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Decaying dark matter from dark instantons

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We construct an explicit, TeV-scale model of decaying dark matter in which the approximate stability of the dark matter candidate is a consequence of a global symmetry that is broken only by instanton-induced operators generated by a non-Abelian dark gauge group. The dominant dark matter decay channels are to standard model leptons. Annihilation of the dark matter to standard model states occurs primarily through the Higgs portal. We show that the mass and lifetime of the dark matter candidate in this model can be chosen to be consistent with the values favored by fits to data from the PAMELA and Fermi-LAT experiments.

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I. INTRODUCTION

Evidence has been accumulating for an electron and positron excess in cosmic rays compared with expectations from known galactic sources. Fermi-LAT [1] and H.E.S.S. [2] have measured an excess in the flux of electrons and positrons up to a TeV or more. The PAMELA satellite is sensitive to electrons and positrons up to a few hundred GeV in energy, and is able to distinguish positrons from electrons and charged hadrons. PAMELA detects an upturn in the fraction of positron events beginning around 7 GeV [3]. This is in contrast to the expected decline in the positron fraction from secondary production mechanisms. Curiously, no corresponding excess of protons or antiprotons has been detected [4].

Although conventional astrophysical sources may ultimately prove the explanation of the anomalous cosmic ray data [5], an intriguing possibility is that dark matter annihilation or decay provides the source of the excess leptons. If dark matter annihilation is responsible for the excess leptons, then the annihilation cross section typically requires a large boost factor $\sim$100–1000 to produce the observed signal [6]. Possible sources of the boost factor include Sommerfeld enchancement from additional attractive interactions in the dark sector [7], WIMP capture [8,9] or Breit-Wigner resonant enhancement [10–12].

Alternatively, decaying dark matter can provide an explanation of the cosmic ray data if the dark matter decay channels favor leptonic over hadronic final states [13]. A typical scenario of this type that is consistent with PAMELA and Fermi-LAT data includes dark matter with a mass of a few TeV that decays to leptons, with an anomalously long lifetime of $\sim 10^{26}$ s [14,15]. From a model-building perspective, an intriguing issue is the origin of this long lifetime, and whether it can be explained with a minimum of theoretical contrivance. With this goal in mind, we present a new model of TeV-scale dark matter, one in which an anomalous global symmetry prevents dark matter decays except through instantons of a non-Abelian gauge field in the dark sector. Instanton-induced decays naturally produce the long required lifetime. Small mixings between standard model leptons and dark fermions give rise to the leptonic final states observed in the cosmic ray data. Dark matter annihilation through the Higgs portal allows for the appropriate dark matter relic abundance, with dark matter masses consistent with the range preferred by PAMELA and Fermi-LAT data.

Superheavy dark matter decays through instantons have been considered before as a possible explanation for ultrahigh energy cosmic ray signals, but those scenarios assumed superheavy dark matter with a mass of 10^{13} GeV or higher [16] which cannot simultaneously explain the lower energy electron and positron flux being considered here. Models of anomaly-induced dark matter decays without a dark gauge sector can also be constructed. For example, a supersymmetric extension of the radiative seesaw model of neutrino masses can explain the PAMELA data through dark matter decays via an anomalous discrete symmetry [17]. The TeV-scale model we present, which is based on the smallest, continuous non-Abelian dark gauge group and smallest set of exotic particles necessary to implement our idea, suggests a prototypical set of new particles and interactions that could perhaps be probed at the LHC.

In Sec. II we present the model and describe the leptonic decay mode via instantons. In Sec. III we consider dark matter annihilation channels and demonstrate that annihilation through the Higgs portal can lead to the measured dark matter relic density. In Sec. IV we consider dark matter interactions with nuclei and find that our model is safely below current direct detection bounds. We conclude in Sec. V.

II. THE MODEL

The gauge group of the dark sector is $SU(2)_D \times U(1)_D$. The matter content consists of four sets of left-handed $SU(2)_D$ doublets and right-handed singlets:

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$$\psi_L \equiv \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}_L \psi_{uR}, \psi_{dR},$$

$$\chi_L^{(i)} \equiv \begin{pmatrix} \chi_u^{(i)} \\ \chi_d^{(i)} \end{pmatrix}_L \chi_{uR}^{(i)}, \chi_{dR}^{(i)}, \qquad (i = 1 \cdots 3). \quad (2.1)$$

We include an $SU(2)_D$ doublet and singlet Higgs field, H_D and η, respectively, that are responsible for completely breaking the dark gauge group. In addition, the Higgs field H_D is responsible for giving Dirac masses to the ψ and χ fields. The model is constructed so that ψ number corresponds to an anomalous global symmetry that is violated by the $\psi\chi^{(1)}\chi^{(2)}\chi^{(3)}$ vertex generated via $SU(2)_D$ instantons, as indicated in Fig. 1. The χ fields are assigned hypercharges so that they mix with standard model leptons, leading to the decay $\psi \to \ell^+\ell^-\nu$. The required lifetime ($\sim 10^{26}$ s) and the appropriate dark matter relic density ($\Omega_D h^2 \sim 0.1$) constrain the free parameters of the model.

