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Nonperturbative calculation of the anomalous magnetic moment in the Yukawa model within truncated Fock space

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Within the covariant formulation of light-front dynamics, we calculate the state vector of a physical fermion in the Yukawa model. The state vector is decomposed in Fock sectors and we consider the first three ones: the single constituent fermion, the constituent fermion coupled to one scalar boson, and the constituent fermion coupled to two scalar bosons. This last three-body sector generates nontrivial and nonperturbative contributions to the state vector, which are calculated numerically. Field-theoretical divergences are regularized using Pauli-Villars fermion and boson fields. Physical observables can be unambiguously deduced using a systematic renormalization scheme we have developed previously. As a first application, we consider the anomalous magnetic moment of the physical fermion.

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I. INTRODUCTION

The understanding of hadronic systems in terms of their elementary degrees of freedom has been, and still is, one of the most challenging problems in particle and nuclear physics over the last ten years. The phenomenological properties of hadrons are now rather well understood in terms of models, like the constituent quark model or the bag model. The understanding of their properties from the original Lagrangian of QCD is, however, still under active development.

In nuclear physics, the properties of nuclear structure in terms of the exchanges of pions are also well known. They are described by using a phenomenological nucleon-nucleon potential expressed in terms of the exchanges of one pion, two correlated pions, and so on. However, their complete description from an effective chiral Lagrangian is still missing.

A common difficulty in both domains is the description of relativistic bound systems. This description should be nonperturbative from the start in order to be able to find, for instance, the physical mass of the bound state from the pole of the scattering amplitude. The problem is especially acute when the interaction coupling constant is large.

One of the most relevant approaches aimed at studying relativistic systems of interacting particles is light-front dynamics (LFD), proposed initially by Dirac [1]. LFD is a form of Hamiltonian dynamics which deals with the state vector defined not at a fixed time moment, but on the light-front plane $t + z = 0$, in its traditional form. The state vector is then usually decomposed in a series of Fock sectors, each containing a fixed number of particles.

The use of LFD to investigate relativistic bound states has been advocated for a long time. However, while the dynamics of few-body systems, based on a phenomenologically constructed interaction, has developed rapidly, the application of LFD to field theory beyond a perturbative framework is not yet under complete theoretical control. This is due to the fact that any practical calculation relies on taking into account only a restricted number of Fock sectors in the state vector decomposition or, in other words, on the Fock space truncation. This approximation strongly complicates the renormalization procedure, in contrast to that in standard perturbation theory. Indeed, the full cancellation of field-theoretical divergences which appear in a given Fock sector requires taking into account contributions from other sectors. If even a part of the latter is beyond our approximation, some divergences may leave uncancelled. Mathematically, it reflects itself in a possible dependence of approximately calculated observables on the regularization parameters (e.g., cutoffs). This prevents us from making any physical predictions if we cannot control the renormalization procedure in one way or another.

In a previous study [2] (see also references therein) we have developed an appropriate renormalization procedure—the so-called Fock sector dependent renormalization (FSDR) scheme—in order to keep the cancellation of field-theoretical divergences under permanent control. Our approach is based on the covariant formulation of LFD (CLFD), where the state vector is defined on an arbitrary light-front plane characterized by a lightlike four-vector ω [3] and given by the equation $\omega \cdot x = 0$. The covariant formulation is necessary in order to control any violation of rotational invariance, including that which is caused by the Fock space truncation. In particular, this is important in order to formulate, in an unambiguous way, the renormalization conditions one should impose on the bare coupling constant (BCC) to relate it to the physical one.

In Ref. [2] we calculated the fermion state vector and the electromagnetic form factors within the Yukawa model and QED in the lowest nontrivial approximation, when the state vector includes only two Fock sectors given by one constituent fermion and one constituent fermion coupled to

one boson. For this two-body Fock space truncation, the electromagnetic form factors are identical to those obtained in the second order of perturbation theory, giving rise to a Schwinger-type correction to the fermion magnetic moment. Note that this result is not surprising, in spite of the fact we have not done any expansion in the powers of the coupling constant, since no other contributions to the fermion electromagnetic vertex, apart from the perturbative ones (resummed to all orders), are generated in the two-body truncation.

We shall present in this work the calculation of the fermion anomalous magnetic moment (AMM) within the same Yukawa model, but for the three-body Fock space truncation, when the state vector includes an additional Fock sector containing one constituent fermion coupled to two scalar bosons. The presence of three-body states gives rise to nontrivial nonperturbative contributions to the AMM, which can not be fully incorporated in perturbation theory. Besides that, the Yukawa model is a quite nontrivial one from the point of view of the renormalization procedure, since it exhibits simultaneously, mass, vertex, and wave function renormalization.

The plan of the article is the following. We recall in Sec. II the main features of our nonperturbative approach and the renormalization procedure. The eigenvalue equations are derived in the three-body truncation in Sec. III. We calculate the fermion electromagnetic form factors in Sec. IV and present our numerical results for the AMM in Sec. V. Our conclusions are drawn in Sec. VI. The appendices collect all necessary details to calculate the AMM.

II. BOUND STATE SYSTEMS IN LIGHT-FRONT DYNAMICS

A. General framework

The state vector $\phi(p)$ describing any relativistic system with a total four-momentum p forms a representation of the Poincaré group. The four-momentum operator squared $\hat{P}^2$ is one of the Casimir operators of this group and the state vector satisfies the equation

$$\hat{P}^2\phi(p) = M^2\phi(p), \tag{1}$$

where M is the mass of the physical system under consideration and $p^2 = M^2$.

LFD serves as an effective and convenient tool to solve this eigenvalue equation. Indeed, one of the main advantages of LFD is that, due to kinematical constraints, the vacuum state of a system of interacting particles coincides with the free vacuum, and all intermediate states result from fluctuations of the physical system. One can thus construct the state vector in terms of combinations of free fields, i.e., decompose it in a series of Fock sectors:

$$\phi(p) = \sum_{n=1}^{\infty} \int dD_n \phi_n(k_1, \ldots, k_n; p)|n\rangle, \tag{2}$$

where $|n\rangle$ is the state containing n free particles with the four-momenta $k_1, \ldots, k_n$ and ϕ_n's are relativistic n-body wave functions, the so-called Fock components. The phase space volume element is represented schematically by dD_n. In the following, we shall restrict our study to a physical system composed of one fermion and $n-1$ bosons. In that case

$$|n\rangle \equiv a^\dagger(k_1)c^\dagger(k_2)\ldots c^\dagger(k_n)|0\rangle, \tag{3}$$

where $a^\dagger$ and $c^\dagger$ are fermion and boson creation operators, respectively, and

$$\phi_n(k_1, \ldots k_n; p) = \bar{u}(k_1)\psi_n(k_1, \ldots k_n; p)u(p), \tag{4}$$

where u's are bispinors. To completely determine the state vector, we normalize it according to

$$\phi(p')^\dagger\phi(p) = 2p_0\delta^{(3)}(\mathbf{p}' - \mathbf{p}). \tag{5}$$

With the decomposition (2), the normalization condition (5) writes

$$\sum_{n=1}^{\infty} I_n = 1, \tag{6}$$

where I_n is the contribution of the n-body Fock sector to the full norm. The explicit formula for I_n can be found in Ref. [2].

In the following, we shall use CLFD [3] as a general framework. The covariance of our approach is due to the invariance of the light-front plane equation. This implies that ω is not the same in any reference frame, but varies according to Lorentz transformations, like the coordinate x. It is not the case in the standard formulation of LFD where ω is fixed to $\omega = (1, 0, 0, -1)$ in any reference frame.

The light-front momentum operator $\hat{P}_\rho$ can be constructed from the energy-momentum tensor. It is decomposed according to

$$\hat{P}_\rho = \hat{P}_\rho^{(0)} + \hat{P}_\rho^{\text{int}}, \tag{7}$$

where the two terms on the right-hand side are, respectively, the free (i.e. independent of the coupling constant and counterterms) and interaction parts of the four-momentum operator. The operator $\hat{P}_\rho^{\text{int}}$ is related to the interaction Hamiltonian $H^{\text{int}}(x)$ on the light front by

$$\hat{P}_\rho^{\text{int}} = \omega_\rho \int H^{\text{int}}(x)\delta(\omega \cdot x)d^4x. \tag{8}$$

From the general transformation properties of the light-front plane $\omega \cdot x = 0$, one can derive the following conservation law [3] for each Fock component:

$$k_1 + k_2 + \cdots + k_n = p + \omega\tau_n, \tag{9}$$

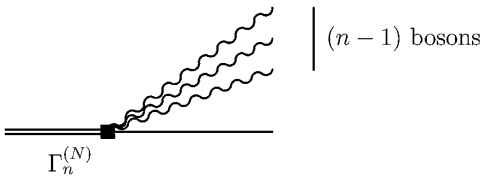

FIG. 1. Vertex function of order n for the Fock space truncation of order N.

where the quantity τ_n is a measure of how far the n-body system is off the energy shell.[1] It is completely determined by the conservation law (9) and the on-mass shell condition for each individual particle momentum:

$$2\omega \cdot p\tau_n = (s_n - M^2), \tag{10}$$

where $s_n = (k_1 + \ldots + k_n)^2$.

It is convenient to introduce, instead of the wave functions ϕ_n, the vertex functions Γ_n (which we will also refer to as Fock components), defined by

$$\bar{u}(k_1)\Gamma_n u(p) = (s_n - M^2)\phi_n \equiv 2\omega \cdot p\tau_n \phi_n. \tag{11}$$

The vertex function Γ_n will be represented graphically by the diagram of Fig. 1. With the definition

$$\mathcal{G}(p) = \sum_{n=1}^{\infty} \int dD_n \bar{u}(k_1)\Gamma_n(k_1, \ldots, k_n, p)u(p)|n\rangle, \tag{12}$$

the eigenvalue equation (1) writes [2]

$$\mathcal{G}(p) = \frac{1}{2\pi} \int [-\tilde{H}^{\text{int}}(\omega\tau)]\frac{d\tau}{\tau}\mathcal{G}(p), \tag{13}$$

where $\tilde{H}^{\text{int}}$ is the interaction Hamiltonian in momentum space:

$$\tilde{H}^{\text{int}}(p) = \int H^{\text{int}}(x)e^{-ip\cdot x}d^4x. \tag{14}$$

With the form (13), the eigenvalue equation can thus be represented graphically, using the same rules as those derived in Ref. [3] for the calculation of matrix elements of the S matrix. This graph technique was developed by Kadyshevsky [4] and transformed to the case of CLFD in Ref. [5].

The substitution of the decomposition (12) into the eigenvalue equation (13) results in an (infinite) system of equations for the Fock components. In order to solve this system in practice, we should make it finite, i.e., truncate the decomposition (12), or equivalently (2), by retaining only those Fock sectors where the number of particles does not exceed some maximal value N. The finite system can be solved numerically and nonperturbatively, that is, for any value of the coupling constant. This approach was developed in a series of papers [2,6–8]. In Ref. [2] it was applied to the case of the two-body truncation, i.e. for $N = 2$.

[1]The term "off the energy shell" is borrowed from the equal-time dynamics where the spatial components of the four-momenta are always conserved, but the energies of intermediate states are not equal to the incoming energy.

B. Renormalization conditions

In order to be able to make definite predictions for physical observables, one should also define a proper renormalization scheme which allows us to express, like in perturbation theory, observables through the physical coupling constant and masses and exclude the bare ones. The basis of the state vector decomposition, i.e. the states $|n\rangle$ in Eq. (3) is constructed from free physical fermion and boson fields, with their physical masses m and μ, respectively. The interaction Hamiltonian contains the corresponding mass counterterms (MCs) δm and $\delta\mu^2$ responsible for the fermion and boson mass renormalization. Since we will not consider here the fluctuations of the boson in terms of fermion-antifermion pairs, we have to set $\delta\mu^2 = 0$. The MC δm is determined from the eigenvalue equation (13) by demanding that the bound state mass M is equal, for the ground state, to the physical mass m of the constituent fermion. For this reason, we will distinguish M and m only when it is necessary. Otherwise, we will set $M = m$.

Besides MCs, the interaction Hamiltonian includes also the BCC g_0. The latter is determined, as in perturbation theory, by relating the on-energy-shell two-body vertex function Γ_2 to the physical coupling constant g. As follows from Eq. (9), taking Γ_2 on the energy shell is equivalent to setting $\tau_2 = 0$. Once M is identified with m, the latter condition reduces to $s_2 = (k_1 + k_2)^2 = m^2$ [see Eq. (10)]. Below, for brevity, we will denote the on-energy-shell two-body vertex function as $\Gamma_2(s_2 = m^2)$ to indicate that its arguments are connected by the corresponding kinematical constraint.

Being a solution of the system of eigenvalue equations (13), Γ_2 depends on the BCC g_0. Hence, relating Γ_2 to g is equivalent to relating g_0 to g, which just means coupling constant renormalization. This can be most easily done starting from the three point function with all undressed on-mass-shell external lines, called $\tilde{\Gamma}_2$. It is connected with the physical coupling constant by the following standard relation (see, e.g., Ref. [9]):

$$\sqrt{Z_f}\tilde{\Gamma}_2(s_2 = m^2)\sqrt{Z_f}\sqrt{Z_b} = g, \tag{15}$$

where the Z factors are the so-called field strength renormalization constants for the fermion (f) (both in the initial and final state) and boson (b) lines, respectively. This condition can also be recovered by demanding that the residue of the fermion-boson elastic scattering amplitude at $s_2 = m^2$ equals g^2. One can thus deduce the expression for Z_f in terms of the full fermion self-energy $\Sigma(\not{p})$:

$$Z_f = \left[1 - \frac{\partial \Sigma(\not{p})}{\partial \not{p}} \bigg|_{\not{p}=m}\right]^{-1}, \tag{16}$$

and similarly for Z_b as a function of the full boson self-energy.

