k}} = n_{\mathbf{k}1} + n_{\mathbf{k}2}$ is the sum of the occupation numbers for modes $\mathbf{k}1$ and $\mathbf{k}2$), and over time scales $\Delta t \equiv 1/\Delta E_{\mathbf{k}} \ll H^{-1}$ (for which a is approximately constant), the above equations reduce to those for a decoupled mode $\mathbf{k}$ on Minkowski spacetime [54]. These limiting equations are in turn just those of pilot-wave dynamics for an ensemble of nonrelativistic particles of constant mass $m = a^3$ in a two-dimensional harmonic oscillator potential of constant angular frequency $\omega = k/a$. The numerical results for this last case [43] show that, in the Minkowski limit, for a decoupled mode $\mathbf{k}$ in a superposition of many different states of definite occupation number, one will obtain relaxation $\rho_{\mathbf{k}}(q_{\mathbf{k}1}, q_{\mathbf{k}2}, t) \to |\psi_{\mathbf{k}}(q_{\mathbf{k}1}, q_{\mathbf{k}2}, t)|^2$ (on a coarse-grained level, assuming appropriate initial conditions), on a time scale $\tau_{\mathbf{k}}$ of order

$$\tau_{\mathbf{k}} \sim \frac{1}{\Delta E_{\mathbf{k}}}.$$

If, in the Minkowski limit, relaxation occurs so efficiently for a single decoupled mode, then we may reasonably expect that for a realistic entangled quantum state—in some preinflationary era—relaxation will occur at least as efficiently. One might then conclude that, even if there is initial nonequilibrium, it will have relaxed away by the time inflation begins. However, before drawing definite conclusions, one must first consider the possible effect of spatial expansion on the relaxation process. One finds, in particular, that the character of the evolution can be very different in the long-wavelength limit.

In the case of a decoupled mode on expanding space, described by Eqs. (21)–(23), it is found [54] that in the long-wavelength limit, $\lambda_{\mathrm{phys}} \gg \Delta n_{\mathbf{k}} \cdot H^{-1}$, the wave function $\psi_{\mathbf{k}}$ is approximately static—or "frozen"—over time scales $\sim H^{-1}$. Furthermore, one expects that the trajectories $(q_{\mathbf{k}1}(t), q_{\mathbf{k}2}(t))$ will be frozen over time scales $\sim H^{-1}$, in which case an arbitrary nonequilibrium distribution $\rho_{\mathbf{k}} \neq |\psi_{\mathbf{k}}|^2$ will also be frozen over time scales $\sim H^{-1}$. (This is of course reminiscent of the freezing of super-Hubble modes in the theory of cosmological perturbations [1,2].) It then begins to appear possible that the normal process of relaxation to quantum equilibrium could be suppressed for long-wavelength modes in a preinflationary era, and that remnants of initial nonequilibrium could survive up to the beginning of inflation.

That this is indeed possible has been shown [54] by deriving a general and rigorous condition for the freezing of quantum nonequilibrium, a condition applicable to an arbitrary time interval $[t_i, t_f]$ and to any (generally entangled) quantum state of a scalar field. (The condition may also be applied to mixed states and to interacting fields.) The condition is obtained by considering the displacements of the de Broglie-Bohm trajectories over the time interval $[t_i, t_f]$. It is found that, for a pure subensemble with (time-dependent) mean occupation numbers $\langle \hat{n}_{\mathbf{k}r}\rangle$, nonequilibrium will be frozen (or at least partially frozen) for modes with wave number k if the time evolution of $\langle \hat{n}_{\mathbf{k}r}\rangle$ satisfies the "freezing inequality" [54]

$$\frac{1}{k} > 4a_f\sqrt{\langle \hat{n}_{\mathbf{k}r}\rangle_f + 1/2}\int_{t_i}^{t_f} dt\frac{1}{a^2}\sqrt{\langle \hat{n}_{\mathbf{k}r}\rangle + 1/2}. \tag{24}$$

For a radiation-dominated expansion on $[t_i, t_f]$, with $a(t) = a_f(t/t_f)^{1/2}$, this inequality implies that (using $\langle \hat{n}_{\mathbf{k}r}\rangle \geq 0$)

$$\lambda_{\mathrm{phys}}(t_f) > 2\pi H_f^{-1}\ln(t_f/t_i), \tag{25}$$

where $H_f^{-1} = 2t_f$ and the right-hand side is larger than H_f^{-1} if $t_f \gtrsim (1.17)t_i$. Thus, in a radiation-dominated expansion, if the freezing inequality (24) is satisfied, the corresponding modes must be super-Hubble [54].

We are now in a position to begin to address the question of whether or not very early nonequilibrium in a preinflationary era could survive until the onset of inflation itself. The above considerations show that, for short-wavelength modes, any initial quantum nonequilibrium is likely to be rapidly destroyed during a preinflationary phase. On the other hand, relaxation can be suppressed for long-wavelength modes—if the freezing inequality (24) is satisfied—and it is then possible that these modes (for whatever fields may be present) will still be in nonequilibrium at the onset of inflation.

Denoting, for a moment, the (approximately) constant Hubble radius during inflation by $H_{\rm inf}^{-1}$, relevant cosmological fluctuations originate from inside $H_{\rm inf}^{-1}$. For some of these modes to be out of equilibrium, they must have evolved from modes that were outside the Hubble radius in the (presumably radiation-dominated) preinflationary phase. Therefore, for this scenario to work, some preinflationary nonequilibrium modes must enter the Hubble radius during the transition to the inflationary phase, and they must avoid complete relaxation to equilibrium by the time inflation begins. (As we shall see in Sec. VI, relaxation does not occur during inflation itself.) Now, modes of physical wavelength $\lambda_{\rm phys} = a\lambda$ can enter the Hubble radius $H^{-1} = a/\dot{a}$ only if $\lambda_{\rm phys}$ increases more slowly than does H^{-1}, that is, only if the comoving Hubble radius $H^{-1}/a = 1/\dot{a}$ increases—as occurs for a decelerating universe, $\ddot{a} < 0$ [which, from the Friedmann equation $\ddot{a}/a = -(4\pi G/3)(\rho + 3p)$, requires that the energy density ρ and pressure p satisfy $\rho + 3p > 0$].

During a decelerating preinflationary phase, then, any frozen nonequilibrium modes at super-Hubble radii can enter the Hubble radius. Once they do so, they are likely to begin to relax to equilibrium. For all modes that are inside $H_{\rm inf}^{-1}$ at the onset of inflation, some time will necessarily have been spent in what might be crudely termed the "relaxation zone," with $\lambda_{\rm phys} \lesssim H^{-1}$, during the preinflationary phase. For example, for a radiation-dominated preinflationary phase (starting at some initial time t_i) that makes an abrupt transition to an inflationary phase at $t = t_f$, we have $a = a_f(t/t_f)^{1/2}$ and $H^{-1} = 2t$ (on $[t_i, t_f]$), and a mode of comoving wavelength λ enters the Hubble radius ($a\lambda \sim H^{-1}$) at a time $t_{\rm enter}(\lambda) \sim a_f^2\lambda^2/t_f$, so that the time spent in the relaxation zone is

$$\Delta t_{\rm relax}(\lambda) = (t_f - t_{\rm enter}) \sim t_f(1 - a_f^2\lambda^2/t_f^2).$$

There can be significant residual nonequilibrium at the beginning of inflation, provided the "no relaxation" condition

$$\Delta t_{\rm relax}(\lambda) \lesssim \tau(\lambda) \qquad (26)$$

is satisfied, where $\tau(\lambda)$ is again a relaxation time scale as defined above [and where $\tau(\lambda)$ may be evaluated at the intermediate time $t_{\rm enter} + \frac{1}{2}(t_f - t_{\rm enter})$]. Because $\tau(\lambda)$ will depend on the wave functional, a proper calculation of $\tau(\lambda)$ requires a specific model of the preinflationary phase.

Given a specific form for the function $\tau(\lambda)$, the condition (26) will determine a range of wavelengths λ for which residual nonequilibrium may reasonably be expected to have survived from the preinflationary era. Because preinflationary modes with larger values of λ enter the Hubble radius later and so spend less time in the relaxation zone, the condition (26) will presumably imply that residual nonequilibrium will be possible for λ larger than some infrared cutoff λ_c. [The scenario might be improved if, during the transition from a preinflationary to an inflationary phase, the Hubble radius was a rapidly increasing function of the scale factor ($dH^{-1}/da \gg H^{-1}/a$). For then super-Hubble nonequilibrium modes could be pushed far inside the Hubble radius in a short time.]

We hope that future work, based on a specific preinflationary model, will yield a prediction for the infrared cutoff λ_c. It is of course possible that λ_c will turn out to be so much larger than today's Hubble radius that it yields a negligible effect on CMB predictions [as could occur if the relaxation time scale $\tau(\lambda)$ is too short, or if the number of inflationary e-folds is too large]. This remains to be seen.

In this paper, the focus is on "phenomenology": we simply assume that some modes could be in quantum nonequilibrium at the beginning of the inflationary phase, and we show how CMB data may be used to set experimental bounds on such nonequilibrium. Still, the above preliminary reasoning already suggests that if there is residual nonequilibrium from a preinflationary phase, then we should expect to find it at large wavelengths, beyond some cutoff λ_c.

B. Possible production of nonequilibrium at the Planck scale

We have discussed whether nonequilibrium might have survived into the inflationary phase, on the assumption that there was nonequilibrium in some preinflationary era. Another question is whether nonequilibrium might be *generated* during (or indeed even before) the inflationary era.