The charge assignments for these fields are summarized in Table I. Let us first discuss the consistency of the charge assignments. Cancellation of the $SU(2)_D^2$ U(1) anomalies requires that the sum of the U(1) charges over all the dark doublet fermion fields must vanish. As one can see from Table I, this is clearly the case for the $U(1)_D$ and $U(1)_Y$ charges of the left-handed doublet ψ and χ fields. Since SU(2) is an anomaly free group and has traceless generators, all other $SU(2)_D$ anomalies vanish trivially. Now consider the $U(1)_D^p U(1)_Y^q$ anomalies (where p and q are non-negative integers satisfying $p + q = 3$). For each field in Table I with a given $U(1)_D \times U(1)_Y$ charge assignment, one notes that there is another with the same charge assignment but opposite chirality. As far as the Abelian groups are concerned, the theory is vectorlike and the corresponding anomalies vanish. Finally, we note that the theory has precisely four $SU(2)_D$ doublets and is free of a Witten anomaly.

The gauge symmetries of the model lead to a global $U(1)_\psi$ symmetry that prevents the decay of the lightest ψ

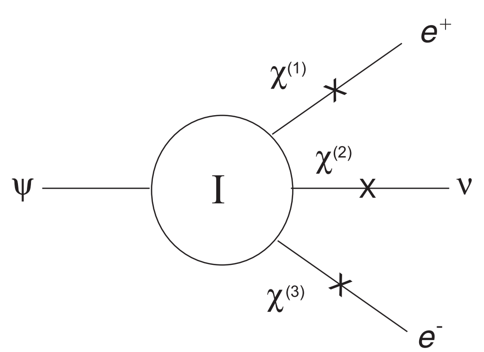

FIG. 1. Dark matter decay vertex. The circle represents the instanton-induced interaction, while X's represent mass mixing between the χ fields and standard model leptons. Note that e and ν represent leptons of any generation.

mass eigenstate at any order in perturbation theory. To confirm this statement, we need to show that all renormalizable interactions that violate this symmetry are forbidden by the dark-sector gauge symmetry. The possible problematic interactions that could violate this global symmetry fall into the following categories:

(1) Terms involving $\bar{\psi}^c\psi$. Here the superscript indicates charge conjugation, $\psi^c \equiv i\gamma^0\gamma^2\bar{\psi}^T$. This combination has $U(1)_\psi$ charge $+2$. However, it also has $U(1)_D$ charge -1. Since we have no Higgs field with the $U(1)_D$ charge ± 1, there are no renormalizable interactions that violate ψ number by two units.

(2) Terms involving a χ fermion and ψ or ψ^c. Such terms violate ψ number by ± 1 unit. However, the possible bilinears involving ψ and any χ have $U(1)_D$ charges $\pm 1/3$ or $\pm 2/3$. Again, we have no Higgs field with the necessary $U(1)_D$ charge to form a renormalizable gauge invariant term of this type.

(3) Terms involving a standard model fermion and ψ or ψ^c. Such an interaction would violate ψ number by ± 1, but would have $U(1)_D$ charge $\pm 1/2$. Again, we have no Higgs fields with charge $\pm 1/2$ that would allow the construction of a renormalizable invariant.

Since the renormalizable interactions of the theory have an unbroken $U(1)_\psi$ symmetry, no perturbative process involving these interactions will violate the global symmetry. However, since the $SU(2)_D^2\, U(1)_\psi$ anomaly is nonzero, nonperturbative interactions due to instantons will generate operators that violate the $U(1)_\psi$ symmetry.

Instantons are gauge field configurations which stationarize the Euclidean action but have a nontrivial winding number around the three-sphere at infinity. Following 't Hooft [18,19], if there are N_f Dirac pairs of chiral fermions which transform in the fundamental representation of a gauge group, then due to the chiral anomaly a one-instanton configuration violates the axial $U(1)_A$ charge by $2N_f$ units. The non-Abelian, $SU(N_f) \times SU(N_f)$ chiral symmetry is nonanomalous, so the instanton process must involve the $2N_f$ chiral fermions in a symmetric fashion. Figure 1 shows the effective $\psi\chi^{(1)}\chi^{(2)}\chi^{(3)}$ interaction induced by the instanton configuration in our model.[1] Given the hypercharge assignments of the χ fields, these states have electric charges $+1$, 0 and -1, the same as standard model leptons, of any generation. After the dark and standard model gauge symmetries are spontaneously broken, there is no symmetry which prevents the χ states and the standard model leptons from mixing. By including a single vectorlike lepton pair, we now show that mixing leading to the decay $\psi \to \ell^+\ell^-\nu$ can arise via purely renormalizable interactions.

[1]In this model, Planck-suppressed operators of this form, if they are present, are negligible compared to the instanton-induced effects.

TABLE I. Particles charged under the dark gauge groups. The $SU(2)_D \times U(1)_D$ charge assignments are indicated in parentheses; the subscripts $+$, $-$ and 0 represent the standard model hypercharges $+1$, -1 and 0, respectively. Note that the ψ and χ states are fermions, while the H_D and η are complex scalars.

