Probing Majorana neutrinos in rare K and D, D_s, B, B_c meson decays

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We study lepton number violating decays of charged K, D, D_s, B, and B_c mesons of the form $M^+ \rightarrow M'^- \ell^+ \ell^+$, induced by the existence of Majorana neutrinos. These processes provide information complementary to neutrinoless double nuclear beta decays, and are sensitive to neutrino masses and lepton mixing. We explore neutrino mass ranges m_N from below 1 eV to several hundred GeV. We find that in many cases the branching ratios are prohibitively small, however in the intermediate range $m_\pi < m_N < m_{B_c}$, in specific channels and for specific neutrino masses, the branching ratios can be at the reach of high luminosity experiments like those at the LHC-b and future super flavor factories, and can provide bounds on the lepton mixing parameters.

I. INTRODUCTION

One of the outstanding issues in neutrino physics today is to clarify the Dirac or Majorana character of neutrino masses. The discovery of neutrino oscillations indicates that neutrinos are massive particles with masses likely to be much smaller than those of charged fermions [1]. This fact provides an important clue to the existence of a more fundamental physics underlying the standard model (SM) of particle physics, because neutrinos are naturally massless in the SM. Although the experimental results on neutrino oscillations can determine the neutrino mixing parameters and their squared mass differences, the absolute magnitudes of the masses as well as their origin remain unknown and constitute fundamental open questions in neutrino physics. Many experiments have been set to search for the absolute magnitude of neutrino masses. Direct methods to determine the mass of the electron neutrino use the end point of the electron spectrum in beta decays. The most sensitive of these experiments uses tritium [2], setting the present upper bound $m_{\nu_e} < 2$ eV [3], and the next experiment is expected to reach a sensitivity of 0.2 eV [4]. Other experiments do direct searches for muon and tau neutrino masses, setting the upper bounds $m_{\nu_\mu} < 190$ keV and $m_{\nu_\tau} < 18.2$ MeV, respectively, at 90% C.L. [3]. To date, the most stringent bound on the sum of all light neutrino masses is obtained from cosmological observations, given by $\sum_i m_{\nu_i} < 0.17$ eV at 95% C.L., a figure which is, to a certain extent, model dependent [5].

If neutrinos are Dirac particles, they must have right-handed electroweak singlet components in addition to the known left-handed modes; in such a case lepton number remains as a conserved quantity. Alternatively, if neutrinos are Majorana particles, then a neutrino is indistinguishable from its antiparticle and lepton number would be violated by two units ($\Delta L = 2$) in some processes that involve neutrinos. The experimental results to date are unable to distinguish between these two alternatives.

There have been several attempts to determine a Majorana nature of neutrinos by studying $\Delta L = 2$ processes. The most prominent of these processes are neutrinoless nuclear double beta decays ($0\nu\beta\beta$), which have been regarded as the most sensitive way to look for lepton number violation (LNV) [6]. The observation of $0\nu\beta\beta$ would indeed be very important not only because it would establish the existence of LNV, implying that neutrinos are Majorana particles, but also because they would provide a scale for the absolute magnitude of light neutrino masses, complementary to the direct searches mentioned above: these nuclear processes are proportional to the square of the *effective* neutrino mass $m_{ee} = |\sum_{i=1}^3 U_{ei}^2 m_i|$, with m_i and U_{ei} being the individual neutrino masses and the $\nu_i - e$ mixing matrix elements, respectively [7]. However, it has long been recognized that, even though the experiments are very sensitive, the extraction of the neutrino mass scale and the Majorana nature of neutrinos from nuclear $0\nu\beta\beta$ is a difficult task, because reliable information on neutrino properties can be inferred only if the nuclear matrix elements for $0\nu\beta\beta$ are calculated correctly. The calculation of the nuclear matrix elements for $0\nu\beta\beta$, usually performed within either the quasiparticle random phase approximation [8] or the nuclear shell model [9] or their variants, is known to be a complex task, sometimes with large differences among the different approaches [10]. Even in the most refined treatments, the estimates of the nuclear matrix elements remain affected by various large uncertainties [11].

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Another avenue to detect the Majorana nature of neutrinos is to study $\Delta L = 2$ processes in rare meson decays [12–14]. In this paper we study $\Delta L = 2$ decays of heavy charged mesons whose signals could be captured at high intensity experiments such as LHC-b and future super B factories as well as advanced K factories. The $\Delta L = 2$ processes we treat in this paper are rare neutrinoless decays of heavy charged mesons into a lighter meson and two charged leptons of the same sign [13]. These processes, just like neutrinoless nuclear double beta decays, can occur only via Majorana neutrino exchange, and thus their experimental observation could establish the Majorana character of the neutrinos and the absolute scale of neutrino masses in much the same way as in nuclear $0\nu\beta\beta$ decays, but there are some essential differences. From a theoretical viewpoint, the uncertainties in meson decays are much easier to handle than in nuclear $0\nu\beta\beta$ decays. However, from the experimental viewpoint, the $\Delta L = 2$ meson decay rates in the case of standard neutrinos ($m_\nu < 2$ eV) are prohibitively small for any experiment, while $0\nu\beta\beta$ decays are more realistic options, due to their macroscopically large samples of decaying nuclei. In contrast, for heavier, nonstandard neutrinos, the meson decay rates are good alternatives to search for, as they can be within reach of future experiments.

In this study it is important to distinguish between standard and sterile neutrinos. From direct searches we know the standard electron-neutrino mass is below 2 eV [5], and neutrino oscillation experiments tell us that all three neutrino masses differ from one another by much less than that value [15]. Therefore all neutrinos with masses above 2 eV are assumed to be nonstandard. Since our work is mainly relevant for neutrinos above this bound, in what follows we will denote them generically by the letter N, instead of ν.

An important motivation to search for sterile (nonstandard) neutrinos with masses of the order of 1 MeV is that their existence has nontrivial observable consequences for cosmology and astrophysics. They are presumed to participate in big-bang nucleosynthesis, supernovae explosions, large scale structure formation and, in general, to be a component of the dark matter in the Universe [16]. Thus, sterile neutrino masses and their mixing with the standard neutrinos must be subjected to cosmological and astrophysical bounds [17]. There are also some laboratory bounds coming from the fact that sterile neutrinos contribute via mixing with the standard neutrinos to various processes which are forbidden in the SM. Those bounds turn out to be much weaker than the cosmological and astrophysical bounds, but useful in cases where the latter become inapplicable [18].

We have separated the analysis into three different cases, depending on the relevant neutrino mass range. If the exchanged neutrino is much lighter than the energy scale in the process, the amplitude of the decay rate is proportional to the square of an effective electron-neutrino mass, $m_{ee}^2 = |\sum_N U_{eN}^2 m_N|^2$, which is anticipated to be of the order of ~ 1 eV^2 or less from current neutrino data and cosmological observations such as WMAP [19], if only standard neutrinos are involved. Instead, if the exchanged neutrino is much heavier than the decaying meson, the decay rate is proportional to $|U_{N\ell_1} U_{N\ell_2}/m_N|^2$, where m_N and $U_{N\ell}$ are the heavy neutrino mass and its mixing with the standard leptons, respectively. In general, in this case $U_{N\ell}$ is small and m_N is large, so the factor constitutes a severe suppression to the decay rate. Finally, for the case of Majorana neutrinos with intermediate masses between that of the initial and the final meson, the decay rate is dominated by a resonantly enhanced s-channel amplitude [12,13,20], where the intermediate neutrino goes on its mass shell.

In Sec. II, we describe the approximation methods for the calculations of rare heavy meson decays of the form $M^+ \rightarrow M'^- l_1^+ l_2^+$ (where M and M' are pseudoscalar mesons). Here we are interested in K^+, D^+, D_s^+, B^+, and B_c^+ decays into $\pi^- \ell^+ \ell^+$, $K^- \ell^+ \ell^+$, $D^- \ell^+ \ell^+$, $D_s^- \ell^+ \ell^+$, and $B^- \ell^+ \ell^+$ (where $\ell = e$, μ, or τ), therefore, we will denote the initial and final mesons generically by M^+ and M'^-, respectively. We separate the analysis for the three cases of light neutrinos ($m_N < m_{M'}$), intermediate neutrinos ($m_{M'} < m_N < m_M$), and heavy neutrinos ($m_N > m_M$). We include the results and discussions in each subsection. In Sec. III we summarize the results and state our conclusions.

II. CALCULATIONS OF $M^+ \rightarrow M'^- l_1^+ l_2^+$

We now describe our approximation methods for the calculations of rare heavy meson decays of the form
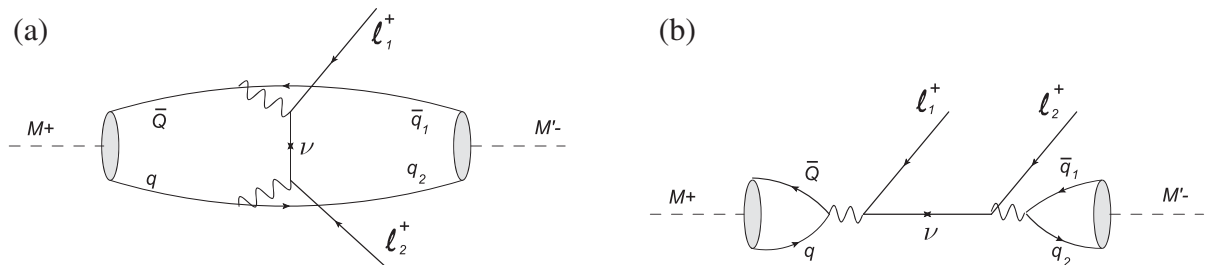

FIG. 1. The t-type and s-type weak amplitudes at the quark level that enter in the process $M^+ \rightarrow M'^- \ell_1^+ \ell_2^+$ (plus the same diagrams with leptons exchanged if they are identical).

TABLE I. Values of input parameters used in our calculations. They correspond to the central values given in Ref. [3], except for f_B and f_{B_s} which are taken from Ref. [21], and V_{cs} which is calculated by imposing the unitarity constraint on the CKM matrix.

