# $B^{(*)}\bar{B}^{(*)}$ intermediate states contribution to $\Upsilon(4S, 5S) \rightarrow \eta_b + \gamma$ radiative decay

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In this work, we investigate the rescattering effects in the radiative decay  $Y(5S) \rightarrow \eta_b + \gamma$ , which were suggested to be crucially important for understanding the anomalous largeness of the branching ratios  $B(Y(5S) \rightarrow Y(1S) + \pi\pi)$  and  $B(Y(5S) \rightarrow Y(1S) + \eta)$ . Our calculations show that the rescattering effects may enhance  $\Gamma(Y(10860) \rightarrow \eta_b + \gamma)$  by four orders, but the tetraquark structure does not. Recently the *BABAR* and CLEO collaborations have measured the mass of  $\eta_b$  and the branching ratios  $\mathcal{B}(Y(2S) \rightarrow \eta_b + \gamma)$ ,  $\mathcal{B}(Y(3S) \rightarrow \eta_b + \gamma)$ . We hope that very soon,  $Y(10860) \rightarrow \eta_b + \gamma$  will be measured and it would be an ideal opportunity for testing whether the rescattering or the tetraquark structure is responsible for the anomaly of  $\mathcal{B}(Y(5S) \rightarrow Y(nS)\pi^+\pi^-(n = 1, 2, 3))$ , i.e., the future measurements on the radiative decays of Y(5S) might be a touchstone of the two mechanisms.

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#### I. INTRODUCTION

In 2008, the Belle Collaboration reported their first observation of  $e^+e^- \rightarrow Y(1S, 2S, 3S)\pi^+\pi^-$  [1] and  $e^+e^- \rightarrow Y(1S)K^+K^-$  near the peak of Y(5S) at  $\sqrt{s} \sim 10.87$  GeV [1]. Assuming that the observed signal events are only from Y(5S), the measured partial widths for the final states  $Y(nS)\pi^+\pi^-(n = 1, 2, 3)$  and  $Y(1S)K^+K^-$  are  $0.52 \sim 0.85$  MeV and 0.067 MeV, respectively, which are larger than the corresponding partial widths of Y(nS)  $(n = 2, 3, 4) \rightarrow Y(1S) + \pi\pi(K\bar{K})$  [1] by more than 2 orders of magnitude. The anomalously large partial widths in  $e^+e^- \rightarrow Y(1S, 2S)\pi^+\pi^-$  at the energy peak of Y(5S) have stimulated theorists' interests for exploring the source, what results in these observations.

The authors of Ref. [2] suggested that the rescattering processes of  $Y(5S) \rightarrow B^{(*)}\bar{B}^{(*)} \rightarrow Y(mS) + \sigma/f_0(980) \rightarrow$  $Y(mS) + \pi\pi$  make a substantial contribution to the observed dipion transition of Y(5S). Furthermore, they applied the same mechanism to the transition  $Y(4S, 5S) \rightarrow$  $Y(1S) + \eta$  [3]. They have found that the obtained ratio of  $\Gamma(Y(4S) \rightarrow Y(1S) + \eta)$  to  $\Gamma(Y(4S) \rightarrow Y(1S) + \pi\pi)$ reaches  $1.8 \sim 4.5$ , which is consistent with the *BABAR* measurement on this ratio [4]. By the same mechanism, Meng and Chao also studied the energy distribution of the dipion in the processes  $Y(5S) \rightarrow Y(1S, 2S, 3S) + \pi^+\pi^-$ , and observed the energy dependence of  $Y(5S) \rightarrow Y(1S, 2S, 3S)\pi^+\pi^-$  to be different from that of  $Y(5S) \rightarrow B^{(*)}\bar{B}^{(*)}$  [5]. Simonov and Veselov investigated the dipion transitions of Y(5S) by using the field correlation method, which is similar to the rescattering mechanism proposed in Ref. [2] in some sense. The obtained  $\Gamma(Y(5S) \rightarrow Y(nS) + \pi^+\pi^-)$  (n = 1, 2, 3) are in a reasonable agreement with the experimental data [6].