ψ_L	$(\mathbf{2}, -1/2)_0$	ψ_{uR}, ψ_{dR}	$(\mathbf{1}, -1/2)_0$
$\chi_L^{(1)}$	$(\mathbf{2}, +1/6)_+$	$\chi_{uR}^{(1)}$, $\chi_{dR}^{(1)}$	$(\mathbf{1}, +1/6)_+$
$\chi_L^{(2)}$	$(\mathbf{2}, +1/6)_0$	$\chi_{uR}^{(2)}$, $\chi_{dR}^{(2)}$	$(\mathbf{1}, +1/6)_0$
$\chi_L^{(3)}$	$(\mathbf{2}, +1/6)_-$	$\chi_{uR}^{(3)}$, $\chi_{dR}^{(3)}$	$(\mathbf{1}, +1/6)_-$
H_D	$(\mathbf{2}, 0)_0$	η	$(\mathbf{1}, 1/6)_0$

We introduce a vectorlike lepton pair, E_L, E_R, with mass M_E and the same quantum numbers as a right-handed electron; in the notation of Table I:

$$E_L \sim E_R \sim (\mathbf{1}, 0)_-. \tag{2.2}$$

In addition, we assume in this model that standard model neutrinos have purely Dirac masses. If the Higgs vacuum expectation values (VEVs) are smaller than the masses of the heavy states, then the mixing to standard model leptons shown in Fig. 1 can be estimated via the diagram in Fig. 2. Otherwise, one has to diagonalize the appropriate fermion mass matrices. We discuss the exact diagonalization in an Appendix for the reader who is interested in the details. Here, the diagrammatic approach is sufficient to establish that the mixing is present, and is no larger than order $\langle\eta\rangle/M_\chi$, $\langle\eta\rangle/M_\chi$, and $\langle\eta\rangle\langle H\rangle/(M_\chi M_E)$, where H is the standard model Higgs, for the $\chi_L^{(1)} - e_R^c$, $\chi_L^{(2)} - \nu_R^c$ and $\chi_L^{(3)} - e_L$ mixing angles, respectively. We take each mixing angle to be 0.01 in the estimates that follow, and demonstrate in the Appendix how this choice can be easily obtained. Further, we assume that decays to the heavy eigenstates are not kinematically allowed, as is also illustrated in the Appendix. Because of the mixing, the $\chi^{(i)}$ particles decay quickly to standard model particles via couplings to the Higgs bosons and standard model electroweak gauge bosons. The heavier ψ mass eigenstate decays to lighter states via $SU(2)_D$ gauge-boson-exchange interactions.

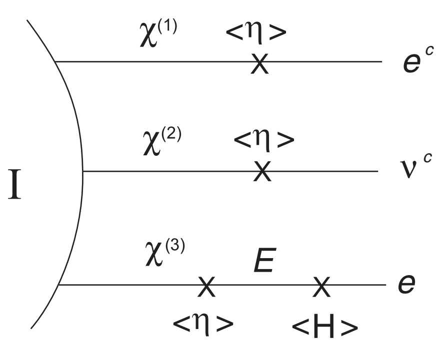

FIG. 2. Diagrammatic interpretation of mixing from χ states to standard model fermions, corresponding to the right-hand side of Fig. 1. Here E represents the vectorlike lepton described in the text, and H is the standard model Higgs.

The instanton-induced vertex in Fig. 1 follows from an interaction of the form

$$\begin{aligned}\mathcal{L}_I = {} & \frac{C}{6g_D^8}\exp\left(-\frac{8\pi^2}{g_D^2}\right)\left(\frac{m_\psi}{v_D}\right)^{35/6}\frac{1}{v_D^2} \\ & \times (2\delta_{\alpha\beta}\delta_{\gamma\sigma} - \delta_{\alpha\sigma}\delta_{\beta\gamma})\cdot[(\overline{\chi_{L\beta}^{(2)c}}\psi_L^\alpha)(\overline{\chi_{L\sigma}^{(1)c}}\chi_L^{(3)\gamma}) \\ & - (\overline{\chi_{L\beta}^{(1)c}}\psi_L^\alpha)(\overline{\chi_{L\sigma}^{(2)c}}\chi_L^{(3)\gamma})] + \text{H.c.},\end{aligned} \tag{2.3}$$

where α, β, γ and σ are $SU(2)_D$ indices [19,20]. The dimensionless coefficient C can be computed using the results in Ref. [19] and one finds $C \approx 7 \times 10^8$. The operators in Eq. (2.3) lead, via mixing, to operators of the form $\bar{\nu}_R\psi_L\bar{e}_R e_L$ and $\bar{e}_R\psi_L\bar{\nu}_R e_L$. Assuming that the product of mixing angles is $\approx 10^{-6}$, as discussed earlier, one may estimate the decay width:

$$\Gamma(\psi \to \ell^+\ell^-\nu) \approx \frac{1}{g_D^{16}}\exp(-16\pi^2/g_D^2)\left(\frac{m_\psi}{v_D}\right)^{47/3} m_\psi. \tag{2.4}$$

For example, for $m_\psi = 3.5$ TeV and $v_D = 4$ TeV, one obtains a dark matter lifetime of 10^{26} s for

$$g_D \approx 1.15, \tag{2.5}$$

where g_D is defined in dimensional regularization and renormalized at the scale m_ψ [19]. For similar parameter choices, one can slightly adjust g_D to maintain the desired lifetime. As mentioned earlier, dimension-six Planck-suppressed operators are much smaller than the operators in Eq. (2.3). Sphaleron-induced interactions are suppressed by $\sim\exp[-4\pi v_D/(g_D T)] \sim \exp(-44\ \text{TeV}/T)$, and become negligible well before the temperature at which dark matter freeze-out occurs.

