

Reappraisal of spontaneous R -parity violation

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In this short reappraisal of spontaneous lepton number violation in a supersymmetric scenario implemented through singlet sneutrino vacuum expectation value (VEV), we contribute two new things in the context where the lepton number symmetry is global: (i) provide explicit expressions of R -parity violating couplings in terms of the neutrino Yukawa couplings and the singlet sneutrino VEV, and (ii) estimate the limit on this VEV using the current knowledge of the light neutrino mass and the astrophysical constraint on the Majoron-electron coupling. Besides, we put updated constraints on the VEV and Yukawa coupling of the singlet superfield when the lepton number is gauged.

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I. INTRODUCTION

It is well known that if the supersymmetric partners of all standard model (SM) particles are introduced in a theory and one constructs the most general Lagrangian that is invariant under supersymmetry and the SM gauge symmetry, the Lagrangian contains terms that violate both lepton number (\mathcal{L}) and baryon number (\mathcal{B}). The *explicit* \mathcal{L} - and \mathcal{B} -violating parts appear in the superpotential as

$$\begin{aligned} \mathcal{W} \supset & \sum_{abc} \lambda_{abc} L_a L_b \hat{E}_c + \sum_{abc} \lambda'_{abc} L_a Q_b \hat{D}_c \\ & + \sum_{abc} \lambda''_{abc} \hat{U}_a \hat{D}_b \hat{D}_c + \sum_a \mu_a L_a H_u. \end{aligned} \quad (1)$$

Above, all superfields are left chiral, and the subscripts a , b , c are generation indices on lepton doublet fields L , quark doublet fields Q , and SU(2)-singlet charged fields E , U , and D , in obvious notations. The hat on a superfield means that the left-chiral fermion part of that superfield is the antiparticle of fermion whose name is suggested in the letter denoting the superfield. These terms also violate R parity, defined by $(-1)^{3\mathcal{B}+\mathcal{L}+2\mathcal{S}}$, where \mathcal{S} is the spin of the particle. The antisymmetry in the first two generation indices of λ and in the last two indices of λ'' suggests $\lambda_{abc} = -\lambda_{bac}$ and $\lambda''_{abc} = -\lambda''_{acb}$. Clearly, there are nine λ -type, 27 λ' -type, and nine λ'' -type trilinear, plus three μ -type bilinear, R -parity violating (RPV) couplings. These 48 new couplings add further twists and complications to phenomenology and bring in more uncertainty to theoretical predictions.

Our aim in this paper is to explore a restrictive scenario in which there will be much fewer RPV couplings, thus offering more predictivity. For definiteness, we assume that these couplings are generated by *spontaneous* \mathcal{L} violation. This immediately rules out the \mathcal{B} -violating λ'' -type couplings. Now, we recall that Aulakh and Mohapatra [1] were the first to have implemented the idea of spontaneous violation of $\mathcal{L} = 1$ global lepton number in a supersymmetric context through the VEV of the sneutrino compo-

nent of a lepton doublet superfield. A testable feature of this realization was a photino mediated contribution to neutrinoless double beta decay. Neutrinos were predicted to be massless at tree level, with a suggestion that very small masses ($m_\nu \sim (10^{-5}-10^{-8})$ eV) are induced at one-loop order through a combination of supersymmetry breaking and lepton number violation. A follow-up study [2] revealed that if supersymmetry breaking terms include trilinear scalar couplings and gaugino Majorana masses, neutrino mass would be generated at the tree level itself, with a special property that even with three generations only one nonvanishing mass eigenvalue would emerge at tree level. The mechanism of [1] would indeed induce two other small masses at one-loop order. Other implications of this scenario were studied in the context of matter-enhanced solar neutrino oscillation [3–5].

An important feature of spontaneous \mathcal{L} violation is the existence of Majoron (J), which is a physical massless Nambu-Goldstone boson arising from the imaginary part of the sneutrino field that acquires a VEV. The mass of the real scalar (ρ), associated with the Majoron, in the doublet Majoron scenario turns out to be very small leading to unacceptably large $Z \rightarrow \rho J$ decay, which is ruled out by the LEP data on Z boson decay width. In fact, gauge nonsinglet Majoron models are all strongly disfavored by electroweak precision measurements [6,7]. Subsequently, *singlet* Majoron scenarios were proposed in the supersymmetric context. In some of these models, lepton number was spontaneously broken by the VEV of a field carrying one unit of lepton number [8–10], and in some others by a field carrying two units of the lepton number [11], like the nonsupersymmetric models of spontaneous lepton number violation [12,13]. Since the first kind, i.e., $\Delta\mathcal{L} = 1$ violation, is a speciality of supersymmetric models that non-supersymmetric models do not have, we take it up for our work here.