Parameter	Value	Parameter	Value
f_π	130.4 (MeV)	V_{ud}	0.9742
f_K	155.5 (MeV)	V_{us}	0.2255
f_{D^+}	205.8 (MeV)	V_{ub}	0.0039
f_{D_s}	273. (MeV)	V_{cd}	-0.230
f_B	196. (MeV)	V_{cs}	0.950
f_{B_c}	322. (MeV)	V_{cb}	0.041

$M^+ \to M'^- l_1^+ l_2^+$ (where M and M' are pseudoscalar mesons) in all three neutrino mass ranges described above. At the quark level, the decay occurs via two types of amplitudes, shown in Fig. 1. We find that in the case of light neutrinos ($m_N < m_\pi$), the amplitude on the left in Fig. 1 ("t-type" diagram) dominates due to long distance contributions, and the decay rate becomes proportional to $m_M^7 \times m_N^2$. For this reason, only the decays of the heavier B mesons are of any importance in this case. In contrast, for intermediate neutrino masses ($m_{M'} < m_N < m_M$), the diagram on the right in Fig. 1 ("s-type" diagram) dominates when the neutrino propagator becomes resonant on its mass shell, in which case the decay rate turns out to be less dependent of the neutrino mass, but very sensitive to the mixing elements. Finally, for heavy neutrinos ($m_N > m_M$), both amplitudes in Fig. 1 are comparable and the decay rate is $\propto 1/m_N^2$.

In Table I we list the numerical values of the input parameters we use in our numerical estimates.

A. The case of light neutrinos ($m_N < m_\pi$)

We find that a neutrinoless decay like $B^+ \to D^- \ell^+ \ell^+$ with light Majorana neutrinos in the intermediate state is dominated at the meson level by the amplitude shown in Fig. 2, when the intermediate state goes on mass shell. This amplitude originates at the quark level from the t-type weak amplitude shown in Fig. 1. We find the s-type amplitude shown in Fig. 1 to be subdominant, or at most
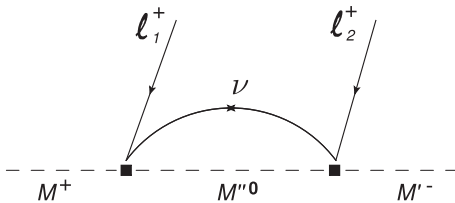

FIG. 2. The main diagram in an effective meson theory for $M^+ \to M'^- \ell^+ \ell^+$ (plus diagram with leptons exchanged if they are identical), mediated by Majorana neutrinos, when the neutrino is much lighter than the final meson. The amplitude is estimated considering the intermediate state on its mass shell.

comparable with the former. In this sense, our treatment differs from that of Ali *et al.* [13], where the s-type amplitude is assumed to dominate [22]. However, since the rate in any case turns out to be too small for any foreseeable experiment, we will just do an order-of-magnitude estimate for it, calculating the absorptive part and assuming that the dispersive part is not much larger. The absorptive part of the amplitude is calculated by setting the intermediate particles on their mass shell and then integrating over their phase space:

$$\mathcal{M}_{\text{abs}}(B^+ \to D^- \ell^+ \ell^+) = \int dps_{DN} A_{B \to DN\ell} A_{DN \to D\ell}, \quad (1)$$

where $A_{B \to DN\ell}$ and $A_{DN \to D\ell}$ are the tree-level amplitudes for the respective subprocesses, and dps_{DN} is the Lorentz-invariant phase space of the intermediate D-N pair, which in the rest frame of the pair is $dps_{DN} = \sum_s (1/16\pi^2) \times (|\mathbf{p}_N|/m_{D\ell})d\Omega_N$. Here $\sum_s$ is the sum over the neutrino spins, $\mathbf{p}_N$ is the 3-momentum of the neutrino in the D-N rest frame, and $m_{D\ell}$ is the invariant mass of the pair. In turn, the amplitudes of the weak subprocesses are

$$A_{B^+ \to D^0 N\ell} = \frac{G_F}{\sqrt{2}} V_{cb} U_{N\ell} \langle \bar{D}^0(p')|J^\mu(0)|B^+(p)\rangle \bar{u}_N(p_N)$$
$$\times \gamma_\mu(1 - \gamma_5)v_\ell(l_1),$$

$$A_{D^0 N \to D^- \ell} = \frac{G_F}{\sqrt{2}} V_{ud} U_{N\ell} \langle D^-(p')|J^\mu(0)|\bar{D}^0(p)\rangle \bar{v}_N(p_N)$$
$$\times \gamma_\mu(1 - \gamma_5)v_\ell(l_2), \quad (2)$$

where V is the Cabbibo-Kobayashi-Maskata (CKM) matrix for quark mixing, and U the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix for lepton mixing. The hadronic matrix elements can be parametrized in terms of phenomenological form factors $F^+(q^2)$ and $F^-(q^2)$ as

$$\langle \bar{D}^0(p')|J^\mu(0)|B^+(p)\rangle = F_{BD}^+(q^2)(p + p')^\mu$$
$$+ F_{BD}^-(q^2)(p - p')^\mu, \quad (3)$$

and similarly for $\langle D^-(p')|J^\mu(0)|\bar{D}^0(p)\rangle$, where q is the corresponding 4-momentum transfer. In our crude estimate, we will neglect the F^- form factors and assume the F^+ to be constants of order unity over the kinematical range.

Now, in the product $A_{B \to DN\ell} \times A_{DN \to D\ell}$, after summing over intermediate spin states, the two lepton lines can be combined into a single one by using in $A_{DN \to D\ell}$ the identity $\bar{v}_N \gamma^\mu (1 - \gamma_5)v_\ell = \bar{u}_{\bar{\ell}} \gamma^\mu (1 + \gamma_5)u_N$. Here $u_{\bar{\ell}}$ is a u spinor for the charged antilepton, and the neutrino is assumed to satisfy the Majorana condition $u_{\bar{N}} = \lambda_N u_N$ (where λ_N is a phase). The result is then

$$\mathcal{M}_{\text{abs}} = \frac{G_F^2}{2} V_{cb} V_{ud} U_{N\ell}^2 \lambda_N \int \frac{d\Omega_N}{16\pi^2} \frac{|\mathbf{p}_N|}{m_{D\ell}} F_{BD}^+ F_{DD}^+ \bar{u}_{\bar{\ell}}(l_2)$$
$$\times (\slashed{p}_D + \slashed{p}_{D^0})(1 + \gamma_5)(\slashed{p}_N + m_N)(\slashed{p}_{D^0} + \slashed{p}_B)$$
$$\times (1 - \gamma_5)v_\ell(l_1). \quad (4)$$

This angular integral is quite simple, because in the *D-N* frame the energy of every particle in the process is fixed. The subsequent steps to obtain the decay rate are straightforward and described in Appendix A. The expression for the rate is thus the integral [see Eq. (A8)]:

$$\Gamma(B^+ \rightarrow D^- \ell^+ \ell^-)$$

$$= \frac{G_F^4}{(16\pi^2)^2} |V_{cb} V_{ud}|^2 F_{BD}^{+2} F_{DD}^{+2} \frac{|U_{N\ell}^2 m_N|^2}{m_B^2}$$

$$\times \int_{(m_D+m_\ell)}^{(m_B-m_\ell)} \frac{dm_{D\ell}}{2\pi} \frac{|\mathbf{p}_N|^2}{m_{D\ell}^2} |\tilde{l}_1||l_2| \times \mathcal{R}, \quad (5)$$

where $|\mathbf{p}_N|$, $|\tilde{l}_1|$, and $|l_2|$ are the 3-momenta of the neutrino and leptons (given in Appendix A) and $\mathcal{R}$ is a quantity of dimension m^6 shown in Eq. (A6). The integral can be easily done numerically, which we do considering a *D* meson in the intermediate and final states ($b \rightarrow c$ transition), or alternatively a pion ($b \rightarrow u$ transition).

Notice that by assuming the form factors to be constant unity we are overestimating the process, while by neglecting the F^- form factors and the dispersive part of the amplitude we may be inducing an uncertainty of an order of magnitude. Within our approximations, in both cases the results for the branching ratios are extremely small:

$$\text{Br}(B^+ \rightarrow D^- \ell^+ \ell^+) \sim 1.2 \times 10^{-31} \left(\frac{U_{N\ell}^2 m_N}{1 \text{ eV}}\right)^2, \quad (6)$$

$$\text{Br}(B^+ \rightarrow \pi^- \ell^+ \ell^+) \sim 2.3 \times 10^{-33} \left(\frac{U_{N\ell}^2 m_N}{1 \text{ eV}}\right)^2, \quad (7)$$

where we used the values of the CKM elements shown in Table I, and also $\Gamma_B = 4.0 \times 10^{-13}$ GeV. We can compare these results with those of Ali *et al.* [13], who considered the *s*-type diagram only. In our notation, their result for $\text{Br}(B^+ \rightarrow \pi^- e^+ e^+)$ becomes $(0.3\text{–}1.8) \times 10^{-35}(U_{N\ell}^2 m_N/\text{eV})^2$, which is 2 orders of magnitude smaller than Eq. (7).

Nevertheless, we expect our results to be just rough estimates within 1 or 2 orders of magnitude, as we have taken the form factors $F_{BD}^+ \sim F_{DD}^+$ to be unity, and we have neglected the form factors F_{BD}^- and F_{DD}^- altogether. In general, the form factors F^+ are expected to be unity at most at the kinematical end point where the two meson wave functions could overlap completely (provided they have the same shape), but it should be smaller for all other q^2 values.

Taking for F^+ an average value of e.g. 0.3 instead of unity, our calculated rates get reduced by a factor $(F^+)^4 \sim 10^{-2}$, reducing Eq. (7) to a value comparable with the result of Ali *et al.*

Accordingly, in the case of light neutrinos, our crude estimate cannot clearly show the dominance of the *t*-type diagram. However, it does show at least that a calculation based purely on the *s*-type diagram may be

an underestimation [22]. It also shows that this potential underestimation is hardly more than 2 orders of magnitude, keeping these branching ratios still beyond the reach of foreseen experiments, as concluded in Ref. [13].