Finally, let us consider whether the choice $v_D = 4$ TeV conflicts with other meaningful constraints on the heavy particle content of the model. In short, a spectrum of $\sim$4 TeV χ and E fermions with order 0.01 mixing angles with standard model leptons presents no phenomenological problems. These states are above all direct detection bounds; they are vectorlike under the standard model gauge group so that the S parameter is small; they mix weakly enough with standard model leptons so that other precision observables are negligibly affected. On this last point, we note that the correction to the muon and Z-boson decay widths due to the fermion mixing is a factor of 10^{-8} smaller than the widths predicted in the standard model, which is within the current experimental uncertainties. The dark-sector gauge bosons are also phenomenologically safe. They do not have couplings that distinguish standard model lepton flavor (since they do not couple directly to standard model leptons) so that tree-level lepton-flavor violating processes are absent. The effective four-standard-model-fermion operators that are induced by dark gauge boson exchanges are suppressed by

$\sim(0.01)^4/v_D^2 \sim 1/(40\,000\ \mathrm{TeV})^2$, which is consistent with the existing contact interaction bounds [21].

We now turn to the question of whether the model provides for the appropriate dark matter relic density.

III. RELIC DENSITY

For the regions of model parameter space considered in this section, dark matter annihilations to standard model particles proceed via mixing between the dark and ordinary Higgs bosons, often described as the Higgs portal [22]. We take into account mixing between the doublet Higgs fields, H_D and H, in our discussion below. This is consistent with a simplifying assumption that the η Higgs does not mix with the others in the scalar potential. Such an assumption is adequate for our purposes since we aim only to show that some parameter region exists in which the correct dark matter relic density is obtained. Consideration of a more general potential would likely provide additional solutions in a much larger parameter space, but would not alter the conclusion that the desired relic density can be achieved.

In this section, ψ will refer to the dark matter mass eigenstate, i.e., the lightest mass eigenstate of the ψ_u-ψ_d mass matrix, which we take as diagonal, for convenience. The potential for the doublet fields has the form:

$$V = -\mu^2 H^\dagger H + \lambda(H^\dagger H)^2 - \mu_D^2 H_D^\dagger H_D + \lambda_D(H_D^\dagger H_D)^2 + \lambda_{\mathrm{mix}}(H^\dagger H)(H_D^\dagger H_D). \tag{3.1}$$

In unitary gauge, H and H_D are given by

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ v + h \end{bmatrix}, \qquad H_D = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ v_D + h_D \end{bmatrix}, \tag{3.2}$$

where v and v_D are the H and H_D VEVs, respectively. At the extrema of this potential,

$$\begin{aligned} v(-\mu^2 + \lambda v^2 + \tfrac{1}{2}\lambda_{\mathrm{mix}} v_D^2) &= 0, \\ v_D(-\mu_D^2 + \lambda_D v_D^2 + \tfrac{1}{2}\lambda_{\mathrm{mix}} v^2) &= 0. \end{aligned} \tag{3.3}$$

The h-h_D mass matrix follows from Eq. (3.1),

$$M_H^2 = \begin{pmatrix} 2\lambda v^2 & \lambda_{\mathrm{mix}} v v_D \\ \lambda_{\mathrm{mix}} v v_D & 2\lambda_D v_D^2 \end{pmatrix}. \tag{3.4}$$

Diagonalizing the mass matrix, one finds the mass eigenvalues

$$m_{1,2}^2 = (\lambda_D v_D^2 + \lambda v^2) \mp (\lambda_D v_D^2 - \lambda v^2)\sqrt{1 + y^2}, \tag{3.5}$$

where

$$y = \frac{\lambda_{\mathrm{mix}} v v_D}{\lambda_D v_D^2 - \lambda v^2}. \tag{3.6}$$

The mass eigenstates h_1 and h_2 are related to h and h_D by a mixing angle

$$\begin{aligned} h_1 &= h\cos\theta - h_D\sin\theta, \\ h_2 &= h\sin\theta + h_D\cos\theta, \end{aligned} \tag{3.7}$$

where

$$\tan 2\theta = y. \tag{3.8}$$

Dark matter annihilations proceed via exchanges of the physical Higgs states h_1 and h_2. We take into account the final states W^+W^-, ZZ, h_1h_1 and $t\bar{t}$, where t represents the top quark. For the parameter choices considered later, final states involving h_2 will be subleading. The relevant annihilation cross sections are given by

$$\sigma_{W^+W^-} = \frac{g^2 m_\psi^2 \sin^2\theta\cos^2\theta}{128\pi m_W^2 v_D^2} s^2 \times \left|\frac{1}{s - m_1^2 + i m_1\Gamma_1} - \frac{1}{s - m_2^2 + i m_2\Gamma_2}\right|^2 \times \sqrt{1 - \frac{4m_\psi^2}{s}}\sqrt{1 - \frac{4m_W^2}{s}}\left(1 - \frac{4m_W^2}{s} + \frac{12 m_W^4}{s^2}\right), \tag{3.9}$$

$$\sigma_{ZZ} = \frac{g^2 m_\psi^2 \sin^2\theta\cos^2\theta}{256\pi m_W^2 v_D^2} s^2 \times \left|\frac{1}{s - m_1^2 + i m_1\Gamma_1} - \frac{1}{s - m_2^2 + i m_2\Gamma_2}\right|^2 \times \sqrt{1 - \frac{4m_\psi^2}{s}}\sqrt{1 - \frac{4m_Z^2}{s}}\left(1 - \frac{4m_Z^2}{s} + \frac{12 m_Z^4}{s^2}\right), \tag{3.10}$$

$$\sigma_{h_1h_1} = \frac{m_\psi^2}{16\pi v_D^2}\left|\frac{3g_{111}\sin\theta}{s - m_1^2 + i m_1\Gamma_1} + \frac{g_{112}\cos\theta}{s - m_2^2 + i m_2\Gamma_2}\right|^2 \times \sqrt{1 - \frac{4m_\psi^2}{s}}\sqrt{1 - \frac{4m_{h_1}^2}{s}}, \tag{3.11}$$

$$\sigma_{t\bar{t}} = \frac{3 m_\psi^2 m_t^2 \sin^2\theta\cos^2\theta}{16\pi v_D^2 v^2} s \times \left|\frac{1}{s - m_1^2 + i m_1\Gamma_1} - \frac{1}{s - m_2^2 + i m_2\Gamma_2}\right|^2 \times \left(1 - \frac{4m_t^2}{s}\right)\left(1 - \frac{4m_\psi^2}{s}\right). \tag{3.12}$$

