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Gravity waves and linear inflation from axion monodromy

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Wrapped branes in string compactifications introduce a monodromy that extends the field range of individual closed-string axions to beyond the Planck scale. Furthermore, approximate shift symmetries of the system naturally control corrections to the axion potential. This suggests a general mechanism for chaotic inflation driven by monodromy-extended closed-string axions. We systematically analyze this possibility and show that the mechanism is compatible with moduli stabilization and can be realized in many types of compactifications, including warped Calabi-Yau manifolds and more general Ricci-curved spaces. In this broad class of models, the potential is linear in the canonical inflaton field, predicting a tensor to scalar ratio $r \approx 0.07$ accessible to upcoming cosmic microwave background observations.

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I. INTRODUCTION: AXION RECYCLING

An important class of inflationary models [1], chaotic inflation [2], involves an inflaton field excursion that is large compared to the Planck scale M_P [3]. These models have a GUT-scale inflaton potential, and are accessible to observational tests via a B-mode polarization signature in the cosmic microwave background (CMB) [4,5].

The Planckian or super-Planckian field excursions required for high-scale inflation may be formally protected by an approximate shift symmetry in effective field theory. A canonical class of examples with a field excursion $\Delta\Phi \simeq M_P$, known as natural inflation, employs a pseudo-Nambu-Goldstone boson mode (an axion) as the inflaton [6,7].

Because inflation is sensitive to Planck-suppressed operators, however, it is still of significant interest to go beyond effective field theory and realize inflation in string theory, a candidate ultraviolet completion of gravity. Conversely, CMB observations which discriminate among different inflationary mechanisms provide an opportunity to probe some basic features of the ultraviolet completion of gravity.

The lightest scalar fields in string compactifications roughly divide into radial and angular moduli. Radial moduli, such as the dilaton and the compactification volume, have an unbounded field range as they go toward weak-coupling limits. In these limits their contributions to the potential are typically very steep, not sourcing large-field inflation in any example yet studied. Angular moduli, such as axions, have potentials that are classically protected by shift symmetries. However, in the case of axions it has been argued that the field range contained within a single period is generally sub-Planckian in string theory [8], leading to proposals to extend the field range by combining many axions [9].

In the present work, we show that in the presence of suitable wrapped branes, the potential energy is no longer a periodic function of the axion. When this *monodromy* in the moduli space is taken into account, a single axion develops a kinematically unbounded field range with a potential energy growing *linearly* with the canonically normalized inflaton field. This implements the monodromy mechanism introduced in [10] in a wide class of string compactifications.

Because the basic idea is very simple, let us indicate it here. Axions arise in string compactifications from integrating gauge potentials over nontrivial cycles. For example, in type IIB string theory, there are axions $b_I = \int_{\Sigma_I^{(2)}} B$ arising from integrating the Neveu-Schwarz (NS) two-form potential B_{MN} over two-cycles $\Sigma_I^{(2)}$, and similarly axions $c_I = \int_{\Sigma_I^{(2)}} C$ arise from the Ramond-Ramond (RR) two-form C_{MN}. In the absence of additional ingredients such as fluxes and space-filling wrapped branes, the potential for these axions is classically flat, and develops a periodic contribution from instanton effects. A Dp-brane wrapping $\Sigma_I^{(2)}$, on the other hand, carries a potential energy that is *not* a periodic function of the axion: in fact, this energy increases without bound as b_I increases. The effective action for such a wrapped brane is the DBI action, given in terms of the embedding coordinates $X^M(\xi)$ as s

$$S_{\rm DBI} = -\int \frac{d^{p+1}\xi}{(2\pi)^p}\,\alpha'^{-(p+1)/2} \times e^{-\Phi}\sqrt{\det(G_{MN}+B_{MN})\partial_\alpha X^M \partial_\beta X^N} \quad (1)$$

where we have omitted the corresponding Chern-Simons term, which will be unimportant for our considerations. A key example is a D5-brane wrapped on a two-cycle $\Sigma^{(2)}$ of size $\ell\sqrt{\alpha'}$, which yields a potential

$$V(b) = \frac{\epsilon}{g_s(2\pi)^5\alpha'^2}\sqrt{\ell^4 + b^2} \quad (2)$$

that is linear in the axion field b at large b. (Here we have included a factor ϵ to represent the effects of warping, which we describe more carefully below.) Similarly, an NS5-brane wrapped on $\Sigma_I^{(2)}$ introduces a monodromy in the c_I direction.

Monodromy is a common phenomenon in string compactifications. In the past, it has been studied extensively in the context of particle states in field theory [11] and the corresponding non-space-filling wrapped branes of string theory [12]. The present case of monodromy in the *potential energy* arises when a would-be periodic direction γ is "unwrapped" by the inclusion of an additional space-filling ingredient whose potential energy grows as one moves in the γ direction, extending the kinematic range of the corresponding scalar field. Because the wrapped branes are space-filling, their charge must be cancelled within the compactification. We will do so with an antibrane wrapped on a distant, homologous two-cycle as depicted in Fig. 2 in Sec. IV below.

In the bulk of this paper, we analyze the conditions under which this yields controlled large-field inflation in string theory. We find a reasonably natural class of viable models. As is usually the case in inflationary model building from string theory, much of the challenge is to gain systematic control of Planck-suppressed corrections to the effective action. After ensuring that our candidate inflaton potential does not destabilize the compactification moduli, and that fluxes do not affect the structure of our candidate inflaton potential, we establish that instanton effects, which produce sinusoidal contributions to the axion potential, can be naturally suppressed. We assess these conditions for both perturbative and nonperturbative stabilization mechanisms, drawing examples based both on Calabi-Yau compactifications and on more general compactifications that break supersymmetry at the Kaluza-Klein scale. In the case of nonperturbative stabilization mechanisms in type IIB string theory, we find a controlled set of models for the RR two-form axions c_I, while perturbative stabilization mechanisms suggest opportunities for inflating in the b_I as well as in the c_I directions. These varied implementations of our axion monodromy mechanism give identical predictions for the overall tilt and tensor to scalar ratio in the CMB, as they are all well-described by a linear potential for a canonically normalized inflaton.[1] Our prediction for these quantities lies well within the exclusion contours from present data [13], and is ultimately distinguishable from the predictions of other canonical models via planned CMB experiments [5,14] (see Fig. 3).

