PHYSICAL REVIEW D **82,** 054022 (2010)

Low-energy interactions of Nambu-Goldstone bosons with D mesons in covariant chiral perturbation theory

L. S. Geng,[1,2] N. Kaiser,[2] J. Martin-Camalich,[3] and W. Weise[2]

[1]*School of Physics and Nuclear Energy Engineering, Beihang University, Beijing 100191, China*
[2]*Physik Department, Technische Universität München, D-85747 Garching, Germany*
[3]*Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC, E-46071 Spain*
(Received 4 August 2010; published 20 September 2010)

We calculate the scattering lengths of Nambu-Goldstone bosons interacting with D mesons in a covariant formulation of chiral perturbation theory, which satisfies heavy-quark spin symmetry and analytical properties of loop amplitudes. We compare our results with previous studies performed using heavy-meson chiral perturbation theory and show that recoil corrections are sizable in most cases.

DOI: 10.1103/PhysRevD.82.054022 PACS numbers: 13.75.Lb, 12.39.Fe

I. INTRODUCTION

In recent years, studies of charmonium and open-charm systems have witnessed a renaissance. This was mainly led by the experimental discoveries of various new particles, either the still largely mysterious X, Y, Z particles or the new open-charm states. Many of these states cannot be easily understood in conventional quark models without introducing new degrees of freedom in addition to their basic $q\bar{q}$ structure, notably multiquark components, i.e., $qq\bar{q}\,\bar{q}$.

An interesting resonance in this context is the $D^*_{s0}(2317)$ with a mass of 2317.8 ± 0.6 MeV and a small width of several MeV ($\Gamma < 3.8$ MeV) [1]. The $D^*_{s0}(2317)$ was first observed by the *BABAR* collaboration in the inclusive $D_s^+\pi^0$ invariant mass distribution from e^+e^- annihilation data at energies near 10.6 GeV [2] and later confirmed by Belle [3] and CLEO [4]. The nature of this state has been extensively discussed in the literature [5–17]. All studies seem to agree that the coupling of the $D^*_{s0}(2317)$ to the nearby DK threshold cannot be ignored. From this perspective, it is particularly interesting to note that the $D^*_{s0}(2317)$ can be "dynamically" generated in coupled-channel unitary approaches with interaction kernels provided by either chiral perturbation theory (χPT) [15,16] or a SU(4) Lagrangian [17]. Such approaches have provided many interesting results in the past few years, e.g., in explaining the nature of some low-lying hadronic states such as the $f_0(600)$ and the $\Lambda(1405)$ (for a comprehensive list of references, see Ref. [18]). In this dynamical picture of the $D^*_{s0}(2317)$, the interactions of DK and coupled channels play a decisive role. A quantity that characterizes the strength of such an interaction at low energies is the scattering length. Although it cannot be measured directly given the short lifetime of the D mesons, it can nevertheless be studied on the lattice [19,20]. The s-wave DK scattering lengths have recently been computed by several authors [21,22]. In Ref. [21], the calculation was performed using a covariant formulation of χPT up to next-to-leading order (NLO) and its unitarized version. In Ref. [22], the calculation was performed using the heavy-meson χPT (HMχPT) [23–27] in the heavy-quark limit up to next-to-next-to-leading order (NNLO), where recoil effects have been neglected. The authors cautioned, however, that since the D mesons are not heavy enough, recoil corrections may not be small and have to be studied.

To our knowledge, recoil corrections have so far not been systematically studied in χPT describing the interactions between heavy-light mesons and Nambu-Goldstone bosons. They have been, however, studied quite extensively in the one-baryon sector with three flavors: u, d, and s. There these corrections were found to be fairly large and play an important role in the studies of many physical observables [28–31]. Because of the large baryon mass that does not vanish in the chiral limit, covariant baryon χPT often faces the so-called power-counting-breaking (PCB) problem [32]. This problem has been traditionally dealt with using a dual expansion in terms of both p/Λ_χ and $1/M_B$, where p is a generic small quantity, M_B the baryon mass, and $\Lambda_\chi = 4\pi f_\pi$ the chiral symmetry breaking scale. This is the celebrated heavy baryon χPT [33,34]. Though very successful in describing many observables, this approach is not covariant and modifies the analyticity of loop amplitudes. More importantly, from a practical point of view it suffers from slow convergence, particularly in the 3-flavor sector. To overcome these problems, several other approaches to deal with the PCB problem have been proposed, which among others include the infrared (IR) [35] and extended-on-mass-shell (EOMS) [36,37] renormalization schemes. While the IR scheme was found to introduce artificial cuts in certain cases (see, e.g., Ref. [38]), the EOMS approach is fully covariant and conserves the analytical properties of loop amplitudes. In a series of applications [28–31], it has been shown that the EOMS approach also improves the convergence behavior of SU(3) baryon χPT.

The main purpose of the present work is to study the scattering lengths of Nambu-Goldstone bosons (ϕ) interacting with D mesons in a covariant formulation of χPT by using the EOMS scheme to remove the PCB terms induced by the large D meson masses. Given the fact that the D meson mass (~ 1.9 GeV) is much larger than the nucleon mass, it is anticipated that the recoil corrections should be smaller than those in the nucleon case. Nevertheless, such recoil corrections may still be sizable as we will show in this work.

This paper is organized as follows. In Sec. II, we introduce the relevant effective Lagrangians and explain briefly the EOMS renormalization scheme. In Sec. III, we show the numerical results and compare them with those of earlier studies by paying special attention to the recoil corrections. A short summary follows in Sec. IV.