To estimate the actual range of these branching ratios we would need to have estimates of the neutrino masses and mixings as well. Using the standard parametrization of the PMNS neutrino mixing matrix multiplied by a 3×3 Majorana phase matrix, the term $U_{N\ell}^2 m_N$ can be explicitly written in terms of three light neutrino masses, three neutrino mixing angles, two Majorana phases, and one Dirac phase. Since the sign of Δm_{31}^2 is not determined from the existing data, there are two possible neutrino mass hierarchies, one called normal ($m_3 > m_{1,2}$) and the other inverted ($m_3 < m_{1,2}$). The size of the term $U_{N\ell}^2 m_N$ in general depends on the mass hierarchy. If we consider standard neutrinos, we know that $m_\nu < 2$ eV, and we can roughly use $U_{\nu\ell} \sim \mathcal{O}(1)$ for either $\ell = e$ or μ, in consistency with oscillation experiments. We then get branching ratios smaller than 10^{-31} and 10^{-33}, respectively, values which are prohibitively small for any foreseen experiment. On the other hand, if we consider heavier neutrinos (but still lighter than m_π), i.e. $m_N \sim 100$ MeV, the results could be more promising, but in those cases we should use the mixings of standard with extra neutrinos, which are suppressed: U_{Ne}^2, $U_{N\mu}^2 < 0.002$ [23], so the resulting branching ratios have the upper bounds 10^{-21} and 10^{-23}, respectively, which are still prohibitively small.

As a final remark, we want to comment on the assumptions involved in this calculation. First, the fundamental process at the quark level (see Fig. 1) with two electroweak vertices has been modeled as a process with hadrons and leptons, where a single long distance contribution (an intermediate state with a meson and a neutrino on shell) is supposed to dominate; we have thus neglected other possible intermediate hadronic states (e.g. excitations of the intermediate meson and multimeson states) as well as a short-distance contribution where both weak vertices coalesce into a single one [24]. We have assumed the dominance of the single *D-N* intermediate channel as it goes on its mass shell. Another assumption was to consider the absorptive part as representative of the full amplitude; since we are only after an order-of-magnitude estimate, this is likely to be a good assumption, again due to the resonant character of the intermediate state as it goes on its mass shell. Within the hadronic approximation for the weak currents, we took into account just one of the form factors of each hadronic current, and assumed it to be constant (unity) within the whole dynamic range. In principle one can expect the form factor to be unity at most, as explained before; taking the q^2 dependence into account one should then obtain a lower value for the rate, but as we have seen, it is unlikely for this effect to change the result by more than 2 orders of magnitude. These approximations are therefore consistent with the level of precision we seek.

B. The case of intermediate mass neutrinos
$(m_\pi < m_N < m_{B_c})$

In contrast to the previous case, the process $M^+ \rightarrow M'^- \ell^+ \ell^+$ in the case of Majorana neutrinos with masses in the intermediate range $m_{M'} < m_N < m_M$ is dominated by the s-type amplitude of Fig. 1, corresponding at the meson level to Fig. 3, as the neutrino in the intermediate s channel goes into its mass shell. As stated in the Introduction, Majorana neutrinos with such masses must be sterile and should originate from new physics beyond the SM.

Since there are two identical leptons in the final state, one must also consider the diagram with crossed leptons and then integrate over half the phase space. However, for the case of the intermediate neutrino on mass shell, the result is the same as using a single diagram, as if the leptons were distinct, as shown in Fig. 3. The effective amplitude then is

$$\mathcal{M} = \frac{G_F^2}{2} U_{N\ell}^{*2} V_{qQ}^* V_{q_2 q_1}^* f_M f_{M'} \frac{\tilde{M}}{(p_N^2 - m_N^2) + i m_N \Gamma_N}, \quad (8)$$

where $U_{N\ell}$ and $V_{q_i q_j}$ are the PMNS lepton mixing and CKM quark mixing elements, respectively, f_M, $f_{M'}$ are the meson decay constants, and we define $\tilde{M}$ as the reduced matrix element that contains all the spinor structure of the amplitude:

$$\tilde{\mathcal{M}} = \lambda_N \bar{u}_{\bar{\ell}}(l_1)\not{p}_M(1 + \gamma_5)\not{p}_N + m_N)\not{p}_{M'}(1 - \gamma_5)v(l_2), \quad (9)$$

where the notation is the same as in Eq. (4).

The decay rate we seek is then given by $\Gamma(M^+ \rightarrow M'^- \ell^+ \ell^+) = (1/2 m_M) \int d\mathrm{ps}_3 |\mathcal{M}|^2$, where $d\mathrm{ps}_3$ is the final 3-particle phase space. The calculation of the squared matrix element and the integration over the final phase space are shown in Appendix B, resulting in the following expression [see Eq. (B3)]:

$$\Gamma(M \rightarrow M'\ell^+\ell^+) = \frac{G_F^4}{32\pi^2 m_M} f_M^2 f_{M'}^2 |V_{qQ}V_{q_2 q_1}|^2$$
$$\times \frac{|U_{N\ell}|^4}{m_N \Gamma_N} \frac{|\tilde{l}_1|}{m_M} \frac{|l_2|}{m_N} \{(m_N^2 + m_\ell^2)m_M^2$$
$$- (m_N^2 - m_\ell^2)^2\} \times \{(m_N^2 - m_\ell^2)^2$$
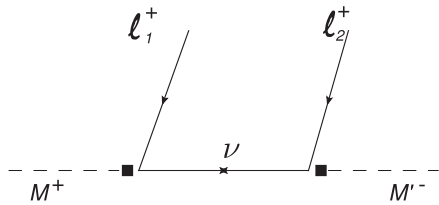$$- (m_N^2 + m_\ell^2)m_{M'}^2\}, \quad (10)$$

FIG. 3. The dominating diagram (plus diagram with leptons exchanged if they are identical) in an effective meson theory for $M^+ \rightarrow M'^- \ell^+ \ell^+$, mediated by Majorana neutrinos with mass in the range between $m_{M'}$ and m_M.

where $|\tilde{l}_1|$ and $|l_2|$ are the 3-momenta of the first electron in the M meson rest frame and of the second electron in the neutrino rest frame, respectively.

Before we can use this expression, we also need a theoretical expression for Γ_N, the total decay width of the intermediate Majorana neutrino, in terms of the same neutrino parameters we have just used. The total width Γ_N can be estimated by comparing the decay modes of N with those of the τ^- lepton, where $\Gamma_\tau \propto m_\tau^5$. Both N and τ^- decay via the same type of diagrams and couplings, but there are a few differences: (a) N has a different mass (thus $\Gamma_N \propto m_N^5$); (b) Γ_N has an additional factor of 2 due to the Majorana character of N (unlike τ^- which is a Dirac particle), because it decays with equal probability into both $(\ell'^- + \text{rest})+$ and $(\ell'^+ + \overline{\text{rest}}^-)$; and (c) Γ_N has an additional mixing factor $|U_{N\ell'}|^2$. Therefore:

$$\Gamma_N \approx 2\sum_{\ell'} |U_{N\ell'}|^2 \left(\frac{m_N}{m_\tau}\right)^5 \times \Gamma_\tau. \quad (11)$$

This expression for Γ_N is a good approximation when m_N is near 2 GeV; in this case the decay channels of N are those of τ, where the virtual W boson produces $e^- \bar{\nu}_e$, $\mu^- \bar{\nu}_\mu$ and $d\bar{u}$ (the last channel is actually a set of three, due to color). However, for $m_N > 2$ GeV, the additional channels $\tau^- \bar{\nu}_\tau$ and $s\bar{c}$ open, increasing the expression in Eq. (11) by up to a factor ≈ 1.5, including phase space suppression due to the masses of the products. Consequently, using Eq. (11) in Eq. (10) may overestimate the rates by at most $\sim 30\%$. We will thus use Eq. (11) in the estimation of the LNV rates, but keeping in mind that a correction in Γ_N should be included in a more refined study.

Accordingly, and if we neglect the charged lepton mass, Eq. (10) turns into

$$\Gamma(M \rightarrow M'\ell^+\ell^+) \approx \frac{1}{128\pi^2} G_F^4 f_M^2 f_{M'}^2 |V_{qQ}V_{q_2 q_1}|^2$$
$$\times \frac{|U_{N\ell}|^4}{\sum_{\ell'} |U_{N\ell'}|^2} \frac{m_M m_\tau^5}{2\Gamma_\tau} \left(1 - \frac{m_{M'}^2}{m_N^2}\right)^2$$
$$\times \left(1 - \frac{m_N^2}{m_M^2}\right)^2. \quad (12)$$

Here we will use $m_\tau = 1.77$ GeV and $\Gamma_\tau = 2.3 \times 10^{-12}$ GeV [3]. Equation (12) is valid for m_N in the range $m_{M'} < m_N < m_M$; it vanishes at the two end points of this range, and reaches its maximum at $m_N = \sqrt{m_M \cdot m_{M'}}$, where $(1 - m_{M'}^2/m_N^2)^2(1 - m_N^2/m_M^2)^2 \rightarrow (1 - m_{M'}/m_M)^4$.

Consequently, these suppressed nonstandard decays can impose more or less stringent bounds on the mixing elements between the standard leptons and extra neutrinos, $|U_{N\ell}|$, depending on the Majorana neutrino mass. In particular, the nonobservation of these processes defines m_N-dependent upper bounds for the corresponding $|U_{N\ell}|$.

In Figs. 4–7 we show the branching ratios for the decays $K^+ \rightarrow \pi^- \ell^+ \ell^+$, $D^+ \rightarrow M'^- \ell^+ \ell^+$, $D_s^+ \rightarrow M'^- \ell^+ \ell^+$, $B^+ \rightarrow D^- \ell^+ \ell^+$, and $B_c^+ \rightarrow M'^- \ell^+ \ell^+$ as functions of
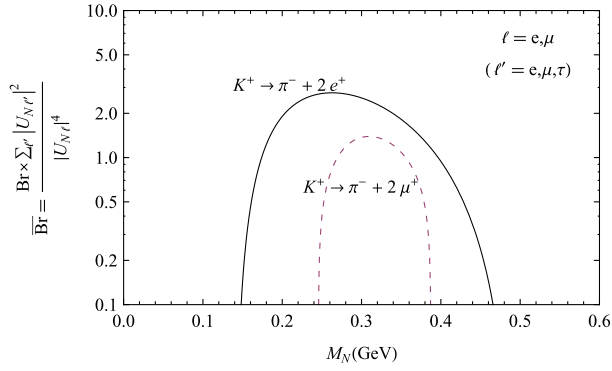
FIG. 4 (color online).   Branching ratios for $K^+ \to \pi^- \ell^+ \ell^+$ ($\ell = e, \mu$) as functions of the exchanged neutrino mass m_N in the range $m_\pi < m_N < m_K$, with the lepton mixing factor, $|U_{Ne}|^4/\sum_{\ell'} |U_{N\ell'}|^2$, divided out.

m_N, where the bounds on the mixings $|U_{N\ell}|$ can be deduced also as functions of m_N.