The two-body Fock component Γ_2 being a solution of the eigenvalue equation (13) does not coincide with $\tilde{\Gamma}_2$. By definition, $\tilde{\Gamma}_2$ has no radiative corrections to any of its three legs, while Γ_2, on the contrary, includes such corrections. The relation between these two vertex functions taken off the energy shell is rather complicated in LFD. Fortunately, we need to know it on the energy shell only, where it simplifies strongly, because the on-shell radiative corrections mentioned above reduce to c-number factors. Indeed, Γ_2 is a particular case of the general vertex function shown in Fig. 1, corresponding to $n = 2$, i.e. two of its legs are represented by single external lines (one for the constituent fermion and one for the constituent boson), while the third leg, for the physical fermion, is shown by a double line. Radiative corrections to each of the two external single lines are given by insertions of self-energy parts with their subsequent summation. The latter, of course, can be done directly within LFD by using the graph techniques rules, but we will choose a simpler way.

Each on-energy-shell amplitude calculated in LFD must coincide with that found in the standard four-dimensional Feynman approach and taken on the mass shell. Hence, $\Gamma_2(s_2 = m^2)$ coincides with its on-mass-shell Feynman counterpart.[2]

The summation of radiative corrections to external lines in the Feynman approach is technically easier than in LFD, since it can be done for each of the two lines independently. We allow the external particle momenta being off the mass shell (in order to avoid intermediate singularities), then sum up the radiative corrections, and finally perform a limiting transition to the mass shell. Thus, summing a chain series of self-energy blocks—together with the mass-counterterm insertion—on the constituent fermionic line with the four-momentum k_1 brings the factor

$$\lim_{\not{k}_1 \to m}\left[1 + \frac{\Sigma_r(\not{k}_1)}{\not{k}_1 - m} + \frac{\Sigma_r(\not{k}_1)}{\not{k}_1 - m} \times \frac{\Sigma_r(\not{k}_1)}{\not{k}_1 - m} + \ldots\right] = \lim_{\not{k}_1 \to m}\left(\frac{\not{k}_1 - m}{\not{k}_1 - m - \Sigma_r(\not{k}_1)}\right) = \left[1 - \frac{\partial \Sigma_r(\not{k}_1)}{\partial \not{k}_1} \bigg|_{\not{k}_1 = m}\right]^{-1}, \tag{17}$$

where $\Sigma_r(\not{k}_1) = \Sigma(\not{k}_1) - \Sigma(m)$. This factor is nothing other than Z_f given by Eq. (16). The analogous procedure for the boson line leads to the factor Z_b. The total factor which appears due to the dressing of the two constituent lines is therefore $Z_f Z_b$.

Concerning the double fermionic line in Γ_2, its renormalization factor is related to the normalization condition for the state vector. The eigenvalue equation (13) transforms into a homogeneous system of linear integral equations for the vertex functions. Hence, the solution is determined up to an arbitrary common factor. In practice, for solving this system of equations, it is convenient to fix the (constant) one-body Fock component ϕ_1, so that the other components (Γ_2, Γ_3, etc.) become proportional to it. Then the double line in Γ_2 brings the factor ϕ_1 which, in its turn, is determined by the normalization condition (5) for the state vector. Since ϕ_1^2 is just the norm of the one-body Fock sector, ϕ_1 is equal to $\sqrt{I_1}$, where I_1 is the first term in the sum (6).

The relation between Γ_2 and $\tilde{\Gamma}_2$ becomes, thereforen on the energy shell

$$\Gamma_2(s_2 = m^2) = \sqrt{I_1}\tilde{\Gamma}_2(s_2 = m^2) Z_f Z_b. \tag{18}$$

Excluding $\tilde{\Gamma}_2(s_2 = m^2)$ from Eqs. (15) and (18), we find that the renormalization condition reads

$$\Gamma_2(s_2 = m^2) = g\sqrt{I_1}\sqrt{Z_b}, \tag{19}$$

where Γ_2 is expressed from the eigenvalue equation (13), through the BCC g_0. A similar discussion of the renormalization condition in terms of the one-body component ϕ_1 was already done in Ref. [10].

When we neglect the boson dressing by fermion-antifermion fluctuations, as we do in this work, the condition (19) finally reduces to

$$\Gamma_2(s_2 = m^2) = g\sqrt{I_1}. \tag{20}$$

Note that there is a simple relation between the one-body normalization factor I_1 and the field strength renormalization factor Z_f:

$$Z_f = I_1, \tag{21}$$

as shown in Appendix A.

C. Renormalization scheme

The above conditions imposed on the BCC and MC are necessary in order to express physical observables, like the electromagnetic form factors, through the measurable coupling constant and masses. As a consequence, one should expect a full cancellation of divergences.

Such a program could be realized in perturbation theory or nonperturbatively if the Fock space is not truncated. The latter case is hardly achieved in practice. Usually, Fock space is truncated to a finite order N of admitted Fock sectors, and the cancellation of divergences is not anymore guaranteed. For instance, looking at Fig. 2 for the calculation of the fermion propagator in the second order of perturbation theory, one immediately realizes that the can-

[2]A three-leg vertex which enters, as an internal sub-block, in physically observed amplitudes is always off-shell. Taking it on-shell, we imply its analytical continuation into a nonphysical kinematical region

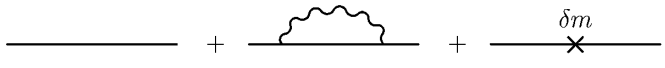

FIG. 2. Renormalization of the fermion propagator in the second order of perturbation theory.

cellation of divergences between the self-energy contribution (of order two in the Fock decomposition) and the fermion MC (of order one) involves two different Fock sectors [2]. This means that, as a necessary condition for the cancellation of divergences, any MC and, more generally, any BCC should be associated with the number of particles present (or ''in flight'') in a given Fock sector. In other words, all MCs and BCCs must depend on the Fock sector under consideration. The original MC, δm, and the fermion-boson BCC, g_0, should thus be extended each to a whole series:

$$g_0 \to g_{0l}, \tag{22a}$$

$$\delta m \to \delta m_l, \tag{22b}$$

with $l = 1, 2, \ldots N$. The quantities g_{0l} and δm_l are calculated by solving the systems of equations for the vertex functions in the $N = 1$, $N = 2$, $N = 3, \ldots$ approximations successively. This FSDR scheme has been proposed initially in Ref. [11] and developed as a full renormalization scheme in Ref. [2]. An alternative approach, also in the Pauli-Villars (PV) regulated Yukawa model with the two-boson truncation, but with a sector-independent renormalization scheme, was developed in Ref. [12].

Note that the series (22) does not imply that we have an infinite number of counterterms or bare parameters. We still have the original ones g_0 and δm in the Hamiltonian we start with, but they have different values according to the level of approximation used in the calculation. In the limit of an infinite N, and if the Fock sector expansion converges, g_{0N} and δm_N should turn to the true BCC and the MC, respectively. This is completely analogous to the case of perturbation theory where, at each order n, one determines $g_0^{(n)}$ and $\delta m^{(n)}$.

Apart from the mass and vertex radiative corrections, the third type of divergences arises from the field renormalization, i.e., from the constants Z_f and Z_b. The values of these constants should also depend on the maximal number N of particles kept in a given truncation. Consider, for instance, the vertex function Γ_2 represented by Fig. 1 for $n = 2$. The dressing of the physical fermion leg (the factor $\sqrt{I_1}$) should be calculated for the truncation to the N-th order. The situation changes, however, for the constituent (single) fermion line. The state in which the constituent fermion is considered already contains one constituent boson. Hence, even if the boson line is not dressed, the dressing of the constituent fermion leg involves radiative corrections of order $(N-1)$. In other words, the dressing factor Z_f for the constituent fermion leg must be calculated for the lower, $(N-1)$-body truncation. Otherwise, we would go beyond our approximation, since the effective number of particles in which the physical fermion can fluctuate would exceed N.

Taking this into account, the relations (15) and (18) for a finite order truncation N (and in the absence of boson dressing) obtain the following form:

$$\sqrt{Z_f^{(N)}}\tilde{\Gamma}_2(s_2 = m^2)\sqrt{Z_f^{(N-1)}} = g, \tag{23a}$$

$$\Gamma_2(s_2 = m^2) = \sqrt{I_1^{(N)}}\tilde{\Gamma}_2(s_2 = m^2)Z_f^{(N-1)}. \tag{23b}$$

The superscripts (N) and $(N-1)$ here and below just indicate the order of the Fock space truncation in which the corresponding quantities are calculated.

It follows from Eqs. (23a) and (21) that the renormalization condition (20) simply writes

$$\Gamma_2(s_2 = m^2) = g\sqrt{I_1^{(N-1)}}, \tag{24}$$

in the absence of boson dressing.

For the simplest case of the two-body truncation, $N = 2$, one thus gets

$$\Gamma_2^{(2)}(s_2 = m^2) = g, \tag{25}$$

since $I_1^{(1)} = 1$. We recover here the condition given in Ref. [2]. This condition is, however, valid only for $N = 2$.

III. YUKAWA MODEL IN THREE-BODY TRUNCATED FOCK SPACE

A. Eigenvalue equations

We consider in this study the Yukawa model: a spin-1/2 fermion interacting with massive spinless bosons. The regularization is provided by the PV method. In addition to physical particles, we introduce therefore one PV fermion and one PV boson with (large) masses m_1 and μ_1, respectively. This amounts to extend the physical Fock space to embrace negatively normalized PV particles [2]. The interaction Hamiltonian in Eq. (13) is given by

$$H^{\text{int}} = -g_0\bar{\psi}'\psi'\varphi' - \delta m\bar{\psi}'\psi',$$

with

$$\psi' = \psi + \psi_{\text{PV}}, \qquad \varphi' = \varphi + \varphi_{\text{PV}}, \tag{26}$$

where ψ and φ are the free physical fermion and boson field operators, while ψ_{PV} and φ_{PV} are their PV partners, with a negative norm. The bosons are supposed to be neutral.

In the three-body truncation, the system of coupled equations for the vertex functions, derived from the eigenvalue equation (13), is shown graphically in Fig. 3. On the right-hand side of the last equation, the sum of the diagrams with permutated boson legs appears, reflecting the symmetrization of the amplitude due to the identity of bosons. Expressing Γ_3 through Γ_2 by means of this equation, and substituting the result into the second equation,

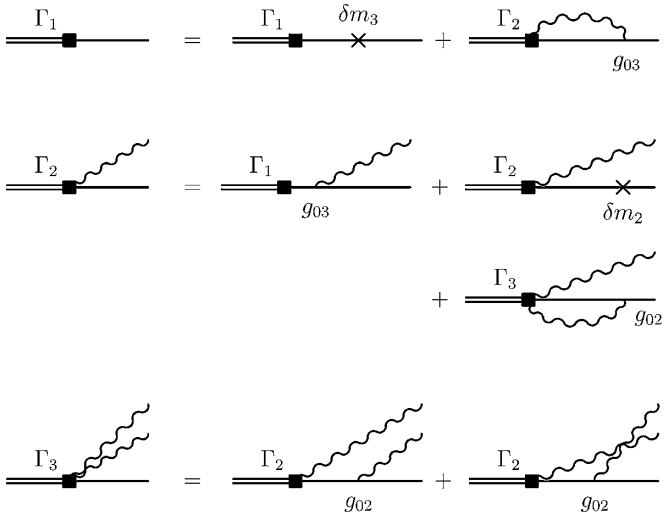

FIG. 3. System of equations for the vertex functions in the case of three-body Fock space truncation.

we can exclude the highest order vertex function Γ_3 from the full system of equations. We thus obtain a reduced equation for the two-body vertex function. Together with the first equation in Fig. 3, it forms a system of equations involving the vertex functions Γ_1 and Γ_2 only, as shown in Fig. 4. Analytically, these equations read

$$\bar{u}(p_{1i})\Gamma_1^i u(p) = \bar{u}(p_{1i})(V_1 + V_2)u(p), \tag{27a}$$

$$\bar{u}(k_{1i})\Gamma_2^{ij} u(p) = \bar{u}(k_{1i})(V_3 + V_{45} + V_6)u(p), \tag{27b}$$

where the indices i and j refer to whether the line of a constituent fermion (i) or a constituent boson (j) corresponds to a physical particle (i, $j = 0$) or to a PV one (i, $j = 1$). The term V_{45} means the sum of V_4 and V_5. The explicit expressions for V_{1-6} are given in Appendix C. Note that the first equation in Fig. 3 is just a constraint which determines δm_3. The contribution V_5 on the right-hand side of Fig. 4 involves the two-body self-energy depending, due to the departure off the energy shell, on a four-momentum k with $k^2 \neq m^2$. This self-energy is decomposed, in CLFD, according to [8]:

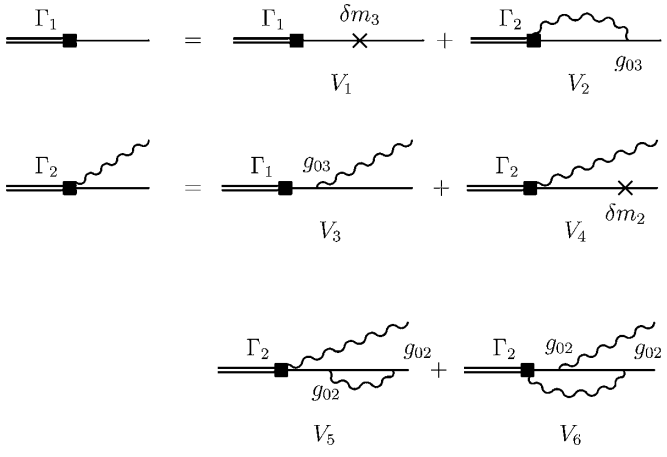

FIG. 4. Reduced equation for the two-body vertex function, obtained from that shown in Fig. 3 after the exclusion of the three-body component.

$$\Sigma(\not{k}) = g_{02}^2\left[\mathcal{A}(k^2) + \mathcal{B}(k^2)\frac{\not{k}}{m} + \mathcal{C}(k^2)\frac{m\not{\omega}}{\omega\cdot k}\right], \tag{28}$$

where the factors m are here for convenience only. The coefficients $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ are calculated in Appendix B.

