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Light scalar mesons and charmless hadronic $B_c \to SP, SV$ decays in the perturbative QCD approach

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The scalar productions in heavy meson decays can provide a good platform to study not only heavy flavor physics but also their own physical properties in a dramatically different way. In this work, based on the assumption of two-quark structure of the scalars, the charmless hadronic $B_c \to SP, SV$ decays (here, S, P, and V denote the light scalar, pseudoscalar, and vector mesons, respectively) are investigated by employing the perturbative QCD (pQCD) factorization approach. In the standard model all these considered B_c meson decays can only occur through the annihilation diagrams. From our numerical evaluations and phenomenological analysis, we find that (a) the pQCD predictions for the CP-averaged branching ratios (BRs) of the considered B_c decays vary in the range of 10^{-5} to 10^{-8}, which will be tested in the ongoing LHCb and forthcoming Super-B experiments, while the CP-violating asymmetries for these modes are absent naturally in the standard model because only one type tree operator is involved; (b) for $B_c \to SP, SV$ decays, the BRs of $\Delta S = 0$ processes are basically much larger than those of $\Delta S = 1$ as generally expected because the different Cabibbo-Kobayashi-Maskawa factors are involved; (c) analogous to $B \to K^* \eta^{(\prime)}$ decays, $\mathrm{Br}(B_c \to \kappa^+ \eta) \sim 5 \times \mathrm{Br}(B_c \to \kappa^+ \eta')$ in the pQCD approach, which can be understood by the constructive and destructive interference between the η_q and η_s contributions to the $B_c \to \kappa^+ \eta$ and $B_c \to \kappa^+ \eta'$ decays, however, $\mathrm{Br}(B_c \to K_0^*(1430)\eta)$ is approximately equal to $\mathrm{Br}(B_c \to K_0^*(1430)\eta')$ in both scenarios because the factorizable contributions from the η_s term play the dominant role in the considered two channels; (d) if $a_0(980)$ and κ are the $q\bar{q}$ bound states, the pQCD predicted BRs for $B_c \to a_0(980)(\pi, \rho)$ and $B_c \to \kappa K^{(*)}$ decays will be in the range of $10^{-6} \sim 10^{-5}$, which are within the reach of the LHCb experiments and could be measured in the near future; and (e) for the $a_0(1450)$ and $K_0^*(1430)$ channels, the BRs for $B_c \to a_0(1450)(\pi, \rho)$ and $B_c \to K_0^*(1430)K^{(*)}$ modes in the pQCD approach are found to be $(5 \sim 47) \times 10^{-6}$ and $(0.7\text{–}36) \times 10^{-6}$, respectively. A measurement of them at the predicted level will favor the $q\bar{q}$ structure and help understand the physical properties of the scalars and the involved QCD dynamics in the modes, especially the reliability of the pQCD approach to these B_c meson decays.

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I. INTRODUCTION

The scalar mesons are especially important to understand because they have the same quantum numbers as the vacuum ($J^{PC} = 0^{++}$). Great efforts have been made by the physicists on both experimental and theoretical aspects to understand the inner structure of the scalars but it is well known that the underlying structure of them is not yet well established (for a review, see e.g. [1–3]). Up to now, many different possible solutions to the scalars have been proposed such as $\bar{q}q$, $\bar{q}\,\bar{q}\,qq$, meson-meson bound states or even supplemented with a scalar glueball. More likely, they are not made of one simple component but are the superpositions of these contents. The different scenarios tend to give very different predictions on the production and decay of the scalar mesons which are helpful to determine the dominant component.

The first charmless B decay into a scalar meson, i.e., $B \to f_0(980)K$, was measured by Belle [4] in 2002 (updated in [5]) and subsequently confirmed by *BABAR* [6] in 2004. After that these two B factories operated at KEK and SLAC respectively have found many decay channels with the scalars as one of the productions in B meson decays [1,7]. These measurements should provide information on the nature of the scalar mesons. It is enough reason to believe that, as a different unique insight to the internal structure of the scalars, the heavy B meson decaying into scalar mesons can provide a good place to explore their physical properties.

Recently, the production of scalar mesons with $q\bar{q}$ structure in the two-body charmless B decays has been intensively studied in Refs. [8–12] theoretically, in which many predictions are within the reach of the current B factory experiments and to be examined in the near future. It is hoped that through the study of $B_c \to SP, SV$ (here, S, P, and V are the light scalar, pseudoscalar, and vector mesons, respectively) decays, old puzzles related to the internal

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structure and related parameters, e.g., the masses and widths, of light scalar mesons can receive new understanding.

Experimentally, the Large Hadron Collider (LHC) experiment at CERN is running now, where the B_c meson could be produced abundantly. Motivated by the forthcoming large number of B_c production and decay events in the ongoing LHCb experiments, the scalar meson spectrum would become one of the most interesting topics for both experimental and theoretical studies in the near future. At that time, more and more channels with scalar mesons will be opened and tested in the experiment, which will help us to further explore the nature of the scalars. On the other hand, for the B_c meson, one can study the two heavy flavors b and c in a meson simultaneously. The B_c meson decays may also provide windows for studying the perturbative and nonperturbative QCD, final state interactions, testing the predictions of the standard model (SM), and can shed light on new physics scenarios beyond the SM [13].

Inspired by the above observations, in this work, we therefore will focus on the two-body charmless hadronic decays $B_c \to SP, SV$, which can only occur through the weak annihilation diagrams. The size of annihilation contributions has been an important issue in B physics for many years. The importance of annihilation contributions has already been tested in the previous predictions of branching ratios of pure annihilation $B \to D_s K$ decays [14], direct CP asymmetries of $B^0 \to \pi^+\pi^-$, $K^+\pi^-$ decays [15–17], and in the explanation of the $B \to \phi K^*$ polarization problem [18,19] though there still exist many different viewpoints.[1] The two-body B decays into the final states with one scalar meson may suffer from large weak annihilation contributions, which have been analyzed in Refs. [10–12] preliminarily. Thus it is very interesting to explore the size of annihilation contributions in these considered $B_c \to SP, SV$ channels, which will also be helpful to investigate the annihilated decay mechanism and the physical properties of the scalars.

In this paper, we will study the CP-averaged branching ratios (BRs) of charmless hadronic $B_c \to SP, SV$ decays by employing the low energy effective Hamiltonian [23] and the perturbative QCD (pQCD) factorization approach [15,16,24]. By keeping the transverse momentum k_T of the quarks, the pQCD approach is free of end-point singularity and the Sudakov formalism makes it more self-consistent. Rather different from the QCD factorization approach [25] and soft-collinear effective theory, the pQCD approach can be used to calculate the annihilation diagrams straightforwardly [26], as has been done, for example, in Refs. [11,12,14–18,27–30].

The paper is organized as follows. In Sec. II, we present a brief review of light scalar mesons and the formalism of the pQCD approach. The wave functions and distribution amplitudes for heavy B_c and light scalar, pseudoscalar, and vector mesons are also given here. Then we perform the perturbative calculations for the considered $B_c \to SP, SV$ decay channels with the pQCD approach in Sec. III. The analytic formulas of the decay amplitudes for all the considered modes are also collected in this section. The numerical results and phenomenological analysis are given in Sec. IV. Finally, Sec. V contains the main conclusions and a short summary.

II. LIGHT SCALAR MESONS, FORMALISM, AND WAVE FUNCTIONS

A. Light scalar mesons

Until now, the people have discovered many scalar states experimentally but know little about their underlying structures, which are not well established theoretically yet (for a review, see Refs. [1–3]). According to the meson particle collected by the Particle Data Group [1], the light scalar mesons below or near 1 GeV, including $a_0(980)$, $K_0^*(800)$ (or κ), $f_0(600)$ (or σ), and $f_0(980)$, are usually viewed to form an SU(3) flavor nonet; while scalar mesons around 1.5 GeV, including $a_0(1450)$, $K_0^*(1430)$, $f_0(1370)$, and $f_0(1500)/f_0(1710)$ form another nonet.[2]

Recently, Cheng, Chua, and Yang [10] proposed two possible scenarios to describe these light scalar mesons in the QCD sum rule method:

(1) In scenario 1(S1), the scalar mesons in the former nonet are treated as the lowest lying states, and in the latter one as the corresponding first excited states, respectively. Based on the naive two-quark model, the flavor structure of the light scalar mesons in S1 read

$$\sigma = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \qquad f_0 = s\bar{s}, \qquad a_0^+ = u\bar{d},$$
$$a_0^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \qquad a_0^- = d\bar{u}, \qquad \kappa^+ = u\bar{s},$$
$$\kappa^0 = d\bar{s}, \qquad \bar{\kappa}^0 = s\bar{d}, \qquad \kappa^- = s\bar{u}. \tag{1}$$

[1] Recently, the authors announced in Ref. [20] that the annihilation contributions in charmless hadronic B decays are real and small in the soft-collinear effective theory [21] at leading power, while the authors in another work [22] discussed that they may be the almost imaginary contributions, which can generate a sizable strong phase. This discrepancy between these two approaches/methods needs to be clarified definitely by the experiments in the future.

[2] For the sake of simplicity, we will use a_0 and f_0 to denote $a_0(980)$ and $f_0(980)$, respectively, unless otherwise stated. We will also adopt the forms a, K_0^*, f, and f' to denote the scalar mesons $a_0(980)$ and $a_0(1450)$, $K_0^*(800)$ and $K_0^*(1430)$, $f_0(600)$ and $f_0(1370)$, and $f_0(980)$ and $f_0(1500)/f_0(1710)$ correspondingly in the following sections, unless otherwise stated.

Here, it is assumed that the lightest σ and heaviest f_0 in the lighter scalar nonet has the ideal mixing. But various experimental data indicate that f_0 should not have the pure $s\bar{s}$ component and the isoscalars σ and f_0 must have a mixing of f_0^q and f_0^s [10], which is analogous to the $\eta - \eta'$ mixing system,

$$\begin{pmatrix} \sigma \\ f_0 \end{pmatrix} = \begin{pmatrix} \cos\theta_0 & -\sin\theta_0 \\ \sin\theta_0 & \cos\theta_0 \end{pmatrix} \begin{pmatrix} f_0^q \\ f_0^s \end{pmatrix}, \tag{2}$$

with $f_0^q = (\bar{u}u + \bar{d}d)/\sqrt{2}$ and $f_0^s = \bar{s}s$, where θ_0 is the mixing angle between σ and f_0. Many works have been made to explore the mixing angle θ_0 [31]: θ_0 lies in the ranges of $25° < \theta_0 < 40°$ and $140° < \theta_0 < 165°$. But the fact that θ_0 tends to be not a unique value, indicates that σ and f_0 may not be purely $q\bar{q}$ states.

While the mixing of the isosinglet scalar mesons $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ has been discussed in detail in the literature (see Ref. [32] and references therein), in this work we will adopt the mixing mechanism as given in Ref. [32]:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.78 & 0.51 & -0.36 \\ -0.54 & 0.84 & 0.03 \\ 0.32 & 0.18 & 0.93 \end{pmatrix} \begin{pmatrix} f_0^q \\ f_0^s \\ f_0^G \end{pmatrix}. \tag{3}$$

As discussed in [32], it is evident that $f_0(1370)$ and $f_0(1500)$ mainly consist of f_0^q and f_0^s, just with small or tiny glueball components; however, $f_0(1710)$ is composed primarily of the scalar glueball, i.e., f_0^G. We will therefore only take the scalar mesons $f_0(1370)$ and $f_0(1500)$ into account in the present work, and leave the contribution from scalar glueball content for future study.

(2) In scenario 2(S2), the scalar mesons in the latter nonet are the lowest lying resonances and the corresponding first excited states lie between (2.0–2.3) GeV. S2 corresponds to the case that light scalar mesons below or near 1 GeV are four-quark bound states, while all scalar mesons are made of two quarks in S1. In order to give quantitative predictions, since we do not know how to deal with the four-quark states in the factorization approach presently, we here just consider the evaluations on the scalar mesons with $q\bar{q}$ structure in S2.

In short, we will investigate these light scalar mesons in the pure annihilation $B_c \to SP, SV$ decays with the assumption of two-quark structure proposed in the above two possible scenarios.

B. Formalism of pQCD approach

Since the b quark is rather heavy, we work in the frame with the B_c meson at rest, i.e., with the B_c meson momentum $P_1 = (m_{B_c}/\sqrt{2})(1, 1, \mathbf{0}_T)$ in the light-cone coordinates. For the charmless hadronic $B_c \to M_2M_3$[3] decays, assuming that the M_2 (M_3) meson moves in the plus (minus) z direction carrying the momentum P_2 (P_3) and the longitudinal polarization vector ϵ_2^L (ϵ_3^L) (if $M_{2(3)}$ is the vector meson). Then the two final state meson momenta can be written as

$$P_2 = \frac{m_{B_c}}{\sqrt{2}}(1 - r_3^2, r_2^2, \mathbf{0}_T), \qquad P_3 = \frac{m_{B_c}}{\sqrt{2}}(r_3^2, 1 - r_2^2, \mathbf{0}_T), \tag{4}$$

respectively, where $r_2 = m_{M_2}/m_{B_c}$ and $r_3 = m_{M_3}/m_{B_c}$. When M_2 or M_3 is a vector meson, the longitudinal polarization vector, ϵ_2^L or ϵ_3^L, can be given by

$$\epsilon_2^L = \frac{m_{B_c}}{\sqrt{2}m_{M_2}}(1 - r_3^2, -r_2^2, \mathbf{0}_T), \quad \text{or} \quad \epsilon_3^L = \frac{m_{B_c}}{\sqrt{2}m_{M_3}}(-r_3^2, 1 - r_2^2, \mathbf{0}_T). \tag{5}$$

Putting the (light-)quark momenta in B_c, M_2, and M_3 mesons as k_1, k_2, and k_3, respectively, we can choose

$$k_1 = (x_1 P_1^+, 0, \mathbf{k}_{1T}), \quad k_2 = (x_2 P_2^+, 0, \mathbf{k}_{2T}), \quad k_3 = (0, x_3 P_3^-, \mathbf{k}_{3T}). \tag{6}$$

Then, for $B_c \to M_2M_3$ decays, the integration over k_1^-, k_2^-, and k_3^+ will conceptually lead to the decay amplitudes in the pQCD approach,

$$\mathcal{A}(B_c \to M_2M_3) \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \times \mathrm{Tr}[C(t)\Phi_{B_c}(x_1, b_1)\Phi_{M_2}(x_2, b_2)\Phi_{M_3}(x_3, b_3) \times H(x_i, b_i, t)S_t(x_i)e^{-S(t)}], \tag{7}$$

where b_i is the conjugate space coordinate of k_{iT}, and t is the largest energy scale in function $H(x_i, b_i, t)$. The large logarithms $\ln(m_W/t)$ are included in the Wilson coefficients $C(t)$. The large double logarithms ($\ln^2 x_i$) are summed by the threshold resummation [33], and they lead to $S_t(x_i)$ which smears the end-point singularities on x_i. The last term, $e^{-S(t)}$, is the Sudakov form factor which suppresses the soft dynamics effectively [34]. Thus, it makes the perturbative calculation of the hard part H applicable at intermediate scale, i.e., m_{B_c} scale. We

[3]For the sake of simplicity, we will use M_2 and M_3 to denote the two final state light mesons, respectively, unless otherwise stated.

will calculate analytically the function $H(x_i, b_i, t)$ for the considered decays at leading order (LO) in α_s expansion and give the convoluted amplitudes in the next section.