Since the resonant peak of  $e^+e^- \rightarrow Y(1S, 2S, 3S)\pi^+\pi^$ appears at  $\sqrt{s} = 10.87$  GeV [1,7] which deviates from the central mass of Y(5S) [8], theorists suggest that this enhancement may be explained by a mixing between the normal 5S state with an exotic component, such as a hybrid state  $b\bar{b}g$  or a tetraquark state  $b\bar{b}q\bar{q}$ .

Let us have a closer look at the different explanations. By the initial state radiation (ISR), the BABAR Collaboration once announced their observation of a charmoniumlike state Y(4260) by studying the  $J/\psi \pi^+ \pi^$ invariant mass spectrum of  $e^+e^-_{\rm ISR} \rightarrow J/\psi \pi^+\pi^-$  [9]. For understanding the data, theorists suggested different exotic structures for Y(4260) [10–17]. Hou then indicated that searching for the bottom counterpart of Y(4260) via  $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$  would be an interesting topic [18]. The observation of an enhancement at 10.87 GeV in  $Y(1S, 2S, 3S)\pi^+\pi^-$  invariant mass spectra seems to advocate the existence of a bottom analogue of Y(4260) [19]. Karliner and Lipkin proposed that the large partial widths of  $Y(5S) \rightarrow Y(1S, 2S, 3S)\pi^+\pi^-$  might be due to an intermediate state  $T^{\pm}_{b\bar{b}}\pi^{\mp}$ , where  $T^{\pm}_{b\bar{b}}$  could be identified as an isovector charged tetraquark  $b\bar{b}u\bar{d}$  or  $b\bar{b}d\bar{u}$  [20]. That is in fact an extension of the tetraquark explanation for Y(4260) given in Ref. [12] to the *b* range. A different tetraquark structure: the lowest lying *P*-wave tetraquark

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 $Y_b = [bq][\bar{b}\bar{q}] (q = u, d)$  of  $J^{PC} = 1^{--}$  with its mass equal to 10 890 MeV, was proposed by Ali *et al.* [21,22]. In their model, the two light flavors in the tetraquark join to constitute a resonant state  $[\sigma(600), f_0(980), \text{ and } f_2(1270)]$ which then decays into two pions. This mechanism can explain the anomalous  $Y(1S, 2S)\pi^+\pi^-$  production near the resonance Y(5S) and the structure at the dipion invariant mass spectrum as well as the  $\cos\theta$  distribution of  $e^+e^- \rightarrow Y_b \rightarrow Y(1S, 2S)\pi^+\pi^-$  by the Belle Collaboration [1], where  $\theta$  is the angle between the momentum of Y(5S) and that of  $\pi^-$  in the center of mass frame of the two pions.

In parallel to the interpretation which invokes the exotic structure of Y(5S), alternative mechanisms have been suggested to stand for the anomalous  $Y(1S, 2S, 3S)\pi^+\pi^-$  production near the Y(5S) in  $e^+e^- \rightarrow Y(1S, 2S, 3S)\pi^+\pi^-$  processes. We cannot rule out any possible mechanisms that interpret the Belle data until more evidence could support or negate some (or just one) of them. Thus, further exploration is extremely necessary for determining the physics behind the observed phenomena.

In this work, we would like to further test the rescattering mechanism proposed by the authors of Refs. [2,3] in the radiative decays of  $\Upsilon(5S)$ , namely,  $\Upsilon(5S) \rightarrow \eta_b + \gamma$ . We suppose that  $\Upsilon(5S) \rightarrow \eta_b + \gamma$  radiative decay occurs via the intermediate state  $B^{(*)}\bar{B}^{(*)}$ . In fact,  $\Upsilon(5S) \rightarrow B^{(*)}\bar{B}^{(*)} \rightarrow$  $\eta_{h} + \gamma$  radiative decay is similar to  $\Upsilon(5S) \rightarrow B^{(*)}\bar{B}^{(*)} \rightarrow$  $\Upsilon(1S) + \eta$ , where one only needs to replace the effective vertices of  $B^{(*)}\bar{B}^{(*)}\Upsilon(1S)$  and  $B^{(*)}\bar{B}^{(*)}\eta$  by the electromagnetic vertices  $B^{(*)}\bar{B}^{(*)}\gamma$  and  $B^{(*)}\bar{B}^{(*)}\eta_b$ , respectively, in the diagrams given in Ref. [3]. The electromagnetic vertex is relatively simple compared to the hadronic one, thus for the low energy processes, if one writes the effective electromagnetic vertex as e times the phenomenologically introduced form factor which is similar to the hadronic cases (see the text for details), the results would be more reliable. In this work, we would take all inputs which were used in Refs. [2,3], except at the electromagnetic vertex.