In contrast to the two-body case, the system of equations for the three-body truncation is rather nontrivial. For example, its iteration generates all the graphs for the self-energy which contain one fermion and two bosons, including overlapping self-energy type diagrams. The number of such irreducible graphs is infinite. Some of them are shown in Fig. 5. The solution of the system of coupled equations incorporates the sum of these contributions to all orders.

Note that due to the covariance of our approach, we can identify the contribution $\sim\not{\omega}$ to the self-energy which explicitly depends on the light-front plane orientation. If not regularized, the coefficient $\mathcal{C}(k^2)$ is quadratically divergent and needs *a priori* both PV fermion and boson regularization. After this regularization however, $\mathcal{C}(k^2) \equiv 0$ for any values of the PV fermion and boson masses. This makes the two-body self-energy identical to the result obtained in perturbation theory in the Feynman approach [7]. The contributions $\mathcal{A}$ and $\mathcal{B}$ do respect chiral symmetry in the sense that they are equal to zero when the constituent mass m, as well as the physical mass M (which, in our case, coincides with m), goes to zero, without the need of an extra PV boson. This is at variance with the standard formulation of LFD where it is claimed that an additional PV boson is needed, if the PV fermion mass is kept finite [13].

The parameters g_{02} and δm_2 are taken from the $N = 2$ calculation [2]. They are given by

$$g_{02}^2 = \frac{g^2}{1 - g^2 J_2}, \tag{29a}$$

$$\delta m_2 = g^2[\mathcal{A}(m^2) + \mathcal{B}(m^2)], \tag{29b}$$

where

$$J_2 = -\frac{\mathcal{B}(m^2)}{m} - z_0, \tag{30}$$

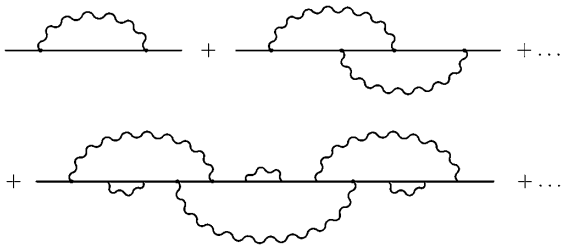

FIG. 5. Radiative corrections to the self-energy.

with $z_0 = 2m[\mathcal{A}'(m^2) + \mathcal{B}'(m^2)]$. The norms of the one- and two-body Fock sectors, entering the normalization condition (6), are

$$I_1^{(2)} = 1 - g^2 J_2, \tag{31a}$$

$$I_2^{(2)} = g^2 J_2. \tag{31b}$$

In the two-body approximation mentioned above and discussed in detail in Ref. [2], the two-body vertex function is automatically independent of ω since $\mathcal{C}(k^2) \equiv 0$. Moreover, it is a constant (i.e. it does not depend on the momenta of the constituent). Because of all these, the renormalization condition (25) directly leads to the relation (29a) between the bare and physical coupling constants. In principle, nothing prevents Γ_2 from being ω dependent, since it is an off-shell object, but this dependence must completely disappear on the energy shell, i.e., for $s_2 = m^2$. It would be indeed so if no Fock space truncation occurs. The latter, in approximations higher than the two-body one (i.e. for $N = 3, 4, \ldots$), may cause some ω dependence of Γ_2 even on the energy shell, which immediately makes the general renormalization condition (19) ambiguous. If so, one has to insert into the light-front interaction Hamiltonian new counterterms which explicitly depend on ω and cancels the ω dependence of $\Gamma_2(s_2 = m^2)$. Its explicit form will be given in the next subsection. Note that the explicit covariance of CLFD allows us to separate the terms which depend on the light-front plane orientation (i.e. on ω) from other contributions and establish the structure of these counterterms. This is not possible in ordinary LFD.

B. Calculation of the two-body fock component

The method of solution is similar to that used in the calculation [2] for $N = 2$. We first decompose the vertex functions in invariant amplitudes. The vertex functions on the left-hand sides of Eqs. (27), being matrices in the spin indices, can be decomposed in a full set of spin matrices. This decomposition is very simple in CLFD and takes the form

$$\begin{aligned} &\bar{u}(p_{1i})\Gamma_1^i u(p) = (m_i^2 - M^2)\psi_1^i \bar{u}(p_{1i})u(p), \\ &\bar{u}(k_{1i})\Gamma_2^{ij} u(p) = \bar{u}(k_{1i})\left[b_1^{ij} + b_2^{ij}\frac{m\phi}{\omega\cdot p}\right]u(p), \end{aligned} \tag{32}$$

where ψ_1^i is a constant, and $b_{1,2}^{ij}$ are invariant functions of particle momenta, with $m_0 \equiv m$ and $\mu_0 \equiv \mu$. We denote temporarily, in the first of the above equations, the physical fermion mass by M in order to avoid singularities, since the equations contain the combination $\Gamma_1^i/(m_i^2 - M^2)$ which becomes indeterminate for $i = 0$ at $M = m$. Using $M \neq m$ allows us to take a smooth limit $\lim_{M\to m}[\Gamma_1^i/(m_i^2 - M^2)] = \psi_1^i$; after that one may set $M = m$.

Each of the functions $b_{1,2}^{ij}$ depends on two invariant kinematical variables. As usual, we define a pair of variables, consisting of the longitudinal and transverse (with respect to the three-vector $\boldsymbol{\omega}$) momenta:

$$x = \frac{\omega\cdot k_2}{\omega\cdot p}, \qquad \mathbf{R}_\perp = \mathbf{k}_{2\perp} - x\mathbf{p}_\perp, \tag{33}$$

where k_2 is the boson four-momentum. Then $b_{1,2}^{ij}$ are functions of x and $R_\perp^2$.

The renormalization condition (24), for $N = 3$, implies two conditions

$$b_1^{00}(s_2 = m^2) = g\sqrt{I_1^{(2)}}, \tag{34a}$$

$$b_2^{00}(s_2 = m^2) = 0, \tag{34b}$$

for the spin components of Γ_2 at $s_2 = m^2$, where the two-body invariant energy squared s_2 is expressed through $R_\perp$ and x as follows:

$$s_2 = \frac{R_\perp^2 + \mu^2}{x} + \frac{R_\perp^2 + m^2}{1 - x}. \tag{35}$$

One should emphasize that the renormalization conditions are imposed on the two-body vertex function Γ_2^{00} corresponding to both physical constituents. The condition (34a) defines unambiguously g_{03}. The condition (34b) is not verified automatically if the Fock space is truncated for $N \geq 3$, unlike the case $N = 2$. We should thus enforce it by introducing an appropriate counterterm, as explained above (see also Ref. [2]). It corresponds to the following additional structure in the interaction Hamiltonian:

$$\delta\mathcal{H}_\omega^{\text{int}} = -Z_\omega \bar{\psi}' \frac{m\phi}{i\omega\cdot\partial}\psi'\varphi', \tag{36}$$

where Z_ω is a constant adjusted to make Eq. (34b) true. The operator $\phi/(i\omega\cdot\partial)$, in momentum space, leads to the appearance of a new three-leg vertex $\phi/(\omega\cdot k)$ on each fermion-boson vertex with total incoming momentum k. In principle, a similar new ω-dependent counterterm should be also added to the Hamiltonian in order to cancel the ω dependence of δm_3, in full analogy with the cancellation of $b_2^{00}(s_2 = m^2)$ [7]. However, as we will see below, δm_3 is needed only for a calculation in the four-body Fock space truncation. For this reason, working within the three-body truncation only, we may not bother about additional counterterms excepting that given by Eq. (36).

To solve the system of equations. (27), we substitute the decompositions (32) into the expressions for V_{1-6} given in Appendix C, then multiply Eqs. (27a) and (27b) by $u(p_{1i})$ and $u(k_{1i})$, respectively, to the left and each of them by $\bar{u}(p)$ to the right, and sum over spin projections. We thus get

$$(m_i^2 - m^2)(\not{p}_{1i} + m_i)\psi_1^i(\not{p} + m) = (\not{p}_{1i} + m_i)(V_1 + V_2)(\not{p} + m), \tag{37a}$$

$$(\not{k}_{1i} + m_i)\left[b_1^{ij} + b_2^{ij}\frac{m\not{\omega}}{\omega \cdot p}\right](\not{p} + m) = (\not{k}_{1i} + m_i)(V_3 + V_{45} + V_6)(\not{p} + m). \tag{37b}$$

The system of matrix equations (37a) and (37b) can be transformed into a homogeneous system of ten linear integral equations for ten unknown functions (two ψ_1^i, four b_1^{ij}, and four b_2^{ij}). These equations are obtained by taking the trace of Eqs. (37a) and (37b) (six equations), and by taking the trace of Eq. (37b) after the multiplication of its both sides by $\not{\omega}$ (four equations).

In order to achieve the limit $m_1 \to \infty$, it is convenient to replace the functions ψ_1^i and $b_{1,2}^{ij}$ by the new functions α_i, h_i^j, and H_i^j according to

$$\psi_1^i = \frac{m}{m_i(m + m_i)}\alpha_i, \qquad b_1^{ij} = \frac{m_i}{m}h_i^j,$$
$$b_2^{ij} = \frac{m_i}{m}\frac{H_i^j - (1 - x + \frac{m_i}{m})h_i^j}{2(1 - x)}. \tag{38}$$

A careful analysis shows that in this limit the PV mass m_1 disappears from the equations written in terms of α_i, h_i^j, and H_i^j. These functions have therefore a finite limit. Below we will imply that the limit $m_1 \to \infty$ is taken and α_i, h_i^j, and H_i^j denote the limiting values.

For further simplification of the equations, it is convenient to introduce new functions $\tilde{h}_{0,1}^j$ and $\tilde{H}_{0,1}^j$ by means of the relation

$$\begin{pmatrix} h_{0,1}^j \\ H_{0,1}^j \end{pmatrix} = \alpha_0\kappa\begin{pmatrix} \tilde{h}_{0,1}^j \\ \tilde{H}_{0,1}^j \end{pmatrix}, \tag{39}$$

with

$$\kappa = g_{03}\frac{1 - g^2J_2}{1 + g^2z_0}. \tag{40}$$

Using the substitution (39) and denoting

$$Z'_\omega = \frac{2Z_\omega}{g_{03}} - \frac{\alpha_1}{\alpha_0}, \tag{41}$$

the initial system of ten equations splits into two subsystems. The first one contains two equations involving the ratio α_1/α_0. The value of δm_3 just ensures that both equations define the same quantity α_1/α_0. It is not interesting for our study in the three-body approximation, since α_1, as will be seen below, drops out from the observables we calculate here, while α_0 is uniquely determined by the normalization condition for the state vector. As already mentioned, we also do not need to calculate δm_3 itself. It is used as an input in the calculation at the next, $N = 4$, truncation.

The second subsystem of eight equations involves the eight functions $\tilde{h}_i^j$ and $\tilde{H}_i^j$ only, since the ratio α_1/α_0 is absorbed into the definition of Z'_ω in Eq. (41). We thus get

$$\tilde{h}_0^j = 1 + g'^2(K_1^j h_0^j + K_2^j\tilde{h}_1^j) + g'^2 i_0^j,$$
$$\tilde{h}_1^j = g'^2(-K_3^j\tilde{h}_0^j + K_4^j\tilde{h}_1^j) + g'^2 i_1^j,$$
$$\tilde{H}_0^j = Z'_\omega(1 - x) + 2 - x + g'^2(K_1^j\tilde{H}_0^j + K_2^j\tilde{H}_1^j) + g'^2 I_0^j,$$
$$\tilde{H}_1^j = 1 + g'^2(-K_3^j\tilde{H}_0^j + K_4^j\tilde{H}_1^j) + g'^2 I_1^j, \tag{42}$$

where

$$g'^2 = \frac{g^2}{(1 + g^2z_0)}, \tag{43}$$

and

$$K_1^j = \frac{1}{m}\left\{\mathcal{B}_r(s_1) - \frac{2[\mathcal{A}_r(s_1) + \mathcal{B}_r(s_1)]m^2}{m^2 - s_1}\right\},$$
$$K_2^j = \frac{\mathcal{A}_r(s_1) + \mathcal{B}_r(s_1)}{m},$$
$$K_3^j = \frac{[\mathcal{A}_r(s_1) + \mathcal{B}_r(s_1)]m}{m^2 - s_1},$$
$$K_4^j = \frac{\mathcal{B}_r(s_1)}{m}.$$

The substracted self-energy contributions $\mathcal{A}_r(s_1)$ and $\mathcal{B}_r(s_1)$, are given by

$$\mathcal{A}_r(s_1) = \mathcal{A}(s_1) - \mathcal{A}(m^2),$$
$$\mathcal{B}_r(s_1) = \mathcal{B}(s_1) - \mathcal{B}(m^2),$$

with

$$s_1 = -\frac{R_\perp^2}{x} + (1 - x)m^2 - \frac{1 - x}{x}\mu_j^2. \tag{44}$$

The functions $\mathcal{A}$ and $\mathcal{B}$ are given in Appendix B, while the integral terms $i_{0,1}^j$ and $I_{0,1}^j$ are given in Appendix D.