Let us consider the decay $B^+ \to D^- e^+ e^+$ as an example. Here we must use $V_{qQ} \to V_{ub}$ and $V_{q_1 q_2} \to V_{cd}$

as inputs, as well as f_B and f_D (see Table I) and $\Gamma_B = 4.0 \times 10^{-13}$ GeV [3]. The branching ratio for this process as a function of m_N is shown in Fig. 6(a), lower dashed line, and reaches a maximum:

$$\mathrm{Br}_{\max}(B^+ \to D^- e^+ e^+) = 3 \times 10^{-7} \times \frac{|U_{Ne}|^4}{\sum_{\ell'} |U_{N\ell'}|^2}$$

$$\text{at } m_N \sim 3 \text{ GeV}. \qquad (13)$$

This expression just gives the maximal possible value of this branching ratio, which occurs only if m_N happens to be near 3 GeV, but for other values of m_N, it could be much smaller, as shown in Fig. 6(a).

Analogous to Eq. (13), the maximal branching ratio of any of the other decays has the form:

$$\mathrm{Br}_{\max}(M^+ \to M'^- \ell^+ \ell^+) = \mathcal{C} \times \frac{|U_{Ne}|^4}{\sum_{\ell'} |U_{N\ell'}|^2}. \qquad (14)$$
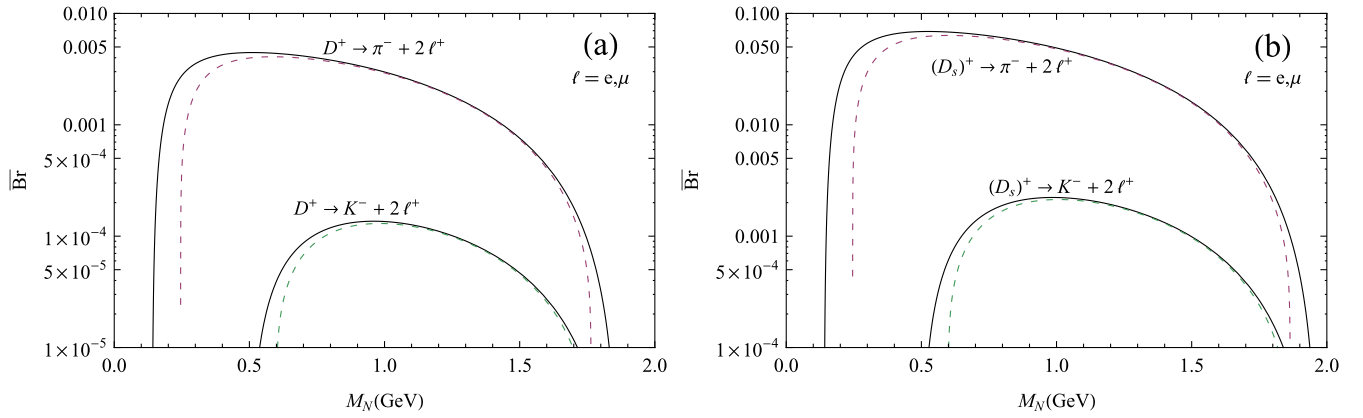

FIG. 5 (color online).   Branching ratios for (a) D^+ decays and (b) D_s^+ decays, as functions of the neutrino mass m_N, with the lepton mixing factor divided out as in Fig. 4. The full lines correspond to $\ell = e$ and the dashed lines to $\ell = \mu$.
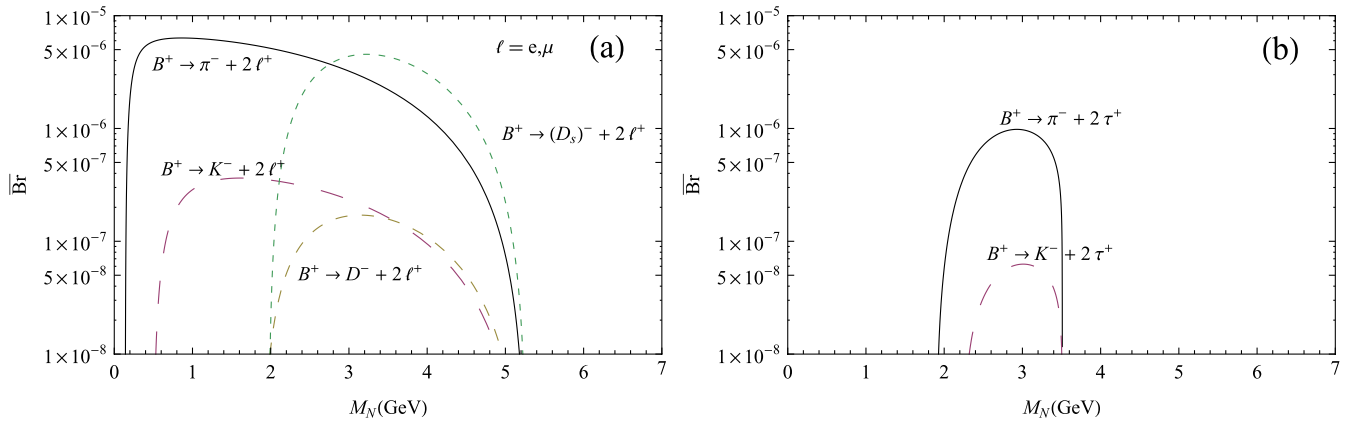

FIG. 6 (color online).   Branching ratios for $B^+ \to M'^- \ell^+ \ell^+$ as functions of the neutrino mass m_N, with the lepton mixing factor divided out as in Fig. 4. The produced pseudoscalars are $M' = \pi$, K, D, and D_s. (a) The case of leptons with negligible mass ($\ell = e, \mu$); (b) the case $\ell = \tau$ (here $M' = D$, D_s are kinematically forbidden).
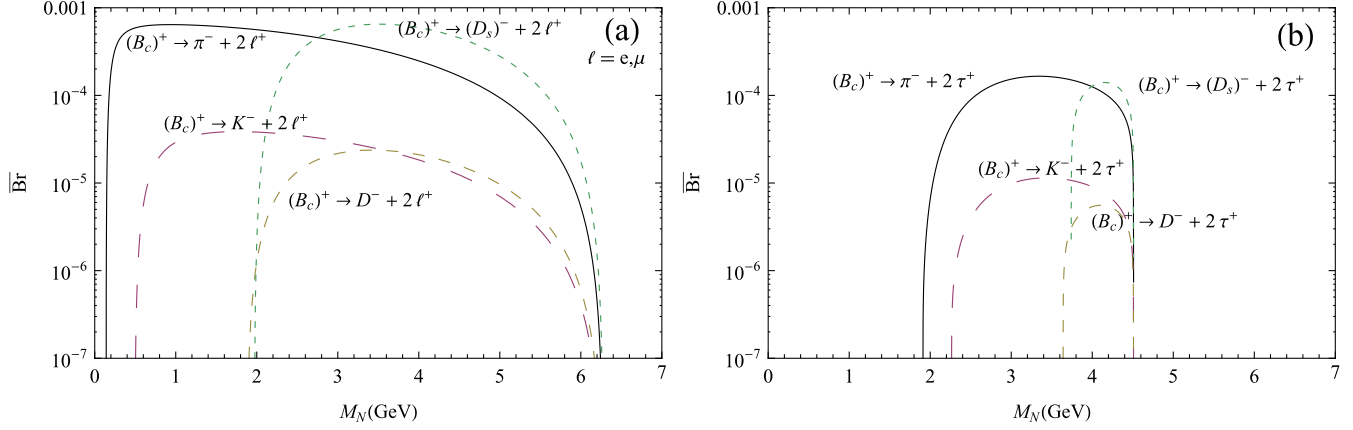
FIG. 7 (color online). Branching ratios for $B_c \to M'^{-}\ell^{+}\ell^{+}$ as functions of the neutrino mass m_N, with the lepton mixing factor divided out as in Fig. 4. The produced pseudoscalars are $M' = \pi$, K, D, and D_s. (a) The case of leptons with negligible mass ($\ell = e$, μ); (b) the case $\ell = \tau$.

Table II shows the coefficient $\mathcal{C}$ appearing in Eq. (14), for the different branching ratios, and the value of the corresponding neutrino mass m_N at which the maximal branching ratio is reached.

Accordingly, an experimental upper bound on the branching ratio for $M^+ \to M'^{-}\ell^{+}\ell^{+}$ imposes an upper bound on the leptonic mixings $|U_{N\ell}|$, a bound that strongly depends on the neutrino mass m_N, and which is most stringent if $m_N \sim \sqrt{m_M \cdot m_{M'}}$, where the branching ratio is maximal. For m_N away from that value, the upper bounds imposed on the mixings become much less stringent.

From the $\mathcal{C}$ values in Table II one can read the potential of different processes to set upper bounds on the lepton

TABLE II. The coefficients $\mathcal{C}$ appearing in Eq. (14) for the maximal branching ratio, and the neutrino mass m_N at which the maximum is reached, for various decays $M^+ \to M'^{-}\ell^{+}\ell^{+}$, where m_ℓ can be neglected. In the last column, the expected upper bound on the branching ratios, provided $|U_{N\ell}|^2 \sim 10^{-6}$ or 10^{-7}, for $m_N \sim 0.1$ GeV or ~ 1 GeV, respectively.