In Eqs. (3.9) and (3.10), g is the standard model SU(2) gauge coupling. In Eq. (3.11), g_{111} and g_{112} represent the h_1^3 and $h_2 h_1^2$ couplings, respectively:

$$\begin{aligned} g_{111} &= (\lambda\cos^3\theta + \tfrac{1}{2}\lambda_{\rm mix}\cos\theta\sin^2\theta)v \\ &\quad - (\lambda_D\sin^3\theta + \tfrac{1}{2}\lambda_{\rm mix}\sin\theta\cos^2\theta)v_D, \\ g_{112} &= [3\lambda\cos^2\theta\sin\theta - \lambda_{\rm mix}(\cos^2\theta\sin\theta - \tfrac{1}{2}\sin^3\theta)]v \\ &\quad + [3\lambda_D\sin^2\theta\cos\theta - \lambda_{\rm mix}(\sin^2\theta\cos\theta - \tfrac{1}{2}\cos^3\theta)]v_D. \end{aligned} \tag{3.13}$$

Finally, in all our annihilation cross sections, Γ_1 (Γ_2) represents the decay width of the Higgs field h_1 (h_2). The width Γ_1 is comparable to that of a standard model Higgs boson and can be neglected without noticeably affecting our numerical results. However, since our eventual parameter choices will place the mass of the heavier Higgs field around $2m_\psi$, we must retain Γ_2; the leading contributions to Γ_2 come from the same final states relevant to the ψ annihilation cross section:

$$\begin{aligned} \Gamma_{h_2\to W^+W^-} &= \frac{g^2 m_2^3}{64\pi m_W^2}\sin^2\theta \\ &\quad \times\sqrt{1-\frac{4m_W^2}{m_2^2}}\left(1-\frac{4m_W^2}{m_2^2}+\frac{12m_W^4}{m_2^4}\right), \\ \Gamma_{h_2\to ZZ} &= \frac{g^2 m_2^3}{128\pi m_W^2}\sin^2\theta\sqrt{1-\frac{4m_Z^2}{m_2^2}}\left(1-\frac{4m_Z^2}{m_2^2}+\frac{12m_Z^4}{m_2^4}\right), \\ \Gamma_{h_2\to h_1h_1} &= \frac{g_{112}^2}{32\pi m_2}\sqrt{1-\frac{4m_1^2}{m_2^2}}, \\ \Gamma_{h_2\to t\bar{t}} &= \frac{3m_2m_t^2}{8\pi v^2}\sin^2\theta\left(1-\frac{4m_t^2}{m_2^2}\right)^{3/2}. \end{aligned} \tag{3.14}$$

The evolution of the ψ number density, n_ψ, is governed by the Boltzmann equation

$$\frac{dn_\psi}{dt} + 3H(t)n_\psi = -\langle\sigma v\rangle[n_\psi^2 - (n_\psi^{\rm EQ})^2], \tag{3.15}$$

where $H(t)$ is the Hubble parameter and $n_\psi^{\rm EQ}$ is the equilibrium number density. The thermally averaged annihilation cross section times relative velocity $\langle\sigma v\rangle$ is given by [23]

$$\langle\sigma v\rangle = \frac{1}{8m_\psi^4 T K_2^2(m_\psi/T)}\int_{4m_\psi^2}^\infty(\sigma_{\rm tot})(s-4m_\psi^2) \times\sqrt{s}K_1(\sqrt{s}/T)ds, \tag{3.16}$$

where $\sigma_{\rm tot}$ is the total annihilation cross section, and the K_i are modified Bessel functions of order i. We evaluate the freeze-out condition [24]

$$\frac{\Gamma}{H(t_F)} \equiv \frac{n_\psi^{\rm EQ}\langle\sigma v\rangle}{H(t_F)} \approx 1, \tag{3.17}$$

to find the freeze-out temperature T_f, or equivalently $x_f \equiv m_\psi/T_f$. We assume the nonrelativistic equilibrium number density

$$n_\psi^{\rm EQ} = 2\left(\frac{m_\psi T}{2\pi}\right)^{3/2}e^{-m_\psi/T}, \tag{3.18}$$

and the Hubble parameter $H = 1.66g_*^{1/2}T^2/m_{\rm Pl}$, appropriate to a radiation-dominated universe. The symbol g_* represents the number of relativistic degrees of freedom and $m_{Pl} = 1.22\times10^{19}$ GeV is the Planck mass. For the parameter choices in Tables II and III, we find $x_f \sim 27$–28. We approximate the relic abundance using [23]

$$\frac{1}{Y_0} = \frac{1}{Y_f} + \sqrt{\frac{\pi}{45}}m_{\rm Pl}m_\psi\int_{x_f}^{x_0}\frac{g_*^{1/2}}{x^2}\langle\sigma v\rangle dx, \tag{3.19}$$