In this context, we present explicit expressions of λ_{abc} , λ'_{abc} , and μ_a couplings in terms of the Yukawa couplings of the general superpotential and the singlet neutrino VEV.

Furthermore, we provide new bounds on the sneutrino VEV and the generic neutrino Yukawa couplings from astrophysical considerations of stellar energy loss.

II. R-PARITY VIOLATING COUPLINGS

Our model has, apart from the superfields in the MSSM, gauge singlet superfield \hat{N}_a , one for each generation. These fields carry lepton number $\mathcal{L} = -1$, and hence their VEVs (V_a) would spontaneously break lepton number. The superpotential of this model can be written as

$$\mathcal{W} = \sum_{ab} h_{ab}^{(u)} Q_a \hat{U}_b H_u + \sum_{ab} h_{ab}^{(d)} Q_a \hat{D}_b H_d + \sum_{ab} h_{ab}^{(l)} L_a \hat{E}_b H_d + \sum_{ab} h_{ab}^{(N)} L_a \hat{N}_b H_u + \mu H_u H_d. \quad (2)$$

We assume that the VEVs of \hat{N}_a are generated by the same mechanism as in [8–10] which require the existence of two more gauge singlet superfields (with $\mathcal{L} = +1$ and 0, respectively). Though we implicitly acknowledge their existence we do not explicitly display how those two additional singlets appear in the superpotential, whose *raison d'être* is to provide the V_a 's. Beside that, Eq. (2) is the most general gauge invariant superpotential that also conserves lepton number before the scalar components of \hat{N}_a go to the vacuum. It is not difficult to realize that the VEVs V_a by themselves cannot break supersymmetry. Also note that apart from the term containing the \hat{N} fields, the superpotential corresponds exactly to that of R -parity conserving minimal supersymmetry.

The \mathcal{L} -violating operators are generated as soon as the VEVs V_a are induced. Figure 1(a) will generate the λ terms, whereas Fig. 1(b) will generate λ' terms of Eq. (1). The important point is that, these couplings will now be determined by Yukawa couplings and the sneutrino VEVs. It should be noted that when \hat{N} acquires a VEV, the internal line in Fig. 1 is necessarily Higgsino. If we assume that the Higgsino mass parameter μ is a few hundred GeV or more, one can effectively write a contact interaction from Fig. 1. When the singlet sneutrinos acquire VEVs, we obtain

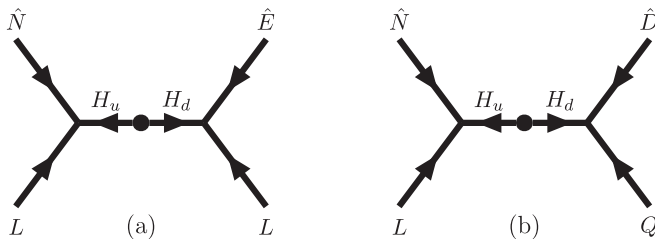


FIG. 1. Generation of $LL\hat{E}$ and $LQ\hat{D}$ operators when the scalar part of \hat{N} acquires a VEV. The thick lines denote superfields. Generation indices have been suppressed. The blob in the middle of the internal line implies that it involves the μ term of the superpotential.

$$\lambda_{abc} \simeq \sum_d \frac{V_d}{\mu} (h_{ad}^{(N)} h_{bc}^{(l)} - h_{bd}^{(N)} h_{ac}^{(l)}), \quad (3)$$

$$\lambda'_{abc} \simeq \sum_d \frac{V_d}{\mu} h_{ad}^{(N)} h_{bc}^{(d)}. \quad (4)$$

