PHYSICAL REVIEW D **82,** 053005 (2010)

Short-baseline electron neutrino disappearance, tritium beta decay, and neutrinoless double-beta decay

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(Received 31 May 2010; published 22 September 2010)

We consider the interpretation of the MiniBooNE low-energy anomaly and the gallium radioactive source experiments anomaly in terms of short-baseline electron neutrino disappearance in the framework of 3 + 1 four-neutrino mixing schemes. The separate fits of MiniBooNE and gallium data are highly compatible, with close best-fit values of the effective oscillation parameters Δm^2 and $\sin^2 2\vartheta$. The combined fit gives $\Delta m^2 \gtrsim 0.1\ \text{eV}^2$ and $0.11 \lesssim \sin^2 2\vartheta \lesssim 0.48$ at 2σ. We consider also the data of the Bugey and Chooz reactor antineutrino oscillation experiments and the limits on the effective electron antineutrino mass in β decay obtained in the Mainz and Troitsk tritium experiments. The fit of the data of these experiments limits the value of $\sin^2 2\vartheta$ below 0.10 at 2σ. Considering the tension between the neutrino MiniBooNE and gallium data and the antineutrino reactor and tritium data as a statistical fluctuation, we perform a combined fit which gives $\Delta m^2 \simeq 2$ eV and $0.01 \lesssim \sin^2 2\vartheta \lesssim 0.13$ at 2σ. Assuming a hierarchy of masses m_1, m_2, $m_3 \ll m_4$, the predicted contributions of m_4 to the effective neutrino masses in β decay and neutrinoless double-β decay are, respectively, between about 0.06 and 0.49 and between about 0.003 and 0.07 eV at 2σ. We also consider the possibility of reconciling the tension between the neutrino MiniBooNE and gallium data and the antineutrino reactor and tritium data with different mixings in the neutrino and antineutrino sectors. We find a 2.6σ indication of a mixing angle asymmetry.

DOI: 10.1103/PhysRevD.82.053005 PACS numbers: 14.60.Pq, 14.60.Lm, 14.60.St

I. INTRODUCTION

Neutrino oscillations have been observed in solar, atmospheric, and long-baseline reactor and accelerator experiments. The data of these experiments are well fitted in the framework of three-neutrino mixing, in which the three flavor neutrinos ν_e, ν_μ, and ν_τ are unitary linear combinations of three massive neutrinos ν_1, ν_2, and ν_3 with the solar (SOL) and atmospheric (ATM) squared-mass differences

$$\Delta m^2_{21} = \Delta m^2_{\text{SOL}} \simeq 8 \times 10^{-5}\ \text{eV}^2, \tag{1}$$

$$|\Delta m^2_{31}| \simeq |\Delta m^2_{32}| = \Delta m^2_{\text{ATM}} \simeq 2 \times 10^{-3}\ \text{eV}^2, \tag{2}$$

where $\Delta m^2_{jk} = m^2_j - m^2_k$ and m_j is the mass of the neutrino ν_j (see Refs. [1–8]).

Besides these well-established observations of neutrino oscillations, there are at least three anomalies which could be signals of short-baseline (SBL) neutrino oscillations generated by a larger squared-mass difference: the LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ signal [9], the gallium radioactive source experiments anomaly [10,11], and the MiniBooNE low-energy anomaly [12]. In this paper we consider the MiniBooNE and gallium anomalies, which can be explained by short-baseline electron neutrino disappearance [13–15] in the effective framework of four-neutrino mixing, as explained in Secs. II and III. On the other hand, the LSND anomaly is disfavored by the results of the MiniBooNE $\nu_\mu \to \nu_e$ experiment [12,16] and may require another explanation [17–23].

In Refs. [13,15] we proposed to explain the MiniBooNE low-energy anomaly [12,16] through the disappearance of electron neutrinos due to very-short-baseline oscillations into sterile neutrinos generated by a squared-mass difference Δm^2 larger than about 20 eV^2. In that case, the analysis of the MiniBooNE data is simplified by the fact that the effective survival probability $P_{\nu_e \to \nu_e}$ is practically constant in the MiniBooNE energy range from 200 to 3000 MeV. In this paper we extend the analysis of MiniBooNE data to lower values of Δm^2, considering the resulting energy dependence of the effective SBL electron neutrino and antineutrino survival probability

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e}(L, E) = 1 - \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right), \tag{3}$$

where L is the neutrino path length and E is the neutrino energy (CPT invariance implies that the survival probabilities of neutrinos and antineutrinos are equal; see Ref. [8]).

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The two-neutrino-like effective short-baseline survival probability in Eq. (3) is obtained in four-neutrino schemes (see Refs. [1,4,6,7]), which are the simplest extension of three-neutrino mixing schemes which can accommodate the two small solar and atmospheric squared-mass differences in Eqs. (2) and (40), and one larger squared-mass difference for short-baseline neutrino oscillations,

$$|\Delta m^2_{41}| = \Delta m^2 \gtrsim 0.1 \text{ eV}^2. \quad (4)$$

The existence of a fourth massive neutrino corresponds, in the flavor basis, to the existence of a sterile neutrino ν_s.

In this paper we consider 3 + 1 four-neutrino schemes, since 2 + 2 four-neutrino schemes are disfavored by the combined constraints on active-sterile transitions in solar and atmospheric neutrino experiments [4]. For simplicity, we consider only 3 + 1 four-neutrino schemes with

$$m_1, m_2, m_3 \ll m_4, \quad (5)$$

which give the Δm^2_{41} in Eq. (4) and appear to be more natural than the other possible 3 + 1 four-neutrino schemes in which either three neutrinos or all four neutrinos are almost degenerate at a mass scale larger than $\sqrt{\Delta m^2}$ (see Refs. [1,4,6,7]).

In 3 + 1 four-neutrino schemes the effective mixing angle in the effective short-baseline electron neutrino survival probability in Eq. (3) is given by (see Refs. [1,4,6,7])

$$\sin^2 2\vartheta = 4|U_{e4}|^2(1 - |U_{e4}|^2). \quad (6)$$

In this paper we assume that the value of $|U_{\mu 4}|^2$ is so small that the effective short-baseline muon neutrino survival probability is practically equal to unity and short-baseline $\overset{(-)}{\nu}_\mu \leftrightarrows \overset{(-)}{\nu}_e$ are negligible.[1] This assumption is justified by the lack of any indication of $\nu_\mu \to \nu_e$ transitions in the MiniBooNE experiment [12,16] and the limits on short-baseline muon neutrino disappearance found in the CDHSW [24], CCFR [25], and MiniBooNE [26] experiments. We do not consider the MiniBooNE antineutrino data [27], which have at present statistical uncertainties which are too large to constrain new physics [15].

The plan of the paper is as follows. In Sec. II we discuss the analysis of MiniBooNE data. In Sec. III we present an update of the analysis of gallium data published in Ref. [14] and the combined analysis of MiniBooNE and gallium data. In Sec. IV we discuss the implications of the measurements of the effective electron neutrino mass in tritium β-decay experiments and their combination with reactor neutrino oscillation data. In Sec. V we present the results of the combined analysis of MiniBooNE, gallium, reactor, and tritium data and in Sec. VI we present the corresponding predictions for the effective masses measured in β-decay and neutrinoless double-β-decay experiments. In Sec. VII we calculate the mixing angle asymmetry between the neutrino and antineutrino sectors which could explain the tension between the neutrino and antineutrino data under our short-baseline ν_e-disappearance hypothesis. In Sec. VIII we draw the conclusions.

II. MINIBOONE

The MiniBooNE experiment was made with the purpose of checking the indication of $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations generated by a $\Delta m^2 \gtrsim 0.1$ eV^2 found in the LSND experiment [9]. The MiniBooNE Collaboration did not find any indication of such oscillations in the $\nu_\mu \to \nu_e$ channel [12,16]. On the other hand, the MiniBooNE Collaboration found an anomalous excess of low-energy ν_e-like events in the data on the search for $\nu_\mu \to \nu_e$ oscillations [12,16], as shown in Fig. 1(a).

As in Refs. [13,15], we consider an explanation of the low-energy MiniBooNE anomaly based on the possible short-baseline disappearance of electron neutrinos, taking into account a possible overall normalization factor f_ν of the calculated ν_e-induced and misidentified ν_μ-induced events which contribute to the observed number of ν_e-like events. The normalization factor f_ν could be due mainly to the uncertainty of the calculated neutrino flux (see Ref. [28]). Since the misidentified ν_μ-induced and ν_e-induced events dominate, respectively, at low and high energies [see Fig. 1(a)], the low-energy excess can be fitted with $f_\nu > 1$ and the high-energy data can be fitted compensating $f_\nu > 1$ with the disappearance of ν_e's.

