CP-violating phases in M theory and implications for electric dipole moments

Gordon Kane,[1] Piyush Kumar,[2] and Jing Shao[1]

[1]*Michigan Center for Theoretical Physics, Ann Arbor, Michigan 48109 USA*

[2]*Department of Physics, University of California, Berkeley, California 94720 USA and Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, California 94720 USA*

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We demonstrate that in effective theories arising from a class of $\mathcal{N} = 1$ fluxless compactifications of M theory on a G_2 manifold with low-energy supersymmetry, *CP*-violating phases do not appear in the soft-breaking Lagrangian except via the Yukawas appearing in the trilinear parameters. Such a mechanism may be present in other string compactifications as well; we describe properties sufficient for this to occur. *CP* violation is generated via the Yukawas since the soft trilinear matrices are generically not proportional to the Yukawa matrices. Within the framework considered, the estimated theoretical upper bounds for electric dipole moments of the electron, the neutron, and mercury are all within the current experimental limits and could be probed in the near future.

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I. INTRODUCTION

The null measurements of the electric dipole moments (EDMs) of the neutron [1], and, recently, heavy atoms like thallium (^{205}Tl) [2,3] and mercury (^{199}Hg) [4,5], have put very strong constraints on the amount of *CP* violation from new physics beyond the standard model (SM). The precision of these measurements is expected to significantly improve in a few years. If an excess above the SM prediction is observed, it requires the presence of new physics beyond the SM. However, since the EDMs, even if observed, are already ''small,'' this strongly suggests that the new physics must be such that it has an underlying mechanism to naturally suppress EDMs.

In general versions of supersymmetric extensions of the standard model, new sources of *CP* violation can arise from complex phases of the soft supersymmetry breaking parameters. These phases are therefore tightly constrained to be small [6,7] (or to have cancellation [8–11]) for TeV-scale superpartners. Thus, from a theoretical perspective, the existence of such small phases has to be explained by some underlying mechanism. Many studies of supersymmetric models from a low-energy phenomenological perspective focus on the mediation mechanism and only parametrize the supersymmetry breaking. Explaining small soft *CP*-violating phases, which requires a dynamical understanding of supersymmetry breaking, is especially challenging as this is not available in such a framework. Without a specification of the supersymmetry breaking mechanism, this problem exists in both gravity and gauge-mediated models of supersymmetry breaking in general.

Put differently, whenever supersymmetry is treated as a general TeV-scale effective theory, both the values and phases of the soft-breaking masses are treated as arbitrary, and EDMs are typically much larger than experimental values. Many people have argued that such large EDMs are implied or required from supersymmetry, and that this is a problem for supersymmetry. Such arguments ignore the fact that any underlying theory will predict and relate phases. This implies that the underlying theory of which low-energy supersymmetry is a low-energy limit has a structure that suppresses or relates the low scale phases.

Substantial progress has been made towards understanding dynamical supersymmetry breaking, especially in recent years. In this work, we will be interested in dynamical mechanisms of supersymmetry breaking with low superpartner masses which can be naturally embedded in the framework of an underlying microscopic theory like string or M theory. In particular, we study the effective four-dimensional theory resulting from fluxless $\mathcal{N} = 1$ compactifications of M theory with chiral matter [12]. These are especially interesting because a hierarchy between the electroweak and Planck scale is generated, and all geometric moduli are stabilized, at the same time [13,14]. We find that the supersymmetry breaking and mediation dynamics is such that it naturally gives rise to vanishing *CP*-violating phases from supersymmetry breaking at leading order, providing an excellent starting point to explain suppressed EDMs. The mechanism is a nontrivial generalization of an old idea [15] (and more recently [16]), and may also apply to other classes of string compactifications where moduli are stabilized in a de Sitter vacuum.

Although the *CP*-violating phases from supersymmetry breaking vanish at leading order, there could still be significant contributions to *CP* violation in the flavor-diagonal sector in principle. First, in the M theory framework, the trilinear matrices are typically not proportional to the Yukawa matrices after moduli are stabilized, which in general leads to nontrivial *CP*-violating phases in the trilinear *A*-terms in the basis of quark and lepton mass eigenstates and therefore generates nonzero EDMs [7,17]. The estimated upper bounds on EDMs are all within the current experimental limits. For some values of parameters, some upper bounds on the EDMs are close to the experimental limits. As will be clear, two features—large sfermion masses and trilinears, and hierarchical Yukawa textures, both natural within the M theory framework—are important for getting viable but interesting EDMs results. In addition, we argue that even though higher-order corrections to the Kähler potential exist, they do not give rise to new CP-violating phases. Finally, it should be remarked that the EDM results are robust anytime the trilinears dominantly acquire their phases from Yukawas and the mass spectrum is as dictated by the M theory framework.

It is worth mentioning that the solution to the supersymmetric CP problem here is largely independent of any particular solution to flavor issues as long as they satisfy a certain criterion. As will be seen, the only feature of the flavor structure used in computing results for EDMs is that the sfermion mass matrices for visible matter in the super-Cabibbo-Kobayashi-Maskawa (CKM) basis are approximately flavor diagonal at low energies. Therefore, the results hold true for any proposed solution to the flavor problem which is consistent with the above feature. For the M theory framework, in particular, this approximately flavor-diagonal structure arises due to the presence of $U(1)$ symmetries under which the chiral matter fields are charged [18]. The spontaneous breaking of these symmetries may introduce small nondiagonal components as long as it occurs at a scale sufficiently below the Planck scale. In the M theory framework, large sfermion masses $\gtrsim 10$ TeV already mitigate the flavor-changing neutral-current problems. In addition, small off-diagonal components (after going to the super-CKM basis) suppressed by an order of magnitude or more could arise from the approximate flavor diagonality of the Kähler metric mentioned above and/or due to family symmetries which could be present in the underlying theory. This would then probably be consistent with all flavor-changing neutral-current constraints. This paper focuses on CP violation arising in the flavor-diagonal sector, which is present in general even if flavor-changing neutral-current problems, arising from off-diagonal terms in the squark mass matrices in the super-CKM basis, are solved; hence it is largely decoupled from flavor physics. A more detailed discussion of flavor issues will appear elsewhere.

The plan of the paper is as follows. In Sec. II, we review the basic mechanism of supersymmetry breaking and its implications for soft CP-violating phases at leading order. In Sec. III, we discuss the connection between Yukawa textures and imaginary parts of the diagonal trilinear matrix components. After a brief discussion of EDMs using the low-energy effective Lagrangian and the present experimental limits in Sec. IV, we compute the detailed predictions for EDMs within this framework in Sec. V. In Sec. VI, we discuss the effect of possible higher-order corrections to the superpotential and Kähler potential in this class of compactifications and argue that the results obtained are robust against these corrections. We conclude in Sec. VII. The appendixes deal with some technical details of the computations.

II. SMALL CP-VIOLATING PHASES FROM SUSY BREAKING

In fluxless compactifications of M theory [13], the moduli superpotential is entirely generated nonperturbatively and, hence, exponentially suppressed relative to the Planck scale. This is crucial for both stabilizing the moduli z_i and generating the hierarchy. The strong gauge dynamics resides in a three-dimensional submanifold of the internal manifold which generically does not intersect the three-dimensional submanifold where the supersymmetric standard model particles live as these three-dimensional manifolds are embedded inside a seven-dimensional internal manifold. For simplicity, we consider two non-Abelian asymptotically free gauge groups with at least one of them assumed to contain light charged matter fields $\mathcal{Q}$ and $\tilde{\mathcal{Q}}$ (with $N_f < N_c$), and an associated meson $\phi = (\tilde{\mathcal{Q}}\mathcal{Q}^T)^{1/2}$ in the low energy. The strong gauge dynamics in the hidden sector stabilizes the moduli of the G_2 manifold, and dynamically generates a supersymmetry breaking scale with $\mathcal{O}(10)$ TeV gravitino mass. The supersymmetry breaking is then mediated to the visible sector through gravitational (m_p suppressed) interactions.

A. Superpotential and Kähler potential

To be self-contained, in the following we briefly discuss the effective action of the fluxless compactifications of M theory studied in detail in [13]. We will emphasize some important features that are crucial for our results. Further details can be found in the above references.

First, the superpotential can be separated into two parts:

$$W = \hat{W} + Y'_{\alpha\beta\gamma}C^\alpha C^\beta C^\gamma, \tag{1}$$

where $\hat{W}$ depends only on the moduli $z_i = s_i + it_i$ and the meson ϕ. Here C_α are the matter fields in the minimal supersymmetric standard model with α being Higgs, quark, or lepton chiral superfields. $Y'_{\alpha\beta\gamma}$ denote the superpotential Yukawa couplings. The effective Yukawa couplings (still not fully normalized) in the minimal supersymmetric standard model are given by $Y_{\alpha\beta\gamma} = e^{K/2}Y'_{\alpha\beta\gamma}$. The connection to the usual convention in the minimal supersymmetric standard model can be made by taking the first index to be the Higgs fields, the second to be the quark doublets, and the third to be the quark singlets, for example, $Y_{H_uQ_iu_j} \equiv Y^u_{ij}$. In the M theory framework, an elegant way to generate Yukawa couplings $Y'_{\alpha\beta\gamma}$ is from membrane instantons [19,20], which also depend holomorphically on the moduli z_i in general as they measure the volume of the manifold which the instanton wraps.

It appears natural to generate a hierarchical Yukawa texture from such effects.

The first term $\hat{W}$ in (1) is the moduli superpotential, and is generated nonperturbatively from gaugino condensation in two hidden sectors, one of which is assumed to also contain matter fields (with $N_f < N_c$) [21]:

$$\hat{W} = A_1(\det(\phi^2))^{a/2}e^{-b_1 f} + A_2 e^{-b_2 f}. \tag{2}$$

Here $b_{1,2}$ are the beta function coefficients of the two hidden sector gauge groups and f are the corresponding gauge kinetic functions given by $f = \sum_{i=1}^{N} N_i z_i$. $\phi = (\tilde{Q} Q^T)^{1/2}$ denotes the meson fields, and we have suppressed the flavor indices for simplicity. The parameter a in the superpotential is a rational number, $a = -2/(N_c - N_f)$.