The two terms in case of λ_{abc} arise depending on whether L_a or L_b appears in the same vertex with the \hat{N}_d superfield in Fig. 1(a). The bilinear lepton number violating terms, shown in Eq. (1), also arise in this model from the $h^{(N)}$ terms of the superpotential in Eq. (2) when the scalar component of \hat{N} acquires a VEV:

$$\mu_a = \sum_b h_{ab}^{(N)} V_b. \quad (5)$$

The origin of the relative minus sign between the two terms in Eq. (3) can be understood by keeping the SU(2) indices. If we denote the SU(2) index carried by L_a and L_b by α and β , respectively, and put the SU(2) indices γ and δ on the internal H_u and H_d superfield lines in Fig. 1, then the diagram with L_a coupling directly to \hat{N}_d will contain the SU(2) factor

$$\varepsilon_{\alpha\gamma} \varepsilon_{\gamma\delta} \varepsilon_{\beta\delta} = \varepsilon_{\beta\alpha}, \quad (6)$$

whereas the other diagram, obtained by interchanging L_a and L_b , should contain

$$\varepsilon_{\beta\gamma} \varepsilon_{\gamma\delta} \varepsilon_{\alpha\delta} = \varepsilon_{\alpha\beta}. \quad (7)$$

Hence, the minus sign in Eq. (3), which makes the coupling antisymmetric in the indices a, b .

We can now estimate how many unknown parameters are present in the \mathcal{L} -violating sector. Without any loss of generality, the couplings $h^{(d)}$ and $h^{(l)}$ can be taken diagonal in the generation indices, and they can be made real. In this case, these couplings are proportional to the masses of the down-type quarks and charged leptons, and are therefore known. The couplings $h^{(u)}$ are irrelevant for our discussion since they do not appear in the expressions of Eqs. (3)–(5). These will contain the up-type quark masses and the parameters of the Cabibbo-Kobayashi-Maskawa matrix. For our purpose, the relevant unknown parameters appear from $h^{(N)}$, and they are nine in number. Besides, there are the three VEVs V_a . To be more precise, there is actually only one independent VEV of the singlet sneutrino fields, since we can always make suitable linear combinations of the three \hat{N} fields leading to the occurrence of a single VEV. This makes a total count of 10, instead of the 39 \mathcal{L} -violating parameters appearing in Eq. (1).

III. SNEUTRINO VEV AND MAJORON-ELECTRON COUPLING

The Majoron-electron coupling arises both at tree level and at one-loop order from different interactions, which could be of similar magnitude. The tree contribution origi-

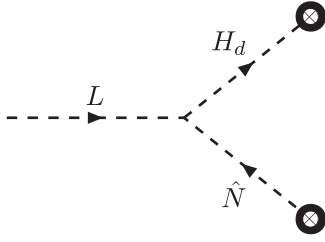


FIG. 2. Diagrammatic way of seeing how a doublet sneutrino gets a VEV. The dashed lines denote the neutral scalar fields contained in the supermultiplets, and the cross-hatched blobs denote their VEVs.

nates from the fact that a nonvanishing V , the generic VEV of a \hat{N} field, must accompany a nonvanishing v_L , the VEV of a doublet sneutrino. This can be seen most easily from the fact that in the scalar potential of the model, the F term of the H_u field contains a term of the form $\mu h^{(N)} L \hat{N} H_d^\dagger$, where in this expression only the scalar components of each superfield is implied and generation indices are suppressed. The diagram in Fig. 2 now clearly shows that the magnitude of v_L should be given by

$$v_L \approx \frac{\mu h^{(N)} V v_d}{M_S^2}, \quad (8)$$

where $h^{(N)}$ is the generic Yukawa coupling involving the \hat{N} fields, v_d is the Higgs VEV contained in H_d , and M_S is a generic doublet sneutrino mass. The above expression can also be appreciated directly from potential minimization. In the tadpole equation $\partial V / \partial v_L = 0$, the trilinear term $\mu h^{(N)} L \hat{N} H_d^\dagger$ will provide a contribution $\mu h^{(N)} V v_d$ for the left-hand side, while the soft mass term $M_S^2 L^\dagger L$ will yield $M_S^2 v_L$. The minimization condition thus gives Eq. (8).