In Refs. [13,15] we considered only very-short-baseline ν_e disappearance due to a $\Delta m^2 \gtrsim 20$ eV^2, which generates a survival probability $P_{\nu_e \to \nu_e}$ which is constant in the MiniBooNE energy range, from 200 to 3000 MeV. In this paper we extend the analysis to lower values of Δm^2, considering the resulting energy dependence of the survival probability. In this case, the theoretical number of ν_e-like events in the jth energy bin is given by

$$N^{\text{the}}_{\nu_e,j} = N^{\nu_e,\text{the}}_{\nu_e,j} + N^{\nu_\mu,\text{the}}_{\nu_e,j}, \quad (7)$$

where

$$N^{\nu_e,\text{the}}_{\nu_e,j} = f_\nu P^{(j)}_{\nu_e \to \nu_e} N^{\nu_e,\text{cal}}_{\nu_e,j} \quad (8)$$

is the number of ν_e-induced events and

$$N^{\nu_\mu,\text{the}}_{\nu_e,j} = f_\nu N^{\nu_\mu,\text{cal}}_{\nu_e,j} \quad (9)$$

is the number of misidentified ν_μ-induced events. Here $N^{\nu_e,\text{cal}}_{\nu_e,j}$ and $N^{\nu_\mu,\text{cal}}_{\nu_e,j}$ are, respectively, the number of ν_e-induced and misidentified ν_μ-induced events calculated

[1] In 3 + 1 four-neutrino schemes the effective short-baseline muon neutrino survival probability has the form in Eq. (3) with $\sin^2 2\vartheta$ replaced by $\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2)$. The effective short-baseline $\overset{(-)}{\nu}_\mu \leftrightarrows \overset{(-)}{\nu}_e$ transition probability is given by $P^{\text{SBL}}_{\overset{(-)}{\nu}_\mu \leftrightarrows \overset{(-)}{\nu}_e}(L, E) = \sin^2 2\vartheta_{e\mu} \sin^2(\frac{\Delta m^2 L}{4E})$, with $\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu 4}|^2$ (see Refs. [1,4,6,7]).

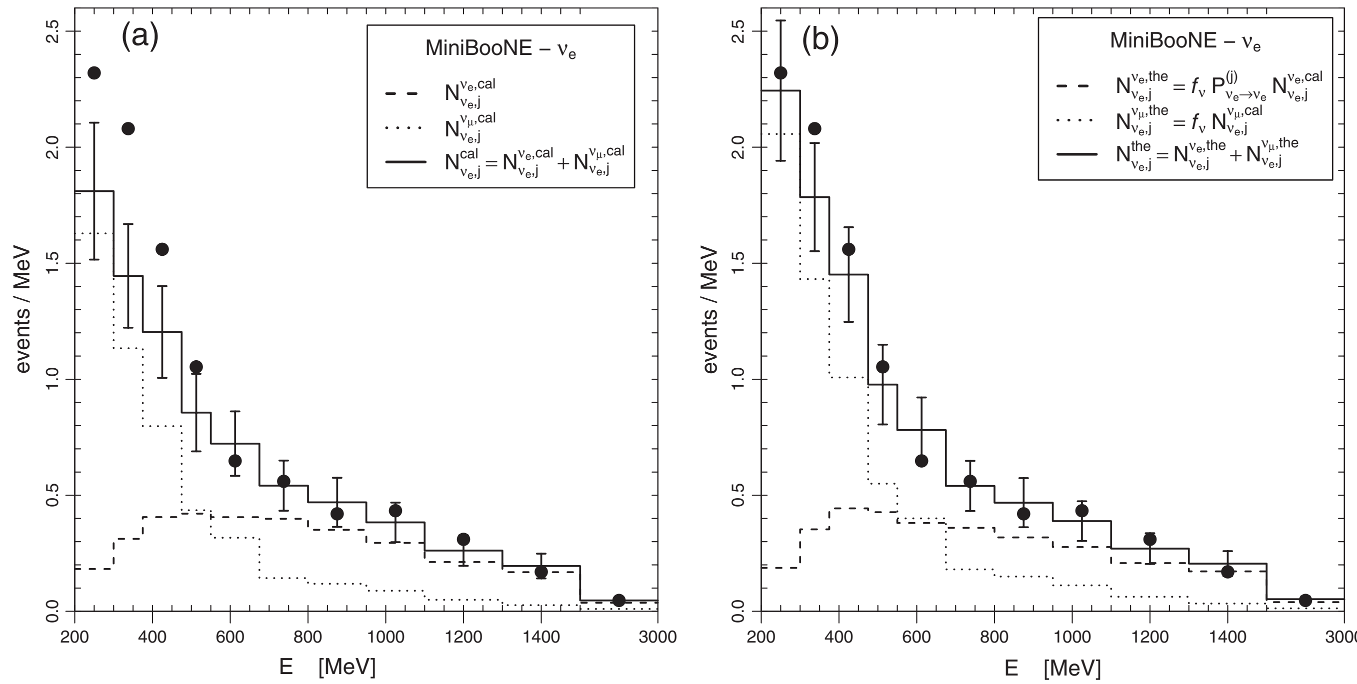

FIG. 1. Expected number of ν_e events compared with MiniBooNE data, represented by the black points. The energy bins are numbered with the index j. The uncertainty is represented by the vertical error bars, which represent the sum of statistical and uncorrelated systematic uncertainties. (a) Expected number of ν_e-like events $N^{\rm cal}_{\nu_e,j}$ calculated by the MiniBooNE Collaboration. $N^{\rm cal}_{\nu_e,j}$ is given by the sum of the ν_e-induced events ($N^{\nu_e,{\rm cal}}_{\nu_e,j}$) and the misidentified ν_μ-induced events ($N^{\nu_\mu,{\rm cal}}_{\nu_e,j}$). (b) Best-fit value of the number of ν_e-like events $N^{\rm the}_{\nu_e,j}$ obtained with the hypothesis of ν_e disappearance. $N^{\rm the}_{\nu_e,j}$ is given by the sum of $N^{\nu_e,{\rm the}}_{\nu_e,j} = f_\nu P^{(j)}_{\nu_e\to\nu_e} N^{\nu_e,{\rm cal}}_{\nu_e,j}$ and $N^{\nu_\mu,{\rm the}}_{\nu_e,j} = f_\nu N^{\rm cal}_{\nu_\mu}$. The best-fit values of f_ν, $\sin^2 2\vartheta$, and Δm^2 are those in the first column of Table I (MB_ν).

by the MiniBooNE Collaboration for the jth energy bin [29,30]. $P^{(j)}_{\nu_e\to\nu_e}$ is the survival probability of electron neutrinos in Eq. (3) averaged in the jth energy bin. The average in each bin is calculated using the ntuple file of 17 037 predicted muon-to-electron neutrino full transmutation events given in Ref. [29], which contains information on reconstructed neutrino energy, true neutrino energy, neutrino baseline, and event weight for each event.

The MiniBooNE measurement of a ratio 1.21 ± 0.24 of detected and predicted charged-current quasielastic ν_μ events [31] allows a value of f_ν as large as about 15%. In Ref. [15] we used this estimate of the uncertainty of f_ν in order to constrain its value in the least-squares analysis. Here we use directly the ν_μ data given in Ref. [29] for the construction of the MiniBooNE least-squares function

$$\chi^2_{\mathrm{MB}\nu} = \sum_{j=1}^{11}\left(\frac{N^{\rm the}_{\nu_e,j} - N^{\rm exp}_{\nu_e,j}}{\sigma_{\nu_e,j}}\right)^2 + \sum_{j,k=1}^{8}(N^{\rm the}_{\nu_\mu,j} - N^{\rm exp}_{\nu_\mu,j}) \times (V^{-1}_{\nu_\mu})_{jk}(N^{\rm the}_{\nu_\mu,k} - N^{\rm exp}_{\nu_\mu,k}). \quad (10)$$

Here $N^{\rm exp}_{\nu_e,j}$ are the numbers of measured ν_e-like events in 11 reconstructed neutrino energy bins and $N^{\rm exp}_{\nu_\mu,j}$ are the numbers of measured ν_μ charged-current quasielastic events in 8 reconstructed neutrino energy bins. The theoretical number of ν_μ events in the jth energy bin is given by

$$N^{\rm the}_{\nu_\mu,j} = f_\nu N^{\rm cal}_{\nu_\mu,j}, \quad (11)$$

where $N^{\rm cal}_{\nu_\mu,j}$ is the number of ν_μ events calculated by the MiniBooNE Collaboration [29]. In order to take into account the correct statistical uncertainty corresponding to the rescaling of the number of ν_μ events due to f_ν in Eq. (11), we used the covariance matrix V_{ν_μ} given by

$$(V_{\nu_\mu})_{jk} = (V^{\rm cal}_{\nu_\mu})_{jk} + (f_\nu - 1)N^{\rm cal}_{\nu_\mu,j}\delta_{jk}, \quad (12)$$

where $V^{\rm cal}_{\nu_\mu}$ is the 8×8 covariance matrix of ν_μ events presented by the MiniBooNE Collaboration in Ref. [29]. We did not use the complete 19×19 covariance matrix of ν_e and ν_μ events given in Ref. [29] because the correlations involving ν_e events have been obtained without taking into account the energy-dependent disappearance of electron neutrinos that we want to test. Assuming the correlations given in that 19×19 covariance matrix would suppress the energy dependence of $P^{(j)}_{\nu_e\to\nu_e}$. Therefore, for the uncertainties $\sigma_{\nu_e,j}$ in Eq. (10) we used only the diagonal elements of the ν_e covariance matrix $V^{\rm cal}_{\nu_e}$ given in Ref. [29], corrected by the change of statistical uncertainty corresponding to the variation of expected events due to f_ν

TABLE I. Values of χ^2, number of degrees of freedom (NDF) and goodness of fit (GoF) for the fit of different combinations of MiniBooNE (MB_ν), gallium (Ga), and reactor (Re) data. The first three lines correspond to the case of $f_\nu = 1$ and no oscillations (Null hyp.). The following six lines correspond to the case $f_\nu > 1$ and ν_e disappearance (Our hyp.). The last three lines give the parameter goodness-of-fit (PG) [32]. In the MB_ν column, the value of χ^2 in the Null. hyp., the number of degrees of freedom in the Null. hyp. (which is equal to the number of energy bins), and the value of χ^2_{min} in Our hyp. are shown as the sum of the contributions of the first three low-energy ν_e bins and the other ν_e and ν_μ energy bins. In the MB_ν + Ga and (MB_ν + Ga) + (Re + ^{3}H) columns the value of χ^2_{min} in Our hyp. is shown as the sum of the contribution of the first three MiniBooNE low-energy ν_e bins and the other contributions.