The limit of infinite PV mass μ_1 is not easy to perform analytically, as it was done for m_1. Setting $\mu_1 \to \infty$ directly in Eqs. (42) makes some integration kernels singular (they decrease too slowly at $R_\perp \to \infty$). The dependence of physical observables, like the AMM, on μ_1 will therefore be studied numerically.

Note that although g_{02}^2 in Eq. (29a) can become infinite (for $J_2 = 1/g^2$) and changes sign from positive to negative at sufficiently large values of the PV boson mass μ_1, the eigenvalue equations (42) do not show any singularity when g_{02}^2 goes to infinity. Indeed, g_{02}^2 does not appear in Eqs. (42). These equations depend only on g'^2, given by Eq. (43), with z_0 strictly positive. Therefore, g'^2 is strictly positive and finite. We shall come back in Sec. V to the interpretation of the limit of large μ_1, when both g_{02}^2 and the norm of the one-body sector $I_1^{(2)}$ are negative, while

the norm of the two-body sector $I_2^{(2)}$ is larger than 1, from Eqs. (31).

The constant Z'_ω entering the system of equations (42) is determined from the renormalization condition (34b), while g_{03}, needed to calculate the renormalized vertex functions in Eqs. (39), is determined from the renormalization condition (34a).

The components $b^{00}_{1,2}(s_2 = m^2)$ entering these renormalization conditions are expressed through the solution of the system of equations (42) by means of Eqs. (38) and (39). The kinematical point $s_2 = m^2$ belongs to a nonphysical region, but there is no need to make an analytical continuation to this region of the solution $\tilde{h}^j_i$ and $\tilde{H}^j_i$ found numerically. Indeed, the integral terms in Eqs. (42) involve integrations within the physical domain only. One can simply set $j = 0$, $R_\perp = R^*_\perp$, $x = x^*$, where $R^*_\perp$ and x^* are determined by the condition $s_2 = m^2$, and calculate the integral terms by substituting there the previously found solution $\tilde{h}^j_i(R_\perp, x)$ and $\tilde{H}^j_i(R_\perp, x)$ for physical values $R_\perp$ and x. After that, Eqs. (42) reduces to a system of four ordinary linear inhomogeneous equations for $\tilde{h}^0_i(R^*_\perp, x^*)$ and $\tilde{H}^0_i(R^*_\perp, x^*)$. Finally, relating the calculated quantities $\tilde{h}^0_0(R^*_\perp, x^*)$ and $\tilde{H}^0_0(R^*_\perp, x^*)$ to $b^{00}_{1,2}(s_2 = m^2)$ we get g_{03} from Eq. (34a)[3] and Z'_ω from Eq. (34b).

The condition $s_2 = m^2$, however, does not determine $R^*_\perp$ and x^* simultaneously. It is convenient to fix x^* somehow and then find $R^*_\perp$ from Eq. (35):

$$R^{*2}_\perp = -[x^{*2}m^2 + (1 - x^*)\mu^2]. \tag{45}$$

Since the two-body vertex function (32) on the energy shell must turn into a constant, the functions $b^{00}_{1,2}(s_2 = m^2)$ also must be constants. In other words, if one relates the arguments of these functions by Eq. (45), their values are independent of the choice of x^*. It would be so in exact calculations, i.e., if Fock space was not truncated. A finite order truncation makes the Fock components, even at $s_2 = m^2$, x^* dependent. As advocated in Ref. [2], we choose $x^* = \frac{\mu}{m+\mu}$. We shall investigate in Sec. V how $b^{00}_{1,2}(s_2 = m^2)$ depends on the choice of x^*.

C. Representation of the three-body component

We can find the three-body component by calculating the amplitude corresponding to the right-hand side of the equation shown by the last line in Fig. 3.

The general form of the relativistic vertex function of a system composed from one constituent fermion and two spinless bosons with total spin 1/2 reads

$$\bar{u}^{\,\alpha}_{\sigma_1}(k_1)\Gamma_{\alpha\beta}(1, 2, 3)u^\beta_\sigma(p), \tag{46}$$

where $\Gamma_{\alpha\beta}(1, 2, 3)$ is a 4×4 matrix in the indices α, β. The arguments of Γ_3, denoted symbolically by numbers, mean three pairs of the standard variables

$$\mathbf{R}_{l\perp} = \mathbf{k}_{l\perp} - x_l\mathbf{p}_\perp, x_l = \frac{\omega \cdot k_l}{\omega \cdot p},$$

with $l = 1$ corresponding to the fermion and $l = 2$, 3 to bosons. Here $\bar{u}^\alpha_{\sigma_1}(k_1)$ is the bispinor of the constituent fermion, $u^\beta_\sigma(p)$ is the bispinor of the physical fermion (of the composite system), σ_1, σ are their spin projections in the corresponding rest frame. Since $\sigma_1 = \pm 1/2$ and $\sigma = \pm 1/2$, we have in general $2 \times 2 = 4$ matrix elements. Usually, parity conservation reduces this number by a factor of 2. However, this is not the case in relativistic calculations, for a n-body wave function with $n \geq 3$ [14]. This wave function is determined by *four independent* matrix elements or, equivalently, by four scalar functions g_{1-4} like

$$\begin{aligned}&\bar{u}(k_1)\Gamma_3(1, 2, 3)u(p)\\ &= \bar{u}(k_1)(g_1S_1 + g_2S_2 + g_3S_3 + g_4S_4)u(p).\end{aligned}$$

For simplicity, we omitted the bispinor indices and the indices marking the particle type (either physical or PV one). It is convenient to construct the four basis spin structures as follows:

$$\begin{aligned}S_1 &= 2x_1 - (m_i + x_1m)\frac{\not{\omega}}{\omega \cdot p},\\ S_2 &= m\frac{\not{\omega}}{\omega \cdot p},\\ S_3 &= iC_{\rm ps}\left[2x_1 - (m_i - x_1m)\frac{\not{\omega}}{\omega \cdot p}\right]\gamma_5,\\ S_4 &= imC_{\rm ps}\frac{\not{\omega}}{\omega \cdot p}\gamma_5,\end{aligned} \tag{47}$$

with $x_1 = \frac{\omega \cdot k_1}{\omega \cdot p}$, and m_i being the internal fermion mass (either the physical or PV one, depending on which type of fermion the momentum k_1 corresponds to), while $C_{\rm ps}$ is the following pseudoscalar:

$$C_{\rm ps} = \frac{1}{m^2\omega \cdot p}e^{\mu\nu\rho\gamma}k_{2\mu}k_{3\nu}p_\rho\omega_\gamma. \tag{48}$$

The function $C_{\rm ps}$ can only be constructed with four independent four-vectors. This is the case in LFD for $n \geq 3$. In the nonrelativistic limit, one would need $n \geq 4$. We can then construct two additional spin structures S_3 and S_4 of the same parity as S_1 and S_2 by combining $C_{\rm ps}$ with parity negative matrices constructed from S_1, S_2, and γ_5 matrices. Instead of $k_{2\mu}k_{3\nu}$ one could have taken any pair of momenta ($k_{1\mu}k_{3\nu}$ or $k_{1\mu}k_{2\nu}$). We take the boson momenta for symmetry. With this definition

$$C^2_{\rm ps} = \frac{1}{m^4}[R^2_{2\perp}R^2_{3\perp} - (\mathbf{R}_{2\perp} \cdot \mathbf{R}_{3\perp})^2]. \tag{49}$$

[3]More precisely, we get not g_{03} alone but the product $g_{03}\alpha_0$. The quantity α_0 is found from the normalization condition for the state vector. This procedure requires knowing the three-body normalization integral which is calculated in the next section.

The three-body vertex function $\Gamma_3(1,2,3)$ is completely determined by the four scalar functions $g_{1-4}(1,2,3)$ in Eq. (47). They depend on $\mathbf{R}_{1-3,\perp}$ in the form of their scalar products among themselves and on x_{1-3}. Since

$$\mathbf{R}_{1\perp} + \mathbf{R}_{2\perp} + \mathbf{R}_{3\perp} = 0, \qquad x_1 + x_2 + x_3 = 1, \quad (50)$$

we can exclude, for instance, $\mathbf{R}_{1\perp}$ and x_1. The functions $g_{1,2}$ are symmetric relative to the permutation $2 \leftrightarrow 3$, whereas $g_{3,4}$ are antisymmetric:

$$g_{1,2}(1,2,3) = g_{1,2}(1,3,2),$$
$$g_{3,4}(1,2,3) = -g_{3,4}(1,3,2),$$

so that the product $C_{\rm ps} g_{3,4}(1,2,3)$ which appears in $S_3 g_3$ and $S_4 g_4$ is symmetric.

Each component g_n is represented as a sum or a difference of two terms:

$$\begin{aligned} g_{1,2}(1,2,3) &= \bar{g}_{1,2}(1,2,3) + \bar{g}_{1,2}(1,3,2), \\ g_{3,4}(1,2,3) &= \bar{g}_{3,4}(1,2,3) - \bar{g}_{3,4}(1,3,2), \end{aligned} \quad (51)$$

where the permutation $2 \leftrightarrow 3$ means

$$\mathbf{R}_{2\perp} \leftrightarrow \mathbf{R}_{3\perp}, \qquad x_2 \leftrightarrow x_3, \qquad \mu_{j_2} \leftrightarrow \mu_{j_3}.$$

In their turn, $\bar{g}_n(1,2,3)$, according to the last line in Fig. 3, are linearly expressed through the functions $\tilde{h}^{j_2}_{0,1}$ and $\tilde{H}^{j_2}_{0,1}$ which form a solution of the equations (42):

$$\begin{aligned} \bar{g}_n(1,2,3) = \alpha_0 \kappa g_{02} [& a_{n0}(1,2,3)\tilde{h}^{j_2}_0(2) + a_{n1}(1,2,3)\tilde{h}^{j_2}_1(2) \\ & + A_{n0}(1,2,3)\tilde{H}^{j_2}_0(2) + A_{n1}(1,2,3)\tilde{H}^{j_2}_1(2)]. \end{aligned} \quad (52)$$

The coefficients a and A in this formula are given in Appendix E.

We can finally calculate the three-body normalization integral. It is given by

$$I_3 = \frac{1}{2} \sum_{j_2, j_3 = 0}^{1} (-1)^{j_2 + j_3} \int \frac{n_3^{j_2 j_3}}{(s_3 - m^2)^2} dD_3, \quad (53)$$

with

$$\begin{aligned} n_3^{j_2 j_3} &= \tfrac{1}{2} \mathrm{Tr}[\bar{\Gamma}_3(\not{k}_1 + m)\Gamma_3(\not{p} + m)] \\ &= 4x_1[R^2_{1\perp} g_1^2 + m^2 g_2^2 + C^2_{\rm ps}(R^2_{1\perp} g_3^2 + m^2 g_4^2)], \end{aligned}$$

and, as usual, $\bar{\Gamma} = \gamma^0 \Gamma^\dagger \gamma^0$. The factor $\frac{1}{2}$ in Eq. (54) results from averaging over initial state spin projections, while the factor $\frac{1}{2}$ in Eq. (53) is the combinatorial factor $\frac{1}{(n-1)!}$ originating from the identity of the two bosons. The contribution of PV fermion is omitted since it disappears in the limit $m_1 \to \infty$. The phase space volume element has the form [see Eq. (3.23) from Ref. [3]]:

$$\begin{aligned} dD_3 = {} & 2(2\pi)^3 \delta^{(2)}(\mathbf{R}_{1\perp} + \mathbf{R}_{2\perp} + \mathbf{R}_{3\perp}) \\ & \times \delta(x_1 + x_2 + x_3 - 1) \prod_{l=1}^{3} \frac{d^2 R_{l\perp} dx_l}{(2\pi)^3 2x_l}. \end{aligned}$$

IV. ELECTROMAGNETIC FORM FACTORS

A. Electromagnetic vertex in CLFD

The electromagnetic vertex contains contributions of one-, two-, and three-body Fock sectors, as shown in Fig. 6. They are expressed, in our FSDR scheme, in terms of the external electromagnetic BCC $\bar{e}_{0l}$, as explained in Ref. [2]. These coupling constants are all identical to the physical fermion charge, i.e., $\bar{e}_{0l} = e$ for all l's. Note that this important property of QED is not preserved in general if FSDR is not used.

The decomposition of the spin-1/2 electromagnetic vertex in CLFD is given by [8,15]

$$\begin{aligned} \bar{u}(p') G^\rho u(p) = e\bar{u}(p') \Big[& F_1 \gamma^\rho + \frac{iF_2}{2m} \sigma^{\rho\nu} q_\nu \\ & + B_1 \Big(\frac{\not{\omega}}{\omega \cdot p} P^\rho - 2\gamma^\rho \Big) + B_2 \frac{m\omega^\rho}{\omega \cdot p} \\ & + B_3 \frac{m^2 \not{\omega} \omega^\rho}{(\omega \cdot p)^2} \Big] u(p), \end{aligned} \quad (54)$$

with $P = p + p'$, and $q = p' - p$. F_1 and F_2 are the physical form factors, while $B_{1,2,3}$ are spurious (nonphysical) contributions which appear if rotational invariance is broken, e.g., by Fock space truncation. The decomposition (54) enables us to separate unambiguously the physical form factors from the nonphysical ones. Under the condition $\omega \cdot q = 0$, all $F_{1,2}$, B_{1-3} depend on $Q^2 \equiv -q^2$ only.