The Kähler potential can be written as

$$K = \hat{K} + \tilde{K}_{\alpha\beta}C^{\alpha\dagger}C^\beta + (Z_{\alpha\beta}C^\alpha C^\beta + \text{H.c.}). \tag{3}$$

Here $\hat{K}$ is the moduli Kähler potential and $\tilde{K}_{\alpha\beta}$ is the Kähler metric of matter fields C^α. $Z_{\alpha\beta}$ is expected to be nonzero only for Higgs field $H_{u,d}$, which is needed to generate μ- and B-terms. In these compactifications, charged chiral matter fields with different flavors are localized at isolated conical singularities [22]. These charged matter fields, in addition to being charged under the relevant non-Abelian gauge group, are also charged under $U(1)$ factors which arise from the Kaluza-Klein reduction of the three-form in 11-dimensional supergravity on two-cycles present in the internal manifold [23]. These $U(1)$'s survive at low energies as good symmetries to all orders in perturbation theory and hence must be respected (up to exponentially suppressed nonperturbative effects). Importantly, it turns out that conical singularities associated to different flavors cannot carry the same charges under the $U(1)$'s in a given basis [18] (at least in local models). This forbids the existence of off-diagonal terms in the Kähler potential of the form $C^{\alpha\dagger}C^\beta$, $\alpha \neq \beta$. Off-diagonal components may be introduced if these symmetries are spontaneously broken, but these will be suppressed as long as this occurs sufficiently (an order of magnitude or more) below the Planck scale.

Thus, the Kähler metric is expected to be approximately flavor diagonal, i.e., $\tilde{K}_{\alpha\beta} \approx \tilde{K}_\alpha \delta_{\alpha\beta}$ at the high scale. As argued in [14], the Kähler potential for localized matter fields C_α in the 11-dimensional frame is canonical, i.e. $C_\alpha^\dagger C_\alpha$ due to the absence of local moduli. Going to the Einstein frame implies that there is an overall dependence on the internal volume V_X, but this still preserves the approximate diagonality. A flavor-diagonal Kähler metric will lead to flavor-diagonal soft scalar mass parameters. Note that the above features hold at the high scale ($\sim$ GUT scale). Renormalization group (RG) effects will in general also lead to small flavor off-diagonal contributions to scalar mass parameters at the electroweak scale.

The Kähler potential for moduli fields contains two pieces:

$$\hat{K} = -3\ln(V_X) + \frac{2}{V_X}\text{Tr}(\phi^\dagger\phi). \tag{4}$$

Here V_X is the volume of the G_2 manifold in units of the 11-dimensional length scale l_{11}. The second term originates from the Kähler potential for vectorlike matter fields Q and $\tilde{Q}$ in the hidden sector, which generally takes the form [14]

$$\hat{K} = \frac{1}{V_X}(Q^\dagger Q + \tilde{Q}^\dagger\tilde{Q}). \tag{5}$$

By using the D-term equations $Q^\dagger Q = \tilde{Q}^T\tilde{Q}^*$ and the definition of the meson field ϕ, it can be rewritten in terms of ϕ as given in the second term in Eq. (4). Of course, there could be additional (higher-order) corrections; these will be discussed in Sec. VI. Now for the simple case $N_f = 1$, we can replace $\det(\phi^2)$ by ϕ^2 and $\text{Tr}(\phi^\dagger\phi)$ by $\bar{\phi}\phi$ in Eqs. (2) and (4), respectively. Furthermore ϕ can be written as $\phi = \phi_0 e^{i\theta}$, with θ as the phase of ϕ.

It has been shown in [13,14] that using the superpotential (2) (with $N_f = 1$) and (4), it is possible to stabilize all moduli, the meson field, and one combination of axions. However, at this level the remaining axions are unfixed and remain massless. At the sufficiently long-lived metastable de Sitter minimum of the potential, supersymmetry is spontaneously broken by the strong gauge dynamics, and soft supersymmetry breaking terms in the visible sector of the following usual form are generated:

$$\mathcal{L}_{\text{soft}} = \tfrac{1}{2}(M_a\lambda\lambda + \text{H.c.}) - m^2_{\bar{\alpha}\beta}\hat{C}^{\bar{\alpha}\dagger}\hat{C}^\beta - \tfrac{1}{6}\hat{A}_{\alpha\beta\gamma}\hat{C}^\alpha\hat{C}^\beta\hat{C}^\gamma + \tfrac{1}{2}(B_{\alpha\beta}\hat{C}^\alpha\hat{C}^\beta + \text{H.c.}), \tag{6}$$

where $\hat{C}^\alpha$'s are the canonically normalized chiral matter fields. The trilinear $\hat{A}_{\alpha\beta\gamma}$ can often be factorized as $A_{\alpha\beta\gamma}Y_{\alpha\beta\gamma}$. In the following, we will be careful in distinguishing between trilinears $\hat{A}$ and A.

B. *CP*-violating phases

Now we turn to analyzing the CP-violating phases in the soft Lagrangian. In order to study the dependence of the soft parameters on complex phases, it is crucial to understand the structure of the superpotential in the relevant supersymmetry breaking vacuum. In the superpotential W in (2), A_1, A_2, z_i, and ϕ are complex variables in general. Without loss of generality, it is possible to choose A_1 and A_2 to be real and positive. Then the superpotential (2) for $N_f = 1$ can be written as

$$W = e^{i\chi_1}(|W_1| + |W_2|e^{-i(\chi_1-\chi_2)}), \tag{7}$$

$$\chi_\alpha \equiv b_\alpha \sum_{i=1}^{N} N^i t_i + \delta_{\alpha 1}a\theta, \qquad \alpha = 1, 2, \tag{8}$$

where $|W_1|$ and $|W_2|$ are the magnitudes of the two terms in (2).

As explicitly shown in [13] and summarized in Appendix A, the relative phase between the first and second terms in W is fixed by the minimization of axions in the vacuum such that

$$\cos(\chi_1 - \chi_2) = -1. \tag{9}$$

This implies that both terms in the superpotential dynamically align with the same phase (up to a negative sign), leaving just one overall phase in the superpotential, $e^{i\chi_1}$. Since it is possible to do a global phase transformation of the superpotential without affecting physical observables, this overall phase χ_1 is not physical and can be rotated away, making the superpotential real. From now on, we will take $\chi_1 = 0$. The Kähler potential, K, as seen from (4), only depends on real fields s_i which determine V_X and the combination $\bar{\phi}\phi$, and so does not contain any explicit phases.

Note that at this level all but one of the axions remain massless since the supergravity scalar potential only depends on one linear combination of axions but depends on all moduli (through the Kähler derivative). So one may worry that other possible terms in the superpotential which eventually stabilize the remaining axions will generically stabilize these axions in such a way that the superpotential is not real in the vacuum, thereby ruining the dynamical alignment of phases. However, it turns out that there exists a class of compactifications in which the additional terms in the superpotential stabilizing the remaining axions are exponentially suppressed relative to the first two terms [24]. In this class of compactifications, all results of the moduli stabilization mechanism in [13,14] are kept intact because the higher-order terms do not perturb the moduli (and one axion) from their expectation values determined by the first two terms. The remaining axions are stabilized in such a way that the superpotential is real in the vacuum up to exponentially suppressed effects. This is explained in Appendix A to which the reader is referred. As shown in [24] this mechanism also solves the strong CP problem in an elegant manner, making this mechanism very attractive. Furthermore, it can be shown that the dynamical alignment of phases also works in certain classes of compactifications in Type IIB string theory considered in [25] which have very similar moduli and axion fixing mechanisms.

We now show that with dynamical alignment of phases in the superpotential, there are no CP-violating phases in the soft terms at the high ($\sim$ GUT) scale. The structure of the F-terms $F^I = \hat{K}^{I\bar{J}} F_{\bar{J}} \equiv \hat{K}^{I\bar{J}}(\partial_{\bar{J}}\bar{W} + (\partial_{\bar{J}}K)\bar{W})$, where I, J run over both z_i and ϕ in general, can be computed as follows. For J corresponding to M theory geometric moduli z_i, it is easy to see that $\partial_{\bar{J}}K$ is real and $\partial_{\bar{J}}\bar{W}$ is real (by rotating away the unphysical γ_W). For J corresponding to meson moduli ϕ, $(\partial_{\bar{J}}K)\bar{W} = \text{real} \times e^{i\gamma_\phi}$, where γ_ϕ is the phase of ϕ. Also, since W depends holomorphically on $\{z_i, \phi\}$ as in the first line in (2), one finds $\partial_{\bar{J}}\bar{W} = A_1 a(\bar{\phi})^{a-1} e^{-b_1 \bar{f}} = a\bar{W}_1/\bar{\phi}$, where $\bar{W}_1$ is the first term in the complex conjugate of the superpotential (2). Since both terms in the superpotential have the same phase, again $\partial_{\bar{J}}\bar{W} = \text{real} \times e^{i\gamma_\phi}$. Therefore, we have $F_{\bar{J}} = \text{real}$ for J corresponding to the moduli and $F_{\bar{J}} = \text{real} \times e^{i\gamma_\phi}$ for J corresponding to meson moduli ϕ.

Now $K^{I\bar{J}}$ is real for I and J both corresponding to either moduli or meson fields, while for one of them corresponding to moduli and the other corresponding to the meson field, one has $K^{i\bar{\phi}} = \text{real} \times e^{-i\gamma_\phi}$; $K^{\phi\bar{j}} = \text{real} \times e^{i\gamma_\phi}$. This can also be verified from the explicit calculation in [14]. Thus, we see that $F^I = \text{real}$ or $F^I = \text{real} \times e^{i\gamma_\phi}$ for I corresponding to z_i or ϕ, respectively. This leads to interesting implications for the soft supersymmetry breaking parameters.