		MB_ν	Ga	MB_ν + Ga	Re + ^{3}H	(MB_ν + Ga) + (Re + ^{3}H)
Null hyp.	χ^2	14.3 + 5.4	9.4		51.5	
	NDF	3 + 16	4		58	
	GoF	0.41	0.051		0.71	
Our hyp.	χ^2_{min}	2.0 + 7.6	1.8	2.2 + 9.2	49.1	4.1 + 63.4
	NDF	16	2	20	56	78
	GoF	0.89	0.40	0.93	0.73	0.80
	$\sin^2 2\vartheta_{bf}$	0.32	0.27	0.28	0.042	0.062
	Δm^2_{bf}	1.84	2.09	1.92	1.85	1.85
	f_ν^{bf}	1.26		1.25		1.17
PG	$\Delta\chi^2_{min}$			0.098	0.01	6.97
	NDF			2	2	2
	GoF			0.95	0.99	0.03

and $P^{(j)}_{\nu_e\to\nu_e}$ in Eqs. (7)–(9):

$$\sigma^2_{\nu_e,j} = (V^{cal}_{\nu_e})_{jj} + N^{the}_{\nu_e,j} - N^{cal}_{\nu_e,j}, \qquad (13)$$

with $N^{cal}_{\nu_e,j} = N^{\nu_e,cal}_{\nu_e,j} + N^{\nu_\mu,cal}_{\nu_e,j}$.

The result of the minimization of $\chi^2_{MB\nu}$ is shown in Fig. 1(b), in which the solid histogram corresponds to the best-fit values of f_ν, $\sin^2 2\vartheta$, and Δm^2 in the first column of Table I. From Fig. 1(b), one can see that the fit is acceptable for all the ν_e energy bins, including the first three bins which are out of fit in Fig. 1(a). In Table I we give separately the contribution to $\chi^2_{MB\nu}$ of the first three low-energy ν_e bins and the sum of the contributions of the other ν_e energy bins and all the ν_μ energy bins. In this way one can see that with $f_\nu = 1$ and $P_{\nu_e\to\nu_e} = 1$ (null hypothesis), although the global value $\chi^2 = 19.7$ is compatible with the number of degrees of freedom, NDF = 19, almost all the χ^2 is due to the anomalous contribution 14.3 of the first three low-energy ν_e bins, whereas the other 16 ν_e and ν_μ energy bins are overfitted, with the excessively small χ^2 contribution of 5.4. This overfitting, which is probably due to an overestimate of the uncertainties, remains in the fit of the data with our hypothesis of $f_\nu > 1$ and ν_e disappearance. On the other hand, our hypothesis clearly explains the low-energy anomaly reducing the χ^2 contribution of the first three low-energy ν_e bins to the acceptable best-fit value of 2.0.

Figure 2 shows the allowed regions in the $\sin^2 2\vartheta - \Delta m^2$ plane and the marginal $\Delta\chi^2 = \chi^2 - \chi^2_{min}$'s for $\sin^2 2\vartheta$ and Δm^2, from which one can infer the corresponding uncorrelated allowed intervals. One can see that the indication in favor of neutrino oscillations is not strong, being at the level of about 79% C.L. (1.2σ). It is interesting to notice that the best-fit value of Δm^2 is about 2 eV^2, which is approximately the same best-fit value obtained in Ref. [14] from the fit of the neutrino data of gallium radioactive source experiments and the antineutrino data of the Bugey and Chooz reactor experiments under the hypothesis of ν_e and $\bar{\nu}_e$ disappearance. The results of the

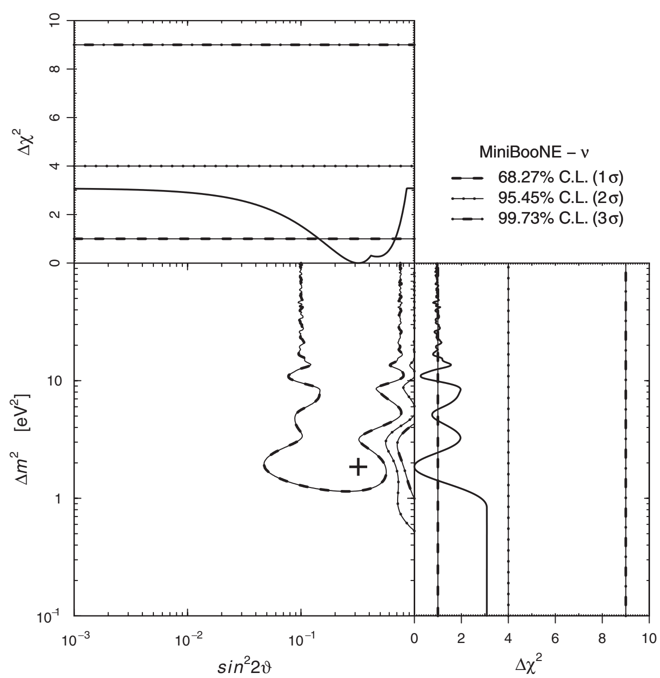

FIG. 2. Allowed regions in the $\sin^2 2\vartheta - \Delta m^2$ plane and marginal $\Delta\chi^2$'s for $\sin^2 2\vartheta$ and Δm^2 obtained from the fit of MiniBooNE neutrino data. The best-fit point is indicated by a +.

combined analysis of MiniBooNE neutrino data and the data of these other experiments is discussed in the following sections.

III. GALLIUM RADIOACTIVE SOURCE EXPERIMENTS

The GALLEX [11,33,34] and SAGE [10,35–37] Collaborations tested the respective gallium solar neutrino detectors in so-called ''gallium radioactive source experiments'' which consist of the detection of electron neutrinos produced by intense artificial ^{51}Cr and ^{37}Ar radioactive sources placed inside the detectors. Taking into account the uncertainty of the cross section of the detection process $\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-$ estimated in Ref. [38], the ratios R of measured and predicted ^{71}Ge event rates are

$$R^{\mathrm{GALLEX}}_{\mathrm{Cr1}} = 0.95^{+0.11}_{-0.12}, \quad (14)$$

$$R^{\mathrm{GALLEX}}_{\mathrm{Cr2}} = 0.81^{+0.10}_{-0.11}, \quad (15)$$

$$R^{\mathrm{SAGE}}_{\mathrm{Cr}} = 0.95^{+0.12}_{-0.12}, \quad (16)$$

$$R^{\mathrm{SAGE}}_{\mathrm{Ar}} = 0.79^{+0.09}_{-0.10}, \quad (17)$$

and the average ratio is

$$R^{\mathrm{Ga}} = 0.86^{+0.05}_{-0.05}. \quad (18)$$

Thus, the number of measured events is about 2.7σ smaller than the prediction.

The theoretical prediction of the rate is based on the calculation of the detection cross section presented in Ref. [38]. It is possible that a part of the observed deficit is due to an overestimation of this cross section [10,37,39], because only the cross section of the transition from the ground state of ^{71}Ga to the ground state of ^{71}Ga is known with precision from the measured rate of electron capture decay of ^{71}Ge to ^{71}Ga. Electron neutrinos produced by ^{51}Cr and ^{37}Ar radioactive sources can be absorbed also through transitions from the ground state of ^{71}Ga to two excited states of ^{71}Ge, with cross sections which are inferred using a nuclear model from $p + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + n$ measurements [40]. This calculation has large uncertainties [41,42]. However, since the contribution of the transitions to the two excited states is only 5% [38], even the complete absence of such transitions would reduce R^{Ga} to about $0.90^{+0.05}_{-0.05}$, leaving an anomaly of about 1.8σ.

Here we consider the electron neutrino disappearance explanation of the gallium radioactive source experiments anomaly [13–15,43–45] (another interesting explanation through quantum decoherence in neutrino oscillations has been proposed in Ref. [21]).

In Ref. [14] we have analyzed the data of the gallium radioactive source experiments in terms of the effective survival probability in Eq. (3). Here we update that analysis taking into account the revised value of $R^{\mathrm{GALLEX}}_{\mathrm{Cr1}}$ in Eq. (14) published recently in Ref. [11] and taking into account the asymmetric uncertainties of $R^{\mathrm{GALLEX}}_{\mathrm{Cr1}}$, $R^{\mathrm{GALLEX}}_{\mathrm{Cr2}}$, and $R^{\mathrm{SAGE}}_{\mathrm{Ar}}$ (which have been symmetrized for simplicity in the analysis presented in Ref. [14]). Following the method described in Ref. [14], we obtained the best-fit values of $\sin^2 2\vartheta$ and Δm^2 in the second column of Table I and the allowed regions in the $\sin^2 2\vartheta - \Delta m^2$ plane shown in Fig. 3. The indication in favor of neutrino oscillations is at the level of about 98% C.L. (2.3σ).