For these considered decays, the related weak effective Hamiltonian $H_{\rm eff}$ [23] can be written as

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}[V_{cb}^* V_{uD}(C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu))], \quad (8)$$

with the local four-quark tree operators $O_{1,2}$:

$$\begin{aligned} O_1 &= \bar{u}_\beta \gamma^\mu (1-\gamma_5) D_\alpha \bar{c}_\beta \gamma^\mu (1-\gamma_5) b_\alpha, \\ O_2 &= \bar{u}_\beta \gamma^\mu (1-\gamma_5) D_\beta \bar{c}_\alpha \gamma^\mu (1-\gamma_5) b_\alpha, \end{aligned} \quad (9)$$

where V_{cb}, V_{uD} are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, "D" denotes the light down quark d or s and $C_i(\mu)$ are Wilson coefficients at the renormalization scale μ. For the Wilson coefficients $C_{1,2}(\mu)$, we will also use the leading order expressions, although the next-to-leading order calculations already exist in the literature [23]. This is the consistent way to cancel the explicit μ dependence in the theoretical formulas. For the renormalization group evolution of the Wilson coefficients from higher scale to lower scale, we use the formulas as given in Ref. [16] directly.

C. Wave functions and distribution amplitudes

In order to calculate the decay amplitude, we should choose the proper wave function of the heavy B_c meson. In principle, there are two Lorentz structures in the $B_q (q = u, d, s)$ or B_c meson wave function. One should consider both of them in the calculations. However, since the contribution induced by one Lorentz structure is numerically small [28,35] and can be neglected approximately, we only consider the contribution from the first Lorentz structure:

$$\Phi_{B_c}(x) = \frac{i}{\sqrt{2N_c}}[(\not{P} + M_{B_c})\gamma_5 \phi_{B_c}(x)]_{\alpha\beta}. \quad (10)$$

Since the B_c meson consists of two heavy quarks and $m_{B_c} \simeq m_b + m_c$, the distribution amplitude ϕ_{B_c} would be close to $\delta(x - m_c/m_{B_c})$ in the nonrelativistic limit. We therefore adopt the nonrelativistic approximation form of ϕ_{B_c} as [36]

$$\phi_{B_c}(x) = \frac{f_{B_c}}{2\sqrt{2N_c}} \delta(x - m_c/m_{B_c}), \quad (11)$$

where f_{B_c} and N_c are the decay constant of B_c meson and the color number, respectively.

The wave function for the scalar meson (S) can generally be defined as

$$\begin{aligned} \Phi_S(x) = \frac{i}{\sqrt{2N_c}}\{&\not{P}\phi_S(x) + m_S \phi_S^S(x) \\ &+ m_S(\not{v}\not{n} - 1)\phi_S^T(x)\}_{\alpha\beta}, \end{aligned} \quad (12)$$

where ϕ_S, $\phi_S^{S,T}$, and m_S are the leading twist and twist-3 distribution amplitudes, and mass of the scalar meson, respectively, while x denotes the momentum fraction carried by quark in the meson, and $n = (1, 0, \mathbf{0}_T)$ and $v = (0, 1, \mathbf{0}_T)$ are dimensionless lightlike unit vectors.

In general, the leading twist light-cone distribution amplitude $\phi_S(x, \mu)$ can be expanded as the Gegenbauer polynomials [10,37]:

$$\begin{aligned} \phi_S(x, \mu) = \frac{3}{\sqrt{2N_c}} x(1-x)\Big\{&f_S(\mu) + \bar{f}_S(\mu) \\ &\times \sum_{m=1}^{\infty} B_m(\mu) C_m^{3/2}(2x-1)\Big\}, \end{aligned} \quad (13)$$

where $f_S(\mu)$, $\bar{f}_S(\mu)$, $B_m(\mu)$, and $C_m^{3/2}(t)$ are the vector and scalar decay constants, Gegenbauer moments, and Gegenbauer polynomials for the scalars, respectively.

Because of the charge conjugation invariance, neutral scalar mesons cannot be produced by the vector current and thus

$$f_\sigma = f_{f_0} = f_{a_0^0} = 0. \quad (14)$$

For other scalar mesons, there exists a relation between the vector and scalar decay constants,

$$\bar{f}_S = \mu_S f_S \quad \text{and} \quad \mu_S = \frac{m_S}{m_2(\mu) - m_1(\mu)}, \quad (15)$$

where m_1 and m_2 are the running current quark masses in the scalars. For the neutral scalar mesons f_0, a_0^0 and σ, f_S vanishes, but the quantity $\bar{f}_S = f_S \mu_S$ remains finite.

The values for scalar decay constants and Gegenbauer moments in the scalar meson distribution amplitudes have been investigated at scale $\mu = 1$ GeV in Ref. [10]:

$$\begin{aligned} &\bar{f}_{a_0} = 0.365 \pm 0.020\ \text{GeV}, \quad B_1 = -0.93 \pm 0.10, \\ &B_3 = 0.14 \pm 0.08\ (\text{S1}), \quad \bar{f}_\kappa = 0.340 \pm 0.020\ \text{GeV}, \\ &B_1 = -0.92 \pm 0.11, \quad B_3 = 0.15 \pm 0.09\ (\text{S1}), \\ &\bar{f}_{f_0} = 0.370 \pm 0.020\ \text{GeV}, \quad B_1 = -0.92 \pm 0.11, \\ &B_3 = 0.15 \pm 0.09\ (\text{S1}); \end{aligned} \quad (16)$$

$$\begin{aligned} &\bar{f}_{a_0(1450)} = -0.280 \pm 0.030\ \text{GeV}, \\ &\quad B_1 = 0.89 \pm 0.20, \quad B_3 = -1.38 \pm 0.18\ (\text{S1}), \\ &\bar{f}_{a_0(1450)} = 0.460 \pm 0.050\ \text{GeV}, \quad B_1 = -0.58 \pm 0.12, \\ &\quad B_3 = -0.49 \pm 0.15\ (\text{S2}); \end{aligned} \quad (17)$$

$$\begin{aligned} &\bar{f}_{K_0^*(1430)} = -0.300 \pm 0.030\ \text{GeV}, \\ &\quad B_1 = 0.58 \pm 0.07, \quad B_3 = -1.20 \pm 0.08\ (\text{S1}), \\ &\bar{f}_{K_0^*(1430)} = 0.445 \pm 0.050\ \text{GeV}, \quad B_1 = -0.57 \pm 0.13, \\ &\quad B_3 = -0.42 \pm 0.22\ (\text{S2}); \end{aligned} \quad (18)$$

$$\bar{f}_{f_0(1500)} = -0.255 \pm 0.030 \text{ GeV},$$
$$B_1 = 0.80 \pm 0.40, \qquad B_3 = -1.32 \pm 0.14 \,(\text{S1}),$$
$$\bar{f}_{f_0(1500)} = 0.490 \pm 0.050 \text{ GeV}, \qquad B_1 = -0.48 \pm 0.11,$$
$$B_3 = -0.37 \pm 0.20 \,(\text{S2}). \tag{19}$$

As for the twist-3 distribution amplitudes ϕ_S^S and ϕ_S^T, we adopt the asymptotic forms:

$$\phi_S^S = \frac{1}{2\sqrt{2N_c}} \bar{f}_S, \qquad \phi_S^T = \frac{1}{2\sqrt{2N_c}} \bar{f}_S(1-2x). \tag{20}$$

Here x stands for the momentum fraction carried by s quark of the relevant strange scalar meson.

For pseudoscalar meson (P), the wave function can be generally defined as

$$\Phi_P(x) = \frac{i}{\sqrt{2N_c}} \gamma_5 \{ \not{P} \phi_P^A(x) + m_0^P \phi_P^P(x) + m_0^P (\not{v} \not{n} - 1) \phi_P^T(x) \}_{\alpha\beta}, \tag{21}$$

where $\phi_P^{A,P,T}$ and m_0^P are the distribution amplitudes and chiral scale parameter of the pseudoscalar meson, respectively.

For the wave functions of vector meson (V), one longitudinal (L) polarization is involved, and can be written as

$$\Phi_V^L(x) = \frac{1}{\sqrt{2N_c}} \{ m_V \not{\epsilon}_V^{*L} \phi_V(x) + \not{\epsilon}_V^{*L} \not{P} \phi_V^t(x) + m_V \phi_V^s(x) \}_{\alpha\beta}, \tag{22}$$

where ϵ_V^L denotes the longitudinal polarization vector of vector mesons, satisfying $P \cdot \epsilon = 0$, ϕ_V, $\phi_V^{t,s}$, and m_V are the leading twist and twist-3 distribution amplitudes, and mass of the vector meson, respectively. For the distribution amplitudes of pseudoscalar $\phi_P^{A,P,T}$, and longitudinal polarization, ϕ_V and $\phi_V^{t,s}$ to be used in this work, we will adopt the same forms as that in the literature (see Ref. [30] and references therein).

III. PERTURBATIVE CALCULATIONS IN THE PQCD APPROACH

From the effective Hamiltonian (8), there are four types of diagrams contributing to the $B_c \to M_2 M_3$ decays as illustrated in Fig. 1, which result in the Feynman decay amplitudes $\mathcal{F}_{fa}^{M_2M_3}$ and $\mathcal{M}_{na}^{M_2M_3}$, where the subscripts fa and na are the abbreviations of factorizable and nonfactorizable annihilation contributions, respectively. Operators $O_{1,2}$ are $(V-A)(V-A)$ currents; we therefore can combine all contributions from these diagrams and obtain the total decay amplitude as

$$\mathcal{A}(B_c \to M_2 M_3) = V_{cb}^* V_{uD} \{ f_{B_c} \mathcal{F}_{fa}^{M_2M_3} a_1 + \mathcal{M}_{na}^{M_2M_3} C_1 \}, \tag{23}$$

where $a_1 = C_1/3 + C_2$. In the next two subsections, we will give the explicit expressions of $\mathcal{F}_{fa}^{M_2M_3}$, $\mathcal{M}_{na}^{M_2M_3}$ and the decay amplitude $\mathcal{A}(B_c \to M_2 M_3)$ for $B_c \to M_2 M_3$ decays, including 32 $B_c \to SP(PS)$ and 30 $B_c \to SV(VS)$ decay modes.

A. $B_c \to SP(PS)$ decays

In this subsection, we will present the factorization formulas for charmless hadronic $B_c \to SP(PS)$ decays. From the first two diagrams of Fig. 1, i.e., (a) and (b), by perturbative QCD calculations, we obtain the decay amplitude for factorizable annihilation contributions as follows:

$$\begin{aligned} \mathcal{F}_{fa}^{SP} = {} & 8\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \\ & \times \{ h_{fa}(1-x_3, x_2, b_3, b_2) E_{fa}(t_a) [x_2 \phi_S(x_2) \phi_P^A(x_3) \\ & + 2 r_S r_0^P \phi_P^P(x_3)((x_2+1)\phi_S^S(x_2) + (x_2-1)\phi_S^T(x_2))] \\ & + h_{fa}(x_2, 1-x_3, b_2, b_3) E_{fa}(t_b) [(x_3-1)\phi_S(x_2)\phi_P^A(x_3) \\ & + 2 r_S r_0^P \phi_S^S(x_2)((x_3-2)\phi_P^P(x_3) - x_3 \phi_P^T(x_3))] \}, \end{aligned} \tag{24}$$

where $\phi_{S(P)}$ corresponds to the distribution amplitudes of mesons $S(P)$, $r_S = m_S/m_{B_c}$, $r_0^P = m_0^P/m_{B_c}$, and $C_F = 4/3$ is a color factor. The function h_{fa}, the scales t_i, and $E_{fa}(t)$ can be found in Appendix B of Ref. [30].

For the nonfactorizable diagrams (c) and (d) in Fig. 1, all three meson wave functions are involved. The integration of b_3 can be performed using δ function $\delta(b_3 - b_2)$, leaving only integration of b_1 and b_2. The corresponding decay amplitude is

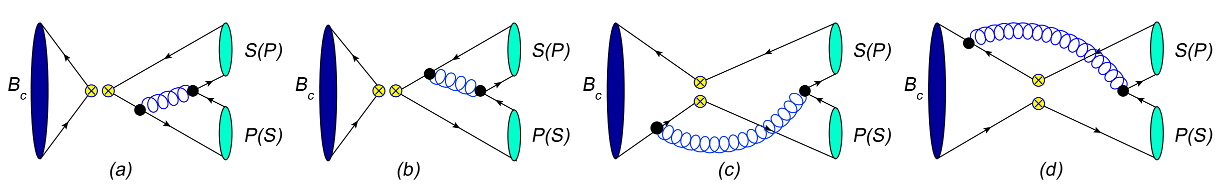

FIG. 1 (color online). Typical Feynman diagrams for two-body charmless hadronic $B_c \to SP(PS)$ decays at leading order. By replacing the pseudoscalar meson P in (a)–(d) with the vector meson V, one will obtain the corresponding Feynman diagrams for $B_c \to SV(VS)$ decay modes.

$$\mathcal{M}_{na}^{SP} = \frac{16\sqrt{6}}{3}\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \{h_{na}^c(x_2, x_3, b_1, b_2) E_{na}(t_c)[(r_c - x_3 + 1)\phi_S(x_2)\phi_P^A(x_3) + r_S r_0^P (\phi_S^S(x_2) \times ((3r_c + x_2 - x_3 + 1)\phi_P^P(x_3) - (r_c - x_2 - x_3 + 1)\phi_P^T(x_3)) + \phi_S^T(x_2)((r_c - x_2 - x_3 + 1)\phi_P^P(x_3) + (r_c - x_2 + x_3 - 1)\phi_P^T(x_3)))] - E_{na}(t_d)[(r_b + r_c + x_2 - 1)\phi_S(x_2)\phi_P^A(x_3) + r_S r_0^P (\phi_S^S(x_2) \times ((4r_b + r_c + x_2 - x_3 - 1)\phi_P^P(x_3) - (r_c + x_2 + x_3 - 1)\phi_P^T(x_3)) + \phi_S^T(x_2)((r_c + x_2 + x_3 - 1)\phi_P^P(x_3) - (r_c + x_2 - x_3 - 1)\phi_P^T(x_3)))] h_{na}^d(x_2, x_3, b_1, b_2)\}, \tag{25}$$

where $r_b = m_b/m_{B_c}$, $r_c = m_c/m_{B_c}$, and $r_b + r_c \approx 1$ in the B_c meson.