The creation of quantum nonequilibrium from a prior equilibrium state is impossible in standard de Broglie-Bohm theory (leaving aside extremely rare fluctuations [15]), though it might occur in alternative hidden-variables models—for example, in models that deviate from quantum theory for processes taking place over very short time scales [55]. But even in de Broglie-Bohm theory, it does not seem entirely clear if we know how to incorporate gravitation [41] (see, however, Ref. [56]). It is therefore conceivable that effects involving gravity are able to upset the equilibrium state. In particular, as has been discussed at length elsewhere, it is not unreasonable to propose that quantum nonequilibrium can be generated by the formation and evaporation of a black hole [22,25].

This proposal is motivated by the (controversial) question of information loss in black holes. In the standard picture of black-hole formation and evaporation, it appears that a closed system can evolve from an initial pure state to a final mixed state, thereby violating ordinary quantum theory [57]. Further, because the final state describes thermal radiation that depends on the initial mass of the hole but not on the details of the initial state, it is impossible even in principle to retrodict the initial state from the final state. While Hawking's original argument for information loss remains controversial, a new approach to avoiding information loss invokes the possible existence of quantum nonequilibrium in the outgoing radiation, which could then carry more information than ordinary radiation can in a conventional (mixed) quantum state. A mechanism for the creation of such nonequilibrium has been outlined [22], involving an assumed nonequilibrium behind the horizon (presumably near the singularity) that is transferred to the exterior region by the entanglement between the ingoing and outgoing modes of the Hawking radiation. A simple rule has been suggested, whereby the decreased "hidden-variable entropy" S_{hv} [equal to minus the subquantum H-function (3)] of the outgoing nonequilibrium radiation balances the increase in von Neumann entropy $S_{\mathrm{von\,N}} = -\mathrm{Tr}(\hat{\rho}\ln\hat{\rho})$ generated by the pure-to-mixed transition:

$$\Delta(S_{\mathrm{hv}} + S_{\mathrm{von\,N}}) = 0. \tag{27}$$

Possible experimental tests of this proposal are discussed in Refs. [22,25].

It is sometimes suggested that, at the Planck scale, processes will occur involving the formation and evaporation of microscopic black holes. If one takes this (rather heuristic) picture seriously and combines it with the above proposal, one is led to the conclusion that quantum nonequilibrium will be generated at the Planck scale. During the inflationary phase, such processes might have an effective description in terms of nonequilibrium modes of the inflaton field at Planckian or trans-Planckian (physical) frequencies. One might reason as follows. If a mode of comoving wavelength λ once had a physical wavelength $\lambda_{\mathrm{phys}} = a\lambda \lesssim l_{\mathrm{P}}$ near the beginning of inflation, one could assume that upon exiting the Planckian regime (that is, once λ_{phys} becomes bigger than l_{P}) the mode will be out of equilibrium, having encountered some gravitational process that generates nonequilibrium while $\lambda_{\mathrm{phys}} \sim l_{\mathrm{P}}$, whereas modes that were never smaller than l_{P} will not encounter any such process. Roughly, one could model this by introducing a cutoff λ'_{c} such that nonequilibrium exists only for comoving wavelengths $\lambda \lesssim \lambda'_{\mathrm{c}}$ (*below* the critical value λ'_{c}, in contrast with the scenario in the preceding section). In addition to providing an estimate for λ'_{c}, one also needs to estimate the degree of nonequilibrium, which for a given mode $\mathbf{k}r$ may be quantified by the relative (or hidden-variable) entropy

$$S_{\mathrm{hv}}(k) \equiv -\int dq_{\mathbf{k}r}\rho_{\mathbf{k}r}\ln(\rho_{\mathbf{k}r}/|\psi_{\mathbf{k}r}|^2). \tag{28}$$

An estimate for $S_{\mathrm{hv}}(k)$ might arise from an application of (27) in some form, though this remains to be studied.

It is to be hoped that further development of this idea will lead to a detailed prediction for the form and magnitude of nonequilibrium for modes emerging from the Planckian regime. Pending such development, again, in this paper we restrict ourselves to using current data to set limits on any hypothetical quantum nonequilibrium that may be present during the inflationary phase.

V. MEASURING PRIMORDIAL QUANTUM FLUCTUATIONS

The above considerations suggest that, during inflation, some field modes may exhibit nonequilibrium fluctuations that violate quantum theory. Our aim in this paper is to show how to use CMB data to set bounds on such violations.

We shall first recall how measurements of the CMB today allow us to infer statistical properties of inflaton fluctuations during the inflationary era. This involves working backwards from the CMB data, first to classical curvature perturbations in the early Universe, and from these, backwards even further to inflaton fluctuations during the inflationary phase. After having highlighted the key assumptions that are made in the standard treatment, we will be in a position to understand exactly how corrections to quantum theory during inflation are able to have an effect on the CMB.

A. CMB observations and primordial curvature perturbations

Employing angular coordinates (θ, ϕ) on the sky, CMB measurements provide us with a temperature function $T(\theta, \phi)$. Writing $\Delta T(\theta, \phi) \equiv T(\theta, \phi) - \bar{T}$, where $\bar{T}$ is the average temperature over the sky, the temperature anisotropy may be decomposed into spherical harmonics,

$$\frac{\Delta T(\theta, \phi)}{\bar{T}} = \sum_{l=2}^{\infty}\sum_{m=-l}^{+l} a_{lm}Y_{lm}(\theta, \phi) \tag{29}$$

(where as usual we omit the dipole term). A mode l corresponds to an angular scale $\approx 60°/l$.

A complete measurement of the microwave sky provides us with one function $T(\theta, \phi)$, or equivalently with one set $\{a_{lm}\}$ of coefficients. In order to carry out a statistical analysis of $\{a_{lm}\}$, it is usually assumed (if only implicitly) that the observed $T(\theta, \phi)$ is a single realization of a stochastic process, whose probability distribution $P[T(\theta, \phi)]$ (which may be thought of as representing a theoretical "ensemble of skies") satisfies the condition of statistical isotropy:

$$P[T(\theta - \delta\theta, \phi - \delta\phi)] = P[T(\theta, \phi)] \tag{30}$$

for arbitrary angular displacements $\delta\theta, \delta\phi$. This condition implies that, for a given l, each a_{lm} has the same (marginal) probability distribution $p_l(a_{lm})$, with variance

$$C_l \equiv \langle |a_{lm}|^2 \rangle \tag{31}$$

(the angular power spectrum, where $\langle \ldots \rangle$ denotes an average over the theoretical ensemble).

Thus, given the assumption (30), it follows that for each l we have what are, in effect, $2l+1$ independent realizations of the same random variable (with the same probability distribution). The observed quantity

$$C_l^{\rm sky} \equiv \frac{1}{2l+1} \sum_{m=-l}^{+l} |a_{lm}|^2$$

(constructed from measurements made on a single sky) then provides an unbiased estimate of the angular power spectrum C_l (that is, $\langle C_l^{\rm sky} \rangle = C_l$), with a "cosmic variance" given by

$$\frac{\Delta C_l^{\rm sky}}{C_l} = \sqrt{\frac{2}{2l+1}}. \tag{32}$$

For large values of l, the quantity $C_l^{\rm sky}$ is an accurate estimate of C_l (that is, we expect to find $C_l^{\rm sky} \approx C_l$). For small values of l, however, $C_l^{\rm sky}$ is an inaccurate estimate of C_l.

The observed CMB anisotropy is caused by classical inhomogeneities on the last scattering surface, when the CMB photons decoupled (together with effects taking place afterwards as the CMB photons propagate through space towards us). These inhomogeneities in turn originate from classical perturbations that were present at much earlier times. In the long "primordial" period between $t_{\rm exit}(k)$ and $t_{\rm enter}(k)$ (during which $k \ll Ha$, or $\lambda_{\rm phys} \gg H^{-1}$), the classical curvature perturbation

$$\mathcal{R}_{\mathbf{k}} \equiv \frac{1}{4}\left(\frac{a}{k}\right)^2 {}^{(3)}R_{\mathbf{k}} \tag{33}$$

is time independent. (Here, ${}^{(3)}R_{\mathbf{k}}$ is the Fourier component of the spatial curvature scalar on comoving hypersurfaces, that is, on hypersurfaces with zero momentum density.) To a good first approximation, we may ignore gravitational waves, in which case $\mathcal{R}_{\mathbf{k}}$ is the only independent degree of freedom for the classical primordial perturbations. In terms of $\mathcal{R}_{\mathbf{k}}$, the a_{lm} may be expressed as [58]

$$a_{lm} = \frac{i^l}{2\pi^2} \int d^3\mathbf{k}\, \mathcal{T}(k,l) \mathcal{R}_{\mathbf{k}} Y_{lm}(\hat{\mathbf{k}}), \tag{34}$$

where the transfer function $\mathcal{T}(k,l)$ encodes the astrophysical processes that generate the temperature anisotropy.

A given primordial curvature perturbation $\mathcal{R}_{\mathbf{k}}$ (for all $\mathbf{k}$) generates one set $\{a_{lm}\}$ of temperature-anisotropy coefficients. A probability distribution $P[\mathcal{R}_{\mathbf{k}}]$ for $\mathcal{R}_{\mathbf{k}}$ will generate a probability distribution $P[\{a_{lm}\}]$ for $\{a_{lm}\}$. If we make the assumption of statistical homogeneity, that $P[\mathcal{R}_{\mathbf{k}}]$ is translationally invariant—that is, in position space, $P[\mathcal{R}(\mathbf{x}-\mathbf{d})] = P[\mathcal{R}(\mathbf{x})]$ for arbitrary displacements $\mathbf{d}$—it follows that $\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle e^{-i(\mathbf{k}+\mathbf{k}')\cdot \mathbf{d}} = \langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle$ and so

$$\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}^*_{\mathbf{k}'} \rangle = \delta_{\mathbf{k}\mathbf{k}'} \langle |\mathcal{R}_{\mathbf{k}}|^2 \rangle. \tag{35}$$

From (34) and (35), the angular power spectrum (31) may be written as

$$C_l = \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} \mathcal{T}^2(k,l) \mathcal{P}_{\mathcal{R}}(k), \tag{36}$$

where

$$\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{4\pi k^3}{V} \langle |\mathcal{R}_{\mathbf{k}}|^2 \rangle \tag{37}$$

is the primordial power spectrum. We shall assume, as is usually done, that $\langle |\mathcal{R}_{\mathbf{k}}|^2 \rangle$ is a function of k only.