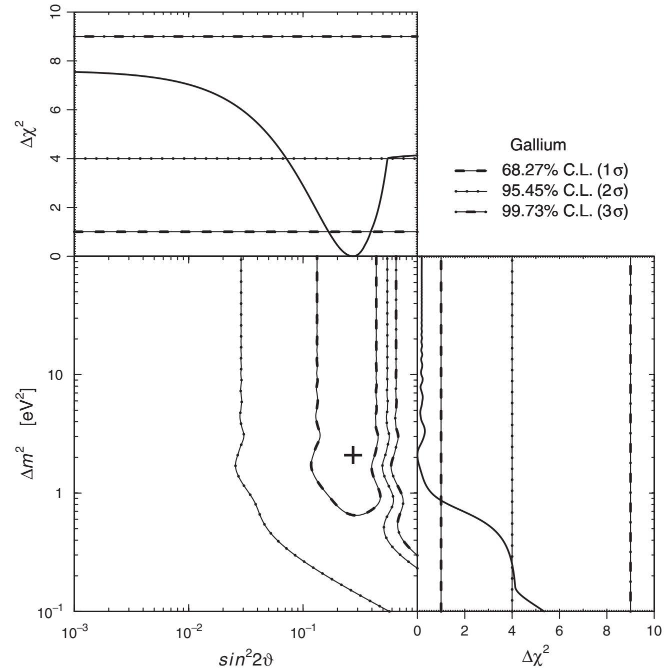

FIG. 3. Allowed regions in the $\sin^2 2\vartheta - \Delta m^2$ plane and marginal $\Delta\chi^2$'s for $\sin^2 2\vartheta$ and Δm^2 obtained from the combined fit of the results of the two GALLEX ^{51}Cr radioactive source experiments and the SAGE ^{51}Cr and ^{37}Ar radioactive source experiments. The best-fit point corresponding to χ^2_{min} is indicated by a +.

From Table 2 and the comparison of Figs. 2 and 3 one can see that the fits of MiniBooNE and gallium data lead to remarkably similar results: the best-fit values of the oscillation parameters are very close and the allowed regions in the $\sin^2 2\vartheta - \Delta m^2$ plane are highly compatible. This is certainly an impressive success of our hypothesis of electron neutrino disappearance.

The results of the combined fit of MiniBooNE and gallium data are shown in the third column of Table I and in Fig. 4. The separate data sets are well fitted by the electron neutrino disappearance hypothesis: the χ^2 contribution of the first three MiniBooNE low-energy ν_e bins is 2.2, that of the other 16 MiniBooNE ν_e and ν_μ energy bins is 7.4, and that of the 4 gallium data is 1.9. The consistency of the combined fit is also supported by the excellent value

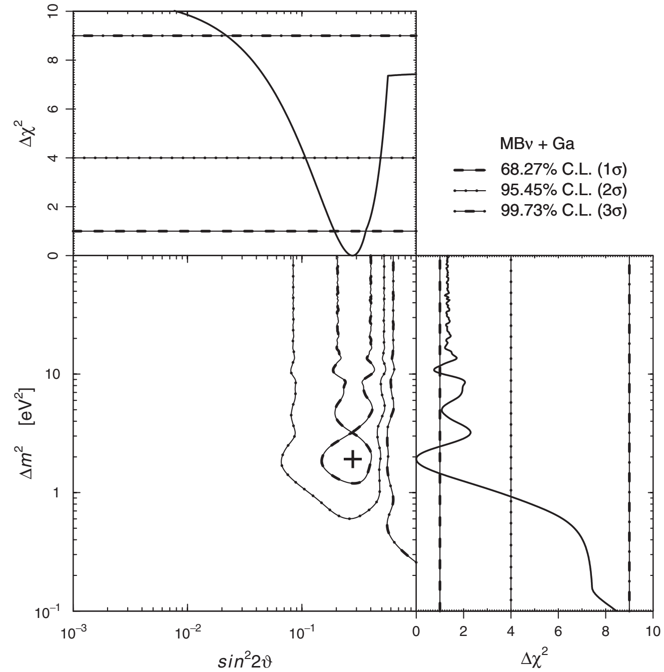

FIG. 4. Allowed regions in the $\sin^2 2\vartheta - \Delta m^2$ plane and marginal $\Delta\chi^2$'s for $\sin^2 2\vartheta$ and Δm^2 obtained from the combined fit of the results of MiniBooNE neutrino data and the data of the gallium radioactive source experiments. The best-fit point corresponding to $\chi^2_{\rm min}$ is indicated by a +.

of the parameter goodness of fit. Combining the two data sets improves the indication in favor of neutrino oscillations to the level of about 99.5% C.L. (2.8σ).

IV. REACTOR AND TRITIUM EXPERIMENTS

The indication of electron neutrino disappearance that we have found from the analysis of MiniBooNE and gallium data must be confronted with the results of reactor electron antineutrino experiments. Assuming *CPT* invariance, the survival probabilities of neutrinos and antineutrinos are equal (see Ref. [8]). Thus, we can combine directly the results presented in the previous section with the results of the analysis of the data of the Bugey and Chooz reactor experiments obtained in Ref. [14]. We are encouraged in this task by the coincidence of the best-fit value of Δm^2 at about 2 eV^2.

In addition to reactor neutrino experiments, also tritium β-decay experiments give information on the masses and mixing of neutrinos through the measurement of the electron energy spectrum in the process

$$^3\text{H} \to {}^3\text{He} + e^- + \bar{\nu}_e. \qquad (19)$$

The most accurate measurements of the effective electron neutrino mass (see Refs. [2,8])

$$m_\beta = \left(\sum_k |U_{ek}|^2 m_k^2\right)^{1/2} \qquad (20)$$

have been performed in the Mainz [46] and Troitsk [47]:

$$m_\beta^2 = -0.6 \pm 2.2 \pm 2.1 \text{ eV}^2 \quad \text{(Mainz)}, \qquad (21)$$

$$m_\beta^2 = -2.3 \pm 2.5 \pm 2.0 \text{ eV}^2 \quad \text{(Troitsk)}. \qquad (22)$$

These measurements can be interpreted and combined in order to derive upper bounds for the effective mass m_β through a χ^2 analysis in the physical region $m_\beta^2 \geq 0$. In Fig. 5 we plotted the corresponding $\Delta\chi^2$'s as a function of m_β. One can see that

$$m_\beta \leq 2.3 \text{ eV} \quad \text{(Mainz, 95\% C.L.)}, \qquad (23)$$

$$m_\beta \leq 2.0 \text{ eV} \quad \text{(Troitsk, 95\% C.L.)}, \qquad (24)$$

in approximate agreement with the corresponding values in Refs. [46,47]. The combined upper bound is

$$m_\beta \leq 1.8 \text{ eV} \quad \text{(Mainz + Troitsk, 95\% C.L.)}. \qquad (25)$$

In 3 + 1 four-neutrino schemes, taking into account that the mass splittings among m_1, m_2, and m_3 in Eqs. (2) and (40) are negligible for a measurement of m_β at the scale of 0.1–1 eV, the effective mass is given by

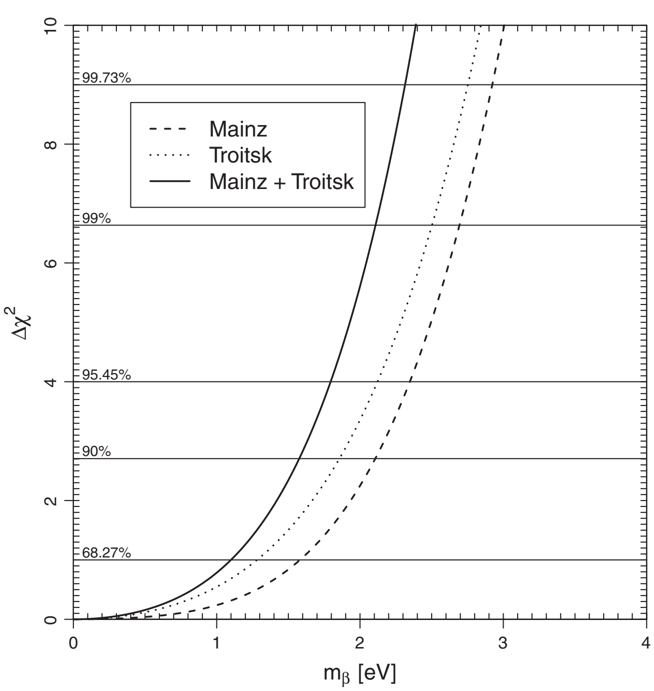

FIG. 5. $\Delta\chi^2$ as a function of m_β. The horizontal lines correspond to the indicated value of confidence level. The dashed and dotted lines have been obtained, respectively, from the results in Eqs. (21) and (22) of the Mainz and Troitsk tritium β-decay experiments. The solid line is the result of the combined fit.