where Y is the ratio of the number to entropy density and the subscript 0 indicates the present time. The ratio of the dark matter relic density to the critical density ρ_c is given by $\Omega_D = Y_0 s_0 m_\psi/\rho_c$, where s_0 is the present entropy density, or equivalently

$$\Omega_D h^2 \approx 2.8\times10^8\ {\rm GeV}^{-1}Y_0 m_\psi. \tag{3.20}$$

In our numerical analysis, we assume that the heavy states are sufficiently nondegenerate, so that we do not have to consider coannihilation processes [25]. In Tables II and III, we show representative points in the model's parameter space, spanning a range of ψ masses, in which we obtain the correct dark matter relic abundance, $\Omega_D h^2 \approx 0.1$, and in which the masses m_1 and m_2 are consistent with the LEP bound $m_{1,2} > 114.4$ GeV [21]. It is common wisdom that

TABLE II. Examples of viable parameter sets for $v_D = 4$ TeV. For each point listed, $\Omega_D h^2 \approx 0.1$ and the Higgs masses are consistent with the LEP bound.

m_ψ (TeV)	$\sqrt{2\lambda v^2}$ (TeV)	$\sqrt{2\lambda_D v_D^2}$ (TeV)	$\lambda_{\rm mix}$	m_1 (GeV)	m_2 (TeV)
1.0	0.19	1.98	0.21	158	1.98
1.5	0.22	2.98	0.28	199	2.98
2.0	0.26	3.97	0.39	241	3.97
2.5	0.27	4.97	0.42	257	4.97
3.0	0.29	5.96	0.52	277	5.96
3.5	0.31	6.96	0.57	299	6.96
4.0	0.35	7.95	0.70	339	7.95

TABLE III. Examples of viable parameter sets for $v_D = 4$ TeV, with m_1 below 130 GeV. For each point listed, $\Omega_D h^2 \approx 0.1$ and the Higgs masses are consistent with the LEP bound.

m_ψ (TeV)	$\sqrt{2\lambda v^2}$ (TeV)	$\sqrt{2\lambda_D v_D^2}$ (TeV)	$\lambda_{\rm mix}$	m_1 (GeV)	m_2 (TeV)
1.0	0.16	1.98	0.21	121	1.98
1.5	0.15	2.98	0.28	118	2.98
2.0	0.16	3.97	0.39	127	3.97
2.5	0.15	4.97	0.42	124	4.97
3.0	0.15	5.96	0.52	122	5.96
3.5	0.15	6.96	0.57	127	6.96
4.0	0.15	7.95	0.70	122	7.95

weakly interacting dark matter candidates with masses of a few hundred GeV typically yield relic densities in the correct ballpark. We have assumed masses above 1 TeV since most fits to the positron excess in PAMELA and Fermi-LAT indicate that a decaying dark matter candidate should have a mass in this range. One would therefore expect that $\Omega_D h^2$ in our model should be larger than desirable. The reason this is not the case is that we have chosen parameters for which the heavier Higgs h_2 is within 1% of $2m_\psi$, leading to a resonant enhancement in the annihilation rate. While we would be happier without this tuning, it is no larger than tuning that exists in, for example, the Higgs sector of the minimal supersymmetric standard model. It is also worth pointing out that this tuning is related to the portal that connects the dark to standard model sectors of the theory and is not strictly tied to the mechanism that we have proposed for dark matter decay. Other portals are possible. For example, one might study the limit of the model in which the $U(1)_D$ gauge boson is lighter and kinetically mixes with hypercharge, a possibility that would lead to other annihilation channels. Finally, we point out that Tables II and III include $m_\psi = 3.5$ TeV, which naively corresponds to the value preferred by a fit to the PAMELA and Fermi-LAT data, assuming a spin-1/2 dark matter candidate that decays to $\ell^+\ell^-\nu$ [15]. However, other masses should not be discounted since astrophysical sources may also contribute to the observed positron excess [5].

IV. DIRECT DETECTION

We now consider whether the parameter choices described in the previous section are consistent with the current bounds from direct detection experiments. The most relevant constraints come from experiments that search for spin-independent, elastic scattering of dark matter off target nuclei. The relevant low-energy effective interaction from t-channel exchanges of the Higgs mass eigenstates is given by

$$\mathcal{L}_{\rm int} = \sum_q \alpha_q \bar{\psi}\psi\bar{q}q, \tag{4.1}$$

where

$$\alpha_q = \frac{m_q m_\psi \sin\theta\cos\theta}{v v_D}\left(\frac{1}{m_1^2} - \frac{1}{m_2^2}\right). \tag{4.2}$$

This interaction is valid for momentum exchanges that are small compared to $m_{1,2}$, which is always the case given that typical dark matter velocities are nonrelativistic. Following the approach of Ref. [26], Eq. (4.1) leads to an effective interaction with nucleons

$$\mathcal{L}_{\rm eff} = f_p\bar{\psi}\psi\bar{p}p + f_n\bar{\psi}\psi\bar{n}n, \tag{4.3}$$

where f_p and f_n are related to α_q through the relation [26]

$$\frac{f_{p,n}}{m_{p,n}} = \sum_{q=u,d,s}\frac{f_{Tq}^{(p,n)}\alpha_q}{m_q} + \frac{2}{27}f_{Tg}^{(p,n)}\sum_{q=c,b,t}\frac{\alpha_q}{m_q}, \tag{4.4}$$