Current measurements of the CMB show that $\mathcal{P}_{\mathcal{R}}(k) \approx$ const (an approximately flat or scale-free spectrum) [59].

B. Inflationary slow-roll predictions

Standard inflation predicts an approximately flat primordial power spectrum $\mathcal{P}_{\mathcal{R}}(k)$. Let us briefly review how this comes about.

An approximately homogeneous inflaton field $\phi_0(t) + \phi(\mathbf{x}, t)$ (where ϕ is a small perturbation), with a potential V, has an energy density $\rho \approx \frac{1}{2}\dot{\phi}_0^2 + V(\phi_0)$. In the slow-roll approximation, $\rho \approx V(\phi_0)$ is approximately constant in time. The Friedmann equation $(\dot{a}/a)^2 = (8\pi G/3)\rho$ then implies an approximate de Sitter expansion, $a \propto e^{Ht}$, where $H = \sqrt{(8\pi G/3)V(\phi_0)}$. The time evolution of ϕ_0 is given by

$$3\frac{\dot{a}}{a}\dot{\phi}_0 + \frac{dV}{d\phi_0} = 0$$

(where in the slow-roll approximation we may neglect the term $\ddot{\phi}_0$). The flatness conditions for V are $\varepsilon \ll 1$, $|\eta| \ll 1$, where

$$\varepsilon \equiv \frac{1}{16\pi G}\left(\frac{1}{V}\frac{dV}{d\phi_0}\right)^2, \qquad \eta \equiv \frac{1}{8\pi G}\frac{1}{V}\frac{d^2V}{d\phi_0^2}. \tag{38}$$

The primordial perturbations are generated by quantum fluctuations during the slow roll. As a first approximation, the quantum fluctuations may be calculated for an eternal de Sitter expansion, and in this approximation one obtains an exactly scale-free primordial power spectrum. Corrections to this approximation yield small corrections to the scale-free result.

The quantum theory of primordial perturbations has been developed in great detail [1,58,60]. In the slow-roll limit ($\dot{H} \to 0$), with V satisfying the flatness conditions, the inflaton perturbation $\phi = \phi(\mathbf{x}, t)$ evolves like a free massless field [until at least a few e-folds after $t_{\rm exit}(k)$ for the mode $\mathbf{k}$]. The quantized field $\hat{\phi}$ is usually assumed to be in the vacuum state. One may then use standard quantum field theory to calculate the probability distribution for the inflaton perturbation ϕ.

It is usually assumed that, a few Hubble times or e-folds after $t_{\rm exit}(k)$ (that is, in the "late-time limit"), the resulting quantum probability distribution for ϕ may be regarded as a classical probability distribution over classical perturbations ϕ. This assumption has been justified by WKB-type classicality at late times [61], by squeezing of the inflationary vacuum state [62,63], and by environmental decoherence [64,65]. The latter, in particular, seems to distinguish the field configuration basis (that is, the basis of eigenstates of the field operator $\hat{\phi}$) as a robust pointer basis, where the relevant interactions are local in field space [64,65]. The resulting distribution of field configurations is then, for practical purposes, indistinguishable from a classical distribution. Recent studies seem to confirm these conclusions: the pointer states consist (more precisely) of narrow Gaussians that approximate eigenstates of $\hat{\phi}$ [66], and the locality of interactions in field space ensures that at late times the density matrix becomes essentially diagonal in the field configuration basis [67]. (See also Ref. [68] for further discussion of WKB classicality in the late-time limit.)

Given a classical inflaton perturbation ϕ, the corresponding curvature perturbation is given by [1]

$$\mathcal{R}_{\mathbf{k}} = -\left[\frac{H}{\dot{\phi}_0}\phi_{\mathbf{k}}\right]_{t=t_*(k)}, \tag{39}$$

where $t_*(k)$ is a time a few e-folds after $t_{\rm exit}(k)$. The perturbation $\mathcal{R}_{\mathbf{k}}$ is time independent between $t_*(k)$ and the approach to $t_{\rm enter}(k)$ (long after inflation ends), and is believed to seed what eventually grow into the dominant perturbations in the CMB.

Note that the inflaton perturbation ϕ is defined on a spatially flat slicing. (The inhomogeneous field ϕ necessarily vanishes on comoving slices, since the momentum density $-\dot{\phi}\nabla\phi$ is by definition zero on such slices.) Then, in the slow-roll limit $\dot{H}\to 0$, the backreaction of metric perturbations on ϕ can be ignored [1]. The curvature perturbation $\mathcal{R}$ is defined on the comoving slicing. Thus, (39) relates quantities defined on different slicings.

The predicted (quantum-theoretical) primordial power spectrum, for $\mathcal{R}_{\mathbf{k}}$ at $t = t_*$, is then given by

$$\mathcal{P}_{\mathcal{R}}^{\rm QT}(k) = \left[\frac{H^2}{\dot{\phi}_0^2}\mathcal{P}_{\phi}^{\rm QT}(k)\right]_{t_*(k)}, \tag{40}$$

where

$$\mathcal{P}_{\phi}^{\rm QT}(k) \equiv \frac{4\pi k^3}{V}\langle|\phi_{\mathbf{k}}|^2\rangle_{\rm QT} \tag{41}$$

is the power spectrum of the inflaton fluctuations.

As we have said, to a first approximation the inflaton fluctuations are usually taken to be quantum vacuum fluctuations in de Sitter spacetime. From the standard field operator expansion

$$\hat{\phi}(\mathbf{x},t) = \sum_{\mathbf{k}}\left(\frac{(k/a+iH)}{k\sqrt{2Vk}}\hat{a}_{\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{x}+k/Ha)} + \frac{(k/a-iH)}{k\sqrt{2Vk}}\hat{a}_{\mathbf{k}}^{\dagger}e^{-i(\mathbf{k}\cdot\mathbf{x}+k/Ha)}\right) \tag{42}$$

in terms of mode functions

$$\phi_+(\mathbf{x},t) \propto \left(\frac{k}{a}+iH\right)e^{i(\mathbf{k}\cdot\mathbf{x}+k/Ha)} \tag{43}$$

[solutions of (7) reducing to positive-frequency Minkowski modes in the short-wavelength limit $k/a \gg H$], the Bunch-Davies vacuum is defined by $\hat{a}_{\mathbf{k}}|0\rangle = 0$ (for all $\mathbf{k}$). In this quantum state, the two-point (equal-time) correlation function is

$$\langle 0|\hat{\phi}(\mathbf{x},t)\hat{\phi}(\mathbf{x}',t)|0\rangle = \sum_{\mathbf{k}}\frac{(k/a)^2+H^2}{2Vk^3}e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \tag{44}$$

(where the first term in the numerator gives a Minkowskian contribution $1/4\pi^2a^2|\mathbf{x}-\mathbf{x}'|^2$). The quantum variance of each mode is given by the Fourier transform of the quantum two-point function,

$$\langle|\phi_{\mathbf{k}}|^2\rangle_{\rm QT} = \frac{V}{(2\pi)^3}\int d^3\mathbf{x}\,e^{-i\mathbf{k}\cdot\mathbf{x}}\langle 0|\hat{\phi}(\mathbf{x}+\mathbf{y})\hat{\phi}(\mathbf{y})|0\rangle,$$

yielding

$$\langle|\phi_{\mathbf{k}}|^2\rangle_{\rm QT} = \frac{V}{2(2\pi)^3}\frac{H^2}{k^3}\left(1+\frac{k^2}{H^2a^2}\right). \tag{45}$$

The width decreases with time, tending to a finite constant. The power spectrum is

$$\mathcal{P}_{\phi}^{\rm QT}(k) = \frac{k^2}{4\pi^2a^2} + \frac{H^2}{4\pi^2}. \tag{46}$$

In the long-wavelength limit $k/a \ll H$ ($\lambda_{\rm phys} \gg H^{-1}$), where the mode is well outside the Hubble radius, we have

$$\mathcal{P}_{\phi}^{\rm QT}(k) = \frac{H^2}{4\pi^2}. \tag{47}$$

[If instead we set $k = Ha$, then $\mathcal{P}_{\phi}^{\rm QT}(k) = H^2/2\pi^2$.]

To a lowest-order approximation, then, the quantum fluctuations of the inflaton field generate a scale-free spectrum of primordial curvature perturbations:

$$\mathcal{P}_{\mathcal{R}}^{\rm QT}(k) = \frac{1}{4\pi^2}\left[\frac{H^4}{\dot{\phi}_0^2}\right]_{t_*(k)}. \tag{48}$$

These perturbations $\mathcal{R}_{\mathbf{k}} \propto \phi_{\mathbf{k}}$ remain frozen outside the Hubble radius until the time $t_{\rm enter}(k)$ is approached.

Because H and $\dot{\phi}_0$ are in fact slowly changing during the inflationary phase, higher-order corrections lead to a small dependence of $\mathcal{P}_{\mathcal{R}}^{\rm QT}(k)$ on k.

VI. TIME EVOLUTION OF NONEQUILIBRIUM VACUA

We now turn to the effect of quantum nonequilibrium on the predictions of inflationary cosmology. (A brief, preliminary account was given in Ref. [25].) Our strategy is to consider nonequilibrium corrections to the lowest-order (scale-free) quantum spectrum, and then to compare these effects with the higher-order quantum corrections.

