Next-to-leading-order QCD correction to inclusive $J/\psi(\Upsilon)$ production in Z^0 decay

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In this paper, we study the $J/\psi(\Upsilon)$ production in Z boson decay in a color-singlet model (CSM). We calculate the next-to-leading-order (NLO) QCD correction to $Z \rightarrow$ quarkonium $+ Q\bar{Q}$, the dominant contribution in the CSM, with the vector and axial-vector parts in the $ZQ\bar{Q}$ vertex being treated separately. The results show that the vector and axial-vector parts have the same K factor (the ratio of the NLO result to the leading-order result) 1.13 with the renormalization scale $\mu = 2m_c$ and $m_c = 1.5$ GeV, and the K factor falls to 0.918 when applying the Brodsky, Lepage, and Mackenzie (BLM) renormalization scale scheme with obtained $\mu_{\text{BLM}} = 2.28$ GeV and $m_c = 1.5$ GeV. By including the contributions from the next-dominant ones, the photon and gluon fragmentation processes, the branching ratio for $Z \rightarrow J/\psi_{\text{prompt}} + X$ is $(7.3–10.0) \times 10^{-5}$ with the uncertainty consideration for the renormalization scale and charm quark mass. The results are about one-half of the central value of the experimental measurement 2.1×10^{-4}. Furthermore, the J/ψ energy distribution in our calculation cannot describe the experimental data. Therefore, even at QCD NLO, the contribution to $Z \rightarrow J/\psi_{\text{prompt}} + X$ from the CSM cannot fully account for the experimental measurement. And there should be contributions from other mechanisms, such as the color-octet (COM) contributions. We define $R_{c\bar{c}} = \frac{\Gamma(Z \rightarrow J/\psi c\bar{c}X)}{\Gamma(Z \rightarrow J/\psi X)}$ and obtain $R_{cc} = 0.84$ for only the CSM contribution and $R_{cc} = 0.49$ for COM and CSM contributions together. Then the R_{cc} measurement could be used to clarify the COM contributions.

I. INTRODUCTION

Heavy quarkonium is an ideal system being used to study the perturbative and nonperturbative aspects of QCD. First, the heavy quark mass sets a large scale for perturbative calculation. Second, the dileptonic decay of heavy quarkonium makes the identification and measurement efficient. In 1995, the nonrelativistic QCD (NRQCD), a rigorous effective theory in describing the production and decay of heavy quarkonium, was proposed [1], and it makes the color-singlet model (CSM) [2] its leading-order approximation in v (the velocity between heavy quark and antiquark in the meson rest frame). More details on NRQCD and heavy quarkonium physics can be found in Ref. [3].

In recent years, there are many works on the next-to-leading-order (NLO) QCD correction for heavy quarkonium productions. To explain the experimental measurement [4,5] of J/ψ production at the B factories, a series of calculations [6,7] have been performed and revealed that the NLO QCD corrections can change the leading-order (LO) theoretical predictions considerably, and the NLO results in the CSM give the main contribution to the related processes. Together with the relativistic correction [8], it seems that all the experimental data for J/ψ production at the B factories could be understood. For J/ψ production in the hadron colliders, there is obviously progress in the theoretical calculation. The NLO QCD correction to the CSM processes [9,10] greatly enhanced the p_t (transverse momentum of J/ψ) distribution of J/ψ production at the large p_t region, and the p_t distribution of J/ψ

polarization is drastically changed from mostly transverse polarization at LO into mostly longitudinal polarization at NLO [10]. It is found that the NLO QCD correction to J/ψ production for color-octet (COM) parts is quite small, about 10% [11]. Even including all this progress, we still cannot obtain a satisfactory explanation for both the p_t distribution of the production and the polarization for the J/ψ hadroproduction. The partial next-to-next-to-leading-order (NNLO) calculations for Υ and J/ψ hadroproduction show that the uncertainty from the QCD higher order correction [12] is much bigger, but still cannot cover the J/ψ or Υ polarization measurements. Recent studies reveal that the NLO QCD correction also plays an important role in J/ψ production at the RHIC [13] and the hadroproduction of χ_c [14]. The J/ψ photoproduction once was considered as a positive example with the p_t and z distribution well described by the NLO calculations in the CSM [15]. But neither the p_t distribution of the production nor the polarization for J/ψ can be well described by the recent NLO calculations in the CSM [16]. It seems that the complete calculation at NLO in the COM [17] can account for the experimental measurements on the p_t distribution. But the complete calculation on J/ψ polarization at NLO in the COM is a real challenge.