where $\langle n|m_q\bar{q}q|n\rangle = m_n f_{Tq}^n$. Numerically, the $f_{Tq}^{(p,n)}$ are given by [27]

$$\begin{aligned} f_{Tu}^p &= 0.020 \pm 0.004, \\ f_{Td}^p &= 0.026 \pm 0.005, \\ f_{Ts}^p &= 0.118 \pm 0.062, \end{aligned} \tag{4.5}$$

and

$$\begin{aligned} f_{Tu}^n &= 0.014 \pm 0.003, \\ f_{Td}^n &= 0.036 \pm 0.008, \\ f_{Ts}^n &= 0.118 \pm 0.062, \end{aligned} \tag{4.6}$$

while $f_{Tg}^{(p,n)}$ is defined by

$$f_{Tg}^{(p,n)} = 1 - \sum_{q=u,d,s} f_{Tq}^{(p,n)}. \tag{4.7}$$

We can approximate $f_p \approx f_n$ since f_{Ts} is larger than other f_{Tq}'s and f_{Tg}. For the purpose of comparing the predicted cross section with existing bounds, we evaluate the cross section for scattering off a single nucleon, which can be approximated

$$\sigma_n \approx \frac{m_r^2 f_p^2}{\pi}, \tag{4.8}$$

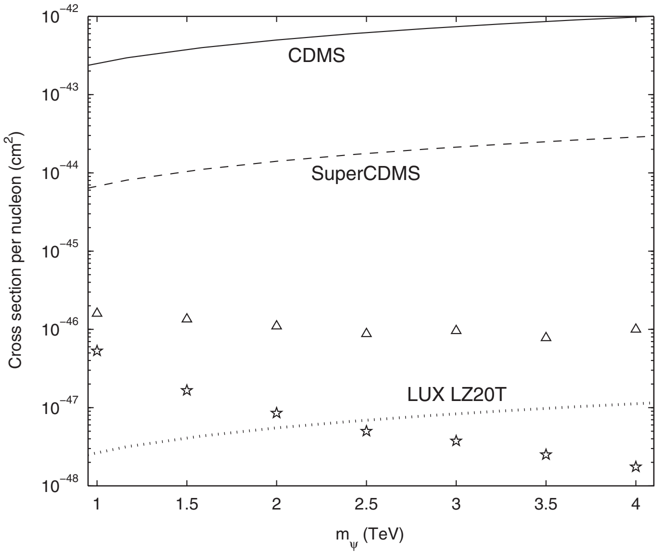

FIG. 3. Dark matter-nucleon elastic scattering cross section for the parameter sets in Table II (stars) and Table III (triangles). The solid line is the current bound from CDMS Soudan 2004–2009 Ge [28]. The dashed line represents the projected bound from SuperCDMS phase A. The dotted line represents the projected reach of the LUX LZ20T experiment, assuming 1 event sensitivity and 13 ton-kilodays. The graph is obtained using the DM Tools software [29].

where m_r is nucleon-dark matter reduced mass $1/m_r = 1/m_n + 1/m_\psi$. Our results are shown in Fig. 3, for the parameter sets given in Tables II and III. The predicted cross sections are far below the current CDMS bounds [28] for dark matter masses between 1 and 4 TeV. However, there is hope that the model can be probed by the future LUX LZ20T experiment [30].

V. CONCLUSIONS

We have presented a new TeV-scale model of decaying dark matter. The approximate stability of the dark matter candidate, ψ, is a consequence of a global U(1) symmetry that is exact at the perturbative level, but is violated by instanton-induced interactions of a non-Abelian dark gauge group. The instanton-induced vertex couples the dark matter candidate to heavy, exotic states that mix with standard model leptons; the dark matter then decays to $\ell^+\ell^-\nu$ final states, where the leptons can be of any generation desired. We have shown that a lifetime of $\sim 10^{26}$ s, which is desirable in decaying dark matter scenarios, can be obtained for perturbative values of the non-Abelian dark gauge coupling. In addition, by studying dark matter annihilations through the Higgs portal, we have provided examples of parameter regions in which the appropriate dark matter relic density may be obtained, assuming dark matter masses that are consistent with fits to the results from the PAMELA and Fermi-LAT experiments. The nucleon-dark matter cross section in our model is lower than the present bound from CDMS, but may be probed in future experiments. It might also be possible to probe the spectrum of our model at the LHC.

The model in this paper provides a concrete, TeV-scale scenario in which dark matter decay is mediated by instantons, and gives a new motivation for the study of non-Abelian dark gauge groups [31]. However, it is by no means the only possible model of this type. One might study variations of the model in which different annihilation channels are dominant, or the dark matter is lighter, or the standard model leptons are directly charged under the new non-Abelian gauge group. It may also be worthwhile to consider how low-scale leptogenesis and baryogenesis might be accommodated in this type of scenario. While we have assumed parameter choices motivated by the observed cosmic ray positron excess, one might incorporate the present model in a multicomponent dark matter scenario if this were required to explain new results from ongoing and future direct detection experiments.