First, the tree-level gaugino masses are given by

$$M_a^{\text{tree}}(\mu) = \frac{g_a^2(\mu)}{8\pi}\left(\sum_I e^{\hat{K}/2} F^I \partial_I f_a^{\text{vis}}\right). \tag{10}$$

Since f_a^{vis} only depends on the geometric moduli z_i with integer coefficient, and as we have found, the auxiliary component F^I of z_i is real, there are no phases generated for the tree-level gaugino masses. In the M theory framework, the tree-level gaugino masses are suppressed relative to the gravitino mass [12,14], and the one-loop anomaly mediated contribution has to be included, which is given by [26]

$$M_a^{AMSB} = -\frac{g_a^2}{16\pi^2}\bigg(b_a e^{\hat{K}/2}\bar{W} - b'_a e^{\hat{K}/2} F^I \hat{K}_I + 2\sum_i C_a^i e^{\hat{K}/2} F^I \partial_I \ln\tilde{K}_i\bigg). \tag{11}$$

This contribution includes terms proportional to either $\bar{W}$ or $F^I \partial_I \hat{K}$ or $F^I \partial_I \tilde{K}_i$. Since the Kähler potential is a real function of z_i, $\partial_{z_i}\hat{K}$ and $\partial_{z_i}\tilde{K}$ are real. In addition, the Kähler potential only depends on $\bar{\phi}\phi$, which implies that the derivative with respective to ϕ is proportional to $\bar{\phi} \sim e^{-i\gamma_\phi}$. Therefore, all these terms are real, which gives rise to real anomaly mediated gaugino masses. Hence, the gaugino masses have no observable phase in the above framework.

The trilinear A-terms (with the Yukawa couplings factored out) are given in general by [27]

$$A_{\alpha\beta\gamma} = e^{\hat{K}/2} F^I \partial_I [\ln(e^{\hat{K}} Y'_{\alpha\beta\gamma}/\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)], \tag{12}$$

where I, J run over both z_i and ϕ. It should be noted that in order to be able to factor out the Yukawa matrices the matter Kähler metric has to be diagonal. This is a good approximation in the M theory framework as we have discussed. Since the moduli Kähler potential $\hat{K}$ and the visible sector Kähler metric $\tilde{K}$ are real functions of $z_i + \bar{z}_i$ and $\bar{\phi}\phi$ and superpotential takes the form in Eq. (1), it is straightforward to check that the contractions $F^I \partial_I \hat{K}$,

$F^I\partial_I\tilde{K}$, and $F^I\partial_I\ln\hat{Y}^l$ are all real, implying that no *CP* phases are generated in the trilinear *A*-terms through supersymmetry breaking. However, there could be phases in the full trilinear couplings $\hat{A}$ coming from the Yukawa couplings, as we shall discuss in the next section.

Finally, we move on to the μ- and *B*-terms. We focus on the case where the superpotential contribution to the overall high scale μ parameter vanishes. This can be easily guaranteed by a symmetry [19]. In this case, μ and *B* parameters of $\mathcal{O}(m_{3/2})$ can be generated by the Giudice-Masiero mechanism [28] via the parameter $Z_{\alpha\beta}$ in Eq. (4). The general result for μ and *B* can be written in terms of *Z*, $F^I\partial_I\hat{K}$, $F^I\partial_I\tilde{K}$, and $F^I\partial_I Z$ [27], all of which have the same phase γ_Z from $Z_{\alpha\beta}$ (complex in general). Therefore, μ and *B* share the same phase e^{γ_Z}. However, this phase is not physical since it can be eliminated by a Peccei-Quinn $U(1)$ rotation [29].

Before ending this section, we would like to summarize our result in a more general fashion. In the previous analysis, we have seen in general $\gamma_B = \gamma_\mu$ and $\gamma_{M_a} = \gamma_{A_f} = 0$. Here γ_{M_a} and γ_{A_f} are defined as the overall phases of M_a and A_f, respectively. Note γ_{A_f} is not a basis-independent definition. From the observational point of view, the relevant physical phases must be reparametrization invariant and basis-independent, which can be built from the following combinations [29]:

$$\gamma_{1\lambda} = \gamma_\mu - \gamma_B + \gamma_{M_a}, \qquad \gamma_{2f} = \gamma_\mu - \gamma_B + \gamma'_{A_f}, \quad (13)$$

where $\gamma'_{A_f} = \frac{1}{3}\mathrm{Arg}[\mathrm{Det}(\hat{A}_f Y^\dagger)]$ is the flavor- and basis-independent *CP* phase of the trilinears. Note that $\gamma'_{A_f} \neq \gamma_{A_f}$ in general unless that the trilinears are universal. So we can easily see $\gamma_{1\lambda} = 0$ and $\gamma_{2f} = \gamma'_{A_f}$. This implies that the only flavor-independent phases that can appear in a physical observable must be those from the trilinears. In addition, there can be flavor-dependent *CP* phases coming from the relative phases of the trilinear matrix elements. In the following section, we will discuss the combination of these phases that are relevant for the calculation of EDMs.

III. *CP*-VIOLATING PHASES FROM YUKAWAS

Although the *CP*-violating phases from supersymmetry breaking are absent or small as found above, there is an additional contribution to *CP* violation if the trilinear $\hat{A}$ parameters are not aligned with the Yukawas. This can be easily seen as follows. Since the Yukawa matrix generically contains $\mathcal{O}(1)$ phases in order to explain the observed CKM phase, the unitary matrices needed to go to the super-CKM basis (in which the Yukawa matrices are real and diagonal) also contain some phases. Therefore, the rotation by itself can induce *CP*-violating phases even if the *A* or $\hat{A}$ matrices are initially completely real as long as $\hat{A}$'s are not proportional to Yukawas in the flavor eigenstate basis (or equivalently *A*'s are flavor nonuniversal and nondiagonal). This implies, in particular, that the diagonal components of trilinear $\hat{A}$ will contain *CP* phases in the super-CKM basis, giving rise to possibly important contributions for EDMs.

In the M theory framework, the Yukawa couplings $Y'_{\alpha\beta\gamma}$ depend holomorphically on the geometric moduli z_i in general which get nonzero *F*-term vacuum expectation values. Hence, from (12) we find that the second term in the expression for trilinears gives rise to an $\mathcal{O}(1)$ misalignment between the Yukawas and the trilinears. If the Yukawa couplings depend on moduli or other hidden sector fields which do not break supersymmetry, then the trilinears can be naturally aligned with the Yukawas [16]. However, within M theory, this does not seem to be a generic situation; hence we will consider the conservative case in which the trilinears are misaligned with the Yukawas.

In the remainder of this section, we will estimate the diagonal *CP* phases in the trilinear $\hat{A}$ in the super-CKM basis since they are directly related to the EDM observables. We consider flavor nonuniversal and nondiagonal trilinear *A* matrices (in the flavor eigenstate basis) at the grand unified theory (GUT) scale with real $\mathcal{O}(1)$ matrix elements. To set the conventions, we write down the soft trilinear terms explicitly

$$\begin{aligned}\mathcal{L}_{\mathrm{soft}} \sim{}& A^u_{ij}Y^u_{ij}\bar{Q}_{Li}H_u u_{Rj} + A^d_{ij}Y^d_{ij}\bar{Q}_{Li}H_d d_{Rj}\\ &+ A^e_{ij}Y^e_{ij}\bar{L}_{Li}H_d e_{Rj},\end{aligned} \quad (14)$$

where $A^{u,d,e}$ are the trilinear matrices in the gauge eigenstate basis of matter fields.

In the M theory framework, chiral matter fields are localized on singular points inside the compact G_2 manifold [19,30–32]. Although a detailed understanding of Yukawas within M theory is not yet available, a hierarchical Yukawa texture seems well motivated. From a phenomenological point of view, therefore, we consider the following Yukawa texture,

$$Y^u_{ij} \sim \epsilon^q_i\epsilon^u_j, \qquad Y^d_{ij} \sim \epsilon^q_i\epsilon^d_j, \qquad Y^e_{ij} \sim \epsilon^l_i\epsilon^e_j, \qquad (15)$$

which can arise naturally from the localization of matter fields in extra dimensional models [33–36] or from a spontaneously broken flavor symmetry (Froggatt-Nielson mechanism) [37]. Then, the fermion mass hierarchy is given by

$$\begin{aligned}m^u_i/m^u_j \sim |\epsilon^q_i\epsilon^u_i|/|\epsilon^q_j\epsilon^u_j|, \quad m^d_i/m^d_j \sim |\epsilon^q_i\epsilon^d_i|/|\epsilon^q_j\epsilon^d_j|,\\ m^e_i/m^e_j \sim |\epsilon^l_i\epsilon^e_i|/|\epsilon^l_j\epsilon^e_j|.\end{aligned} \quad (16)$$

It is straightforward to check that the observed fermion mass hierarchy can be accommodated by a set of properly chosen ϵ_i with the hierarchy $|\epsilon_1| \lesssim |\epsilon_2| \lesssim |\epsilon_3|$. The above Yukawa couplings can have $\mathcal{O}(1)$ phases in order to explain the *CP* phase in the CKM matrix. To simplify the discussion, we eliminate the phases in the diagonal elements by a redefinition of the quark and lepton fields. Therefore, the diagonal elements $(\hat{A}^\psi)_{11,22,33}$ with $\psi = u, d, e$ are all real at the GUT scale.