II. THEORETICAL FRAMEWORK

In this section we introduce the chiral Lagrangians relevant to the present study up to NNLO and explain briefly the EOMS renormalization scheme used to remove the PCB pieces appearing in the covariant loop calculation.

A. Chiral Lagrangians

To introduce the chiral effective Lagrangians, one must specify a power-counting rule. In the present case the light meson masses m_ϕ and the field gradients $\partial_\mu \phi$ are counted as $\mathcal{O}(p)$, while $\partial_\mu P$, $\partial_\nu P^*_\mu$, m_P, and m_{P^*} are counted as $\mathcal{O}(1)$, where ϕ denotes the Nambu-Goldstone bosons, and $P = (D^0, D^+, D_s^+)$ and $P^*_\mu = (D^{*0}, D^{*+}, D_s^{*+})_\mu$ are the D and D^* meson fields. The Nambu-Goldstone boson propagator, $\frac{i}{q^2-m_\phi^2}$, is counted as $\mathcal{O}(p^{-2})$, while the heavy-light pseudoscalar and vector meson propagators, $\frac{i}{q^2-m_P^2}$ and $\frac{i}{q^2-m_{P^*}^2}(-g^{\mu\nu}+\frac{q^\mu q^\nu}{m_{P^*}^2})$, are counted as $\mathcal{O}(p^{-1})$.

The leading order chiral Lagrangian describing the self-interaction of Nambu-Goldstone bosons has the standard form:

$$\mathcal{L}^{(2)} = \frac{1}{48 f_0^2}\langle ((\partial_\mu \Phi)\Phi - \Phi \partial_\mu \Phi)^2 + \mathcal{M}\Phi^4\rangle, \qquad (1)$$

where Φ collects the pseudoscalar octet fields

$$\Phi = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, \qquad (2)$$

$\mathcal{M} = \mathrm{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2)$ is the mass matrix, and f_0 is the pseudoscalar decay constant in the chiral limit. Here and in the following, $\langle \cdots \rangle$ always denotes the trace in the respective flavor space.

The lowest-order chiral Lagrangian for the heavy-light pseudoscalar and vector mesons is[1]

$$\begin{aligned}\mathcal{L}^{(1)} = {}& \langle \mathcal{D}_\mu P \mathcal{D}^\mu P^\dagger \rangle - \mathring{m}_D^2 \langle P P^\dagger \rangle - \langle \mathcal{D}_\mu P^{*\nu} \mathcal{D}^\mu P^{*\dagger}_\nu \rangle \\ & + \mathring{m}_D^2 \langle P^{*\nu} P^{*\dagger}_\nu \rangle + ig\langle P^*_\mu u^\mu P^\dagger - P u^\mu P^{*\dagger}_\mu \rangle \\ & + \frac{g}{2\mathring{m}}\langle (P^*_\mu u_\alpha \partial_\beta P^{*\dagger}_\nu - \partial_\beta P^*_\mu u_\alpha P^{*\dagger}_\nu)\epsilon^{\mu\nu\alpha\beta}\rangle, \end{aligned} \qquad (3)$$

where $\mathcal{D}_\mu P_a = \partial_\mu P_a - \Gamma^{ba}_\mu P_b$ and $\mathcal{D}^\mu P_a^\dagger = \partial^\mu P_a^\dagger + \Gamma^\mu_{ab} P_b^\dagger$ with a (b) the SU(3) flavor index, g is the heavy-light pseudoscalar-vector coupling constant of dimension 1, and $\mathring{m}$ is the mass of the heavy-light meson in the chiral limit. The vector and axial-vector currents, Γ_μ and u_μ, are defined as

$$\begin{aligned} \Gamma_\mu &= \tfrac{1}{2}(u^+ \partial_\mu u + u \partial_\mu u^+) \quad \text{and} \\ u_\mu &= i(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger) \end{aligned} \qquad (4)$$

with $u^2 = U = \exp(\frac{i\Phi}{f_0})$. The numerical value of g can be fixed by reproducing the $D^{*+} \to D^0 \pi^+$ decay width. Using the PDG average, $\Gamma_{D^{*+}} = 96 \pm 22$ keV and $\mathrm{BR}_{D^{*+}\to D^0\pi^+} = (67.7 \pm 0.5)\%$ [1], one obtains $\Gamma_{D^{*+}\to D^0\pi^+} = \frac{1}{12\pi}(g^2/f_\pi^2)(|q_\pi|^3/m_{D^{*+}}^2) = 65 \pm 15$ keV and accordingly $g = 1177 \pm 137$ MeV.

The NLO Lagrangian relevant to our study is[2]

$$\begin{aligned}\mathcal{L}^{(2)} = {}& -2[c_0\langle PP^\dagger\rangle\langle\chi_+\rangle - c_1\langle P\chi_+P^\dagger\rangle - c_2\langle PP^\dagger\rangle\langle u^\mu u_\mu\rangle \\ & - c_3\langle P u^\mu u_\mu P^\dagger\rangle], \end{aligned} \qquad (5)$$

where $\chi_\pm = u^\dagger \mathcal{M} u^\dagger \pm u\mathcal{M}u$. Here we have adopted a convention consistent with that of Ref. [22] for the purpose of later comparison. In general, there are more terms contributing to ϕP scattering, e.g., $\langle \partial_\mu P \partial_\nu P^\dagger\rangle\langle u^\mu u^\nu\rangle$, $\langle \partial_\mu P u^\mu u^\nu \partial_\nu P^\dagger\rangle$, $\langle \partial_\mu P u^\nu u^\mu \partial_\nu P^\dagger\rangle$. However, at the ϕP threshold, it can be easily shown that these terms lead to the same structure as those proportional to c_2 and c_3 and therefore can be neglected. The low-energy constant (LEC) c_1 can be determined from the mass splitting between strange and nonstrange heavy-light mesons within the same doublet, i.e.,

$$-8c_1(m_K^2 - m_\pi^2) = (m_{D_s}^2 - m_D^2 + m_{D_s^*}^2 - m_{D^*}^2)/2. \qquad (6)$$

Using the masses given in Table I, we obtain $c_1 = -0.225$.