With both the successful and unsuccessful aspects for theoretical progress in heavy quarkonium production, it is worthwhile to study more cases in detail, such as the J/ψ production associated with a photon [18], QED contributions in J/ψ hadroproduction [19], inclusive J/ψ production from Υ decay [20], and J/ψ production from

Z decay. Heavy quarkonium production in Z decay has been widely studied in the CSM and the COM at LO [21–24], and the measurement at the LEP by the L3 Collaboration gives the branching ratio as [25]

$$\text{Br}\,(Z \to J/\psi_{\text{prompt}} + X)$$
$$= (2.1 \pm 0.6(\text{stat}) \pm 0.4(\text{sys})^{+0.4}_{-0.2}(\text{theo})) \times 10^{-4}, \quad (1)$$

$$\text{Br}\,(Z \to \Upsilon(1S) + X) < 4.4 \times 10^{-5}. \quad (2)$$

Theoretical investigation on this process indicates that even the dominant channel $Z \to J/\psi + c + \bar{c}$ in all the CSM ones at LO only gives a $1/3$ prediction to the total branching ratio of the experimental measurement. Including the contribution of the gluon fragmentation process in the COM can enhance the theoretical results about 3 times [22]. This once is an evidence for the effect of the COM. It also has been studied in the color-evaporation model (CEM) in Ref. [26] and obtained consistent results with the experimental data. But the CEM always gives unpolarized J/ψ in conflict with experimental measurements. By resuming the large logarithm from the large difference of J/ψ and Z mass, the COM prediction on J/ψ energy distribution ($d\Gamma/dz$ with $z = 2E_{J/\psi}/M_z$) is roughly consist with the data [27]. Considering the larger impact of the NLO QCD corrections to the production of heavy quarkonium, it is necessary to investigate the NLO QCD correction to $Z \to J/\psi + c\bar{c} + X$. In this paper, we calculate the NLO QCD correction to $Z \to J/\psi + c\bar{c} + X$, and also include the contributions from the gluon and photon fragmentation processes as well with only the CSM in consideration. The study could provide more insight into the effect of the color-octet mechanism and put more constrains on the value of the color-octet matrix elements.

This paper is organized as follows. In Sec. II, we study the NLO QCD correction to the heavy quark association process with different schemes on the choice of the renormalization scale. In Sec. III, we investigated dominant fragmentation processes and give the total results on prompt J/ψ production in Z decay. In Sec. IV, the summary and conclusion are presented.

II. THE HEAVY QUARK ASSOCIATION PROCESS

For the calculation on $Z \to J/\psi + c\bar{c} + X$ at NLO, there are virtual and real correction parts as

$$Z \to J/\psi + c + \bar{c}, \quad (3)$$

$$Z \to J/\psi + c + \bar{c} + g. \quad (4)$$

There are vector and axial-vector parts in the coupling of the Z boson to fermions, but the interference between them is a P-parity violation contribution and is unobservable after averaging out at the space distribution. Therefore, it does not contribute in our calculation on the partial decay width and energy distribution of heavy quarkonium, and

we can study them separately. There are 4 Feynman diagrams for both parts at LO, 80 for the vector part and 76 for the axial-vector part at NLO. The typical diagrams are presented in Fig. 1. The dimensional regularization is used to regulate the ultraviolet (UV) and infrared (IR) divergences, and the Coulomb singularity is regulated by introducing a small relative velocity between the quark pair in the quarkonium and absorbed into the wave function of the quarkonium. In calculating the axial-vector part, we have to face the γ_5 problem. The structure of all the amplitude squared diagrams could be classified into four cases shown in Fig. 2.

Case 1. There are only one fermion-loop and two γ_5 matrices appearing in it. Then γ_5's can be moved together and give an identity matrix by $\gamma_5^2 = 1$.

Case 2. There are two fermion-loops and the two γ_5 matrices appearing in one of them. It is the same as case 1.

Case 3. There are two fermion loops. Each of them has a γ_5. Because there are no UV and IR divergences in the loops, the dimension can be set as 4 safely.

Case 4. The only special case are the two triangle anomalous diagrams. In this case we use the scheme described in Ref. [28] to handle it, which fixes the starting point to write down all the amplitude and abandon the cyclicity in calculating the trace of the fermion loop with an odd number of γ_5. These two triangle anomalous diagrams will not contribute at all according to Yang's theorem [29] when the two gluon lines are on mass shell, but will contribute in our case since the two connected gluons are off mass shell.

The on-mass-shell (OS) scheme is used to define the renormalization constants Z_m, Z_2, and Z_3, which
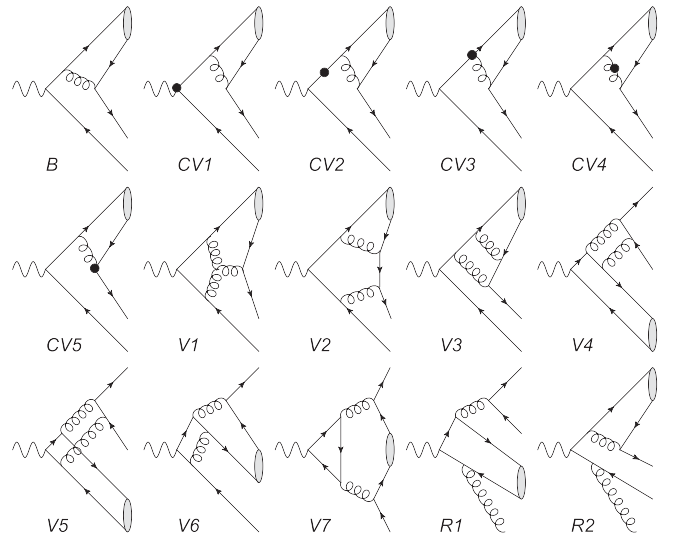

