Energy and momentum relaxation of heavy fermion in dense and warm plasma

Sreemoyee Sarkar* and Abhee K. Dutt-Mazumder†

High Energy Physics Division, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata-700 064, India

(Received 13 May 2010; published 7 September 2010)

We determine the drag and the momentum diffusion coefficients of heavy fermion in dense plasma. It is seen that in degenerate matter the drag coefficient at the leading order mediated by the transverse photon is proportional to $(E - \mu)^2$ while for the longitudinal exchange this goes as $(E - \mu)^3$. We also calculate the longitudinal diffusion coefficient to obtain the Einstein relation in a relativistic degenerate plasma. Finally, finite temperature corrections are included both for the drag and the diffusion coefficients.

DOI: 10.1103/PhysRevD.82.056003 PACS numbers: 11.10.Wx, 12.20.Ds, 12.38.Mh

I. INTRODUCTION

Recent years have witnessed significant progress in understanding the properties of hot and/or dense relativistic plasma [1,2]. Such studies draw their motivations both from the theory and the experiments. In particular, the possibility of creating high temperature quark gluon plasma by colliding heavy ions in the laboratory mimicking the conditions of the microsecond old universe has been a matter of intense research activities in the past decades. Further impetus to these studies comes from astrophysics where it is important to know the properties of such plasma at high density, which, for example, might exist in the core of neutron stars or in white dwarfs.

One of the interesting quantities which has assumed special interest recently is the study of partonic energy loss in relativistic plasma. Several calculations [3–7] have been performed over the last decades to estimate such energy loss in a plasma. Similarly, there exists several studies in which the momentum diffusion coefficient of a heavy fermion has been estimated [8–11]. These two quantities are of utmost importance to understand the equilibration of fermions in a plasma. So far, these calculations were largely confined to the case of hot plasma with zero chemical potential due to their relevance to the experiments performed at the Relativistic Heavy Ion Collider or the ones to be performed at the Large Hadron Collider.

There still exists another domain of the quantum chromodynamic (QCD) phase diagram where the chemical potential (μ) might be higher compared to the temperature (T). This is the region of interest of the upcoming experiments on compressed baryonic matter to be performed at FAIR/GSI [12–14]. Partially motivated by these proposed experiments and partly by another theoretical work on the fermion damping rate [15], we calculate here the drag (η) and the longitudinal momentum diffusion coefficient ($\mathcal{B}$) of a heavy fermion in quantum electrodynamic (QED) plasma. It is known that the former and the latter are related to the energy loss and the momentum relaxation of the fermion in a plasma. Moreover, in equilibrating plasma, these two quantities viz. η and $\mathcal{B}$ are related to each other via the Einstein relation (ER) which at finite temperature reads as $\mathcal{B} = 2ET\eta$. As indicated above, such calculations, for dense ($T = 0$) and/or warm ($T \ll \mu$) plasma are rather limited. In fact, we are aware of only one calculation of energy loss where the effects of finite chemical potential has been considered, although the temperature considered there is still high [16]. We, on the contrary, first consider the extreme case of zero temperature and then incorporate finite temperature corrections to our result both for the drag (energy loss) and the diffusion coefficient in the limit $\mu \gg T$. We also determine the relationship between η and $\mathcal{B}$, i.e., ER at zero temperature, which shows some interesting behavior due to the finite density plasma effect.

Before we proceed further, it would be worthwhile to draw our attention to [15]. This is an interesting work in many ways. First, it is known that the fermion damping rate (γ) in hot plasma is plagued with divergences which cannot be removed by the ordinary screening effect [17]. This is because the magnetic interaction is screened only dynamically [18] and the problem remains for the static photons (or gluons in QCD). Therefore, to obtain a finite result, a suitable resummation has to be performed. This was first done in [19,20]. Reference [15] shows that at zero temperature due to Pauli blocking, a finite result can be obtained without performing further resummation. This is consistent with the conclusion drawn in [21]. Second, in the relativistic plasma γ is dominated by the magnetic exchange and is proportional to $(E - \mu)$, while the electric photon exchange gives a contribution proportional to $(E - \mu)^2$. Here, it is important to note that the dynamical screening in the transverse sector enhances the damping rate compared to its longitudinal counterpart. It might be recalled also that for nonrelativistic Coulomb plasma the damping rate goes as $(E - \mu)^2$ [22]. Thus, it would be interesting to see how do the drag and the diffusion coefficient depend on $(E - \mu)$ in degenerate plasma.