The nonzero value of the doublet sneutrino VEV induces, from the supergraph shown in Fig. 1(a), a tree-level electron-Majoron coupling. Assuming, for the sake of illustration, that the soft mass of H_d is of the same order as μ , this coupling is given by

$$g_{eeJ}^{\text{tree}} \approx \frac{1}{M_S^2} (h^{(N)})^2 V m_e, \quad (9)$$

where the factor m_e , equal to $h^{(l)}$ times v_d , ensures a chirality-flipping coupling. The loop induced contribution to the electron-Majoron coupling arises from the diagrams shown in Fig. 3, when one of the external \hat{N} lines obtains a VEV. A rough estimate of the coupling thus generated is of the order

$$g_{eeJ}^{\text{loop}} \approx \frac{g^2}{16\pi^2 M_0^2} (h^{(N)})^2 V m_e, \quad (10)$$

where M_0 is the heaviest mass in the diagram, either of the Z boson or of the neutralino (through its Higgsino component). The magnitude of the tree and loop contributions to

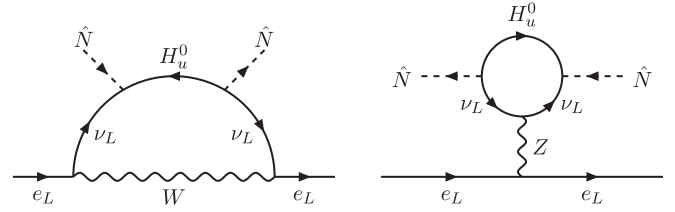


FIG. 3. Effective operators that give rise to a coupling between the electron and the Majoron when one of the external scalar lines goes to the vacuum.

g_{eeJ} could be of the same order, or one may dominate over the other, depending on the magnitude of the parameters M_S and M_0 . A cancellation between the two contributions is unlikely and we avoid any such fine-tuning.

IV. COMBINED ASTROPHYSICAL AND NEUTRINO MASS CONSTRAINTS

There are stringent astrophysical constraints on any Majoron model. Majoron emission leads to stellar energy loss, and singlet Majorons may be emitted via Compton-like processes $\gamma + e \rightarrow e + J$. The allowed leaking of stellar energy can be translated into a bound on the singlet sneutrino VEV. In fact, it turns out that Majoron coupling to the electron should be less than about 10^{-10} [14–17] from a study of the main sequence stars. Putting $M_S \sim 100$ GeV in the tree-level contribution Eq. (9), the Majoron emission bound implies

$$(h^{(N)})^2 V \lesssim 2 \text{ MeV}, \quad (11)$$

while from the loop contribution Eq. (10), for $M_0 = 100$ GeV, the bound turns out to be

$$(h^{(N)})^2 V \lesssim 1 \text{ GeV}. \quad (12)$$

Although the bounds in Eqs. (11) and (12) are quite different, both are independently significant, as the scalar mass (M_S) involved in Eq. (9) can be much larger than the neutralino (or, the Z boson) mass (M_0) in Eq. (10). The above bounds can be more stringent if, instead of main sequence stars, we use red giant stars, which give $g_{eeJ} \lesssim 3 \times 10^{-13}$ [18]. However, we use the constraints from main sequence stars which are more reliably understood.

It is interesting to note from Eqs. (9) and (10) that the electron coupling to the Majoron is *directly* proportional to the lepton number breaking VEV. In contrast, the same coupling is *inversely* proportional to the lepton number breaking VEV in the nonsupersymmetric singlet Majoron model [12,13]. The reason for the difference is that in the nonsupersymmetric case, where lepton number symmetry is broken by the VEV of a scalar field carrying two units of lepton number, heavy singlet neutrinos whose mass is of the same order as their VEVs float in the loop causing propagator suppression. In our case, V appears only in the numerator when an \hat{N} is replaced by its VEV.

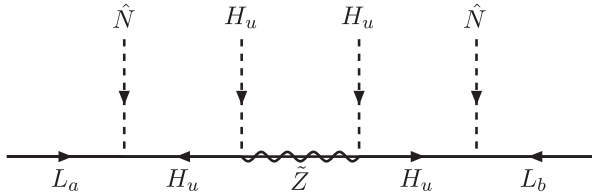


FIG. 4. Schematic diagram showing how a neutrino acquires a tree-level mass in our model after the \hat{N} fields and the neutral H_u acquires VEVs.