Decay	$\mathcal{C}$	m_N at maximum	Br$<$
$K^+ \to \pi^-\ell^+\ell^+$	2.8	0.26 GeV	2.8×10^{-6}
$D^+ \to \pi^-\ell^+\ell^+$	4.5×10^{-3}	0.51 GeV	4.5×10^{-10}
$D^+ \to K^-\ell^+\ell^+$	1.4×10^{-4}	0.96 GeV	1.4×10^{-11}
$D_s^+ \to \pi^-\ell^+\ell^+$	6.9×10^{-2}	0.53 GeV	6.9×10^{-9}
$D_s^+ \to K^-\ell^+\ell^+$	2.2×10^{-3}	0.99 GeV	2.2×10^{-10}
$D_s^+ \to D^-\ell^+\ell^+$	8.5×10^{-8}	1.92 GeV	8.5×10^{-15}
$B^+ \to \pi^-\ell^+\ell^+$	6.3×10^{-6}	0.86 GeV	6.3×10^{-13}
$B^+ \to K^-\ell^+\ell^+$	3.6×10^{-7}	1.61 GeV	3.6×10^{-14}
$B^+ \to D^-\ell^+\ell^+$	1.7×10^{-7}	3.14 GeV	1.7×10^{-14}
$B^+ \to D_s^-\ell^+\ell^+$	4.5×10^{-6}	3.23 GeV	4.5×10^{-13}
$B_c^+ \to \pi^-\ell^+\ell^+$	6.4×10^{-4}	0.94 GeV	6.4×10^{-11}
$B_c^+ \to K^-\ell^+\ell^+$	3.9×10^{-5}	1.76 GeV	3.9×10^{-12}
$B_c^+ \to D^-\ell^+\ell^+$	2.4×10^{-5}	3.43 GeV	2.4×10^{-12}
$B_c^+ \to D_s^-\ell^+\ell^+$	6.5×10^{-4}	3.52 GeV	6.5×10^{-11}
$B_c^+ \to B^-\ell^+\ell^+$	1.6×10^{-11}	5.76 GeV	1.6×10^{-18}

mixing elements $|U_{N\ell}|$, for different neutrino masses m_N. For a given experimental upper bound of a branching ratio, the larger the $\mathcal{C}$ coefficient, the more stringent the upper bound that can be imposed on $|U_{N\ell}|$, provided the neutrino mass is near the indicated value where the theoretical branching ratio is maximal.

From Eq. (14) it is clear that the bounds on the mixings imposed from these decays appear in the combination

$$\frac{|U_{N\ell}|^4}{|U_{Ne}|^2 + |U_{N\mu}|^2 + |U_{N\tau}|^2}, \qquad \ell = e, \mu, \text{ or } \tau, \quad (15)$$

and not just $|U_{N\ell}|$. Only if $|U_{N\ell}|$ is much larger than the other mixings, then this expression reduces to $|U_{N\ell}|^2$. Otherwise, one must use the bounds on Br($M \to M'^{-}\ell^{+}\ell^{+}$) for a given meson pair M and M', but for *all* lepton flavors $\ell = e$, μ, and τ, in order to disentangle the bounds for each of the mixings $|U_{N\ell}|$. Moreover, these bounds will depend on m_N, since the relation between the branching ratios and the mixings depends on m_N, as it was already mentioned and shown in Figs. 4–7.

On the other hand, to explore the prospects of experimentally observing any of these processes, one needs at least an estimate of the $|U_{N\ell}|$ elements. Present upper bounds on the heavy-to-light neutrino mixing $|U_{N\ell}|^2$ for $\ell = e$, μ, vary considerably with the neutrino mass, but are typically in the range $|U_{N\ell}|^2 < 10^{-4}$, 10^{-6}, 10^{-7}, for $m_N \sim 10$ MeV, 100 MeV, 1 GeV, respectively (pp. 546–548 in Ref. [3]). We have then listed in the last column of Table II the expected upper bound on the branching ratios, provided the mixing elements have the values just mentioned.

C. The case of heavy neutrinos ($m_N > m_{B_c}$)

If neutrinos happen to be much heavier than the decaying meson, then in general both diagrams in Fig. 1 contribute with more or less the same strength, and reduce at the meson level to a single pointlike interaction diagram as

shown in Fig. 8. The vertex in Fig. 8 represents the double weak interaction shown in Fig. 1, where the neutrino line as well as all other internal lines have been reduced to a point. At the meson level, the specific tensor structure of this four-particle vertex cannot be selected among all the general possibilities, so we start from the fundamental quark and lepton interactions as shown in Fig. 1 and exhibit the approximations involved to get to the leading term at the meson level. These details are presented in Appendix C. Our model of the dynamics in this case is equivalent to that of Ali *et al.* [13]. In summary, if we can approximate the hadronic tensor by the product of two currents, factorized by a vacuum insertion, the squared amplitude is then given in terms of the mesons' decay constants and the kinematics of mesons and leptons separate into independent factors [see Eq. (C5)]:

$$|\mathcal{M}|^2 \sim f_M^2 f_{M'}^2 (p_M \cdot p_{M'})^2 (\ell_1 \cdot \ell_2).$$

The decay rate then becomes [see Eq. (C6)]

$$\Gamma(M^+ \to M'^- \ell^+ \ell^+)$$
$$= \frac{G_F^4}{128\pi^3} \left| \frac{U_{N\ell}^{*2}}{m_N} \right|^2 \left| V_{qQ}^* V_{q_1 q_2}^* + \frac{V_{q_1 Q}^* V_{qq_2}^*}{N_c} \right|^2$$
$$\times f_M^2 f_{M'}^2 m_M^3 \int_{4m_\ell^2}^{(m_M - m_{M'})^2} dm_{\ell\ell}^2 \lambda^{1/2}\left(1, \frac{m_{M'}^2}{m_M^2}, \frac{m_{\ell\ell}^2}{m_M^2}\right)$$
$$\times \lambda^{1/2}\left(1, \frac{m_\ell^2}{m_{\ell\ell}^2}, \frac{m_\ell^2}{m_{\ell\ell}^2}\right)\left(1 + \frac{m_{M'}^2}{m_M^2} - \frac{m_{\ell\ell}^2}{m_M^2}\right)^2 (m_{\ell\ell}^2 - 2m_\ell^2),$$

(16)

where the function $\lambda(x, y, z)$ is defined in Eq. (A2). This expression exactly coincides with the expression obtained in Ref. [13] for the heavy neutrino cases.

The integral above can be easily done numerically. In order to do the phenomenology, we set a fiducial value for the neutrino mass $m_N = 100$ GeV, and a corresponding mixing element $|U_{N\ell}|^2 = 10^{-2}$ and express the branching fraction of this decay in terms of a dimensionless quantity $\mathcal{B}$, whose value, according to Eq. (16), is determined by the masses of the external particles:
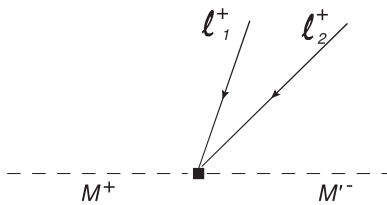

FIG. 8. The diagram in an effective meson theory for $M^+ \to M'^- e^+ e^+$, when the neutrino mass is much larger than that of the decaying meson, which is resulted from the four amplitudes of Fig. 1.

$$\text{Br}(M^+ \to M'^- \ell^+ \ell^+) \equiv \frac{\Gamma(M^+ \to M'^- \ell^+ \ell^+)}{\Gamma_M}$$
$$= \mathcal{B} \times \left(\frac{100 \text{ GeV}}{m_N}\right)^2 \left(\frac{|U_{N\ell}|^2}{10^{-2}}\right)^2. \quad (17)$$

For the case $B^+ \to D^- \ell^+ \ell^+$ we must use $f_M = f_{B^+}$ and $f_{M'} = f_{D^-}$. Using the values shown in Table I, as well as $\Gamma_{B^+} = 4.0 \times 10^{-13}$ [3], the result is

$$\text{Br}(B^+ \to D^- \ell^+ \ell^+) \approx 1.1 \times 10^{-22} \times \left(\frac{100 \text{ GeV}}{m_N}\right)^2$$
$$\times \left(\frac{|U_{N\ell}|^2}{10^{-2}}\right)^2. \quad (18)$$

Similar results can be obtained for decays of other mesons $M^+ = K^+$, D^+, D_s^+, and B_c^+. The coefficients $\mathcal{B}$ for various decays are given in Table III. The present bounds on the PMNS mixing elements $|U_{N\ell}|$ for heavy Majorana neutrinos ($m_N \gtrsim 100$ GeV) are [23]

$$\sum_N |U_{Ne}|^2 \equiv (s_L^{\nu_e})^2 \leq 0.005,$$
$$(s_L^{\nu_\mu})^2 \leq 0.002, \quad (s_L^{\nu_\tau})^2 \leq 0.010. \quad (19)$$

So, Eq. (17) and the bounds in Eq. (19) allow us to interpret the $\mathcal{B}$ values in Table III as upper bound estimates of the corresponding branching ratios, for the case of heavy Majorana neutrinos with masses above 100 GeV.

As expected, our results in Table III coincide with those of Ref. [13], with discrepancies within 10% due to variations in the input parameters (a bit larger discrepancies are found in D^+ and D_s decays due to the different values we used for f_D and f_{D_s}). The conversion of their theoretical estimates into our notation is a simple factor 10^{-14} due to different units used. In the cases where Ref. [13] quotes a range, our agreements are with their central values. The sole exception occurs in $B^+ \to \pi^- \tau^+ \tau^+$, where their result is almost exactly a factor of 10 larger. We can attribute this discrepancy only to a misprint in the power of 10 of their result.

From Table III we see that the highest upper bounds ($\sim 10^{-20}$) are for the branching ratios of $B_c^+ \to D_s^- \ell^+ \ell^+$, with $\ell = e$ or μ. Yet, these bounds are several orders of magnitude too small to be detected in current and future experiments, like those at the LHC-*b* and super flavor factories.