It is known that at finite temperature the calculation for the energy loss and diffusion coefficients are plagued with infrared divergences [17]. To deal with this problem, in hot

*sreemoyee.sarkar@saha.ac.in
†abhee.dm@saha.ac.in

plasma one separates the integration into two domains: one involving the exchange of hard photons (or gluons), i.e., the momentum transfer $(q) \sim T$ and the other involving soft photons (or gluons) when $q \sim eT$ ($e \ll 1$). In the case of the former, one uses a bare propagator and introduces an arbitrary cutoff (q^*) [23] parameter to regularize the integration. For the latter, on the other hand, one uses the hard thermal loop (HTL) corrected propagator. These two parts, upon addition, yield results independent of this intermediate scale. In the case of degenerate plasma, one also encounters similar infrared divergences and following Ref. [15] one can proceed along the same way as finite temperature [using the hard dense loop (HDL) corrected propagator] and show that both for the drag and diffusion coefficient the final result becomes independent of the intermediate cutoff parameter. This, however, as we shall see, is not required in the case of dense plasma. Here, the dominant or the leading order contribution comes entirely from the soft sector and the hard photon exchange contributes only to the higher order. It might be mentioned here that although we calculate these quantities for QED, with appropriate color factors the results can easily be extended to the case of QCD with the addition of one more diagram involving a triple gluon vertex [4]. Furthermore, it might be noted that the quark energy loss calculations in general should also include a Bremsstrahlung radiation of the gluons. However, in the present context we are concerned with only the two body scatterings and therefore restrict ourselves to the collisional energy loss alone.

Furthermore, expressions derived for the degenerate plasma, wherever possible, have been directly compared with their finite temperature counterparts (with zero chemical potential). This brings the similarities and the differences of these two extreme scenarios into clearer relief.

The plan of the paper is as follows. First in Sec. II we calculate drag and diffusion coefficients in degenerate plasma and discuss Einstein relation. In Sec. III, the finite temperature corrections have been incorporated both for η and $\mathcal{B}$. The results are then summarized in Sec. IV. An Appendix has also been added to understand the origin of difference in ER in a cold medium than from that of a hot plasma.

II. HEAVY FERMION AT ZERO TEMPERATURE

A. Drag coefficient

In this section we first calculate the drag coefficient of a heavy fermion in a degenerate QED plasma. For this we consider the scattering of a heavy fermion having energy (E) (which we assume to be hard), with the constituents of the plasma viz. the electrons. Incidentally, this drag coefficient (η) is related to the to the energy loss by the following equation:

$$\eta = \frac{1}{Ev_i}\left(-\frac{dE}{dx}\right), \tag{1}$$

where, $\mathbf{v}_i = \frac{\mathbf{p}}{E}$ is the velocity of the incident fermion, (dE/dx) is the energy loss, and $\mathbf{p}$ is the three momentum of the incident fermion. Thus, the calculation of the drag coefficient boils down to the calculation of collisional energy loss in a plasma [3–7]. Now the energy loss (dE/dx) can be obtained by averaging over the interaction rate times the energy transfer per scattering ω and dividing by the velocity of the incoming particle [3],

$$\frac{dE}{dx} = \frac{1}{v_i}\int d\Gamma\omega. \tag{2}$$

This expression is quite general and valid for both the finite temperature and/or density where only the phase space will be different due to the modifications of the distribution functions depending upon the values of μ and T.

The scattering rate, which is essential for the calculation of η as evident from Eqs. (1) and (2), is related to the imaginary part of the fermion self-energy (Σ) by the following equation [24]:

$$\Gamma(E) = -\frac{1}{2E}\,\mathrm{Tr}[\mathrm{Im}\Sigma(p_0 + i\eta, \mathbf{p})(P + m)]|_{p_0=E}. \tag{3}$$

In the last equation, m is the mass of the incoming heavy fermion. The full fermion self-energy represented in Fig. 1 can be written explicitly as

$$\Sigma(P) = e^2T\sum_s\int\frac{d^3q}{(2\pi)^3}\gamma_\mu \times S_f(i(\omega_n - \omega_s), \mathbf{p} - \mathbf{q})\gamma_\nu\Delta_{\mu\nu}(i\omega_s, \mathbf{q}), \tag{4}$$

where, $p_0 = i\omega_n + \mu$, $q_0 = i\omega_s$. $\omega_n = \pi(2n + 1)T$, and $\omega_s = 2\pi sT$ are the Matsubara frequencies for fermion and boson, respectively, with integers n and s. After performing the sum over the Matsubara frequency in Eq. (4), $i\omega_n + \mu$ is analytically continued to the Minkowski space $i\omega_n + \mu \to p_0 + i\eta$, with $\eta \to 0$. The blob in Fig. 1 here represents the HTL/HDL corrected photon propagator which is in the Coulomb gauge is given by [1]

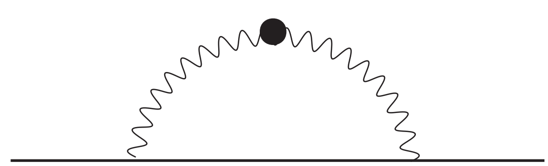

FIG. 1. Fermion self-energy with resummed photon propagator.

