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QCD phase transition in a strong magnetic background

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We investigate the properties of the deconfining/chiral restoring transition for two flavor QCD in the presence of a uniform background magnetic field. We adopt standard staggered fermions and a lattice spacing of the order of 0.3 fm. We explore different values of the bare quark mass, corresponding to pion masses in the range 200–480 MeV, and magnetic fields up to $|e|B \sim 0.75\ \text{GeV}^2$. The deconfinement and chiral symmetry restoration temperatures remain compatible with each other and rise very slightly ($< 2\%$ for our largest magnetic field) as a function of the magnetic field. On the other hand, the transition seems to become sharper as the magnetic field increases.

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I. INTRODUCTION

The study of strong interactions in presence of magnetic background fields is relevant to many phenomenological contexts. Large magnetic fields ($B \sim 10^{16}$ Tesla, i.e. $\sqrt{|e|B} \sim 1.5$ GeV) may have been produced at the cosmological electroweak phase transition [1] and they may have influenced subsequent QCD transitions. Slightly lower fields are expected to be produced in noncentral heavy ion collisions, reaching up to 10^{14} Tesla at the Relativistic Heavy Ion Collider and up to $\sim 10^{15}$ Tesla at the LHC [2,3]. Large magnetic fields, of the order of 10^{10} Tesla, are also expected in some neutron stars known as magnetars [4].

The influence of electric and magnetic fields on the chiral properties of the vacuum has been studied for some time, using various approximations or effective models of QCD [5–12], predicting an enhancement of chiral symmetry breaking as a magnetic field is switched on. Recently, new interesting phenomenology has been proposed, consisting in the appearance of an electric current parallel to the magnetic field in presence of deconfined quarks and local *CP* violations, induced, e.g. by topological charge fluctuations [13,14]. That is usually known as the chiral magnetic effect and experimental confirmations of it are currently being searched by heavy ion experiments [15].

An important issue is the influence of the magnetic field on the structure of the QCD phase diagram, in particular, on the location and the nature of deconfinement and chiral symmetry restoration. Clarifying that in the case of strong magnetic fields is essential to correctly predict the phenomenological consequences of the QCD transition on the evolution of the Universe during its early stages. Some computations exist, based on different approximations and QCD-like models [16–22], which predict the possibility of a quite rich phenomenology, ranging from a possible splitting of deconfinement and chiral symmetry restoration to a sizable increase in the strength of the transition. However, the various model predictions are not always consistent among themselves.

A clarification of these issues may come from lattice QCD computations. A magnetic background field, contrary to an electric field or a finite baryon density, does not give rise to technical difficulties such as a sign problem. The phase diagram in presence of a chromo-magnetic background field has been investigated in Refs. [23,24]: the transition temperature decreases as a function of the external field, with deconfinement and chiral symmetry restoration staying strictly related to each other. Investigations in presence of (electro-) magnetic (e.m.) fields have been done since long with the purpose of studying the magnetic properties of hadrons [25,26], while some recent studies [27–30] have reported mostly on the chiral properties of the theory and about numerical evidence for the chiral magnetic effect.

In this paper we report on a first investigation of the QCD phase transition in presence of an (electro-) magnetic background field. In order to do that, it is essential to include dynamical quark contributions, since only quark fields, being electrically charged, are influenced by the magnetic field. We have considered $N_f = 2$ QCD with standard staggered fermions and different values of the quark masses, to appreciate how the effects of the magnetic field change as the mass spectrum changes (in the heavy quark limit the magnetic field becomes irrelevant). In Sec. II, we give some details about lattice QCD in the presence of a background field and about our numerical setup. In Sec. III, we present our numerical results and finally, in Sec. IV, we give our conclusions.