FIG. 1. Typical Feynman diagrams for $J/\psi + c\bar{c} + X$ production in Z decay. B is the Born diagram. $CV1$–$CV5$ represent the counterterm diagrams and corresponding loop diagrams. $V1$–$V7$ represent the box and the anomalous triangle diagrams. $R1$ and $R2$ are the real parts.
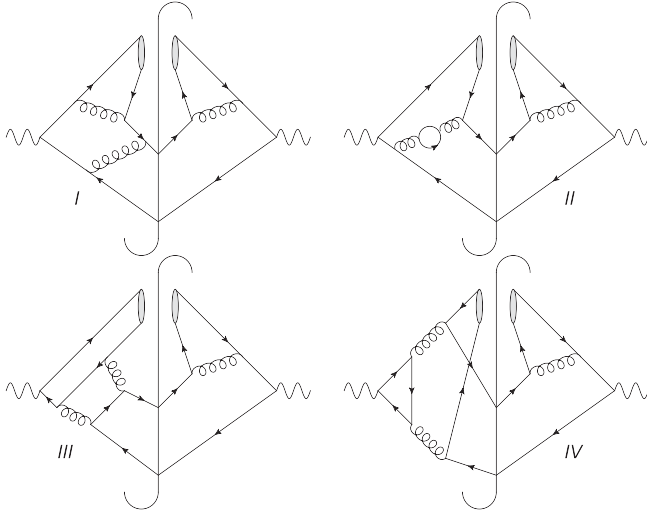
FIG. 2. The typical amplitude squared diagrams for $J/\psi + c\bar{c} + X$ production in Z decay.

correspond to charm quark mass m_c, charm field ψ_c, and gluon field A^a_μ, while Z_g for the QCD gauge coupling α_s is defined in the modified-minimal-subtraction ($\overline{\text{MS}}$) scheme:

$$\delta Z_m^{\text{OS}} = -3C_F \frac{\alpha_s}{4\pi}\left[\frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln\frac{4\pi\mu^2}{m_c^2} + \frac{4}{3}\right],$$

$$\delta Z_2^{\text{OS}} = -C_F \frac{\alpha_s}{4\pi}\left[\frac{1}{\epsilon_{\text{UV}}} + \frac{2}{\epsilon_{\text{IR}}} - 3\gamma_E + 3\ln\frac{4\pi\mu^2}{m_c^2} + 4\right],$$

$$\delta Z_3^{\text{OS}} = \frac{\alpha_s}{4\pi}\left[(\beta_0' - 2C_A)\left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}}\right)\right.$$
$$\left. - \frac{4}{3}T_F\left(\frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln\frac{4\pi\mu^2}{m_c^2}\right)\right],$$

$$\delta Z_g^{\overline{\text{MS}}} = -\frac{\beta_0}{2}\frac{\alpha_s}{4\pi}\left[\frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln(4\pi)\right], \quad (5)$$

where μ is the renormalization scale, γ_E is Euler's constant, $\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$ is the one-loop coefficient of the QCD beta function, and n_f is the number of active quark flavors. There are three massless light quarks u, d, and s, and heavy quark c, so $n_f = 4$. In $SU(3)_c$, color factors are given by $T_F = \frac{1}{2}$, $C_F = \frac{4}{3}$, and $C_A = 3$. And $\beta_0' \equiv \beta_0 + (4/3)T_F = (11/3)C_A - (4/3)T_F n_{lf}$, where $n_{lf} \equiv n_f - 1 = 3$ is the number of light quark flavors. Actually in the NLO total amplitude level, the terms proportion to δZ_3^{OS} cancel each other, and thus the result is independent of the renormalization scheme of the gluon field. The above renormalization scheme and constant are similar to those in Ref. [30]. The bottom quark should be considered for the calculation of Υ production.