Likewise, we can get the analytic factorization formulas of the contributions from $B_c \to PS$ decays easily:

$$\mathcal{F}_{fa}^{PS} = 8\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \{h_{fa}(1 - x_3, x_2, b_3, b_2) E_{fa}(t_a)[x_2 \phi_P^A(x_2)\phi_S(x_3) - 2r_0^P r_S \phi_S^S(x_3)((x_2 + 1)\phi_P^P(x_2) + (x_2 - 1)\phi_P^T(x_2))] + h_{fa}(x_2, 1 - x_3, b_2, b_3) E_{fa}(t_b)[(x_3 - 1)\phi_P^A(x_2)\phi_S(x_3) - 2r_0^P r_S \phi_P^P(x_2)((x_3 - 2)\phi_S^S(x_3) - x_3 \phi_S^T(x_3))]\}, \tag{26}$$

$$\mathcal{M}_{na}^{PS} = \frac{16\sqrt{6}}{3}\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \{h_{na}^c(x_2, x_3, b_1, b_2) E_{na}(t_c)[(r_c - x_3 + 1)\phi_P^A(x_2)\phi_S(x_3) - r_0^P r_S (\phi_P^P(x_2) \times ((3r_c + x_2 - x_3 + 1)\phi_S^S(x_3) - (r_c - x_2 - x_3 + 1)\phi_S^T(x_3)) + \phi_P^T(x_2)((r_c - x_2 - x_3 + 1)\phi_S^S(x_3) + (r_c - x_2 + x_3 - 1)\phi_P^T(x_3)))] - E_{na}(t_d)[(r_b + r_c + x_2 - 1)\phi_P^A(x_2)\phi_S(x_3) - r_0^P r_S (\phi_P^P(x_2) \times ((4r_b + r_c + x_2 - x_3 - 1)\phi_S^S(x_3) - (r_c + x_2 + x_3 - 1)\phi_S^T(x_3)) + \phi_P^T(x_2)((r_c + x_2 + x_3 - 1) \times \phi_S^S(x_3) - (r_c + x_2 - x_3 - 1)\phi_S^T(x_3)))] h_{na}^d(x_2, x_3, b_1, b_2)\}. \tag{27}$$

Based on Eqs. (23)–(27), we can write down the total decay amplitudes for 32 $B_c \to SP(PS)$ decays straightforwardly:

$$\mathcal{A}(B_c \to a^+\pi^0) = V_{cb}^* V_{ud}\{[f_{B_c}\mathcal{F}_{fa}^{a\pi} a_1 + \mathcal{M}_{na}^{a\pi} C_1] - [f_{B_c}\mathcal{F}_{fa}^{\pi a} a_1 + \mathcal{M}_{na}^{\pi a} C_1]\}/\sqrt{2}, \tag{28}$$

$$\mathcal{A}(B_c \to a^0\pi^+) = V_{cb}^* V_{ud}\{[f_{B_c}\mathcal{F}_{fa}^{\pi a} a_1 + \mathcal{M}_{na}^{\pi a} C_1] - [f_{B_c}\mathcal{F}_{fa}^{a\pi} a_1 + \mathcal{M}_{na}^{a\pi} C_1]\}/\sqrt{2}; \tag{29}$$

$$\mathcal{A}(B_c \to a^+\eta) = V_{cb}^* V_{ud}\{[f_{B_c}\mathcal{F}_{fa}^{a\eta_q} a_1 + \mathcal{M}_{na}^{a\eta_q} C_1] + [f_{B_c}\mathcal{F}_{fa}^{\eta_q a} a_1 + \mathcal{M}_{na}^{\eta_q a} C_1]\}\cos\phi, \tag{30}$$

$$\mathcal{A}(B_c \to a^+\eta') = V_{cb}^* V_{ud}\{[f_{B_c}\mathcal{F}_{fa}^{a\eta_q} a_1 + \mathcal{M}_{na}^{a\eta_q} C_1] + [f_{B_c}\mathcal{F}_{fa}^{\eta_q a} a_1 + \mathcal{M}_{na}^{\eta_q a} C_1]\}\sin\phi; \tag{31}$$

$$\mathcal{A}(B_c \to f\pi^+) = V_{cb}^* V_{ud}\{[f_{B_c}\mathcal{F}_{fa}^{\pi f_0^q} a_1 + \mathcal{M}_{na}^{\pi f_0^q} C_1] + [f_{B_c}\mathcal{F}_{fa}^{f_0^q \pi} a_1 + \mathcal{M}_{na}^{f_0^q \pi} C_1]\}\cos\theta_0, \tag{32}$$

$$\mathcal{A}(B_c \to f'\pi^+) = V_{cb}^* V_{ud}\{[f_{B_c}\mathcal{F}_{fa}^{\pi f_0^q} a_1 + \mathcal{M}_{na}^{\pi f_0^q} C_1] + [f_{B_c}\mathcal{F}_{fa}^{f_0^q \pi} a_1 + \mathcal{M}_{na}^{f_0^q \pi} C_1]\}\sin\theta_0; \tag{33}$$

$$\mathcal{A}(B_c \to K_0^{*+}\bar{K}^0) = V_{cb}^* V_{ud}\{f_{B_c}\mathcal{F}_{fa}^{K_0^*\bar{K}} a_1 + \mathcal{M}_{na}^{K_0^*\bar{K}} C_1\}, \tag{34}$$

$$\mathcal{A}(B_c \to \bar{K}_0^{*0}K^+) = V_{cb}^* V_{ud}\{f_{B_c}\mathcal{F}_{fa}^{\bar{K}_0^* K^+} a_1 + \mathcal{M}_{na}^{\bar{K}_0^* K^+} C_1\}; \tag{35}$$

$$\mathcal{A}(B_c \to K_0^{*0}\pi^+) = V_{cb}^* V_{us}\{f_{B_c}\mathcal{F}_{fa}^{K_0^*\pi} a_1 + \mathcal{M}_{na}^{K_0^*\pi} C_1\}, \tag{36}$$

$$= \sqrt{2}\mathcal{A}(B_c \to K_0^{*+}\pi^0); \tag{37}$$

$$\mathcal{A}(B_c \to a^+K^0) = V_{cb}^* V_{us}\{f_{B_c}\mathcal{F}_{fa}^{Ka} a_1 + \mathcal{M}_{na}^{Ka} C_1\}, \tag{38}$$

$$= \sqrt{2}\mathcal{A}(B_c \to K^+a^0); \tag{39}$$

$$\mathcal{A}(B_c \to K_0^{*+}\eta) = V_{cb}^* V_{us}\{f_{B_c}[\mathcal{F}_{fa}^{K_0^*\eta_q}\cos\phi - \mathcal{F}_{fa}^{\eta_s K_0^*}\sin\phi] a_1 + [\mathcal{M}_{na}^{K_0^*\eta_q}\cos\phi - \mathcal{M}_{na}^{\eta_s K_0^*}\sin\phi] C_1\}, \tag{40}$$

$$\mathcal{A}(B_c \to K_0^{*+}\eta') = V_{cb}^* V_{us}\{f_{B_c}[\mathcal{F}_{fa}^{K_0^*\eta_q}\sin\phi + \mathcal{F}_{fa}^{\eta_s K_0^*}\cos\phi] a_1 + [\mathcal{M}_{na}^{K_0^*\eta_q}\sin\phi + \mathcal{M}_{na}^{\eta_s K_0^*}\cos\phi] C_1\}; \tag{41}$$

$$\mathcal{A}(B_c \to fK^+) = V_{cb}^* V_{us}\{f_{B_c}[\mathcal{F}_{fa}^{Kf_0^q}\cos\theta_0 - \mathcal{F}_{fa}^{f_0^s K}\sin\theta_0]a_1 + [\mathcal{M}_{na}^{Kf_0^q}\cos\theta_0 - \mathcal{M}_{na}^{f_0^s K}\sin\theta_0]C_1\}, \quad (42)$$

$$\mathcal{A}(B_c \to f'K^+) = V_{cb}^* V_{us}\{f_{B_c}[\mathcal{F}_{fa}^{Kf_0^q}\sin\theta_0 + \mathcal{F}_{fa}^{f_0^s K}\cos\theta_0]a_1 + [\mathcal{M}_{na}^{Kf_0^q}\sin\theta_0 + \mathcal{M}_{na}^{f_0^s K}\cos\theta_0]C_1\}. \quad (43)$$

B. $B_c \to SV(VS)$ decays

After the replacement of the pseudoscalar meson P with the vector meson V in Fig. 1, we will get the Feynman diagrams for pure annihilation $B_c \to SV(VS)$ modes at leading order. By following the same procedure as stated in the above subsection, we can obtain the analytic decay amplitudes for $B_c \to SV$ decays:

$$\mathcal{F}_{fa}^{SV} = -8\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \{h_{fa}(1-x_3, x_2, b_3, b_2)E_{fa}(t_a)[x_2\phi_S(x_2)\phi_V(x_3) - 2r_S r_V \phi_V^s(x_3)((x_2+1)\phi_S^S(x_2) + (x_2-1)\phi_S^T(x_2))] + h_{fa}(x_2, 1-x_3, b_2, b_3)E_{fa}(t_b)[(x_3-1)\phi_S(x_2)\phi_V(x_3) - 2r_S r_V \phi_S^S(x_2)((x_3-2)\phi_V^s(x_3) - x_3\phi_V^t(x_3))]\}, \quad (44)$$

$$\mathcal{M}_{na}^{SV} = -\frac{16\sqrt{6}}{3}\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \{h_{na}^c(x_2, x_3, b_1, b_2)E_{na}(t_c)[(r_c - x_3 + 1)\phi_S(x_2) \times \phi_V(x_3) - r_S r_V(\phi_S^S(x_2)((3r_c + x_2 - x_3 + 1)\phi_V^s(x_3) - (r_c - x_2 - x_3 + 1)\phi_V^t(x_3)) + \phi_S^T(x_2) \times ((r_c - x_2 - x_3 + 1)\phi_V^s(x_3) + (r_c - x_2 + x_3 - 1)\phi_V^t(x_3)))] - E_{na}(t_d)[(r_b + r_c + x_2 - 1)\phi_S(x_2)\phi_V(x_3) - r_S r_V(\phi_S^S(x_2)((4r_b + r_c + x_2 - x_3 - 1)\phi_V^s(x_3) - (r_c + x_2 + x_3 - 1)\phi_V^t(x_3)) + \phi_S^T(x_2) \times ((r_c + x_2 + x_3 - 1)\phi_V^s(x_3) - (r_c + x_2 - x_3 - 1)\phi_V^t(x_3)))]h_{na}^d(x_2, x_3, b_1, b_2)\}, \quad (45)$$

with $r_V = m_V/m_{B_c}$.

Similarly, the factorization formulas for $B_c \to VS$ decays can be easily obtained but with the simple replacements in Eqs. (44) and (45) as follows:

$$\phi_S \leftrightarrow \phi_V, \quad \phi_S^S \leftrightarrow \phi_V^s, \quad \phi_S^T \leftrightarrow \phi_V^t, \quad r_S \leftrightarrow r_V. \quad (46)$$

The total decay amplitudes of the 30 $B_c \to SV(VS)$ decays can therefore be written as

$$\mathcal{A}(B_c \to a^+\rho^0) = V_{cb}^* V_{ud}\{[f_{B_c}\mathcal{F}_{fa}^{a\rho}a_1 + \mathcal{M}_{na}^{a\rho}C_1] - [f_{B_c}\mathcal{F}_{fa}^{\rho a}a_1 + \mathcal{M}_{na}^{\rho a}C_1]\}/\sqrt{2}, \quad (47)$$

$$\mathcal{A}(B_c \to a^0\rho^+) = V_{cb}^* V_{ud}\{[f_{B_c}\mathcal{F}_{fa}^{\rho a}a_1 + \mathcal{M}_{na}^{\rho a}C_1] - [f_{B_c}\mathcal{F}_{fa}^{a\rho}a_1 + \mathcal{M}_{na}^{a\rho}C_1]\}/\sqrt{2}; \quad (48)$$

$$\mathcal{A}(B_c \to a^+\omega) = V_{cb}^* V_{ud}\{[f_{B_c}\mathcal{F}_{fa}^{a\omega}a_1 + \mathcal{M}_{na}^{a\omega}C_1] + [f_{B_c}\mathcal{F}_{fa}^{\omega a}a_1 + \mathcal{M}_{na}^{\omega a}C_1]\}/\sqrt{2}; \quad (49)$$

$$\mathcal{A}(B_c \to f\rho^+) = V_{cb}^* V_{ud}\{[f_{B_c}\mathcal{F}_{fa}^{\rho f_0^q}a_1 + \mathcal{M}_{na}^{\rho f_0^q}C_1] + [f_{B_c}\mathcal{F}_{fa}^{f_0^q\rho}a_1 + \mathcal{M}_{na}^{f_0^q\rho}C_1]\}\cos\theta_0, \quad (50)$$

$$\mathcal{A}(B_c \to f'\rho^+) = V_{cb}^* V_{ud}\{[f_{B_c}\mathcal{F}_{fa}^{\rho f_0^q}a_1 + \mathcal{M}_{na}^{\rho f_0^q}C_1] + [f_{B_c}\mathcal{F}_{fa}^{f_0^q\rho}a_1 + \mathcal{M}_{na}^{f_0^q\rho}C_1]\}\sin\theta_0; \quad (51)$$

$$\mathcal{A}(B_c \to K_0^{*+}\bar{K}^{*0}) = V_{cb}^* V_{ud}\{f_{B_c}\mathcal{F}_{fa}^{\bar{K}^* K_0^*}a_1 + \mathcal{M}_{na}^{\bar{K}^* K_0^*}C_1\}, \quad (52)$$

$$\mathcal{A}(B_c \to \bar{K}_0^{*0}K^{*+}) = V_{cb}^* V_{ud}\{f_{B_c}\mathcal{F}_{fa}^{\bar{K}_0^* K^{*+}}a_1 + \mathcal{M}_{na}^{\bar{K}_0^* K^{*+}}C_1\}; \quad (53)$$

$$\mathcal{A}(B_c \to K_0^{*0}\rho^+) = V_{cb}^* V_{us}\{f_{B_c}\mathcal{F}_{fa}^{K_0^*\rho}a_1 + \mathcal{M}_{na}^{K_0^*\rho}C_1\}, \quad (54)$$

$$= \sqrt{2}\mathcal{A}(B_c \to K_0^{*+}\rho^0); \quad (55)$$

$$\mathcal{A}(B_c \to a^+K^{*0}) = V_{cb}^* V_{us}\{f_{B_c}\mathcal{F}_{fa}^{K^* a}a_1 + \mathcal{M}_{na}^{K^* a}C_1\}, \quad (56)$$

$$= \sqrt{2}\mathcal{A}(B_c \to K^{*+}a^0); \quad (57)$$

$$\mathcal{A}(B_c \to K_0^{*+}\omega) = V_{cb}^* V_{us}\{f_{B_c}\mathcal{F}_{fa}^{K_0^*\omega}a_1 + \mathcal{M}_{na}^{K_0^*\omega}C_1\}/\sqrt{2}, \quad (58)$$

$$\mathcal{A}(B_c \to K_0^{*+}\phi) = V_{cb}^* V_{us}\{f_{B_c}\mathcal{F}_{fa}^{\phi K_0^*} a_1 + \mathcal{M}_{na}^{\phi K_0^*} C_1\}; \tag{59}$$

$$\mathcal{A}(B_c \to fK^{*+}) = V_{cb}^* V_{us}\{f_{B_c}[\mathcal{F}_{fa}^{K^* f_0^q}\cos\theta_0 - \mathcal{F}_{fa}^{f_0^s K^*}\sin\theta_0]a_1 + [\mathcal{M}_{na}^{K^* f_0^q}\cos\theta_0 - \mathcal{M}_{na}^{f_0^s K^*}\sin\theta_0]C_1\}, \tag{60}$$

$$\mathcal{A}(B_c \to f'K^{*+}) = V_{cb}^* V_{us}\{f_{B_c}[\mathcal{F}_{fa}^{K^* f_0^q}\sin\theta_0 + \mathcal{F}_{fa}^{f_0^s K^*}\cos\theta_0]a_1 + [\mathcal{M}_{na}^{K^* f_0^q}\sin\theta_0 + \mathcal{M}_{na}^{f_0^s K^*}\cos\theta_0]C_1\}. \tag{61}$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will make the theoretical predictions on the CP-averaged BRs for those considered $B_c \to SP, SV$ decay modes. First of all, the central values of the input parameters to be used are given in the following:

(i) Masses (GeV):

$$m_W = 80.41, \quad m_{B_c} = 6.286, \quad m_b = 4.8, \quad m_c = 1.5; \quad m_\phi = 1.02,$$
$$m_{K^*} = 0.892, \quad m_\rho = 0.770, \quad m_\omega = 0.782; \quad m_{a_0} = 0.985, \quad m_\kappa = 0.800,$$
$$m_\sigma = 0.600, \quad m_{f_0} = 0.980; \quad m_{a_0(1450)} = 1.474, \quad m_{K_0^*(1430)} = 1.425, \quad m_{f_0(1370)} = 1.350,$$
$$m_{f_0(1500)} = 1.505; \quad m_0^\pi = 1.4, \quad m_0^K = 1.6, \quad m_0^{\eta_q} = 1.08, \quad m_0^{\eta_s} = 1.92. \tag{62}$$

(ii) Decay constants (GeV):

$$f_\phi = 0.231, \quad f_\phi^T = 0.200, \quad f_{K^*} = 0.217,$$
$$f_{K^*}^T = 0.185; \quad f_\rho = 0.209, \quad f_\rho^T = 0.165,$$
$$f_\omega = 0.195, \quad f_\omega^T = 0.145; \quad f_\pi = 0.131,$$
$$f_K = 0.16, \quad f_{B_c} = 0.489. \tag{63}$$

(iii) QCD scale and B_c meson lifetime:

$$\Lambda_{\overline{\rm MS}}^{(f=4)} = 0.250 \text{ GeV}, \qquad \tau_{B_c} = 0.46 \text{ ps}. \tag{64}$$

Here, we adopt the Wolfenstein parametrization, and the updated parameters $A = 0.814$, $\lambda = 0.2257$, $\bar\rho = 0.135$, and $\bar\eta = 0.349$ [1] for the CKM matrix. In numerical calculations, central values of the input parameters will be used implicitly unless otherwise stated.

For $B_c \to SP, SV$ decays, the decay rate can be written as

$$\Gamma = \frac{G_F^2 m_{B_c}^3}{32\pi}(1 - r_S^2)|\mathcal{A}(B_c \to M_2 M_3)|^2, \tag{65}$$

where the corresponding decay amplitudes $\mathcal{A}$ have been given explicitly in Eqs. (28)–(43) and (47)–(61). Using the decay amplitudes obtained in the last section, it is straightforward to calculate the CP-averaged BRs with uncertainties as presented in Tables I, II, III, IV, V, VI, VII, and VIII. The dominant errors come from the uncertainties of charm quark mass $m_c = 1.5 \pm 0.15$ GeV, the scalar decay constants $\bar f_S$, the Gegenbauer moments a_i of the relevant pseudoscalar or vector meson distribution amplitudes, the Gegenbauer moments B_i of the scalar meson distribution amplitudes, and the chiral enhancement factors $m_0^\pi = 1.4 \pm 0.3$ GeV and $m_0^K = 1.6 \pm 0.1$ GeV, respectively.

Among the considered $B_c \to SP, SV$ decays, the pQCD predictions for the CP-averaged BRs of those $\Delta S = 0$ processes are basically much larger than those of $\Delta S = 1$ channels (one of the two final state mesons is a strange one), the main reason is the enhancement of the large CKM factor $|V_{ud}/V_{us}|^2 \sim 19$ for those $\Delta S = 0$ decays as generally expected. Maybe there exist no such large differences for certain decays, which is just because the enhancement arising from the CKM factor is partially canceled by the difference between the magnitude of individual decay amplitudes. The pQCD predictions for the CP-averaged BRs of considered B_c decays vary in the range of 10^{-5} to 10^{-8}. For $B_c \to a_0(1450)^+\pi^0$ decay with a rate of 10^{-5}–10^{-6} for example, we show the decay amplitudes arising from both factorization and nonfactorization annihilation contributions explicitly (in units of 10^{-3} GeV^3),

$$\begin{aligned}\mathcal{A}_{fa}(B_c \to a_0(1450)^+\pi^0) &= 0.292 + i2.489;\\ \mathcal{A}_{na}(B_c \to a_0(1450)^+\pi^0) &= 6.717 + i7.508;\end{aligned} \tag{66}$$

in S1, while

$$\begin{aligned}\mathcal{A}_{fa}(B_c \to a_0(1450)^+\pi^0) &= 0.553 - i0.356;\\ \mathcal{A}_{na}(B_c \to a_0(1450)^+\pi^0) &= 3.161 - i5.137\end{aligned} \tag{67}$$

in S2, where the central values are quoted for clarification. One can find that the dominant nonfactorizable decay amplitude governs this channel and subsequently results in the large branching ratio in both scenarios, which can be seen in Table III. The other modes with large decay rates can be analyzed similarly.

TABLE I. The pQCD predictions of branching ratios (BRs) for the $\Delta S = 0$ processes of charmless hadronic $B_c \to (a_0, \kappa, \sigma, f_0)(\pi, K, \eta, \eta')$ decays in S1. The source of the dominant errors is explained in the text.

$\Delta S = 0$ Decay modes	BRs (10^{-6})
$B_c \to a_0^+ \pi^0$	$6.5^{+2.3}_{-1.5}(m_c)^{+0.9}_{-0.6}(\bar{f}_S)^{+2.1}_{-1.4}(a_2^{\pi})^{+1.4}_{-1.1}(B^S_{1,3})^{+0.4}_{-0.7}(m_0)$
$B_c \to a_0^0 \pi^+$	$3.5^{+1.6}_{-1.0}(m_c)^{+0.4}_{-0.4}(\bar{f}_S)^{+1.0}_{-0.6}(a_2^{\pi})^{+1.1}_{-0.9}(B^S_{1,3})^{+0.7}_{-0.4}(m_0)$
$B_c \to a_0^+ \eta \times 10$	$3.6^{+3.4}_{-0.9}(m_c)^{+0.4}_{-0.3}(\bar{f}_S)^{+1.7}_{-0.4}(a_2^{\eta})^{+1.6}_{-0.6}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)$
$B_c \to a_0^+ \eta' \times 10$	$2.4^{+2.2}_{-0.6}(m_c)^{+0.3}_{-0.2}(\bar{f}_S)^{+1.1}_{-0.3}(a_2^{\eta'})^{+1.1}_{-0.4}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)$
$B_c \to \bar{\kappa}^0 K^+$	$3.4^{+2.1}_{-1.1}(m_c)^{+0.5}_{-0.4}(\bar{f}_S)^{+1.8}_{-1.4}(a^K_{1,2})^{+1.4}_{-0.9}(B^S_{1,3})^{+0.2}_{-0.0}(m_0)$
$B_c \to \bar{K}^0 \kappa^+$	$2.1^{+0.1}_{-0.0}(m_c)^{+0.3}_{-0.2}(\bar{f}_S)^{+1.5}_{-0.1}(a^K_{1,2})^{+0.7}_{-0.4}(B^S_{1,3})^{+0.3}_{-0.1}(m_0)$
$B_c \to \pi^+ \sigma \times 10$	$3.2^{+1.9}_{-0.0}(m_c)^{+0.3}_{-0.3}(\bar{f}_S)^{+1.1}_{-1.3}(a_2^{\pi})^{+0.9}_{-0.7}(B^S_{1,3})^{+0.1}_{-0.3}(m_0)(f_0^q)$
$B_c \to \pi^+ f_0 \times 10$	$1.8^{+1.1}_{-0.0}(m_c)^{+0.2}_{-0.2}(\bar{f}_S)^{+1.6}_{-0.6}(a_2^{\pi})^{+0.7}_{-0.2}(B^S_{1,3})^{+0.7}_{-0.0}(m_0)(f_0^q)$

As discussed in Ref. [38], the B_c decays with the branching ratio of 10^{-6} can be measured at the LHC. Hence, our pQCD predicted BRs with 10^{-6} or larger for these $B_c \to SP, SV$ decays are expected to be measured in the ongoing LHCb experiments, which will be very helpful to study the physical contents of the scalars and the involved QCD dynamics and annihilation mechanism in the considered channels. Moreover, there is no CP violation for all these decays within the SM, since there is only one kind of tree operator involved in the decay amplitude of all considered B_c decays, which can be seen from Eq. (23).

A. $B_c \to a_0(P, V)$ and $B_c \to a_0(1450)(P, V)$ decays

In this subsection, we will make some discussions on the $B_c \to a(P, V)$ decays involving 14 $\Delta S = 0$ and 8 $\Delta S = 1$ processes, respectively.

From the numerical results for considered modes as given in Tables I, III, V, and VII, one can find that the CP-averaged BRs for all the $\Delta S = 0$ $B_c \to a(P, V)$ processes are in the range of 10^{-6}–10^{-5} within the theoretical errors except for $B_c \to a_0^+ \eta^{(\prime)}$ decays, which are expected to be tested by the ongoing LHCb measurements and the forthcoming Super-B experiments. Since we make the perturbative calculations based on the assumption of two-quark structure for the scalars, once these theoretical predictions could be verified by the related experiments, then these results will help us to explore the underlying structure of the scalar a meson.

For $B_c \to a_0(\pi, \rho)$ decays, their BRs can be read from Tables I and V (in units of 10^{-6}),

$$\mathrm{Br}(B_c \to a_0^+ \pi^0) = 6.5^{+3.6}_{-2.5}, \quad \mathrm{Br}(B_c \to a_0^0 \pi^+) = 3.5^{+2.3}_{-1.6}, \tag{68}$$

$$\mathrm{Br}(B_c \to a_0^+ \rho^0) = 12.7^{+6.1}_{-5.6}, \quad \mathrm{Br}(B_c \to a_0^0 \rho^+) = 10.6^{+5.5}_{-3.5}, \tag{69}$$

TABLE II. Same as Table I but for the $\Delta S = 1$ processes of charmless hadronic $B_c \to (a_0, \kappa, \sigma, f_0)(\pi, K, \eta, \eta')$ decays in S1.

$\Delta S = 1$ Decay modes	BRs (10^{-7})
$B_c \to a_0^+ K^0$	$4.0^{+0.4}_{-0.9}(m_c)^{+0.4}_{-0.5}(\bar{f}_S)^{+1.7}_{-1.7}(a^K_{1,2})^{+0.7}_{-1.0}(B^S_{1,3})^{+0.0}_{-0.2}(m_0)$
$B_c \to a_0^0 K^+$	$2.0^{+0.2}_{-0.5}(m_c)^{+0.2}_{-0.3}(\bar{f}_S)^{+0.9}_{-0.9}(a^K_{1,2})^{+0.4}_{-0.5}(B^S_{1,3})^{+0.0}_{-0.1}(m_0)$
$B_c \to \kappa^+ \eta$	$4.5^{+1.3}_{-0.9}(m_c)^{+0.5}_{-0.5}(\bar{f}_S)^{+0.9}_{-0.6}(a_2^{\eta})^{+0.8}_{-0.9}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)$
$B_c \to \kappa^+ \eta' \times 10$	$8.8^{+2.4}_{-0.1}(m_c)^{+1.3}_{-0.8}(\bar{f}_S)^{+0.6}_{-0.7}(a_2^{\eta'})^{+3.7}_{-2.5}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)$
$B_c \to \kappa^0 \pi^+$	$2.1^{+1.1}_{-0.6}(m_c)^{+0.2}_{-0.2}(\bar{f}_S)^{+0.6}_{-0.3}(a_2^{\pi})^{+0.6}_{-0.4}(B^S_{1,3})^{+0.1}_{-0.0}(m_0)$
$B_c \to \kappa^+ \pi^0$	$1.1^{+0.6}_{-0.3}(m_c)^{+0.1}_{-0.1}(\bar{f}_S)^{+0.3}_{-0.2}(a_2^{\pi})^{+0.4}_{-0.2}(B^S_{1,3})^{+0.1}_{-0.0}(m_0)$
$B_c \to K^+ \sigma$	$1.6^{+0.2}_{-0.3}(m_c)^{+0.1}_{-0.2}(\bar{f}_S)^{+0.6}_{-0.7}(a^K_{1,2})^{+0.4}_{-0.4}(B^S_{1,3})^{+0.0}_{-0.2}(m_0)(f_0^q)$
	$0.9^{+0.5}_{-0.5}(m_c)^{+0.1}_{-0.1}(\bar{f}_S)^{+0.3}_{-0.5}(a^K_{1,2})^{+0.2}_{-0.4}(B^S_{1,3})^{+0.0}_{-0.1}(m_0)(f_0^s)$
$B_c \to K^+ f_0$	$1.8^{+0.4}_{-0.3}(m_c)^{+0.2}_{-0.2}(\bar{f}_S)^{+0.6}_{-0.6}(a^K_{1,2})^{+0.3}_{-0.4}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)(f_0^q)$
	$0.3^{+0.5}_{-0.2}(m_c)^{+0.0}_{-0.1}(\bar{f}_S)^{+0.4}_{-0.2}(a^K_{1,2})^{+0.1}_{-0.1}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)(f_0^s)$

TABLE III. Same as Table I but for the $\Delta S = 0$ processes of charmless hadronic $B_c \to (a_0(1450), K_0^*(1430), f_0(1370), f_0(1500))(\pi, K, \eta, \eta')$ decays in S1 and S2, respectively.