Our mechanism relies on specific additional ingredients—branes—intrinsic in the ultraviolet completion of gravity afforded by string theory. Although string theory restricts the range of the original axion period in the first place, it then recycles a single axion via monodromy, providing a simple generalization of [2,6] with its own distinctive predictions. The subject of axion inflation has thus almost come full circle.[2]

II. AXIONS AND THE CANDIDATE INFLATON ACTION

Axions in string theory arise from integrating gauge potentials over nontrivial cycles in the compactification manifold X. Let Σ_I, $I = 1, \ldots h^{1,1}(X)$ be an integral basis of the homology $H_2(X, \mathbb{Z})$, and let ω^I be a dual basis of the cohomology $H^2(X, \mathbb{Z})$, with $\int_{\Sigma_I} \omega^J = \alpha' \delta_I{}^J$. Then for the Neveu-Schwarz two-form potential $B^{(2)}$, let us write

$$B^{(2)} = b_I(x)\omega_2^I \tag{3}$$

with x the four-dimensional spacetime coordinate.

In the case of type II theories, additional axions arise from integrating the RR p-form potentials over p-cycles. Taking ω^α, $\alpha = 1, \ldots b^p(X)$, to be a basis of $H^p(X, \mathbb{Z})$ dual to an integral homology basis, we can write

$$C^{(p)} = c_\alpha^{(p)}(x)\omega_p^\alpha. \tag{4}$$

In type IIB string theory, for example, we have an RR two-form $C^{(2)}$ which will play a key role in the case of Calabi-Yau compactifications.

At the perturbative level in the string coupling and the inverse string tension, and in the absence of spacetime-filling branes wrapping the corresponding cycles, axions enjoy a continuous shift symmetry. This symmetry is broken down to a discrete shift symmetry by spacetime and world-sheet instantons. The resulting period of these axions, collectively denoted by $a = \{b \text{ or } c\}$, is

$$a \rightarrow a + (2\pi)^2 \tag{5}$$

as can be seen from the world-sheet coupling $(i/2\pi\alpha') \times \int_{\Sigma_I^{(2)}} B$ in the case of $B^{(2)}$.

A. Axion kinetic terms

In order to analyze the possibility of inflation with axions, we will need their kinetic and potential terms. The classical kinetic term[3] for the b_I fields descends from the $|H_3|^2$ term in the ten-dimensional action, with $H_3 = dB$. In terms of the metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + g_{ij}dy^i dy^j \tag{6}$$

we have

[1] There may also be novel signatures from finer details of the power spectrum originating in the repeated circuits of the fundamental axion period, as we discuss further below.

[2] Though we hope to have added something to the subject this time around.

[3] The kinetic terms are in general corrected by world-sheet instantons or D-instantons, in the cases of b and c, respectively. In our examples below we will ensure that these instanton effects are negligible in our inflationary solutions.

$$\int d^{10}x \frac{\sqrt{g}}{(2\pi)^7 g_s^2 \alpha'^4} \frac{1}{2}|H|^2 \Rightarrow \mathcal{S}_{\text{kin},b} = \int \frac{d^{10}x}{12(2\pi)^7 g_s^2 \alpha'^4} \sqrt{g} g^{\mu\nu} \partial_\mu b_I \times \partial_\nu b_J \omega^I_{ij} \omega^J_{i'j'} g^{ii'} g^{jj'} \tag{7}$$

and similarly for the $C^{(p)}$ fields, with $F^{(p+1)} = dC^{(p)}$:

$$\int d^{10}x \frac{\sqrt{g}}{(2\pi)^7 \alpha'^4} \frac{1}{2}|F^{(p+1)}|^2 \Rightarrow \mathcal{S}_{\text{kin},c} = \int \frac{d^{10}x}{2(2\pi)^7 (p+1)! \alpha'^4} \times \sqrt{g} g^{\mu\nu} \partial_\mu c_I \partial_\nu c_J \times \omega^I_{i_1 \ldots i_p} \omega^J_{i'_1 \ldots i'_p} g^{i_1 i'_1} \ldots g^{i_p i'_p}. \tag{8}$$

To simplify the presentation we will now restrict attention to b_I and to $c^{(2)}_\alpha \equiv c_I$, but the extension to other $c^{(p)}_\alpha$ is immediate. The four-dimensional kinetic terms for our axions b_I, c_I, collectively denoted as $a_I = \{b_I \text{ or } c_I\}$, may then be written

$$\begin{aligned} S_{\text{kin}} &= \frac{1}{2} \int d^4x \sqrt{g_4} \gamma^{IJ} g^{\mu\nu} \partial_\mu a_I \partial_\nu a_J \\ &\equiv \frac{1}{2} \int d^4x \sqrt{g_4} \sum_I f^2_{a_I} (\partial a'_I)^2 \\ &\equiv \frac{1}{2} \int d^4x \sqrt{g_4} \sum_I (\partial \phi_{a_I})^2, \end{aligned} \tag{9}$$

where in the second equality we have diagonalized the metric γ^{IJ}, and in the third equality we have defined the canonically normalized axion field ϕ_{a_I} for the Ith axion of type $a = \{b \text{ or } c\}$. In much of this paper, we will focus on a single axion at a time, and use the notation ϕ_a for its canonically normalized field. The canonically normalized inflaton field has periodicity

$$\phi_a \rightarrow \phi_a + (2\pi)^2 f_a \tag{10}$$

corresponding to (5).

Using (7) and (8), the axion kinetic term depends on the geometry of the compactification via

$$\gamma^{IJ} = \frac{1}{6(2\pi)^7 g_s^2 \alpha'^4} \int \omega_I \wedge \star \omega_J \tag{11}$$

for b_I, and

$$\gamma^{IJ} = \frac{1}{6(2\pi)^7 \alpha'^4} \int \omega_I \wedge \star \omega_J \tag{12}$$

for c_I. To express these results in terms of the four-dimensional reduced Planck mass M_P, we use

$$\alpha' M_P^2 = \frac{2}{(2\pi)^7} \frac{\mathcal{V}}{g_s^2}, \tag{13}$$

where $\mathcal{V}\alpha'^3$ is the volume of the compactification.