We shall represent $q = (q_0, \mathbf{\Delta}, q_\parallel)$, where $q_\parallel$ and $\mathbf{\Delta}$ are, respectively, the longitudinal and transverse components of the momentum transfer with respect to the three-vector ω. Since $\omega \cdot q = \omega_0(q_0 - q_\parallel) = 0$, we have $Q^2 = \mathbf{\Delta}^2$.

After construction of the matrix

$$O^\rho = \frac{1}{4m^2}(\not{p}' + m) G^\rho (\not{p} + m), \quad (55)$$

and calculation of the traces

$$\begin{aligned} c_4 &= \mathrm{Tr}[O_\rho \omega^\rho] m/\omega \cdot p, \\ c_5 &= \mathrm{Tr}[O_\rho \omega^\rho \not{\omega}] m^2/(\omega \cdot p)^2, \end{aligned} \quad (56)$$

the electromagnetic form factors write [7]

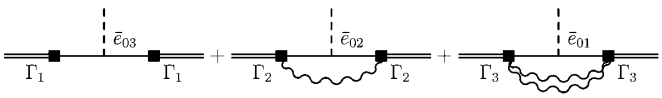

FIG. 6. One-, two-, and three-body contributions to the electromagnetic vertex.

$$F_1 = \frac{1}{2}c_5, \qquad F_2 = \frac{2m^2}{Q^2}(c_5 - c_4). \tag{57}$$

The value $F_1(Q^2 = 0)$ equals one, since it coincides with the norm of the state vector. The value $F_2(Q^2 = 0)$ is just the AMM.

B. One-body contribution

The one-body contribution to the form factor F_1 is given by the first diagram in Fig. 6. It does not depend on Q^2 and coincides with the norm of the one-body sector:

$$F_{1,1b} = \alpha_0^2 \equiv I_1. \tag{58}$$

There is no one-body contribution to the form factor F_2:

$$F_{2,1b} = 0. \tag{59}$$

C. Two-body contribution

The two-body contribution to the electromagnetic vertex, as given by the second diagram in Fig. 6, writes

$$\bar{u}(p')G^{\rho}_{2b}u(p) = \frac{1}{(2\pi)^3}\sum_{i,i',j=0}^{1}(-1)^{i+i'+j}\int d^2R_\perp \int_0^1 \frac{dx}{2x(1-x)^2}\frac{\bar{u}(p')\bar{\Gamma}_2^{i'j}(\not{k}'_{1i'} + m_{i'})\gamma^\rho(\not{k}_{1i} + m_i)\Gamma_2^{ij}u(p)}{(s_2^{i'j} - m_{i'}^2)(s_2^{ij} - m_i^2)},$$

where Γ_2^{ij} is given by Eq. (32) with $b_{1,2}^{ij} = b_{1,2}^{ij}(R_\perp^2, x)$, k_{1i} ($k'_{1i'}$) is the momentum of the constituent fermion incoming to (outgoing from) the elementary electromagnetic vertex, $s_2^{ij} = (k_{1i} + k_{2j})^2$, $s_2^{i'j} = (k'_{1i'} + k_{2j})^2$, and k_{2j} is the constituent boson momentum. $\Gamma_2^{i'j}$ has the same decomposition as Γ_2^{ij}, with the replacement $b_{1,2}^{ij}(R_\perp^2, x) \to b_{1,2}^{i'j}(R_\perp'^2, x)$ with $\mathbf{R}'_\perp = \mathbf{R}_\perp - x\boldsymbol{\Delta}$.

Using the relations (38) and (39), we can calculate the two-body contribution to the electromagnetic form factors in terms of the solutions $\tilde{h}_i^j$, $\tilde{H}_i^j$ of the system of eigenvalue equations (42). After taking the limit $m_1 \to \infty$ the result is as follows:

$$F_{1,2b} = \frac{\alpha_0^2\kappa^2}{16\pi^3}\sum_{j=0}^{1}(-1)^j\int d^2R_\perp\int_0^1 dx\frac{x[(\mathbf{R}_\perp\cdot\mathbf{R}'_\perp)\tilde{h}_0^j\tilde{h}_0^{\prime j} + m^2\tilde{H}_0^j\tilde{H}_0^{\prime j}]}{[R_\perp^2 + x^2m^2 + (1-x)\mu_j^2][R_\perp'^2 + x^2m^2 + (1-x)\mu_j^2]}, \tag{60}$$

$$F_{2,2b} = \frac{\alpha_0^2\kappa^2m^2}{4\pi^3\boldsymbol{\Delta}^2}\sum_{j=0}^{1}(-1)^j\int d^2R_\perp\int_0^1 dx\frac{x(\mathbf{R}_\perp\cdot\boldsymbol{\Delta})\tilde{h}_0^j\tilde{H}_0^{\prime j}}{[R_\perp^2 + x^2m^2 + (1-x)\mu_j^2][R_\perp'^2 + x^2m^2 + (1-x)\mu_j^2]}. \tag{61}$$

Functions with primes depend on $R_\perp'^2$ and x. The value $F_{1,2b}(Q^2 = 0)$ coincides with the two-body contribution to the normalization integral,

$$I_2 = \frac{\alpha_0^2\kappa^2}{16\pi^3}\sum_{j=0}^{1}(-1)^j\int_0^\infty d^2R_\perp\int_0^1 dxx \times\frac{R_\perp^2(\tilde{h}_0^j)^2 + m^2(\tilde{H}_0^j)^2}{[R_\perp^2 + x^2m^2 + (1-x)\mu_j^2]^2}. \tag{62}$$

To calculate the two-body contribution to the AMM, which is given by the value $F_{2,2b}(Q^2 = 0)$, one should go over to the limit $\boldsymbol{\Delta} \to 0$ in Eq. (61). The corresponding analytic formula includes derivatives over $R_\perp$ from the Fock components. For numerical calculations it is however more convenient to find $F_{2,2b}$ at small but finite Q^2 and then, decreasing the latter, to reach desired accuracy. The result of this numerical limiting procedure is very stable.

D. Three-body contribution

The three-body contribution to the electromagnetic vertex reads

$$\bar{u}(p')G^{\rho}_{3b}u(p) = \frac{1}{2}\sum_{j_2,j_3=0}^{1}(-1)^{j_2+j_3}\int\frac{\bar{u}(p')\bar{\Gamma}'_3(\not{k}'_1 + m)\gamma^\rho(\not{k}_1 + m)\Gamma_3u(p)}{4x_1^2x_2x_3(s_3 - m^2)(s'_3 - m^2)}dD_3,$$

where dD_3 is defined by Eq. (54) and

$$s_3 = \frac{R_{1\perp}^2 + m^2}{x_1} + \frac{R_{2\perp}^2 + \mu_{j_2}^2}{x_2} + \frac{R_{3\perp}^2 + \mu_{j_3}^2}{x_3}. \tag{63}$$

s'_3 differs from s_3 by the following shift of the arguments:

$$\begin{aligned}\mathbf{R}_{1\perp} &\to \mathbf{R}'_{1\perp} = \mathbf{R}_{1\perp} + (1-x_1)\boldsymbol{\Delta},\\ \mathbf{R}_{2\perp} &\to \mathbf{R}'_{2\perp} = \mathbf{R}_{2\perp} - x_2\boldsymbol{\Delta},\\ \mathbf{R}_{3\perp} &\to \mathbf{R}'_{3\perp} = \mathbf{R}_{3\perp} - x_3\boldsymbol{\Delta}.\end{aligned} \tag{64}$$

$\bar{\Gamma}'_3$ is obtained from Γ_3 by the same shift of arguments. From the decomposition (47), we can calculate G^ρ, and construct the matrix O^ρ by means of Eq. (55). The form factors are thus given by Eqs. (57) and read

$$F_{1,3b} = \int (C^{(1)}_{11} g_1 g'_1 + C^{(1)}_{22} g_2 g'_2 + C^{(1)}_{33} g_3 g'_3 + C^{(1)}_{44} g_4 g'_4 + 2C^{(1)}_{31} g_3 g'_1) \frac{dD_3}{d_1}, \tag{65a}$$

$$F_{2,3b} = 2\int (C^{(2)}_{12} g_1 g'_2 + C^{(2)}_{41} g_4 g'_1 + C^{(2)}_{32} g_3 g'_2 + C^{(2)}_{34} g_3 g'_4) \frac{dD_3}{d_2}, \tag{65b}$$

where

$$d_1 = m^4 x_2 x_3 (m^2 - s_3)(m^2 - s'_3), \qquad d_2 = \frac{2\Delta^2}{m^2} d_1.$$

g'_n differs from g_n by the shift of the arguments (64). The coefficients $C^{(1,2)}_{nk}$ in Eqs. (65a) and (65b) are given in Appendix F.

The value $F_{1,3b}(Q^2 = 0)$ coincides with the norm, I_3, of the three-body sector given by Eq. (53). The quantity α_0 which has been unknown up to now, is determined from the normalization condition for the state vector:

$$\alpha_0^2 + I_2 + I_3 = 1. \tag{66}$$

Since both I_2 and I_3 are proportional to α_0^2, then, denoting $I_{2,3} \equiv \alpha_0^2 \kappa^2 \tilde{I}_{2,3}$, where κ is defined by Eq. (40), we immediately get

$$\alpha_0^2 = \frac{1}{1 + \kappa^2(\tilde{I}_2 + \tilde{I}_3)}. \tag{67}$$

V. NUMERICAL RESULTS

The solution of Eqs. (42) is found by a matrix inversion after the discretization of the integrals, using the Gaussian method. All integrals are finite at finite PV boson mass μ_1. As already mentioned, the limit of infinite PV fermion mass m_1 has been done analytically, while the Fock components $\tilde{h}^j_i$, $\tilde{H}^j_i$, and, hence, $b^{ij}_{1,2}$ in Eq. (32) do depend on the PV boson mass μ_1. The numerical calculations have been performed on an ordinary modern laptop.

The AMM is calculated for a typical set of physical parameters $m = 0.938$ GeV, $\mu = 0.138$ GeV, and two values of the coupling constant $\alpha \equiv \frac{g^2}{4\pi} = 0.2$ and 0.5. This mimics, to some extent, a physical nucleon coupled to scalar "pions." The typical pion-nucleon coupling constant is given by $g = \frac{g_A}{2F_\pi}\langle k \rangle$ where $\langle k \rangle$ is a typical momentum scale, and g_A and F_π are the axial coupling constant and the pion decay constant, respectively. For $\langle k \rangle = 0.2$ GeV we just get $\alpha \simeq 0.2$.

We plot in Fig. 7 the AMM as a function of $\log[\frac{\mu_1^2}{\mu^2}]$, for the two different values of α pointed out above. We show also on each of these plots the value of the AMM calculated for the $N = 2$ truncation, which coincides with the AMM obtained in the second order of perturbation theory. The results for $\alpha = 0.2$ show rather good convergence as $\mu_1 \to \infty$. The contribution of the three-body Fock sector to the AMM is sizeable but small, indicating that the Fock decomposition (2) converges rapidly. This may show that once higher Fock components are small, we can achieve a practically converging calculation of the AMM. Note that this value of α is not particularly small: it is about 30 times the electromagnetic coupling, and is about the size of the typical pion-nucleon coupling in a nucleus.

When α increases, we see that the contribution of the three-body sector considerably increases. For $\alpha = 0.5$ the three-body contribution to the AMM starts to dominate at large values of μ_1. The dependence of the AMM on the PV boson mass μ_1 becomes more appreciable, although it keeps rather small.

In order to have a more physical insight into the relative importance of different Fock sectors in the decomposition (2) for the state vector, we plot in Fig. 8 the contributions of the one-, two-, and three-body Fock sectors to the norm of

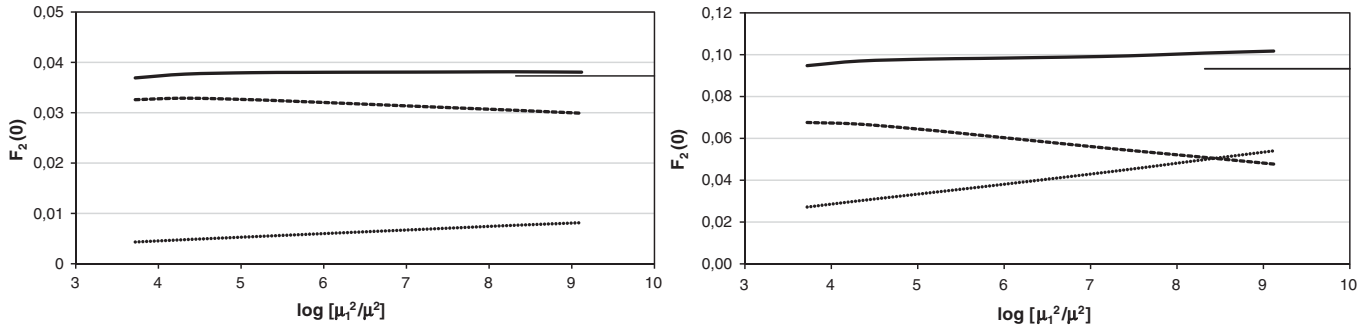

FIG. 7. The anomalous magnetic moment in the Yukawa model as a function of the PV mass μ_1, for two different values of the coupling constant, $\alpha = 0.2$ (left plot) and 0.5 (right plot). The dashed and dotted lines are, respectively, the two- and three-body contributions, while the solid line is the total result. The AMM value calculated in the $N = 2$ approximation is shown by the thin line on the right axis.