First, we point out that the RG corrections to the trilinear couplings typically mix the phases between different flavors. This will lead to phases in the diagonal elements of the trilinear matrices. It can be understood from the RG equation for Yukawa couplings and trilinear couplings, e.g., for Y^u and $\hat{A}^u$, which are given by

$$\begin{aligned} \frac{dY^u}{dt} &\sim \frac{1}{16\pi^2} Y^u[3\,\mathrm{Tr}(Y^u Y^{u\dagger}) + 3Y^{u\dagger}Y^u + Y^{d\dagger}Y^d], \\ \frac{d\hat{A}^u}{dt} &\sim \frac{1}{16\pi^2}\hat{A}^u[3\,\mathrm{Tr}(Y^u Y^{u\dagger}) + 5Y^{u\dagger}Y^u + Y^{d\dagger}Y^d] \\ &\quad + \frac{Y^u}{16\pi^2}[6\,\mathrm{Tr}(\hat{A}^u Y^{u\dagger}) + 4Y^{u\dagger}\hat{A}^u + 2Y^{d\dagger}\hat{A}^d], \end{aligned} \tag{17}$$

where only terms involving Yukawas are explicitly shown. From the above equations, we notice that the phases in $\hat{A}^u$ evolve during the RG running. To illustrate this, one can examine the following term which contributes to the running of $\hat{A}^u_{11}$:

$$\frac{d\hat{A}^u_{11}}{dt} \sim \frac{5}{16\pi^2}\hat{A}^u_{13}Y^{u\dagger}_{33}Y^u_{31} + \frac{4}{16\pi^2}Y^u_{13}Y^{u\dagger}_{33}\hat{A}^u_{31}. \tag{18}$$

From the equation, one can see that the phases of Y^u_{31}, Y^u_{13}, $\hat{A}^u_{13}$, and $\hat{A}^u_{31}$ can enter that of $\hat{A}^u_{11}$ through RG effects, although it was real at the high scale. The magnitude of this correction can be significant since the magnitude of the right-hand side of the above equation is proportional to $\frac{1}{16\pi^2}|Y^u_{33}|^2$ given the factorizable Yukawa matrices as in Eq. (15). This indicates that the RG corrections to $\hat{A}^u_{11}$ can give $\mathcal{O}(1)$ phases. This is also true for other elements in the trilinear matrices and Yukawa matrices. The only exception is for the third generation Y^u_{33} and $\hat{A}^u_{33}$, for which the largest RG corrections come from the terms involving only Y^u_{33} and $\hat{A}^u_{33}$ with additional flavor mixing terms typically suppressed by $\epsilon_2^2/\epsilon_3^2$. Since $\hat{A}^u_{33}$ has the same phase as Y^u_{33}, the corresponding A-term $A^u_{33} = \hat{A}^u_{33}/Y^u_{33}$ remains real up to corrections of the order $(\epsilon_2/\epsilon_3)^2$.

Starting from the Yukawa matrices $Y^{u,d,e}_{ij}$ defined in the flavor eigenstate basis, the super-CKM basis can be achieved by unitary rotations of the matter fields so that the Yukawa matrices are real and diagonal. In the super-CKM basis, the trilinear couplings become

$$(\hat{A}^\psi_{\mathrm{SCKM}})_{ij} = (V^{\psi\dagger}_L)_{il}A^\psi_{lk}Y^\psi_{lk}(V^\psi_R)_{kj}, \tag{19}$$

where $\psi = u, d, e$. To be concrete, we focus on the up-type trilinear in the discussion below. Given the hierarchical Yukawa matrices in Eq. (15), the unitary transformation matrices are given by

$$\begin{aligned} (V^u_L)_{ij} &\sim (V^u_L)_{ji} \sim \epsilon^q_i/\epsilon^q_j, \quad \text{for } i<j, \\ (V^u_R)_{ij} &\sim (V^u_R)_{ji} \sim \epsilon^u_i/\epsilon^u_j, \quad \text{for } i<j. \end{aligned} \tag{20}$$

One can now perform the same transformation for the trilinear terms to get the diagonal elements in the super-CKM basis, which can be schematically written as

$$(\hat{A}^u_{\mathrm{SCKM}})_{11} \approx \epsilon^q_1\epsilon^u_1 \sum_{i,j=1,2,3}\xi_{ij}A^u_{ij}, \tag{21}$$

$$(\hat{A}^u_{\mathrm{SCKM}})_{22} \approx \epsilon^q_2\epsilon^u_2 \sum_{i,j=2,3}\eta_{ij}A^u_{ij}, \tag{22}$$

$$(\hat{A}^u_{\mathrm{SCKM}})_{33} \approx Y^u_{33}A^u_{33} \sim \epsilon^q_3\epsilon^u_3 A^u_{33}, \tag{23}$$

where ξ_{ij} and η_{ij} are possibly $\mathcal{O}(1)$ coefficients arising from those implicit coefficients in the Yukawa matrices in (15), and are complex in general. In the above equations, we have neglected subleading terms suppressed by ϵ_i/ϵ_j for $i<j$. Since the off-diagonal components in A_{ij} can be $\mathcal{O}(1)$ within our framework, the summations in Eqs. (21) and (22) can be of $\mathcal{O}(1)$ in magnitude with $\mathcal{O}(1)$ phases. The $(\hat{A})_{33}$ component, however, does not mix with other components and is proportional to Y^u_{33}, so no phase is generated at leading order for $\hat{A}^u_{33}$.

Therefore, we conclude that the first two diagonal components of the complete trilinear coupling in the super-CKM basis can contain order one phases, while the third diagonal component is real up to small corrections, i.e.,

$$\begin{gathered} \mathrm{Im}(\hat{A}^\psi_{\mathrm{SCKM}})_{11} \sim A_0 Y^\psi_{11}, \qquad \mathrm{Im}(\hat{A}^\psi_{\mathrm{SCKM}})_{22} \sim A_0 Y^\psi_{22}, \\ \mathrm{Im}(\hat{A}^\psi_{\mathrm{SCKM}})_{33} \sim A_0\left(\frac{\epsilon_2}{\epsilon_3}\right)^2 Y^\psi_{33}, \end{gathered} \tag{24}$$

where A_0 is the characteristic magnitude of the trilinear A-terms. Here $(\epsilon_2/\epsilon_3)^2$ is roughly the ratio of the second and third generation fermion masses, which depends on the choice of $\psi = u, d, e$. In the discussion of EDMs in Sec. V B, we will be taking $\epsilon_2/\epsilon_3 \sim 0.1$ as an order of magnitude estimate compatible with the quark mass hierarchy.

Although the result in Eq. (24) is derived for the specific class of Yukawa matrices in Eq. (15), it is nevertheless more generic than that. In fact, for any Yukawa matrices such that the linear combination of terms giving the diagonal component of Y_{SCKM} do not have any large cancellation, the first two relations in Eq. (24) would still be valid. An additional requirement that the flavor mixing contribution to $(Y_{\mathrm{SCKM}})_{33}$ is suppressed would lead to a similar relation as the last one in Eq. (24). These conditions can be easily accommodated for more general Yukawa textures. Other Yukawa textures that do not satisfy these conditions are possible, but seem less generic and natural in order to get the hierarchical fermion masses and mixings. This result has important implications for EDM predictions, which we shall discuss in Sec. V.

IV. ELECTRIC DIPOLE MOMENTS AND THE EXPERIMENTAL LIMITS

Before starting our calculation of EDMs, we briefly summarize some general results relevant for the calculation of EDMs. In the minimal supersymmetric standard models, the important CP-odd terms in the Lagrangian are

$$\begin{aligned}\delta\mathcal{L} = &-\sum_{q=u,d,s} m_q\bar{q}(1+i\theta_q\gamma_5)q + \theta_G\frac{\alpha_s}{8\pi}G\tilde{G}\\ &-\frac{i}{2}\sum_{f=u,d,s}(d_q^E\bar{q}F_{\mu\nu}\sigma_{\mu\nu}\gamma_5 q + \tilde{d}_q^C\bar{q}g_s t^a G^a_{\mu\nu}\sigma_{\mu\nu}\gamma_5 q)\\ &-\frac{1}{6}d_q^G f_{\alpha\beta\gamma}G_{\alpha\mu\rho}G^{\rho}_{\beta\nu}G_{\gamma\lambda\sigma}\epsilon^{\mu\nu\lambda\sigma}, \qquad (25)\end{aligned}$$

where θ_G is the QCD θ angle. The terms in the second line in (25) are dimension five operators, which are generated by CP violation in the supersymmetry breaking sector and evolved down to $\sim$1 GeV. The coefficients $d_q^{E,C}$ correspond to quark electric dipole moment and chromoelectric dipole moment (CEDM), respectively. The last line in (25) contains the gluonic dimension six Weinberg operator. The CP-odd four-fermion interactions are not important here, and so have not been included above.

Now let us briefly summarize the EDM results for electrons, neutrons, and mercury in terms of the coefficients of these operators. The electron EDM in minimal supersymmetric models is given by

$$d_e^E = d_e^{\chi^+} + d_e^{\chi^0} + d_e^{\mathrm{BZ}},$$

where $d_e^{\chi^\pm}$ and $d_e^{\chi^0}$ are one-loop contributions from the neutralino and chargino while d_e^{BZ} is the two-loop Barr-Zee type contribution [38–46]. It should be noted that what is actually measured is the atomic EDM d_{Tl}, which receives contributions mainly from the electron EDM and the CP-odd electron-nucleon couplings [47]:

$$d_{Tl} = -585\times d_e^E - 8.5\times 10^{-19}e\ \mathrm{cm}(C_S\ \mathrm{TeV}^2) + \cdots,$$

where C_S is the coefficient of the operator $\bar{e}i\gamma_5 e\bar{N}N$. The C_S coefficient could be generated from a new scalar particle coupled to quarks and leptons through a CP-odd Higgs-like coupling [47]. However, this is independent of CP-odd interactions originating from the soft terms. Given the current experimental limit $|d_{Tl}| < 9\times 10^{-25}e$ cm, we obtain an upper limit on electron EDM

$$|d_e^E| < 2\times 10^{-27}e\ \mathrm{cm}.$$

For the neutron, there exist several different approaches to compute the corresponding EDM. In the following discussion, we shall follow a simple approach, i.e., the naive dimensional analysis (NDA) [48–50]. The neutron EDM can be calculated as

$$d_n = \tfrac{4}{3}d_d - \tfrac{1}{3}d_u. \qquad (26)$$

In this expression, the quark EDMs can be estimated via NDA as

$$d_q = \eta^E d_q^E + \eta^C\frac{e}{4\pi}d_q^C + \eta^G\frac{e\Lambda}{4\pi}d^G$$

with $d_q^{E,C} = d_q^{\tilde{g}(E,C)} + d_q^{\tilde{\chi}^+(E,C)} + d_q^{\tilde{\chi}^0(E,C)}$. The QCD correction factors are given by $\eta^E = 1.53$, $\eta^C \sim \eta^G \sim 3.4$ [9], and $\Lambda \sim 1.19$ GeV is the chiral symmetry breaking scale. The current experimental limit on neutron EDM is given by

$$|d_n| < 3\times 10^{-26}e\ \mathrm{cm}.$$

The current theoretical estimate for the mercury EDM induced by dimension 5 operators is given by [51]

$$d_{H_g} = -7.0\times 10^{-3}e(d_d^C - d_u^C - 0.012d_s^C) + 10^{-2}\times d_e,$$

where we have included the contribution from the strange quark CEDM [52]. The recent experimental result on mercury EDM [5] significantly tightens the bound

$$|d_{Hg}| < 3.1\times 10^{-29}e\ \mathrm{cm}.$$

In the standard model, the primary source of hadronic EDMs comes from the QCD θ-term in (25). This gives the following results [47,53–55]:

$$d_n \sim 3\times 10^{-16}\theta e\ \mathrm{cm}, \qquad d_D \sim -1\times 10^{-16}\theta e\ \mathrm{cm},$$
$$|d_{Hg}| \sim \mathcal{O}(10^{-18}\text{–}10^{-19})\theta e\ \mathrm{cm}. \qquad (27)$$

On the other hand, the electron EDM is induced by the SM electroweak interactions, and is typically of order $10^{-38}e$ cm [56,57]. The results in (26) together with the suppressed leptonic EDMs provide a correlation pattern for the θ-induced electric dipole moments. The current upper bound on the neutron EDM implies $\theta < \mathcal{O}(10^{-10})$, which leads to the strong CP problem. Once EDMs are observed for n, Hg, and Tl, it will be essential to separate the strong and weak contributions, by combining data on different nuclei and d_e^E.