At NNLO one has[3]

$$\begin{aligned}\mathcal{L}^{(3)} = {}& -\frac{i}{2}\kappa\langle \partial^\mu P(x_\mu^-)P^\dagger - P(x_\mu^-)\partial^\mu P^\dagger\rangle \\ & + \frac{\gamma_0}{2}\langle \partial^\mu P\Gamma_\mu P^\dagger - P\Gamma_\mu\partial^\mu P^\dagger\rangle\langle\chi_+\rangle \\ & + \gamma_1\langle \partial^\mu P\chi_+\Gamma^\mu P^\dagger - P\Gamma_\mu\chi_+\partial^\mu P^\dagger\rangle \\ & + \gamma_2\langle \mathcal{D}^\mu PP^\dagger - P\mathcal{D}^\mu P^\dagger\rangle\langle\mathcal{D}_\mu\chi_+\rangle, \end{aligned} \qquad (7)$$

[1]Because of heavy-quark spin symmetry, the pseudoscalar and vector mesons can be assigned to the same multiplet.

[2]D^* mesons in NLO and NNLO Lagrangians do not contribute to the NNLO $D\phi$ scattering lengths and therefore will not be explicitly shown in this work.

[3]In Ref. [22] only one such term, that proportional to κ, was considered.

TABLE I. Numerical values of (isospin-averaged) masses [1] and decay constants [39] (in units of MeV) used in the present study. The f_K/f_π ratio is consistent with the latest determination [40], while the f_η/f_π ratio is in agreement with that determined in a number of other approaches (see, e.g., Ref. [41]). The eta meson mass is calculated using the Gell-Mann-Okubo mass relation: $m_\eta^2 = (4m_K^2 - m_\pi^2)/3$.

$\mathring{m}_D$	$m_{D_s^*}$	m_{D^*}	m_{D_s}	m_D	m_π	m_K	m_η	f_π	f_K	f_η
1972.1	2112.3	2008.6	1968.5	1867.2	138.0	495.6	566.7	92.4	$1.22f_\pi$	$1.31f_\pi$

where $x_\mu^- = [\chi_-, u_\mu]$. Although the number of relevant LECs at NNLO is considerably smaller than that present in meson-baryon scattering processes [42,43], it is still relatively large considering the scarcity of lattice data. Therefore in the present study we follow the approach adopted in Refs. [44,45] in studies of meson-baryon scatterings and put the NNLO LECs to zero. This is acceptable in the present work because we focus on the recoil corrections and not on the absolute values of the scattering lengths.

B. Power-counting restoration and the EOMS renormalization scheme

In a covariant formulation of χPT describing the interactions between heavy-light mesons and Nambu-Goldstone bosons, one has to face the PCB problem. That is to say, in the calculation of a loop diagram one may find terms with a chiral order lower than that determined by the naive power counting as prescribed in the previous subsection. Such analytical PCB terms can be removed, just as in baryon χPT, by using the heavy-meson expansion, the IR, or the EOMS renormalization prescriptions. The essence of the EOMS approach lies in the fact that χPT, by construction, contains all the structures allowed by symmetry. Therefore, the PCB pieces appearing in a loop calculation can always be removed by redefining the corresponding LECs. This is equivalent to removing the finite PCB pieces directly from the loop results. In practice, this can be achieved in two slightly different ways: (1) one can first perform the loop calculation analytically, and then remove the PCB terms, or (2) one can first perform an expansion in terms of the inverse heavy-meson mass, $1/m_H$, calculate the PCB terms, and then subtract them from the full results. It should be noticed that the second prescription is different from the heavy-meson (baryon) expansion because in general integration and expansion may not commute. But since the PCB terms are finite and analytical, the second prescription should always work.

In the present study, we have explicitly checked that all the PCB terms appearing in our loop calculation can be removed by redefining the LECs introduced in the previous subsection.

III. RESULTS AND DISCUSSION

Figure 1 shows the tree-level diagrams contributing to the (dimensionless) threshold T matrices at LO [Figs. 1(a)–1(c)], NLO [Fig. 1(d)], and NNLO [Fig. 1(e)]. Among the three LO diagrams, the contact term [Fig. 1(a)] yields the following results:

$$T_1^{(1)} = -\frac{\mathring{m}_D m_K}{f_0^2}, \qquad T_2^{(1)} = 0, \qquad T_3^{(1)} = 0,$$
$$T_4^{(1)} = \frac{2\mathring{m}_D m_K}{f_0^2}, \qquad T_5^{(1)} = 0, \qquad T_6^{(1)} = -\frac{m_\pi \mathring{m}_D}{f_0^2}, \tag{8}$$