$$\Delta_{\mu\nu}(Q) = \delta_{\mu 0}\delta_{\nu 0}\Delta_l(Q) + P^t_{\mu\nu}\Delta_t(Q), \tag{5}$$

with $P^t_{ij} = (\delta_{ij} - \hat{q}_i\hat{q}_j)$, $\hat{q}^i = \mathbf{q}^i/|\mathbf{q}|$, $P^t_{i0} = P^t_{0i} = P^t_{00} = 0$, and Δ_l, Δ_t are given by [1]

$$\Delta_l(q_0, q) = \frac{-1}{q^2 + \Pi_l}, \tag{6}$$

$$\Delta_t(q_0, q) = \frac{-1}{q_0^2 - q^2 - \Pi_t}. \tag{7}$$

For subsequent calculations it is convenient here to introduce the spectral functions $\rho_{l,t}$ [1]:

$$\rho_{l,t}(q_0, \mathbf{q}) = 2\,\mathrm{Im}\Delta_{l,t}(q_0 + i\eta, \mathbf{q}). \tag{8}$$

At the leading order these are derived from the one-loop photon self-energy where the loop momenta are assumed to be hard in comparison to the photon momentum [25,26]. In the literature the formalism is known as the HTL/HDL approximation as discussed in [1],

$$\rho_l(q_0, q) = \frac{2\pi m_D^2 x\Theta(1-x^2)}{2[q^2 + m_D^2(1 - \frac{x}{2}\ln|\frac{x+1}{x-1}|)]^2 + \frac{m_D^4\pi^2x^2}{2}},$$
$$\rho_t(q_0, q) = \frac{2\pi m_D^2 v_f^2 x(1-x^2)\Theta(1-x^2)}{[2q^2(x^2v_f^2 - 1) - m_D^2x^2v_f^2(1 + \frac{(1-x^2)}{2x}\ln|\frac{x+1}{x-1}|)]^2 + \frac{m_D^4v_f^4\pi^2x^2(1-x^2)^2}{4}}, \tag{9}$$

where v_f is the Fermi velocity and $x = \frac{q_0}{qv_f}$. For a ultrarelativistic plasma ($v_f \to 1$) the Debye mass is $m_D^2 = \frac{e^2}{\pi^2} \times (\mu^2 + \frac{\pi^2T^2}{3})$.

In Eq. (4), the fermion propagator has the following spectral representation with the notation $\mathbf{k} = (\mathbf{p} - \mathbf{q})$ [1]:

$$S_f(i\omega_n, \mathbf{k}) = \int_{-\infty}^{\infty}\frac{\mathrm{d}k_0}{2\pi}\frac{(K + m)\rho_f(K)}{k_0 - i\omega_n - \mu}. \tag{10}$$

Hence, for $\rho_f(K)$ we use the free spectral density given by

$$\rho_f(K) = \frac{\pi}{E_k}[\delta(k_0 - E_k) - \delta(k_0 + E_k)]. \tag{11}$$

One can take the imaginary part of Eq. (4) to calculate the scattering rate with the help of Eq. (3). For the calculation of the drag coefficient, one then inserts the energy exchange ω in the expression of Γ and calculates dE/dx from Eq. (2) to obtain

$$\begin{aligned}-\frac{dE}{dx} = \frac{\pi e^2}{Ev_i}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}k_0}{2\pi}\rho_f(k_0) \\ \times\int_{-\infty}^{\infty}\frac{\mathrm{d}q_0}{2\pi}q_0(1 + n(q_0) - \bar{n}(k_0))\delta(E - k_0 - q_0) \\ \times[p_0k_0 + \mathbf{p}\cdot\mathbf{k} + m^2]\rho_l(q_0, q) \\ + 2[p_0k_0 - (\mathbf{p}\cdot\hat{\mathbf{q}})(\mathbf{k}\cdot\hat{\mathbf{q}}) - m^2]\rho_t(q_0, q).\end{aligned} \tag{12}$$

It is to be mentioned here that the scattering process involves space-like photons. Hence, here only the cut of the spectral function contributes. In the above equation n and $\bar{n}$ are the Bose-Einstein and the Fermi-Dirac distribution functions:

$$n(q_0) = \frac{1}{e^{\beta q_0} - 1}, \qquad \bar{n}(k_0) = \frac{1}{e^{\beta(k_0-\mu)} + 1}. \tag{13}$$

From now onwards in this section, we exclusively focus on the ultradegenerate plasma. The finite temperature corrections which might be important for dense and warm plasma will be incorporated in the next section. For the $T = 0$, $\mu \neq 0$ limit, $(1 + n(q_0)) = \Theta(q_0)$ and $\bar{n}(k_0) = \Theta(\mu - E + q_0)$, where Θ represents the step function. These functions, as we shall see, restrict the phase space of the q_0 integration severely. The zero temperature spectral functions $\rho_{l,t}$ now involve the Debye mass $m_D^2 = e^2v_f\mu^2/\pi^2$.

Note that, the delta function in Eq. (12) sets $q_0 = qv\cos\theta$ and the theta functions impose further restrictions on q_0. We consider quasiparticles, with a velocity close to the Fermi velocity, which undergoes collisions with the particles near the Fermi surface. Hence, we can make an approximation here as $v \approx v_f$.