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APPENDIX A: MASS MIXING EXAMPLE

In Sec. II, we presented a diagrammatic representation of the mixing that takes the χ states to standard model leptons. Here we study the numerical diagonalization of the corresponding fermion mass matrices, to demonstrate that mixing angles of the size assumed in our analysis are easily obtained. To simplify the discussion, we focus on mixing with standard model leptons of a single generation, which we denote by e and ν. We include (1) Dirac masses for the χ fields:

$$\mathcal{L} \supset \sum_i [a_i \bar{\chi}_L^{(i)} \langle H_D \rangle \chi_{uR}^{(i)} + b_i \bar{\chi}_L^{(i)} \langle H_D \rangle \chi_{dR}^{(i)} + c_i \bar{\chi}_L^{(i)} \langle \tilde{H}_D \rangle \chi_{uR}^{(i)} + d_i \bar{\chi}_L^{(i)} \langle \tilde{H}_D \rangle \chi_{dR}^{(i)}] + \text{H.c.}, \quad \text{(A1)}$$

where $\tilde{H}_D \equiv i\sigma^2 H_D^*$. These terms generate a completely general two-by-two Dirac mass matrix for the χ fermions. (2) Mixing between the χ fields and standard model leptons:

$$\mathcal{L} \supset g_1 \langle \eta \rangle \bar{\chi}_{dR}^{(1)} e_R^c + g_2 \langle \eta \rangle \bar{\chi}_{uR}^{(1)} e_R^c + \lambda_e \bar{L} \langle H \rangle e_R + g_3 \langle \eta \rangle \bar{\chi}_{dR}^{(2)} \nu_R^c + g_4 \langle \eta \rangle \bar{\chi}_{uR}^{(2)} \nu_R^c + \lambda_\nu \bar{L} \langle \tilde{H} \rangle \nu_R + \text{H.c.} \quad \text{(A2)}$$

(3) Mixing involving the vectorlike leptons E_L and E_R:

$$\mathcal{L} \supset g_5 \langle \eta \rangle \bar{\chi}_{dR}^{(3)} E_L + g_6 \langle \eta \rangle \bar{\chi}_{uR}^{(3)} E_L + M_E \bar{E}_L E_R + g_7 \bar{L} \langle H \rangle E_R + \text{H.c.} \quad \text{(A3)}$$

We now write down the mass matrices which follow from Eqs. (A1)–(A3). For the neutral states, we work in the basis $f_L^0 = (\chi_{uL}^{(2)}, \chi_{dL}^{(2)}, \nu_R^c)$ and $f_R^0 = (\chi_{uR}^{(2)}, \chi_{dR}^{(2)}, \nu_L^c)$. The neutral mass terms can be written as $\bar{f}_L^0 M_0 f_R^0$ + H.c., where

$$M_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} c_2 v_D & d_2 v_D & 0 \\ a_2 v_D & b_2 v_D & 0 \\ g_4 v_\eta & g_3 v_\eta & \sqrt{2} m_\nu \end{pmatrix}, \tag{A4}$$

assuming, for simplicity, that the VEVs and couplings are real. Similarly, the mass terms for the charged states may be written $\bar{f}_L^- M_c f_R^-$ + H.c., where we assume the basis $f_L^- = (\chi_{uR}^{(1)c}, \chi_{dR}^{(1)c}, \chi_{uL}^{(3)}, \chi_{dL}^{(3)}, E_L, e_L)$ and $f_R^- = (\chi_{uL}^{(1)c}, \chi_{dL}^{(1)c}, \chi_{uR}^{(3)}, \chi_{dR}^{(3)}, E_R, e_R)$. In this case,

$$M_c = \frac{1}{\sqrt{2}} \begin{pmatrix} c_1 v_D & a_1 v_D & 0 & 0 & 0 & g_2 v_\eta \\ d_1 v_D & b_1 v_D & 0 & 0 & 0 & g_1 v_\eta \\ 0 & 0 & c_3 v_D & d_3 v_D & 0 & 0 \\ 0 & 0 & a_3 v_D & b_3 v_D & 0 & 0 \\ 0 & 0 & g_6 v_\eta & g_5 v_\eta & \sqrt{2} M_E & 0 \\ 0 & 0 & 0 & 0 & g_7 v & \sqrt{2} m_e \end{pmatrix}. \tag{A5}$$

Given a choice of parameters, it is now a simple matter to compute the relevant mixing angles numerically. As an example, let us work in units of the dark scale v_D, which we will assume is 4 TeV. In addition we take $v_\eta = v_D$, $M_E = 1.5 v_D$ and set the standard model lepton masses to zero (the conclusions do not change if we require realistic standard model lepton masses). If one assumes that only the following parameters are nonzero:

$$\{b_1, c_1, b_2, c_2, b_3, c_3, g_1, g_2, g_3, g_4, g_5, g_6, g_7\} = \{1.9, 1.8, 1.8, 1.7, 2.1, 2.0, 0.02, 0.02, 0.02, 0.02, 0.7, 0.6, 1.0\}, \tag{A6}$$

then one finds

$$\chi_{uL}^{(1)} = 0.011 e_{R0}^c + \cdots, \qquad \chi_{dL}^{(1)} = 0.011 e_{R0}^c + \cdots,$$
$$\chi_{uL}^{(2)} = 0.012 \nu_{R0}^c + \cdots, \qquad \chi_{dL}^{(2)} = 0.011 \nu_{R0}^c + \cdots,$$
$$\chi_{uL}^{(3)} = 0.009 e_{L0} + \cdots, \qquad \chi_{dL}^{(3)} = 0.010 e_{L0} + \cdots,$$

where the fields on the right represent mass eigenstates. In addition, the nonzero mass eigenvalues are all larger than the ψ mass if $m_\psi < 1.2 v_D$, so that only decays to standard model leptons via the instanton vertex are kinematically allowed. Given the number of free parameters involved, one sees that the mixing angles are highly model dependent and can be easily set to the values assumed in Sec. II.

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