Thus, one can expect that the corresponding mechanism should enhance the ratio of  $\Upsilon(5S) \rightarrow \eta_b + \gamma$ . As a by-product, we will extend the rescattering mechanism in  $\Upsilon(5S) \rightarrow \eta_b + \gamma$  to study the radiative decay  $\Upsilon(4S) \rightarrow \eta_b + \gamma$ .

The relevant phenomenological study of  $\eta_b$  via the transitions  $\Upsilon(3S) \rightarrow \eta_b + \gamma$  and  $\Upsilon(2S) \rightarrow \eta_b + \gamma$  is carried out in Refs. [23–31]. In our recent theoretical work,  $\Upsilon(nS) \rightarrow \eta_b + \gamma$  without including the rescattering effect, was calculated in the light-cone quark model, which indicated that the decay widths of  $\Upsilon(4S) \rightarrow \eta_b + \gamma$  and  $\Upsilon(5S) \rightarrow \eta_b + \gamma$  are of the same order of magnitude. After performing  $\Upsilon(5S) \rightarrow \eta_b + \gamma$  via the intermediate state  $B^{(*)}\bar{B}^{(*)}$ , we can compare the results  $\Upsilon(5S) \rightarrow \eta_b + \gamma$  with and without including the rescattering effect.

Recently the *BABAR* Collaboration [32,33] and the CLEO Collaboration [34] have measured the mass of  $\eta_b$ 

via  $Y(3S) \rightarrow \eta_b + \gamma$ , which makes us believe that  $Y(4S, 5S) \rightarrow \eta_b + \gamma$  can be measured in the near future. Whether the rescattering effect plays an important role in  $Y(4S, 5S) \rightarrow \eta_b + \gamma$ , radiative decays will be tested by the future experimental measurement. Moreover, the rescattering mechanism for Y(4S, 5S) proposed in Refs. [2,3] can be tested.

This paper is organized as follows. In Sec. II we study the possible rescattering effects on  $\Upsilon(4S, 5S) \rightarrow \eta_b + \gamma$ and present the numerical result. Section III is devoted to the conclusion and the discussion.

# II. RESCATTERING EFFECT ON $\Upsilon(4S, 5S) \rightarrow \eta_b + \gamma$

As indicated in Refs. [2,3], the rescattering effect may remarkably enhance the rates of  $Y(5S) \rightarrow Y(1S) + \pi\pi$ and  $Y(5S) \rightarrow Y(1S) + \eta$ . Thus, in this work we apply the same mechanism to study on  $Y(4S, 5S) \rightarrow \eta_b + \gamma$ radiative decay, where the transitions  $Y(5S) \rightarrow \eta_b + \gamma$ can occur via rescattering subprocesses with the intermediate states being  $B^{(*)}\bar{B}^{(*)}$ . The corresponding schematic diagrams are depicted in Fig. 1.

The remaining diagrams can be obtained by the charge conjugation transformation  $B^{(*)} \leftrightarrow \overline{B}^{(*)}$  to diagrams (a)–(f) and the isospin transformation  $B^{(*)0} \leftrightarrow B^{(*)+}$  and  $\overline{B}^{(*)0} \leftrightarrow B^{(*)-}$  to diagrams (a), (c), and (e). We need to emphasize that the diagrams corresponding to diagrams (b), (d), (f) after the isospin transformation are absent, since the electromagnetic interactions of  $B^0 B^0 \gamma$  and  $B^{*0} B^{*0} \gamma$  do not exist.



FIG. 1. The schematic diagrams for  $\Upsilon(nS) \rightarrow B^{(*)+}B^{(*)-} \rightarrow \eta_b + \gamma$ .