We will use (11) and (12), guided by [8,15], to determine the decay constants in our specific examples below. To provide intuition, we now record the result in the simplified case in which all length scales $L\sqrt{\alpha'}$ in the compactification are the same (and $\mathcal{V} \equiv L^6$). From (7) and (8) we obtain

$$\begin{aligned} \phi_b^2 &\sim \frac{L^2}{3 g_s^2 (2\pi)^7 \alpha'} b^2, \\ \phi^2_{c^{(p)}_\alpha} &\sim \frac{L^{6-2p}}{3(2\pi)^7 \alpha'} (c^{(p)}_\alpha)^2 \quad \text{(one scale)}. \end{aligned} \tag{14}$$

Using (13) this gives

$$\frac{\phi_b^2}{M_P^2} \sim \frac{b^2}{6L^4}, \qquad \frac{\phi_{c_I}^2}{M_P^2} \sim \frac{g_s^2 c_I^2}{6L^4} \quad \text{(one scale)}. \tag{15}$$

B. Wrapped five-brane action

As discussed in the introduction, wrapping appropriate branes on cycles threaded by $B^{(2)}$ and $C^{(p)}$ introduces a nonperiodic potential for the axions b and c. This follows immediately from the DBI action (1) in the case of D5-branes on 2-cycles with $B^{(2)}$-fields, and can be seen by duality to apply to (p, q)-five branes on cycles with both $B^{(2)}$- and $C^{(2)}$-fields.

For D5-branes on a two-cycle $\Sigma^{(2)}$ of size $\ell\sqrt{\alpha'}$ with b axions turned on, or NS5-branes on a two-cycle with a c axion, we have

$$\begin{aligned} V(b) &= \frac{\epsilon}{g_s (2\pi)^5 \alpha'^2} \sqrt{\ell^4 + b^2}, \\ V(c) &= \frac{\epsilon}{g_s^2 (2\pi)^5 \alpha'^2} \sqrt{\ell^4 + c^2 g_s^2}, \end{aligned} \tag{16}$$

where ϵ encodes warp-factor dependence to be discussed in Sec. IV. A similar contribution arises from an anti–five-brane wrapped on a distant, homologous two-cycle as depicted in Fig. 2 below.

In the large-field regime of interest, this potential is linear in the axion a, and hence in the canonically normalized field ϕ_a:

$$V(\phi_a) \approx \mu_a^3 \phi_a \tag{17}$$

with μ_a a function of the parameters of the compactification that depends on the model. We will analyze its structure in detail in several specific models in Sec. IV and V.

Let us also note a useful dual formulation of (2) and (16) which elucidates the monodromy effect introduced by the wrapped brane. Consider a D5-brane in type IIB string theory wrapped on a two-cycle arising as the blowup cycle

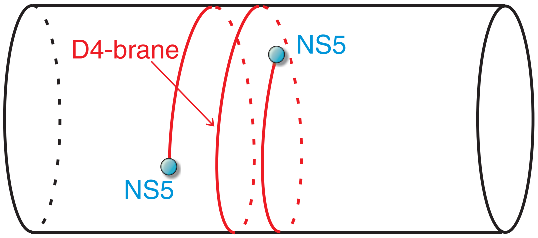

FIG. 1 (color online). T-dual, "brane box", description of this configuration, in which the fractional D3-brane becomes a D4-brane stretched between two NS5-branes on a T-dual circle. Moving in the b direction through multiple periods in closed-string moduli space in the original description corresponds to moving one of the NS5-branes around the circle, dragging the D4-brane around with it so as to introduce multiple wrappings.

of a supersymmetric $\mathbb{R}^3 \times S^1/\mathbb{Z}_2$ orbifold; this is equivalent to a fractional D3-brane at the orbifold singularity. There is a T-dual, "brane box," description of this configuration, in which the fractional D3-brane becomes a D4-brane stretched between two NS5-branes on a T-dual circle (see e.g. [16]). Moving in the b direction through multiple periods in closed-string moduli space in the original description corresponds to moving one of the NS5-branes around the circle, dragging the D4-brane around with it so as to introduce multiple wrappings. This T-dual description makes the linear potential manifest; see Fig. 1.

C. Basic phenomenological requirements

Our candidate inflaton action takes the form

$$S = \int d^4x\sqrt{g}\left(\frac{1}{2}(\partial\phi_a)^2 - \mu_a^3\phi_a\right) + \text{corrections}, \qquad (18)$$

where we indicated corrections which we will analyze below, suppressing them using symmetries, warping, and the natural exponential suppression of nonperturbative effects.

In order to obtain 60 e-folds of accelerated expansion, inflation must start at $\phi_a \sim 11M_P$. In addition, the quantum fluctuations of the inflaton must generate a level of scalar curvature perturbation $\Delta_{\mathcal{R}}|_{60} \simeq 5.4 \times 10^{-5}$, with

$$\Delta_{\mathcal{R}}|_{N_e} = \sqrt{\frac{1}{12\pi^2}\frac{V^3}{M_P^6 V'^2}}\Bigg|_{N_e}. \qquad (19)$$

This requires

$$\mu_a \sim 6 \times 10^{-4} M_P. \qquad (20)$$

Given $f_a = \phi_a/a$ and the above results, the *number of circuits* of the fundamental axion period $(2\pi)^2 f_a$ required for inflation is

$$N_w = 11\frac{M_P}{f_a(2\pi)^2}. \qquad (21)$$

We will compute this number of circuits in each of the specific models below. In the very simple case with all cycles of the same size, this gives, using (15),

$$N_w \sim 11\sqrt{6}\frac{L^2}{(2\pi)^2} \quad \text{(one scale)} \qquad (22)$$

for b, while the requisite number of circuits for an RR inflaton c is larger by a factor $1/g_s$.