$$T_7^{(1)} = \frac{2m_\pi \mathring{m}_D}{f_0^2}, \qquad T_8^{(1)} = 0, \qquad T_9^{(1)} = \frac{\mathring{m}_D m_K}{f_0^2},$$
$$T_{10}^{(1)} = -\frac{\mathring{m}_D m_K}{f_0^2}, \qquad T_{11}^{(1)} = \frac{\mathring{m}_D m_K}{f_0^2}, \tag{9}$$

where $1, \cdots 11$ denote the D_sK, $DK(1)$, $D_s\pi$, $DK(0)$, $D_s\eta$, $D\pi(3/2)$, $D\pi(1/2)$, $D\eta$, $D_s\bar{K}$, $D\bar{K}(1)$, and $D\bar{K}(0)$ channels, respectively. In labeling the 11 channels, we have explicitly shown their isospin in parentheses whenever necessary. In the above results, f_0 is the Nambu-Goldstone boson decay constant in the chiral limit. The contributions given by Figs. 1(b) and 1(c) also count as $\mathcal{O}(p)$, but at threshold they are in fact $\mathcal{O}(p^2)$ and have the same structure as those provided by the NLO Lagrangians and therefore can be effectively taken into account by

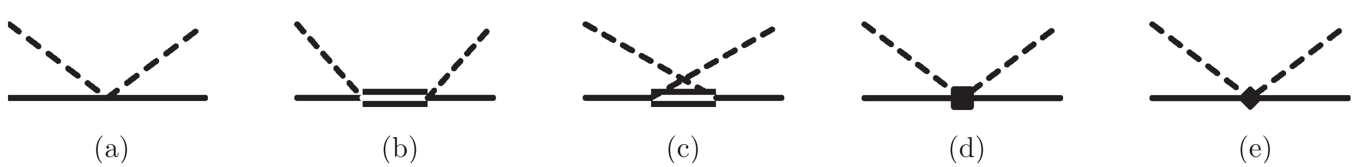

FIG. 1. Tree-level contributions at LO [(a), (b), (c)], NLO (d), and NNLO (e).

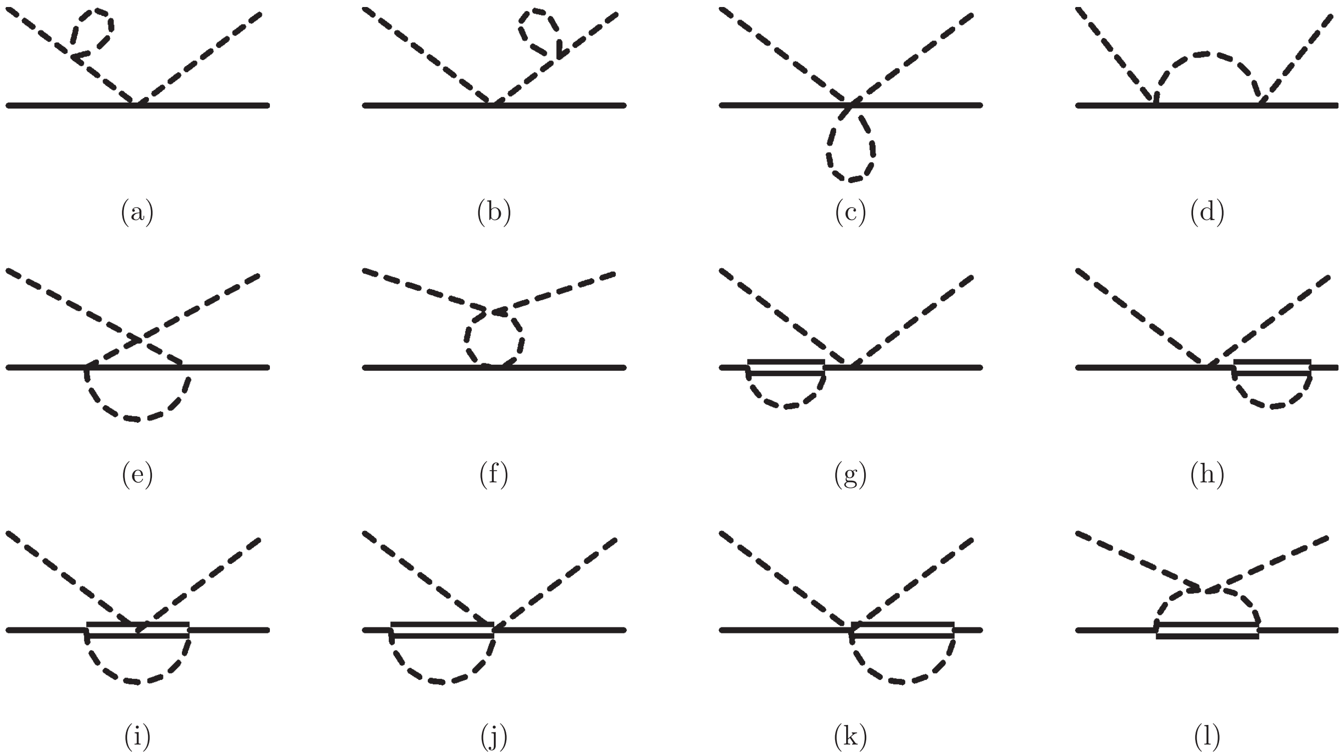

FIG. 2. NNLO loop contributions that survive in the infinite heavy-meson mass ($m_H \to \infty$) limit.

redefining the LECs C_1 and C_0 (see below). Hence we will not calculate them explicitly in the present work. Following the same arguments, we will also neglect similar diagrams at NNLO with one of the LO vertices replaced by a NLO vertex.