Indeed, since the intermediate states  $B^{(*)}\bar{B}^{(*)}$  can be on shell as described in Fig. 1, both the dispersive (real) and absorptive (imaginary) parts of the loop contribute to the amplitudes of  $\Upsilon(4S, 5S) \rightarrow B^{(*)}\bar{B}^{(*)} \rightarrow \eta_b + \gamma$ . In our earlier work [35], we investigated the contributions of the final state interaction (FSI) to the decay amplitudes of  $J/\psi \rightarrow$ VP, where V and P stand as light vector and pseudoscalar mesons. Interferences of the FSI contribution and the tree diagram result in the decay widths. However, for that case, the on-shell  $D\bar{D}$  channels are not open because of the energy-momentum conservation (the other channels with light mesons are highly Okubo-Zweig-Iizuka suppressed); therefore, for the processes  $J/\psi \rightarrow D\bar{D} \rightarrow VP$  there is no contribution from the absorptive part, but only from the dispersive one. Thus we only need to evaluate the real part of the loop. By fitting the decay widths of two channels we determine the model parameters, one of which is for the form factor at the effective vertex and another for the interference. Then we predict the widths of other channels and obtained results which are very close to the data. As is well known, since the form factor is introduced, renormalization is automatically realized and this corresponds to the Pauli-Villas renormalization. Fitting data of a few channels is just like the on-shell scheme. The key point is that once we have data to fit, we may more accurately estimate the contributions of (maybe) both dispersive and absorptive parts. In general, when not enough data are available, accurate calculation of the FSI contributions is impossible. Namely, one can only estimate their order of magnitude of FSI. That is our present case. By general arguments, if the absorptive part exists, its contribution might exceed that of the dispersive part. Anyhow, one can argue that they should have the same order of magnitude. Moreover, it is noticed, the masses of  $\Upsilon(4S, 5S)$  are much above the thresholds of  $B^{(*)}\bar{B}^{(*)}$ , and it implies that the imaginary part may be dominant. In Refs. [3,5] the authors made a clearer discussion on it. Thus, in this work we only consider the contribution from the absorptive part to the decay amplitude, namely, neglecting the real part would just be an estimate of the lower bound of FSI. The purpose of this work is to find an effective probe for the two mechanisms (tetraquark structure or FSI) which can explain the largeness of branching ratios of  $B(\Upsilon(4S, 5S) \rightarrow \Upsilon(mS) + \pi\pi)$ and  $B(\Upsilon(mS) + \eta)$  with  $m \leq 3$ . Since they lead to very distinct results for the widths of  $\Upsilon(4S, 5S) \rightarrow \eta_b + \gamma$  by orders, one can be content with the estimate of the only lower bound.

The absorptive part of the decay amplitude of  $\Upsilon(5S) \rightarrow \eta_b + \gamma$  is expressed as

Abs 
$$[\Upsilon(5S) \rightarrow B^{(*)}\bar{B}^{(*)} \rightarrow \gamma \eta_b]$$
  
=  $2(M_C^{(a)} + M_C^{(b)} + M_C^{(c)} + M_C^{(d)} + M_C^{(e)} + M_C^{(f)}) + 2(M_N^{(a)} + M_N^{(c)} + M_N^{(e)}),$  (1)

where the subscripts *C* and *N* denote the decay amplitudes relevant to the intermediate  $B^{(*)+}B^{(*)-}$  and  $B^{(*)0}\overline{B}^{(*)0}$ , respectively. Factor 2 in Eq. (1) is from their charge conjugation.

According to the Cutkosky rules [36], the general expression of the absorptive part of the amplitude corresponding to diagrams (a)–(f) in Fig. 1 is expressed as

$$M^{(i)} = \frac{|\mathbf{p}_1|}{32\pi^2 m_{\Upsilon(nS)}} \int d\Omega \,\mathcal{A}_i[\Upsilon(nS) \to B^{(*)}\bar{B}^{(*)}] \\ \times \,\mathcal{C}_i[B^{(*)}\bar{B}^{(*)} \to \eta_b + \gamma] \cdot \mathcal{F}(m_i, q^2)$$
(2)