V. PREDICTIONS FOR EDMS

For an explicit computation of the EDMs, it is important to specify the general structure of supersymmetry breaking parameters, in particular, the structure of the trilinear parameters (especially the imaginary part of the diagonal components), as well as that of the scalar and gaugino masses, since all of these appear in the final expression for the EDMs. This is the subject of this section.

Within the M theory framework, the general structure of supersymmetry breaking parameters is as follows. For the choice of microscopic parameters with a vanishingly small positive cosmological constant, the gravitino mass naturally turns out to be in the range 10–100 TeV [13]. The gravitino mass is essentially $\sim F_\phi/m_p$. However, as mentioned earlier, the F-terms of the moduli are suppressed compared to F_ϕ. Since the gauge kinetic function for the visible sector depends only on the moduli, from (10) it is easy to check that the gaugino masses are suppressed relative to that of the gravitino. However, this suppression does not hold for the scalar masses, trilinears, and μ and $B\mu$ parameters unless the visible sector is sequestered from the supersymmetry breaking sector. Sequestering does not seem to be generic in M theory [14], so scalar masses, trilinears, and μ and $B\mu$ parameters typically turn out to be of $\mathcal{O}(m_{3/2}) \sim \mathcal{O}(10)$ TeV. The third generation squarks, however, could be significantly lighter because of the RG effects.

As we have discussed in Sec. II, within the M theory framework it is natural to expect that the Kähler metric for visible matter fields is approximately diagonal in the flavor indices. Then, the scalar mass matrix turns out to be roughly diagonal with suppressed off-diagonal contributions. The estimates for the EDMs then depend on the overall scale of the squark masses. So for concreteness we consider gauginos with masses $\lesssim 600$ GeV, nonuniversal but flavor-diagonal scalar mass matrices with masses $\sim$20 TeV, and μ, $B\mu$, and trilinear parameters of the same order as scalar masses. Some contributions to EDMs depend primarily on third generation sfermion masses, so we also mention the situation when third generation scalars are much lighter, i.e., $\mathcal{O}(1)$ TeV.

We now estimate the contribution to the EDMs of the electron, neutron, and mercury from dimension 5 and 6 operators [Eq. (25)] in the M theory framework. As we have seen in the Sec. III, the CP-violating phases appear only in the trilinear $\hat{A}$ parameters. After renormalization group evolution and the super-CKM rotation of the trilinear matrices, these phases appear in the off-diagonal elements in the squark mass matrices, leading to imaginary parts of the following mass-insertion parameters:

$$(\delta_q^{ii})_{LR} = \frac{v_q((\hat{A}^q_{\rm SCKM})_{ii} - \mu^* Y^q_{ii} R_q)}{(m^2_{\tilde{q}})_{ii}}, \qquad (28)$$

where $R_{u(d)} = \cot\beta$ ($\tan\beta$) and $v_{u(d)} = v\sin\beta(v\cos\beta)$. As explained above, $\hat{A}_{\rm SCKM}$ is in general a 3×3 matrix in the super-CKM basis and its diagonal components contain CP-violating phases. Thus, these insertion parameters contribute to EDMs through the dimension 5 and 6 operators in (25).

A. Leading contributions

The dimension five electric and chromoelectric couplings can be generated at leading order [6,8,9] at one-loop through the vertices $f\tilde{f}\tilde{\chi}^0_i$, $f\tilde{f}'\tilde{\chi}^\pm_i$, and $q\tilde{q}\,\tilde{g}$ as can be seen in Fig. 1.

First consider the quark CEDM which contributes to both the mercury and neutron EDMs. Since there exists a hierarchy between gauginos and squarks in the M theory framework [12,13], one can expand using the small ratio

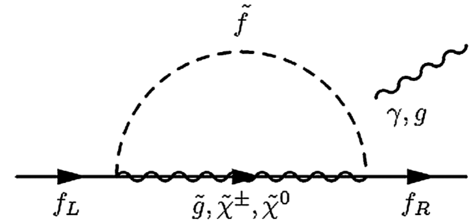

FIG. 1. One-loop contributions to fermion (C)EDMs.

$r \equiv m_i^2/m_{\tilde{q}}^2$, where m_i is the corresponding neutralino, chargino, or gluino masses in the diagram. One then obtains the following result:

$$d_q^C \sim \frac{g_s \alpha_s}{4\pi} \frac{m_q}{m_i^3} \,{\rm Im}(A^q_{\rm SCKM}) r^2 G(r), \qquad (29)$$

where $A^q_{\rm SCKM}$ is the diagonal element of the corresponding trilinear matrix (factoring out the Yukawa coupling) in the super-CKM basis. In the expression, the function $G(r) = C(r) + rC'(r)$ for gluinos and $G(r) = B(r) + rB'(r)$ for charginos and neutralinos. The function $B(r)$ and $C(r)$ are loop functions defined in Appendix B. One can see that d_q^C decreases rapidly as $m_{\tilde{q}}^{-4}$ when the squark masses increase. However, the function $G(r)$ behaves differently for different particles ($\tilde{g}$, $\tilde{\chi}^\pm$, $\tilde{\chi}^0$) in the loop. Due the gaugino and squark mass hierachy, r is small. From Fig. 2, we can see that $C(r) + rC'(r)$ is enhanced in the small r region compared to other functions which remain small. Therefore, the gluino contribution dominates the quark CEDM. For the quark EDM, it is given by a similar expression as (29) but now the quantity $G(r)$ is determined only by $A(r)$ and $B(r)$. In particular, $G(r)$ is determined solely by $B(r)$ for $\tilde{g}$ and $\tilde{\chi}^0$ in the loop, and by a combination of $A(r)$ and $B(r)$ for $\tilde{\chi}^\pm$ in the loop. Since $A(r) + rA'(r)$ and $B(r) + rB'(r)$ are much smaller than $C(r) + rC'(r)$ as seen from Fig. 2, the quark EDM contributions to the neutron EDM are negligible compared to that of the quark CEDM contributions. Therefore, we only need to calculate the quark CEDM, for which the gluino diagram

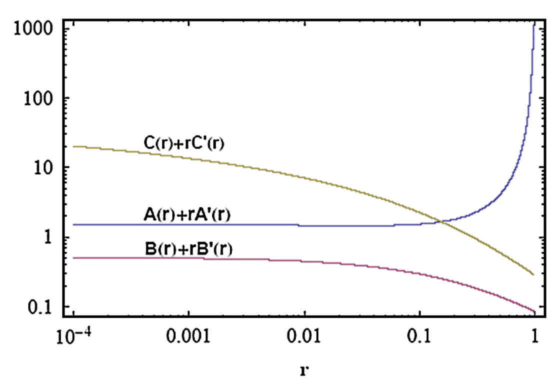

FIG. 2 (color online). Comparison of the one-loop functions $A(r)$, $B(r)$, and $C(r)$. The x coordinate is the ratio $r \equiv m_i^2/m_{\tilde{q}}^2$.

gives the dominant contribution as explained above. Since $A^q_{\rm SCKM} \sim m_{\tilde{q}}$ in the M theory framework, one obtains

$$d_q^C \sim 10^{-28} \times \left(\frac{m_q}{1\ \mathrm{MeV}}\right)\left(\frac{m_{\tilde{g}}}{600\ \mathrm{GeV}}\right)\left(\frac{20\ \mathrm{TeV}}{m_{\tilde{u}}}\right)^3 e\ \mathrm{cm}. \quad (30)$$

Based on the quark EDM and CEDM, the neutron EDM can be computed from (26):

$$d_n^{\rm NDA} \sim 3 \times 10^{-28} \times \left(\frac{m_{\tilde{g}}}{600\ \mathrm{GeV}}\right)\left(\frac{20\ \mathrm{TeV}}{m_{\tilde{u}}}\right)^3 e\ \mathrm{cm}. \quad (31)$$

Similarly, the mercury EDM is

$$|d_{H_g}| \sim 10^{-30} \times \left(\frac{m_{\tilde{g}}}{600\ \mathrm{GeV}}\right)\left(\frac{20\ \mathrm{TeV}}{m_{\tilde{u}}}\right)^3 e\ \mathrm{cm}. \quad (32)$$

Moving on to the electron EDM, it can be computed at leading order from the one-loop neutralino and chargino diagrams. First, we notice that the chargino diagram does not contribute in our framework. This is because of the fact that the μ-term phase[1] and the gaugino phases are zero and there is no phase in the chargino mixing matrix, as can be seen from Eq. (B7) in Appendix B. The neutralino contribution, on the other hand, gives rise to a nonzero contribution because of a dependence on the selectron mixing parameters (which contains *CP*-violating phases) in its couplings [see Eq. (B8)]. Given the fact that the Higgsino coupling to the electron and selectron is suppressed by the small electron Yukawa coupling, and the wino does not couple to right-handed fermions and sfermions, the dominant contribution is from the diagram with $\tilde{\chi}^0_2$ (almost pure bino in the M theory framework), which can be calculated using Eq. (B11) in Appendix B. Thus, the electron EDM is given by

$$d_e^E \sim \left(\frac{m_{\tilde{\chi}^0_2}}{200\ \mathrm{GeV}}\right)\left(\frac{20\ \mathrm{TeV}}{m_{\tilde{e}}}\right)^3 \times 10^{-31} e\ \mathrm{cm}. \quad (33)$$