II. NUMERICAL SETUP

We consider $N_f = 2$ QCD, with quarks carrying different electric charges and coupled to a background e.m. field. The background field affects quark propagation and corresponds to a modification of the Dirac operator. In the continuum the covariant derivative changes by inclusion of the e.m. A_μ field; on the lattice one has to add appropriate $U(1)$ fields to the gauge link variables which parallel transport quarks fields from one lattice site to the other. In the case of a uniform magnetic field B, with different electric charges for the two flavors, $q_u = 2|e|/3$ and $q_d = -|e|/3$ ($|e|$ being the elementary charge), the partition function of the (rooted) staggered fermion discretized version of the theory is

$$Z(T,B) \equiv \int \mathcal{D}U e^{-S_G} \det M^{1/4}[B,q_u] \det M^{1/4}[B,q_d], \quad (1)$$

$$M_{i,j}[B,q] = am\delta_{i,j} + \frac{1}{2}\sum_{\nu=1}^{4} \eta_{i,\nu}(u(B,q)_{i,\nu}U_{i,\nu}\delta_{i,j-\hat{\nu}} - u^*(B,q)_{i-\hat{\nu},\nu}U^\dagger_{i-\hat{\nu},\nu}\delta_{i,j+\hat{\nu}}). \quad (2)$$

$\mathcal{D}U$ is the functional integration over the gauge link variables $U_{n,\mu}$, S_G is the discretized pure gauge action (we consider a standard Wilson action). The subscripts i and j refer to lattice sites, $\hat{\nu}$ is a unit vector on the lattice and $\eta_{i,\nu}$ are the staggered phases. Periodic (antiperiodic) boundary conditions (b.c.) must be taken for gauge (fermion) fields along the Euclidean time direction, while spatial periodic b.c. are chosen for all fields. $u(B,q)_{i,\nu}$ are the gauge links corresponding to the background $U(1)$ magnetic field. We shall consider a constant magnetic field $\vec{B} = B\hat{z}$ and the following choice for the gauge field:

$$A_y = Bx; \qquad A_\mu = 0 \quad \text{for } \mu = x, z, t. \quad (3)$$

The corresponding $U(1)$ links on the lattice are

$$u(B,q)_{n,y} = e^{ia^2qBn_x}; \qquad u(B,q)_{n,\mu} = 1 \quad \text{for } \mu = x, z, t. \quad (4)$$

This choice corresponds to a magnetic flux a^2B going through each plaquette in the $x-y$ plane, except at the boundary (L_x, y, z, t), due to the periodic b.c. in the spatial directions. In order to guarantee the smoothness of the background field across the boundary and the gauge invariance of the fermion action the $U(1)$ gauge fields must be modified at the boundary of the x direction:

$$u(B,q)_{n,x=L_x} = e^{-ia^2qL_xBn_y} \quad (5)$$

and the magnetic field must be quantized, $a^2qB = 2\pi b/L_xL_y$, where b is an integer. That corresponds to taking the appropriate gauge invariant b.c. for fermion fields on the torus [31] (with the possible additional free phases θ_x and θ_y [31] set to zero). Given the two different values of q_u and q_d, the quantization of B in our case is set by the d quark charge $q_d = -|e|/3$,

$$|e|B = 6\pi T^2\left(\frac{N_t}{L_s}\right)^2 b, \quad (6)$$

$T = 1/(N_ta)$ is the temperature and $L_x = L_y \equiv L_s$.

Our simulations have been carried out on $16^3 \times 4$ lattices and for three different bare quark masses $am = 0.01335$, 0.025 and 0.075. The corresponding (Goldstone) pion masses are $am_\pi = 0.307(3)$, 0.417(3), and 0.707(3). The temperature $T = 1/(N_ta)$ is changed by varying the lattice spacing through the inverse gauge coupling β.

Zero T estimates of the string tension, done at the same β values where the $B = 0$ transition takes place, lead to a ranging from 0.29 to 0.31 fm as am is decreased, corresponding to $T_c(B=0)$ ranging from 170 to 160 MeV. The corresponding values of the (Goldstone) pion mass are $m_\pi \approx 195$, 275 and 480 MeV. For each quark mass we have done simulations using magnetic field corresponding to $b = 0$, 8, 16 and 24, i.e. for $|e|B = 0$, $3\pi T^2$, $6\pi T^2$ and $9\pi T^2$, respectively. Thus, for the lightest pion mass our magnetic field reaches values up to $|e|B \approx 19m_\pi^2$, i.e. $\sqrt{|e|B} \approx 850$ MeV in physical units. Note that, since we are working with a fixed value of N_t, B changes while changing temperature as $a^{-2} \propto T^2$. However, for all the quark masses the range of couplings (hence of a) that we explore corresponds to a <2% change in T and hence the magnetic field only changes at most by a few percent.