We use the Feynman Diagram Calculation (FDC) package [31] to generate the Feynman diagram and amplitude, to do the tensor reduction and scalar integration, and to give the FORTRAN code for numerical calculation finally.
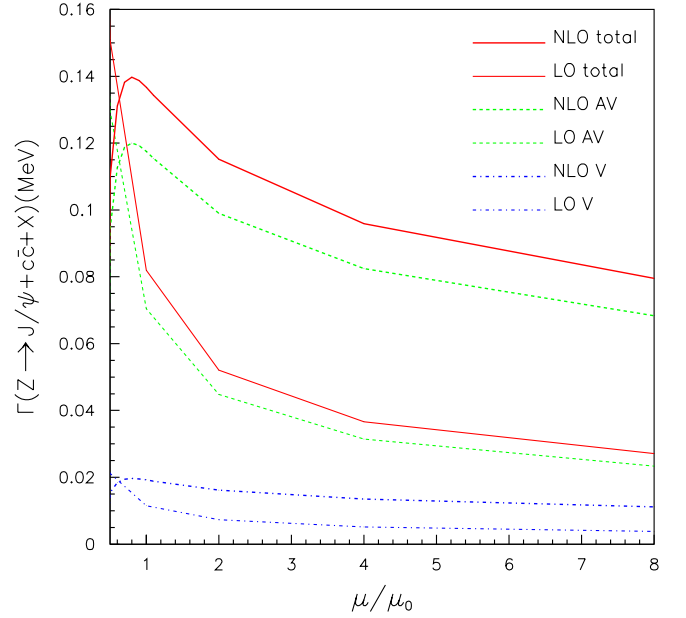

FIG. 3 (color online). The μ dependence of the partial decay width for $Z \to J/\psi c\bar{c} + X$ with $m_c = 1.5$ GeV and $\mu_0 = 2m_c$. Here the LO results are calculated with the wave function at origin at LO and the α_s are fixed by one-loop running, and these choices are also applied for the LO plots in Figs. 4–6. In all the figures, V presents the vector part result, A–V presents the axial-vector part, and total presents the sum of these two parts.

Because there are some large numbers generated in the program, the quadruple precision FORTRAN source is used.

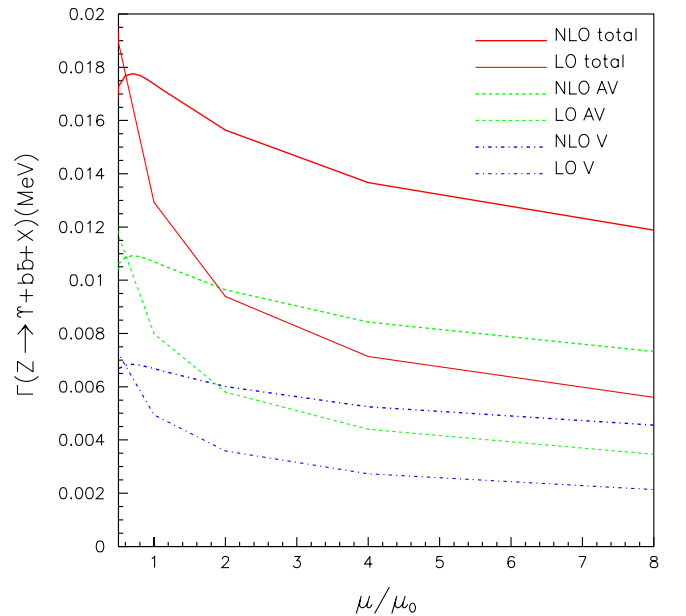The leptonic width of $J/\psi(\Upsilon)$ is used to extract their wave functions at origin $R_s^{J/\psi(\Upsilon)}$, which is

FIG. 4 (color online). The μ dependence of the partial decay width for $Z \to \Upsilon b\bar{b} + X$ with $m_b = 4.75$ GeV and $\mu_0 = 2M_b$.
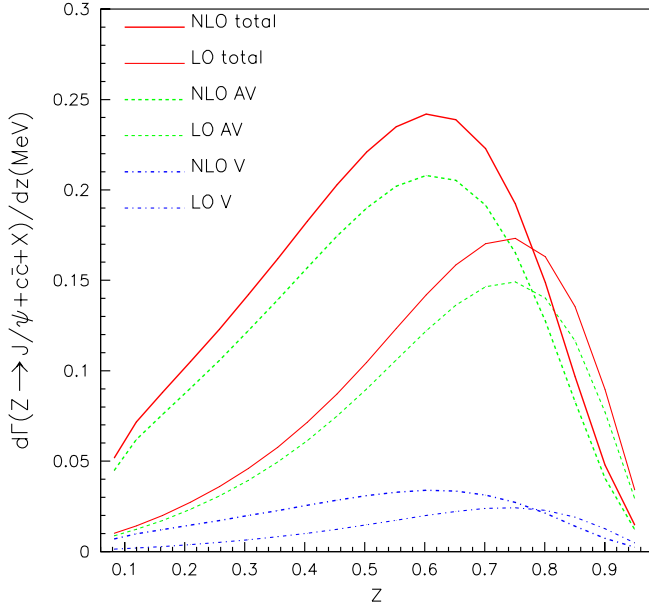
FIG. 5 (color online).   The J/ψ energy distribution in $Z \to J/\psi c\bar{c}$ with $m_c = 1.5$ GeV and $\mu = 2M_c$.