As a final remark in this section, we want to compare these results with what would be expected if lepton flavor violation came from other sources, namely, supersymmetry with *R*-parity violation (RPV), or left-right symmetric electroweak theories. One can anticipate that such LNV processes should involve mass scales well above the electroweak scale, typically around the TeV scale, so they can be compared with the same LNV processes mediated by heavy Majorana neutrinos. Even though the experimental observation of the LNV meson decays would strongly support the hypothesis that neutrinos are

TABLE III. Branching ratio coefficients $\mathcal{B}$ appearing in Eq. (17), for various decays $M^+ \to M'^- \ell^+ \ell^+$, if the process is dominated by heavy neutrinos ($m_N \gg m_M$). $\mathcal{B}$ values correspond to branching ratios if $m_N = 100$ GeV and $|U_{N\ell}|^2 = 10^{-2}$. All three lepton flavors are considered ($\ell = e, \mu, \tau$). Entries with $\cdots$ are decays that are kinematically forbidden.

Decay	$\mathcal{B}(\ell = e)$	$\mathcal{B}(\ell = \mu)$	$\mathcal{B}(\ell = \tau)$
$K^+ \to \pi^- \ell^+ \ell^+$	8.47×10^{-24}	2.44×10^{-24}	$\cdots$
$D^+ \to \pi^- \ell^+ \ell^+$	1.90×10^{-23}	1.78×10^{-23}	$\cdots$
$D^+ \to K^- \ell^+ \ell^+$	1.58×10^{-23}	1.47×10^{-23}	$\cdots$
$D_s^+ \to \pi^- \ell^+ \ell^+$	2.14×10^{-22}	2.02×10^{-22}	$\cdots$
$D_s^+ \to K^- \ell^+ \ell^+$	2.46×10^{-23}	2.30×10^{-23}	$\cdots$
$D_s^+ \to D^- \ell^+ \ell^+$	6.99×10^{-28}	$\cdots$	$\cdots$
$B^+ \to \pi^- \ell^+ \ell^+$	1.13×10^{-23}	1.12×10^{-23}	7.42×10^{-25}
$B^+ \to K^- \ell^+ \ell^+$	8.44×10^{-25}	8.37×10^{-25}	5.01×10^{-26}
$B^+ \to D^- \ell^+ \ell^+$	1.02×10^{-22}	1.01×10^{-22}	$\cdots$
$B^+ \to D_s^- \ell^+ \ell^+$	5.02×10^{-23}	4.96×10^{-23}	$\cdots$
$B_c^+ \to \pi^- \ell^+ \ell^+$	1.76×10^{-21}	1.75×10^{-21}	3.04×10^{-22}
$B_c^+ \to K^- \ell^+ \ell^+$	1.73×10^{-22}	1.72×10^{-22}	2.89×10^{-23}
$B_c^+ \to D^- \ell^+ \ell^+$	3.20×10^{-22}	3.17×10^{-22}	2.14×10^{-23}
$B_c^+ \to D_s^- \ell^+ \ell^+$	9.17×10^{-21}	9.10×10^{-21}	5.17×10^{-22}
$B_c^+ \to B^- \ell^+ \ell^+$	3.31×10^{-28}	2.96×10^{-28}	$\cdots$

Majorana particles, these other sources could produce the same signals without involving Majorana neutrinos *directly*, just as it occurs in neutrinoless double beta decays.

In a supersymmetric extension of the standard model that includes RPV, the exchange of charged lepton or quark superpartners and neutralinos or gluinos rather than W bosons and Majorana neutrinos can also induce these LNV meson decays. RPV supersymmetry allows for additional trilinear terms in the superpotential, of the form:

$$W = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c, \quad (20)$$

where i, j, and k denote the families, L and Q denote lepton and quark isodoublet chiral superfields, and E^c, U^c, and D^c charged lepton and quark isosinglet chiral superfields. Of these terms, only the second leads to LNV meson decays [25]. The effective Lagrangian for these decays induced by the RPV terms can be written as [26]

$$L_{\rm eff}^{\Delta L=2} = \frac{G_F^2}{2m_p} \bar{e}(1 + \gamma_5)e^c \left[\eta_{PS} J_{PS} J_{PS} - \frac{1}{4} \eta_T J_T^{\mu\nu} J_{T\mu\nu} \right],$$
$$(21)$$

where the hadronic currents are $J_{PS} = \bar{u}^\alpha (1 + \gamma_5) d_\alpha$ and $J_T^{\mu\nu} = \bar{u}^\alpha \sigma^{\mu\nu}(1 + \gamma_5) d_\alpha$, with color index α and $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$. Here m_p is the proton mass and the explicit forms of the parameters η_{PS} and η_T are given in [27]. We should add that, in the case of LNV decays induced by heavy neutrinos, the effective Lagrangian can also be put in the form above if we use $\eta_N J_{VA}^\mu J_{VA\mu}$, where $J_{VA}^{\mu\nu} = \bar{u}^\alpha \gamma^\mu (1 - \gamma_5) d_\alpha$ and $\eta_N = \frac{|U_{N\ell}|^2}{10^{-2}} / \frac{100 \text{ GeV}}{m_N}$ (see Sec. II C and Appendix C). If we assume that the contribution induced by either gluinos or neutralinos is dominant over the others and the masses of the sfermions are almost equal, the parameters η_{PS} and η_T are of the order of

$$\frac{\pi \lambda'_{ijk} \lambda'_{ij'k'}}{G_F^2 m_{\tilde{f}}^4} \left(\frac{\alpha_s m_p}{6 m_{\tilde{g}}} + \frac{\alpha_2 m_p}{2 m_\chi} \right), \quad (22)$$

where α_s, α_2, m_χ, $m_{\tilde{g}}$, and $m_{\tilde{f}}$ denote the strong coupling, $SU(2)$ weak coupling, neutralino mass, gluino mass, and sfermion mass, respectively, and $\lambda'_{ijk} \lambda'_{ij'k'}$ actually depends on the process ($\lambda'_{123} \lambda'_{111}$ for $B \to Dee$, $\lambda'_{113} \lambda'_{112}$ for $B \to Kee$, $\lambda'_{113} \lambda'_{111}$ for $B \to \pi ee$, $\lambda'_{122} \lambda'_{111}$ for $D \to Kee$, $\lambda'_{121} \lambda'_{111}$ for $D \to \pi ee$, and $\lambda'_{112} \lambda'_{111}$ for $K \to \pi ee$). Besides $0\nu\beta\beta$ decay, the electron electric dipole moment experiments lead to the most stringent bounds on single λ'_{111}, which are 5.5×10^{-5} for $m_{\tilde{f}} = 100$ GeV and 2.4×10^{-7} for $m_{\tilde{f}} = 1$ TeV [28,29]. There are also several bounds on single RPV couplings λ'_{11k} and λ'_{12k} coming from experimental results for forward-backward asymmetries in the fermion pair production reactions measured at LEP and SLC, and leptonic π decays, respectively; $\lambda'_{11k} \lesssim 0.02$ and $\lambda'_{12k} \lesssim 0.21$ for $m_{\tilde{d}_{kR}} = 100$ GeV [29]. Imposing those bounds on $\lambda'_{111}, \lambda'_{11k}(k = 2, 3)$ and $\lambda'_{12k}(k = 1, 2, 3)$, the magnitudes of the corresponding terms in Eq. (22) are of the order of 10^{-8}–10^{-9} for $m_{\tilde{g},\chi,\tilde{f}} = 100$ GeV. These should be compared with the parameter η_N for the case of LNV induced by Majorana neutrinos. For $|U_{N\ell}|^2 \sim 10^{-2}$ and $m_N \sim 100$ GeV, the magnitude of η_N becomes unity. Thus, the above bounds on λ'_{ijk} would imply that the RPV supersymmetric contribution to the corresponding LNV decays should be much smaller than those induced by heavy neutrinos for $m_N = 100$ GeV (and maximally allowed $|U_{N\ell}|$). Conversely, the LNV meson decay experiments can be used to put bounds on the corresponding λ'_{ijk} parameters, especially in those cases where the bounds are very loose or still nonexistent.

Alternatively, a left-right symmetric model also involves a large mass scale which may characterize LNV mediated by heavy physics [30]. In $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, this gauge group breaks down to $SU(2)_L \times U(1)_Y$ via an extended Higgs sector containing a bidoublet Φ and two triplets $\Delta_{L,R}$ whose leptonic couplings generate Majorana neutrino masses and thus lepton number violation. The $\Delta_{L,R}$-lepton interactions are not suppressed by lepton masses and have the structure $L \sim h_{ij}\Delta_{L,R}^{++}\bar{l}_i^c(1 \pm \gamma_5)l_j +$ H.c., where the couplings h_{ij} are in general diagonal and associated with the heavy neutrino mixing matrix. In this model, short-distance contributions to LNV decays arise from the exchange of both heavy right-handed Majorana neutrinos and $\Delta_{L,R}$, which can be parametrized by [31]

$$\frac{g_2^4}{M_{W_R}^4}\frac{1}{M_{\nu_R}}, \qquad \frac{g_2^3}{M_{W_R}^3}\frac{h_{ij}}{M_\Delta^2}, \qquad (23)$$

where g_2, M_{W_R}, M_Δ, and M_{ν_R} denote the weak gauge coupling, the $SU(2)_R$ gauge boson mass, the triplet scalar mass, and the right-handed neutrino mass, respectively. These terms are to be compared with $\eta_N G_F^2/m_p$ corresponding to the LNV decays induced by heavy neutrinos. Imposing the current lower bound of 715 GeV on M_{W_R} [3] and taking $M_{\nu_R} \sim M_\Delta \sim 1$ TeV, those terms multiplied by m_p/G_F^2 are of the order of 10^{-9}–10^{-10}, which are again very small compared with $\eta_N^{max} \sim 1$ for $m_N = 100$ GeV.

III. SUMMARY AND CONCLUSIONS

We have studied lepton number violating decays of charged K, D, D_s, B, and B_c mesons of the form $M^+ \rightarrow M'^- \ell^+ \ell^+$, induced by the existence of Majorana neutrinos. These decays violate lepton number by two units, and therefore can occur only if neutrinos are of Majorana type. The decays are sensitive to neutrino masses and lepton mixing, and can also provide information complementary to neutrinoless double beta decays. We explore neutrino mass ranges m_N from below 1 eV to several hundred GeV.