Lagrangian (5) provides the NLO tree-level contributions [Fig. 1(d)],[4]

$$T_1^{(2)} = \frac{C_1 m_K^2}{f_0^2}, \qquad T_2^{(2)} = \frac{(C_0 + C_1)m_K^2}{2f_0^2},$$
$$T_3^{(2)} = \frac{(C_0 + C_1)m_\pi^2}{2f_0^2}, \qquad T_4^{(2)} = \frac{(3C_1 - C_0)m_K^2}{2f_0^2}, \tag{10}$$

$$T_5^{(2)} = \frac{(7C_1 - C_0)m_\eta^2 - 16c_1(m_\eta^2 - m_\pi^2)}{6f_0^2},$$
$$T_6^{(2)} = \frac{C_1 m_\pi^2}{f_0^2}, \qquad T_7^{(2)} = \frac{C_1 m_\pi^2}{f_0^2}, \tag{11}$$

$$T_8^{(2)} = \frac{4c_1(m_\eta^2 - m_\pi^2) + (C_0 + 2C_1)m_\eta^2}{3f_0^2},$$
$$T_9^{(2)} = \frac{C_1 m_K^2}{f_0^2}, \qquad T_{10}^{(2)} = \frac{C_1 m_K^2}{f_0^2}, \qquad T_{11}^{(2)} = \frac{C_0 m_K^2}{f_0^2}, \tag{12}$$

where we have introduced two combinations of the 4 LECs: $C_1 = 4(2c_0 - c_1 + 2c_2 + c_3)$ and $C_0 = 4(2c_0 + c_1 + 2c_2 - c_3)$.

The NNLO loop contributions can be separated into two groups: those that survive in the infinite heavy-meson mass ($m_H \to \infty$) limit (Fig. 2) and those that vanish in the $m_H \to \infty$ limit (Fig. 3). For the first group, our results recover those of Ref. [22] in the $m_H \to \infty$ limit. For the second group, our calculation shows that they indeed vanish in the $m_H \to \infty$ limit but are not negligible in a covariant calculation, as shown below. It should be noted that all the loop contributions can be calculated analytically except the box diagrams [Fig. 3(l) and its crossed counterpart]. However, the analytical results are quite involved and therefore we refrain from showing them explicitly. As explained earlier, in the present work we are going to neglect all the NNLO counterterms. Accordingly we have removed from our loop results all the NNLO analytical terms. Therefore our loop results contain only NNLO nonanalytical and higher-order terms. This way the differences between our covariant loop results and those of HMχPT are strictly recoil corrections.

The s-wave scattering lengths are related to the threshold T-matrix elements through

$$a = \frac{1}{8\pi(m_\phi + m_P)} T_{\rm thr} = \frac{\mathring{m}_D}{8\pi(m_\phi + m_P)} \tilde{T}_{\rm thr}, \tag{13}$$

where m_ϕ is the Nambu-Goldstone mass induced by ex-

[4]These results are the same as those of Ref. [22] except that there the expression for $T_4^{(2)}$ is incorrect [46].