FIG. 6 (color online).   The Υ energy distribution in $Z \to \Upsilon b\bar{b}$ with $m_b = 4.75$ GeV and $\mu = 2M_b$.

$$\Gamma_{ee} = \left(1 - \frac{16\alpha_s}{3\pi}\right)\frac{4\alpha^2 e_{c(b)}^2}{M_{J/\psi(\Upsilon)}^2}|R_s^{J/\psi(\Upsilon)}|^2.$$

Here the values of the parameters are chosen as $\Gamma_{ee}^{J/\psi} = 5.55$ keV, $\Gamma_{ee}^{\Upsilon} = 1.34$ keV [32], $\alpha = 1/137$, and $\alpha_s = \alpha_s^{2\,\mathrm{loop}}(2m_Q)$. The one-loop and two-loop running programs of CTEQ6 are used to fix the LO and NLO values of α_s. The LO wave functions of heavy quarkonium are used to obtain the LO results in Figs. 3–6. In the following calculation, $\alpha = 1/128$ is used, and the central value of the heavy quark mass is chosen as $m_c = 1.5$ GeV ($m_b = 4.75$ GeV). We also use $m_c = 1.4, 1.6$ GeV ($m_b = 4.65, 4.85$ GeV) for the uncertainty estimate. The default choice of the renormalization scale is $2m_c(2m_b)$ for J/ψ (Υ).

The LO and NLO partial decay widths of $Z \to J/\psi + c\bar{c} + X$ are presented in Table I. The difference between
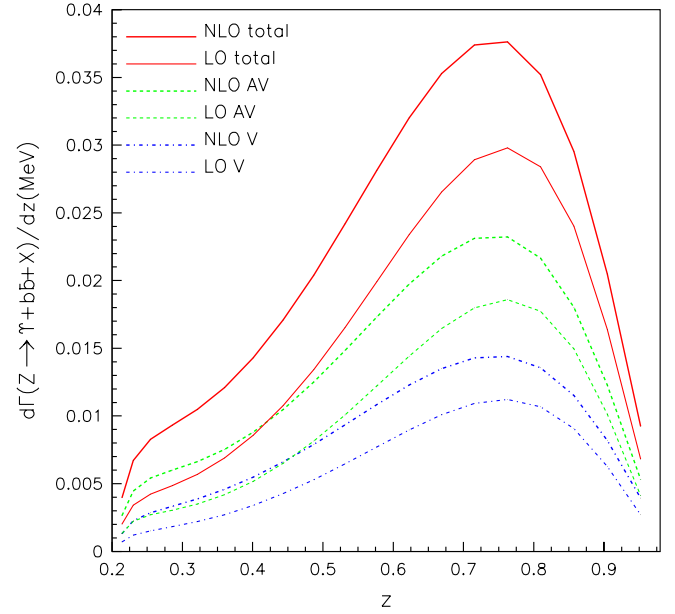
our LO results and the other LO results in literature is mainly due to the different choice of the wave functions at origin. The QCD correction enhances the partial decay width about 13% for both the vector part and the axial-vector part when the same wave function at origin is used. This may provide a hint that the picture of heavy quark fragmentation into quarkonium works at the energy scale at NLO. It can also be seen that the K factors are insensitive to the variance of the quark mass. For the Υ production, similar results are presented in Table II. And it is easy to find that there is a very small difference in K factors between the vector part and the axial-vector part. It could be thought that the large bottom quark mass makes the fragmentation picture less effective than that in the J/ψ production process.

The renormalization scale dependences of the partial decay widths for J/ψ and Υ are shown in Figs. 3 and 4.

TABLE I.   The partial decay width for J/ψ with the renormalization scale $\mu = 2m_c$ and different charm quark mass m_c.

m_c (GeV)	$\alpha_s(\mu)$	$\Gamma_V^{(0)}$ (keV)	$\Gamma_V^{(1)}$ (keV)	$\Gamma_V^{(1)}/\Gamma_V^{(0)}$	$\Gamma_{AV}^{(0)}$ (keV)	$\Gamma_{AV}^{(1)}$ (keV)	$\Gamma_{AV}^{(1)}/\Gamma_{AV}^{(0)}$	$\Gamma_{\mathrm{tot}}^{(1)}/\Gamma_{\mathrm{tot}}^{(0)}$
1.4	0.266	19.6	22.2	1.13	120	136	1.13	1.13
1.5	0.259	16.9	19.1	1.13	103	117	1.13	1.13
1.6	0.252	14.8	16.7	1.13	90.0	102	1.13	1.13

TABLE II.   The partial decay width for Υ with the renormalization scale $\mu = 2m_b$ and different bottom quark mass m_b.