Now, consider the case of the hard photon exchange where the medium effects on the photon propagator can be ignored. In this case using the bare propagator we get

$$\begin{aligned}\left(-\frac{dE}{dx}\right) \simeq \frac{e^2m_D^2}{8\pi v_f}\int dq\int_0^{E-\mu}dq_0\left\{\frac{q_0^2}{v_f^2q^4} + \frac{v_f^2q_0^2}{2q^4}\right\} \\ \simeq \frac{e^2(E-\mu)^3m_D^2}{24\pi v_f}\left(\frac{1}{v_f^2} + \frac{v_f^2}{2}\right)\int\frac{dq}{q^4}.\end{aligned} \tag{14}$$

This actually is the leading hard contribution that comes from the diagram, when, the blob of Fig. 1 is replaced with one fermion loop. Evidently, the above integral is infrared divergent and unlike the finite temperature here, higher powers of q appear in the denominator. We shall remark on this later once we have expressions both for η and $\mathcal{B}$.

To deal with this infrared divergence in the soft domain, one uses the HDL corrected photon propagator [25,26] given by Eq. (7), with the Debye mass $m_D^2 = e^2v_f\mu^2/\pi^2$ as mentioned earlier, to obtain

$$\left(-\frac{dE}{dx}\right)\Big|_{\rm soft}(E) \simeq \frac{e^2}{2v_f}\int\frac{d^3q}{(2\pi)^3}qv_f\cos\theta(\Theta(q_0) - \Theta(\mu - E + q_0))\Theta(q^* - q)\{\rho_l(q_0, q) + v_f^2(1-\cos^2\theta)\rho_t(q_0, q)\}$$
$$\simeq \frac{e^2}{8\pi^2 v_f}\int_D dqq\int dq_0q_0 \times\left\{\rho_l(q_0, q) + \left(v_f^2 - \frac{q_0^2}{q^2}\right)\rho_t(q_0, q)\right\}. \quad (15)$$

The integration domain (D) above is limited by the Θ functions,

$$D\text{: } 0 \le q_0 \le E - \mu; \qquad q_0 \le q \le q^*. \quad (16)$$

With these we get

$$\left(-\frac{dE}{dx}\right)\Big|_{\rm soft}(E) \simeq \frac{e^2m_D^2}{4\pi v_f}\int_D dq_0dq \times\left\{\frac{q_0^2}{v_f^2\{2[q^2 + m_D^2Q_l(\frac{q_0}{q})]^2 + \frac{m_D^4\pi^2q_0^2}{2q^2}\}} + \frac{v_f^2q_0^2}{[2q^2 + m_D^2v_f^2Q_t(\frac{q_0}{q})]^2 + \frac{m_D^4v_f^4\pi^2q_0^2}{4q^2}}\right\}, \quad (17)$$

where

$$Q_l(x) = 1 - \frac{x}{2}\ln\frac{1+x}{1-x}, \qquad Q_t(x) = -Q_l(x) + \frac{1}{1-x^2}. \quad (18)$$

We are mainly interested in the energy loss of a quasiparticle which is currently close to the Fermi surface, hence, $(E-\mu) \ll m_D$ is the physically interesting region where the quasiparticle concept is meaningful. The denominator of Eq. (17) can now be expanded in powers of q_0. We replace $s^* = (q^*/m_D)^2$ and compute separately the electric (l) and the magnetic part (t),

$$\left(-\frac{dE}{dx}\right)\Big|^l_{\rm soft} \simeq \frac{e^2(E-\mu)^3}{48\pi m_D v_f^3}\int_0^{s^*}\frac{ds}{\sqrt{s}(s+1)^2}, \quad (19)$$

$$\left(-\frac{dE}{dx}\right)\Big|^t_{\rm soft} \simeq \frac{e^2m_D^2v_f^2}{4\pi v_f}\int_D dq_0dq\frac{q_0^2}{4q^4 + \frac{\pi^2m_D^4v_f^4q_0^2}{4q^2}}. \quad (20)$$

After explicit calculation, the electric and magnetic contributions to the expression of energy loss take the following form:

$$\left(-\frac{dE}{dx}\right)\Big|^l_{\rm soft} \simeq \frac{e^2(E-\mu)^3}{96v_f^3m_D} - \frac{e^2(E-\mu)^3m_D^2}{72\pi v_f^3q^{*3}}, \quad (21)$$

$$\left(-\frac{dE}{dx}\right)\Big|^t_{\rm soft} \simeq \frac{e^2(E-\mu)^2}{48\pi v_f} - \frac{e^2(E-\mu)^3v_fm_D^2}{144\pi q^{*3}}. \quad (22)$$

It is worthwhile to note here that the leading order terms in the last two equations are finite and independent of the cutoff parameter. Here the q^* dependent term appears only at $O(e^4)$. Therefore, we write

$$\left(-\frac{dE}{dx}\right)\Big|^l_{\rm soft} \simeq \frac{e^2(E-\mu)^3}{96v_f^3m_D} + O(e^4), \quad (23)$$

$$\left(-\frac{dE}{dx}\right)\Big|^t_{\rm soft} \simeq \frac{e^2(E-\mu)^2}{48\pi v_f} + O(e^4). \quad (24)$$