The rational hybrid Monte Carlo algorithm has been used to simulate rooted staggered fermions: we need to treat separately each flavor and thus take the fourth root of the fermion determinant. Typical statistics are of the order of 10 k molecular dynamics trajectories.

III. NUMERICAL RESULTS

In Figs. 1–3 we show the behavior of $\langle\bar{\psi}\psi\rangle$ (average of the u and d quark condensates) and of the Polyakov loop for different magnetic fields and $am = 0.075$, 0.025, and 0.01335. Results are presented as a function of the inverse gauge coupling β: we recall that T is an increasing function of β, a conversion into physical units will be presented later. For all temperatures $\langle\bar{\psi}\psi\rangle$ increases as a function of B, as expected from analytic predictions. Interpreting the drop of the condensate as the signal for chiral symmetry restoration, we infer that the transition temperature increases as a function of B. A sharper drop is observed at

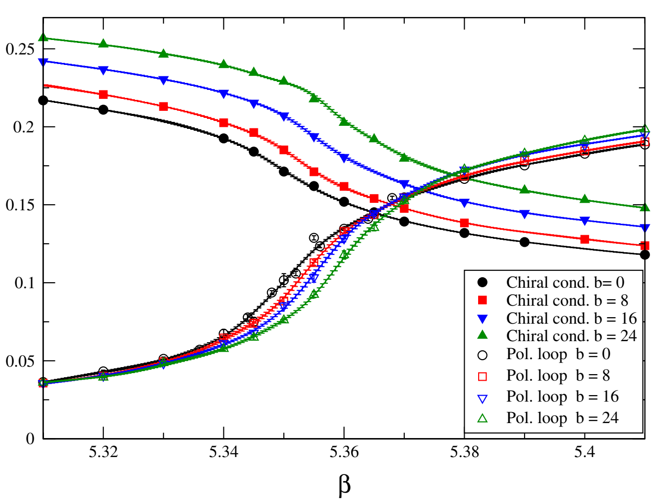

FIG. 1 (color online). Chiral condensate and Polyakov loop for $am = 0.075$.

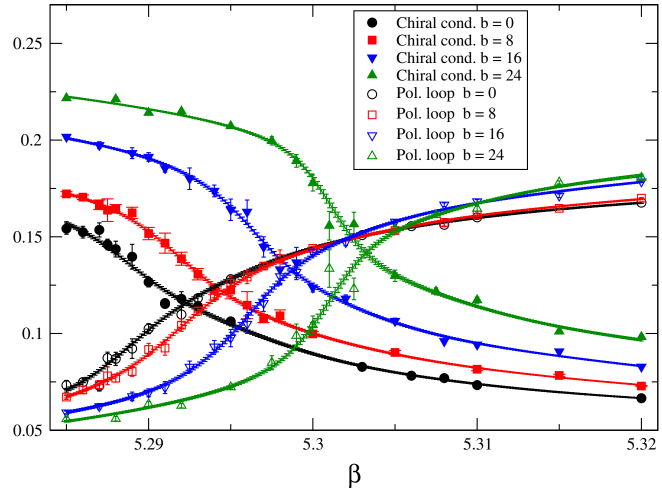

FIG. 2 (color online). Same as in Fig. 1 for $am = 0.025$.

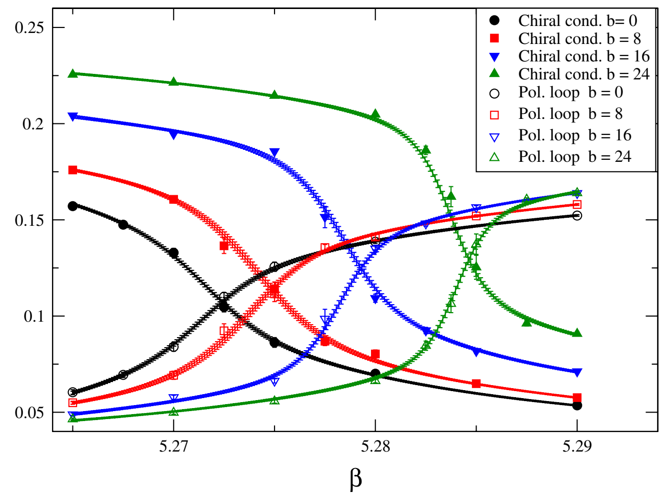

FIG. 3 (color online). Same as in Fig. 1 for $am = 0.01335$.

the highest fields explored, especially for the lowest quark masses, indicating a sizable increase in the strength of the transition; that is visible from the behavior of the disconnected chiral susceptibility (Fig. 4).