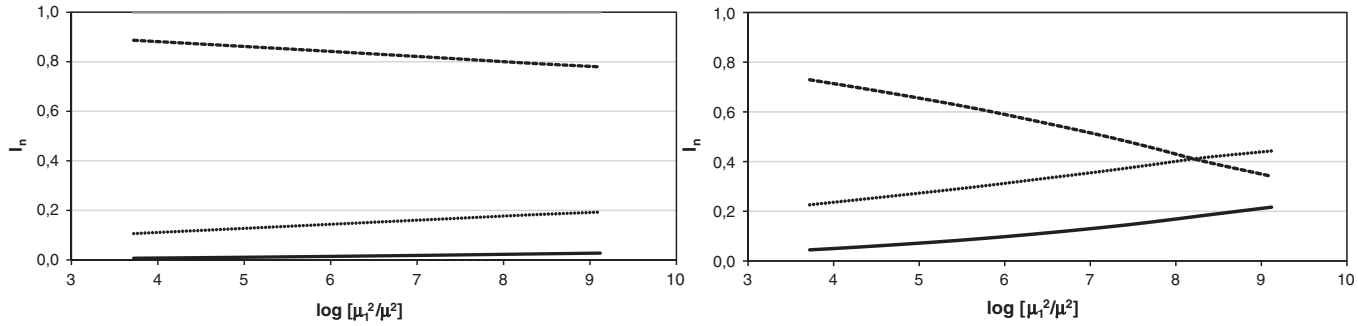

FIG. 8. Individual contributions of the one- (dashed line), two- (dotted line), and three-body (solid line) Fock sectors to the norm of the state vector as a function of the PV boson mass μ_1, for $\alpha = 0.2$ (left plot) and $\alpha = 0.5$ (right plot).

the state vector for the two values of the coupling constant, considered in this work. We see again that at $\alpha = 0.2$ the three-body contribution to the norm is small, while it is not negligible, and increases with μ_1, when $\alpha = 0.5$.

At very large values of μ_1, and for large α, I_1 becomes negative. As already mentioned, we still get a well-defined solution of Eqs. (42), and there is no discontinuity whatsoever in the value of the AMM. As shown in Fig. 7, the convergence of the AMM as a function of the PV boson mass is expected in any case to settle much before we enter into this regime. According to renormalization theory, the mass of the PV boson should be much larger than any intrinsic momentum scale present in the calculation of physical observables. With this limitation, physical observables should be independent of any variation of the PV boson mass, within an accuracy which can be increased at will. This is what we found in our numerical calculation for small enough values of α.

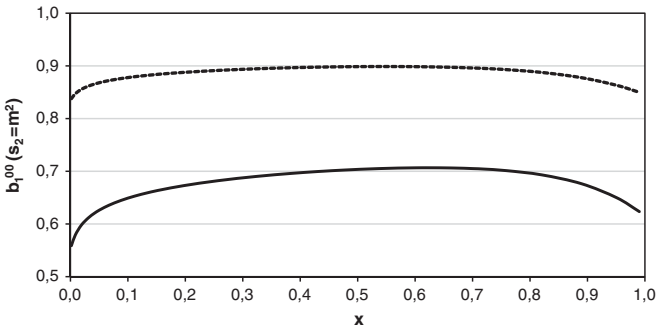

FIG. 9. The spin component b_1^{00} of the two-body vertex function (32) calculated at $s_2 = m^2$, as a function of x, for $\alpha = 0.2$ (dashed line) and $\alpha = 0.5$ (solid line), for a typical value of $\mu_1 = 100$ GeV.

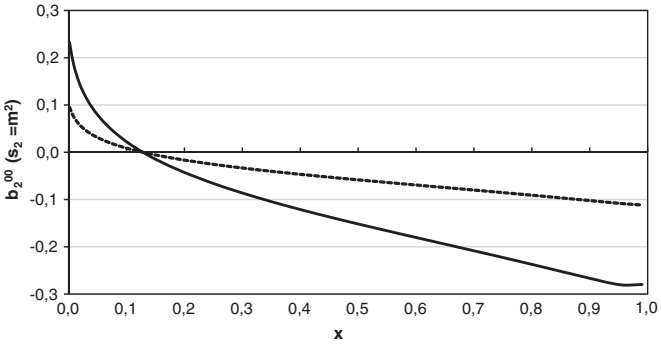

FIG. 10. The same as in Fig. 9, but for the spin component b_2^{00}.

In order to understand the possible origin of the residual dependence of the AMM on μ_1, we plot in Figs. 9 and 10 the two-body spin components b_1^{00} and b_2^{00} calculated at $s_2 = m^2$, as a function of x. As we already mentioned in Sec. III, $b_1^{00}(s_2 = m^2)$ and $b_2^{00}(s_2 = m^2)$ should be independent of x in an exact calculation. Moreover, b_2^{00} should be zero. It is here fixed to zero at a given value of $x = x^* \equiv \frac{\mu}{m+\mu}$, by the adjustment of the constant Z'_ω in the system of equations (42). We clearly see in these figures that b_1^{00} is not a constant, although its dependence on x is always weak, while b_2^{00} is not identically zero, although its value is relatively smaller than that of b_1^{00} for $\alpha = 0.2$, and starts to be not negligible for $\alpha = 0.5$.

We plot in Figs. 11 and 12 the two physical components h_0^0 and H_0^0 as a function of $R_\perp$, at $x = \frac{\mu}{m+\mu}$. As expected from the system of eigenvalue equations (42), the functions $\tilde{h}_0^0$ and $\tilde{H}_0^0$ tend to constants at large $R_\perp$. Hence, the functions h_0^0 and H_0^0 related to them by Eq. (39) tend to constants too. Note that the two-body wave function ϕ_2

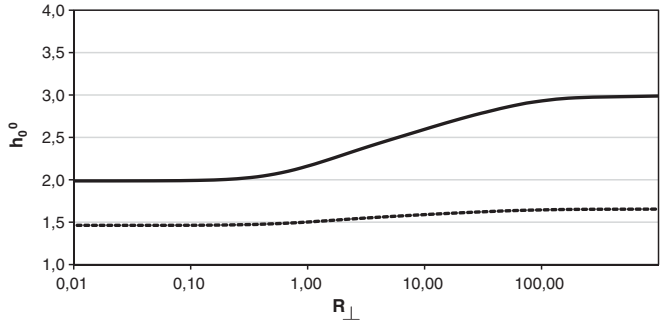

FIG. 11. The component h_0^0 defined by Eq. (39), as a function of $R_\perp$ at $x = \frac{\mu}{\mu+m}$, for a typical value of $\mu_1 = 100$ GeV and for $\alpha = 0.2$ (dashed line) and $\alpha = 0.5$ (solid line).

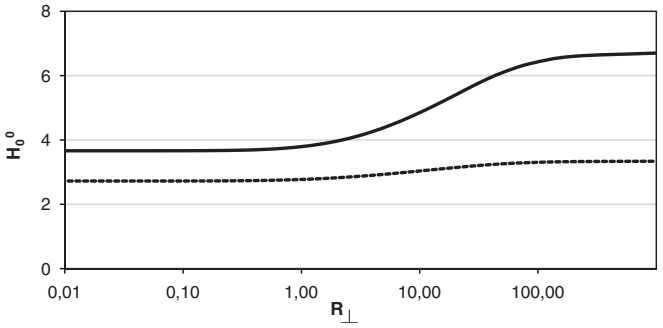

FIG. 12. The same as in Fig. 11, but for the component H_0^0.

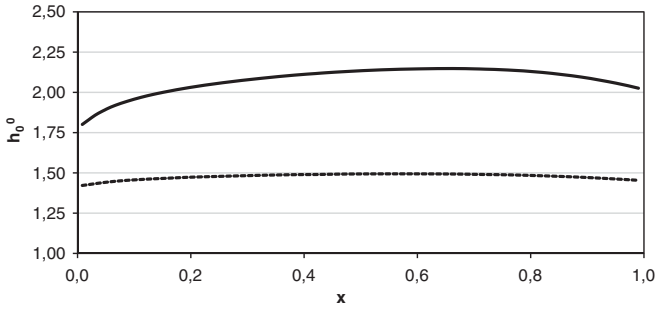

FIG. 13. The component h_0^0 defined by Eq. (39), as a function of x at $R_\perp = 0$, for a typical value of $\mu_1 = 100$ GeV and for $\alpha = 0.2$ (dashed line) and $\alpha = 0.5$ (solid line).

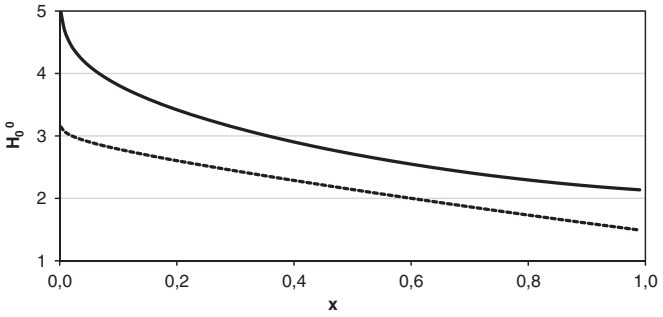

FIG. 14. The same as in Fig. 13, but for the component H_0^0.

related to Γ_2 by Eq. (11) goes to zero at large momenta due to the rapidly decreasing kinematical factor $(s_2 - m^2)^{-1}$.

For completness, we plot in Figs. 13 and 14 the two physical components h_0^0 and H_0^0 as a function of x, at $R_\perp = 0$.

VI. CONCLUSION

We calculated the anomalous magnetic moment of a fermion in the Yukawa model, in the first nontrivial approximation, incorporating a constituent fermion coupled to zero, one, and two scalar bosons, i.e., within the three-body Fock state truncation. We applied a general formalism based on the covariant formulation of light-front dynamics and an appropriate Fock sector dependent renormalization scheme which enables us to control uncancelled divergences when Fock space is truncated. We paid particular attention to the renormalization conditions necessary to relate the bare coupling constant to the physical one. To do this, we need to identify all spurious contributions originating from the violation of rotational invariance, coming from the Fock space truncation. This is possible in our covariant formulation.

The anomalous magnetic moment shows a very nice convergence as a function of the regularization scale (the Pauli-Villars boson mass μ_1), for the coupling constant value $\alpha = 0.2$ which mimics a nucleon coupled to a scalar "pion." For this value of α, the two-body component gives a dominant contribution to the anomalous magnetic moment. As α increases, we see the onset of higher Fock components.

This shows up in the large contribution of the three-body component, and in the dependence of the anomalous magnetic moment as a function of the regularization scale. We believe that this latter dependence should be largely, if not completely, removed by incorporating the relevant fermion-antifermion contributions to the three-body Fock components. We are currently investigating these contributions [16].

As we have seen in our study, the calculation of nonperturbative properties of bound state systems demand that we control all approximations in a quantitative way, in order to be able to make physical predictions order by order in the Fock space truncation. We think that the combination of the covariant formulation of light-front dynamics with an appropriate Fock sector dependent renormalization scheme is a quite promising method to investigate these properties in a very elegant way, with a minimum of Fock components and computational time.

ACKNOWLEDGMENTS

Two of us (V. A. K. and A. V. S.) are sincerely grateful for the warm hospitality of the Laboratoire de Physique Corpusculaire, Université Blaise Pascal, in Clermont-Ferrand, where part of the present study was performed.

APPENDIX A: RELATION BETWEEN THE FOCK COMPONENT NORMALIZATION AND THE FIELD STRENGTH RENORMALIZATION FACTORS

We shall prove here the relation (21) in the general case, i.e., without Fock space truncation. We omit for a moment antifermion contributions generated by the process $b \to f\bar{f}$. They will be incorporated below. The fermion self-energy is given by a sum of irreducible graphs with all possible intermediate states:

$$\Sigma(\not{p}) = \sum_{n=1}^{\infty} \Sigma_n(\not{p}). \tag{A1}$$

Here $\Sigma_n(\not p)$ is the contribution from a graph with n intermediate states. For example, the first graph in Fig. 5 for the self-energy contains one intermediate state, the second one contains three intermediate states, while the third one contains 11 intermediate states.

For the calculations we will use the 3D light-front graph techniques exposed in Ref. [3]. It can be represented in different equivalent forms. We use the form in which an amplitude is represented as a product of energy denominators (one denominator for each intermediate state) multiplied by appropriate spin matrices. The amplitude (with a conventional minus sign for the self-energy) corresponding to a graph with n intermediate states has the following form:

$$\Sigma_n(\not p) = -g_0^{n+1}\int dD \prod_{i=1}^{n}\frac{\not k_i + m_i}{s_i - p^2}. \tag{A2}$$

Since we do not truncate Fock space, we use the same BCC in all vertices. s_i is the square of the invariant energy of the i-th intermediate state. The product $\prod_{i=1}^{n}$ runs through all n intermediate states. The integration in Eq. (A2) is performed over all independent variables.