$$m_\beta^2 \simeq (1 - |U_{e4}|^2) m_1^2 + |U_{e4}|^2 m_4^2 = m_1^2 + |U_{e4}|^2 \Delta m_{41}^2. \tag{26}$$

In 3 + 1 schemes of the type in Eq. (5), we have

$$m_\beta \geq |U_{e4}| \sqrt{\Delta m^2}. \tag{27}$$

The effective mixing angle in short-baseline electron neutrino disappearance experiments is related to $|U_{e4}|$ by Eq. (6). Inverting this relation and taking into account that the value of $|U_{e4}|$ must be small in order to fit the data of solar neutrino experiments with neutrino oscillations, we have

$$|U_{e4}|^2 = \tfrac{1}{2}(1 - \sqrt{1 - \sin^2 2\vartheta}). \tag{28}$$

In Fig. 6 we show the limits in the $\sin^2 2\vartheta - \Delta m^2$ plane obtained from Eqs. (27) and (28) and the results in Eqs. (21) and (22) of the Mainz and Troitsk tritium β-decay experiments. Notice that for small values of $\sin^2 2\vartheta$ the bounds are practically linear in the log-log plot in Fig. 6, because in this case $|U_{e4}|^2 \simeq \sin^2 2\vartheta/4$ and the inequality in Eq. (27) leads to

$$\log\Delta m^2 \lesssim 2\log 2 + 2\log m_\beta^{\rm ub} - \log\sin^2 2\vartheta, \tag{29}$$

where $m_\beta^{\rm ub}$ is the upper bound for m_β.

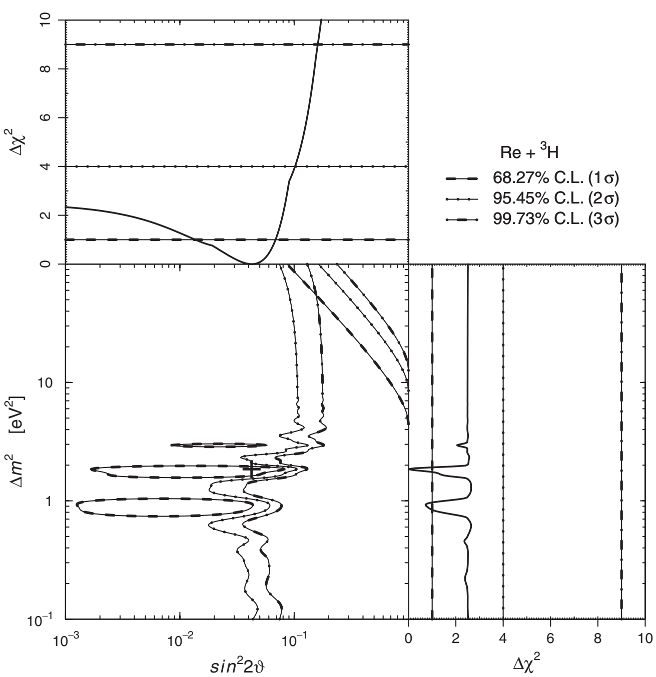

FIG. 6. Allowed regions in the $\sin^2 2\vartheta - \Delta m^2$ plane and marginal $\Delta\chi^2$'s for $\sin^2 2\vartheta$ and Δm^2 obtained from the combined fit of the results of the Bugey and Chooz reactor experiments and the results of the Mainz and Troitsk tritium β-decay experiments. The three lines in the upper-right corner are the exclusion curves obtained from the results of the Mainz and Troitsk tritium β-decay experiments alone. The best-fit point corresponding to $\chi^2_{\rm min}$ is indicated by a +.

In the fourth column of Table I and in Fig. 6 we report the results of the analysis of the data of the Bugey and Chooz reactor experiments presented in Ref. [14],[2] with the addition in the analysis of the results of the Mainz and Troitsk tritium β-decay experiments, which affect the high-Δm^2 region. As already commented on in Ref. [14], the reactor data are compatible with both the null hypothesis of the absence of electron antineutrino disappearance and our hypothesis of electron antineutrino disappearance, with a hint in favor of electron antineutrino oscillations due to a Δm^2 of about 2 eV.

V. COMBINED ANALYSIS

The results of the combined analysis of MiniBooNE, gallium, reactor, and tritium data are presented in the last column of Table I and in Fig. 7. One can see that the goodness of fit is high. The separate data sets are fitted fairly well by the electron neutrino disappearance hypothesis: the χ^2 contribution of the first three MiniBooNE low-energy ν_e bins is 4.1, that of the other 16 MiniBooNE ν_e and ν_μ energy bins is 7.5, that of the 4 gallium data is 6.3, that of the 56 reactor degrees of freedom is 49.0, and that of the 2 tritium degrees of freedom is 0.57. On the other hand, the 3% parameter goodness of fit of the combined analysis of neutrino MiniBooNE and gallium data and antineutrino reactor and tritium data is rather low.

This low compatibility of the neutrino and antineutrino data sets is illustrated in Fig. 8, where we have plotted the marginal $\Delta\chi^2$'s for $\sin^2 2\vartheta$ obtained with the analysis of different data sets.[3] One can see that

(1) The neutrino MiniBooNE and gallium data agree to indicate a value of $\sin^2 2\vartheta$ between about 0.11 and 0.48 at 2σ.
(2) The antineutrino reactor data indicate a value of $\sin^2 2\vartheta$ smaller than about 0.10 at 2σ. The tritium data are practically irrelevant for the determination of $\sin^2 2\vartheta$.
(3) The combined analysis is dominated by the reactor data and indicates a value of $\sin^2 2\vartheta$ between about 0.01 and 0.13 at 2σ.

The discrepancy between the neutrino and antineutrino determinations of $\sin^2 2\vartheta$ is about 2σ, in rough agreement with the above-mentioned 3% parameter goodness of fit of the combined analysis. In fact, the 2σ disagreement be-

[2]As an erratum, let us notice that in the fourth column of Table III in Ref. [14] there is a small mistake in the evaluation of the parameter goodness of fit. The correct values are $\Delta\chi^2_{\rm min} = 0.52$ and GoF $= 0.47$. We also notice that in the version of Ref. [14] published in Phys. Rev. D the value of $\sin^2 2\vartheta_{\rm bf}$ for the Ga + Bu + Ch analysis (last column of Table III in Ref. [14]) is different from the correct one, which is 0.054 (see the arXiv version of Ref. [14]).

[3]We thank the anonymous referee of Phys. Rev. D for suggesting this interesting figure.

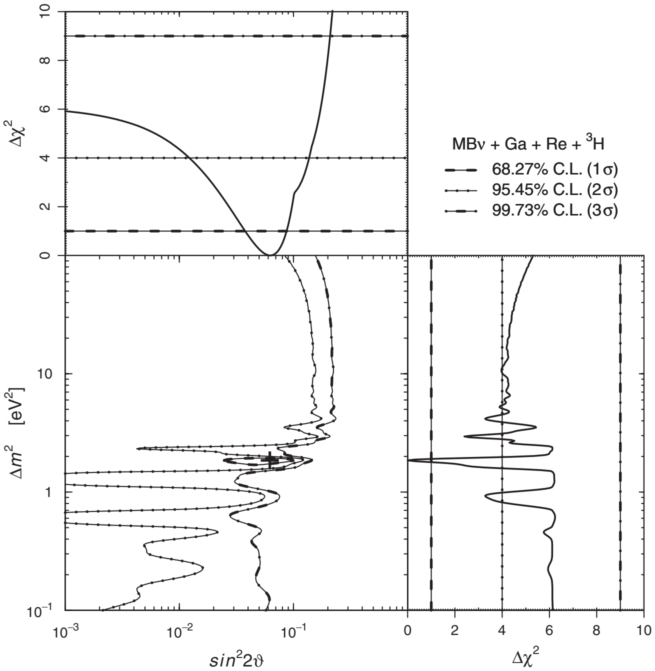

FIG. 7. Allowed regions in the $\sin^2 2\vartheta - \Delta m^2$ plane and marginal $\Delta\chi^2$'s for $\sin^2 2\vartheta$ and Δm^2 obtained from the combined fit of the results of MiniBooNE, gallium, reactor, and tritium experiments. The best-fit point corresponding to $\chi^2_{\rm min}$ is indicated by a +. The three lines in the upper-right corner give the 1σ, 2σ, and 3σ limits in the $\sin^2 2\vartheta - \Delta m^2$ plane obtained from Eqs. (20) and (28) and the results in Eqs. (21) and (22) of the Mainz and Troitsk tritium β-decay experiments.