D. Constraints on corrections to the slow-roll parameters

Our next task is to ensure that the inflaton potential $V_{\text{inf}} \approx \mu_a^3\phi_a$ is the primary term in the axion potential. All other contributions to the axion potential must make negligible contributions to the slow-roll parameters

$$\epsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta = M_P^2\frac{V''}{V}. \qquad (23)$$

A good figure of merit to keep in mind is that Planck-suppressed dimension-six operators such as $V(\phi - \phi_*)^2/M_P^2$, with ϕ_* a constant, contribute $\mathcal{O}(1)$ corrections to η, for any inflaton potential V. In what follows, we will analyze the conditions for sufficiently suppressing corrections to the slow-roll parameters.

Our specific setups discussed below will include reasonably generic examples which naturally suppress these corrections well below the 1% level, as is required in standard slow-roll inflation. In other examples, instanton-induced sinusoidal corrections to the potential lead to *oscillating* shifts in η of order one. Let us pause to assess the conditions on the slow-roll parameters in monodromy-driven inflation. In this class of models, the brane-induced inflaton potential is the leading effect breaking the approximate shift symmetry in the inflaton direction; other effects—in particular, instantons, in the case of our axion models—produce *periodic* corrections to the potential. In general, such models can tolerate larger *oscillating* contributions to η, as we now explain.

In the present situation, the corrections $\Delta\epsilon$ and $\Delta\eta$ to the slow-roll parameters oscillate as a periodic function of $a = \phi_a/f_a$ with period $(2\pi)^2$. The potential becomes steeper and flatter repeatedly during the evolution, and because these two effects can compensate each other, it is worth analyzing carefully what level of suppression of the amplitude of $\Delta\eta$ is really necessary to ensure 60 e-folds of inflation overall.

Let us simply give order-of-magnitude, parametric estimates for the net effect of the steeper and flatter regions. It would be interesting to study this in more detail, with an eye toward ancillary observational signatures which might arise in the power spectrum of density perturbations.

The potential takes the form[4]

$$V = \mu_a^3 \phi_a + \Lambda^4 \cos\left(\frac{\phi_a}{2\pi f_a}\right) \tag{24}$$

with Λ a constant determined by the instanton action.

The second term yields an oscillating contribution to η given, for $\Lambda \ll \mu_a^3 \phi_a$, by

$$\eta = M_p^2 \left(\frac{1}{2\pi f_a}\right)^2 \frac{\Lambda^4}{\mu_a^3 \phi_a} \cos\left(\frac{\phi_a}{2\pi f_a}\right). \tag{25}$$

The condition that the slope $V'(\phi_a)$ be non-negative can be written as

$$11\eta \frac{2\pi f_a}{M_P} \leq 1, \tag{26}$$

where we used (25) and the fact that $\phi_a \leq 11 M_P$ during the 60 e-folds of inflation in our linear potential.

Let us assume that averaging over the oscillations, the system remains in its slow-roll regime, and check the conditions for this to be self-consistent. The average field velocity is then

$$\dot{\phi}_a \simeq -\frac{\mu_a^3}{3H}, \tag{27}$$

and the time Δt during a period $\Delta\phi_a \sim (2\pi)^2 f_a$ is of order $\frac{\Delta\phi_a}{\dot{\phi}_a} \sim 3(2\pi)^2 f_a H/\mu_a^3$. Using this and the fact that η is of order $\ddot{\phi}_a / H\dot{\phi}_a$, we obtain the change $\Delta\dot{\phi}_+$ in the field velocity during the (half-)period in which the potential is relatively steep:

$$\Delta\dot{\phi}_+ \sim |\eta| (2\pi)^2 f_a H. \tag{28}$$

Similarly, on the flat regions of the potential, $\ddot{\phi}_a + 3H\dot{\phi}_a \simeq 0$, and we obtain

$$\Delta\dot{\phi}_- \sim -(2\pi)^2 f_a H. \tag{29}$$

Thus, we see that the kinetic energy does not build up over each full period of oscillation between steeper and flatter potential energy—which ensures that potential-energy dominated inflation proceeds—as long as $|\eta| \lesssim 1$. Again, many of the specific examples realizing axion monodromy inflation described below naturally yield much smaller corrections to η, but this possibility of larger oscillations in other examples is an intriguing new element worth investigating further in future work.

[4]Here for simplicity we neglect terms proportional to $\phi_a \cos(\phi_a/2\pi f_a)$, as they produce subdominant corrections to the slow-roll parameters. Such corrections may arise from instanton effects tied to the brane, hence the power of ϕ_a in front of the cosine.

III. NECESSARY CONDITIONS FOR CONTROLLED INFLATION

So far, we have a candidate for inflation along the direction ϕ_a, with potential $V_{\rm inf} \approx \mu_a^3 \phi_a$. We must now ensure that the proposed inflaton action (18) indeed arises in a consistent and controllable string compactification. This entails a series of nontrivial conditions dictated not directly by observations, but by our goal of producing a consistent and *computable* string realization. We first briefly summarize these requirements, then, in the following subsections, show how each of them can be met. As in [6], we will use the natural exponential suppression of instanton corrections to the axion potential.

The first, rather obvious condition is that the axion a which is to serve as the inflaton is actually part of the spectrum. This constrains the structure of the orientifold action used in moduli stabilization; however, we expect that some suitable modes do survive a generic orientifold projection. Next, we must demonstrate that the proposed inflaton potential is in fact the dominant contribution to the total potential for a: additional effects in the compactification must make subleading contributions to the axion potential. Specifically, couplings to fluxes and periodic contributions from instantons (world-sheet instantons and D-brane instantons, in the cases of b and c, respectively) must therefore be controlled or eliminated. Next, we must show that the energy stored in the axion does not source excessive distortion of the local geometry near the wrapped branes. Finally, the inflaton potential must remain subdominant to the moduli-stabilizing potential, and shifts in the moduli during inflation must not give large corrections to the inflaton potential.

A. Axions and the orientifold projection

We must first ensure that the axions b, c of interest are part of the spectrum. That is, the orientifolds which are crucially used in moduli stabilization (or their generalizations in F-theory) must project in the required modes. Some of the conditions for this in the case of type IIB Calabi-Yau O3/O7 orientifolds appear in [17,18], where the corresponding multiplets consist of b and c fields descending from Kähler moduli hypermultiplets in the "parent" unorientifolded Calabi-Yau manifold.