In the Bunch-Davies vacuum, a mode $\mathbf{k}r$ has wave function $\psi_{\mathbf{k}r} = \psi_{\mathbf{k}r}(q_{\mathbf{k}r}, t) = |\psi_{\mathbf{k}r}|e^{is_{\mathbf{k}r}}$ with a Gaussian amplitude

$$|\psi_{\mathbf{k}r}|^2 = \frac{1}{\sqrt{2\pi\Delta_k^2}} e^{-q_{\mathbf{k}r}^2/2\Delta_k^2} \tag{49}$$

of width

$$\Delta_k^2 = \frac{H^2}{2k^3}\left(1 + \frac{k^2}{H^2a^2}\right) \tag{50}$$

(contracting in time, and independent of r and of the direction of $\mathbf{k}$) and with a phase

$$s_{\mathbf{k}r} = -\frac{ak^2q_{\mathbf{k}r}^2}{2H(1 + k^2/H^2a^2)} + h(t), \tag{51}$$

where

$$h(t) = \frac{1}{2}\left(\frac{k}{Ha} - \tan^{-1}\left(\frac{k}{Ha}\right)\right)$$

is independent of $q_{\mathbf{k}r}$. [It is readily verified that the above wave function $\psi_{\mathbf{k}r}(q_{\mathbf{k}r}, t)$ satisfies the Schrödinger equation (17) for a mode $\mathbf{k}r$, and that in the limit $H \to 0$, $a \to 1$ one recovers the wave function $\psi_{\mathbf{k}r}(q_{\mathbf{k}r}, t) \propto e^{-kq_{\mathbf{k}r}^2}e^{-i(1/2)kt}$ for the Minkowski vacuum.]

In the quantum vacuum, the $q_{\mathbf{k}r}$ are independent random variables, each with a Gaussian distribution of zero mean. The width of each Gaussian decreases with time, approaching the asymptotic value $H/\sqrt{2k^3}$ (in the long-wavelength limit $k/a \ll H$). In the nonequilibrium (de Broglie-Bohm) vacuum, in contrast, each $q_{\mathbf{k}r}$ evolves deterministically in time, and the probability distribution for each $q_{\mathbf{k}r}$ depends on what the probability distribution was at some ''initial'' time.

The phase (51) implies a de Broglie velocity field

$$\frac{dq_{\mathbf{k}r}}{dt} = \frac{1}{a^3}\frac{\partial s_{\mathbf{k}r}}{\partial q_{\mathbf{k}r}} = -\frac{k^2Hq_{\mathbf{k}r}}{k^2 + H^2a^2}. \tag{52}$$

To solve (52) for the trajectories $q_{\mathbf{k}r}(t)$, it is convenient to introduce the conformal time η, defined by $d\eta = dt/a$. (For $a \propto e^{Ht}$ we have $\eta = -1/Ha$; as t runs from $-\infty$ to $+\infty$, η runs from $-\infty$ to 0.) In terms of η, the equation of motion for $q_{\mathbf{k}r}$ reads

$$\frac{dq_{\mathbf{k}r}}{d\eta} = \frac{k^2\eta q_{\mathbf{k}r}}{1 + k^2\eta^2}, \tag{53}$$

which has the solution

$$q_{\mathbf{k}r}(\eta) = q_{\mathbf{k}r}(0)\sqrt{1 + k^2\eta^2}. \tag{54}$$

The width of the packet is given by

$$\Delta_k^2 = \frac{H^2}{2k^3}(1 + k^2\eta^2). \tag{55}$$

An arbitrary distribution $\rho_{\mathbf{k}r}(q_{\mathbf{k}r}, \eta)$ [generally $\neq |\psi_{\mathbf{k}r}(q_{\mathbf{k}r}, \eta)|^2$] necessarily satisfies the continuity equation

$$\frac{\partial \rho_{\mathbf{k}r}}{\partial \eta} + \frac{\partial}{\partial q_{\mathbf{k}r}}\left(\rho_{\mathbf{k}r}\frac{dq_{\mathbf{k}r}}{d\eta}\right) = 0,$$

which for the velocity field (53) has the solution

$$\rho_{\mathbf{k}r}(q_{\mathbf{k}r}, \eta) = \frac{1}{\sqrt{1 + k^2\eta^2}}\rho_{\mathbf{k}r}(q_{\mathbf{k}r}/\sqrt{1 + k^2\eta^2}, 0) \tag{56}$$

for any given $\rho_{\mathbf{k}r}(q_{\mathbf{k}r}, 0)$.

The time evolution amounts to a simple (homogeneous) contraction of both $|\psi_{\mathbf{k}r}|^2$ and $\rho_{\mathbf{k}r}$. At times $\eta < 0$, $|\psi_{\mathbf{k}r}|^2$ is a contracting Gaussian packet of width $\Delta_k(\eta) = \Delta_k(0)\sqrt{1 + k^2\eta^2}$, and in the late-time limit $\eta \to 0$, $|\psi_{\mathbf{k}r}|^2$ approaches a static Gaussian of width $\Delta_k(0) = H/\sqrt{2k^3}$. At times $\eta < 0$, $\rho_{\mathbf{k}r}$ is a contracting arbitrary distribution of width $D_{\mathbf{k}r}(\eta) = D_{\mathbf{k}r}(0)\sqrt{1 + k^2\eta^2}$ [with arbitrary $D_{\mathbf{k}r}(0)$], and in the late-time limit $\eta \to 0$, $\rho_{\mathbf{k}r}$ approaches a static packet of width $D_{\mathbf{k}r}(0)$ [where the asymptotic packet differs from the earlier packet by a homogeneous rescaling of $q_{\mathbf{k}r}$, as in (56)].

For simplicity, we assume that (like Δ_k) the nonequilibrium width $D_{\mathbf{k}r}$ is independent of r and of the direction of $\mathbf{k}$, so that $D_{\mathbf{k}r} = D_k(t)$. We then have the result

$$\frac{D_k(t)}{\Delta_k(t)} = (\text{const in time}) \equiv \sqrt{\xi(k)}. \tag{57}$$

Note that, for each mode, the ''nonequilibrium factor'' $\xi(k)$ may be defined at any convenient fiducial time (in particular, not necessarily at the same time for every k). At least in this lowest-order approximation for the quantum state Ψ, it makes no difference whether we set the initial conditions for nonequilibrium at the same time for all values of k, or at different times for different values of k [for example, at $t(k)$ such that $\lambda_{\text{phys}}(k)$ exceeds some critical value].

VII. NONEQUILIBRIUM POWER SPECTRUM

The above result for the nonequilibrium Bunch-Davies vacuum may be written as

$$\langle|\phi_{\mathbf{k}}|^2\rangle = \langle|\phi_{\mathbf{k}}|^2\rangle_{\text{QT}}\xi(k). \tag{58}$$

This implies that the nonequilibrium power spectrum for the inflaton fluctuations takes the form

$$\mathcal{P}_{\phi}(k) = \mathcal{P}_{\phi}^{\rm QT}(k)\xi(k), \tag{59}$$

which for $k/a \ll H$ reads

$$\mathcal{P}_{\phi}(k) = \frac{H^2}{4\pi^2}\xi(k). \tag{60}$$

The primordial power spectrum for the curvature perturbations is then

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R}}^{\rm QT}(k)\xi(k),$$

where $\mathcal{P}_{\mathcal{R}}^{\rm QT}(k)$ is given by (48). Thus we have

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{\xi(k)}{4\pi^2}\left[\frac{H^4}{\dot{\phi}_0^2}\right]_{t_*(k)}. \tag{61}$$

In general, $\xi(k) \neq 1$ and scale invariance is broken. In future work, along the lines outlined in Sec. IV, we hope to be able to predict features of the function $\xi(k)$. For the purposes of this paper, $\xi(k)$ is (in principle) an arbitrary function to be constrained by observation.

VIII. GENERAL REMARKS

Before considering how CMB data may be used to constrain the nonequilibrium function $\xi(k)$, we make some general remarks on the above scenario.

A. Transfer of microscopic nonequilibrium to cosmological scales

We saw in Sec. VI that, for each mode $\mathbf{k}$ during the inflationary phase, the respective widths $D_k(t)$ and $\Delta_k(t)$ of the nonequilibrium and equilibrium distributions remain in a fixed ratio $D_k(t)/\Delta_k(t) = \sqrt{\xi(k)}$ over time. This holds in the approximation where the inflationary phase is treated as an exact de Sitter expansion. At least to a first approximation, then, we may conclude that quantum nonequilibrium (if it exists) will not relax during the inflationary phase, but is instead preserved over time.

Furthermore, because of the exponential expansion of physical wavelengths $\lambda_{\rm phys}$ during inflation, nonequilibrium (if there is any to start with) will not only be preserved but will also be transferred from microscopic to macroscopic scales. This "magnification" of the nonequilibrium length scale is particularly striking in the late-time or large-wavelength limit $\lambda_{\rm phys} \gg H^{-1}$, where the de Broglie velocity field tends to zero for each mode, $\dot{q}_{\mathbf{k}r} \to 0$. In this limit, which takes effect a few e-foldings after the mode exits the Hubble radius, both $\rho_{\mathbf{k}r}$ and $|\psi_{\mathbf{k}r}|^2$ become *frozen*. Once this happens, any difference between $\rho_{\mathbf{k}r}$ and $|\psi_{\mathbf{k}r}|^2$ is preserved, and is transferred to larger and larger length scales as the physical wavelength $\lambda_{\rm phys} = a(t)(2\pi/k) \propto e^{Ht}$ of the mode gets larger and larger. The frozen nonequilibrium then exists at a physical length scale that grows exponentially with time, from microscopic to macroscopic scales.