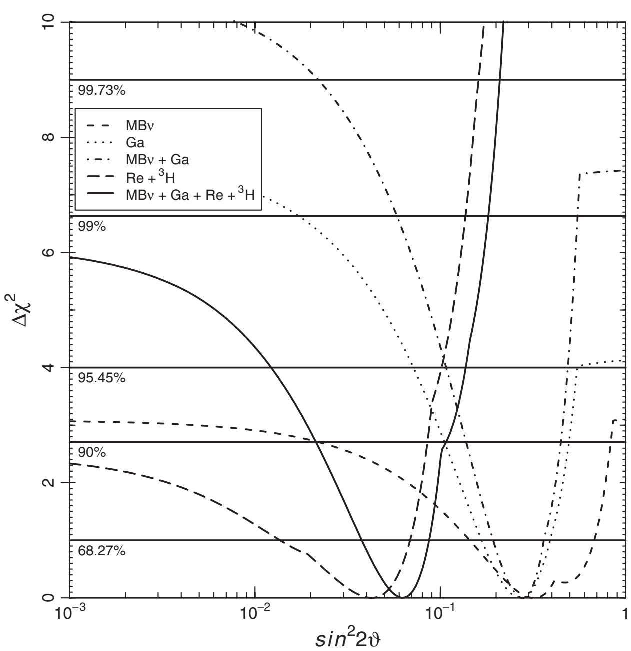

FIG. 8. Marginal $\Delta\chi^2 = \chi^2 - \chi^2_{\rm min}$ for $\sin^2 2\vartheta$ obtained from the analysis of different combinations of MiniBooNE, gallium, reactor, and tritium data. The marginal $\Delta\chi^2$ obtained from the analysis of reactor data alone and that obtained from the combined analysis of reactor and tritium data are shown by the same line, since they practically coincide.

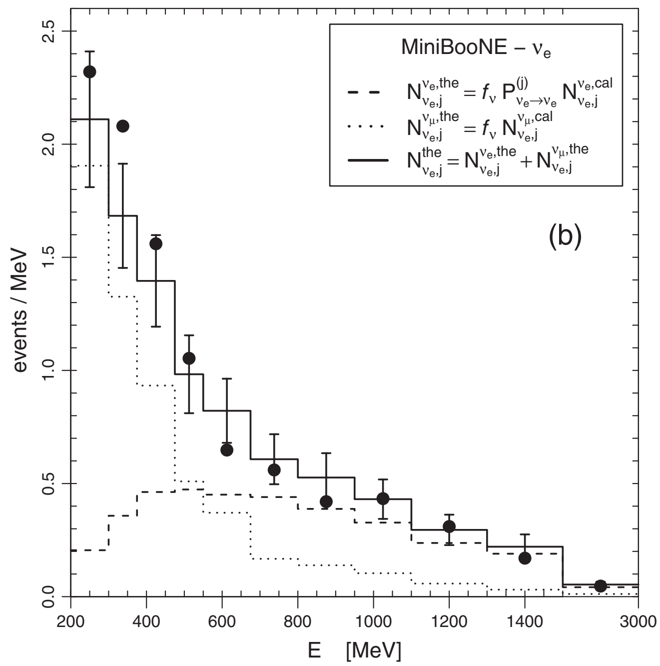

FIG. 9. Expected number of MiniBooNE ν_e events in the best-fit result of the combined analysis of MiniBooNE, gallium, reactor, and tritium data (last column in Table I). The notation is the same as in Fig. 1.

tween the neutrino and antineutrino data sets is entirely due to the different requirements on the value of $\sin^2 2\vartheta$, whereas they nicely agree on a best-fit value of Δm^2 at about 2 eV^2.

Since the 3% parameter goodness of fit of the combined analysis shows a tension between the neutrino and antineutrino data (under our ν_e-disappearance hypothesis) but is not sufficiently small to reject with confidence the compatibility of the neutrino and antineutrino data sets,[4] in the following part of this section and in Sec. VI we consider the results and implications of the combined analysis. In Sec. VII we consider a possible difference between the effective mixing angles in the neutrino and antineutrino sectors.

Although the combined analysis of neutrino and antineutrino data favors smaller values of $\sin^2 2\vartheta$ than those obtained from the analysis of MiniBooNE and gallium data alone, the fit of the MiniBooNE and gallium data remains better than in the case of no oscillations and $f_\nu = 1$.

Figure 9 shows the fit of MiniBooNE ν_e data corresponding to the best-fit result of the combined analysis. One can see that the fit of the first three low-energy bins is not as good as that in Fig. 1(b), but it is nevertheless acceptable and much better than that in Fig. 1(a).

[4]For example, the review on Statistics in the 2000 edition of the Review of Particle Physics [48] says that if the goodness of fit "is larger than an agreed-upon value (0.001, 0.01, or 0.05 are common choices), the data are consistent with the assumptions."

For the gallium source experiments, the best-fit values of the oscillation parameters give $R_{\rm Cr1}^{\rm GALLEX} = R_{\rm Cr2}^{\rm GALLEX} = 0.97$, $R_{\rm Cr}^{\rm SAGE} = 0.96$, and $R_{\rm Ar}^{\rm SAGE} = 0.96$. Therefore, the experimental values of $R_{\rm Cr1}^{\rm GALLEX}$ and $R_{\rm Cr}^{\rm SAGE}$ in Eqs. (14) and (16) are fitted very well and the gallium χ^2 contribution of 6.3 is almost equally due to the loose fits of $R_{\rm Cr2}^{\rm GALLEX}$ and $R_{\rm Ar}^{\rm SAGE}$ in Eqs. (15) and (17).

Considering the combined fit of the results of MiniBooNE, gallium, reactor, and tritium data as a fair indication in favor of a possible short-baseline electron neutrino disappearance generated by the effective mixing parameters $\Delta m^2 \simeq 2$ eV and $0.01 \lesssim \sin^2 2\vartheta \lesssim 0.13$, in the next section we present the corresponding predictions for the effective neutrino masses in β-decay and neutrinoless double-β-decay experiments which could be measured in future experiments.

VI. PREDICTIONS FOR BETA-DECAY AND NEUTRINOLESS DOUBLE-BETA-DECAY EXPERIMENTS

In this section we present predictions for the effective neutrino masses in β-decay and neutrinoless double-β-decay experiments obtained as a consequence of the combined fit of MiniBooNE, gallium, reactor, and tritium data discussed in the previous section.

Figure 10 shows the residual $\Delta\chi^2 = \chi^2 - \chi^2_{\rm min}$ as a function of the contribution $|U_{e4}|\sqrt{\Delta m^2}$ to the effective mass m_β in β-decay experiments [see Eq. (27)]. Since from the last column of Table I we have $\sin^2 2\vartheta_{\rm bf} \ll 1$, we obtain

$$|U_{e4}|^2_{\rm bf} \simeq \frac{\sin^2 2\vartheta_{\rm bf}}{4} = 0.016, \qquad (30)$$

and the best-fit value of $|U_{e4}|\sqrt{\Delta m^2}$ is

$$(|U_{e4}|\sqrt{\Delta m^2})_{\rm bf} = 0.17 \text{ eV}, \qquad (31)$$

and

$$0.06 \leq |U_{e4}|\sqrt{\Delta m^2} \leq 0.49 \text{ eV} \quad \text{at } 2\sigma. \qquad (32)$$

This prediction is relevant for the KATRIN experiment [49], which is under construction and scheduled to start in 2012. The expected sensitivity of about 0.2 eV at 90% C.L. may be sufficient to observe a positive effect if $|U_{e4}|\sqrt{\Delta m^2}$ is sufficiently large, as allowed by $\Delta\chi^2$ in Fig. 10.

If massive neutrinos are Majorana particles, neutrinoless double-β decay is possible, with a decay rate proportional to the effective Majorana mass (see Refs. [2,8,50–52])

$$m_{2\beta} = |\sum_k U_{ek}^2 m_k|. \qquad (33)$$

The results of the combined fit of MiniBooNE, gallium, reactor, and tritium data discussed in Sec. IV allow us to estimate the contribution of the heaviest massive neutrino ν_4 to $m_{2\beta}$, which is approximately given by $|U_{e4}|^2\sqrt{\Delta m^2}$, taking into account the mass hierarchy in Eq. (5).

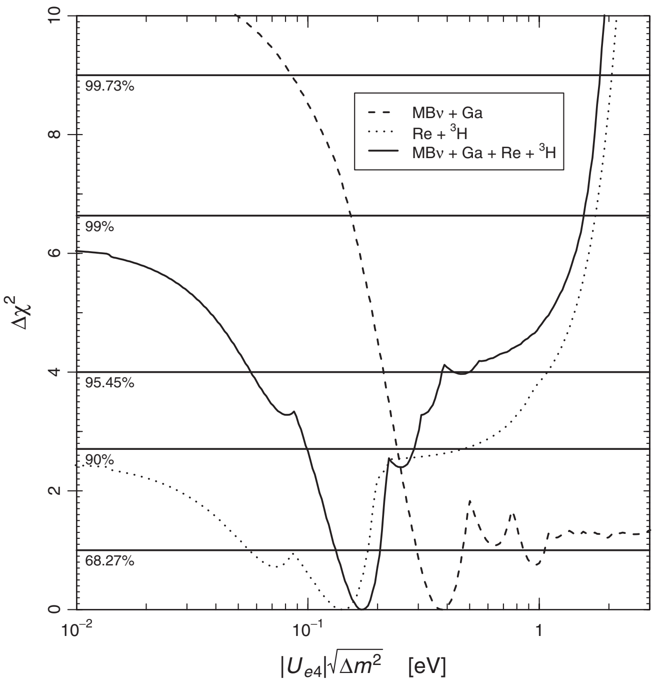

FIG. 10. $\Delta\chi^2 = \chi^2 - \chi^2_{\rm min}$ as a function of the contribution $|U_{e4}|\sqrt{\Delta m^2}$ to the effective β-decay electron neutrino mass m_β in four-neutrino schemes obtained from the analysis of MiniBooNE and gallium data (dashed line), from the analysis of reactor and tritium data (dotted line), and from the combined analysis of the two sets of data (solid line).