The decay rates are dominated by different weak amplitudes, depending on the mass of the neutrinos involved in the intermediate states.

If the mass of the neutrino that dominates the process is below the mass of the produced meson, we find that the main contribution to the branching ratio should come from a two-particle intermediate state that goes on shell, formed by a meson (with the correct flavor) and the neutrino. These cases have a topology similar to neutrinoless double beta decay. However, the branching ratios obtained in these cases are far too small to be detected in foreseen experiments. Indeed, if the neutrinos involved are standard (masses below 1 eV, albeit Majorana) we find the branching ratios to be below 10^{-31}, and if they are heavier (up to the order of 100 MeV), the branching ratios to be below 10^{-21}, which are too small to be detected in the foreseen future, so we do not go into more refined calculations in these cases.

Instead, if the neutrino mass is in the range between the masses of the initial and final meson, the process is dominated by an intermediate state with just the neutrino, which goes on shell. In this "long distance" process, the neutrino is essentially produced and then it decays. Some of the branching ratios in this case are now within or near the reach of current or foreseen experiments, as shown in Table II. For example, $\mathcal{B}(K^+ \rightarrow \pi^- e^+ e^+)$ can be up to 10^{-6} and $\mathcal{B}(B_c \rightarrow \pi^- e^+ e^+)$ up to 10^{-11}. Experimental exploration of these decays can then at least provide upper bounds for the lepton mixing elements of the standard charged leptons with exotic Majorana neutrinos, bounds that will be dependent on the mass of the neutrino involved.

Finally, if the process is dominated by a neutrino that is considerably heavier than the decaying meson, the branching ratio is again far too suppressed to be experimentally observed, as shown in Table III and previously predicted in Ref. [13]. Indeed, for neutrino masses near 100 GeV or above, the branching ratios are all below 10^{-20}. In this case we also explore other underlying physics sources that could induce these LNV decays without involving Majorana neutrinos directly, namely, RPV supersymmetric models and left-right symmetric models. Concerning RPV SUSY, if we impose the current bounds on the relevant parameters λ'_{ijk}, the effect of these interactions on the decays would be lower than the effect of neutrinos. Otherwise, in general the experimental bound on each of the decays will impose bounds on its corresponding λ' parameter. Finally, concerning left-right symmetric models, their effect seems to fall far below the contributions of heavy Majorana neutrinos, and thus these decays may not be useful to put bounds on those models.

APPENDIX A: THE CASE OF LIGHT NEUTRINOS

In this Appendix we present the calculation of the decay rate for the process $M^+ \rightarrow M'^- \ell^+ \ell^+$ in the case in which neutrinos are lighter than the mesons in the process. The calculation is done assuming that the transition matrix element can be approximated by its absorptive part, which is given in Eq. (4).

After doing the integral in Eq. (4), the square of the amplitude, summed over the final lepton spins, becomes

$$|\mathcal{M}_{abs}|^2 = \frac{G_F^4}{16\pi^2}|V_{cb}V_{ud}|^2 F_{BD}^{+2}F_{DD}^{+2}|U_{N\ell}^2 m_N|^2 \frac{|\mathbf{p}_N|^2}{m_{D\ell}^2} \times \mathcal{T},$$
(A1)

where we have defined

$$\mathcal{T} = 2\,\text{Tr}[l_2(\not{p}_D\not{p}_B + m_{D^0}^2 + E_{D^0}(\gamma^0\not{p}_B \\ + \not{p}_D\gamma^0))l_1(\not{p}_B\not{p}_D + m_{D^0}^2 + E_{D^0}(\not{p}_B\gamma^0 + \gamma^0\not{p}_D))],$$

and where $|\mathbf{p}_N|$ is the neutrino 3-momentum in the rest frame of the D-N pair. Using the well-known expression

$$\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2yz - 2xz,$$
(A2)

it can be written as $|\mathbf{p}_N| = \lambda^{1/2}(m_{D\ell}^2, m_{D_0}^2, m_N^2)/2m_{D\ell}$.

Since the expression for $|\mathcal{M}_{abs}|^2$ in Eq. (A1) is not explicitly covariant, it is convenient to separate the phase space integral over the D-ℓ-ℓ final state (dps_3) into the 2-body phase spaces for $B \to \ell_1 + X_{D\ell}$ and $X_{D\ell} \to D + \ell_2$, with the invariant mass of the pair $X_{D\ell}$ integrated over its physical range:

$$\int dps_3 \equiv \int \prod_{i=1}^{3} \frac{d^3 p_i}{(2\pi)^3 2E_i}(2\pi)^4 \delta^4(\Sigma pi - p_M)$$

$$= \int \frac{dm_{D\ell}^2}{2\pi} \int dps_{(B\to\ell_1 X_{D\ell})} \int dps_{(X_{D\ell}\to D\ell_2)},$$
(A3)

where the 2-body phase spaces in their respective rest frames reduce to

$$dps_{(B\to\ell_1 X_{D\ell})} = \frac{1}{16\pi^2}\frac{|\tilde{l}_1|}{m_B}d\Omega_{\ell_1},$$

$$dps_{(X_{D\ell}\to D\ell_2)} = \frac{1}{16\pi^2}\frac{|l_2|}{m_{D\ell}}d\Omega_{\ell_2},$$

and the 3-momenta in the respective cases are

$$|\tilde{l}_1| = \frac{\lambda^{1/2}(m_B^2, m_{D\ell}^2, m_\ell^2)}{2m_B} \quad \text{and}$$

$$|l_2| = \frac{\lambda^{1/2}(m_{D\ell}^2, m_D^2, m_\ell^2)}{2m_{D\ell}}.$$
(A4)

Now, the integration over $d\Omega_{\ell_2}$ of the nontrivial factor $\mathcal{T}$ in Eq. (A1) can be expressed as

$$\int dps_{(X_{D\ell}\to D\ell_2)}\mathcal{T} = \frac{1}{16\pi^2}\frac{|l_2|}{m_{D\ell}}4\pi\mathcal{R},$$
(A5)

where $\mathcal{R}$ is a long expression of dimension m^6:

$$\mathcal{R} \equiv \{8(m_{D^0}^2 + 2E_{D^0}^2)^2 E_1 E_2 + 16(m_{D^0}^2 + 2E_{D^0}^2)(E_D E_2 + |l_2|^2)(E_B E_1 - |l_1|^2) + 16(m_{D^0}^2 + 2E_{D^0}^2)E_{D^0}(E_2(E_B E_1 - |l_1|^2) \\ - E_1(E_D E_2 + |l_2|^2)) + 16E_{D^0}(2E_B(E_B E_1 - |l_1|^2)(E_D E_2 + |l_2|^2) - m_D^2 E_2(E_B E_1 - |l_1|^2) - m_B^2 E_1(E_D E_2 + |l_2|^2)) \\ + 8E_{D^0}^2(E_1 E_2([E_B - E_D]^2 + |l_1|^2 + |l_2|^2) + 2(E_B - E_D)(E_1|l_2|^2 - E_2|l_1|^2) - 2|l_1|^2|l_2|^2) + 8(m_D^2 E_2 - 2E_D^2 E_2 \\ - 2E_D|l_2|^2)(m_B^2 E_1 - 2E_B^2 E_1 + 2E_B|l_1|^2)\},$$
(A6)

where all kinematical variables here are defined in the rest frame of the D-ℓ pair and are functions of its invariant mass $m_{D\ell}$:

$$E_D = \frac{m_{D\ell}^2 + m_D^2 - m_\ell^2}{2m_{D\ell}}, \qquad E_2 = \frac{m_{D\ell}^2 - m_D^2 + m_\ell^2}{2m_{D\ell}},$$

$$E_{D^0} = \frac{m_{D\ell}^2 + m_{D^0}^2 - m_N^2}{2m_{D\ell}}, \qquad E_B = \frac{m_B^2 + m_{D\ell}^2 - m_\ell^2}{2m_{D\ell}},$$

$$E_1 = \frac{m_B^2 - m_{D\ell}^2 - m_\ell^2}{2m_{D\ell}}, \qquad |l_1| = \frac{\lambda^{1/2}(m_B^2, m_{D\ell}^2, m_\ell^2)}{2m_{D\ell}}.$$
(A7)

Finally, since the result in Eq. (A5) is independent of angles, the subsequent integration over $d\Omega_{\ell_1}$ simply brings a factor 4π. The decay rate $\Gamma(B^+ \to D^-\ell^+\ell^-)$ then results in the expression:

$$\Gamma(B^+ \to D^-\ell^+\ell^-)$$

$$= \frac{G_F^4}{(16\pi^2)^2}|V_{cb}V_{ud}|^2 F_{BD}^{+2}F_{DD}^{+2}\frac{|U_{N\ell}^2 m_N|^2}{m_B^2}$$

$$\times \int_{(m_D+m_\ell)}^{(m_B-m_\ell)}\frac{dm_{D\ell}}{2\pi}\frac{|\mathbf{p}_N|^2}{m_{D\ell}^2}|\tilde{l}_1||l_2| \times \mathcal{R},$$
(A8)

where $|\mathbf{p}_N|$, $|\tilde{l}_1|$, $|l_2|$, and $\mathcal{R}$ were defined above and are explicit functions of $m_{D\ell}$. The integral in the expression above can be easily done numerically.

APPENDIX B: THE CASE OF INTERMEDIATE MASS NEUTRINOS

Here we present the meson decay rate $M^+ \to M'^-\ell^+\ell^+$ in the case in which neutrinos have a mass in the intermediate range $m_{M'} < m_N < m_M$.