Once inflation has ended, there will be a frozen nonequilibrium distribution of curvature perturbations $\mathcal{R}_{\mathbf{k}}$ at macroscopic length scales. These perturbations are then transferred to cosmological length scales by the subsequent (post-inflationary) Friedmann expansion.

B. Quantum measurement of the inflaton field

As we saw in Sec. V B, in the standard quantum theory of inflationary cosmology it is usual to assume that, during inflation, when the physical wavelength of a mode significantly exceeds the Hubble radius, the corresponding inflaton perturbation effectively "becomes classical"—in the sense that the final quantum probability distribution for inflaton (and hence curvature) perturbations behaves, to a good approximation, like a classical probability distribution. As mentioned in Sec. V B, various studies seem to confirm the validity of this assumption [61–68]. In particular, the basis of eigenstates of the field operator $\hat{\phi}$ (suitably smeared with narrow Gaussians) seems to act as a robust pointer basis, so that the quantum distribution of field configurations is, for practical purposes, indistinguishable from a classical distribution [64–66].

In the pilot-wave formulation of inflationary cosmology, there is a well-defined inflaton configuration or "beable" (in Bell's terminology [69]) at all times, even before Hubble exit. In writing the formulas (60) and (61), we have tacitly identified the inflaton beable after Hubble exit with the "classical" inflaton field after Hubble exit—where the latter generates the primordial curvature perturbation via Eq. (39). This identification merits some comment.

Generally speaking, in pilot-wave theory, it is the precise value of the total beable configuration that (together with the wave function) determines the outcome of a subsequent quantum measurement. However, as a rule, one must be cautious about identifying beable values with quantum measurement values: their relationship must be established on a case-by-case basis, through analysis of the particular measurement process that is taking place. As is well known [29,69], there are circumstances where a quantum measurement outcome does not provide a faithful record of the actual prior value of the beable (in which case the so-called quantum "measurement" is in fact not a true measurement). For instance, in the pilot-wave theory of nonrelativistic particles, while the outcome of a quantum position measurement usually has the same value as the actual particle position prior to the measurement, for a quantum momentum measurement the outcome usually does not simply coincide with the prior particle momentum given by de Broglie's velocity formula. Instead, the quantum momentum outcome depends on the initial particle position in a way that depends on the details of the measurement process. Thus, while quantum position measurements are usually "faithful," quantum momentum measurements are usually not.

Similarly, we expect that in the pilot-wave theory of fields, a quantum measurement of the field configuration will (usually) provide a faithful record of the value of the actual field beable appearing in the de Broglie-Bohm dynamics. For other measurements, however, this simple identification will not hold: instead, the outcomes will depend on the initial field beable in a way that depends on the details of the "measurement" process.

Now, in the case at hand, conventional analysis of the quantum-to-classical transition during inflation indicates that the environment effects a quantum measurement of the inflaton field in the basis of field configurations [64–67]. If this is correct, then we are indeed justified in our above identification of the de Broglie-Bohm inflaton field after Hubble exit with the classical inflaton field after Hubble exit.

Should the conventional analysis (for some reason) turn out to be incorrect—in particular, if the quantum-to-classical transition involves effective quantum measurements of the inflaton field in a basis different from the field configuration basis—then there will be a more complicated relationship between the de Broglie-Bohm inflaton field and the emergent classical inflaton field, a relationship that will depend on the details of the effective measurement process. There would then also be a more complicated relationship between the nonequilibrium distribution for the inflaton beable and the nonequilibrium distribution for the primordial curvature perturbations.

C. Weak dependence on pilot-wave dynamics

It is worth noting that the above results for the time evolution of nonequilibrium vacua are only weakly dependent on the details of the de Broglie-Bohm dynamics. The results are in fact determined by just two features: (a) there is a field beable $\phi(\mathbf{x}, t)$ whose time evolution is continuous and differentiable, and (b) the dynamics is "separable," in the sense that for a product quantum state $\Psi[\phi, t] = \prod_{\mathbf{k}r} \psi_{\mathbf{k}r}(q_{\mathbf{k}r}, t)$ the velocity of each component $q_{\mathbf{k}r}$ is independent of the other $q_{\mathbf{k}r}$'s.

To see this, note that from (b) the evolution reduces to that of a collection of independent one-dimensional systems. Then, in each one-dimensional configuration space with coordinate $q_{\mathbf{k}r}$, the local conservation of quantum equilibrium

$$\frac{\partial |\psi_{\mathbf{k}r}|^2}{\partial t} + \frac{\partial(|\psi_{\mathbf{k}r}|^2 v_{\mathbf{k}r})}{\partial q_{\mathbf{k}r}} = 0, \tag{62}$$

for some velocity field $v_{\mathbf{k}r} = v_{\mathbf{k}r}(q_{\mathbf{k}r}, t)$, uniquely fixes $v_{\mathbf{k}r}$ as

$$v_{\mathbf{k}r}(q_{\mathbf{k}r}, t) = \frac{1}{|\psi_{\mathbf{k}r}(q_{\mathbf{k}r}, t)|^2} \int_{q_{\mathbf{k}r}}^{\infty} dq'_{\mathbf{k}r} \frac{\partial |\psi_{\mathbf{k}r}(q'_{\mathbf{k}r}, t)|^2}{\partial t}$$

(assuming that $|\psi_{\mathbf{k}r}|^2 v_{\mathbf{k}r}$ vanishes at infinity), as follows immediately by integrating (62) with respect to the coordinate $q_{\mathbf{k}r}$, from some fixed value $q_{\mathbf{k}r}$ to ∞.

Thus, for the case at hand, the assumption of a differentiable and separable evolution fixes the de Broglie-Bohm velocity field uniquely. Note that this uniqueness arises only because the system reduces to a collection of independent one-dimensional systems. It is only in one dimension that the local conservation of quantum equilibrium fixes the velocity field. In two or more dimensions, other velocity fields are possible, distinct from that of de Broglie and Bohm [70].

The conditions (a) and (b) could certainly be violated in other hidden-variables theories. There might, for example, be no field beable $\phi(\mathbf{x}, t)$ at all. Also, it is possible to have a pilot-wave-type theory with a nonseparable dynamics [71]. Still, property (a) might well emerge in some limit from a deeper hidden-variables theory. And property (b) seems desirable, even if not strictly necessary. In any case, our point here is to emphasize that (a) and (b) are the only features of pilot-wave dynamics that really enter into our considerations.

IX. BOUND ON PRIMORDIAL QUANTUM NONEQUILIBRIUM

Let us now illustrate how the available data may be used to constrain the nonequilibrium function $\xi(k)$ appearing in the result (61) for the primordial power spectrum, where the observed spectrum $\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R}}^{\mathrm{QT}}(k)\xi(k)$ consists of the usual quantum contributions together with possible nonequilibrium corrections ($\xi \neq 1$).

It is currently a very active field of research to determine the k dependence of the observed spectrum $\mathcal{P}_{\mathcal{R}}(k)$, and to compare the results with the k dependence of the quantum-theoretical prediction $\mathcal{P}_{\mathcal{R}}^{\mathrm{QT}}(k)$. It is straightforward to reinterpret these studies as effectively providing constraints on the nonequilibrium function $\xi(k)$.

The observed spectrum $\mathcal{P}_{\mathcal{R}}(k)$ is usually parametrized in terms of the spectral index $n(k)$, defined by

$$n(k) - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}, \tag{63}$$

and the running of the spectral index, $n'(k) \equiv dn/d\ln k$. For $n(k)$ approximately constant, it is convenient to write the power spectrum in the form

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R}}(k_0)\left(\frac{k}{k_0}\right)^{n(k)-1}, \tag{64}$$

where k_0 is some chosen reference or pivot point. [Note that the definitions (63) and (64) of $n(k)$ generally agree at $k = k_0$ only, and they agree for all k if $dn(k)/dk = 0$.] The index $n(k)$ may be written as a Taylor expansion

$$n(k) = n_0 + \frac{1}{2} \ln\left(\frac{k}{k_0}\right) n'_0 + \ldots,$$

where $n_0 \equiv n(k_0)$ is the spectral index at $k = k_0$, and $n'_0 \equiv (dn/d\ln k)_0$ is the running of the spectral index at $k = k_0$.

The observed values of $n(k)$, $n'(k)$ may be used to set bounds on early quantum nonequilibrium. To illustrate this, we shall consider a best-fit value of n_0,

$$n_0 = 0.960^{+0.014}_{-0.013} \tag{65}$$

at $k_0 = 0.002\ \mathrm{Mpc}^{-1}$ [59]. [Adding nonequilibrium parameters would of course affect the best-fitting procedure, but the value (65) suffices here for illustration. A best fitting of nonequilibrium inflationary models to CMB data is outside the scope of this paper.]

We have $\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}^{\mathrm{QT}}_{\mathcal{R}}(k)\xi(k)$, where $\mathcal{P}^{\mathrm{QT}}_{\mathcal{R}}(k)$ is predicted by standard inflationary theory. One may adopt the following parametrization:

$$\mathcal{P}^{\mathrm{QT}}_{\mathcal{R}}(k) = \mathcal{P}^{\mathrm{QT}}_{\mathcal{R}}(k_0)\left(\frac{k}{k_0}\right)^{n^{\mathrm{QT}}(k)-1}, \tag{66}$$

where $n^{\mathrm{QT}}(k)$ is the usual (quantum-theoretical) spectral index, and

$$\xi(k) = \xi(k_0)\left(\frac{k}{k_0}\right)^{\nu(k)-1}, \tag{67}$$

where $\nu(k)$ is the "nonequilibrium spectral index." The observed index (minus 1) is then a sum

$$(n-1) = (n^{\mathrm{QT}} - 1) + (\nu - 1) \tag{68}$$

of contributions from quantum theory and from nonequilibrium corrections.