The world-sheet orientation reversal Ω which is part of every orientifold projection acts with a (-1) on the Neveu-Schwarz two-form potential B_{MN}. However, orientifolds typically include a geometric projection—a reflection I_{9-p} on some $9-p$ directions—at the same time. Two simple situations in which axions are projected in are the following. First, a B_{MN} field with one leg along the orientifold p-plane and the other transverse to it will be projected in by the full ΩI_{9-p} action. Second, the orientifold may exchange two separate cycles Σ_1 and Σ_2, independent in homology in the covering space, into each other. This projects in one combination of the two axions of the parent theory.

B. Conditions on the potential

A generic string compactification will generate additional contributions to the potential for ϕ_a going beyond the candidate inflaton potential (16) and (17). In this subsection, we will describe the conditions for these corrections to be consistent with inflation.

1. Conditions on flux couplings

We must first ensure that background fluxes do not couple to the putative inflaton in such a way as to introduce problematic contributions to the potential. Ramond-Ramond fluxes $\tilde{F}_q$ include Chern-Simons corrections of the form $B_2 \wedge F_{q-2}$ and $C_{q-3} \wedge H_3$. These contributions, if present in the flux compactification being used to stabilize the moduli, yield masses for the corresponding components of b and c through the terms proportional to $|\tilde{F}_q|^2$ in the ten-dimensional Lagrange density.

The extra contributions to the generalized field strengths give contributions of the form

$$\int d^{10}x \frac{\sqrt{g}}{16(2\pi)^7 \alpha'^4} |B_2 \wedge F_p|^2 \tag{30}$$

or, in the $C^{(p)}$ case,

$$\int d^{10}x \frac{\sqrt{g}}{16(2\pi)^7 \alpha'^4} |C_p \wedge H_3|^2 \tag{31}$$

to the effective action (for definiteness we have given the normalizations for the case of $|\tilde{F}_5|^2$ in type IIB). It is worth emphasizing that in type IIB flux compactifications on Calabi-Yau orientifolds, the class of fluxes that are consistent with the no-scale structure derived in [17,19], namely, imaginary self-dual fluxes, do not contribute to the axion potential: the axionic fields enjoy a no-scale cancellation of their contribution to the flux-induced potential [17].

In more general models we will have to ensure that we can make analogous choices of fluxes to remove flux contributions to the axion potential. If the wedge products (30) and (31) are nonzero, and if the relevant flux F_p or H_3 contributes leading moduli-stabilizing terms of order the barriers in $\mathcal{U}_{\rm mod}$, then the corresponding axion may be obstructed from being the inflaton. As an example, consider the case of a product manifold. The coupling (30) scales like

$$\int d^{10}x \frac{\sqrt{g}}{16(2\pi)^7 \alpha'^4} |F_p|^2 |b/L^2|^2 \approx \int d^{10}x \sqrt{g} \frac{3|F_p|^2}{8(2\pi)^7 \alpha'^4} \frac{\phi_b^2}{M_P^2} \tag{32}$$

while the contribution of the F_p flux to the moduli potential scales like

$$\int d^{10}x \frac{\sqrt{g}}{2(2\pi)^7 \alpha'^4} |F_p|^2 \sim \int d^4x \sqrt{g_4}\, \mathcal{U}_{\rm mod}. \tag{33}$$

Thus, a super-Planckian excursion of the ϕ_b field would lead to a contribution (32) which would overwhelm the moduli-stabilizing barriers.[5] Similar comments apply to curvature couplings and generalized fluxes.

2. Effects of instantons

The effective action for axions is corrected by instanton effects. Worldsheet instantons depend periodically on b type axions, while Euclidean D-branes (D-brane instantons) introduce periodic dependence on the c type axions (and nonperiodic, exponentially damped dependence on b/g_s). Both types of instantons are exponentially suppressed in the size of the cycle wrapped by the Euclidean world sheet or world volume.[6]

First, consider the kinetic terms in the effective action. These take the form

$$\frac{1}{2} \int d^4x \sqrt{g} f_a^2 (\partial a)^2 (1 + \epsilon_1 f_{\rm per}(a)), \tag{34}$$

where $f_{\rm per}(a)$ is a periodic function of $a \simeq a + (2\pi)^2$ normalized to have amplitude 1. This changes the canonically normalized field to be

$$\phi_a = f_a \int^a da' \sqrt{1 + \epsilon_1 f_{\rm per}(a')}. \tag{35}$$

Suppressing corrections to the slow roll parameters requires sufficiently small ϵ_1. In terms of the bare canonically normalized field $\phi_a^{(0)}$, our periodic function varies on a scale of order $(2\pi)^2 f_a$: $f_{\rm per} = f_{\rm per}(\phi_a^{(0)}/f_a)$. Thus for small ϵ_1, the potential expanded about a local minimum ϕ_* of $f_{\rm per}$ is of the form

$$V_{\rm inf}(\phi_a) \simeq \mu_a^3 \phi_a \left(1 + \epsilon_1 \frac{(\phi_a - \phi_*)^2}{(2\pi f_a)^2}\right) = \mu^3 \phi_a \left(1 + \epsilon_1 \left(\frac{M_P}{2\pi f_a}\right)^2 \frac{(\phi_a - \phi_*)^2}{M_P^2}\right). \tag{36}$$

Thus if

$$\epsilon_1 \lesssim 10^{-2} \left(\frac{2\pi f_a}{M_P}\right)^2 \approx \frac{1}{4\pi^2 N_w^2} \tag{37}$$

[5] There are interesting ideas for obtaining a large field range via large-N gauge theory [20], which on the gravity side might involve warped-down flux-induced monodromy. This may provide a way to use flux couplings to introduce an inflationary axion potential consistent with moduli stabilization, but this question requires further analysis.

[6] One may also consider nonperturbative effects arising in Euclidean quantum gravity, as explored in [21]; these are exponentially suppressed in the controlled regime of weak coupling and weak curvature.

then the instanton corrections to the kinetic terms do not affect inflation, since the slow roll parameters $\epsilon = \frac{M_P^2}{2}(\frac{V'}{V})^2$ and $\eta = M_P^2 \frac{V''}{V}$ remain of order 10^{-2}.