with i = a, b, c, d, e, f. Here,  $d\Omega$  and  $\mathbf{p}_1$  are the solid angle and linear momentum of the on-shell  $B^{(*)}$  in the rest frame of  $\Upsilon(nS)$ , respectively.  $\mathcal{A}_i$  and  $\mathcal{C}_i$  are the amplitudes describing  $\Upsilon(5S) \rightarrow B^{(*)}\bar{B}^{(*)}$  and  $B^{(*)}\bar{B}^{(*)} \rightarrow \eta_b \gamma$  by exchanging  $B^{(*)}$  meson. The off-shell effect of the meson exchanged at the *t* channel is compensated by a monepole form factor which reflects the inner structures of the mesons at the effective vertex [35,37–39]

$$\mathcal{F}(m_i, q^2) = \frac{(\Lambda + m_i)^2 - m_i^2}{(\Lambda + m_i)^2 - q^2},$$
(3)

where q and  $m_i$  are the momentum and the mass of the exchanged meson, respectively. The cutoff can be parameterized as  $\Lambda = \alpha \Lambda_{\rm QCD}$  with  $\Lambda_{\rm QCD} = 220$  MeV and the dimensionless parameter  $\alpha$  being the order of unit. Later we will show the dependence of decay width of  $\Upsilon(5S) \rightarrow B^{(*)}\bar{B}^{(*)} \rightarrow \eta_b + \gamma$  on  $\alpha$ .

For obtaining  $\mathcal{A}_i$  and  $\mathcal{C}_i$  in Eq. (2), we adopt the effective Lagrangian approach. The effective couplings for  $\Upsilon BB$ ,  $\Upsilon B^*B$ , and  $\Upsilon B^*B^*$  adopted in this work are directly borrowed from Refs. [2,3]

$$\mathcal{L}_{YBB} = g_{YBB} \Upsilon_{\mu} (\partial^{\mu} BB^{\dagger} - B \partial^{\mu} B^{\dagger}),$$
  

$$\mathcal{L}_{YB^{*}B} = \frac{g_{YB^{*}B}}{m_{Y}} \varepsilon^{\mu\nu\alpha\beta} \partial_{\mu} \Upsilon_{\nu} \times (B^{*}_{\alpha} \overleftrightarrow{\partial}_{\beta} B^{\dagger} - B \overleftrightarrow{\partial}_{\beta} B^{*\dagger}_{\alpha}),$$
  

$$\mathcal{L}_{YB^{*}B^{*}} = g_{YB^{*}B^{*}} (-\Upsilon^{\mu} B^{*\nu} \overleftrightarrow{\partial}_{\mu} B^{*\dagger}_{\nu} + \Upsilon^{\mu} B^{*\nu} \partial_{\nu} B^{*\dagger}_{\mu} - \Upsilon_{\mu} \partial_{\nu} B^{*\mu} B^{*\nu\dagger}),$$
(4)

where  $\vec{\partial} = \vec{\partial} - \vec{\partial}$  and the coupling constants were determined as [2,3]

$$g_{\Upsilon(5S)BB} = 2.5,$$
  
 $g_{\Upsilon(5S)B^*B} = 1.4 \pm 0.3,$   
 $g_{\Upsilon(5S)B^*B^*} = 2.5 \pm 0.4.$ 

Following the strategy of Refs. [3,40,41], we list the Lagrangian describing the electromagnetic interaction  $B^{(*)}B^{(*)}\gamma$ 

$$\mathcal{L}_{\gamma BB} = eA_{\mu} (\partial^{\mu} BB^{\dagger} - B\partial^{\mu} B^{\dagger}), \qquad (5a)$$
$$\mathcal{L}_{\gamma B^{*}B^{*}} = e(-A^{\mu} B^{*\nu} \overleftrightarrow{\partial}_{\mu} B^{*\dagger}_{\nu} + A^{\mu} B^{*\nu} \partial_{\nu} B^{*\dagger}_{\mu} - A_{\mu} \partial_{\nu} B^{*\mu} B^{*\nu\dagger}), \qquad (5b)$$

$$\mathcal{L}_{\gamma B^* B} = \frac{g_{\gamma B^* B}}{m_{B^*}} e \varepsilon^{\mu \nu \alpha \beta} \partial_{\mu} A_{\nu} \\ \times (B^*_{\alpha} \overleftrightarrow{\partial}_{\beta} B^{\dagger} - B \overleftrightarrow{\partial}_{\beta} B^{*\dagger}_{\alpha}).$$
(5c)

In terms of the theoretically evaluated value of  $\Gamma(B^{*+} \rightarrow B^+ \gamma) = 0.40 \pm 0.03$  keV and  $\Gamma(B^{*0} \rightarrow B^0 \gamma) = 0.13 \pm 0.03$  keV [42,43], one obtains  $g_{\gamma B^{*+}B^+} \approx 3.47$  and  $g_{\gamma B^{*0}B^0} \approx 1.97$ .