Figure 11 shows $\Delta\chi^2 = \chi^2 - \chi^2_{\rm min}$ as a function of the contribution $|U_{e4}|^2\sqrt{\Delta m^2}$ in four-neutrino schemes to $m_{2\beta}$. The best-fit value is

$$(|U_{e4}|^2\sqrt{\Delta m^2})_{\rm bf} = 0.02 \text{ eV}, \qquad (34)$$

and

$$0.003 \leq |U_{e4}|^2\sqrt{\Delta m^2} \leq 0.07 \text{ eV} \quad \text{at } 2\sigma. \qquad (35)$$

This range must be confronted with the expected contributions to $m_{2\beta}$ coming from the three light massive neutrinos ν_1, ν_2, and ν_3. Assuming a hierarchy of masses,

$$m_1 \ll m_2 \ll m_3 \ll m_4, \qquad (36)$$

which is the most natural case compatible with the hierarchy in Eq. (5), we have

$$m_{2\beta} \simeq |U_{e2}^2\sqrt{\Delta m^2_{\rm SOL}} + U_{e3}^2\sqrt{\Delta m^2_{\rm ATM}} + U_{e4}^2\sqrt{\Delta m^2}|, \qquad (37)$$

where we have neglected the contribution of the lightest massive neutrino ν_1. From the 3σ upper limits of the three-neutrino mixing parameters given in Ref. [4], we obtain

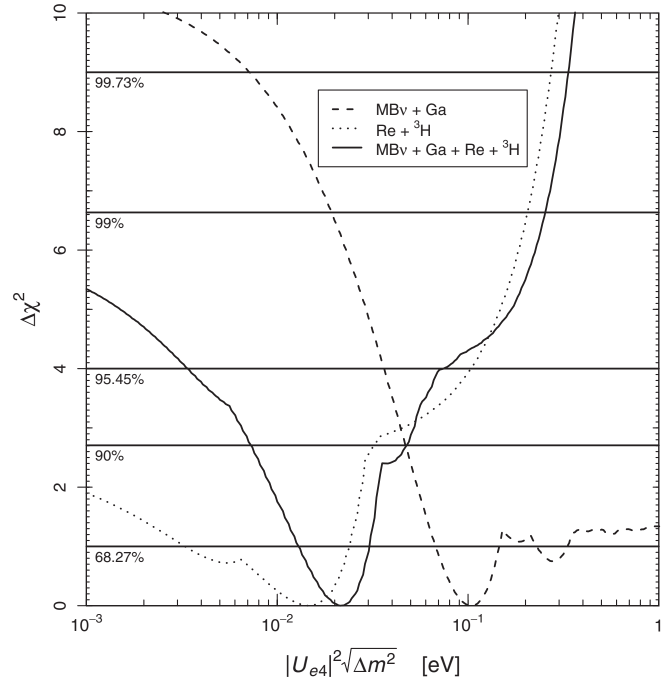

FIG. 11. $\Delta\chi^2 = \chi^2 - \chi^2_{\min}$ as a function of the contribution $|U_{e4}|^2\sqrt{\Delta m^2}$ to the effective neutrinoless double-β decay Majorana mass $m_{2\beta}$ in four-neutrino schemes obtained from the analysis of MiniBooNE and gallium data (dashed line), from the analysis of reactor and tritium data (dotted line), and from the combined analysis of the two sets of data (solid line).

$$2\times 10^{-3} \lesssim |U_{e2}|^2\sqrt{\Delta m^2_{\rm SOL}} \lesssim 4\times 10^{-3}\ \text{eV}, \qquad (38)$$

$$|U_{e3}|^2\sqrt{\Delta m^2_{\rm ATM}} \lesssim 3\times 10^{-3}\ \text{eV}. \qquad (39)$$

Therefore, strong cancellations between the contributions of ν_2 and ν_3 are possible (albeit not likely [53]), whereas the range in Eq. (35) disfavors strong cancellations between the contributions of ν_2 and ν_3 and the contribution of ν_4. In this case, $m_{2\beta} \simeq |U_{e4}|^2\sqrt{\Delta m^2}$ leading to a possible observation of neutrinoless double-β decay in future experiments which will be sensitive to values of $m_{2\beta}$ smaller than 10^{-1} eV (e.g. CUORE [54], EXO [55], SuperNEMO [56]; see the review in Ref. [52]).

On the other hand, if neutrinoless double-β decay experiments which are sensitive to values of $m_{2\beta}$ of the order of 10^{-1} eV (e.g. CUORICINO [57], GERDA [58], Majorana [59]; see the review in Ref. [52]) will see a positive signal, maybe compatible with the signal asserted in Ref. [60], the mass hierarchy in Eq. (36) will become unlikely and the favorite 3 + 1 four-neutrino schemes will be those in which the three light neutrinos ν_1, ν_2, and ν_3 are almost degenerate at the mass scale of $m_{2\beta}$.

VII. MIXING ANGLE ASYMMETRY?

The tension between neutrino and antineutrino data discussed in Sec. V could be due to a difference of the effective mixing angles in the neutrino and antineutrino sectors. Such a difference could be due to a violation of the fundamental *CPT* symmetry or to another unknown mechanism. Phenomenological analyses of different masses and mixings for neutrinos and antineutrinos have been presented in several publications [61–74].

In this section we consider the possibility that neutrinos and antineutrinos have different effective masses and mixings in short-baseline ν_e and $\bar\nu_e$ disappearance experiments. We fit the neutrino and antineutrino data with the survival probabilities

$$P^{\rm SBL}_{\nu_e\to\nu_e}(L,E) = 1 - \sin^2 2\vartheta_\nu \sin^2\left(\frac{\Delta m^2_\nu L}{4E}\right), \qquad (40)$$

$$P^{\rm SBL}_{\bar\nu_e\to\bar\nu_e}(L,E) = 1 - \sin^2 2\vartheta_{\bar\nu} \sin^2\left(\frac{\Delta m^2_{\bar\nu} L}{4E}\right). \qquad (41)$$

The results for the two fits are those presented in Fig. 4 and the third column in Table I for neutrinos (MB_ν + Ga) and Fig. 6 and the fourth column in Table I for antineutrinos (Re + ^{3}H).

Since the fit of the data does not require a difference of Δm^2_ν and $\Delta m^2_{\bar\nu}$, we consider only the mixing angle asymmetry

$$A_{\sin^2 2\vartheta} = \sin^2 2\vartheta_\nu - \sin^2 2\vartheta_{\bar\nu}. \qquad (42)$$

Figure 12 shows the marginal $\Delta\chi^2$ as a function of $A_{\sin^2 2\vartheta}$.

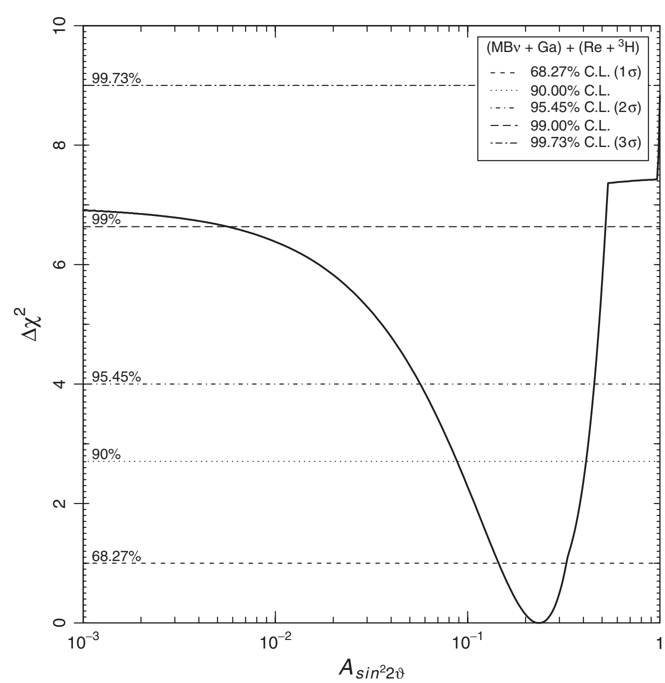

FIG. 12. Marginal $\Delta\chi^2 = \chi^2 - \chi^2_{\min}$ as a function of the mixing angle asymmetry $A_{\sin^2 2\vartheta}$.

The best-fit value of $A_{\sin^2 2\vartheta}$ is

$$A^{\mathrm{bf}}_{\sin^2 2\vartheta} = 0.23, \quad (43)$$

and the 2σ allowed range of $A_{\sin^2 2\vartheta}$ is

$$0.06 \leq A_{\sin^2 2\vartheta} \leq 0.45, \quad (44)$$

but there is no limit on the asymmetry at 3σ. The statistical significance of $A_{\sin^2 2\vartheta} > 0$ is 99.14% C.L. (2.6σ).