The decomposition of $\Sigma(\not p)$ is similar to that given by Eq. (28):

$$\Sigma(\not p) = \mathcal{A}(p^2) + \mathcal{B}(p^2)\frac{\not p}{m} + \mathcal{C}(p^2)\frac{m\not\omega}{\omega\cdot p} + \mathcal{C}_1(p^2)\sigma, \tag{A3}$$

where

$$\sigma = \frac{1}{4\omega\cdot p}(\not p\not\omega - \not\omega\not p).$$

Equation (A3) is the most general form of the fermion self-energy in CLFD. The term with the function $C_1(p^2)$ does not appear for the two-body ($N = 2$) Fock space truncation, but it may appear for $N \geq 3$. We give here the coefficient $\mathcal{C}$ which will be used below:

$$\mathcal{C} = \frac{1}{4m}\,\mathrm{Tr}\left[\left(\not p - \not\omega\frac{p^2}{\omega\cdot p}\right)\Sigma(\not p)\right]. \tag{A4}$$

To find $\frac{\partial\Sigma(\not p)}{\partial\not p}|_{\not p=M}$, we first replace in Eq. (A3) p^2 by M^2, $\not p$ by M (that is, replace γ^ν by p^ν/M), and then calculate the derivative over M. It is convenient to make this replacement by using the formula

$$\Sigma(M) = \frac{1}{4M}\,\mathrm{Tr}[(\not p + M)\Sigma(\not p)]_{p^2=M^2}.$$

We get

$$\frac{\partial\Sigma(\not p)}{\partial\not p}\bigg|_{\not p=M} = \frac{\partial}{\partial M}\left\{\frac{1}{4M}\,\mathrm{Tr}[(\not p + M)\Sigma(\not p)]_{p^2=M^2}\right\}. \tag{A5}$$

We substitute here $\Sigma(\not p)$ from Eqs. (A1) and (A2).

The contribution of the derivative from the j-th factor $\frac{1}{s_j-M^2}$ of the denominator, called $\Sigma^{\mathrm{den}}_{\mathrm{nj}}$, which results from Eq. (A2), reads

$$\frac{\partial\Sigma^{\mathrm{den}}_{\mathrm{nj}}(\not p)}{\partial\not p}\bigg|_{\not p=M} = -g_0^{n+1}2M\int dD\,\mathrm{Tr}\left\{\frac{1}{4M}(\not p + M)\left[\prod_{i_1=1}^{j-1}\frac{(\not k_{i_1}+m_{i_1})}{(s_{i_1}-M^2)}\right]\frac{(\not k_j+m_j)}{(s_j-M^2)^2}\left[\prod_{i_2=j+1}^{n}\frac{(\not k_{i_2}+m_{i_2})}{(s_{i_2}-M^2)}\right]\right\}. \tag{A6}$$

The factor

$$\Gamma_j = g_0^j\int dD'\prod_{i_1=1}^{j-1}\frac{(\not k_{i_1}+m_{i_1})}{(s_{i_1}-M^2)} \tag{A7}$$

is a contribution of the graph with $j-1$ intermediate states into the vertex function, and similarly for the second product. In contrast to Eq. (A6), where the integration dD runs over the phase volumes of all the intermediate states, the integration dD' in Eq. (A7) runs over the phase volumes of the intermediate states $i_1 = 1,\ldots,j-1$ only.

Since all the four-momenta are on the corresponding mass shells $k_j^2 = m_j^2$, we have

$$(\not k_j + m_j) = \sum_{\sigma=\pm1/2} u_\sigma(k_j)\bar u_\sigma(k_j),$$

$$\frac{1}{2}\,\mathrm{Tr}[(\not p + M)O] = \frac{1}{2}\sum_{\lambda=\pm1/2}\bar u_\lambda(p)Ou_\lambda(p),$$

for an arbitrary matrix O. The factor $\frac{1}{2}$ in the last equation is introduced for averaging over initial spin projections.

We therefore get

$$\begin{aligned}\frac{\partial\Sigma^{\mathrm{den}}_{\mathrm{nj}}(\not p)}{\partial\not p}\bigg|_{\not p=M} &= -\frac{1}{2}\sum_{\lambda,\sigma}\int dD_j\frac{\bar u_\lambda(p)\Gamma_j u_\sigma(k_j)}{(s_j-M^2)}\\ &\quad\times\frac{[\bar u_\lambda(p)\Gamma_j u_\sigma(k_j)]^\dagger}{(s_j-M^2)}\\ &= -\frac{1}{2}\sum_{\lambda,\sigma}\int dD_j\,\phi^\lambda_{j,\sigma}(p)\phi^{\lambda\dagger}_{j,\sigma}(p).\end{aligned} \tag{A8}$$

Here the integration dD_j runs over the phase volume of the j-th intermediate state not included in the integral for Γ_j. The vertex function Γ_j may correspond to any fixed number of particles in the state j allowed by a given graph. We took into account that the factor $1/(s_j - M^2)$ turns each Γ into ϕ according to Eq. (11). Taking the sum over all the graphs and over all the intermediate states j, we recover in Eq. (A8) the contribution to the normalization integral $I_{N\geq2}$ from all the N-body states with $N \geq 2$ (each intermediate state in irreducibles graphs for Σ contains at least

two particles). Since the rules of the graph techniques used to calculate Σ imply that the one-body states are normalized to 1, this means that $I_{N\geq 2}$ corresponds to a state vector normalized by the condition $I_1 = 1$. If $I_1 \neq 1$, then Eq. (A6) determines the ratio $-I_{N\geq 2}/I_1$. That is

$$\left.\frac{\partial \Sigma^{\rm den}(\not p)}{\partial \not p}\right|_{\not p = M} = -\frac{I_{N\geq 2}}{I_1}. \tag{A9}$$

The contribution of the derivative of other factors in Eq. (A5), except for $\prod_i \frac{1}{s_i - M^2}$, which is called $\Sigma^{\rm num}$, reads

$$\left.\frac{\partial \Sigma^{\rm num}(\not p)}{\partial \not p}\right|_{\not p = M} = -g_0^{n+1} \int dD \frac{\frac{\partial}{\partial M}\{\frac{1}{4M}\,{\rm Tr}[(\not p + M)\prod_{i=1}^n (\not k_i + m_i)]_{p^2=M^2}\}}{\prod_{i=1}^n (s_i - M^2)}. \tag{A10}$$

Consider first the case when the products in Eq. (A10) contain only one factor with a fixed i. Then Σ corresponds to the first graph in Fig. 5. We calculate the trace in Eq. (A10), using the following explicit expression for the scalar product:

$$k_i \cdot p = \frac{1}{2x_i}(\mathbf{R}_{i\perp}^2 + m_i^2 + x_i^2 M^2),$$

where the variables $\mathbf{R}_{i\perp}$ and x_i are constructed according to Eq. (33). Calculating then the derivative over M, we find

$$\left.\frac{\partial \Sigma^{\rm num}(\not p)}{\partial \not p}\right|_{\not p = M} = -g_0^2 \int dD_2 \frac{\frac{\partial}{\partial M}\{\frac{1}{4M}\,{\rm Tr}[(\not p + M)(\not k_i + m_i)]\}}{(s_i - M^2)} = -g_0^2 \int dD_2 \frac{\frac{1}{2x_i M^2}(\mathbf{R}_{i\perp}^2 + m_i^2 - x_i^2 M^2)}{(s_i - M^2)}, \tag{A11}$$

where $dD_2 = \frac{d^2 R_{i\perp}\, dx_i}{(2\pi)^3 2x_i(1-x_i)}$ is the two-body phase space volume element.

Let us calculate now the value of the coefficient $\mathcal{C}$ in Eq. (A3). It is given by Eq. (A4). We still consider a particular case and keep one factor only with fixed i. Then $\mathcal{C}$ obtains the form

$$\begin{aligned}\mathcal{C} &= -g_0^2 \int dD_2 \frac{\frac{1}{4m}\,{\rm Tr}[(\not p - \not\omega \frac{p^2}{\omega\cdot p})(\not k_i + m_i)]}{(s_i - M^2)} \\ &= -g_0^2 \int dD_2 \frac{(-\frac{M^2}{m})\frac{1}{2x_i M^2}(\mathbf{R}_{i\perp}^2 + m_i^2 - x_i^2 M^2)}{(s_i - M^2)}.\end{aligned} \tag{A12}$$

Comparing Eq. (A11) with Eq. (A12), we find the relation

$$\left.\frac{\partial \Sigma^{\rm num}(\not p)}{\partial \not p}\right|_{\not p = M} = -\frac{m}{M^2}\mathcal{C}. \tag{A13}$$

It turns out that Eq. (A13) is valid in the most general case. In the latter case, but still without antifermions, we get in the numerator in Eqs. (A11) and (A12) a product of the matrices $\prod_{i=1}^n (\not k_i + m_i)$, instead of the single term $(\not k_i + m_i)$. This product can be decomposed in the full set of the 4×4 matrices as follows:

$$\begin{aligned}\prod_{i=1}^n (\not k_i + m_i) = G_0 + \sum_i G_1^i \not k_i + \sum_{i_1, i_2} G_2^{i_1 i_2} \sigma(k_{i_1}, k_{i_2}) \\ + G_3 \gamma_5 + \sum_i G_4^i \gamma_5 \not k_i,\end{aligned} \tag{A14}$$

where $\sigma(k_{i_1}, k_{i_2}) = \frac{i}{2}(\not k_{i_1} \not k_{i_2} - \not k_{i_2} \not k_{i_1})$. The coefficients G_{1-3} depend on the scalar products of the four-momenta $k_1, \ldots k_n$:

$$k_i \cdot k_j = \frac{1}{2x_i x_j}[x_i^2 m_j^2 + (x_j \mathbf{R}_{i\perp} - x_i \mathbf{R}_{j\perp})^2 + x_j^2 m_i^2].$$

It is important that these scalar products and, hence, the functions G_{1-3} do not depend on M. Therefore, G_{1-3} can be extracted from the operator $\frac{\partial}{\partial M}$. We replace $(\not k_i + m_i)$ in Eqs. (A11) and (A12) by the product $\prod_{i=1}^n (\not k_i + m_i)$ represented in the form (A14) (and take the product of the denominators). The matrices γ_5 and $\gamma_5 \not k_i$ give zero contributions to both Eqs. (A11) and (A12), whereas with the matrices 1, $\not k_i$, and $\sigma(k_{i_1}, k_{i_2})$ we reproduce the relation (A13).

The incorporation of antifermions (say, the $ff\bar f$ intermediate state, in addition to bosons) does not change the form of the denominator (though the energies s_i incorporate now the antifermion momenta). That results in Eq. (A9). The corresponding numerator contains now the spin matrices of all the fermions + antifermions [we get one factor $1/(s_i - p^2)$ and a product of three matrices $(\pm \not k_i + m_i)$ for the $ff\bar f$ state; the signs plus and minus stand for fermions and antifermions, respectively]. We still can decompose the full numerator according to Eq. (A14) and again reproduce the formula (A13).

In this way, taking the sum of Eqs. (A9) and (A13), we finally find

$$\left.\frac{\partial \Sigma(\not p)}{\partial \not p}\right|_{\not p = M} = -\frac{I_{N\geq 2}}{I_1} - \frac{m}{M^2}\mathcal{C}. \tag{A15}$$

If rotational invariance is preserved (it can be violated for instance by omitting some time-ordered graphs or by using rotationally noninvariant cutoffs), $\mathcal{C}$ is zero. It is indeed zero, for example, in the two-body approximation with the PV regularization [see Eq. (B6) in Appendix B]. If $\mathcal{C} = 0$,

substituting Eq. (A15) into Eq. (16) and taking into account that $I_1 + I_{N\geq 2} = 1$, we prove the relation (21).

APPENDIX B: SELF-ENERGY COEFFICIENTS

We give here explicit formulas for the coefficients $\mathcal{A}(k^2)$, $\mathcal{B}(k^2)$, and $\mathcal{C}(k^2)$ entering Eq. (28) for the two-body self-energy in the Yukawa model. If $\Sigma(\not{k})$ is known, these coefficients can be found as follows:

$$g_{02}^2\mathcal{A}(k^2) = \tfrac{1}{4}\mathrm{Tr}[\Sigma(\not{k})], \tag{B1}$$

$$g_{02}^2\mathcal{B}(k^2) = \frac{m}{4\omega\cdot k}\,\mathrm{Tr}[\Sigma(\not{k})\not{\omega}], \tag{B2}$$

$$g_{02}^2\mathcal{C}(k^2) = \frac{1}{4m}\,Tr\left[\Sigma(\not{k})\left(\not{k} - \not{\omega}\frac{k^2}{\omega\cdot k}\right)\right]. \tag{B3}$$

In the Yukawa model, the self-energy regularized by one PV boson and one PV fermion reads, in CLFD,

$$\Sigma(\not{k}) = -\frac{g_{02}^2}{(2\pi)^3}\int d^2R_\perp \int_0^1 \frac{dx}{2x(1-x)} \times \sum_{i,j=0}^{1}(-1)^{i+j}\frac{(\not{k}_1 + m_i)}{(s_{ij} - k^2)}, \tag{B4}$$

where k_1 is the internal fermion four-momentum. The light-front variables are, as usual, $\mathbf{R}_\perp = \mathbf{k}_{2\perp} - x\mathbf{k}_\perp$, $x = \omega\cdot k_2/\omega\cdot k$ (k_2 is the boson four-momentum), and

$$s_{ij} = \frac{R_\perp^2 + \mu_j^2}{x} + \frac{R_\perp^2 + m_i^2}{1-x}. \tag{B5}$$

The coefficients $\mathcal{A}$ and $\mathcal{B}$ converge without the PV fermion (i.e., they have finite limit when $m_1 \to \infty$). Substituting Eq. (B4) into Eqs. (B1) and (B2), integrating over $d^2R_\perp$, and omitting the PV fermion, we get

$$\mathcal{A}(k^2) = \frac{m}{16\pi^2}\int_0^1 \log\left[\frac{xm^2 - x(1-x)k^2 + (1-x)\mu^2}{xm^2 - x(1-x)k^2 + (1-x)\mu_1^2}\right]dx,$$

$$\mathcal{B}(k^2) = \frac{m}{16\pi^2}\int_0^1 (1-x)\log\left[\frac{xm^2 - x(1-x)k^2 + (1-x)\mu^2}{xm^2 - x(1-x)k^2 + (1-x)\mu_1^2}\right]dx.$$

Notice that in the limit $\mu_1 \to \infty$ and for fixed k^2 the values of $\mathcal{A}_r(k^2) = \mathcal{A}(k^2) - \mathcal{A}(m^2)$ and $\mathcal{B}_r(k^2) = \mathcal{B}(k^2) - \mathcal{B}(m^2)$ are finite.