m_b (GeV)	$\alpha_s(\mu)$	$\Gamma_V^{(0)}$ (keV)	$\Gamma_V^{(1)}$ (keV)	$\Gamma_V^{(1)}/\Gamma_V^{(0)}$	$\Gamma_{AV}^{(0)}$ (keV)	$\Gamma_{AV}^{(1)}$ (keV)	$\Gamma_{AV}^{(1)}/\Gamma_{AV}^{(0)}$	$\Gamma_{\mathrm{tot}}^{(1)}/\Gamma_{\mathrm{tot}}^{(0)}$
4.65	0.184	5.50	6.88	1.24	8.95	11.1	1.25	1.24
4.75	0.183	5.33	6.68	1.24	8.61	10.7	1.25	1.25
4.85	0.182	5.17	6.49	1.24	8.29	10.3	1.26	1.25

The QCD correction improves the scale dependence in the small μ region and there is similar behavior for LO and NLO results in the other region. In Figs. 5 and 6, the energy distributions of J/ψ and Υ are shown with z defined as $2E_{J/\psi(\Upsilon)}/M_Z$. The NLO QCD correction shifts the maximum point of J/ψ energy distribution from the large z region to the middle z region. But for Υ, the shift is not manifest.

To study the uncertainty from different choices of the renormalization scale, in addition to our default choice $2m_c(m_b)$ for $J/\psi(\Upsilon)$ in the calculation, we use two other schemes to fix the renormalization scale. At first, the decay width could be expressed as

$$\Gamma^{\mathrm{NLO}}(\mu) = \Gamma^{\mathrm{LO}}(\mu)\left[1 + \frac{\alpha_s(\mu)}{\pi}\left(A + \beta_0' \ln\frac{\mu}{2m_Q} + Bn_f\right)\right].$$
(6)

Here the LO results depend on the renormalization scale through the running of the coupling constant. A and B are independent of the scale and $\beta_0' = 11 - 2n_{lf}/3$. We extract the parameters in Eq. (6) and present them in Table III.

Scheme I: From Figs. 3 and 4, it can be seen that there are the μ points where the partial decay widths reach their maximum values. By using Eq. (6), the values of μ and partial decay widths can be obtained and presented in Table IV.

Scheme II: In the Brodsky, Lepage, and Mackenzie (BLM) scheme [33], the n_{lf} (light quark flavor) dependence of the QCD correction is absorbed into the running

TABLE III. The extracted parameters for Eq. (6).

m_c (GeV)	$\Gamma^{\mathrm{LO}}_{J/\psi}$ (keV)	A	B
1.40	176	2.08	-0.178
1.50	151	2.12	-0.182
1.60	131	2.16	-0.186
m_b (GeV)	$\Gamma^{\mathrm{LO}}_{\Upsilon}$ (keV)		
4.65	17.8	4.97	-0.273
4.75	17.2	5.05	-0.275
4.85	16.6	5.12	-0.278

TABLE IV. The maximum partial decay width for $Z \to J/\psi(\Upsilon) + c\bar{c}(b\bar{b}) + X$ in scheme I.

m_c (GeV)	μ (GeV)	$\Gamma^{\mathrm{NLO}}_{J/\psi}$ (keV)
1.40	2.26	162
1.50	2.42	139
1.60	2.58	120
m_b (GeV)		$\Gamma^{\mathrm{NLO}}_{\Upsilon}$ (keV)
4.65	6.48	18.4
4.75	6.57	17.8
4.85	6.66	17.2

TABLE V. The partial decay width with different charm quark mass m_c and renormalization scale $\mu = \mu^*$ in the BLM scheme.

m_c (GeV)	μ^* (GeV)	$\alpha_s(\mu^*)$	$\Gamma^{(0)}$ (keV)	$\Gamma^{(1)}$ (keV)	$\Gamma^{(1)}/\Gamma^{(0)}$
1.4	2.14	0.298	176	162	0.919
1.5	2.28	0.290	151	139	0.918
1.6	2.42	0.282	131	120	0.918

TABLE VI. The partial decay width with different bottom quark mass m_b and renormalization scale $\mu = \mu^*$ in the BLM scheme.

m_b (GeV)	μ^* (GeV)	$\alpha_s(\mu^*)$	$\Gamma^{(0)}$ (keV)	$\Gamma^{(1)}$ (keV)	$\Gamma^{(1)}/\Gamma^{(0)}$
4.65	6.18	0.204	17.8	18.3	1.03
4.75	6.29	0.203	17.2	17.7	1.03
4.85	6.39	0.202	16.6	17.1	1.03

of α_s by shifting the renormalization scale. An improved result on the process $e^+e^- \to J/\psi c\bar{c}$ has been obtained in Ref. [7]. So we also try this scheme in our calculation and the results are presented in Tables V and VI. It can be seen that the convergences of the perturbative expansions are all improved and the K factor is even lower than 1 for the J/ψ production.

The above two schemes give almost the same results for both the J/ψ and Υ processes. In the following discussion we will adopt the results from the BLM scheme.