B. Two-loop contributions

So far, we have considered the one-loop contribution to quark and electron EDMs (and/or CEDMs). In addition, there are two-loop Barr-Zee type contributions [38–46] such as the one in Fig. 3. In general, the Barr-Zee type diagrams can involve squarks, charginos, or neutralinos in the inner loop, and Higgs bosons (neutral or charged) and/or gauge bosons in the outer loop (the two-loop diagram considered in split supersymmetry is not relevant here, since there the *CP* violation is not from trilinear couplings, but instead from the chargino sector). Since only the trilinear couplings contain *CP*-violating phases in our framework, we consider those diagrams with squarks running in the inner loop as seen in Fig. 3. One might wonder whether there are any two-loop diagrams that would contribute if there were phases in the gaugino masses or μ-term such as

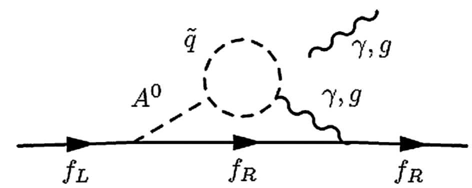

FIG. 3. Two-loop Barr-Zee type diagrams contributing to fermion (C)EDMs.

in the split supersymmetry scenario [45,58]. Since the Higgsino in the M theory framework is very heavy with mass $\mu \sim m_{3/2}$ and hence decoupled from the low-energy theory, the only diagram which might contribute is the one in Fig. 4. However, it turns out that the *CP* phases in the 2 W-chargino-neutralino couplings cancel out (up to a small correction due to the heavy Higgsino) in the final result giving no EDM contribution.

When the mass splitting between the two third generation squarks is not particularly large, the diagram to the quark CEDM can be estimated as (see Appendix C)

$$\begin{aligned} d_f^{\rm CBZ} &\approx \frac{g_s\alpha_s}{64\pi^3}\frac{m_f R_f \mu}{M_A^4}\sum_{q=\tilde{t},\tilde{b}} y_q^2\, \mathrm{Im}(A^q_{\rm SCKM})F'(r_q) \\ &\sim 10^{-32} \times R_f\left(\frac{m_f}{1\ \mathrm{MeV}}\right)\left(\frac{20\ \mathrm{TeV}}{m_{\tilde{q}}}\right)^2 e\ \mathrm{cm}, \end{aligned} \quad (34)$$

where $R_f = \cot\beta\ (\tan\beta)$ for $I_3 = 1/2\ (-1/2)$, and $r_q \equiv m^2_{\tilde{q}}/M^2_A$ with $m_{\tilde{q}}$ third generation squark mass and M_A the pseudoscalar mass of A_0. Here we have used (24) for $\mathrm{Im}(A^q_{\rm SCKM})$. For simplicity, we also take $\mu \sim M_A \sim m_{\tilde{t},\tilde{b}} \sim m_{\tilde{u}}$. It can be seen that the result of the Barr-Zee diagram to quark CEDM (similar for EDM) is negligibly small. One of the reasons is that *CP* violation in the third generation is suppressed by about 2 orders of magnitude as in (24). Similarly, for the electron EDM the result is

$$d_e^{\rm EBZ} \sim 10^{-33} \times \left(\frac{20\ \mathrm{TeV}}{m_{\tilde{u}_3}}\right)^2 \tan\beta e\ \mathrm{cm} \quad (35)$$

which is again quite suppressed. This contribution may be enhanced for large $\tan\beta$ as seen from above. The M theory framework, however, generically predicts $\tan\beta = \mathcal{O}(1)$ [14].

The neutron EDM could also get a contribution from the dimension six pure gluonic operator (Weinberg operator), which can be generated from the two-loop gluino-top-stop and gluino-bottom-sbottom diagrams (see Fig. 5). For the

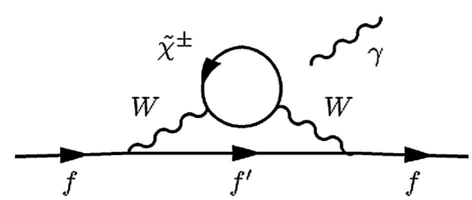

FIG. 4. Two-loop Barr-Zee type diagrams which do not involve a sfermion in the loop.

[1]Here we have set $\gamma_B = 0$ by a $U(1)_{\rm PQ}$ rotation.

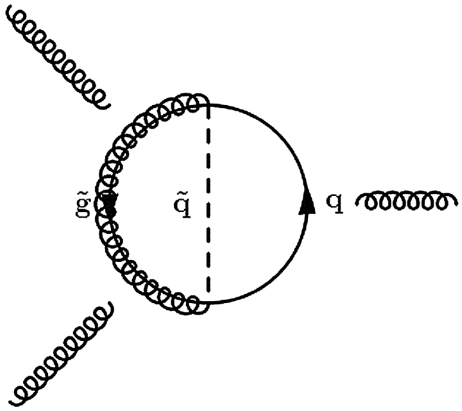

FIG. 5. Two-loop diagrams contributing to the Weinberg operator.

case where CP violation only comes from the soft trilinear couplings, the result can be estimated by [59]

$$d^G \approx -3\alpha_s\left(\frac{g_s}{4\pi}\right)^3 \frac{1}{m_{\tilde{g}}^3} \sum_{q=t,b} \mathrm{Im}(A_q^{\mathrm{SCKM}}) z_q H(z_1, z_2, z_q), \quad (36)$$

where $z_i = m_{\tilde{q}_i}^2/m_{\tilde{g}}^2$ for $i = 1, 2$, and $z_q = m_q^2/m_{\tilde{g}}^2$ for $q = t, b$. The two-loop function $H(z_1, z_2, z_t)$ is given in [59]. This gives a contribution to the neutron EDM $d_n^G \sim 10^{-30} e$ cm for $m_{\tilde{t},\tilde{b}} \approx 20$ TeV, $m_{\tilde{g}} = 600$ GeV, and $A_q = 20$ TeV. Thus the neutron EDM from the Weinberg operator is smaller than the one-loop CEDM contribution. However, when the masses of the third generation squarks and trilinears are around 1 TeV, the contribution to the neutron EDM can be significantly larger, and be comparable to the one-loop result.

To summarize our results, we have calculated the EDMs arising from the CP-violating phases in the trilinear terms in a general framework with light gauginos and heavy sfermions, and the results are within current experimental bounds. We find that the one-loop diagram is typically the dominant contribution to EDMs. However, in contrast to the situation in which gaugino and sfermion masses are comparable, the one-loop diagram with gluino is enhanced over the one with neutralino. This leads to a larger ratio between neutron EDM and electron EDM of $\gtrsim 10^3$. In typical supersymmetric models with gaugino and sfermion masses of the same order, this ratio is $\lesssim 10^2$ [60]. This seems to be a robust feature of models in which the gauginos are suppressed relative to the squarks and the trilinears and the trilinears are not proportional to Yukawas. Finally, it is easy to see that the mercury EDM provides the most stringent limit on the squark masses. For squark masses around 10 TeV, the mercury EDM will increase to $\sim 10^{-29} e$ cm, which could be tested in the near future with better experimental precision. Basically, we have found that the upper bounds on the EDMs in the M theory framework result from two of its generic features—large scalar masses and trilinears [$\mathcal{O}(10)$ TeV], and CP-violating phases only in the trilinear couplings and those arising only from the Yukawas.

VI. HIGHER-ORDER CORRECTIONS

In Sec. II, we found that there are no CP-violating phases from supersymmetry breaking at leading order in the framework of M theory compactifications considered. It is therefore important to check if corrections to the Kähler potential and superpotential lead to further contributions to CP-violating phases in the soft parameters and in turn to the EDMs. Although the detailed form of possible corrections is not known in M theory, some general arguments can nevertheless be made, which strongly suggest that higher-order corrections still naturally suppress CP-violating phases.

The corrections to the soft parameters may arise in general from corrections in the superpotential and the Kähler potential. In the zero flux sector which we have considered, the superpotential may only receive additional nonperturbative corrections from strong gauge dynamics or from membrane instantons. As mentioned in Sec. II B and explained in Appendix A, the dynamical alignment of phases still works if these additional terms are subdominant compared to the first two terms. The subdominance of these additonal terms is required anyway to keep the results of the moduli stabilization mechanism intact and hence the consistency of the whole approach. Moreover, it also provides an elegant dynamical solution to the strong CP problem within string theory [24].

The Kähler potential for the hidden sector comprising the moduli and hidden matter fields on the other hand receives perturbative corrections such as terms with higher powers of ϕ as there is no nonrenormalization theorem for the Kähler potential. However, the field ϕ is composed of elementary quark fields Q, $\tilde{Q}$ which are charged under the hidden gauge group. Therefore, higher-order corrections must be functions of $Q^\dagger Q$ or $\tilde{Q}^\dagger \tilde{Q}$ in order to be gauge invariant. When written in terms of ϕ, these corrections are always functions of $\phi^\dagger \phi$. This structure is important for our claim of small CP-violating phases since it does not introduce any new phases in the soft parameters. In addition, the perturbative corrections to the Kähler potential are always functions of $z_i + \bar{z}_i$, which do not lead to any CP-violating phases in the soft terms as argued in Sec. II B. The dependence on $z_i + \bar{z}_i$ is a reflection of the shift (Peccei-Quinn) symmetry of the axion $t_i \rightarrow t_i + \delta$, where $t_i = \mathrm{Im}(z_i)$. This symmetry is only broken by exponentially suppressed nonperturbative effects [61,62]. This implies that up to exponentially suppressed contributions, the corrections to the Kähler potential do not give rise to any CP-violating phases in the soft terms. Thus, although it is very hard in general to compute the form of corrections in M theory, the result that the CP-violating phases in soft parameters are highly suppressed should be quite robust as it only relies on symmetries.