The effective couplings for  $\eta_b B^* B$ ,  $\eta_b B^* B^*$  can be expressed as

$$\mathcal{L}_{B^*B\eta_b} = ig_{B^*B\eta_b}B^*_{\mu}\partial^{\mu}\eta_b B^{\dagger}, 
\mathcal{L}_{B^*B^*\eta_b} = i\frac{g_{B^*B^*\eta_b}}{m_{R^*}}\varepsilon^{\mu\nu\alpha\beta}\partial_{\mu}B^*_{\nu}B^{*\dagger}_{\alpha}\partial_{\beta}\eta_b.$$
(6)

If considering the heavy quark spin symmetry [44],  $g_{\eta_b B^*B}$ and  $g_{\eta_b B^*B^*}$  are related to  $g_{Y(1S)BB}$ , which shows

$$g_{\eta_b B^* B} = g_{\eta_b B^* B^*} = g_{\Upsilon(1S)BB},\tag{7}$$

where  $g_{\Upsilon(1S)BB} = 15$  [2,3].

Applying the rescattering mechanism to study  $\Upsilon(4S) \rightarrow \eta_b + \gamma$  radiative decay, one obtains

Abs 
$$[\Upsilon(4S) \rightarrow B\bar{B} \rightarrow \gamma \eta_b] = 2(M_C^{(a)} + M_N^{(a)}),$$
 (8)

where only the diagram (a) in Fig. 1 contributes to  $\Upsilon(4S) \rightarrow \eta_b + \gamma$  due to the mass of  $\Upsilon(4S)$  being just above the threshold of  $B\bar{B}$ . Factor 2 comes from the isospin symmetry and the charge conjugate. The subscripts *C* and *N* denote the decay amplitudes relevant to the intermediate  $B^+B^-$  and  $B^0\bar{B}^0$ , respectively.

With the above preparation, we obtain the dependence of the decay widths of  $\Upsilon(5S) \rightarrow B^{(*)}\bar{B}^{(*)} \rightarrow \eta_b + \gamma$  and  $\Upsilon(4S) \rightarrow B\bar{B} \rightarrow \eta_b + \gamma$  on  $\alpha = 1 \sim 3$ , as shown in Fig. 2.



FIG. 2 (color online). The dependence of decay widths of  $\Upsilon(5S) \rightarrow B^{(*)}\bar{B}^{(*)} \rightarrow \eta_b + \gamma$  and  $\Upsilon(4S) \rightarrow B\bar{B} \rightarrow \eta_b + \gamma$  on  $\alpha$ .

With all the parameters we can obtain  $\Gamma(\Upsilon(5S) \rightarrow \eta_b + \gamma) = 4.77$  keV which is 4 orders bigger than the direct transition in Refs. [45,46], where  $\Upsilon(5S)$  is regarded as a pure 5S state and  $\Gamma(\Upsilon(5S) \rightarrow \eta_b + \gamma)$  is not anomalous compared to  $\Gamma(\Upsilon(1S, 2S, 3S, 4S) \rightarrow \eta_b + \gamma)$  as long as the rescattering is not taken into account. We explore the dependence of the width of  $\Gamma(\Upsilon(5S) \rightarrow \eta_b + \gamma)$  on the cutoff  $\Lambda$  and the results are depicted in Fig. 2, where we can find that the width increases with the increase of  $\Lambda$ .

## **III. DISCUSSION AND CONCLUSION**

The anomalous largeness of the branching ratio of  $Y(5S) \rightarrow Y(1S, 2S) + \pi\pi$  stimulates a hot discussion from different theoretical groups. There are two possible interpretations which are based on different physics scenarios. The first is that the observed Y(10860) is a tetraquark  $b\bar{q} \, \bar{b} \, q$  or has a sizable tetraquark component. In this scenario, the two light ingredients join to constitute a resonant state which later decays into two pions. This picture can explain the structure of the dipion invariant mass spectra observed by the Belle Collaboration. However, since the mechanism for the tetraquark decay is governed by the nonperturbative QCD which is not fully understood so far, the transition matrix element cannot be reliably estimated. Even though the picture seems reasonable, one is unable to quantitatively obtain the large rate.