$\Delta S = 0$ Decay modes	BRs (10^{-6})
$B_c \to a_0(1450)^+\pi^0$	$21.0^{+6.9}_{-5.7}(m_c)^{+4.7}_{-4.3}(\bar{f}_S)^{+4.9}_{-5.6}(a_2^\pi)^{+4.0}_{-5.7}(B^S_{1,3})^{+0.0}_{-0.2}(m_0)$ (S1)
	$6.3^{+4.4}_{-2.6}(m_c)^{+1.4}_{-1.3}(\bar{f}_S)^{+0.8}_{-0.3}(a_2^\pi)^{+2.8}_{-1.8}(B^S_{1,3})^{+0.5}_{-0.2}(m_0)$ (S2)
$B_c \to a_0(1450)^0\pi^+$	$11.9^{+3.8}_{-3.1}(m_c)^{+2.7}_{-2.4}(\bar{f}_S)^{+1.3}_{-2.4}(a_2^\pi)^{+2.8}_{-2.1}(B^S_{1,3})^{+0.5}_{-0.9}(m_0)$ (S1)
	$4.9^{+3.6}_{-2.3}(m_c)^{+1.1}_{-1.0}(\bar{f}_S)^{+0.4}_{-0.4}(a_2^\pi)^{+2.6}_{-1.2}(B^S_{1,3})^{+0.0}_{-0.4}(m_0)$ (S2)
$B_c \to a_0(1450)^+\eta$	$2.7^{+0.4}_{-0.1}(m_c)^{+0.6}_{-0.4}(\bar{f}_S)^{+0.5}_{-0.0}(a_2^\eta)^{+1.1}_{-0.7}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)$ (S1)
	$1.0^{+0.2}_{-0.3}(m_c)^{+0.2}_{-0.2}(\bar{f}_S)^{+0.2}_{-0.1}(a_2^\eta)^{+0.7}_{-0.5}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)$ (S2)
$B_c \to a_0(1450)^+\eta'$	$1.1^{+0.2}_{-0.1}(m_c)^{+0.4}_{-0.3}(\bar{f}_S)^{+0.3}_{-0.0}(a_2^{\eta'})^{+0.7}_{-0.5}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)$ (S1)
	$0.6^{+0.2}_{-0.2}(m_c)^{+0.2}_{-0.1}(\bar{f}_S)^{+0.1}_{-0.1}(a_2^{\eta'})^{+0.5}_{-0.3}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)$ (S2)
$B_c \to \bar{K}_0^*(1430)^0K^+$	$19.2^{+4.7}_{-5.4}(m_c)^{+4.1}_{-3.9}(\bar{f}_S)^{+2.5}_{-3.8}(a^K_{1,2})^{+2.2}_{-1.9}(B^S_{1,3})^{+0.3}_{-0.5}(m_0)$ (S1)
	$0.7^{+0.7}_{-0.2}(m_c)^{+0.2}_{-0.2}(\bar{f}_S)^{+0.8}_{-0.5}(a^K_{1,2})^{+1.1}_{-0.9}(B^S_{1,3})^{+0.0}_{-0.1}(m_0)$ (S2)
$B_c \to \bar{K}^0K_0^*(1430)^+$	$4.3^{+0.8}_{-0.9}(m_c)^{+0.9}_{-0.9}(\bar{f}_S)^{+1.4}_{-2.2}(a^K_{1,2})^{+1.0}_{-0.8}(B^S_{1,3})^{+0.2}_{-0.0}(m_0)$ (S1)
	$9.2^{+1.4}_{-2.0}(m_c)^{+2.0}_{-2.0}(\bar{f}_S)^{+0.7}_{-0.9}(a^K_{1,2})^{+3.8}_{-4.3}(B^S_{1,3})^{+0.0}_{-0.4}(m_0)$ (S2)
$B_c \to f_0(1370)\pi^+$	$3.6^{+1.2}_{-0.7}(m_c)^{+0.9}_{-0.8}(\bar{f}_S)^{+1.9}_{-1.3}(a_2^\pi)^{+0.9}_{-1.2}(B^S_{1,3})^{+0.3}_{-0.0}(m_0)(f_0^q$, S1)
	$1.0^{+0.2}_{-0.3}(m_c)^{+0.2}_{-0.2}(\bar{f}_S)^{+0.4}_{-0.4}(a_2^\pi)^{+1.0}_{-0.9}(B^S_{1,3})^{+0.1}_{-0.1}(m_0)(f_0^q$, S2)
$B_c \to f_0(1500)\pi^+$	$3.7^{+1.1}_{-1.0}(m_c)^{+0.9}_{-0.8}(\bar{f}_S)^{+1.8}_{-1.5}(a_2^\pi)^{+0.6}_{-1.3}(B^S_{1,3})^{+0.2}_{-0.3}(m_0)(f_0^q$, S1)
	$0.9^{+0.2}_{-0.3}(m_c)^{+0.2}_{-0.2}(\bar{f}_S)^{+0.5}_{-0.2}(a_2^\pi)^{+1.1}_{-0.6}(B^S_{1,3})^{+0.1}_{-0.0}(m_0)(f_0^q$, S2)

TABLE IV. Same as Table I but for the $\Delta S = 1$ processes of charmless hadronic $B_c \to (a_0(1450), K_0^*(1430), f_0(1370), f_0(1500))(\pi, K, \eta, \eta')$ decays in S1 and S2, respectively.

$\Delta S = 1$ Decay modes	BRs (10^{-7})
$B_c \to a_0(1450)^+K^0$	$2.3^{+1.0}_{-0.7}(m_c)^{+0.5}_{-0.5}(\bar{f}_S)^{+2.3}_{-1.2}(a^K_{1,2})^{+1.2}_{-0.8}(B^S_{1,3})^{+0.1}_{-0.1}(m_0)$ (S1)
	$6.7^{+2.6}_{-1.8}(m_c)^{+1.6}_{-1.4}(\bar{f}_S)^{+2.3}_{-0.8}(a^K_{1,2})^{+2.5}_{-1.9}(B^S_{1,3})^{+0.1}_{-0.2}(m_0)$ (S2)
$B_c \to a_0(1450)^0K^+$	$1.2^{+0.5}_{-0.4}(m_c)^{+0.3}_{-0.3}(\bar{f}_S)^{+1.2}_{-0.6}(a^K_{1,2})^{+0.6}_{-0.4}(B^S_{1,3})^{+0.1}_{-0.1}(m_0)$ (S1)
	$3.4^{+1.3}_{-0.9}(m_c)^{+0.8}_{-0.7}(\bar{f}_S)^{+1.2}_{-0.4}(a^K_{1,2})^{+1.3}_{-1.0}(B^S_{1,3})^{+0.1}_{-0.1}(m_0)$ (S2)
$B_c \to K_0^*(1430)^+\eta$	$5.4^{+2.8}_{-1.9}(m_c)^{+1.1}_{-1.1}(\bar{f}_S)^{+1.5}_{-1.6}(a_2^\eta)^{+0.7}_{-0.8}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)$ (S1)
	$6.2^{+2.6}_{-1.9}(m_c)^{+1.5}_{-1.3}(\bar{f}_S)^{+0.3}_{-0.0}(a_2^\eta)^{+1.4}_{-2.5}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)$ (S2)
$B_c \to K_0^*(1430)^+\eta'$	$3.3^{+0.0}_{-0.6}(m_c)^{+0.5}_{-0.7}(\bar{f}_S)^{+0.2}_{-0.3}(a_2^{\eta'})^{+0.0}_{-0.5}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)$ (S1)
	$5.1^{+0.2}_{-0.3}(m_c)^{+1.2}_{-1.1}(\bar{f}_S)^{+0.7}_{-0.3}(a_2^{\eta'})^{+2.7}_{-1.7}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)$ (S2)
$B_c \to K_0^*(1430)^0\pi^+$	$6.5^{+3.5}_{-2.3}(m_c)^{+1.2}_{-1.3}(\bar{f}_S)^{+0.4}_{-0.8}(a_2^\pi)^{+0.9}_{-0.8}(B^S_{1,3})^{+0.1}_{-0.2}(m_0)$ (S1)
	$0.8^{+0.9}_{-0.5}(m_c)^{+0.2}_{-0.2}(\bar{f}_S)^{+0.0}_{-0.1}(a_2^\pi)^{+0.5}_{-0.4}(B^S_{1,3})^{+0.0}_{-0.1}(m_0)$ (S2)
$B_c \to K_0^*(1430)^+\pi^0$	$3.2^{+1.8}_{-1.2}(m_c)^{+0.6}_{-0.6}(\bar{f}_S)^{+0.2}_{-0.4}(a_2^\pi)^{+0.5}_{-0.4}(B^S_{1,3})^{+0.1}_{-0.1}(m_0)$ (S1)
	$0.4^{+0.5}_{-0.3}(m_c)^{+0.1}_{-0.1}(\bar{f}_S)^{+0.0}_{-0.1}(a_2^\pi)^{+0.3}_{-0.2}(B^S_{1,3})^{+0.0}_{-0.1}(m_0)$ (S2)
$B_c \to f_0(1370)K^+$	$0.9^{+0.4}_{-0.3}(m_c)^{+0.3}_{-0.2}(\bar{f}_S)^{+0.9}_{-0.4}(a^K_{1,2})^{+0.5}_{-0.3}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)(f_0^q$, S1)
	$2.7^{+0.9}_{-0.8}(m_c)^{+0.6}_{-0.5}(\bar{f}_S)^{+0.6}_{-0.3}(a^K_{1,2})^{+1.2}_{-1.1}(B^S_{1,3})^{+0.1}_{-0.1}(m_0)(f_0^q$, S2)
	$7.7^{+1.8}_{-1.7}(m_c)^{+1.9}_{-1.7}(\bar{f}_S)^{+3.5}_{-2.2}(a^K_{1,2})^{+1.6}_{-2.0}(B^S_{1,3})^{+0.1}_{-0.0}(m_0)(f_0^s$, S1)
	$0.7^{+0.2}_{-0.0}(m_c)^{+0.1}_{-0.2}(\bar{f}_S)^{+0.8}_{-0.4}(a^K_{1,2})^{+0.6}_{-0.5}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)(f_0^s$, S2)
$B_c \to f_0(1500)K^+$	$0.9^{+0.4}_{-0.3}(m_c)^{+0.2}_{-0.2}(\bar{f}_S)^{+0.8}_{-0.4}(a^K_{1,2})^{+0.5}_{-0.3}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)(f_0^q$, S1)
	$2.8^{+1.0}_{-0.8}(m_c)^{+0.5}_{-0.6}(\bar{f}_S)^{+0.4}_{-0.4}(a^K_{1,2})^{+1.1}_{-1.1}(B^S_{1,3})^{+0.1}_{-0.1}(m_0)(f_0^q$, S2)
	$7.9^{+1.5}_{-1.3}(m_c)^{+2.0}_{-1.7}(\bar{f}_S)^{+3.5}_{-2.3}(a^K_{1,2})^{+1.6}_{-2.0}(B^S_{1,3})^{+0.4}_{-0.0}(m_0)(f_0^s$, S1)
	$0.8^{+0.2}_{-0.0}(m_c)^{+0.2}_{-0.1}(\bar{f}_S)^{+0.8}_{-0.4}(a^K_{1,2})^{+0.5}_{-0.5}(B^S_{1,3})^{+0.0}_{-0.0}(m_0)(f_0^s$, S2)

TABLE V. Same as Table I but for the $\Delta S = 0$ processes of charmless hadronic $B_c \to (a_0, \kappa, \sigma, f_0)(\rho, K^*, \omega, \phi)$ decays in S1.

$\Delta S = 0$ Decay modes	BRs (10^{-6})
$B_c \to a_0^+ \rho^0$	$12.7^{+4.4}_{-3.8}(m_c)^{+1.5}_{-1.3}(\bar{f}_S)^{+2.9}_{-2.9}(a_2^\rho)^{+2.7}_{-2.7}(B_{1,3}^S)$
$B_c \to a_0^0 \rho^+$	$10.6^{+4.4}_{-2.6}(m_c)^{+1.2}_{-1.1}(\bar{f}_S)^{+1.5}_{-1.0}(a_2^\rho)^{+2.7}_{-1.9}(B_{1,3}^S)$
$B_c \to a_0^+ \omega \times 10$	$9.8^{+9.2}_{-3.0}(m_c)^{+0.8}_{-1.3}(\bar{f}_S)^{+3.3}_{-1.9}(a_2^\omega)^{+3.5}_{-1.7}(B_{1,3}^S)$
$B_c \to \bar{\kappa}^0 K^{*+}$	$8.8^{+4.5}_{-2.5}(m_c)^{+1.1}_{-0.9}(\bar{f}_S)^{+2.2}_{-1.0}(a_{1,2}^{K^*})^{+3.0}_{-1.7}(B_{1,3}^S)$
$B_c \to \bar{K}^{*0} \kappa^+$	$4.9^{+0.8}_{-0.7}(m_c)^{+0.6}_{-0.5}(\bar{f}_S)^{+0.6}_{-1.0}(a_{1,2}^{K^*})^{+1.7}_{-1.6}(B_{1,3}^S)$
$B_c \to \rho^+ \sigma \times 10$	$1.6^{+2.1}_{-0.0}(m_c)^{+0.2}_{-0.2}(\bar{f}_S)^{+1.5}_{-1.1}(a_2^\rho)^{+0.6}_{-0.9}(B_{1,3}^S)(f_0^q)$
$B_c \to \rho^+ f_0 \times 10$	$0.8^{+1.4}_{-0.0}(m_c)^{+0.1}_{-0.1}(\bar{f}_S)^{+0.6}_{-0.6}(a_2^\rho)^{+0.7}_{-0.5}(B_{1,3}^S)(f_0^q)$

where the various errors as specified have been added in quadrature. One could find the rather different decay patterns from these theoretical predictions, i.e., Eqs. (68) and (69) that $\mathrm{Br}(B_c \to a_0^+ \pi^0) > \mathrm{Br}(B_c \to a_0^0 \pi^+)$ while $\mathrm{Br}(B_c \to a_0^+ \rho^0) \sim \mathrm{Br}(B_c \to a_0^0 \rho^+)$ within the theoretical uncertainties. Because $f_\rho(f_\rho^T) \sim 1.6(1.3) \times f_\pi$, it is evident that $\mathrm{Br}(B_c \to a_0 \rho) > \mathrm{Br}(B_c \to a_0 \pi)$. Based on these pQCD predictions of BRs for $B_c \to a_0(\pi, \rho)$ decays, which are within the reach of LHCb experiments [38], it is expected that if the observation or the experimental upper limit on the decay modes $B_c \to a_0 \pi (a_0 \rho)$ is much smaller than the expectation, this might rule out the $q\bar{q}$ structure for the a_0.