VII. CONCLUSIONS

In this paper, we have discussed *CP* violation in theories arising from fluxless M theory compactifications with low-energy supersymmetry and all moduli stabilized. We have found that the supersymmetry breaking dynamics is *CP* conserving (up to exponentially suppressed effects). The gaugino masses, scalar masses, and μ and $B\mu$ parameters are real at the GUT scale and remain real at the electroweak scale since RG evolution does not introduce new *CP*-violtaing phases. However, the full trilinear couplings $\hat{A}_{\alpha\beta\gamma} = A_{\alpha\beta\gamma}Y_{\alpha\beta\gamma}$ manage to pick up *CP*-violating phases from the Yukawa couplings as the trilinear matrices are not proportional to the Yukawa matrices. In addition, RG effects mix the phases between different flavors.

Given a model of Yukawas, therefore, one can estimate the effects of these phases of Yukawas on the trilinear couplings, and therefore on the EDMs. Since hierarchical Yukawa textures are well motivated within this framework, we compute EDMs for such textures. The other relevant feature of the low-energy theory is that the scalar masses and trilinears are naturally of $\mathcal{O}(m_{3/2})$ due to the absence of sequestering generically. Moreover, these scalar masses and trilinears are naturally of $\mathcal{O}(10\ \text{TeV})$ since this is the natural scale of the gravitino mass within the M theory framework. These two features naturally give rise to small *CP*-violating effects consistent with experimental limits.

We have estimated the electron, neutron, and mercury electric dipole moments utilizing the above features, and found that the estimated upper bounds of the EDMs are all within current experimental limits. The estimated upper bound for the mercury EDM is close to the current experimental limit and could be probed in the near future. A robust prediction of the framework is the existence of a hierarchy of about 3 orders of magnitude between the neutron and electron EDMs. This essentially results from the mass hierarchy between gauginos and scalars as predicted within M theory [13], and provides an additional means to test the framework.

It should be emphasized that our results for EDMs are based on the result that the *CP*-violating phases are entirely from the Yukawas, and therefore, any experimental result which indicates other significant sources of phases would contradict and rule out this approach. We also discuss effects of possible corrections to the Kähler potential and superpotential, and the generalization to other string compactifications.

Note that our results are largely independent of a full solution to the flavor problem. Our results have been derived using the fact that the squark matrices are approximately flavor diagonal at low energies which is naturally predicted within the M theory framework, as explained in Sec. II A. Therefore, any solution to the flavor problem consistent with the above feature is consistent with our results.

The quark Yukawas give the CKM phases, and the lepton Yukawas the Pontecorvo-Maki-Nakagawa-Sakata phases. The latter can provide the phases needed for baryogenesis via leptogenesis consistent with the above framework, for example, as described in [63]. So even with no phases from the soft supersymmetry breaking this framework can give a complete description of all known *CP* violation.

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APPENDIX A: DYNAMICAL ALIGNMENT OF PHASES IN THE SUPERPOTENTIAL

The dynamical alignment of phases is crucial for solving the SUSY *CP* problem in the M theory framework. It means that terms in the moduli superpotential dynamically align to acquire the same phase in the vacuum (perhaps up to exponentially suppressed effects), implying that the superpotential in the vacuum becomes real after rotating away the unphysical overall phase.

It was shown in [13,14] that with just two nonperturbative contributions (''double condensate terms'') in the superpotential, all N moduli, the meson field, and one axion can be fixed in the supergravity regime with both terms in the superpotential acquiring a common phase in the vacuum. At this level, however, all but one of the axions remain unfixed. This is because the supergravity scalar potential only depends on one linear combination of axions, while it depends on all the moduli (through the Kähler derivative). It is important to stabilize the remaining N axions in a way such that the superpotential becomes (dominantly) real in the vacuum.

Precisely such a mechanism to stabilize the axions has been recently studied in [24]. There, it was shown that higher-order terms in the superpotential, which depend on the remaining linear combinations of axions, can stabilize these axions. The superpotential is then given by

$$W = A_1\phi^a e^{-b_1 f} + A_2 e^{-b_2 f} + \sum_{k>2} A_k e^{-b_k f^k}, \qquad \text{(A1)}$$

where we have assumed that the subdominant terms in (A1) arise from string instantons or gaugino condensates in pure super-Yang-Mills hidden sectors. It is possible to include matter in the hidden sectors as well, but that will not change the qualitative results obtained. In order to keep the original moduli stabilization mechanism and the resulting analysis of [13,14] intact, the ''double condensate'' terms must be parametrically larger than these higher-order terms. This can be naturally obtained when $b_1 \sim b_2 < b_k$, $k > 2$. It turns out that this will also make the superpotential real in the vacuum (up to exponentially suppressed effects), as shown below.

It is easy to see that in the $\mathcal{N} = 1$ supergravity potential, the dominant terms arise from the first two terms in (A1). Following [13], the dependence of the dominant potential on the axions is given by

$$V_{\rm dom}(t_i) = \frac{1}{\mathcal{V}^3}\bigg[\sum_{\alpha=1}^{2}\bigg(\sum_{i=1}^{N} V_{\alpha i}^{(1)} + V_{\alpha}^{(2)}\bigg)e^{-2b_\alpha \, {\rm Re}(f)} + \sum_{i=1}^{N}(V_i^{(3)} + V^{(4)})e^{-(b_1+b_2)\,{\rm Re}(f)} \times \cos(\chi_1 - \chi_2)\bigg], \tag{A2}$$

where $\chi_i \equiv b_i\vec{N}^i \cdot \vec{t} + \delta_{i1}a\theta$, ${\rm Im}(f) \equiv \vec{N}\cdot\vec{t}$, $\phi = \phi_0 e^{i\theta}$, and $V_{\alpha i}^{(1)}$, $V_{\alpha}^{(2)}$, $V_i^{(3)}$, $V^{(4)}$ are positive coefficients independent of the axions. Thus, the axion combination $\chi_1 - \chi_2$ is minimized at

$$\cos(\chi_1 - \chi_2) = \cos[(b_1 - b_2)\vec{N}\cdot\vec{t} + a\theta] = -1 \tag{A3}$$

while the remaining axion combinations stay unfixed.

Now, including other terms in the superpotential such that the "double condensate" approximation is valid, one finds that the remaining N axions are stabilized by the next N largest terms in the scalar potential depending on those N linearly independent combinations of axions. These terms are generated by the product of one of the two dominant terms and one of the subdominant terms. To compute the stabilized values of the axion combinations $\chi_1 - \chi_k$, $k > 2$, it is useful to write the effective potential for light axions after integrating out the moduli, meson field, and one axion combination which receive a mass of $\mathcal{O}(m_{3/2})$. This is given by [24]

$$V_{\rm eff}(\chi_i) \approx V_0 - m_{3/2}e^{K/2}\sum_{k=3}^{N+2} D_k e^{-b_k V_k}\cos(\chi_1 - \chi_k), \quad \forall k\colon\, b_k V_k < b_{k+1}V_{k+1}, \tag{A4}$$

where D_k are $\mathcal{O}(1)$ coefficients. This implies that the N axion combinations $\chi_1 - \chi_k$ are all stabilized such that

$$\cos(\chi_1 - \chi_k) = \pm 1, \qquad k = 3, \ldots, N+2, \tag{A5}$$

depending on whether D_k is negative or positive, respectively. In terms of χ_i, the full superpotential can thus be written as

$$W = e^{i\chi_1}\bigg(|W_1| + |W_2|e^{-i(\chi_1-\chi_2)} + \sum_{k=3}^{N+2}|W_k|e^{-i(\chi_1-\chi_k)} + \ldots\bigg) = e^{i\chi_1}\bigg(|W_1| - |W_2| \pm \sum_{k=3}^{N+2}|W_k|\bigg) + \ldots$$

which is (dominantly) real up to one overall phase. The phases of other possible terms present in the superpotential (denoted by "…" above) will be completely determined by the stabilized axions above, and may be different from 0 or π. However, since these terms are exponentially suppressed relative to the dominant terms, one finds that after rotating away the unphysical overall phase the superpotential is real in the vacuum up to exponentially suppressed effects. Note that for the above mechanism to work it is crucial that the terms in the superpotential which stabilize all moduli and one combination of axions are dominant compared to the remaining terms. In the case studied in this paper this amounts to having the first two terms in the superpotential dominant compared to other ones. If many or all terms are comparable to each other, the analysis becomes more complicated and the axion combinations $\chi_1 - \chi_k$, $k = 3, 4, \ldots$, are then generically stabilized at a nontrivial value other than 0 or π, implying that the superpotential is not real in the vacuum. Another consequence of the above result which has been studied in [24] in detail is that these light axion mass eigenstates are exponentially lighter compared to the moduli, meson, and heavy axion which are $\mathcal{O}(m_{3/2})$. This allows for a beautiful dynamical solution to the strong *CP* problem within string theory and explicitly realizes the string axiverse scenario considered in [64]. It is remarkable that the mechanism which stabilizes the axions in the framework considered in [24] and solves the strong *CP* problem automatically stabilizes them in such a way that the superpotential is (dominantly) real in the vacuum. The above result also applies to other classes of string compactifications. For example, it was shown in [25] that within certain classes of Type IIB flux compactifications, all Kähler moduli and one of their axion partners can be stabilized with a superpotential with a constant term and just one nonperturbative contribution which depends on a linear combination of all moduli, i.e., for

$$W = W_0 + Ae^{-bf}, \qquad f = \sum_{i}^{N} N^i T_i. \tag{A6}$$

All but one of the axions remain massless at this level. If these axions are stabilized by subdominant terms in the superpotential, this would give rise to the same conclusion in these compactifications as well.

APPENDIX B: THE LEADING ONE-LOOP CONTRIBUTIONS TO EDM

The fermion EDMs can be generated at one-loop in supersymmetric models with *CP*-violating phases in the soft supersymmetry breaking sector. Within the framework considered in this paper, the *CP*-violating phases only reside in the trilinear terms and therefore appear in the mass mixing terms of the left- and right-handed sfermions. Therefore, the main contribution to the quark EDM and CEDM comes from diagrams involving gluinos because of the large gauge coupling. For the electron EDM, the dominant contribution comes from the diagram involving neutralinos. This is because the diagrams with charginos in the loop require CP-violating phases in the chargino sector which do not arise within the M theory framework considered.