The square of $\tilde{\mathcal{M}}$ in Eq. (9) and sum over external lepton spins of this reduced amplitude, after some algebra, can be written as

$$|\tilde{\mathcal{M}}|^2 = 32m_N^2\{(m_N^2 - m_\ell^2)^2(l_1 \cdot l_2) \\ + m_\ell^2((m_N^2 - m_\ell^2)^2 - m_M^2 m_{M'}^2)\}. \quad (B1)$$

The final 3-body phase space can again be separated into two 2-body integrals, and another over the invariant mass of the intermediate state (which in this case is the neutrino momentum squared, p_N^2):

$$\int dps_3 = \int \frac{dp_N^2}{2\pi} \int dps_{(M\to l_1 N)} \int dps_{(N\to l_2 M')}, \quad (B2)$$

where $dps_{(M\to l_1 N)} = 1/(16\pi^2)(|\tilde{l}_1|/m_M)d\Omega_1$ and $dps_{(N\to l_2 M')} = 1/(16\pi^2)(|l_2|/m_N)d\Omega_2$. This time, the propagator of the intermediate neutrino in the matrix element [see Eq. (8)] can be approximated by a delta function, since it is a narrow state:

$$\frac{1}{(p_N^2 - m_N^2)^2 + m_N^2\Gamma_N^2} \to \frac{\pi}{m_N\Gamma_N}\delta(p_N^2 - m_N^2).$$

The only term in $|\mathcal{M}|^2$ that depends on an integration angle is the one that contains $(l_1 \cdot l_2)$ [see Eq. (B1)]. In the neutrino rest frame, $\int d\Omega_2(l_1 \cdot l_2) = 4\pi E_1 E_2$, where $E_1 = (m_M^2 - m_N^2 - m_\ell^2)/2m_N$ and $E_2 = (m_N^2 + m_\ell^2 - m_{M'}^2)/2m_N$ are the respective energies of the external leptons. All other solid angle integrals give just a factor 4π. Putting everything together, we obtain the decay rate:

$$\Gamma(M \to M'\ell^+\ell^+) = \frac{1}{2m_M}\frac{G_F^4}{4}f_M^2 f_{M'}^2 |V_{qQ}V_{q_2 q_1}|^2 |U_{N\ell}|^4 \int dps_3 \frac{|\tilde{\mathcal{M}}|^2}{(p_N^2 - m_N^2)^2 + m_N^2\Gamma_N^2}$$

$$= \frac{G_F^4}{32\pi^2 m_M}f_M^2 f_{M'}^2 |V_{qQ}V_{q_2 q_1}|^2 \frac{|U_{N\ell}|^4}{m_N\Gamma_N}\frac{|\tilde{l}_1|}{m_M}\frac{|l_2|}{m_N}\{(m_N^2 + m_\ell^2)m_M^2 - (m_N^2 - m_\ell^2)^2\}$$

$$\times \{(m_N^2 - m_\ell^2)^2 - (m_N^2 + m_\ell^2)m_{M'}^2\}, \quad (B3)$$

where $|\tilde{l}_1| = \lambda^{1/2}(m_M^2, m_N^2, m_\ell^2)/2m_M$ and $|l_2| = \lambda^{1/2}(m_N^2, m_{M'}^2, m_\ell^2)/2m_N$.

APPENDIX C: THE CASE OF HEAVY NEUTRINOS

Here we present the derivation of the meson decay rate for $M^+ \to M'^-\ell^+\ell^+$ ($\ell = e, \mu, \tau$) when the exchanged Majorana neutrino is heavier than the decaying meson. In this case, both weak amplitudes shown in Fig. 1 contribute with similar strength to the process.

In order to see the approximations involved, let us first consider the neutrino mass m_N not to be necessarily high. Let us first recall the valence quark content of the decaying meson M as $(\bar{Q}q)$ and of the produced meson M' as $(\bar{q}_2 q_1)$. Let us also denote by $J_{(\bar{Q}q)}^\mu \equiv \bar{Q}\gamma^\mu(1 - \gamma_5)q$ the weak $V - A$ quark current with flavor change $q \to Q$. In much the same way as before, we rearrange the lepton line using charged-conjugated spinors and the Majorana character of the neutrino, thus appearing an irrelevant Majorana phase $\lambda_N = \exp(i\delta_N)$. Then the contribution of Fig. 1(a) (and diagram with exchanged antileptons $\ell_1^+ \leftrightarrow \ell_2^+$) can be written as

$$\mathcal{M}_{1a} = (-1)\frac{G_F^2}{2}U_{N\ell}^{*2}\lambda_N^*(V_{q_1 Q}^* V_{qq_2}^*)\langle M'|J_{(\bar{q}_2 q)}^\mu J_{(\bar{Q}q_1)}^\nu|M\rangle$$

$$\times \bar{u}_{\bar{\ell}}(l_2)\gamma_\mu(1 + \gamma_5)\left(\frac{k_N + m_N}{k_N^2 - m_N^2 + i\Gamma_N m_N}\right.$$

$$+ \left.\frac{k_N' + m_N}{k_N'^2 - m_N^2 + i\Gamma_N m_N}\right)\gamma_\nu(1 - \gamma_5)v_\ell(l_1). \quad (C1)$$

Here we called k_N and k_N' the corresponding neutrino momenta for the two crossings of the external lepton lines. Now, if the neutrino is heavy, in the denominators we neglect all except m_N^2. In addition, using the orthogonality of the chiral projectors and the relation $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, the expression reduces to

$$\mathcal{M}_{1a} = \frac{G_F^2}{2}U_{N\ell}^{*2}\lambda_N^*(V_{q_1 Q}^* V_{qq_2}^*)\langle M'|J_{(\bar{q}_2 q)}^\mu J_{(\bar{Q}q_1)\mu}|M\rangle$$

$$\times \frac{4}{m_N}[\bar{u}_{\bar{\ell}}(l_2)(1 - \gamma_5)v_\ell(l_1)]. \quad (C2)$$

In much the same way one can treat the contribution of Fig. 1(b), but now the roles of the flavors q and q_1 are interchanged, thus the quark currents and CKM elements are different:

$$\mathcal{M}_{1b} = \frac{G_F^2}{2}U_{N\ell}^{*2}\lambda_N^*(V_{qQ}^* V_{q_1 q_2}^*)\langle M'|J_{(\bar{q}_2 q_1)}^\mu J_{(\bar{Q}q)\mu}|M\rangle$$

$$\times \frac{4}{m_N}[\bar{u}_{\bar{\ell}}(l_2)(1 - \gamma_5)v_\ell(l_1)]. \quad (C3)$$

The largest uncertainty here is in the determination of the hadronic matrix element. As a first estimate, reasonable for the level of accuracy we seek, we can separate the currents inserting the vacuum and assuming it saturates the expression (vacuum saturation approximation). We must then do a Fierz rearrangement in M_{1a} to match the flavors, which then mismatches the color, inducing a suppression factor $1/N_c$. The hadronic currents then

reduce to their corresponding decay constants, f_M and $f_{M'}$, and the total amplitude for heavy neutrino exchange ("h") becomes

$$
\begin{aligned}
\mathcal{M}_{\mathrm{h}} &= \mathcal{M}_{1a} + \mathcal{M}_{1b} \\
&= \frac{G_F^2}{2} U_{N\ell}^{*2} \lambda_N^* \left[V_{qQ}^* V_{q_1q_2}^* + \frac{V_{q_1Q}^* V_{qq_2}^*}{N_c} \right] f_M f_{M'} (p_M \cdot p_{M'}) \\
&\quad \times \frac{4}{m_N} [\bar{u}_{\bar{\ell}}(l_2)(1-\gamma_5)v_\ell(l_1)].
\end{aligned} \tag{C4}
$$

One may neglect the $1/N_c$ term, except if the other term is much more CKM suppressed. The square and sum over final polarizations of this amplitude are

$$
|\mathcal{M}_{\mathrm{h}}|^2 = |\mathcal{K}_{\mathrm{h}}|^2 32(p_M \cdot p_{M'})^2 (l_1 \cdot l_2), \tag{C5}
$$

where we have gathered all constant factors under the symbol

$$
|\mathcal{K}_{\mathrm{h}}|^2 = G_F^4 |\frac{U_{N\ell}^{*2}}{m_N}|^2 |V_{qQ}^* V_{q_1q_2}^* + \frac{V_{q_1Q}^* V_{qq_2}^*}{N_c}|^2 f_M^2 f_{M'}^2.
$$

The decay width $\Gamma(M^+ \to M'^- \ell^+ \ell^+)$ can now be calculated explicitly, by integrating $|\mathcal{M}_{\mathrm{h}}|^2$ over the final phase space. This time we express the M'-ℓ-ℓ phase space as

$$
\int d\mathrm{ps}_3 = \int \frac{dm_{\ell\ell}^2}{2\pi} \int d\mathrm{ps}_{(M \to M' X_{\ell\ell})} \int d\mathrm{ps}_{(X_{\ell\ell} \to \ell\ell)},
$$

where $d\mathrm{ps}_{(M \to M' X_{\ell\ell})} = (1/16\pi^2)(|\mathbf{p}_{M'}|/m_M)d\Omega_{M'}$ and $d\mathrm{ps}_{(X_{\ell\ell} \to \ell\ell)} = (1/16\pi^2)(|l_2|/m_{\ell\ell})d\Omega_{\ell_2}$. In the frame of the lepton pair, the integrand (C5) is independent of angles, so the integral over $d\Omega_{\ell_2}$ is a simple factor of 4π, and equally for the subsequent integral over $d\Omega_{M'}$.

The decay rate is then

$$
\begin{aligned}
&\Gamma(M^+ \to M'^- \ell^+ \ell^+) \\
&= \frac{1}{2!} \frac{1}{8\pi^2} \frac{|\mathcal{K}_{\mathrm{h}}|^2}{m_M} \int_{4m_\ell^2}^{(m_M-m_{M'})^2} \frac{dm_{\ell\ell}^2}{2\pi} \frac{|\mathbf{p}_{M'}|}{m_M} \frac{|l_2|}{m_{\ell\ell}} \\
&\quad \times (m_M^2 + m_{M'}^2 - m_{\ell\ell}^2)^2 (m_{\ell\ell}^2 - 2m_\ell^2),
\end{aligned} \tag{C6}
$$

where $|\mathbf{p}_{M'}| = \lambda^{1/2}(m_M^2, m_{M'}^2, m_{\ell\ell}^2)/2m_M$ and $|l_2| = \lambda^{1/2}(m_{\ell\ell}^2, m_\ell^2, m_\ell^2)/2m_{\ell\ell}$. The factor $1/2!$ appears because there are two identical particles in the final state. This integral can be easily done numerically.

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