On the other hand, the isovector scalar meson $a_0(1450)$ has been confirmed to be a conventional $q\bar{q}$ meson in lattice calculations [39–43] recently. Hence, the calculations for the $a_0(1450)$ channels should be more trustworthy. Our results shown in Tables III and VII indicate that $B_c \to a_0(1450)\pi$ and $B_c \to a_0(1450)\rho$ have large branching ratios, of order $(5\text{–}20) \times 10^{-6}$ and $(15\text{–}47) \times 10^{-6}$, respectively. A measurement of them at the predicted level will reinforce the $q\bar{q}$ nature for the $a_0(1450)$.

For those $B_c \to a(P, V)$ decay modes with $a_0(1450)$ as one of the final states, the pQCD predictions in Tables III and VII show that for the $\Delta S = 0$ processes $B_c \to a_0(1450)(\pi, \eta^{(\prime)}, \rho, \omega)$ the BRs in S1 are much larger than that in S2; however, for the $\Delta S = 1$ processes $B_c \to a_0(1450)K^{(*)}$, the BRs in S1 are much smaller than that in S2, which will be confronted with the ongoing and forthcoming related experiments. It is hoped that the precision measurements could help us to determine which scenario is favored by the experiments, then the inner quark structure definitely.

For $B_c \to a(\eta, \eta')$ decays, the numerical results grouped in Tables I and III indicate the small differences between $B_c \to a\eta$ and $B_c \to a\eta'$ modes, which is mainly because the relevant final state mesons, $\eta^{(\prime)}$, contain the same component $\bar{u}u + \bar{d}d$, just with different coefficients, i.e., $\cos\phi$ and $\sin\phi$. This pattern is very similar to that of $B_c \to \rho\eta^{(\prime)}$ decays [30].

For the $\Delta S = 1$ $B_c \to aK^{(*)}$ processes, the pQCD predicted BRs are in the order of 10^{-7}, which is below the reach of the LHCb experiments ($\sim 10^{-6}$). From these numerical results as displayed in Tables II, IV, VI, and VIII, one can find that $\mathrm{Br}(B_c \to a^+ K^{(*)0}) \approx 2 \times \mathrm{Br}(B_c \to a^0 K^{(*)+})$ although for the a^0 meson the vector decay constant $f_{a^0} = 0$, which exhibits clearly that the contribution is dominated by the odd Gegenbauer moments in the leading twist distribution amplitude of the scalar a meson. This pattern is well consistent with that stressed by the authors in Ref. [10].

TABLE VI. Same as Table I but for the $\Delta S = 1$ processes of charmless hadronic $B_c \to (a_0, \kappa, \sigma, f_0)(\rho, K^*, \omega, \phi)$ decays in S1.

$\Delta S = 1$ Decay modes	BRs (10^{-7})
$B_c \to a_0^+ K^{*0}$	$3.5^{+0.8}_{-0.4}(m_c)^{+0.4}_{-0.3}(\bar{f}_S)^{+0.8}_{-0.5}(a_{1,2}^{K^*})^{+1.2}_{-0.8}(B_{1,3}^S)$
$B_c \to a_0^0 K^{*+}$	$1.7^{+0.4}_{-0.2}(m_c)^{+0.2}_{-0.2}(\bar{f}_S)^{+0.4}_{-0.3}(a_{1,2}^{K^*})^{+0.6}_{-0.4}(B_{1,3}^S)$
$B_c \to \kappa^+ \omega$	$1.9^{+1.0}_{-0.6}(m_c)^{+0.2}_{-0.2}(\bar{f}_S)^{+0.2}_{-0.2}(a_2^\omega)^{+0.6}_{-0.5}(B_{1,3}^S)$
$B_c \to \kappa^+ \phi$	$2.9^{+0.7}_{-0.6}(m_c)^{+0.3}_{-0.3}(\bar{f}_S)^{+0.2}_{-0.6}(a_2^\phi)^{+0.9}_{-1.0}(B_{1,3}^S)$
$B_c \to \kappa^0 \rho^+$	$4.5^{+2.4}_{-1.4}(m_c)^{+0.5}_{-0.5}(\bar{f}_S)^{+0.7}_{-0.3}(a_2^\rho)^{+1.4}_{-0.9}(B_{1,3}^S)$
$B_c \to \kappa^+ \rho^0$	$2.3^{+1.2}_{-0.7}(m_c)^{+0.3}_{-0.3}(\bar{f}_S)^{+0.4}_{-0.2}(a_2^\rho)^{+0.8}_{-0.4}(B_{1,3}^S)$
$B_c \to K^{*+} \sigma$	$1.6^{+0.3}_{-0.1}(m_c)^{+0.2}_{-0.2}(\bar{f}_S)^{+0.3}_{-0.2}(a_{1,2}^{K^*})^{+0.4}_{-0.4}(B_{1,3}^S)(f_0^q)$
	$2.0^{+1.7}_{-0.8}(m_c)^{+0.2}_{-0.2}(\bar{f}_S)^{+0.5}_{-0.0}(a_{1,2}^{K^*})^{+0.7}_{-0.3}(B_{1,3}^S)(f_0^s)$
$B_c \to K^{*+} f_0$	$1.5^{+0.4}_{-0.2}(m_c)^{+0.2}_{-0.1}(\bar{f}_S)^{+0.2}_{-0.1}(a_{1,2}^{K^*})^{+0.5}_{-0.4}(B_{1,3}^S)(f_0^q)$
	$1.9^{+1.2}_{-0.8}(m_c)^{+0.2}_{-0.2}(\bar{f}_S)^{+0.2}_{-0.1}(a_{1,2}^{K^*})^{+0.4}_{-0.4}(B_{1,3}^S)(f_0^s)$

B. $B_c \to \kappa(P, V)$ and $B_c \to K_0^*(1430)(P, V)$ decays

In this type of the considered decays, there are eight $B_c \to K_0^* K^{(*)}$ ($\Delta S = 0$) modes and 16 $B_c \to K_0^*(\pi, \eta^{(\prime)}, \rho, \omega, \phi)$ ($\Delta S = 1$) channels.

In the $\Delta S = 0$ processes, we have four $B_c \to \kappa^+ \bar{K}^{(*)0}$, $\bar{\kappa}^0 K^{(*)+}$ channels in S1 and four $B_c \to K_0^*(1430)^+ \bar{K}^{(*)0}$, $\bar{K}_0^*(1430)^0 K^{(*)+}$ decays in both S1 and S2, respectively. From the pQCD predictions for these considered modes as given in Tables I, III, V, and VII, one can observe that all the BRs are in the range of 10^{-6}–10^{-5} within the theoretical errors, which could be measured by the near future LHCb and Super-B experiments operated at CERN and KEK, respectively.

Here, it is very interesting to note that for $B_c \to K_0^* K^{(*)}$ channels $\mathrm{Br}(B_c \to \bar{K}_0^{*0} K^+) > \mathrm{Br}(B_c \to K_0^{*+} \bar{K}^0)$ and $\mathrm{Br}(B_c \to \bar{K}_0^{*0} K^{*+}) > \mathrm{Br}(B_c \to K_0^{*+} \bar{K}^{*0})$ in S1, respectively, while the situation is quite the contrary for $B_c \to K_0^*(1430) K^{(*)}$ decays in S2. One can also find that for the $B_c \to \bar{K}_0^*(1430)^0 K^{(*)+}$ decays in both scenarios $\mathrm{Br}(B_c \to \bar{K}_0^*(1430)^0 K^{(*)+})_{\mathrm{S1}} \gg \mathrm{Br}(B_c \to \bar{K}_0^*(1430)^0 K^{(*)+})_{\mathrm{S2}}$, while

TABLE VII. Same as Table I but for the $\Delta S = 0$ processes of charmless hadronic $B_c \to (a_0(1450), K_0^*(1430), f_0(1370), f_0(1500))(\rho, K^*, \omega, \phi)$ decays in S1 and S2, respectively.

$\Delta S = 0$ Decay modes	BRs (10^{-6})
$B_c \to a_0(1450)^+ \rho^0$	$47.0^{+16.5}_{-12.5}(m_c)^{+11.2}_{-9.4}(\bar{f}_S)^{+7.3}_{-8.3}(a_2^\rho)^{+13.5}_{-7.5}(B_{1,3}^S)$ (S1)
	$15.3^{+11.9}_{-6.3}(m_c)^{+3.5}_{-3.1}(\bar{f}_S)^{+0.8}_{-0.2}(a_2^\rho)^{+7.8}_{-3.7}(B_{1,3}^S)$ (S2)
$B_c \to a_0(1450)^0 \rho^+$	$27.4^{+8.9}_{-6.4}(m_c)^{+6.2}_{-5.5}(\bar{f}_S)^{+2.4}_{-3.5}(a_2^\rho)^{+6.3}_{-6.1}(B_{1,3}^S)$ (S1)
	$15.5^{+9.3}_{-6.4}(m_c)^{+3.5}_{-3.2}(\bar{f}_S)^{+0.2}_{-0.0}(a_2^\rho)^{+5.8}_{-4.8}(B_{1,3}^S)$ (S2)
$B_c \to a_0(1450)^+ \omega$	$6.5^{+2.0}_{-1.1}(m_c)^{+1.4}_{-1.4}(\bar{f}_S)^{+0.6}_{-0.0}(a_2^\omega)^{+2.2}_{-1.8}(B_{1,3}^S)$ (S1)
	$1.1^{+0.3}_{-0.1}(m_c)^{+0.3}_{-0.2}(\bar{f}_S)^{+0.2}_{-0.0}(a_2^\omega)^{+3.1}_{-0.7}(B_{1,3}^S)$ (S2)
$B_c \to \bar{K}_0^*(1430)^0 K^{*+}$	$35.7^{+18.8}_{-12.3}(m_c)^{+8.3}_{-6.5}(\bar{f}_S)^{+2.9}_{-3.5}(a_{1,2}^{K^*})^{+4.3}_{-3.1}(B_{1,3}^S)$ (S1)
	$5.4^{+5.5}_{-2.8}(m_c)^{+1.4}_{-1.0}(\bar{f}_S)^{+0.7}_{-0.4}(a_{1,2}^{K^*})^{+5.6}_{-1.9}(B_{1,3}^S)$ (S2)
$B_c \to \bar{K}^{*0} K_0^*(1430)^+$	$5.0^{+0.9}_{-1.6}(m_c)^{+1.1}_{-0.7}(\bar{f}_S)^{+1.0}_{-0.8}(a_{1,2}^{K^*})^{+1.6}_{-0.8}(B_{1,3}^S)$ (S1)
	$8.2^{+2.9}_{-2.0}(m_c)^{+2.0}_{-1.9}(\bar{f}_S)^{+1.8}_{-1.7}(a_{1,2}^{K^*})^{+6.1}_{-4.0}(B_{1,3}^S)$ (S2)
$B_c \to f_0(1370)\rho^+$	$6.1^{+3.9}_{-2.1}(m_c)^{+1.6}_{-1.3}(\bar{f}_S)^{+2.8}_{-1.8}(a_2^\rho)^{+2.2}_{-1.5}(B_{1,3}^S)(f_0^q$, S1)
	$1.7^{+0.3}_{-0.2}(m_c)^{+0.3}_{-0.4}(\bar{f}_S)^{+0.5}_{-0.4}(a_2^\rho)^{+1.1}_{-1.3}(B_{1,3}^S)(f_0^q$, S2)
$B_c \to f_0(1500)\rho^+$	$6.1^{+3.7}_{-2.4}(m_c)^{+1.5}_{-1.3}(\bar{f}_S)^{+2.4}_{-1.8}(a_2^\rho)^{+2.2}_{-1.7}(B_{1,3}^S)(f_0^q$, S1)
	$1.7^{+0.5}_{-0.1}(m_c)^{+0.4}_{-0.3}(\bar{f}_S)^{+0.6}_{-0.4}(a_2^\rho)^{+0.9}_{-1.4}(B_{1,3}^S)(f_0^q$, S2)

TABLE VIII. Same as Table I but for the $\Delta S = 1$ processes of charmless hadronic $B_c \to (a_0(1450), K_0^*(1430), f_0(1370), f_0(1500))(\rho, K^*, \omega, \phi)$ decays in S1 and S2, respectively.