Next, let us consider instanton corrections arising directly in the potential energy term in the effective action. These we can write as (using similar notation to that above)

$$V_{\rm inf}(\phi_a) \sim \mu_a^3 \phi_a (1 + \epsilon_2 g_{\rm per}(\phi_a/f_a)) + \epsilon_3 \frac{h_{\rm per}(\phi_a/f_a)}{\alpha'^2}. \tag{38}$$

As before, let us assess sufficient conditions on ϵ_2 and ϵ_3 to ensure that instanton corrections to the slow-roll parameters are negligible. From the first term in (38), we see that

$$\epsilon_2 \lesssim 10^{-2}\left(\frac{2\pi f_a}{M_P}\right)^2 \approx \frac{1}{4\pi^2 N_w^2}. \tag{39}$$

From the second term, we find

$$\epsilon_3 \lesssim 10^{-2}\left(\frac{2\pi f_a}{M_P}\right)^2 (V_{\rm inf}\alpha'^2) \approx \frac{V_{\rm inf}\alpha'^2}{4\pi^2 N_w^2}. \tag{40}$$

Note that the conditions we have imposed here may be relaxed, as discussed in Sec. IV D, because of the oscillatory nature of the corrections. We will obtain negligibly small corrections to η in a simple subset of our specific examples below, but it is worth keeping in mind the possibility of a larger oscillating contribution in other examples.

So far we have enumerated conditions on the amplitudes ϵ_i, $i = 1, 2, 3$ of various instanton contributions to the effective action. In order to implement these conditions, we need to relate the ϵ_i to parameters of the stabilized string compactification in a given model. An *exponentially* small coefficient ϵ_i arises automatically if the instanton wraps a cycle larger than the string scale. For instantons wrapping small cycles, ϵ_i may still be small if the kinetic term is protected by local supersymmetry in the region near the cycle, or if the instanton dynamics is warped down. We will consider several of these cases in the specific models discussed below.

C. Constraints from backreaction on the geometry

We obtained the effective potential from our wrapped five-brane using standard results from ten-dimensional string theory. A basic condition for control of our models is the absence of backreaction of the brane on the ambient geometry, so that this ten-dimensional analysis is valid to a good approximation. In particular, the core size $r_{\rm core}$ of our wrapped brane, including the effects of the axion, must be smaller than the smallest curvature radius $R_\perp$ transverse to it in the compactification.

A single D5-brane is pointlike at weak string coupling, and a single NS5-brane is string-scale in size. However, in our regime of interest the branes in effect carry $N_w \sim a/(2\pi)^2$ units of D3-brane charge. N_w D3-branes produce a backreaction at a length scale $r_{\rm core}$ of order [22]

$$r_{\rm core}^4 \sim 4\pi\alpha'^2 g_s N_w. \tag{41}$$

Thus in order to avoid significant backreaction on our compactification geometry, we require

$$N_w \ll \frac{R_\perp^4}{4\pi g_s \alpha'^2}. \tag{42}$$

The one-scale expression for N_w derived in Sec. III C suggests that this condition will be straightforward to satisfy, since the right hand side of (42) is $\propto R^4$, while the expression (22) scales like two powers of the relevant length scale in the problem. However, fitting GUT-scale inflation into a stabilized compactification requires high moduli-stabilizing potential barriers, which puts constraints on how large the ambient compactification may be. We will implement this condition in the specific models to follow.

D. Constraint from the number of light species

A related but slightly more subtle condition concerns new light species that arise in our brane configuration at large b or c. The effectively large D3-brane charge N_w introduces of order N_w^2 light species [23]. It is important to check the contribution this makes to the renormalized four-dimensional Planck mass. In the regime $g_s N_w > 1$, the effect of the D3-brane charge is best estimated using the gravity side of the (cutoff) AdS/CFT correspondence, following Randall and Sundrum [24]. As just discussed, in the regime (42), the size $r_{\rm core}$ of the gravity solution for the D3-branes is smaller than the ambient size $L\sqrt{\alpha'}$ of the compactification. This leads to a negligible contribution to M_P^2.

E. Consistency with moduli stabilization

A further condition is that our inflaton potential, which depends on the moduli as well as on ϕ_a, not exceed the scale of the potential barriers $\mathcal{U}_{\rm mod}$ separating the system from weak-coupling and large-volume runaway directions in moduli space:

$$V_{\rm inf}(\phi_a) \ll \mathcal{U}_{\rm mod}. \tag{43}$$

Here $\mathcal{U}_{\rm mod}$ denotes the scale of the barrier height in the moduli potential. Since our large-field inflation model has a GUT-scale inflaton potential, this requires high moduli-stabilizing potential barriers.

One must also ensure that the shifts in the moduli induced by the inflaton potential do not appreciably change the shape of the inflaton potential: in other words, the moduli-stabilizing potential must not only have high barriers, it must also have adequate curvature at its minimum. A self-consistent way to analyze such shifts is to use an adiabatic approximation, in which the moduli σ adjust to sit in instantaneous minima $\sigma_*(\phi)$ determined by the inflaton VEV:

$$\partial_\sigma (V_{\rm inf}(\phi, \sigma) + \mathcal{U}_{\rm mod}(\sigma))|_{\sigma=\sigma_*(\phi)} = 0. \quad (44)$$

One then computes the correction this introduces in the inflaton potential $V(\phi, \sigma_*(\phi))$ and checks whether this correction is negligible.