The Polyakov loop P is a pure gauge quantity, coupled to the magnetic field only through quark loops, hence its behavior is less trivial to predict. From Figs. 1–3 we see that, while at low T it decreases as a function of B (as one would expect qualitatively from the fact that $\langle \bar{\psi} \psi \rangle$ increases), at high T it increases. Such behavior is qualitatively similar to what obtained in Ref. [20] by a Polyakov-Nambu-Jona-Lasinio model analysis and should be further investigated, e.g. by determining the renormalized Polyakov loop. If we interpret the rise of P as the onset of deconfinement, we infer that the shift and the increase in strength of the deconfinement transition is similar to what observed for the chiral transition. Data obtained for the Polyakov loop susceptibility lead to similar conclusions (see Fig. 5).

In Table I we report the pseudocritical couplings β_c for deconfinement and chiral restoration obtained by fitting

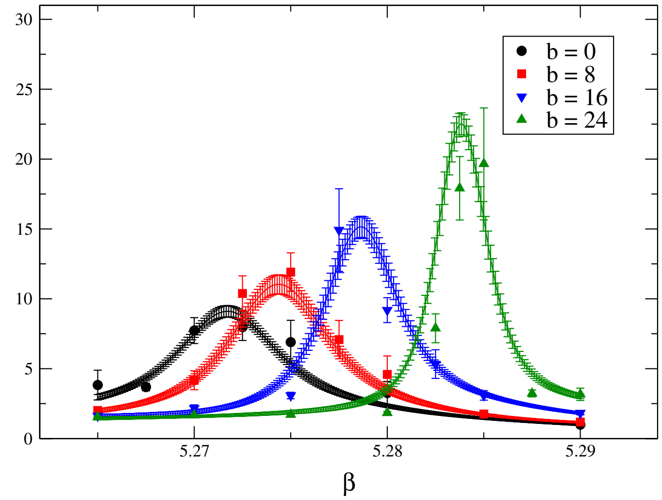

FIG. 4 (color online). Disconnected $\langle \bar{\psi} \psi \rangle$ susceptibility for $am = 0.01335$.

the peak of the susceptibilities by a quadratic function. We have also determined β_c looking for the inflection point of observables, by means of polynomial fits, obtaining compatible results within errors. Data obtained for β_c confirm that no appreciable separation of chiral restoration and deconfinement is induced by the background field, at least for the explored field strengths.

From the values of β_c we obtain the ratio $T_c(B)/T_c(0)$ as a function of the dimensionless ratio eB/T^2, as reported in Fig. 6; the 2-loop β function has been used for the conversion. A direct determination of the physical scale on $T = 0$ lattices is preferable but would require very precise measurements to appreciate T variations of the order of percent. On the other hand, given the small scale range explored, the approximation is reasonable and no qualitative change is expected.

Figure 6 shows that the change in T_c is small and of the order of a few percent at the highest explored fields.

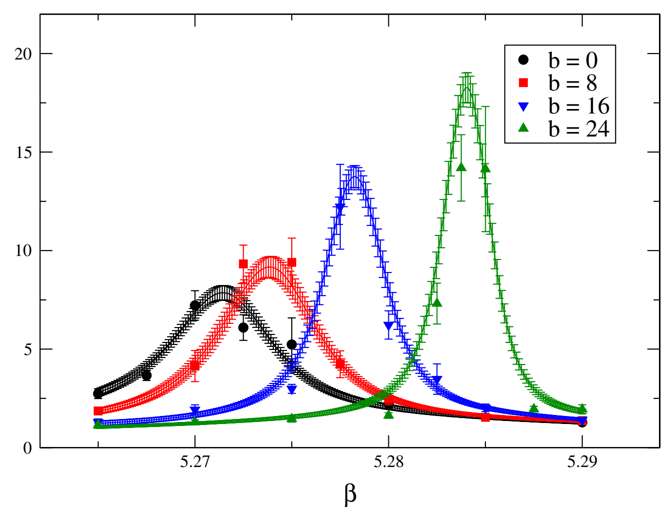

FIG. 5 (color online). Polyakov loop susceptibility for $am = 0.013\,35$.