$\Delta S = 1$ Decay modes	BRs (10^{-7})
$B_c \to a_0(1450)^+ K^{*0}$	$2.7^{+0.4}_{-0.6}(m_c)^{+0.5}_{-0.5}(\bar{f}_S)^{+0.1}_{-0.3}(a_{1,2}^{K^*})^{+0.9}_{-0.6}(B_{1,3}^S)$ (S1)
	$7.0^{+1.9}_{-1.7}(m_c)^{+1.7}_{-1.6}(\bar{f}_S)^{+1.2}_{-0.8}(a_{1,2}^{K^*})^{+1.8}_{-3.5}(B_{1,3}^S)$ (S2)
$B_c \to a_0(1450)^0 K^{*+}$	$1.4^{+0.2}_{-0.3}(m_c)^{+0.3}_{-0.3}(\bar{f}_S)^{+0.1}_{-0.1}(a_{1,2}^{K^*})^{+0.5}_{-0.3}(B_{1,3}^S)$ (S1)
	$3.5^{+1.0}_{-0.9}(m_c)^{+0.9}_{-0.8}(\bar{f}_S)^{+0.6}_{-0.4}(a_{1,2}^{K^*})^{+0.9}_{-1.8}(B_{1,3}^S)$ (S2)
$B_c \to K_0^*(1430)^+ \omega$	$7.1^{+3.3}_{-2.7}(m_c)^{+1.5}_{-1.3}(\bar{f}_S)^{+0.7}_{-0.8}(a_2^\omega)^{+0.8}_{-0.8}(B_{1,3}^S)$ (S1)
	$1.3^{+1.2}_{-0.6}(m_c)^{+0.3}_{-0.2}(\bar{f}_S)^{+0.1}_{-0.1}(a_2^\omega)^{+1.0}_{-0.3}(B_{1,3}^S)$ (S2)
$B_c \to K_0^*(1430)^+ \phi$	$2.8^{+0.4}_{-0.6}(m_c)^{+0.8}_{-0.5}(\bar{f}_S)^{+0.3}_{-0.6}(a_2^\phi)^{+0.8}_{-0.5}(B_{1,3}^S)$ (S1)
	$4.3^{+1.7}_{-1.3}(m_c)^{+1.2}_{-1.0}(\bar{f}_S)^{+0.7}_{-0.4}(a_2^\phi)^{+3.8}_{-2.1}(B_{1,3}^S)$ (S2)
$B_c \to K_0^*(1430)^0 \rho^+$	$16.5^{+8.9}_{-5.9}(m_c)^{+3.2}_{-3.1}(\bar{f}_S)^{+0.9}_{-2.0}(a_2^\rho)^{+1.6}_{-1.8}(B_{1,3}^S)$ (S1)
	$2.9^{+3.1}_{-1.6}(m_c)^{+0.8}_{-0.5}(\bar{f}_S)^{+0.2}_{-0.0}(a_2^\rho)^{+2.6}_{-0.8}(B_{1,3}^S)$ (S2)
$B_c \to K_0^*(1430)^+ \rho^0$	$8.2^{+4.5}_{-3.0}(m_c)^{+1.6}_{-1.6}(\bar{f}_S)^{+0.5}_{-1.0}(a_2^\rho)^{+0.8}_{-0.9}(B_{1,3}^S)$ (S1)
	$1.5^{+1.5}_{-0.8}(m_c)^{+0.4}_{-0.3}(\bar{f}_S)^{+0.1}_{-0.0}(a_2^\rho)^{+1.3}_{-0.4}(B_{1,3}^S)$ (S2)
$B_c \to f_0(1370) K^{*+}$	$1.0^{+0.3}_{-0.2}(m_c)^{+0.3}_{-0.2}(\bar{f}_S)^{+0.2}_{-0.0}(a_{1,2}^{K^*})^{+0.6}_{-0.1}(B_{1,3}^S)(f_0^q$, S1)
	$2.4^{+0.8}_{-0.5}(m_c)^{+0.6}_{-0.4}(\bar{f}_S)^{+0.6}_{-0.2}(a_{1,2}^{K^*})^{+1.4}_{-1.4}(B_{1,3}^S)(f_0^q$, S2)
	$14.3^{+5.8}_{-4.0}(m_c)^{+3.5}_{-3.2}(\bar{f}_S)^{+2.8}_{-3.5}(a_{1,2}^{K^*})^{+3.1}_{-3.3}(B_{1,3}^S)(f_0^s$, S1)
	$3.2^{+2.5}_{-1.3}(m_c)^{+0.7}_{-0.6}(\bar{f}_S)^{+0.5}_{-0.4}(a_{1,2}^{K^*})^{+2.3}_{-1.0}(B_{1,3}^S)(f_0^s$, S2)
$B_c \to f_0(1500) K^{*+}$	$1.1^{+0.2}_{-0.3}(m_c)^{+0.3}_{-0.2}(\bar{f}_S)^{+0.0}_{-0.1}(a_{1,2}^{K^*})^{+0.4}_{-0.3}(B_{1,3}^S)(f_0^q$, S1)
	$2.5^{+0.7}_{-0.7}(m_c)^{+0.5}_{-0.5}(\bar{f}_S)^{+0.4}_{-0.3}(a_{1,2}^{K^*})^{+1.1}_{-1.6}(B_{1,3}^S)(f_0^q$, S2)
	$14.9^{+5.6}_{-5.0}(m_c)^{+3.7}_{-3.3}(\bar{f}_S)^{+2.3}_{-3.9}(a_{1,2}^{K^*})^{+2.4}_{-3.7}(B_{1,3}^S)(f_0^s$, S1)
	$3.5^{+2.6}_{-1.5}(m_c)^{+0.8}_{-0.7}(\bar{f}_S)^{+0.4}_{-0.5}(a_{1,2}^{K^*})^{+2.2}_{-1.1}(B_{1,3}^S)(f_0^s$, S2)

for the $B_c \to K_0^*(1430)^+\bar{K}^{(*)0}$ modes, $\mathrm{Br}(B_c \to K_0^*(1430)^+\bar{K}^{(*)0})_{\mathrm{S1}} < \mathrm{Br}(B_c \to K_0^*(1430)^+\bar{K}^{(*)0})_{\mathrm{S2}}$. It should be stressed that once these predicted BRs and the relevant relations could be tested by the experiments in the near future, this could provide the great opportunities for us to explore the physical properties of the scalars K_0^* and the corresponding annihilation decay mechanism.

For the $\Delta S = 1$ channels $B_c \to K_0^*(\pi, \eta^{(\prime)})$ and $B_c \to K_0^*(\rho, \omega, \phi)$, all the theoretical BRs in the pQCD approach are in the range of 10^{-8}–10^{-7} within the theoretical errors except for $\mathrm{Br}(B_c \to K_0^*(1430)(\rho, \omega))_{\mathrm{S1}} \sim 10^{-6}$ though they are CKM suppressed ($V_{us} = 0.22$), which will be confronted by the ongoing and forthcoming relevant experimental measurements. Because of the contributions from the same component, i.e., $u\bar{u}$, and few differences of the decay constants and masses between ρ^0 and ω, which result in the similar BRs for $B_c \to K_0^*\rho^0$ and $B_c \to K_0^*\omega$ in the considered scenarios. Moreover, we find that the simple relations $\mathrm{Br}(B_c \to K_0^{*0}(\pi^+, \rho^+)) \approx 2 \times \mathrm{Br}(B_c \to K_0^{*+}(\pi^0, \rho^0))$ exist in our pQCD perturbative calculations exactly and $\mathrm{Br}(B_c \to K_0^*(1430)(\pi, \rho, \omega))_{\mathrm{S1}} > \mathrm{Br}(B_c \to K_0^*(1430)(\pi, \rho, \omega))_{\mathrm{S2}}$. However, $\mathrm{Br}(B_c \to K_0^*(1430)^+\phi)_{\mathrm{S1}} < \mathrm{Br}(B_c \to K_0^*(1430)^+\phi)_{\mathrm{S2}}$, whose pattern agrees well with that obtained by Kim, Li, and Wang in Ref. [11].

For $B_c \to K_0^{*+}(\eta, \eta')$ decay modes, based on the pQCD numerical results, we have the following remarks: In this sector, both of the components η_q and η_s in η and η' contribute to these channels but with different coefficients even opposite sign. For $B_c \to \kappa^+\eta^{(\prime)}$ decays, the two parts of contributions make a constructive interference to the branching ratio of $B_c \to \kappa^+\eta$, while a destructive interference to that of $B_c \to \kappa^+\eta'$, which eventually results in $\mathrm{Br}(B_c \to \kappa^+\eta) \approx 5 \times \mathrm{Br}(B_c \to \kappa^+\eta')$. This pattern is very like that of $B \to K^*\eta$ and $K^*\eta'$ decay channels [1,7]. For $B_c \to K_0^*(1430)^+\eta^{(\prime)}$ modes, unlike the $B_c \to \kappa^+\eta^{(\prime)}$, both of them are determined mainly by the factorizable contributions of the η_s term, which leads to $\mathrm{Br}(B_c \to K_0^*(1430)^+\eta) \sim \mathrm{Br}(B_c \to K_0^*(1430)^+\eta')$within the theoretical errors in both scenarios. Meanwhile, it is interesting to note that $\mathrm{Br}(B_c \to K_0^*(1430)^+\eta^{(\prime)})_{\mathrm{S1}} < \mathrm{Br}(B_c \to K_0^*(1430)^+\eta^{(\prime)})_{\mathrm{S2}}$ while $\mathrm{Br}(B_c \to K_0^*(1430)^+\eta) > \mathrm{Br}(B_c \to K_0^*(1430)^+\eta')$ in both scenarios, where only the central values are quoted for comparison. Because of the small BRs ($< 10^{-6}$) for $B_c \to K_0^{*+}\eta^{(\prime)}$ decays, all the above theoretical pQCD predictions of the BRs and the physical relations are expected to be examined in the forthcoming Super-B experiments.

C. $B_c \to f(P, V)$ and $B_c \to f'(P, V)$ decays

As mentioned in the above sections, it is well known that the identification of the structure of these neutral scalar mesons f and f' is very difficult, which is a long-standing puzzle not yet resolved either by experimentalists or by theorists. Although various scenarios on their component have been proposed, by considering the feasibility of factorization approach, we here assume these considered scalars to be only $q\bar{q}$ bound states.

For the considered 16 $B_c \to (f, f')(P, V)$ decays, the numerical pQCD predictions have been displayed in Tables I, II, III, IV, V, VI, VII, and VIII. For the f and f', the quarkonia component has been proposed, which can be seen in Eqs. (2) and (3). For the $\Delta S = 0$ processes $B_c \to (f, f')(\pi^+, \rho^+)$, we use the pure $q\bar{q}$ states f_0^q in f and f' to calculate the BRs in the pQCD approach and obtain the numerical results, in which one can find that the BRs of $B_c \to (\pi^+, \rho^+)(f_0(1370), f_0(1500))(q\bar{q})$ are in the order of 10^{-6} within the theoretical errors in both scenarios and within the reach of the LHCb experiments [38], while the BRs of $B_c \to (\pi^+, \rho^+)(\sigma, f_0)(q\bar{q})$ are highly below the experimental reach of LHCb at CERN. Here, we have assumed that σ and $f_0(1370)$ have the similar decay constant and light-cone distribution amplitudes as f_0 and $f_0(1500)$, respectively.

As mentioned in Sec. II, since the experimental constraints indicate that the mixing angle θ_0 between σ and f_0 lies in the range of $[25°, 40°]$ or $[140°, 165°]$ [31], then the pQCD predictions of the BRs for $B_c \to \pi^+(\sigma, f_0)$ decays with mixing patterns can be read

$$\mathrm{Br}(B_c \to \pi^+\sigma) \approx \begin{cases} (1.9 \sim 2.6) \times 10^{-7} & \text{for } 25° < \theta_0 < 40° \\ (1.9 \sim 3.0) \times 10^{-7} & \text{for } 140° < \theta_0 < 165°, \end{cases} \tag{70}$$

$$\mathrm{Br}(B_c \to \pi^+ f_0) \approx \begin{cases} (0.3 \sim 0.8) \times 10^{-7} & \text{for } 25° < \theta_0 < 40° \\ (0.1 \sim 0.8) \times 10^{-7} & \text{for } 140° < \theta_0 < 165°, \end{cases} \tag{71}$$

where only the central values are quoted, so are the similar cases in the following text unless otherwise stated. Likewise, the pQCD predictions of the BRs for $B_c \to \rho^+(\sigma, f_0)$ decays are as follows:

$$\mathrm{Br}(B_c \to \rho^+\sigma) \approx \begin{cases} (0.9 \sim 1.3) \times 10^{-7} & \text{for } 25° < \theta_0 < 40° \\ (0.9 \sim 1.5) \times 10^{-7} & \text{for } 140° < \theta_0 < 165°, \end{cases} \tag{72}$$

$$\mathrm{Br}(B_c \to \rho^+ f_0) \approx \begin{cases} (0.1 \sim 0.3) \times 10^{-7} & \text{for } 25° < \theta_0 < 40° \\ (0.05 \sim 0.3) \times 10^{-7} & \text{for } 140° < \theta_0 < 165°. \end{cases} \tag{73}$$

According to Ref. [32], $f_0(1370)$ and $f_0(1500)$ mixing has the following form:

$$f_0(1370) = 0.78 f_0^q + 0.51 f_0^s,$$
$$f_0(1500) = -0.54 f_0^q + 0.84 f_0^s, \tag{74}$$

where we neglect the possible small or tiny scalar glueball components in the present paper and leave them for future study. Then the pQCD predictions of the BRs for $B_c \to \pi^+(f_0(1370), f_0(1500))$ decays can be read,

$$\mathrm{Br}(B_c \to \pi^+ f_0(1370)) \approx \begin{cases} 2.2 \times 10^{-6}\,(\mathrm{S1}), \\ 6.0 \times 10^{-7}\,(\mathrm{S2}); \end{cases} \tag{75}$$

$$\mathrm{Br}(B_c \to \pi^+ f_0(1500)) \approx \begin{cases} 1.1 \times 10^{-6}\,(\mathrm{S1}), \\ 2.7 \times 10^{-7}\,(\mathrm{S2}). \end{cases} \tag{76}$$

Likewise, the pQCD predictions of the BRs for $B_c \to \rho^+(f_0(1370), f_0(1500))$ decays are

$$\mathrm{Br}(B_c \to \rho^+ f_0(1370)) \approx \begin{cases} 3.7 \times 10^{-6}\,(\mathrm{S1}), \\ 1.0 \times 10^{-6}\,(\mathrm{S2}); \end{cases} \tag{77}$$

$$\mathrm{Br}(B_c \to \rho^+ f_0(1500)) \approx \begin{cases} 1.8 \times 10^{-6}\,(\mathrm{S1}), \\ 5.0 \times 10^{-7}\,(\mathrm{S2}). \end{cases} \tag{78}$$

For the $\Delta S = 1$ processes $B_c \to K^{(*)+}(f, f')$ decays, the BRs in the pQCD approach based on the pure $q\bar{q}$ state f_0^q or pure $s\bar{s}$ one f_0^s of the scalars f and f' are given in Tables II, IV, VI, and VIII. One can observe straightforwardly from the tables that all the BRs for $B_c \to K^{(*)+}(f, f')$ channels are in the order of 10^{-8}–10^{-7} except for $B_c \to K^{*+} f_0(1500)$ in S1 though which is CKM suppressed. But, if the branching ratio of 10^{-6} for $B_c \to K^{*+} f_0(1500)$ decay can be detected by the experiments, it is doubtless that the scalar meson $f_0(1500)$ is dominated by the $s\bar{s}$ component. When we consider the mixing form for the scalars f and f', the CP-averaged BRs for $B_c \to K^{(*)+}(f, f')$ decays within the pQCD approach have been calculated and shown in Eqs. (79)–(86):

$$\mathrm{Br}(B_c \to K^+ \sigma) \approx \begin{cases} (2.0 \sim 2.0) \times 10^{-7} & \text{for } 25° < \theta_0 < 40° \\ (0.5 \sim 1.1) \times 10^{-7} & \text{for } 140° < \theta_0 < 165°, \end{cases} \tag{79}$$

$$\mathrm{Br}(B_c \to K^+ f_0) \approx \begin{cases} (0.2 \sim 0.5) \times 10^{-7} & \text{for } 25° < \theta_0 < 40° \\ (0.6 \sim 1.4) \times 10^{-7} & \text{for } 140° < \theta_0 < 165°; \end{cases} \tag{80}$$

$$\mathrm{Br}(B_c \to K^{*+} \sigma) \approx \begin{cases} (3.0 \sim 3.5) \times 10^{-7} & \text{for } 25° < \theta_0 < 40° \\ (0.06 \sim 0.8) \times 10^{-7} & \text{for } 140° < \theta_0 < 165°, \end{cases} \tag{81}$$

$$\mathrm{Br}(B_c \to K^{*+} f_0) \approx \begin{cases} (0.1 \sim 0.6) \times 10^{-7} & \text{for } 25° < \theta_0 < 40° \\ (2.7 \sim 3.4) \times 10^{-7} & \text{for } 140° < \theta_0 < 165°; \end{cases} \tag{82}$$

$$\mathrm{Br}(B_c \to K^+ f_0(1370)) \approx \begin{cases} 1.4 \times 10^{-7}\,(\mathrm{S1}), \\ 1.8 \times 10^{-7}\,(\mathrm{S2}); \end{cases} \tag{83}$$

$$\mathrm{Br}(B_c \to K^+ f_0(1500)) \approx \begin{cases} 7.1 \times 10^{-7}\,(\mathrm{S1}), \\ 1.3 \times 10^{-7}\,(\mathrm{S2}); \end{cases} \tag{84}$$

$$\mathrm{Br}(B_c \to K^{*+} f_0(1370)) \approx \begin{cases} 1.8 \times 10^{-7}\,(\mathrm{S1}), \\ 6.3 \times 10^{-8}\,(\mathrm{S2}); \end{cases} \tag{85}$$

$$\mathrm{Br}(B_c \to K^{*+} f_0(1500)) \approx \begin{cases} 1.4 \times 10^{-6}\,(\mathrm{S1}), \\ 5.2 \times 10^{-7}\,(\mathrm{S2}). \end{cases} \tag{86}$$

Hence, based on the numerical results shown in Tables I, II, V, and VI, and Eqs. (70)–(73) and (79)–(82), it is evident that the theoretical implications on the components of σ and f_0 in the light scalar nonet cannot be provided by the small pQCD predictions on the short-distance contributions of $B_c \to (\pi^+, K^+, \rho^+, K^{*+})(\sigma, f_0)$ decays. However, once the large BRs above 10^{-6} for $\Delta S = 0$ processes $B_c \to (\pi^+, \rho^+)(f_0(1370), f_0(1500))$ in both scenarios and $\Delta S = 1$ $B_c \to K^{*+} f_0(1500)$ decay in scenario 1 could be measured in the ongoing LHCb or the forthcoming Sper-B experiments, they may help determine the components, the ratios of quarkonia, and the preferred scenario by the experiments for these two considered scalar $f_0(1370)$ and $f_0(1500)$ mesons, respectively.