In the exact limit $\dot{H} \to 0$, we have $n_{\mathrm{QT}} - 1 = 0$; and in exact quantum equilibrium, we have $\nu - 1 = 0$. Slow-roll inflation predicts a small tilt [1,58]

$$n^{\mathrm{QT}}(k) - 1 = -6\varepsilon + 2\eta \tag{69}$$

where, in the definitions (38) of ε and η, the quantities V and $dV/d\phi_0$ are evaluated at $t_{\mathrm{exit}}(k)$ (for which $k = aH$).

Defining $\nu_0 \equiv \nu(k_0)$ and $n_0^{\mathrm{QT}} \equiv n^{\mathrm{QT}}(k_0)$, we obtain a bound for $|\nu_0 - 1|$ on the assumption that $|n_0^{\mathrm{QT}} - 1|$ is indeed significantly less than 1 (as predicted by inflation). Otherwise, in principle, both $n_0^{\mathrm{QT}} - 1$ and $\nu_0 - 1$ could be large—with comparable magnitudes and opposite signs—and the observed small value of their sum $n_0 - 1 = -0.04^{+0.014}_{-0.013}$ could be an accident. We assume here that the observed small value $|n_0 - 1| \lesssim 0.1$ is not due to such a "conspiratorial" cancellation. Then, roughly, we may write (again at $k_0 = 0.002\ \mathrm{Mpc}^{-1}$)

$$|n_0^{\mathrm{QT}} - 1| \lesssim 0.1, \qquad |\nu_0 - 1| \lesssim 0.1. \tag{70}$$

The bound $|\nu_0 - 1| \lesssim 0.1$ on the nonequilibrium index may be converted into a bound on the hidden-variable entropy $S_{\mathrm{hv}}(k)$—defined by (28)—for modes with k close to $k_0 = 0.002\ \mathrm{Mpc}^{-1}$. [As we have seen, $S_{\mathrm{hv}}(k)$ is the relative entropy of $\rho_{\mathbf{k}r}$ with respect to $|\psi_{\mathbf{k}r}|^2$, and is a natural measure of the difference between $\rho_{\mathbf{k}r}$ and $|\psi_{\mathbf{k}r}|^2$.] We have $\xi(k) \equiv D_k^2/\Delta_k^2$, where D_k and Δ_k are the widths of $\rho_{\mathbf{k}r}$ and $|\psi_{\mathbf{k}r}|^2$, respectively. We know that $|\psi_{\mathbf{k}r}|^2$ is a Gaussian packet, and that in the late-time limit $\Delta_k^2 = H^2/2k^3$. For the purposes of illustration, let us model $\rho_{\mathbf{k}r}$ as a Gaussian (of width D_k). We then have

$$S_{\mathrm{hv}}(k) = \tfrac{1}{2}(1 - \xi(k) + \ln\xi(k)), \tag{71}$$

with $\xi(k)$ parametrized by (67). If $\nu(k)$ varies slowly, then close to k_0 we may write $\nu(k) \approx \nu_0$. Taking $\xi(k_0) = 1$ and assuming that $|\nu_0 - 1|$ is small, we have

$$S_{\mathrm{hv}}(k) \approx -\tfrac{1}{4}(\nu_0 - 1)^2 \ln^2(k/k_0).$$

Restricting ourselves to a range of k close to k_0, such that $|\ln(k/k_0)| \lesssim O(1)$, we then have

$$|S_{\mathrm{hv}}(k)| \lesssim \tfrac{1}{4}(\nu_0 - 1)^2 \lesssim 10^{-2}. \tag{72}$$

Note that approximate equilibrium in this region (close to k_0) does not preclude large departures from equilibrium at much smaller or at much larger values of k.

X. POSSIBLE LOW-POWER ANOMALY AT SMALL l

In the low-l region (say $l \lesssim 20$), the angular power spectrum is dominated by the Sachs-Wolfe effect (resulting from nonuniformities in the local gravitational potential on the last scattering surface).

In this region, $\mathcal{T}^2(k, l) = \pi H_0^4 j_l^2(2k/H_0)$ [1], where H_0 is the Hubble constant today, so that [using (36)]

$$C_l = \frac{H_0^4}{2\pi}\int_0^\infty \frac{dk}{k} j_l^2(2k/H_0)\mathcal{P}_{\mathcal{R}}(k).$$

For $\mathcal{P}_{\mathcal{R}}(k) = \mathrm{const}$ we then have

$$C_l \propto \int_0^\infty \frac{dk}{k} j_l^2(k) = \frac{1}{2l(l+1)},$$

so that $l(l+1)C_l = \mathrm{const}$ at low l—the Sachs-Wolfe plateau —as seems to be approximately observed. (The integrated Sachs-Wolfe effect, taking place along the line of sight, adds a small "rise" at very small l.)

It has been suggested that the data contain anomalously low power at small l, though this is controversial. If there is such low power, it could of course be due to some inadequate processing of the data (such as in the modeling of foregrounds) or to some local astrophysical effect. Otherwise, the signal could be primordial in origin, reflecting an anomaly in the underlying spectrum $\mathcal{P}_{\mathcal{R}}(k)$ of curvature perturbations. In the latter case, the explanation might lie in some modification of the standard inflationary scenario, or in new physics.

If there is a low-power signal at small l requiring new physics, then quantum nonequilibrium provides a possible candidate. Taking $\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}^{\mathrm{QT}}_{\mathcal{R}}(k)\xi(k)$, and assuming (to a first approximation) that $\mathcal{P}^{\mathrm{QT}}_{\mathcal{R}}(k) = \mathrm{const}$, we may write

$$\frac{C_l}{C_l^{\mathrm{QT}}} = 2l(l+1)\int_0^\infty \frac{dk}{k} j_l^2(2k/H_0)\xi(k). \tag{73}$$

If $\xi(k) = 1$ everywhere, then $C_l/C_l^{\rm QT} = 1$. A low-power anomaly, $C_l < C_l^{\rm QT}$, could be explained by having $\xi(k) < 1$ in some suitable region of k space. Because the integral is dominated by the scale $k \approx lH_0/2$, a significant drop in C_l requires $\xi(k) < 1$ for k in this region, that is, $\xi(k) < 1$ for wavelengths $\lambda \approx (4\pi/l)H_0^{-1}$ (comparable to today's Hubble radius).

To have $\xi(k) < 1$ for a primordial perturbation mode $\mathbf{k}$ means that the width D_k of the nonequilibrium distribution for the corresponding inflaton mode is less than the quantum equilibrium width Δ_k. It is reasonable to expect this, if one accepts the scenario of Sec. II, according to which quantum noise arises from statistical relaxation processes (presumably taking place in the very early Universe). On this view, it is natural to assume that early nonequilibrium would have a less-than-quantum dispersion, or $\xi(k) < 1$—as opposed to an early larger-than-quantum dispersion [$\xi(k) > 1$] which, while possible in principle, seems less natural. Furthermore, we saw in Sec. IV A that, in a supposed preinflationary era, relaxation to quantum equilibrium can be suppressed on large spatial scales, and one expects that at the onset of inflation nonequilibrium is most likely to have survived at wavelengths $\lambda \gtrsim \lambda_{\rm c}$, where the value of $\lambda_{\rm c}$ remains to be estimated (pending the development of an appropriate preinflationary model). Therefore, it appears that a dip $\xi(k) < 1$ in the power spectrum below some critical wave number $k_{\rm c} = 2\pi/\lambda_{\rm c}$ could be naturally explained in terms of early quantum nonequilibrium surviving from a very early preinflationary era, though this possibility remains to be developed in detail.

As for the possible production of nonequilibrium in the Planckian regime (Sec. IV B), in the absence of a more detailed model we are unable to give any strong argument for $\xi(k) < 1$, as opposed to $\xi(k) > 1$, for modes with $\lambda \lesssim \lambda'_{\rm c}$ (where $\lambda'_{\rm c}$ remains to be estimated; see Sec. IV B). Again modeling $\rho_{\mathbf{k}r}$ as a Gaussian, the hidden-variable entropy $S_{\rm hv}(k)$ for a single mode is given in terms of $\xi(k)$ by (71). For a given value of $S_{\rm hv}(k)$—perhaps set by some application of (27)—Eq. (71) possesses two solutions for $\xi(k)$, one with $\xi < 1$ and one with $\xi > 1$. That is, the same nonequilibrium entropy can be achieved by both a less-than-quantum and a larger-than-quantum dispersion. On the other hand, from the behavior of the function $1 - \xi + \ln\xi$, one sees that the solution with $\xi < 1$ always has a smaller value of $|\xi - 1|$ than does the solution with $\xi > 1$; that is, the solution with $\xi < 1$ has a dispersion that is closer to the quantum value. On this (weak) basis, one might suggest that $\xi < 1$ will be preferred. A stronger argument for $\xi < 1$ might come from a detailed understanding of the preservation of information by means of nonequilibrium noise suppression in the outgoing quantum state of an evaporating black hole.

In any case, focusing here on the comparison with observation, let us consider the effect at low l of some simple examples of functions $\xi(k)$.

As a first example, motivated by a possible long-wavelength suppression of relaxation at very early (preinflationary) times, we take $\xi(k) = 0$ for $k < k_{\rm c}$ and $\xi(k) = 1$ for $k > k_{\rm c}$, where the simple cutoff is used to model a suppression of quantum noise at wavelengths $\lambda > \lambda_{\rm c} = 2\pi/k_{\rm c}$. We then have

$$\frac{C_l - C_l^{\rm QT}}{C_l^{\rm QT}} = -2l(l+1)\int_0^{k_{\rm c}} \frac{dk}{k} j_l^2(2k/H_0). \tag{74}$$

Again, the dominant scale is $k \approx lH_0/2$, and the correction to C_l will be significant only if the range of integration $(0, k_{\rm c})$ overlaps substantially with this scale—that is, $k_{\rm c}$ cannot be much smaller than $lH_0/2$.