TABLE I. Pseudocritical couplings obtained by fitting the peak of the chiral condensate or Polyakov loop susceptibilities.

am_q	b	β_c (Pol. loop)	β_c $(\bar{\psi}\psi)$
0.013 35	0	5.2714(4)	5.2716(3)
0.013 35	8	5.2739(4)	5.2741(4)
0.013 35	16	5.2783(3)	5.2785(3)
0.013 35	24	5.2836(2)	5.2838(2)
0.025	0	5.2893(2)	5.2898(3)
0.025	8	5.2925(3)	5.2925(3)
0.025	16	5.2961(3)	5.2966(3)
0.025	24	5.3014(4)	5.3018(4)
0.075	0	5.351(1)	5.351(2)
0.075	8	5.353(1)	5.353(2)
0.075	16	5.355(1)	5.357(2)
0.075	24	5.358(1)	5.360(1)

Moreover, there seems to be a saturation as the chiral limit is approached: results for $am = 0.013\,35$ and $am = 0.025$ stay onto each other. Notice that this is true if we plot results as a function of $|e|B/T^2$: had we used $|e|B/m_\pi^2$ results at different masses would have been different: the highest B is about $20m_\pi^2$ for $am = 0.013\,35$ and $10m_\pi^2$ for $am = 0.025$. This suggests that, at least for the strong fields and for the pion masses explored, the relevant scale governing the effect of the magnetic field on the shift of the transition is T itself and not m_π.

Trying to understand the dependence of T_c on B, we have fitted our data for $am = 0.013\,35$ according to

$$\frac{T_c(B)}{T_c(0)} = 1 + A\left(\frac{|e|B}{T^2}\right)^\alpha, \qquad (7)$$

finding that $\alpha = 1.45(20)$ and $A \sim 1.3 \times 10^{-4}$.

Finally we discuss about the nature of the transition. At $B = 0$ it is still unclear if a weak first order transition is present in the chiral limit [32,33], however no clear signal for a finite latent heat has been found on available lattice sizes, hence the first order transition, even if present, is so weak to be of poor phenomenological relevance. On the other hand, our results show that the introduction of a

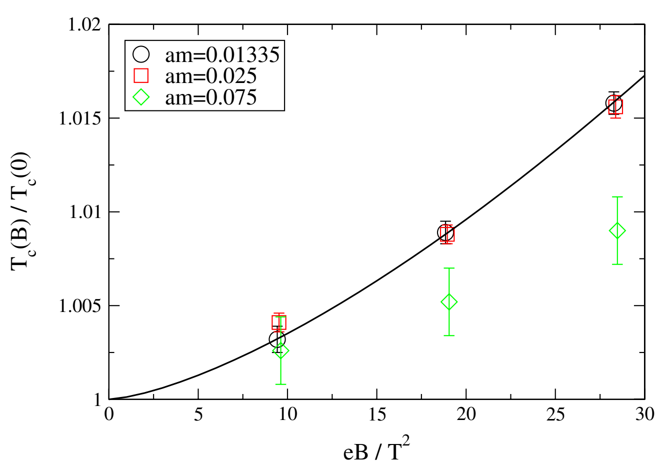

FIG. 6 (color online). $T_c(B)$ for different quark masses. The solid curve is a power law fit to the lightest quark data (see text).

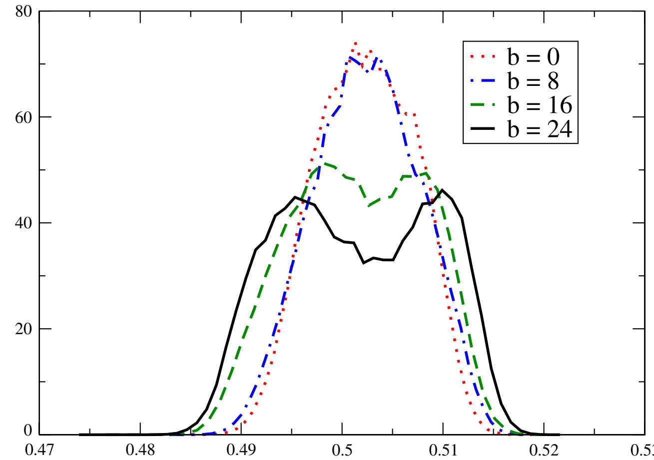

FIG. 7 (color online). Reweighted plaquette distribution at β_c as a function of the external field at $am = 0.013\,35$ on a $16^3 \times 4$ lattice.