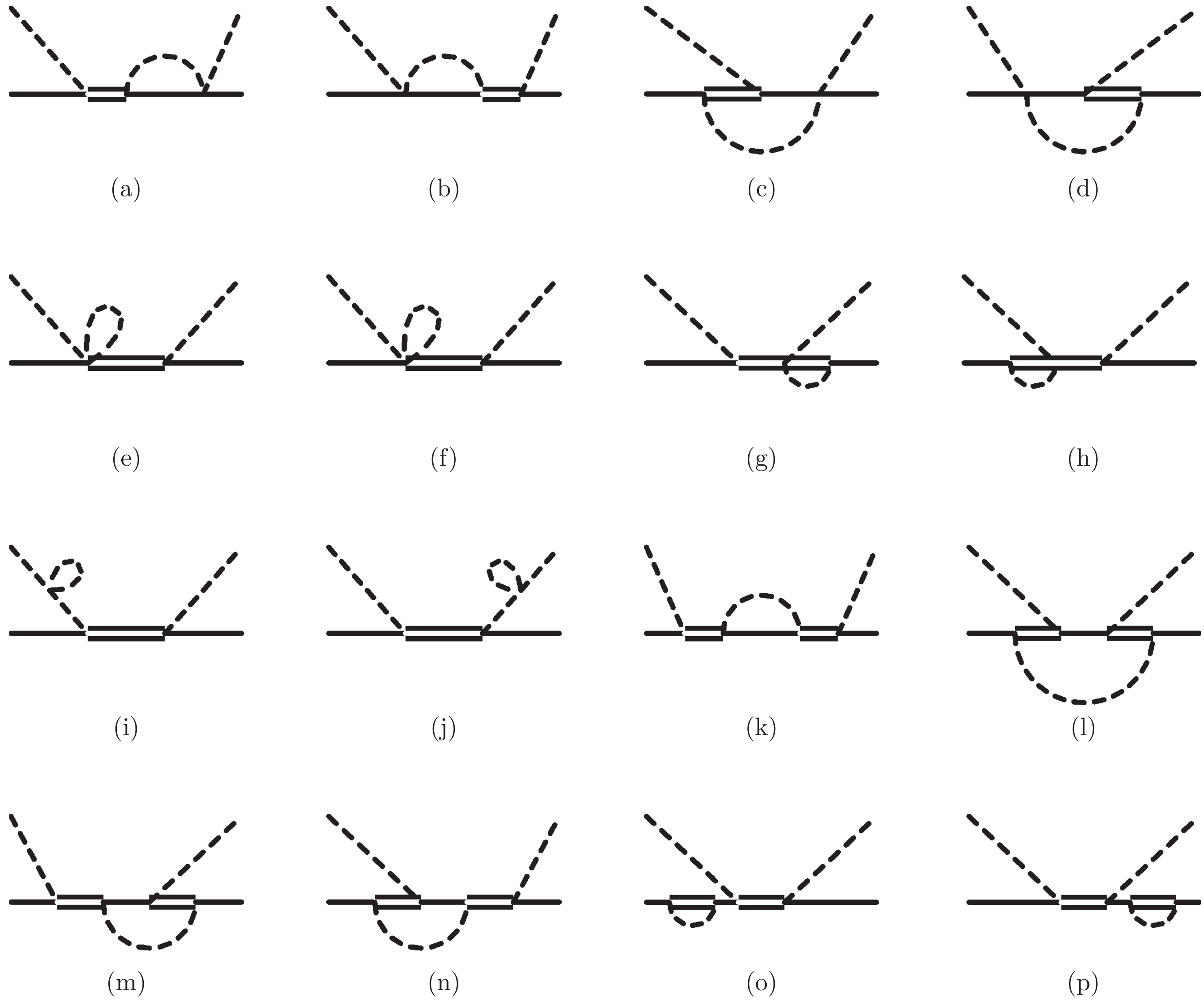

FIG. 3. NNLO loop contributions that vanish in the $m_H \to \infty$ limit. Their crossed counterparts are not shown here but are included in the calculation.

plicit chiral symmetry breaking and m_P is the D meson mass of a particular channel. It should be noted that although in the calculation of T_{thr} we have used the average D meson mass, in evaluating the scattering lengths [Eq. (13)] we use the physical masses as given in Table I. We also use the physical decay constants instead of the chiral limit ones since the differences between them are of higher order. Furthermore, we have introduced $\tilde{T}$ with dimension of a length for a more transparent comparison with the HMχPT results of Ref. [22].

The unknown low-energy constants, C_1 and C_0, could in principle be fixed by reproducing data, which are however not yet available. As in Ref. [22], one can fix them by reproducing the preliminary lattice QCD data (in units of fm): $a_{D\pi(3/2)} = -0.16(4)$, $a_{D\bar{K}(1)} = -0.23(4)$, $a_{D_s\pi} = 0.00(1)$, and $a_{D_sK} = -0.31(2)$.

In Table II, we list the T-matrix elements order by order ($\tilde{T}^{(1)}$ for LO and $\tilde{T}^{(2)}$ for NLO) and the scattering lengths for the 11 independent (strangeness, isospin) channels computed in the $m_H \to \infty$ limit. Since we have neglected the NNLO counterterms, we have denoted the NNLO results by $\tilde{T}_L^{(3)}$ stressing the fact that they contain only loop contributions. We have fixed the LECs C_1 and C_0 by a least-squares fit of the four lattice points [20]. We obtain a $\chi^2/\mathrm{dof} = 5.0$ and the following values for the LECs: $C_1 = 0.4 \pm 1.2$ and $C_0 = 9.6 \pm 10.4$ at the 95% confidence level. Clearly, the four lattice data do not constrain well the two LECs.

In Table III, we show the values of the T-matrix elements and scattering lengths found in the covariant approach. The fit yields a $\chi^2/\mathrm{dof} = 4.5$ and the following values for the two LECs: $C_1 = 2.0 \pm 1.2$ and

TABLE II. Threshold T-matrix elements $\tilde{T}$ and scattering lengths a (in units of fm) in the nonrelativistic χPT up to NNLO ($\tilde{T}^{(1)}$ for LO, $\tilde{T}^{(2)}$ for NLO, and $\tilde{T}^{(3)}$ for NNLO including only loop contributions). The preliminary lattice QCD results [20] are denoted by a(lQCD) and have been fitted to fix the two LECs C_1 and C_0.

(S, I)	$(2, 1/2)$	$(1, 1)$		$(1, 0)$		$(0, 3/2)$	$(0, 1/2)$			$(-1, 1)$	$(-1, 0)$
Channels	D_sK	DK	$D_s\pi$	DK	$D_s\eta$	$D\pi$	$D\pi$	$D\eta$	$D_s\bar{K}$	$D\bar{K}$	$D\bar{K}$
$\tilde{T}^{(1)}$	-7.7	0	0	15.4	0	-3.2	6.4	0	7.7	-7.7	7.7
$\tilde{T}^{(2)}$	0.8	9.7	1.1	-8.0	-1.2	0.1	0.1	7.0	0.8	0.8	18.5
$\tilde{T}^{(3)}_L$	-1.9	$-1.6 + 5.7i$	-1.1	3.5	$0.1 + 9.7i$	-0.8	0.3	$0.8 + 4.8i$	$0.2 + 8.5i$	-3.4	4.8
a	-0.28	$0.27 + 0.19i$	0.00	0.36	$-0.04 + 0.30i$	-0.15	0.26	$0.25 + 0.16i$	$0.28 + 0.27i$	-0.34	1.03
a(lQCD)	$-0.31(2)$		0.00(1)			$-0.16(4)$				$-0.23(4)$	

TABLE III. Same as Table II, but with the threshold T-matrix elements and scattering lengths calculated in relativistic χPT.