So far we have discussed about the soft part and have seen that in the limit $q^* \to \infty$ the cutoff parameter dependent term trivially vanishes. Similarly, if we recall the expression for the hard part, i.e., Eq. (14), after performing the integration in the limit $[q^*, \mu]$ we get

$$\left(-\frac{dE}{dx}\right)\Big|_{\rm hard} \simeq \frac{e^2(E-\mu)^3m_D^2}{72\pi v_f}\left(\frac{1}{v_f^2} + \frac{v_f^2}{2}\right)\left[\frac{1}{q^{*3}} - \frac{1}{\mu^3}\right] \simeq O(e^4). \quad (25)$$

Clearly it fails to contribute at the leading order where the entire contribution comes from the soft sector. This is a distinctive feature of degenerate plasma not encountered at finite temperature ($\mu = 0$). There both the hard and the soft part contribute to the leading order in e^2 and the divergence is only logarithmic. To deal with such divergences in hot plasma one invokes Braaten and Yuan's prescription [23] where an intermediate cutoff is introduced to separate the hard and the soft domains. It is seen that such an intermediate cutoff parameter disappears from the final expressions when both the contributions are added. At zero temperature, a similar approach was adopted for the calculation of fermion damping rate [15] where it was shown that such cancellation takes place also in degenerate plasma. It is obvious from Eqs. (21), (22), and (25) that same thing happens for η also.

From Eq. (25) it is clear that the result obtained from the hard region is suppressed with respect to the soft one [Eq. (21) and (22)]. Hence, the whole contribution to leading order comes from the soft sector alone. The final expression for the drag coefficient at zero temperature becomes

$$\eta \simeq \frac{e^2(E-\mu)^3}{96m_Dv_f^4E} + \frac{e^2(E-\mu)^2}{48\pi v_f^2E} + O(e^4). \quad (26)$$

The first term above corresponds to the electric photon and the latter to the magnetic one, i.e., l or t mode behaves differently. The dominant contribution to η comes from the magnetic sector in the ultrarelativistic case $v_f \to 1$ and the electric sector when $v_f \ll 1$. Results for the light fermion can be obtained from Eq. (26) with the substitution of $v_f \to 1$.

B. Diffusion coefficient

Apart from η, the quantity which could be of importance in the study of heavy fermion propagating in the plasma is the momentum diffusion coefficient (B_{ij}) [8–11]. In fact, we know for Coulomb plasma η and the longitudinal momentum diffusion coefficient ($\mathcal{B}$) are related via ER. The momentum diffusion coefficient B_{ij} can be defined as follows [8,10]:

$$B_{ij} = \int d\Gamma q_i q_j. \tag{27}$$

Decomposing B_{ij} into longitudinal (B_l) and transverse components (B_t) we get the following expression:

$$B_{ij} = B_t\left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) + B_l \frac{p_i p_j}{p^2}. \tag{28}$$

These coefficients $B_{l,t}$ are the longitudinal, transverse squared momentum acquired by the particle through collision with the plasma. Using the above definition, like the energy loss [Eq. (12)], the longitudinal momentum diffusion coefficient ($B_l = \mathcal{B}$, suppressing the index l) can be written as follows:

$$\begin{aligned}\mathcal{B} = {}& \frac{\pi e^2}{E}\int \frac{\mathrm{d}^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}k_0}{2\pi}\rho_f(k_0) \\ &\times \int_{-\infty}^{\infty}\frac{\mathrm{d}q_0}{2\pi} q_{||}^2(1 + n(q_0) - \bar{n}(k_0))\delta(E - k_0 - q_0) \\ &\times [p_0 k_0 + \mathbf{p}\cdot\mathbf{k} + m^2]\rho_l(q_0, q) \\ &+ 2[p_0 k_0 - (\mathbf{p}\cdot\hat{\mathbf{q}})(\mathbf{k}\cdot\hat{\mathbf{q}}) - m^2]\rho_t(q_0, q).\end{aligned} \tag{29}$$

Here, $q_{||} = q\cos\theta$. For the exchange of hard photons using the bare propagator we obtain

$$\begin{aligned}\mathcal{B} &\simeq \frac{e^2 m_D^2}{8\pi v_f^2}\left(\frac{1}{v_f^2} + \frac{v_f^2}{2}\right)\int dq \int_0^{E-\mu} dq_0 \frac{q_0^3}{q^4}, \\ &\simeq \frac{e^2 m_D^2 (E-\mu)^4}{32\pi v_f^2}\left(\frac{1}{v_f^2} + \frac{v_f^2}{2}\right)\int \frac{dq}{q^4}.\end{aligned} \tag{30}$$

Comparing Eq. (14) with (30) it is seen that like η, $\mathcal{B}$ is also infrared divergent involving the fourth power of q in the denominator. At the finite T case, both the quantities are proportional to dq/q at the leading order [10]. To understand the origin of this difference, we focus on the q_0 integration. It is shown in the Appendix that in the medium at finite T, there involves a quadratic power of q_0 in both cases with the limits $-vq$ to $+vq$ giving rise to a term proportional to q^3 in the numerator. This cancels with some of the powers of q coming from the propagator. Whereas in cold matter, from Eqs. (14) and (30) we find that the same integrations appear with q_0^2 and q_0^3 in the numerator while the limits are independent of q, forbidding the cancellation with q's coming from the propagator as before. We note here that the drag and diffusion coefficients are related through $\mathcal{B} = \frac{3E(E-\mu)}{4}\eta$ when we deal with the bare propagator. We shall see in the next paragraph that the same powers of q_0 appear in the numerator when one takes the plasma effects into account but such common scaling behavior is lost.