We now discuss how neutrino mass is generated in our scenario. Figure 4 is a diagrammatic representation of $\Delta\mathcal{L} = 2$ Majorana mass generation at tree level. It gives

$$m_{ab} \propto h_{ac}^{(N)} h_{bd}^{(N)} V_c V_d. \quad (13)$$

Since the mass matrix is of rank one, only one nonvanishing eigenvalue will emerge. This is not surprising, as one can always perform a basis rotation in flavor space to put the VEV only along one direction. In fact, what we discussed is nothing but the neutrino mass generation through bilinear RPV couplings. Accurate expressions of the tree-level neutrino mass induced by bilinear RPV couplings can be found, for example, in [9]. For our purpose, it is enough to use the approximate expression of neutrino mass suggested by Fig. 4:

$$m_\nu \sim g^2 (h^{(N)})^2 \frac{v_u^2 V^2}{M_{\tilde{\chi}}^3}, \quad (14)$$

where v_u denotes the VEV of the scalar component of H_u , and $M_{\tilde{\chi}}$ is a generic neutralino mass capturing the effects of the Zino and Higgsino propagators in Fig. 4, where it is implicitly assumed that $M_{\tilde{\chi}} \sim \mu$. Even though we do not yet precisely know the absolute magnitude of neutrino mass, we make a reasonable guess by putting $m_\nu = 0.1$ eV in Eq. (14). We further assume that $M_{\tilde{\chi}} \sim v_u \sim 100$ GeV, and obtain

$$h^{(N)} V \sim 2 \times 10^{-4} \text{ GeV}. \quad (15)$$

If we compare Eq. (15) with the astrophysical bound in Eq. (11) obtained from the tree-level contribution to electron-Majoron coupling, assuming that the neutrino Yukawa couplings involved with Majoron emission and neutrino mass generation are of the same order, we obtain two limits:

$$V \gtrsim 20 \text{ keV}, \quad h^{(N)} \lesssim 10. \quad (16)$$

On the other hand, if we compare Eq. (15) with Eq. (12), the bound on the electron-Majoron coupling from the loop process, we obtain

$$V \gtrsim 40 \text{ eV}, \quad h^{(N)} \lesssim 5 \times 10^3. \quad (17)$$

The upper bound on $h^{(N)}$ is not particularly useful if the theory has to be perturbative. However, it is inter-

esting to observe that while the astrophysical bounds in Eqs. (11) and (12) are *upper* bounds on a combination of the neutrino Yukawa coupling and the \mathcal{L} -violating VEV, finally we obtain *lower* bounds on the latter—see Eqs. (16) and (17)—if we assume some reasonable value of the light neutrino mass. This is because Eqs. (9), (10), and (14), imply an order-of-magnitude relation between Majoron coupling and neutrino mass:

$$g_{eeJ}^{\text{tree}} \sim \frac{M_{\tilde{\chi}}^3}{g^2 v_u^2 M_S^2} \frac{m_e m_\nu}{V}, \quad g_{eeJ}^{\text{loop}} \sim \frac{1}{16\pi^2} \frac{M_{\tilde{\chi}}^3}{v_u^2 M_0^2} \frac{m_e m_\nu}{V}, \quad (18)$$

where, we recall from Eq. (10) that M_0 is either $M_{\tilde{\chi}}$ or M_Z , whichever is larger. Such a proportionality between neutrino mass and Majoron-electron coupling occurs also in other singlet Majoron models [19].

It should be noted that the electron-Majoron coupling in Eqs. (9) and (10) contains $(h^{(N)})^2$. In writing this, we have suppressed the generation indices. More explicitly, the combination that actually appears is $\sum_i h_{ei}^{(N)} h_{ei}^{(N)}$. The same combination is constrained from the electron-neutrino mass, whose expression appears in Eq. (14). This implies further constraints on other combinations, involving different charged leptons, through neutrino mixing parameters [20].