III. PHOTON AND GLUON FRAGMENTATION PROCESSES AND THE TOTAL RESULTS

There are some QED processes which can give contributions comparable to that of the QCD ones in heavy quarkonium production [34]. The contribution from the photon fragmentation processes was investigated in Ref. [24], and it gives a contribution to the inclusive J/ψ production in Z boson decay that cannot be ignored. Therefore, we further investigate the QCD correction to this photon fragmentation process. At leading order, the following processes must be included:

$$Z \to J/\psi + l^+ + l^-,$$
(7)

$$Z \to J/\psi + q + \bar{q}.$$
(8)

Here $l(q)$ is the lepton (quark) and the final results must be summed over e, μ, and $\tau(u, d, c, s, b)$. We only pick out the photon fragmentation diagrams to calculate. These diagrams form a gauge invariance subgroup. All the typical Feynman diagrams at LO and NLO are shown in Fig. 7.

There are also the gluon fragmentation processes in CSM,

$$Z \to J/\psi + q + \bar{q} + g + g.$$
(9)

Here the $q\bar{q}$ in the final states will be summed over u, d, c, s, and b. Although they are at order $\alpha\alpha_s^4$, the contribution
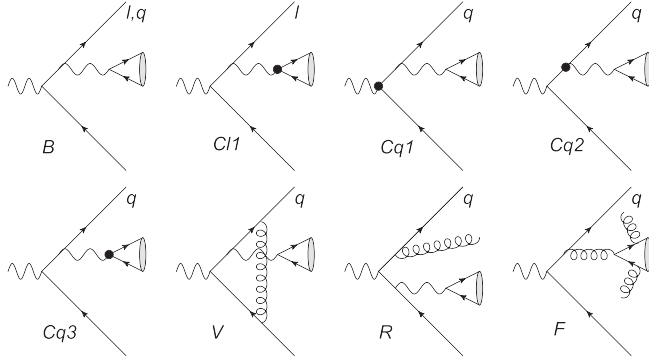
FIG. 7. The typical Feynman diagrams for the fragmentation processes on $Z \to J/\psi + X$. B is the Born diagram. $Cl1$, $Cq1$–$Cq3$ are the counterterm diagrams with their corresponding loop diagrams for $J/\psi + l^+ l^- + X$ and $J/\psi + q\bar{q} + X$ separately, V is the box diagram, R is the diagram for the real correction, and F is the gluon fragmentation process.

of them is not too small [22,23]. The typical Feynman diagrams are shown in Fig. 7.

In evaluating these fragmentation processes, we set the renormalization scale as $2m_c(2m_b)$. The NLO α_s and wave function for quarkonium are also used. Taking all the above processes into account, we get the full results on the partial widths in Table VII and the energy distribution in Fig. 8.

Combining all the above results together and timing a factor of 1.29 to include the contribution from the ψ' feed-down, we obtain the branching ratio of J/ψ production in the Z decay as follows:

$$\text{Br}_{\text{NLO}}^{J/\psi + c\bar{c} + X} = (6.20\text{–}8.38) \times 10^{-5}, \qquad (10)$$

$$\text{Br}^{\text{frag.pro.}} = (1.13\text{–}1.57) \times 10^{-5}, \qquad (11)$$

$$\text{Br}^{\text{total}} = (7.33\text{–}9.95) \times 10^{-5}. \qquad (12)$$

Here we give the range of the branching ratio with the charm mass changing from 1.4 to 1.6 GeV. The total theoretical result is almost one-half of the central value of the experimental measurement in Eq. (1). It is shown in Fig. 8 that the photon and gluon fragmentation processes contribute more in the lower energy region and the energy distribution cannot fit the experimental data.

TABLE VII. The mass of the charm quark is chosen as 1.4 and 1.6 GeV, $\mu = \mu_{\text{BLM}}$ for $J/\psi + c\bar{c}$ and $\mu = 2m_c$ for other processes. $\Gamma_{\text{QCD}}^{\text{gluon}}$ and Γ_{QED} present the contributions of the photon and gluon fragmentation processes, respectively (unit of decay widths: KeV).