A similar calculation of $\mathcal{C}(k^2)$ requires, for convergence, not only one PV boson, but also one PV fermion. We thus find

$$\mathcal{C}(k^2) = -\frac{1}{32m\pi^2}\int_0^1\frac{dx}{1-x}\int_0^\infty dR_\perp^2\sum_{i,j=0}^{1}(-1)^{i+j}\frac{R_\perp^2 - (1-x)^2k^2 + m_i^2}{R_\perp^2 - x(1-x)k^2 + (1-x)\mu_j^2 + xm_i^2} \equiv 0. \tag{B6}$$

APPENDIX C: RIGHT-HAND SIDES OF THE EIGENVALUE EQUATIONS (27)

The system of equations (27) determines the one- and two-body Fock components Γ_1^i, Γ_2^{ij}. The right-hand sides of these equations are denoted by $\bar{u}(p_{1i})(V_1 + V_2)u(p)$ and $\bar{u}(k_{1i})(V_3 + V_{45} + V_6)u(p)$, respectively. The explicit form of V_{1-6} is the following:

$$V_1 = \delta m_3\sum_{i'}(-1)^{i'}\frac{(\not{p}_{i'} + m_{i'})}{m_{i'}^2 - M^2}\Gamma_1^{i'}, \tag{C1a}$$

$$V_2 = g'_{03}\sum_{i',j'}(-1)^{i'+j'}\int\frac{d^2R'_\perp}{(2\pi)^3}\int_0^1\frac{dx'}{2x'}\frac{(\not{k}'_{1i'} + m_{i'})}{2(\omega\cdot p)\tau_{i'j'}}\Gamma_2^{i'j'}, \tag{C1b}$$

$$V_3 = g'_{03}\sum_{i'}(-1)^{i'}\frac{(\not{p}_{i'} + m_{i'})}{m_{i'}^2 - M^2}\Gamma_1^{i'}, \tag{C1c}$$

$$V_{45} = [-\Sigma(\not{p} - \not{k}_{2j}) + \delta m_2]\sum_{i'}(-1)^{i'}\frac{(\not{k}_{1i'} + m_{i'})}{2(\omega\cdot p)(1-x)\tau_{i'j}}\Gamma_2^{i'j}, \tag{C1d}$$

$$V_6 = g_{02}^2\sum_{i',j',i''}(-1)^{i'+j'+i''}\int\frac{d^2R'_\perp}{(2\pi)^3}\int_0^{1-x}\frac{dx'}{2x'}\frac{(\not{k}'_{1i''} + m_{i''})(\not{k}'_{1i'} + m_{i'})}{4(\omega\cdot p)^2(1-x')(1-x-x')\tau_{i'j'}\tau_{i''jj'}}\Gamma_2^{i'j'}, \tag{C1e}$$

with $g'_{03} = g_{03} + Z_\omega \frac{m\phi}{\omega\cdot p}$ and obvious notations for the momenta of the particles in the intermediate states. The term V_{45} stands for the sum of the contributions of the graphs V_4 and V_5 in Fig. 5. The two-body vertex functions Γ_2 inside the integrands depend on $\mathbf{R}'_\perp$ and x', while those which are not integrated depend on $\mathbf{R}_\perp$ and x. After calculating the traces taken from Eqs. (37a) and (37b), we obtain scalar products which are expressed through the variables $\mathbf{R}_\perp, x$ and $\mathbf{R}'_\perp, x'$. Examples are given in Appendix C of Ref. [2].

The values of τ's, which appear in the above formulas, are related to the invariant energies in the corresponding intermediate states. For example, $\tau_{i''jj'}$ in Eq. (C1d) for V_6 has the form

$$\tau_{i''jj'} = \frac{s_{i''jj'} - m^2}{2\omega\cdot p},$$

where

$$s_{i''jj'} = (k_{1i''} + k_{2j} + k'_{2j'})^2.$$

$k_{1i''}$, k_{2j}, and $k'_{2j'}$ are the four-momenta in the intermediate states while s, for any intermediate state, is expressed through the light-front variables as follows:

$$s = \left(\sum_i k_i\right)^2 = \sum_i \frac{R^2_{i\perp} + m_i^2}{x_i},$$

where $\mathbf{R}_{i\perp}$ and x_i are constructed according to Eq. (33). They satisfy the conservation laws similar to Eq. (50).

APPENDIX D: THE INTEGRAL TERMS IN THE EQUATIONS (42)

The numerators and denominators of the kernels in the integrals in Eqs. (42) are linear functions of the scalar products $\mathbf{R}_\perp\cdot\mathbf{R}'_\perp = R_\perp R'_\perp\cos\phi'$, where $\mathbf{R}'_\perp$ is the integration variable. We can therefore analytically integrate over $d\phi'$, using the formulas

$$J_0 = \int_0^{2\pi}\frac{d\phi'}{2\pi D(A + B\cos\phi')} = \frac{\mathrm{sign}(A)}{D\sqrt{A^2 - B^2}},$$
$$J_1 = \int_0^{2\pi}\frac{\cos\phi' d\phi'}{2\pi D(A + B\cos\phi')} = \frac{1}{DB} - \frac{A}{B}J_0.$$

One should substitute here

$$A = R'^2_\perp(1-x)x + x'[x(x+x')m^2 + R^2_\perp(1-x')] - (x+x'-1)(x'\mu_j^2 + x\mu_{j'}^2),$$
$$B = 2R'_\perp R_\perp x'x,$$
$$D = -8\pi^2(1-x').$$

With

$$\eta_1 = m^2x'^2 + (1-x')\mu_{j'}^2 + R'^2_\perp,$$

the integral terms obtain the form

$$i_n^j = \int_0^\infty R'_\perp dR'_\perp\int_0^{1-x}dx'\sum_{i,j'=0}^{1}(-1)^{j'}(c_{ni}\tilde{h}_i^{j'} + C_{ni}\tilde{H}_i^{j'}), \tag{D1}$$

$$I_n^j = \int_0^\infty R'_\perp dR'_\perp\int_0^{1-x}dx'\sum_{i,j'=0}^{1}(-1)^{j'}(c'_{ni}\tilde{h}_i^{j'} + C'_{ni}\tilde{H}_i^{j'}), \tag{D2}$$

for $n = 0, 1$. These integrals converge due to the PV regularization (the sum over j'). The 16 coefficients c, C, c', and C' depend on j'. They are given below.

$$c_{00} = \frac{R'_\perp}{R_\perp\eta_1}\{R'_\perp R_\perp xx'J_0 + J_1[-R'^2_\perp(x-1)x + x'[x(-x(x'-3) + 3x' - 4)m^2 + R^2_\perp(1-x')] + (x-1)(x'-1)(x'\mu_j^2 + x\mu_{j'}^2)]\},$$
$$c_{01} = -\frac{R'_\perp}{R_\perp}x(2x + x' - 2)J_1,$$
$$C_{00} = \frac{m^2}{R_\perp\eta_1}xx'[J_0R_\perp(3x'-2) + R'_\perp(2-3x)J_1],$$
$$C_{01} = \frac{x}{R_\perp}[R'_\perp(x-1)J_1 - R_\perp x'J_0],$$
$$c_{10} = \frac{R'_\perp m^2}{R_\perp\eta_1}xx'(x + 2x' - 2)J_1,$$
$$c_{11} = -\frac{R'_\perp}{R_\perp}x(x + x' - 1)J_1,$$
$$C_{10} = -\frac{m^2}{R_\perp\eta_1}xx'[R_\perp(1-x')J_0 + R'_\perp xJ_1],$$
$$C_{11} = 0,$$
$$c'_{00} = \frac{R'_\perp}{\eta_1}xx'[R'_\perp(3x-2)J_0 + R_\perp(2-3x')J_1],$$
$$c'_{01} = -\frac{R'_\perp}{m^2}x[R'_\perp(x-1)J_0 - R_\perp x'J_1],$$
$$C'_{00} = -\frac{1}{\eta_1}\{R'^2_\perp(x-1)xJ_0 - R'_\perp R_\perp xx'J_1 + [x(x(x'-3) - 3x' + 4)m^2 + R^2_\perp(x'-1)]x'J_0 - (x-1)(x'-1)(x'\mu_j^2 + x\mu_{j'}^2)J_0\},$$
$$C'_{01} = -x(2x + x' - 2)J_0,$$
$$c'_{10} = \frac{R'_\perp}{\eta_1}xx'[R'_\perp xJ_0 - R_\perp(x'-1)J_1],$$
$$c'_{11} = 0,$$
$$C'_{10} = \frac{m^2}{\eta_1}xx'(x + 2x' - 2)J_0,$$
$$C'_{11} = -x(x + x' - 1)J_0.$$

APPENDIX E: COEFFICIENTS IN THE EQUATIONS (52)

The three-body Fock component (one fermion + two bosons) is decomposed in four spin structures by Eq. (47) with the coefficients g_{1-4} being scalar functions. These coefficients are linear combinations (52) of the functions $\tilde{h}$ and $\tilde{H}$ which are the solution of the equations (42). The coefficients of these linear combinations are given below.

With the notation

$$\eta_2 = m^2 x_2^2 + (1-x_2)\mu_{j_2}^2 + R_{2\perp}^2,$$

we have

$$a_{10} = -\frac{m(\mathbf{R}_{1\perp}\cdot\mathbf{R}_{2\perp})x_2(1+x_1-x_2)}{2R_{1\perp}^2 x_1(1-x_2)\eta_2},$$

$$a_{11} = \frac{(\mathbf{R}_{1\perp}\cdot\mathbf{R}_{2\perp})}{2mR_{1\perp}^2(1-x_2)},$$

$$A_{10} = \frac{mx_2[R_{1\perp}^2(1-x_2)+(\mathbf{R}_{1\perp}\cdot\mathbf{R}_{2\perp})x_1]}{2R_{1\perp}^2 x_1(1-x_2)\eta_2},$$

$$a_{20} = \frac{x_2[(1-x_2)(\mathbf{R}_{1\perp}\cdot\mathbf{R}_{2\perp})+R_{2\perp}^2 x_1]}{2mx_1(1-x_2)\eta_2},$$

$$A_{20} = \frac{mx_2(1+x_1-x_2)}{2x_1(1-x_2)\eta_2},$$

$$A_{21} = -\frac{1}{2m(1-x_2)},$$

$$a_{30} = -\frac{m^3 x_2(1+x_1-x_2)}{2R_{1\perp}^2 x_1(1-x_2)\eta_2},$$

$$a_{31} = \frac{m}{2R_{1\perp}^2(1-x_2)},$$

$$A_{30} = \frac{m^3 x_2}{2R_{1\perp}^2(1-x_2)\eta_2},$$

$$a_{40} = \frac{mx_2}{2x_1\eta_2},$$

$$A_{11} = a_{21} = A_{31} = a_{41} = A_{40} = A_{41} = 0.$$

APPENDIX F: COEFFICIENTS ENTERING EQ. (65)

The three-body contributions to the form factors F_1 and F_2 are integrals from bi-linear combinations of the four spin components g_{1-4}. The coefficients determining the three-body contribution to the form factor F_1, Eq. (65a), have the form

$$C_{11}^{(1)} = m^4[R_{1\perp}^2 + (\mathbf{R}_{1\perp}\cdot\Delta)(1-x_1)],$$

$$C_{22}^{(1)} = m^6,$$

$$C_{44}^{(1)} = -m^2\{(\mathbf{R}_{1\perp}\cdot\mathbf{R}_{2\perp})^2 + [(1-x_1)(\mathbf{R}_{2\perp}\cdot\boldsymbol{\Delta}) - x_2(\mathbf{R}_{1\perp}\cdot\boldsymbol{\Delta})](\mathbf{R}_{1\perp}\cdot\mathbf{R}_{2\perp}) + (\mathbf{R}_{1\perp}\cdot\boldsymbol{\Delta})R_{2\perp}^2(x_1-1) + R_{1\perp}^2[(\mathbf{R}_{2\perp}\cdot\boldsymbol{\Delta})x_2 - R_{2\perp}^2]\},$$

$$C_{33}^{(1)} = \frac{1}{m^6}C_{11}^{(1)}C_{44}^{(1)},$$

$$C_{31}^{(1)} = m^2[(\mathbf{R}_{1\perp}\cdot\boldsymbol{\Delta})(\mathbf{R}_{1\perp}\cdot\mathbf{R}_{2\perp}) - R_{1\perp}^2(\mathbf{R}_{2\perp}\cdot\boldsymbol{\Delta})] \times (x_1-1).$$

The coefficients determining the three-body contribution to the form factor F_2, Eq. (65b), have the form

$$C_{12}^{(2)} = -4m^4(\mathbf{R}_{1\perp}\cdot\Delta),$$

$$C_{41}^{(2)} = -4m^2[(\mathbf{R}_{1\perp}\cdot\boldsymbol{\Delta})(\mathbf{R}_{1\perp}\cdot\mathbf{R}_{2\perp}) - R_{1\perp}^2(\mathbf{R}_{2\perp}\cdot\boldsymbol{\Delta})],$$

$$C_{32}^{(2)} = -C_{41}^{(2)},$$

$$C_{34}^{(2)} = \frac{4}{m^2}(\mathbf{R}_{1\perp}\cdot\boldsymbol{\Delta})C_{44}^{(1)}.$$

We recall that $Q^2 = \boldsymbol{\Delta}^2$.

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