Now we turn our attention to the λ and λ' couplings in Eqs. (3) and (4) and see what information on $h^{(N)}$ we get from them. Using the neutrino mass constraint in Eq. (15), the dimensionless prefactor $h^{(N)} V / \mu \sim 10^{-6}$, for $\mu \sim 100$ GeV, provides sufficient suppression to λ and λ' couplings, in addition to those coming from charged lepton (or, down-type quarks) Yukawa couplings, to meet all experimental constraints [21–25]. As a result, we can keep the $h^{(N)}$ matrix elements to be all order unity. We note that, unlike the trilinear λ or λ' couplings, the bilinear μ_a parameters do not pick up the extra suppression from charged lepton Yukawa couplings, and we may expect that the corresponding bilinear soft terms are not suppressed either. This observation helps us to face an important question at this stage: how do we produce an acceptably large *second* neutrino mass eigenvalue? This could be induced by Grossman-Haber loops [26] which contribute to neutrino mass through the \mathcal{L} -violating bilinear soft terms. In these loops there are gauge couplings at the neutrino vertices, and there are two types of \mathcal{L} -violating interactions (e.g., slepton-Higgs or neutrino-neutralino mixing) inside the loop giving rise to $\Delta\mathcal{L} = 2$ interactions. Addition of these loops to Eqs. (13) and (14) breaks the rank one structure of the mass matrix, but one eigenvalue still remains zero. This is very much consistent with the neutrino oscillation data, which do not any way compel us to consider a nonvanishing *third* mass eigenvalue. The generation of the latter requires the relevant λ or λ' couplings to be $\sim (10^{-3} - 10^{-4})$ for superparticle masses of

order 100 GeV, but in our scenario these couplings are significantly more suppressed (For a detailed discussion of how RPV couplings generate neutrino masses and mixing, see, for example, [27–33]).

V. SCENARIO WITH GAUGED LEPTON NUMBER

Equation (2) can also be interpreted as the superpotential of a model where the lepton number symmetry is gauged. Of course, lepton number is anomalous, but the combination $\mathcal{B} - \mathcal{L}$ is not. So, as a simplest example, we can think of a model where the gauge symmetry consists, apart from the standard model gauge group, of another factor of $U(1)_{\mathcal{B}-\mathcal{L}}$. This is the same as the model presented in Ref. [34], where a different combination of the weak hypercharge and $\mathcal{B} - \mathcal{L}$ had been used to denote this extra symmetry.

Without any loss of generality, we assume that only one singlet sneutrino acquires VEV, and to avoid confusion with the global symmetry case, we denote this VEV by v_R . There will be no Majoron in this case: the Goldstone boson will be eaten up by the extra neutral gauge boson that is present in this model. The strength of R -parity violation will be related to the strength of this new gauge force. It has been shown [34] in the context of an E_6 -inspired model that

$$M_{Z'}^2 = \frac{4}{3}\tan^2\theta_W M_W^2 + \frac{25}{12}g'^2 v_R^2. \quad (19)$$

Using the current experimental lower limit $M_{Z'} > 900$ GeV [35] from the Tevatron $p\bar{p}$ collider, we obtain the limit

$$v_R > 1.7 \text{ TeV}. \quad (20)$$

Although we cannot use the astrophysical bounds for this model since there is no Majoron, the neutrino mass formula given in Eq. (14) still holds, where V should be read as v_R . Using $m_\nu = 0.1$ eV as before and putting the lower bound on v_R from Eq. (20), we obtain

$$M_{\tilde{\chi}} \sim (h^{(N)})^{2/3} \times 4500 \text{ TeV}. \quad (21)$$

Unlike in the previous example with global lepton number symmetry, the elements of the Yukawa matrix $h^{(N)}$ will now have to be very small in order to keep the neutralino

mass $M_{\tilde{\chi}}$ in the phenomenologically interesting range of a few hundred GeV to a TeV.

VI. CONCLUSION

We have done a few new things in this paper. In spite of the existence of a vast literature on spontaneous R -parity violation, explicit expressions of lepton number violating couplings in terms of the sneutrino VEV and neutrino Yukawa couplings were somehow missing. Also, only bilinear RPV terms were discussed in the context of Majoron models so far. As we have shown, trilinear RPV terms would be generated too. We displayed them in Eqs. (3)–(5). These might be particularly useful if one attempts to construct some flavor models relating different entries of the $h^{(N)}$ matrix, which can comfortably be of order unity. Note that using 10 parameters we can predict 39 R -parity violating couplings. The other new thing that we have presented is an explicit estimate of the bounds on the singlet sneutrino VEV and the generic $h^{(N)}$ by using the astrophysical constraint on Majoron-electron coupling and the knowledge of the neutrino mass: see Eqs. (16) and (17).

We have also noted that when lepton number is gauged, the nonobservation of any additional gauge boson in any collider experiment puts a strong bound on the singlet sneutrino VEV, which in turn demands that entries of the $h^{(N)}$ matrix have to be small to keep the neutralino masses in the accessible range.

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