The infrared divergence of Eq. (30) can be removed by using the dressed photon propagator [25,26] and providing the upper cutoff as in the case of η. With the HDL corrected propagator one gets

$$\begin{aligned}\mathcal{B}|_{\mathrm{soft}}(E) \simeq {}& \frac{e^2}{2v_f^2}\int\frac{\mathrm{d}^3q}{(2\pi)^3} q_0^2(\Theta(q_0) \\ &- \Theta(\mu - E + q_0))\Theta(q^* - q)\{\rho_l(q_0, q) \\ &+ v_f^2(1 - \cos^2\theta)\rho_t(q_0, q)\} \\ \simeq {}& \frac{e^2}{8\pi^2 v_f^2}\int_D \mathrm{d}q q \int \mathrm{d}q_0 q_0^2 \\ &\times\left\{\rho_l(q_0, q) + \left(v_f^2 - \frac{q_0^2}{q^2}\right)\rho_t(q_0, q)\right\}.\end{aligned} \tag{31}$$

Here the integration domain (D) is the same as before. So,

$$\begin{aligned}\mathcal{B}|_{\mathrm{soft}}(E) \simeq {}& \frac{e^2 m_D^2}{4\pi v_f^4}\int_D dq_0 dq \\ &\times\left\{\frac{q_0^3}{\{2[q^2 + m_D^2 Q_l(\frac{q_0}{q})]^2 + \frac{m_D^4\pi^2 q_0^2}{2q^2}\}}\right. \\ &+ \left.\frac{v_f^4 q_0^3}{[2q^2 + m_D^2 v_f^2 Q_t(\frac{q_0}{q})]^2 + \frac{m_D^4 v_f^4 \pi^2 q_0^2}{4q^2}}\right\}.\end{aligned} \tag{32}$$

Since, we know from energy loss that the dominant contribution to the expression comes from the soft region alone we write the expression for $\mathcal{B}$ as follows:

$$\mathcal{B} \simeq \frac{e^2(E-\mu)^4}{128 m_D v_f^4} + \frac{e^2(E-\mu)^3}{72\pi v_f^2} + O(e^4), \tag{33}$$

which is finite. Now from Eqs. (26) and (33) it can be seen that there is no common scaling factor between η and $\mathcal{B}$. But as ER is formulated in the region where $v_f \ll 1$, in this nonrelativistic region the exchange of the magnetic photons are suppressed in comparison with the electric one. Hence, considering only the electric part we get the same ER, $\mathcal{B} = \frac{3E(E-\mu)}{4}\eta$, as in the case of bare perturbation theory.

III. FINITE TEMPERATURE CORRECTION

The results of the previous section can easily be extended to the case of a hot and dense ($T \ll \mu$) plasma. This could be relevant for heavy ion collision to be performed at GSI where the chemical potential is expected to be much higher than the temperature. Now, while calculating the soft part we replace the zero temperature distribution functions with the finite temperature one in Eq. (15) and write

$$\left(-\frac{dE}{dx}\right)\bigg|_{\rm soft}(E) \simeq \frac{e^2}{2v_f}\int\frac{d^3q}{(2\pi)^3}qv_f\cos\theta(1+n(q_0) - \bar{n}(E-q_0))\Theta(q^*-q)\{\rho_l(q_0,q) + v_f^2(1-\cos^2\theta)\rho_t(q_0,q)\}. \quad (34)$$

With small T and large μ, the above equation can be calculated according to Ref. [27]. In this approach we can write any function $g(\varepsilon)$ along with the fermion distribution function as follows:

$$\int_0^\infty \frac{g(\varepsilon)}{e^{\beta(\varepsilon-\mu)}+1}d\varepsilon = \int_0^\mu g(\varepsilon)d\varepsilon + \frac{\pi^2T^2}{6}g'(\mu). \quad (35)$$

The contributions coming from soft region (l and t) using Eqs. (34), (35), and (9) are found to be given by

$$\left(-\frac{dE}{dx}\right)\bigg|^{l}_{\rm soft} \simeq \frac{e^2(E-\mu)^3}{96v_f^3m_D} - \frac{e^2(E-\mu)^3m_D^2}{72\pi q^{*3}v_f^3} - \frac{e^2(E-\mu)T^2\pi^2}{96v_f^3m_D} + \frac{e^2(E-\mu)T^2m_D^2\pi}{72q^{*3}v_f^3}, \quad (36)$$

$$\left(-\frac{dE}{dx}\right)\bigg|^{t}_{\rm soft} \simeq \frac{e^2(E-\mu)^2}{48\pi v_f} - \frac{e^2(E-\mu)^3v_fm_D^2}{144\pi q^{*3}} - \frac{e^2\pi T^2}{72v_f} + \frac{e^2(E-\mu)m_D^2v_fT^2\pi}{144q^{*3}}. \quad (37)$$