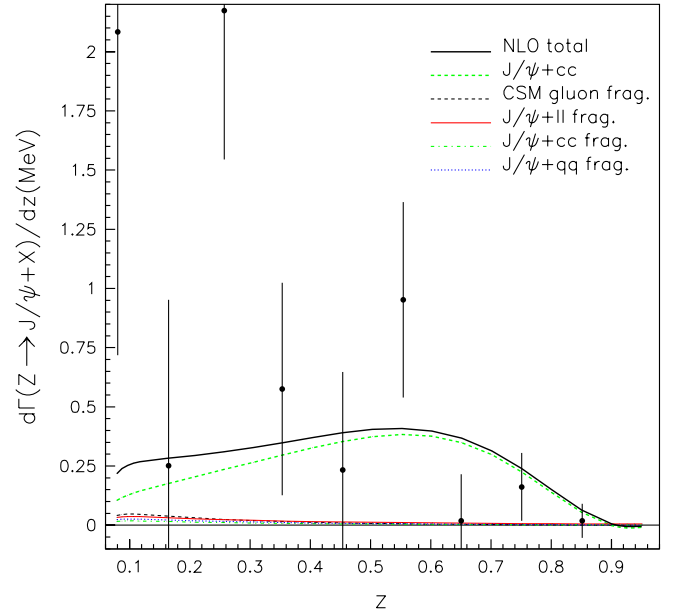| m_c (GeV)	$\Gamma_{J/\psi+c\bar{c}}^{\text{BLM}}$	$\Gamma_{\text{QCD}}^{\text{gluon}}$	$\Gamma_{\text{QED}}^{e,\mu,\tau}$	$\Gamma_{\text{QED}}^{u,d,s}$	Γ_{QED}^{c}
1.4	162	9.21	10.5	6.26	4.36
1.6	120	5.41	8.12	4.91	3.43

FIG. 8 (color online). The J/ψ energy distribution in $Z \to J/\psi + X$ with $m_c = 1.4$ GeV, and $\mu = \mu_{\text{BLM}}$ for $J/\psi + c\bar{c}$ and $\mu = 2m_c$ for other processes. The experimental data are taken from Refs. [27,35].

Furthermore, we defined a ratio as

$$R_{c\bar{c}} = \frac{\Gamma(Z \to J/\psi + c\bar{c} + X)}{\Gamma(Z \to J/\psi + X)}. \qquad (13)$$

Using the theoretical results obtained in the CSM, the ratio is about $R_{c\bar{c}}^{\text{CSM}} = 0.84$ (0.85) with $m_c = 1.4$ (1.6) GeV. If we assume that the derivation of the theoretical prediction from the central value of the experimental results is from gluon fragmentation processes in the COM that was investigated in Refs. [22,23], the ratio can be modified as

$$R_{c\bar{c}} = \frac{1}{\Gamma_{EX}(Z \to J/\psi + X)} \{\Gamma_{\text{CSM}}(Z \to J/\psi + c\bar{c}) + R_{c\bar{c}}^o[\Gamma_{EX}(Z \to J/\psi X) - \Gamma_{\text{CSM}}(Z \to J/\psi X)]\}, \qquad (14)$$

where $R_{c\bar{c}}^o$ from the gluon fragmentation processes in the COM is defined as

$$R_{c\bar{c}}^o = \frac{\Gamma^{g \to {}^3S_1(8)}(Z \to c\bar{c} + J/\psi + X)}{\sum_q \Gamma^{g \to {}^3S_1(8)}(Z \to q\bar{q} + J/\psi + X)}, \qquad (15)$$

and the $q\bar{q}$ in the denominator are summed over u, d, c, s, and b, and $R_{c\bar{c}}^o = 0.17$ is obtained from Ref. [23]. Then we obtain $R_{c\bar{c}}^{\text{CSM+COM}} = 0.49$ (0.41) for $m_c = 1.4$ (1.6) GeV. The above analysis indicates that R_{cc} can be used to clarify the COM contribution.

IV. SUMMARY AND CONCLUSION

We have investigated all the processes that give main contributions to J/ψ inclusive production in Z boson decay in the CSM. The results with the NLO QCD correction are obtained. For the $Z \to J/\psi + c\bar{c}$ process, the NLO results only change the leading-order results slightly, and the K factor is 1.13 with $\mu = 2m_c$ and is insensitive to the charm quark mass. We also use two methods to estimate the dependence of the results on the choice of the renormalization scale, and these two methods give almost the same partial decay width. The K factor even falls to 0.918 by using the BLM scheme. We also include the contributions of the main fragmentation processes. The total branching ratio for $Z \to J/\psi + X$ in CSM is $(7.3\text{–}10.0) \times 10^{-5}$, about one-half of the central value of the experimental data 2.1×10^{-4}. We define $R_{c\bar{c}} = \frac{\Gamma(Z \to J/\psi c\bar{c}X)}{\Gamma(Z \to J/\psi X)}$ and obtain $R_{cc} = 0.84$ for only the CSM contribution and $R_{cc} = 0.49$ for COM and CSM contributions together. Then the R_{cc} measurement could be used to clarify the COM contributions. In addition, the J/ψ energy distribution is inconsistent with the experimental data too. But there are large uncertainties in the experiment results on the inclusive production of J/ψ in Z decay, not only the total branching ratio but also the J/ψ energy distribution. Further experimental measurement with more sample data is needed to clarify the situation. Maybe in the future Z factory these processes could obtain a detailed investigation. In the calculation, the K factors for vector and axial-vector parts of $Z \to J/\psi + c\bar{c} + X$ are almost the same. It may indicate that the mechanism of heavy quark fragmentation into quarkonium is dominant in this process even at NLO.

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