For moduli stabilization mechanisms which use perturbative effects, this condition is satisfied provided (43) holds, as explained in Sec. 2.4.2 of [10]. Let us briefly summarize this here. The volume and string coupling are exponentials in the canonically normalized fields σ; for example, the volume is $\mathcal{V}\alpha'^3 = \mathcal{V}_* e^{\sqrt{3}\sigma_v/M_P}\alpha'^3$ where $\mathcal{V}_*$ is the stabilized value of the volume and σ_v is the canonically normalized field describing volume fluctuations. In perturbative stabilization mechanisms, the leading terms in the moduli potential scale like powers of $L \equiv \mathcal{V}^{1/6}$: schematically,

$$\mathcal{U}_{\rm mod} \sim \sum_n \frac{c_n}{L^n}, \quad (45)$$

where the coefficients c_n depend on other moduli in a similar way. Therefore, as the dependence is exponential in the canonically normalized fields σ, we see that derivatives of the inflaton potential $V_{\rm inf}$ and the moduli-stabilizing potential $\mathcal{U}_{\rm moduli}$ with respect to σ/M_P scale like the potential terms themselves. Combining the tadpole from the inflaton potential $V_{\rm inf}$ with the mass squared from the moduli potential $\mathcal{U}_{\rm mod}$ yields the moduli shifts

$$\frac{\sigma}{M_P} \sim \frac{V_{\rm inf}}{\mathcal{U}_{\rm mod}}. \quad (46)$$

Plugging this back into the potential yields corrections which change its shape. However, these are small, giving corrections to η of order $\eta V_{\rm inf}/\mathcal{U}_{\rm mod}$.

For mechanisms we will study which employ exponential (e.g. instanton) effects to stabilize the volume [25,26], the structure of the potential is schematically [17,18]

$$\mathcal{U}_{\rm mod} \sim \sum_{n,m} \frac{c_n}{L^n} \exp(-C_m(L^4/g_s + \tilde{\gamma} b^2)), \quad (47)$$

where $\tilde{\gamma}$ is a factor of order unity. From this we can analyze two important consistency conditions.

First, let us discuss the moduli shifts. In the expression (47), there are still power-law prefactors in the potential (arising from the rescaling of the potential to Einstein frame) which lead to a similar suppression in the tadpoles for the volume and the string coupling as in the perturbatively stabilized case. Furthermore, there are additional contributions to the masses of the moduli from differentiating the exponential terms which can enhance the masses relative to the perturbatively stabilized case, further suppressing the tadpoles.

Second, it will be important to keep track of which combinations of geometrical moduli and axions are stabilized by a given moduli-stabilization mechanism in calculating the slow-roll parameters. In the scenario [25], a combination of the volume and b type axions of the form $L^4/g_s + \tilde{\gamma} b^2$ is what is stabilized by $\mathcal{U}_{\rm mod}$. This leads to an η problem for inflation along the direction of any Neveu-Schwarz axion b, analogous to the η problem identified in [27]. In this class of models, we will therefore be led to consider instead RR two-form axion inflation.

IV. SPECIFIC MODELS I: WARPED IIB CALABI-YAU COMPACTIFICATIONS

In this section and in Sec. V, we implement our basic strategy in several reasonably concrete models, imposing the consistency conditions delineated above. This is an important exercise, necessary in order to ensure that it is indeed possible to satisfy all the conditions together. Needless to say, there are many ways to generalize—and potentially simplify—these constructions, and we will indicate along the way some further directions for model building.

There are two classes of examples which differ in which combinations of scalar fields are stabilized by the moduli-fixing potential. In the case of mechanisms such as those outlined in [19,25,26,28] which employ nonperturbative effects in a low-energy supersymmetric formulation, the moduli potential stabilizes a combination of the geometric and axion modes. In perturbative stabilization mechanisms such as those outlined in [29–32], the volume and other geometrical moduli are directly stabilized. These latter cases will be discussed in Sec. V.

One canonical class of examples arises in warped flux compactifications of type IIB string theory on orientifolds of Calabi-Yau threefolds. After a telegraphic review of the resulting low-energy supergravity, we show that nonperturbative stabilization of the Kähler moduli leads to an η problem for a candidate inflaton b descending from $B^{(2)}$. We then demonstrate that this problem is absent when $C^{(2)}$ is the inflaton, and furthermore show that the leading remaining dependence of the potential on c, from Euclidean D1-branes, may be naturally exponentially suppressed. Inflation driven by a wrapped NS5-brane which introduces monodromy in the RR two-form axion direction is therefore a reasonably robust and natural occurrence in warped IIB compactifications.

A. Multiplet structure, orientifolds, and fluxes

Consider a compactification of type IIB string theory on a Calabi-Yau threefold, with $h^{1,1}$ Kähler moduli. The resulting four-dimensional $\mathcal{N} = 2$ supergravity contains $h^{1,1} + 1$ hypermultiplets, one of which is the universal hypermultiplet containing the axio-dilaton τ. The remaining hypermultiplets have as bosonic components b^A, c^A, $\mathrm{Re}T^A$, $\mathrm{Im}T^A \equiv \theta^A$, where $\theta^A = \int_{\Sigma_A^{(4)}} C_4$, and T^A is the $\mathcal{N} = 1$ complexified Kähler modulus, defined more carefully below. The axions suitable for monodromy inflation with wrapped five-branes are in the (b^A, c^A) half of these hypermultiplets. The overall volume and other size moduli are contained in the T^A.

We now consider orientifold actions, which break $\mathcal{N} = 2 \to \mathcal{N} = 1$ and play an important role in moduli stabilization. We will particularly focus on orientifold actions whose fixed loci give O3-planes and O7-planes. For the orientifold action we take

$$\mathcal{O} = (-1)^{F_L}\Omega\sigma, \tag{48}$$

where σ is a holomorphic involution of the Calabi-Yau, under whose action the cohomology groups split as:

$$H^{(r,s)} = H_+^{(r,s)} \oplus H_-^{(r,s)}. \tag{49}$$

We correspondingly divide the basis $\omega_A, A = 1, \ldots h^{1,1}$ into $\omega_\alpha, \alpha = 1, \ldots h_+^{1,1}$ and $\omega_I, I = 1, \ldots h_-^{1,1}$. As explained in detail in [17], half of the fields are invariant under the orientifold projection and are kept in the four-dimensional theory. Specifically, Kähler moduli T^α corresponding to even cycles and axionic moduli $G^I = \frac{b^I}{g_s} + i(c^I - C_0 b^I)$ corresponding to odd cycles survive the projection.