We see from the above two equations that the terms containing the separation scale are subleading in comparison with the others, the same behavior also obtained in the zero temperature case. The term with the bare propagator comes as

$$\left(-\frac{dE}{dx}\right)\bigg|_{\rm hard} \simeq \frac{e^2(E-\mu)^3m_D^2}{72\pi v_f}\left(\frac{1}{v_f^2}+\frac{v_f^2}{2}\right)\left[\frac{1}{q^{*3}}-\frac{1}{\mu^3}\right] - \frac{e^2(E-\mu)m_D^2\pi T^2}{72v_f}\left(\frac{1}{v_f^2}+\frac{v_f^2}{2}\right) \times\left[\frac{1}{q^{*3}}-\frac{1}{\mu^3}\right]. \quad (38)$$

The above expression is also suppressed in contrast to the soft one. Hence, with finite temperature correction, drag and diffusion coefficients become

$$\eta \simeq \frac{e^2(E-\mu)^3}{96m_Dv_f^4E} + \frac{e^2(E-\mu)^2}{48\pi v_f^2E} - \frac{e^2\pi^2T^2(E-\mu)}{96m_Dv_f^4E} - \frac{e^2\pi T^2}{72v_f^2E} + O(e^4), \quad (39)$$

$$\mathcal{B} \simeq \frac{e^2(E-\mu)^4}{128m_Dv_f^4} + \frac{e^2(E-\mu)^3}{72\pi v_f^2} - \frac{e^2(E-\mu)^2\pi^2T^2}{64m_Dv_f^4} - \frac{e^2\pi T^2(E-\mu)}{48v_f^2} + O(e^4). \quad (40)$$

One notes here with the thermal correction the ER cannot be established even for the electric sector alone.

IV. SUMMARY

In this work we calculate the energy loss and momentum diffusion of the heavy fermion in dense and warm QED matter and highlight some of the differences that exist between the hot ($\mu = 0$) and the cold ($T = 0$) plasma. Unlike finite temperature, where one encounters logarithmic divergences in calculating η or $\mathcal{B}$, here we come across nonlogarithmic divergences. Furthermore, we see that at the leading order in coupling, the entire contribution comes from the soft sector and this is finite, i.e,. the physics here is dominated by the excitations near the Fermi surface. The exchange of hard photons on the other hand contribute only at $O(e^4)$. It is to be noted that in a thermal medium with vanishing chemical potential both the soft and hard photons or gluons for QCD matter contribute at the same order. Moreover, for ultrarelativistic particles both η and $\mathcal{B}$ receive dominant contributions from the magnetic sector while the electric parts are found to be subleading in $(E-\mu)$. This is consistent with the fermion damping rate calculation [15,21,25] in degenerate plasma. Quantitatively, we find that for the transverse or magnetic interaction η is proportional to $(E-\mu)^2$ while for the electric interaction, it goes as $(E-\mu)^3$. Similar differences for $\mathcal{B}$ are also seen where one more extra power of $(E-\mu)$ is involved in each case. The other important finding of the present investigation is the ER for the drag and diffusion coefficient. In hot plasma, it is known that $\mathcal{B} = 2TE\eta$ [9–11]. At zero temperature, we find $\mathcal{B} = \frac{3E(E-\mu)}{4}\eta$ by considering only the bare propagator, i.e., when we do not take the plasma effects into account. However, we see that this common scale behavior is lost for soft photon exchange where the plasma effects are included and both the magnetic and electric contributions are retained. However, by retaining only the electric contribution for the cold plasma, one arrives at the same relations as obtained by using the bare propagator. For $T \ll \mu$ again we see that, with plasma effects incorporated, η and $\mathcal{B}$ fail to show such common scale behavior even when the magnetic interaction is ignored.

As a last remark, we note that here the entire calculation has been done for QED plasma. This can easily be extended to QCD matter with appropriate modifications like the inclusion of diagrams involving three gluon interaction and proper vertex factors coming from the QCD color algebra. Such studies are in progress and shall be reported in the future.

APPENDIX

To understand the difference of the results between the cold and hot plasma, we first recall the expression for the drag coefficient (η):

$$\eta = \frac{1}{Ev_i}\left(-\frac{dE}{dx}\right).$$

The above relation with Eqs. (10)–(12) can be further simplified to yield

$$\eta \simeq \frac{e^2}{8\pi^2 v^2 E}\int \mathrm{d}qq\int_{-vq}^{vq}\mathrm{d}q_0 q_0(1+n(q_0)-\bar{n}(k_0))\Theta(q^*-q) \times\left\{\rho_l(q_0,q)+\left(v^2-\frac{q_0^2}{q^2}\right)\rho_t(q_0,q)\right\}. \quad \text{(A1)}$$