(S, I)	$(2, 1/2)$	$(1, 1)$		$(1, 0)$		$(0, 3/2)$	$(0, 1/2)$			$(-1, 1)$	$(-1, 0)$
Channels	D_sK	DK	$D_s\pi$	DK	$D_s\eta$	$D\pi$	$D\pi$	$D\eta$	$D_s\bar{K}$	$D\bar{K}$	$D\bar{K}$
$\tilde{T}^{(1)}$	-7.7	0	0	15.4	0	-3.2	6.4	0	7.7	-7.7	7.7
$\tilde{T}^{(2)}$	3.9	5.8	0.7	1.9	4.9	0.4	0.4	5.2	3.9	3.9	7.8
$\tilde{T}^{(3)}_L$	-5.1	$-2.1 + 4.9i$	-0.7	2.5	$-4.4 + 5.8i$	-0.6	0.3	$-0.4 + 3.8i$	$-0.9 + 4.4i$	-6.2	7.7
a	-0.28	$0.12 + 0.16i$	0.00	0.66	$0.02 + 0.18i$	-0.13	0.28	$0.16 + 0.12i$	$0.34 + 0.14i$	-0.33	0.77
a(lQCD)	$-0.31(2)$		0.00(1)			$-0.16(4)$				$-0.23(4)$	

TABLE IV. Decomposition of the relativistic NNLO threshold T-matrix elements $\tilde{T}^{(3)}_L$ [part A from Figs. 2(a)–2(f); part B from Figs. 2(g)–2(l); part C from Fig. 3].

(S, I)	$(2, 1/2)$	$(1, 1)$		$(1, 0)$		$(0, 3/2)$	$(0, 1/2)$			$(-1, 1)$	$(-1, 0)$
Channels	D_sK	DK	$D_s\pi$	DK	$D_s\eta$	$D\pi$	$D\pi$	$D\eta$	$D_s\bar{K}$	$D\bar{K}$	$D\bar{K}$
A	-6.6	$-2.1 + 4.2i$	-0.6	0.7	$-3.5 + 6.7i$	-0.8	0.5	$-1.8 + 3.4i$	$-1.9 + 6.3i$	-7.2	6.7
B	1.9	-0.6	-0.1	1.9	0.2	0.2	-0.2	0.7	0.7	1.5	-1.8
C	-0.4	$0.7 + 0.8i$	0.1	0.0	$-1.0 - 0.9i$	0.0	0.0	$0.7 + 0.5i$	$0.3 - 1.9i$	-0.5	2.8
$\tilde{T}^{(3)}_L = A + B + C$	-5.1	$-2.1 + 4.9i$	-0.7	2.5	$-4.4 + 5.8i$	-0.6	0.3	$-0.4 + 3.8i$	$-0.9 + 4.4i$	-6.2	7.7

$C_0 = 4.0 \pm 10.4$. Comparing the relativistic and nonrelativistic results, one can easily see that the recoil corrections are sizable. For instance, $\tilde{T}^{(3)}_L$ for D_sK changes from -1.9 fm to -5.1 fm, and $\tilde{T}^{(3)}_L$ for $D_s\eta$ changes from $0.1 + 9.7i$ fm to $-4.4 + 5.8i$ fm when going from HMχPT to covariant χPT.

In obtaining the numbers shown in Tables II and III, we have removed the ultraviolet divergences[5] by the modified minimal subtraction ($\widetilde{MS}$) renormalization scheme and have set the renormalization scale μ to $4\pi f_\pi$. In the covariant χPT, there are two heavy scales, Λ_χ and $\mathring{m}_D$. We have checked that our results would remain qualitatively the same if we had set μ to $\mathring{m}_D$.

In Table IV, the loop contributions $\tilde{T}^{(3)}_L$ calculated in covariant χPT are decomposed into three parts: part A comes from Figs. 2(a)–2(f); part B comes from Figs. 2(g)–2(l); part C comes from Fig. 3. It is clear that Figs. 2(a)–2(f) provide the most important contributions, while those from Fig. 3 are similar in size to those from Figs. 2(g)–2(l), which is different in HMχPT, where the contributions from Fig. 3 vanish.

It should be pointed out that since we have neglected all the NNLO counterterm contributions, we are not in a position to comment on the convergence behavior of either the covariant or the HMχPT results. Furthermore, because of the nearby resonance $D^*_{s0}(2317)$, a pure χPT calculation, such as ours, in the $DK(0)$ and $D_s\eta$ channels should be taken with care, where coupled-channel unitarity may play an important role (for a relevant discussion, see Ref. [21]).

IV. SUMMARY

We have studied the scattering lengths of Nambu-Goldstone bosons interacting with D mesons using a covariant formulation of χPT. In particular, we have studied the recoil corrections by comparing the relativistic with the

[5]They appear in both HMχPT (see also Ref. [22]) and in covariant χPT. The HMχPT results could have been made renormalization scale independent if we had kept the NNLO counterterms.

nonrelativistic results. Our studies show that the recoil corrections are sizable, which should be kept in mind in future studies and in using the HMχPT results. Based on available information we cannot conclude which framework is better, although in principle one should trust more covariant results, particularly when recoil corrections are large.

Up to now, χPT describing the interactions between heavy-light mesons and Nambu-Goldstone bosons has often been used in the nonrelativistic limit. With more precise data and lattice QCD results becoming available, one may have to study more carefully the effects of recoil corrections. The present work should be seen as a first step in this direction.