Let us first consider the diagrams contributing to quark EDM and CEDM with gluino running in the loop

$$d_q^{\tilde{g}(E)} = \frac{-2e\alpha_s}{3\pi}\sum_{k=1}^{2}\mathrm{Im}(\Gamma_q^{1k})\frac{m_{\tilde{g}}}{m_{\tilde{q}_k}^2}Q_{\tilde{q}}B\left(\frac{m_{\tilde{g}}^2}{m_{\tilde{q}_k}^2}\right), \tag{B1}$$

$$d_q^{\tilde{g}(C)} = \frac{g_s\alpha_s}{4\pi}\sum_{k=1}^{2}\mathrm{Im}(\Gamma_q^{1k})\frac{m_{\tilde{g}}}{m_{\tilde{q}_k}^2}C\left(\frac{m_{\tilde{g}}^2}{m_{\tilde{q}_k}^2}\right), \tag{B2}$$

where $\Gamma_q^{1k} = D_{q2k}D_{q1k}^*$ and D_q is the 2×2 matrix which diagonalizes the squark mass matrix $m_{\tilde{q}}^2$

$$D_q^\dagger m_q^2 D_q = \mathrm{Diag}(m_{\tilde{q}1}^2, m_{\tilde{q}2}^2). \tag{B3}$$

More explicitly

$$\tilde{q}_L = D_{q11}\tilde{q}_1 + D_{q12}\tilde{q}_2, \quad \tilde{q}_R = D_{q21}\tilde{q}_1 + D_{q22}\tilde{q}_2. \tag{B4}$$

Here $B(r)$ and $C(r)$ are loop functions defined as

$$B(r) = \frac{1}{2(r-1)^2}\left(1 + r + \frac{2r\ln(r)}{1-r}\right),$$
$$C(r) = \frac{1}{6(r-1)^2}\left(10r - 26 + \frac{2r\ln(r)}{1-r} - \frac{18\ln(r)}{1-r}\right).$$

In the above equations, we assume no flavor mixing in the squark mass matrices as argued in the main body of the paper. Using the fact that $\mathrm{Im}(\Gamma_q^{11}) = -\mathrm{Im}(\Gamma_q^{12})$, we have

$$d_q^{\tilde{g}(E)} \approx \frac{-2e\alpha_s Q_{\tilde{q}}}{3\pi}\frac{\mathrm{Im}(m_{\tilde{q}}^2)_{LR}}{m_{\tilde{g}}^3}r^2(B(r) + rB'(r)). \tag{B5}$$

Similarly

$$d_q^{\tilde{g}(C)} \approx \frac{g_s\alpha_s}{4\pi}\frac{\mathrm{Im}(m_{\tilde{q}}^2)_{LR}}{m_{\tilde{g}}^3}r^2(C(r) + rC'(r)). \tag{B6}$$

In the calculation above, we assume the mass splitting of squarks is small compared to the squark mass. This is usually true since we are only interested in the up and down squarks. When $r = m_{\tilde{g}}^2/M_{\tilde{q}}^2 \ll 1$, one finds that $C(r) \gg A(r)$, $B(r)$. It is easy to see that $d_q^{\tilde{g}(C)} \gg d_q^{\tilde{g}(E)}$. For other diagrams which involve neutralinos and charginos, the structure is very similar. However, they are much smaller than $d_q^{\tilde{g}}$ and can be neglected.

Now let us turn to the one-loop diagrams contributing to the electron EDM:

$$d_e^{\tilde{\chi}^+} = \frac{e\alpha_{em}}{4\pi\sin^2\theta_W}\sum_{k=1}^{2}\mathrm{Im}(\Gamma_{ei})\frac{m_{\tilde{\chi}^+}}{m_{\tilde{\nu}}^2}A\left(\frac{m_{\tilde{\chi}^+}^2}{m_{\tilde{\nu}}^2}\right), \tag{B7}$$

$$d_e^{\tilde{\chi}^0} = \frac{e\alpha_{em}}{4\pi\sin^2\theta_W}\sum_{k,i=1,1}^{2,4}\mathrm{Im}(\eta_{eik})\frac{m_{\tilde{\chi}^0}}{m_{\tilde{e}_k}^2}B\left(\frac{m_{\tilde{\chi}^0}^2}{m_{\tilde{e}_k}^2}\right), \tag{B8}$$

where $\Gamma_{ei} = U_{i2}^*V_{i1}^*$, and

$$\eta_{eik} = [-\sqrt{2}\{\tan\theta_W(Q_e - T_{3e})X_{1i} + T_{3e}X_{2i}\}D_{e1k}^* + \kappa_e X_{bi}D_{e2k}^*](\sqrt{2}\tan\theta_W Q_e X_{1i}D_{e2k} - \kappa_e X_{bi}D_{e1k}).$$

Here we have

$$\kappa_e = \frac{m_e}{\sqrt{2}m_W\cos\beta}. \tag{B9}$$

The loop function $A(r)$ is given by

$$A(r) = \frac{1}{2(1-r)^2}\left(3 - r + \frac{2\ln(r)}{1-r}\right). \tag{B10}$$

In the above equations, $U(V)$, X, and D_e are the conventional chargino, neutralino, and selectron mixing matrices. It is easy to see that the chargino diagram does not contribute to the electron EDM in the framework considered, since there is no CP-violating phases in the chargino sector. In the absence of the neutralino mixing, the expression of $d_e^{\tilde{\chi}^0}$ can be significantly simplified:

$$d_e^E \approx \frac{e\alpha_{em}}{4\pi\cos^2\theta_W}\frac{\mathrm{Im}(m_{\tilde{e}}^2)_{LR}}{m_{\tilde{B}}^3}r_1^2[B(r_1) + r_1B'(r_1)], \tag{B11}$$

where $r_1 = m_{\tilde{B}}^2/m_{\tilde{e}}^2$ with $m_{\tilde{e}}$ denoting the average mass of the selectrons. In the above result, the Higgsino contribution is neglected since it is suppressed by the small Y_e^2.

APPENDIX C: BARR-ZEE DIAGRAM

As we have discussed in Sec. V B, we are concerned with the Barr-Zee diagram with the third generation squarks, i.e., $\tilde{t}$ and $\tilde{b}$, running in the inner loop. Here we give the detailed derivation of Eqs. (34) and (35). We start with the general results of EDM and CEDM for the Barr-Zee diagram [40]:

$$\begin{aligned} d_f^E &= Q_f\frac{3e\alpha_{em}}{32\pi^3}\frac{R_f m_f}{M_A^2}\sum_{q=t,b}\xi_q Q_q^2[F(r_1) - F(r_2)],\\ d_f^C &= \frac{g_s\alpha_s}{64\pi^3}\frac{R_f m_f}{M_A^2}\sum_{q=t,b}\xi_q[F(r_1) - F(r_2)], \end{aligned} \tag{C1}$$

where M_A is the mass of pseudoscalar Higgs A_0, $r_{1,2} = m^2_{\tilde{q}_{1,2}}/M_A^2$, $R_f = \cot\beta(\tan\beta)$ for $I_3 = 1/2(-1/2)$, and $F(z)$ is the two-loop function

$$F(z) = \int_0^1 dx \frac{x(1-x)}{z - x(1-x)} \ln\left[\frac{x(1-x)}{z}\right]. \tag{C2}$$

The CP-violating couplings are given by

$$\begin{aligned} \xi_t &= -\frac{\sin 2\theta_{\tilde{t}} m_t \,\mathrm{Im}(\mu e^{i\delta_t})}{2v^2 \sin^2\beta}, \\ \xi_b &= -\frac{\sin 2\theta_{\tilde{b}} m_b \,\mathrm{Im}(A_b e^{-i\delta_b})}{2v^2 \sin\beta\cos\beta}, \end{aligned} \tag{C3}$$

where $\theta_{\tilde{t},\tilde{b}}$ are the stop and sbottom mixing angles, and $\delta_q = \mathrm{Arg}(A_q + R_q\mu^*)$. The mixing angle of the squark sector is given by

$$\begin{aligned} \tan(2\theta_q) &= -\frac{2m_q|\mu R_q + A_q^*|}{M^2_{\tilde{Q}} - M^2_{\tilde{q}} + \cos 2\beta M_Z^2(T_z^q - 2e_q s_w^2)} \\ &\approx -\frac{2m_q|\mu R_q + A_q^*|}{M^2_{\tilde{Q}} - M^2_{\tilde{q}}}. \end{aligned} \tag{C4}$$

Therefore, Eq. (C3) becomes

$$\begin{aligned} \xi_t &\approx \frac{y_t^2|A_t^* + \mu\cot\beta| \,\mathrm{Im}(\mu e^{i\delta_t})}{M^2_{\tilde{Q}} - M^2_{\tilde{t}}}, \\ \xi_b &\approx \cot\beta \frac{y_b^2|A_b^* + \mu\tan\beta| \,\mathrm{Im}(A_b e^{-i\delta_b})}{M^2_{\tilde{Q}} - M^2_{\tilde{b}}}. \end{aligned} \tag{C5}$$

Using Eqs. (C3) and (C4), we can rewrite Eq. (C1) as

$$\begin{aligned} d_f^E &\approx Q_f \frac{3e\alpha_{em}}{32\pi^3} \frac{R_f m_f}{M_A^4} \mathrm{Im}\left[\frac{4y_t^2}{9}\mu(A_t + \mu^*\cot\beta)F'(r_1) \right. \\ &\quad \left. + \frac{y_b^2}{9} A_b(A_b^* + \mu\tan\beta)\cot\beta F'(r_2)\right], \\ d_f^C &\approx \frac{g_s\alpha_s}{64\pi^3} \frac{R_f m_f}{M_A^4} \mathrm{Im}[y_t^2\mu(A_t + \mu^*\cot\beta)F'(r_1) \\ &\quad + y_b^2 A_b(A_b^* + \mu\tan\beta)\cot\beta F'(r_2)], \end{aligned} \tag{C6}$$

where $r_1 \equiv m^2_{\tilde{t}}/M_A^2$ and $r_2 \equiv m^2_{\tilde{b}}/M_A^2$ with $m_{\tilde{t},\tilde{b}}$ being the average masses of the stops and sbottoms, respectively.

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