The corresponding expression for the diffusion coefficient from Eq. (29) is

$$\mathcal{B} \simeq \frac{e^2}{8\pi^2 v^2}\int \mathrm{d}qq\int_{-vq}^{vq}\mathrm{d}q_0 q_0^2(1+n(q_0)-\bar{n}(k_0))\Theta(q^*-q) \times\left\{\rho_l(q_0,q)+\left(v^2-\frac{q_0^2}{q^2}\right)\rho_t(q_0,q)\right\}. \quad \text{(A2)}$$

In the high temperature limit, $(1+n(q_0)-\bar{n}(E-q_0)) \simeq \frac{T}{q_0}+\frac{1}{2}$. It is to be noted that the above integration limits are symmetric in q_0. Hence, for the drag, the factor of $\frac{1}{2}$ and for the diffusion $\frac{T}{q_0}$ contribute. Inserting these in Eqs. (A1) and (A2) we get

$$\eta \simeq \frac{e^2}{16\pi^2 v^2 E}\int \mathrm{d}qq\int_{-vq}^{vq}\mathrm{d}q_0 q_0\Theta(q^*-q) \times\left\{\rho_l(q_0,q)+\left(v^2-\frac{q_0^2}{q^2}\right)\rho_t(q_0,q)\right\}, \quad \text{(A3)}$$

and

$$\mathcal{B} \simeq \frac{e^2 T}{8\pi^2 v^2}\int \mathrm{d}qq\int_{-vq}^{vq}\mathrm{d}q_0 q_0\Theta(q^*-q) \times\left\{\rho_l(q_0,q)+\left(v^2-\frac{q_0^2}{q^2}\right)\rho_t(q_0,q)\right\}. \quad \text{(A4)}$$

In the case of bare interaction, one can show that both η and $\mathcal{B}$ are proportional to $\mathrm{d}q/q$ [10] and even without performing the integration $\mathcal{B} = 2TE\eta$. If we compare Eqs. (A3) and (A4) with Eqs. (15) and (31) we find that the q_0 integration for cold matter is not symmetric, and the limits are independent of q. Here lies the difference of cold and hot plasma.

ACKNOWLEDGMENTS

We would like to thank the referees for their useful comments and S. Mrowczynski for fruitful communications. We also acknowledge helpful discussions with K. Pal and P. Roy.

[1] M. Le Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, England, 1996).
[2] J. I. Kapusta, *Finite Temperature Field Theory* (Cambridge University Press, Cambridge, England, 1989).
[3] E. Braaten and M. H. Thoma, Phys. Rev. D **44**, 1298 (1991).
[4] E. Braaten and M. H. Thoma, Phys. Rev. D **44**, R2625 (1991).
[5] S. Peigne and A. Peshier, Phys. Rev. D **77**, 014015 (2008).
[6] S. Peigne and A. Peshier, Phys. Rev. D **77**, 114017 (2008).
[7] A. K. Dutt-Mazumder, Jan-e Alam, P. Roy, and B. Sinha, Phys. Rev. D **71**, 094016 (2005).
[8] B. Svetitsky, Phys. Rev. D **37**, 2484 (1988).
[9] G. D. Moore and D. Teaney, Phys. Rev. C **71**, 064904 (2005).
[10] P. Roy, A. K. Dutt-Mazumder, and Jan-e Alam, Phys. Rev. C **73**, 044911 (2006).
[11] A. Beraudo, A. De Pace, W. M. Alberico, and A. Molinari, Nucl. Phys. **A831**, 59 (2009).
[12] L. Tolos, D. Cabrera, D. Gamermann, C. Garcia-Recio, R. Molina, J. Nieves, E. Oset, and A. Ramos, Acta Phys. Pol. B **41**, 329 (2010).
[13] P. Senger, J. Phys. G **30**, S1087 (2004).
[14] P. Senger, Prog. Part. Nucl. Phys. **62**, 375 (2009).
[15] M. Le Bellac and C. Manuel, Phys. Rev. D **55**, 3215 (1997).
[16] H. Vija and M. H. Thoma, Phys. Lett. B **342**, 212 (1995).
[17] E. Braaten and R. D. Pisarski, Nucl. Phys. **B337**, 569 (1990).
[18] H. A. Weldon, Phys. Rev. D **26**, 1394 (1982).
[19] J. P. Blaziot and E. Iancu, Phys. Rev. D **55**, 973 (1997).
[20] J. P. Blaziot and E. Iancu, Phys. Rev. Lett. **76**, 3080 (1996).
[21] B. Vanderheyden and J. Ollitrault, Phys. Rev. D **56**, 5108 (1997).
[22] J. J. Quinn and R. A. Ferrell, Phys. Rev. **112**, 812 (1958).
[23] E. Braaten and T. C. Yuan, Phys. Rev. Lett. **66**, 2183 (1991).
[24] H. A. Weldon, Phys. Rev. D **28**, 2007 (1983).
[25] C. Manuel, Phys. Rev. D **62**, 076009 (2000).
[26] C. Manuel, Phys. Rev. D **53**, 5866 (1996).
[27] R. P. Feynman, *Statistical Mechanics, A Set of Lectures* (The Benjamin/Cummings Publishing Company, Reading, MA, 1982).