It is interesting to note that a difference between neutrino and antineutrino mixings can be tested in β-decay experiments by searching for different effective neutrino masses in β^- and β^+ decays. The prediction for the contribution $(|U_{e4}|\sqrt{\Delta m^2})_{\bar{\nu}_e}$ to the effective electron antineutrino mass in β^- decays from the analysis of antineutrino reactor and tritium data can be obtained from the dotted line in Fig. 10: the best fit is

$$(|U_{e4}|\sqrt{\Delta m^2})^{\mathrm{bf}}_{\bar{\nu}_e} = 0.14 \text{ eV}, \quad (45)$$

and

$$(|U_{e4}|\sqrt{\Delta m^2})_{\bar{\nu}_e} \leq 1.07 \text{ eV} \quad \text{at } 2\sigma. \quad (46)$$

For the contribution $(|U_{e4}|\sqrt{\Delta m^2})_{\nu_e}$ to the effective electron neutrino mass in β^+ decays we must consider the dashed line in Fig. 10, which has been obtained from the analysis of MiniBooNE and gallium neutrino data: the best fit is

$$(|U_{e4}|\sqrt{\Delta m^2})^{\mathrm{bf}}_{\nu_e} = 0.38 \text{ eV}, \quad (47)$$

and

$$(|U_{e4}|\sqrt{\Delta m^2})_{\nu_e} \geq 0.21 \text{ eV} \quad \text{at } 2\sigma. \quad (48)$$

Unfortunately the existing and foreseen experiments are β^--decay experiments for which the contribution $(|U_{e4}|\sqrt{\Delta m^2})_{\bar{\nu}_e}$ to the effective electron antineutrino mass is expected to be small. The future β^--decay experiment will use either tritium (KATRIN [49]) or ^{187}Re (MARE [75]). If the mixing difference between the neutrino and antineutrino sectors will be confirmed with highest confidence by future neutrino oscillation data it will be interesting to study the possibility of making β^+-decay experiments for the search of the effective electron antineutrino mass, for which the dashed line in Fig. 10 and Eq. (48) give a reachable lower limit.

We do not consider here neutrinoless double-β decay in the case of a neutrino-antineutrino mixing difference, since the Majorana nature of neutrinos requires a treatment which goes well beyond the purposes of this paper (see Ref. [76]).

VIII. CONCLUSIONS

In this paper we have discussed a neutrino oscillation interpretation of the MiniBooNE low-energy anomaly and the gallium radioactive source experiments anomaly in the framework of 3 + 1 four-neutrino mixing schemes. We have shown that the combined fit of MiniBooNE and gallium data indicates a possible short-baseline electron neutrino disappearance generated by effective oscillation parameters $\Delta m^2 \gtrsim 0.1 \text{ eV}^2$ and $0.11 \leq \sin^2 2\vartheta \leq 0.48$ at 2σ, with the best fit at $\Delta m^2 \simeq 2 \text{ eV}^2$ and $\sin^2 2\vartheta \simeq 0.3$ (see Fig. 4).

We have also considered the data of the Bugey and Chooz reactor neutrino oscillation experiments and the results of the Mainz and Troitsk tritium β-decay experiments, which imply an upper bound on the effective electron neutrino mass of about 2 eV [see Fig. 5 and the combined upper bound in Eq. (25)]. As already discussed in Ref. [14], the Bugey data give a faint indication of a possible short-baseline electron neutrino disappearance generated by effective oscillation parameters $\Delta m^2 \simeq 2 \text{ eV}^2$ and $\sin^2 2\vartheta \simeq 0.04$, which is compatible with Chooz and Tritium data (see Fig. 6).

In Sec. V we have discussed the tension between the neutrino MiniBooNE and gallium data and the antineutrino reactor and tritium data. Considering such tension as a statistical fluctuation, we have presented the results of the combined analysis of MiniBooNE, gallium, reactor, and tritium data: $\Delta m^2 \simeq 2 \text{ eV}^2$ and $0.01 \leq \sin^2 2\vartheta \leq 0.13$ at 2σ, with the best fit at $\Delta m^2 \simeq 2 \text{ eV}^2$ and $\sin^2 2\vartheta \simeq 0.06$ (see Fig. 7).

In Sec. VI, we have presented predictions for the effective neutrino masses in β-decay and neutrinoless double-β-decay experiments obtained as a consequence of the combined analysis of MiniBooNE, gallium, reactor, and tritium data, assuming the hierarchy of masses in Eq. (5). The predicted interval for the contribution of m_4 to the effective neutrino mass in β decay is between about 0.06 and 0.49 eV at 2σ. The upper part of this interval may be reached by the KATRIN experiment [49]. For neutrinoless double-β decay we obtained a prediction for the contribution of m_4 to the effective neutrino mass between about 0.003 and 0.07 eV at 2σ, which may be reached in future experiments (see Ref. [52]).

We also considered, in Sec. VII, the possibility of reconciling the tension between the neutrino MiniBooNE and gallium data and the antineutrino reactor and tritium data discussed in Sec. V with different mixings in the neutrino and antineutrino sectors. We found a 2.6σ indication of a mixing angle asymmetry (99.14% C.L.). We pointed out the possibility of checking the mixing difference between the neutrino and antineutrino sectors by measuring different effective electron antineutrino and neutrino masses in β^-- and β^+-decay experiments.

The indication in favor of short-baseline disappearance of electron neutrinos implies the possible existence of a light sterile neutrino which could have important consequences in physics [18–20,22,77–83], astrophysics [84–92], and cosmology [93–98].

As far as the effective number of neutrino species in cosmology, N_{eff}, is concerned, the analysis of 7-years WMAP data has provided the following result: $N_{\mathrm{eff}} = 4.34^{+0.86}_{-0.88}$ (68% C.L.) [99]. In 2011 the Planck experiment will measure N_{eff} with a factor of 4 improvement in accuracy with respect to present data [100,101]. In other words, the possibility of the existence of a fourth light sterile neutrino could be pursued with 5σ significance.

Finally, we would like to encourage all experiments which can investigate the hypothesis of short-baseline electron neutrino disappearance.

Starting from 2010, at the same L/E of MiniBoone, the magnetic off-axis near detector at 280 m of the T2K experiment [102] will count ν_e events with expected higher statistics and similar ν_μ background contamination. A test of short-baseline oscillations may be done, although the accuracy suffers from the scarce knowledge of the neutrino flux and of the neutrino cross section at 1 GeV energies [15,103].

A better measurement will be possible with the new CERN-PS neutrino beam [104,105], thanks to the presence of 2 detectors at 140 m (NEAR) and 885 m (FAR). At $\Delta m^2 \approx 2\ \mathrm{eV}^2$, the oscillation length is about 1 km for 1 GeV neutrino energies. Therefore, one can reduce the systematic error of the Monte Carlo predictions by normalizing the high-energy part of the ν_e spectrum at the NEAR location. In addition, a better ν_μ background rejection will be possible using the liquid argon technology. The interesting possibility of a ν_e tagging in the CERN-PS beam was also studied in this context [106].

New measurements with a radioactive source could be made in the SAGE experiment [10], with the Borexino detector and with the future LENS detector [79]. At $\Delta m^2 \approx 2\ \mathrm{eV}^2$, the oscillation length is about 1 m for 1 MeV neutrino energies. Therefore Borexino could measure the oscillation pattern over a distance of 4 m (the Borexino radius) using the well-known ν_e-e scattering process and with a vertex resolution that at the moment is about 15 cm [107].

At the Gran Sasso laboratories a very interesting measurement could be realized by using the ICARUS 600 ton detector and new low-cost and high-power proton cyclotrons under development for commercial use [108]. These provide electron neutrino beams with energy up to 52 MeV from muon decay at rest. A low-energy ν_e disappearance experiment (as well as $\overset{(-)}{\nu}_\mu$ disappearance and $\overset{(-)}{\nu}_\mu \overset{(-)}{\nu}_e$ measurements) can be performed with such devices due to the full efficiency of the ICARUS detector at 20 MeV energies. The expected event rate is about 400 charged-current electron neutrino events per year per ton with the ICARUS detector located at 50 m from the source [109]. The number of events is calculated assuming $10^{15}\,\nu_e$'s per year and a fully efficient detector. Since at $\Delta m^2 \approx 2\ \mathrm{eV}^2$ the oscillation length is about 20 m for a 20 MeV neutrino energy, it is possible to measure the full oscillation pattern along the beam direction inside the ICARUS volume.

The disappearance of electron neutrinos can be investigated with high accuracy in future near-detector beta-beam [110] and neutrino factory [73,111] experiments in which the neutrino fluxes will be known with high precision.

Furthermore, the MiniBooNE low-energy anomaly may be clarified by the ArgoNeuT, MicroBooNE [112], and BooNE experiments [113], and the magnetic off-axis near detector at 280 m of the T2K experiment has the unique opportunity to measure the charge of the events of the low-energy anomaly [114].

ACKNOWLEDGMENTS

We would like to thank E. Bellotti, C. Giganti, A. Longhin, F. Pietropaolo, and A. Rubbia for interesting discussions and suggestions.

Note added.—After the completion of this work, two important experimental results were presented at the Neutrino 2010 Conference:

(1) The MiniBooNE Collaboration presented updated results on the search for short-baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations which are compatible with the LSND signal [115,116]. The inclusion of these data in our framework will require a separate analysis in which the assumption of negligible $|U_{\mu 4}|^2$ is relaxed [117].

(2) The MINOS Collaboration presented an indication of a possible difference between the effective mixings of neutrinos and antineutrinos in long-baseline ν_μ and $\bar{\nu}_\mu$ disappearance [118]. This indication is analogous to that discussed in Sec. VII.

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