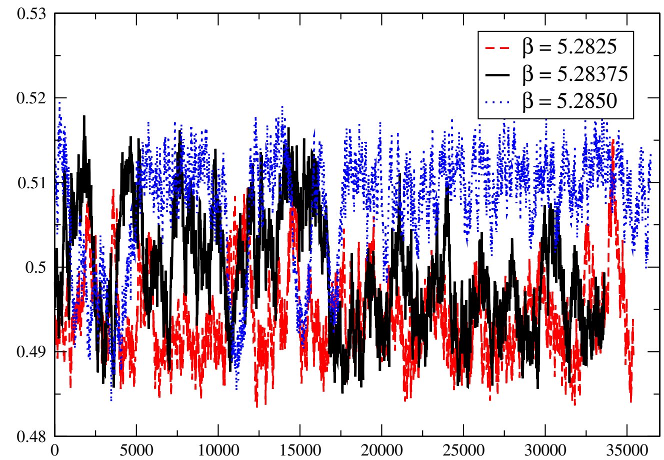

FIG. 8 (color online). Monte Carlo histories of the plaquette at 3 different β values around the transition for $b = 24$ and $am = 0.01335$.

magnetic field makes the transition sharper. The question is if large fields can turn the transition into a first order strong enough to be clearly detectable.

To that aim we have analyzed the reweighted plaquette distribution at the critical couplings and for different values of B: results are shown in Fig. 7. The single peak distribution, which is present at zero or small magnetic field, turns into a double peak distribution, typical of a first order transition, for the largest B explored; also the Monte Carlo histories of the plaquette, Fig. 8, present signals of a metastable behavior. We can consider that as an indication but not as a final answer: numerical simulations on larger lattice sizes are necessary to clarify if the double peak structure survives the thermodynamical limit and for a proper finite size scaling analysis.

IV. CONCLUSIONS

We have presented results from an investigation of the $N_f = 2$ QCD phase diagram in presence of a magnetic background field. We have explored different quark masses, corresponding to m_π ranging from 200 MeV to 480 MeV, and different magnetic fields, with $\sqrt{|e|B}$ up to about 850 MeV ($|e|B \sim 20m_\pi^2$ for the lightest mass).

Main results can be summarized as follows: the transition temperature increases slightly ($< 2\%$ at the highest field) and no evidence is found, within the range of explored fields, for a disentanglement of chiral symmetry restoration and deconfinement. $T_c(B)/T_c(0)$ as a function of $|e|B/T^2$ shows negligible dependence on m_π for the two lowest masses, and is well described by a power law $T_c(B)/T_c(0) = 1 + A(|e|B/T^2)^\alpha$ with $\alpha \sim 1.45(20)$. The transition becomes sharper with some preliminary evidence for a first order transition, in the form of double peak distributions, at the highest fields explored: such indications should be clarified by future studies on larger spatial volumes and by a finite size scaling analysis.

Regarding the comparison with model predictions, our results show partial agreement with some of the results reported in Ref. [21] and in Ref. [20]: the deconfinement and chiral restoring temperatures both increase, even if we do not see any sign for a faster grow and splitting of the chiral transition till $|e|B \sim 20m_\pi^2$. Also, the observed increase in the strength of the transition is common to some models [18,21]. We stress the qualitatively different behavior which is observed in numerical simulations with a background chromo-magnetic field, where T_c decreases as a function of the external field [23,24].

Our results have been obtained using a standard staggered discretization and a coarse lattice, with a lattice spacing $a \sim 0.3$ fm. Apart from possible systematic effects related to the fourth root trick, flavour breaking discretization effects may play an important role, with a distorted hadron spectrum that could partially modify the effect of the magnetic field on QCD thermodynamics. For this reason it will be important to confirm our results in the future by using different lattice discretizations.

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