The alternative interpretation for the largeness is due to the rescattering effects which occur at the hadron level. The dynamics of the rescattering is clear, but the effective vertices must be determined by fitting relevant experimental data. Moreover, for estimating the concerned Feynman diagrams, a form factor which compensates the off-shell effects of the exchanged mesons must be introduced. All these uncertainties must manifest themselves in the theoretical predictions. Even though the two scenarios suffer from theoretical uncertainties, they all offer possible interpretations for the largeness of  $Y(5S) \rightarrow Y(1S, 2S) + \pi\pi$ and  $Y(5S) \rightarrow Y(1S, 2S) + \eta$ . Thus one should regard them in relevant processes. Our strategy is exactly based on this thought.

The rescattering mechanism proposed by the authors of Ref. [2] can greatly enhance the decay rates of  $Y(5S) \rightarrow Y(1S, 2S) + \pi\pi$  and  $Y(5S) \rightarrow Y(1S, 2S) + \eta$  compared to the transition among lower resonances. In this work, we further demonstrate the mechanism at the radiative decay of  $Y(5S) \rightarrow \eta_b + \gamma$ , where the effective electromagnetic vertex is relatively simple. Our result, which is obtained in terms of the light-front quark model, indicates the branching ratio of  $Y(5S) \rightarrow \eta_b + \gamma$  is not enhanced compared to that of  $Y(mS) \rightarrow \eta_b + \gamma$  (m = 1, 2, 3, 4) as long as the rescattering effect is not taken into account. However, there could be a four-order enhancement in magnitude for  $B(Y(5S) \rightarrow \eta_b + \gamma)$  which is induced by the rescattering effects. Thus, measurement of  $Y(5S) \rightarrow \eta_b + \gamma$  would be an ideal probe for the rescattering

mechanism which successfully explains the data of  $Y(5S) \rightarrow Y(1S, 2S) + \pi\pi$ . By contrast, in the tetraquark scenario, the two light quark-antiquark would merge into an energetic photon. Since a real photon cannot be produced by annihilation of a massive quark and a massive antiquark, the quark and antiquark in the tetraquark must be much off shell or exchange gluons with *b* and  $\bar{b}$ ; thus a suppression should be expected. Thus the measurement on  $Y(5S) \rightarrow \eta_b + \gamma$  may distinguish the contributions of the two proposed scenarios. This is one of the tasks of the LHCb which will be operating very soon. If their results give a rather large decay rate on  $Y(5S) \rightarrow \eta_b + \gamma$ , it would strongly support the rescattering mechanism. Otherwise the tetraquark structure scenario would be more favorable.

Recently our experimental colleagues have made great progress. The *BABAR* and CLEO collaborations succeeded to measure the mass  $m_{\eta_b}$  and the  $B(\Upsilon(3S) \rightarrow \eta_b + \gamma)$ and  $B(\Upsilon(2S) \rightarrow \eta_b + \gamma)$  which offer an opportunity for us to study  $B(\Upsilon(5S) \rightarrow \eta_b + \gamma)$ . We expect that our