Note that if, instead, we did take $k_{\rm c} \ll lH_0/2$, the correction to C_l would not only be small, it would be unobservable even in principle, because it would be smaller than the cosmic variance (32). For $k \ll lH_0/2$ we have approximately $j_l^2(2k/H_0) \approx (2^l l!/(2l+1)!)^2 (2k/H_0)^{2l}$, so that

$$\frac{C_l - C_l^{\rm QT}}{C_l^{\rm QT}} \approx -(l+1)\left(\frac{2^l l!}{(2l+1)!}\right)^2 \left(\frac{2k_{\rm c}}{H_0}\right)^{2l}.$$

This correction falls off rapidly with increasing l, and is very small even for the lowest values of l: for example, even taking $2k_{\rm c}/H_0 \approx 1$, we find $(C_4 - C_4^{\rm QT})/C_4^{\rm QT} \approx -6 \times 10^{-6}$. Because such corrections are much smaller than the cosmic variance $\Delta C_l^{\rm sky}/C_l = \sqrt{2/(2l+1)}$, they cannot be measured meaningfully, even in principle. To obtain a measurable effect, the cutoff $k_{\rm c}$ in (74) must not be small compared to $lH_0/2$.

A second example is motivated by the possibility of gravitationally induced nonequilibrium at small scales, at wavelengths $\lambda \lesssim \lambda'_{\rm c}$. If we assume that the nonequilibrium takes the form of noise suppression ($\xi < 1$), one might model this again with a simple cutoff, taking $\xi(k) = 1$ for $k < k'_{\rm c}$ and $\xi(k) = 0$ for $k > k'_{\rm c}$, where $k'_{\rm c} = 2\pi/\lambda'_{\rm c}$. We then have

$$\frac{C_l - C_l^{\rm QT}}{C_l^{\rm QT}} = -2l(l+1)\int_{k'_{\rm c}}^{\infty} \frac{dk}{k} j_l^2(2k/H_0). \tag{75}$$

For a significant effect, the range of integration $(k'_{\rm c}, \infty)$ must again overlap substantially with the dominant region $k \approx lH_0/2$—which now implies that $k'_{\rm c}$ cannot be much larger than $lH_0/2$.

As a third example, we consider a power law

$$\xi(k) = \xi(k_0)\left(\frac{k}{k_0}\right)^{\nu_0 - 1} \tag{76}$$

(with constant index ν_0). From (73) we then have

$$\frac{C_l}{C_l^{\rm QT}} = 2l(l+1)\xi(k_0)\left(\frac{H_0}{2k_0}\right)^{\nu_0 - 1} \int_0^{\infty} dx\, j_l^2(x) x^{\nu_0 - 2}, \tag{77}$$

where

$$\int_0^\infty dx j_l^2(x) x^{\nu_0 - 2} = \frac{\sqrt{\pi}}{4} \frac{\Gamma[(3-\nu_0)/2]\Gamma[l+(\nu_0-1)/2]}{\Gamma[(4-\nu_0)/2]\Gamma[l+(5-\nu_0)/2]}.$$

Should the existence of a low-power anomaly be confirmed, one might try to match the anomaly with one of the above nonequilibrium spectra (74), (75), or (77).

According to the analysis in Ref. [72], cutting off the power below a wave number $k_c \sim 3 \times 10^{-4}$ Mpc^{-1} (comparable to the inverse Hubble scale $H_0 = 2.4 \times 10^{-4}$ Mpc^{-1}) slightly improves the fit to the three-year Wilkinson Microwave Anisotropy Probe data, but the improvement does not seem large enough to justify any conclusion that such a cutoff really exists. Still, the possibility of reduced power at large scales is worth exploring, since it could originate from an early nonequilibrium suppression of quantum noise (as discussed in Sec. IVA).

XI. NONRANDOM PHASES AND INTERMODE CORRELATIONS

So far, we have considered only the angular power spectrum C_l of the microwave sky, and how this could be affected by nonequilibrium corrections to the primordial (scalar) power spectrum $\mathcal{P}_{\mathcal{R}}(k)$. Here, we consider how primordial non-Gaussianity could arise from early quantum nonequilibrium.

The primordial curvature perturbations $\mathcal{R}_{\mathbf{k}}$ are usually assumed to constitute a Gaussian random field, for which the power spectrum provides a complete characterization of the statistical properties. The phases of Gaussian perturbations are randomly distributed, and there are no intermode correlations.

In standard inflationary scenarios, the Gaussianity of $\mathcal{R}_{\mathbf{k}}$ arises directly from the Gaussianity of the quantum vacuum fluctuations of the inflaton perturbation $\phi_{\mathbf{k}}$. (The Gaussianity of $\mathcal{R}_{\mathbf{k}}$ is not, as is sometimes claimed, a mere consequence of the central limit theorem.) In the quantum Bunch-Davies vacuum, the inflaton probability distribution at conformal time η is given by

$$P^{\rm QT}[\phi, \eta] = |\Psi[\phi, \eta]|^2 = \prod_{\mathbf{k}r} |\psi_{\mathbf{k}r}(q_{\mathbf{k}r}, \eta)|^2,$$

where, as we saw in Sec. VI, each $|\psi_{\mathbf{k}r}|^2$ is a Gaussian of zero mean and width $\Delta_k^2 = (H^2/2k^3)(1 + k^2\eta^2)$. The two-point function $\langle 0|\hat{\phi}(\mathbf{x}, \eta)\hat{\phi}(\mathbf{x}', \eta)|0\rangle$ is given by (44). The three-point function $\langle 0|\hat{\phi}(\mathbf{x}, \eta)\hat{\phi}(\mathbf{x}', \eta)\hat{\phi}(\mathbf{x}'', \eta)|0\rangle$ vanishes, as do all odd-point functions. Higher n-point functions (for n even) reduce to sums of products of the two-point function, as expected for a Gaussian random field. In quantum equilibrium, then, the generation of primordial curvature perturbations $\mathcal{R}_{\mathbf{k}} \propto \phi_{\mathbf{k}}$ by inflaton perturbations is a Gaussian random process.

However, as a general matter of principle, the primordial perturbations could be non-Gaussian. And if one considers quantum nonequilibrium for the inflaton field, there is no particular reason why the nonequilibrium inflaton fluctuations should be Gaussian.

We have already seen that, in quantum nonequilibrium, the probability distribution $\rho_{\mathbf{k}r}(q_{\mathbf{k}r}, \eta)$ for a single mode of the inflaton field need not take the quantum Gaussian form (49). Simple forms of non-Gaussianity include a nonzero skewness or kurtosis of $\rho_{\mathbf{k}r}(q_{\mathbf{k}r}, \eta)$ [where the marginal $\rho_{\mathbf{k}r}(q_{\mathbf{k}r}, \eta)$ for $q_{\mathbf{k}r}$ may, in general, be obtained from a correlated joint distribution $P[q_{\mathbf{k}r}, \eta]$, as discussed further below]. But non-Gaussianity can take on a wide variety of forms, and various measures of it have been proposed. Some workers have reported significant primordial non-Gaussianity in the CMB data [73], while others maintain that the data are consistent with primordial Gaussianity [59].

Let us show how quantum nonequilibrium can result in nonrandom phases and intermode correlations for the primordial perturbations.

The coefficients $a_{lm} = |a_{lm}|e^{i\varphi_{lm}}$ in the spherical harmonic expansion (29) are of course generally complex numbers, and their phases φ_{lm} contain a lot of information about the morphology of the temperature anisotropy $\Delta T(\theta, \phi)$ (see, for example, Ref. [74]). Assuming again that the underlying "ensemble of skies" is statistically rotationally invariant, the probability distribution for each φ_{lm} must be independent of m. For a fixed value of l, we then have $2l + 1$ phases φ_{lm} with the same probability distribution $p_l(\varphi_{lm})$, and for large l we may use the measured values of the φ_{lm} to probe $p_l(\varphi_{lm})$. At least to a first approximation, current data are consistent with $p_l(\varphi_{lm})$ being uniform on the unit circle. According to the basic formula (34), each a_{lm} is a linear combination of all the curvature perturbation components $\mathcal{R}_{\mathbf{k}}$. And according to the inflationary result (39), each $\mathcal{R}_{\mathbf{k}}$ is proportional to the late-time inflaton perturbation $\phi_{\mathbf{k}}$. Thus, the phase φ_{lm} of each a_{lm} is ultimately determined by the phases $\theta_{\mathbf{k}}$ of all the inflaton perturbation components $\phi_{\mathbf{k}} = |\phi_{\mathbf{k}}|e^{i\theta_{\mathbf{k}}}$.

In quantum equilibrium, the inflaton phases $\theta_{\mathbf{k}}$ have a time-independent distribution $\rho_{\mathbf{k}}^{\rm QT}(\theta_{\mathbf{k}})$ that is uniform on the unit circle:

$$\rho_{\mathbf{k}}^{\rm QT}(\theta_{\mathbf{k}}) = \frac{1}{2\pi}.$$

This follows immediately from (49): the real and imaginary parts of $\phi_{\mathbf{k}} = \frac{\sqrt{V}}{(2\pi)^{3/2}}(q_{\mathbf{k}1} + iq_{\mathbf{k}2})$ have a joint Gaussian distribution $\propto e^{-(q_{\mathbf{k}1}^2 + q_{\mathbf{k}2}^2)/2\Delta_k^2}$ that is always constant on circles centered on the origin in the complex $\phi_{\mathbf{k}}$ plane.