Frankly speaking, for many considered pure annihilation B_c decays with BRs of or below 10^{-7}, it is still hard to observe them even in LHC due to their tiny decay rates. Their observation at LHC, however, would mean a large nonperturbative contribution or a signal for exotic new physics beyond the SM. It is worth stressing that the theoretical predictions in the pQCD approach still have large theoretical errors induced by the still large uncertainties of many input parameters. Any progress in reducing the error of input parameters, such as the Gegenbauer moments a_i of the pseudoscalar or vector mesons distribution amplitudes, B_i of the scalar mesons distribution amplitudes and the charm quark mass m_c, will help us to improve the precision of the pQCD predictions. We do not consider the possible long-distance contributions, such as the rescattering effects, although they should be present, and they may be large and affect the theoretical predictions. It is beyond the scope of this work and expected to be studied in the future work.

V. SUMMARY

In summary, we studied the two-body charmless hadronic $B_c \to SP, SV$ decays by employing the pQCD factorization approach based on the k_T factorization theorem. These considered decay channels can occur only via the annihilation diagrams and they will provide an important testing ground for the magnitude of the annihilation contributions and implications to the mechanism of annihilation decays. Based on the assumption of two-quark structure of the light scalars, we make the theoretical predictions on the CP-averaged branching ratios of considered $B_c \to SP, SV$ channels. In turn, we could obtain the implications on the component and physical properties of the light scalar mesons through the experimental measurements on these considered charmless hadronic B_c decays. Furthermore, these decay modes might also reveal the existence of the exotic new physics scenario or nonperturbative QCD effects.

The pQCD predictions for CP-averaged branching ratios are displayed in Tables I, II, III, IV, V, VI, VII, and VIII. From our numerical evaluations and phenomenological analysis, we found the following results:

(i) The pQCD predictions for the branching ratios vary in the range of 10^{-5} to 10^{-8}. Many decays with a decay rate at 10^{-6} or larger could be measured at the LHCb experiment.

(ii) For $B_c \to SP, SV$ decays, the branching ratios of $\Delta S = 0$ processes are basically larger than those of the $\Delta S = 1$ ones. Such differences are mainly induced by the CKM factors involved: $V_{ud} \sim 1$ for the former decays while $V_{us} \sim 0.22$ for the latter ones.

(iii) Analogous to $B \to K^* \eta^{(\prime)}$ decays, we find $\mathrm{Br}(B_c \to \kappa^+ \eta) \sim 5 \times \mathrm{Br}(B_c \to \kappa^+ \eta')$. This difference can be understood by the destructive and constructive interference between the η_q and η_s contribution to the $B_c \to \kappa^+ \eta'$ and $B_c \to \kappa^+ \eta$ decay, respectively.

(iv) For $B_c \to K_0^*(1430) \eta^{(\prime)}$ channels, the branching ratios for these two decays are similar to each other in both scenarios, which is mainly because the factorizable contributions of the η_s term play the dominant role and are expected to be tested by the forthcoming Super-B experiments.

(v) If a_0 and κ are the $q\bar{q}$ bound states, the pQCD predicted BRs for $B_c \to a_0(\pi, \rho)$ and $B_c \to \kappa K^{(*)}$ decays will be in the range of 10^{-6}–10^{-5}, which are within the reach of the LHCb experiments and expected to be measured.

(vi) For the $a_0(1450)$ and $K_0^*(1430)$ channels, the BRs for $B_c \to a_0(1450)(\pi, \rho)$ and $B_c \to K_0^*(1430) K^{(*)}$ modes in the pQCD approach are found to be of order $(5\text{–}47) \times 10^{-6}$ and $(0.7 \sim 36) \times 10^{-6}$, respectively. A measurement of them at the predicted level will favor the structure of $q\bar{q}$ for the $a_0(1450)$ and $K_0^*(1430)$ and identify which scenario is preferred.

(vii) Because only tree operators are involved, the CP-violating asymmetries for these considered B_c decays are absent naturally.

(viii) The pQCD predictions still have large theoretical uncertainties, induced by the uncertainties of input parameters.

(ix) We here calculated the branching ratios of the pure annihilation $B_c \to SP, SV$ decays by employing the pQCD approach. We do not consider the possible long-distance contributions, such as the rescattering effects, although they may be large and affect the theoretical predictions. It is beyond the scope of this work.

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[1] S. Spanier, N. A. Törnqvist, and C. Amsler (Particle Data Group), Phys. Lett. B **667**, 1 (2008).

[2] S. Godfrey and J. Napolitano, Rev. Mod. Phys. **71**, 1411 (1999).

[3] F. E. Close and N. A. Törnqvist, J. Phys. G **28**, R249 (2002).

[4] A. Garmash *et al.* (Belle Collaboration), Phys. Rev. D **65**, 092005 (2002).

[5] A. Garmash *et al.* (Belle Collaboration), Phys. Rev. D **71**, 092003 (2005); K. Abe *et al.* (Belle Collaboration), arXiv:hep-ex/0509001; J. Dragic, at the HEP2005, Lisboa, Portugal, 2005 [Proc. Sci. HEP2005 (2005) 248].

[6] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **70**, 092001 (2004).

[7] E. Barberio *et al.* (Heavy Flavor Averaging Group), arXiv:0808.1297; online update at http://www.slac.stanford.edu/xorg/hfag.

[8] C. H. Chen, Phys. Rev. D **67**, 014012 (2003); **67**, 094011 (2003); C. H. Chen and C. Q. Geng, Phys. Rev. D **75**, 054010 (2007).

[9] H. Y. Cheng and K. C. Yang, Phys. Rev. D **71**, 054020 (2005); P. Minkowski and W. Ochs, Eur. Phys. J. C **39**, 71 (2005); A. K. Giri, B. Mawlong, and R. Mohanta, Phys. Rev. D **74**, 114001 (2006).

[10] H. Y. Cheng, C. K. Chua, and K. C. Yang, Phys. Rev. D **73**, 014017 (2006), and references therein; **77**, 014034 (2008).

[11] W. Wang, Y. L. Shen, Y. Li, and C. D. Lü, Phys. Rev. D **74**, 114010 (2006); Y. L. Shen, W. Wang, J. Zhu, and C. D. Lü, Eur. Phys. J. C **50**, 877 (2007); C. S. Kim, Y. Li, and W. Wang, Phys. Rev. D **81**, 074014 (2010); P. Colangelo, F. DeFazio, and W. Wang, Phys. Rev. D **81**, 074001 (2010).

[12] X. Liu and Z. J. Xiao, Commun. Theor. Phys. **53**, 540 (2010); X. Liu, Z. Q. Zhang, and Z. J. Xiao, Chin. Phys. C **34**, 157 (2010); Z. Q. Zhang and Z. J. Xiao, Chin. Phys. C **34**, 528 (2010); arXiv:0812.2314.

[13] N. Brambilla *et al.* (Quarkonium Working Group), arXiv:hep-ph/0412158.

[14] C. D. Lü and K. Ukai, Eur. Phys. J. C **28**, 305 (2003).

[15] Y. Y. Keum, H. N. Li, and A. I. Sanda, Phys. Lett. B **504**, 6 (2001); Phys. Rev. D **63**, 054008 (2001).

[16] C. D. Lü, K. Ukai, and M. Z. Yang, Phys. Rev. D **63**, 074009 (2001).

[17] B. H. Hong and C. D. Lü, Sci. China Ser. G **49**, 357 (2006).

[18] H. N. Li and S. Mishima, Phys. Rev. D **71**, 054025 (2005); H. N. Li, Phys. Lett. B **622**, 63 (2005).

[19] A. V. Gritsan, in *Proceedings of the 5th International Conference on Flavor Physics and CP Violation*, econf C070512, 001 (2007); A. L. Kagan, Phys. Lett. B **601**, 151 (2004).

[20] C. M. Arnesen, Z. Ligeti, I. Z. Rothstein, and I. W. Stewart, Phys. Rev. D **77**, 054006 (2008).

[21] C. W. Bauer, S. Fleming, and M. E. Luke, Phys. Rev. D **63**, 014006 (2000); C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, Phys. Rev. D **63**, 114020 (2001); C. W. Bauer and I. W. Stewart, Phys. Lett. B **516**, 134 (2001); C. W. Bauer, D. Pirjol, and I. W. Stewart, Phys. Rev. D **65**, 054022 (2002); C. W. Bauer, S. Fleming, D. Pirjol, I. Z. Rothstein, and I. W. Stewart, Phys. Rev. D **66**, 014017 (2002).

[22] J. Chay, H. N. Li, and S. Mishima, Phys. Rev. D **78**, 034037 (2008).

[23] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).

[24] H. N. Li, Prog. Part. Nucl. Phys. **51**, 85 (2003), and reference therein.

[25] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. **B591**, 313 (2000).

[26] H. N. Li, Proc. Sci., FPCP2009 (2009) 026 [arXiv:0907.4940].

[27] X. Liu, H. S. Wang, Z. J. Xiao, L. B. Guo, and C. D. Lü, Phys. Rev. D **73**, 074002 (2006); H. S. Wang, X. Liu, Z. J. Xiao, L. B. Guo, and C. D. Lü, Nucl. Phys. **B738**, 243 (2006); Z. J. Xiao, X. F. Chen, and D. Q. Guo, Eur. Phys. J. C **50**, 363 (2007); Z. J. Xiao, D. Q. Guo, and X. F. Chen, Phys. Rev. D **75**, 014018 (2007); Z. J. Xiao, X. Liu, and H. S. Wang, Phys. Rev. D **75**, 034017 (2007); Z. J. Xiao, X. F. Chen, and D. Q. Guo, arXiv:0701146.

[28] A. Ali, G. Kramer, Y. Li, C. D. Lü, Y. L. Shen, W. Wang, and Y. M. Wang, Phys. Rev. D **76**, 074018 (2007).

[29] Z. J. Xiao, Z. Q. Zhang, X. Liu, and L. B. Guo, Phys. Rev. D **78**, 114001 (2008).

[30] X. Liu, Z. J. Xiao, and C. D. Lü, Phys. Rev. D **81**, 014022 (2010); X. Liu and Z. J. Xiao, Phys. Rev. D **81**, 074017 (2010); arXiv:1003.3929.

[31] M. Alford and R. L. Jaffe, Nucl. Phys. **B578**, 367 (2000); H. Y. Cheng, Phys. Rev. D **67**, 034024 (2003); A. V. Anisovich, V. V. Anisovich, and V. A. Nikonov, Eur. Phys. J. A **12**, 103 (2001); Phys. At. Nucl. **65**, 497 (2002); A. Gokalp, Y. Sarac, and O. Yilmaz, Phys. Lett. B **609**, 291 (2005).

[32] H. Y. Cheng, C. K. Chua, and K. F. Liu, Phys. Rev. D **74**, 094005 (2006).

[33] H. N. Li, Phys. Rev. D **66**, 094010 (2002).

[34] H. N. Li and B. Tseng, Phys. Rev. D **57**, 443 (1998).

[35] C. D. Lü and M. Z. Yang, Eur. Phys. J. C **28**, 515 (2003).

[36] J. F. Cheng, D. S. Du, and C. D. Lü, Eur. Phys. J. C **45**, 711 (2006).

[37] R. H. Li, C. D. Lü, W. Wang, and X. X. Wang, Phys. Rev. D **79**, 014013 (2009).

[38] S. Descotes-Genon, J. He, E. Kou, and P. Robbe, Phys. Rev. D **80**, 114031 (2009).

[39] N. Mathur, A. Alexandru, Y. Chen, S. J. Dong, T. Draper, I. Horváth, F. X. Lee, K. F. Liu, S. Tamhankar, and J. B. Zhang, Phys. Rev. D **76**, 114505 (2007).

[40] W. Lee and D. Weingarten, Phys. Rev. D **61**, 014015 (1999); M. Göckeler, R. Horsley, H. Perlt, P. Rakow, G. Schierholz, A. Schiller, and P. Stephenson, Phys. Rev. D **57**, 5562 (1998); S. Kim and S. Ohta, Nucl. Phys. B, Proc. Suppl. **53**, 199 (1997); A. Hart, C. McNeile, and C. Michael, Nucl. Phys. B, Proc. Suppl. **119**, 266 (2003); T. Burch, C. Gattringer, L. Y. Glozman, C. Hagen, C. B. Lang, and A. Schäfer (BGR [Bern-Graz-Regensburg] Collaboration), Phys. Rev. D **73**, 094505 (2006).

[41] W. Bardeen, A. Duncan, E. Eichten, N. Isgur, and H. Thacker, Phys. Rev. D **65**, 014509 (2001).

[42] T. Kunihiro, S. Muroya, A. Nakamura, C. Nonaka, M. Sekiguchi, and H. Wada (SCALAR Collaboration), Phys. Rev. D **70**, 034504 (2004).

[43] S. Prelovsek, C. Dawson, T. Izubuchi, K. Orginos, and A. Soni, Phys. Rev. D **70**, 094503 (2004).