- K. F. Chen *et al.* (Belle Collaboration), Phys. Rev. Lett. 100, 112001 (2008).
- [2] C. Meng and K.T. Chao, Phys. Rev. D 77, 074003 (2008).
- [3] C. Meng and K.T. Chao, Phys. Rev. D 78, 074001 (2008).
- [4] N. Arnaud (BABAR Collaboration), arXiv:0805.2591.
- [5] C. Meng and K. T. Chao, Phys. Rev. D 78, 034022 (2008).
- [6] Yu. A. Simonov and A. I. Veselov, Phys. Lett. B 671, 55 (2009).
- [7] I. Adachi et al. (Belle Collaboration), arXiv:0808.2445.
- [8] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B 667, 1 (2008).
- [9] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. 95, 142001 (2005).
- [10] S.L. Zhu, Phys. Lett. B 625, 212 (2005).
- [11] F.J. Llanes-Estrada, Phys. Rev. D 72, 031503 (2005).
- [12] L. Maiani, V. Riquer, F. Piccinini, and A. D. Polosa, Phys. Rev. D 72, 031502 (2005).
- [13] E. Kou and O. Pene, Phys. Lett. B 631, 164 (2005).
- [14] X. Liu, X. Q. Zeng, and X. Q. Li, Phys. Rev. D 72, 054023 (2005).
- [15] F.E. Close and P.R. Page, Phys. Lett. B 628, 215 (2005).
- [16] C.F. Qiao, Phys. Lett. B 639, 263 (2006).
- [17] C. Z. Yuan, P. Wang, and X. H. Mo, Phys. Lett. B 634, 399 (2006).
- [18] W.S. Hou, Phys. Rev. D 74, 017504 (2006).
- [19] S. Godfrey and S. L. Olsen, Annu. Rev. Nucl. Part. Sci. 58, 51 (2008).
- [20] M. Karliner and H. J. Lipkin, arXiv:0802.0649.
- [21] A. Ali, C. Hambrock, I. Ahmed, and M. J. Aslam, Phys. Lett. B 684, 28 (2010).
- [22] A. Ali, C. Hambrock, and M. J. Aslam, Phys. Rev. Lett. 104, 162001 (2010).

experimental colleagues will carry out the measurement on  $\Upsilon(5S) \rightarrow \eta_b + \gamma$  pretty soon.

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- [23] S. Recksiegel and Y. Sumino, Phys. Lett. B 578, 369 (2004).
- [24] E. J. Eichten and C. Quigg, Phys. Rev. D 49, 5845 (1994).
- [25] L. Motyka and K. Zalewski, Eur. Phys. J. C 4, 107 (1998).
- [26] X. Liao and T. Manke, Phys. Rev. D 65, 074508 (2002).
- [27] A. Gray, I. Allison, C. T. H. Davies, E. Dalgic, G. P. Lepage, J. Shigemitsu, and M. Wingate, Phys. Rev. D 72, 094507 (2005).
- [28] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D 67, 014027 (2003).
- [29] G. Hao, C. F. Qiao, and P. Sun, Phys. Rev. D 76, 125013 (2007).
- [30] G. Hao, Y. Jia, C. F. Qiao, and P. Sun, J. High Energy Phys. 02 (2007) 057.
- [31] H. W. Ke, J. Tang, X. Q. Hao, and X. Q. Li, Phys. Rev. D 76, 074035 (2007).
- [32] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. 101, 071801 (2008); 102, 029901(E) (2009).
- [33] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. 103, 161801 (2009).
- [34] G. Bonvicini *et al.* (CLEO Collaboration), Phys. Rev. D 81, 031104 (2010).
- [35] X. Liu, X. Q. Zeng, and X. Q. Li, Phys. Rev. D 74, 074003 (2006).
- [36] M. E. Peskin and D. V. Schroeder, An Introduction to Quantum Field Theory (Addison-Wesley, Reading, MA, 1995), p. 842.
- [37] H. Y. Cheng, C. K. Chua, and A. Soni, Phys. Rev. D 71, 014030 (2005).
- [38] X. Liu, B. Zhang, and S. L. Zhu, Phys. Lett. B 645, 185 (2007).
- [39] X. Liu, B. Zhang, and X. Q. Li, Phys. Lett. B 675, 441 (2009).

## HONG-WEI KE, XUE-QIAN LI, AND XIANG LIU

- [40] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. 281, 145 (1997).
- [41] D. Y. Chen, Y. B. Dong, and X. Liu, arXiv:1005.0066.
- [42] H. M. Choi, Phys. Rev. D 75, 073016 (2007).
- [43] H. M. Choi, J. Korean Phys. Soc. 53, 1205 (2008).
- [44] N. Isgur and M.B. Wise, Phys. Lett. B 232, 113 (1989)
- [45] H. W. Ke, X. Q. Li, and X. Liu, arXiv:1002.1187.
- [46] H. W. Ke, X. Q. Li, Z. T. Wei, and X. Liu, Phys. Rev. D 82, 034023 (2010).