ACKNOWLEDGMENTS

This work was supported in part by the Alexander von Humboldt foundation and the Fundamental Research Funds for the Central Universities (China). L. S. G thanks M. J. Vicente Vacas, J. Nieves, J. A. Oller, and M. F. M. Lutz for valuable discussions and Y. R. Liu for useful communications about the HMχPT results.

[1] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008) and 2009 partial update for the 2010 edition.
[2] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **90**, 242001 (2003).
[3] P. Krokovny *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 262002 (2003).
[4] D. Besson *et al.* (CLEO Collaboration), Phys. Rev. D **68**, 032002 (2003); **75**, 119908(E) (2007).
[5] W. A. Bardeen, E. J. Eichten, and C. T. Hill, Phys. Rev. D **68**, 054024 (2003).
[6] M. A. Nowak, M. Rho, and I. Zahed, Acta Phys. Pol. B **35**, 2377 (2004).
[7] E. van Beveren and G. Rupp, Phys. Rev. Lett. **91**, 012003 (2003).
[8] Y. B. Dai, C. S. Huang, C. Liu, and S. L. Zhu, Phys. Rev. D **68**, 114011 (2003).
[9] S. Narison, Phys. Lett. B **605**, 319 (2005).
[10] Y. Q. Chen and X. Q. Li, Phys. Rev. Lett. **93**, 232001 (2004).
[11] A. P. Szczepaniak, Phys. Lett. B **567**, 23 (2003).
[12] T. E. Browder, S. Pakvasa, and A. A. Petrov, Phys. Lett. B **578**, 365 (2004).
[13] H. Y. Cheng and W. S. Hou, Phys. Lett. B **566**, 193 (2003).
[14] T. Barnes, F. E. Close, and H. J. Lipkin, Phys. Rev. D **68**, 054006 (2003).
[15] E. E. Kolomeitsev and M. F. M. Lutz, Phys. Lett. B **582**, 39 (2004).
[16] F. K. Guo, P. N. Shen, H. C. Chiang, and R. G. Ping, Phys. Lett. B **641**, 278 (2006).
[17] D. Gamermann, E. Oset, D. Strottman, and M. J. Vicente Vacas, Phys. Rev. D **76**, 074016 (2007).
[18] C. Garcia-Recio, L. S. Geng, J. Nieves, and L. L. Salcedo, arXiv:1005.0956.
[19] J. M. Flynn and J. Nieves, Phys. Rev. D **75**, 074024 (2007).
[20] L. Liu, H. W. Lin, and K. Orginos, Proc. Sci., LATTICE2008 (2008) 112.
[21] F. K. Guo, C. Hanhart, and U. G. Meissner, Eur. Phys. J. A **40**, 171 (2009).
[22] Y. R. Liu, X. Liu, and S. L. Zhu, Phys. Rev. D **79**, 094026 (2009).
[23] M. B. Wise, Phys. Rev. D **45**, R2188 (1992).
[24] T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin, and H. L. Yu, Phys. Rev. D **46**, 1148 (1992); **55**, 5851(E) (1997).
[25] G. Burdman and J. F. Donoghue, Phys. Lett. B **280**, 287 (1992).
[26] E. E. Jenkins, Nucl. Phys. **B412**, 181 (1994).
[27] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. **281**, 145 (1997).
[28] L. S. Geng, J. Martin Camalich, L. Alvarez-Ruso, and M. J. V. Vacas, Phys. Rev. Lett. **101**, 222002 (2008).
[29] L. S. Geng, J. Martin Camalich, and M. J. Vicente Vacas, Phys. Rev. D **80**, 034027 (2009).
[30] L. S. Geng, J. Martin Camalich, and M. J. Vicente Vacas, Phys. Rev. D **79**, 094022 (2009).
[31] J. Martin-Camalich, L. S. Geng, and M. J. V. Vacas, arXiv:1003.1929 [Phys. Rev. D (to be published)].
[32] J. Gasser, M. E. Sainio, and A. Svarc, Nucl. Phys. **B307**, 779 (1988).
[33] E. E. Jenkins and A. V. Manohar, Phys. Lett. B **255**, 558 (1991).
[34] V. Bernard, N. Kaiser, and U. G. Meissner, Int. J. Mod. Phys. E **4**, 193 (1995).
[35] T. Becher and H. Leutwyler, Eur. Phys. J. C **9**, 643 (1999).
[36] J. Gegelia and G. Japaridze, Phys. Rev. D **60**, 114038 (1999).
[37] T. Fuchs, J. Gegelia, G. Japaridze, and S. Scherer, Phys. Rev. D **68**, 056005 (2003).
[38] B. R. Holstein, V. Pascalutsa, and M. Vanderhaeghen, Phys. Rev. D **72**, 094014 (2005).
[39] G. Amoros, J. Bijnens, and P. Talavera, Nucl. Phys. **B602**, 87 (2001).
[40] J. L. Rosner and S. Stone, arXiv:1002.1655.
[41] M. Kolesar and J. Novotny, Fiz. B **17**, 57 (2008).
[42] J. A. Oller, M. Verbeni, and J. Prades, J. High Energy Phys. 09 (2006) 079.
[43] M. Frink and U. G. Meissner, Eur. Phys. J. A **29**, 255 (2006).
[44] M. Mai, P. C. Bruns, B. Kubis, and U. G. Meissner, Phys. Rev. D **80**, 094006 (2009).
[45] Y. R. Liu and S. L. Zhu, Phys. Rev. D **75**, 034003 (2007).
[46] Y. R. Liu (private communication).