In quantum nonequilibrium, the inflaton phases can at some initial (conformal) time η_i have an arbitrary distribution $\rho_{\mathbf{k}}(\theta_{\mathbf{k}}, \eta_i)$. Will the subsequent time evolution generate a late-time distribution that tends towards uniformity on the unit circle? Not in the approximation considered here. The trajectories $q_{\mathbf{k}r}(\eta) = q_{\mathbf{k}r}(0)\sqrt{1 + k^2\eta^2}$ obtained in Sec. VI imply that

$$\theta_{\mathbf{k}}(\eta) = \tan^{-1}(q_{\mathbf{k}2}(\eta)/q_{\mathbf{k}1}(\eta)) = \tan^{-1}(q_{\mathbf{k}2}(0)/q_{\mathbf{k}1}(0)).$$

Thus, during inflation, the phase $\theta_{\mathbf{k}}$ of each inflaton mode is static, so that any initial nonequilibrium distribution (with nonrandom phases) will remain unchanged over time, $\rho_{\mathbf{k}}(\theta_{\mathbf{k}}, \eta) = \rho_{\mathbf{k}}(\theta_{\mathbf{k}}, \eta_i)$ for all values of conformal time η. [In the complex $\phi_{\mathbf{k}}$ plane, the evolution of the joint probability distribution for $q_{\mathbf{k}1}$, $q_{\mathbf{k}2}$ amounts to a purely radial contraction with time, so that the distribution $\rho_{\mathbf{k}}(\theta_{\mathbf{k}}, \eta)$ of phases is time independent.]

We conclude that the time evolution during the inflationary era does not scramble the phases of the inflaton perturbations. Any initial nonuniformity (or nonrandomness) in the phase distribution will remain frozen, all the way to the late-time limit $\eta \to 0$. It would be interesting, in future work, to explore how this could affect the phases φ_{lm} of the measured coefficients a_{lm} in the temperature anisotropy.

We now consider nonequilibrium intermode correlations. In Sec. VI we assumed, for simplicity, that the nonequilibrium distribution satisfied the factorizability condition (19), so that the modes were uncorrelated even in nonequilibrium. However, in principle, correlations among modes are possible: in quantum nonequilibrium, the inflaton modes can be correlated even though $|\Psi|^2$ (for the Bunch-Davies vacuum) is a product.

In terms of conformal time η, an arbitrary correlated joint distribution $P[q_{\mathbf{k}r}, \eta]$ will evolve according to the continuity equation

$$\frac{\partial P}{\partial \eta} + \sum_{\mathbf{k}r} \frac{\partial}{\partial q_{\mathbf{k}r}}\left(P \frac{dq_{\mathbf{k}r}}{d\eta}\right) = 0. \tag{78}$$

Because the wave functional is still that of the Bunch-Davies vacuum, the velocity field $dq_{\mathbf{k}r}/d\eta$ is still given by (53) and the trajectories in configuration space are still given (mode by mode) by the result (54). Given the trajectories, the general solution of (78) may be constructed using the property that $P/|\Psi|^2$ is constant along trajectories (where this follows from the fact that P and $|\Psi|^2$ obey the same continuity equation). Replacing the labels $\mathbf{k}r$ by a single index n, we may equate

$$\frac{P(q_1(0), q_2(0), \ldots, q_n(0), \ldots, 0)}{|\psi_1(q_1(0), 0)|^2 |\psi_2(q_2(0), 0)|^2 \ldots |\psi_n(q_n(0), 0)|^2 \ldots}$$

with

$$\frac{P(q_1(\eta), q_2(\eta), \ldots, q_n(\eta), \ldots, \eta)}{|\psi_1(q_1(\eta), \eta)|^2 |\psi_2(q_2(\eta), \eta)|^2 \ldots |\psi_n(q_n(\eta), \eta)|^2 \ldots}.$$

Using the trajectories $q_n(\eta) = q_n(0)\sqrt{1 + k_n^2 \eta^2}$ and

$$\frac{|\psi_n(q_n, \eta)|^2}{|\psi_n(q_n/\sqrt{1 + k_n^2\eta^2}, 0)|^2} = \frac{1}{\sqrt{1 + k_n^2\eta^2}} = \frac{\Delta_n(0)}{\Delta_n(\eta)}$$

[where the width $\Delta_n(\eta)$ is given by (55)], we deduce that

$$P(q_1, q_2, \ldots, q_n, \ldots, \eta) = P\left(\frac{\Delta_1(0)}{\Delta_1(\eta)} q_1, \frac{\Delta_2(0)}{\Delta_2(\eta)} q_2, \ldots, \frac{\Delta_n(0)}{\Delta_n(\eta)} q_n, \ldots, 0\right) \prod_n \frac{\Delta_n(0)}{\Delta_n(\eta)}.$$

This is an exact solution for the evolution of an arbitrary distribution, expressed in terms of the distribution $P(q_1, q_2, \ldots, q_n, \ldots, 0)$ at conformal time $\eta = 0$.

The possibility of nonequilibrium allows the distribution $P[q_{\mathbf{k}r}, \eta_i]$ at some initial time η_i to be, in principle, anything at all. To narrow down the range of possibilities, one might impose the requirement of statistical homogeneity, $P[\phi(\mathbf{x} - \mathbf{d}), \eta_i] = P[\phi(\mathbf{x}), \eta_i]$ (for arbitrary spatial displacements $\mathbf{d}$).

Clearly, allowing nonrandom phases and intermode correlations in the inflationary vacuum opens up a number of novel possibilities. Indeed, the subject of non-Gaussianity for quantum nonequilibrium states deserves to be developed in more detail.

XII. CONCLUSION

We have shown how inflationary cosmology (assuming it to be essentially correct) may be used to test the validity of quantum theory at very short distances and at very early times. In particular, we have considered the possible effects of quantum nonequilibrium, as described by the hidden-variables theory of de Broglie and Bohm, during the inflationary phase. We have shown, by means of simple examples, how CMB data may be used to set bounds on nonequilibrium deviations from quantum theory.

As for the possible origin of such deviations, we have outlined a scenario where quantum nonequilibrium during the inflationary phase arises from relaxation suppression (for long-wavelength modes) in a preinflationary era. This scenario suggests that primordial nonequilibrium could set in above some infrared cutoff λ_c (though the value of λ_c remains to be estimated). We have also considered the more speculative possibility that nonequilibrium could be generated during the inflationary era, by novel gravitational effects at the Planck scale.

We have, for the most part, discussed quantum nonequilibrium corrections to the primordial (scalar) power spectrum $\mathcal{P}_{\mathcal{R}}(k)$. A preliminary discussion was also given showing how primordial non-Gaussianity (in particular, nonrandom phases and intermode correlations) could also arise from early quantum nonequilibrium.

In this paper we have, for simplicity, considered only the (dominant) scalar part of the primordial perturbations. The standard quantum theory of perturbations around a classical background includes tensor contributions (transverse-traceless metric perturbations, or gravitational waves), as well as the scalar part considered here [1,58,60]. It would be straightforward to extend the present treatment to include tensor perturbations. The standard formalism may be written in the functional Schrödinger picture and converted into a de Broglie-Bohm theory in the usual way, by reinterpreting the quantum probability current in configuration space in terms of an equilibrium ensemble of trajectories. [As mentioned in Sec. II, a de Broglie-Bohm velocity field (2) may be defined by this means for any system with a Hamiltonian given by a differential operator on configuration space [32].] Once the velocity field for the trajectories has been identified, one can consider the evolution of an arbitrary nonequilibrium ensemble. The extended configuration would now include the transverse-traceless metric perturbations (with two independent components, corresponding to the two possible states of polarization, each with approximately the same action as a free massless scalar field). A de Broglie-Bohm velocity field would be defined for these degrees of freedom as well.

An important topic that should be examined is how quantum nonequilibrium would affect the consistency relation between the power spectra for the scalar and tensor perturbations. This relation is especially interesting because it relates fluctuations for distinct degrees of freedom, and because it is independent of the form of the inflaton potential. Presumably, quantum nonequilibrium would, in general, have different effects on different degrees of freedom, resulting in a violation of the consistency relation.

If standard inflationary cosmology is essentially correct, then observations of the CMB have already confirmed—to a first approximation—the validity of the quantum (Born-rule) prediction for the inflaton power spectrum during the inflationary phase. More accurate measurements of the primordial power spectrum will enable us to set unprecedented bounds on violations of quantum theory, at very short distances and at very early times. And close scrutiny of other possible features, such as various forms of non-Gaussianity, will provide further tests of basic quantum predictions.

Should inflation be very firmly established, and should it be found that the predictions of quantum theory continue to hold well at all accessible length scales during the inflationary era, then this would constitute considerable evidence against the hypothesis of quantum nonequilibrium at the big bang (though, of course, nonequilibrium from an earlier era might simply have not survived into the inflationary phase). Furthermore, it would rather undermine the view that quantum theory is merely an effective description of an equilibrium state. In principle, one could still believe that hidden variables exist, and that the hidden-variables distribution is restricted to quantum equilibrium even at the shortest distances and earliest times. But in the complete absence of nonequilibrium, the detailed behavior of the hidden variables (such as the precise form of the trajectories in de Broglie-Bohm theory) would be forever untestable. While exact equilibrium always and everywhere may constitute a logically possible world, from a general scientific point of view it seems unacceptable, and the complete ruling out of quantum nonequilibrium by experiment would suggest that hidden-variables theories should be abandoned.

On the other hand, a positive detection of quantum nonequilibrium phenomena in the early Universe (or indeed elsewhere [25]) would be of fundamental interest, opening up a new and deeper level of nature to experimental investigation.

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