Let us indicate two classes of odd cycles we can project in by orientifolding Calabi-Yau manifolds. The first, considered in [17,18], consists of zero-size cycles which intersect the orientifold fixed plane in a locus of real dimension one. In this case, the orientifold can project in B_{MN} and C_{MN} with their legs oriented so that M (say) is parallel and N transverse to the orientifold fixed plane. The size modulus for the two-cycle is projected out in this case. The second construction arises when the orientifold maps two separate cycles Σ_1 and Σ_2, independent in homology in the covering space, into each other. In this situation, the size modulus T_+ of the even combination Σ_+ of the two two-cycles is projected in, while the G modulus G_- of the odd combination Σ_- of the two-cycles is projected in. It is important to note that this requires the sizes v_1 and v_2 of the two-cycles in the covering Calabi-Yau space to be the same: $v_1 = v_2 \equiv \frac{1}{2}v_+$. The odd volume modulus $v_- = v_1 - v_2$—the difference in size of the two-cycles—is projected out. Note that this allows for a situation in which there are no small geometrical sizes anywhere in the orientifold, if v_+ is large. In particular, as long as v_+ is large, the fact that v_- is zero does not indicate the presence of any small curvature radii or small geometrical sizes in the compactification.

Let us consider the latter "free exchange" case for definiteness. In order to straightforwardly satisfy Gauss' law in the compactification, it is simplest to consider two families of two-cycles Σ_1 and Σ_2, extending into warped regions of the parent Calabi-Yau. Within each family, place a five brane in a local minimum of the warp factor, and an anti–five-brane at a distant local minimum of the warp factor. The orientifold exchanges the two families, yielding families of (anti)invariant two-cycles $\Sigma_+(\Sigma_-)$. The warped five brane, with its monodromy in the axion direction, provides our candidate inflationary potential energy. This is illustrated schematically in Fig. 2.

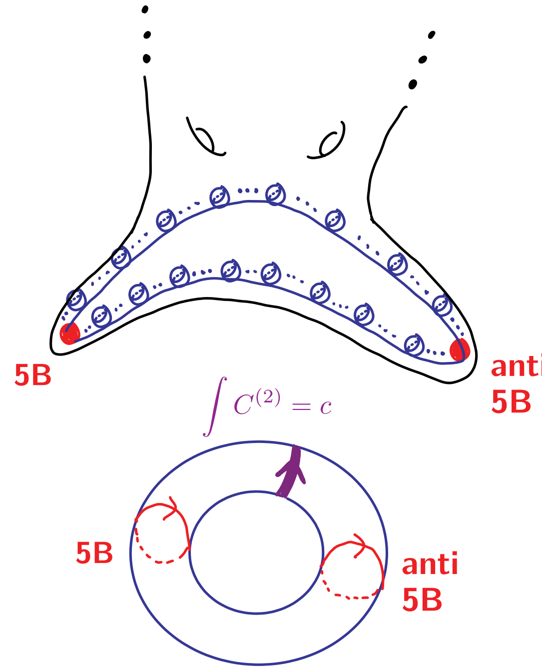

FIG. 2 (color online). Schematic of tadpole cancellation. Blue: Two-real-parameter family of two cycles Σ_1, drawn as spheres, extending into warped regions of the Calabi-Yau. Red: We have placed a five-brane in a local minimum of the warp factor, and an anti–five-brane at a distant local minimum of the warp factor. In the lower figure, Σ_1 is drawn as the cycle threaded by $C^{(2)}$, and global tadpole cancellation is manifest.

As a standard example, we may consider a warped throat which is approximately given by $AdS_5 \times X_5$, where X_5 is an Einstein space, the two factors have common curvature radius $R \sim L\sqrt{\alpha'}$, and the throat is cut off in the IR and UV [19,24,33].

$$ds^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(r)}(dr^2 + r^2 ds_{X_5}^2) \tag{50}$$

with warp factor $e^{A(r)} \sim r/R$.

To complete the definition of the Kähler moduli, we first define the Kähler form $J = v_\alpha \omega^\alpha$. The compactification volume $\mathcal{V}\alpha'^3$ satisfies[7]

$$\mathcal{V} = \frac{(2\pi)^6}{6} c^{\alpha\beta\gamma} v_\alpha v_\beta v_\gamma, \tag{51}$$

where $c^{\alpha\beta\gamma}$ are the triple intersection numbers. Then the complexified Kähler modulus is given by [17]

$$T_\alpha = \tfrac{3}{4}c^{\alpha\beta\gamma}v_\beta v_\gamma + \tfrac{3}{2}i\theta_\alpha - \tfrac{3}{8}e^\phi c^{\alpha IJ}G_I(G+\bar{G})_J. \tag{52}$$

[7]This formula follows our convention (13); another common convention is to define the volume in units of $l_s = 2\pi\sqrt{\alpha'}$ (see the appendix of [34]).

In the instructive simple case where $h_+^{(1,1)} = 1$ (so that the index α takes a single value, L, corresponding to the overall volume modulus), the classical Kähler potential for the sector descending from nonuniversal hypermultiplets takes the form

$$\mathcal{K} = -3\log(T_L + \bar{T}_L + \tfrac{3}{2}e^{-\phi}c^{LIJ}b_I b_J) + \ldots \tag{53}$$

The quantity inside the logarithm depends only on the overall volume and string coupling.

Moduli stabilization is essential for any realization of inflation in string theory, and we must check its compatibility with inflation in each class of examples. In type IIB compactifications on Calabi-Yau threefolds, inclusion of generic three-form fluxes stabilizes the complex structure moduli and dilaton [19]. A subset of these three-form fluxes—imaginary self-dual fluxes—respect a no-scale structure [18,19]. This suffices to cancel the otherwise dangerous flux couplings described in Sec. III A 1.

B. An eta problem for B

In this class of compactifications, however, the stabilization of the Kähler moduli leads to an η problem in the b direction. This problem arises because the nonperturbative effects (e.g. from Euclidean D3-branes or strong dynamics on wrapped seven-branes) stabilize the Kähler moduli [25] T^α rather than directly stabilizing the overall volume $\mathcal{V}\alpha'^3$.

Consider a setup with one or more D5-branes wrapping a curve $\Sigma_I^{(2)}$; as already explained, b_I is the candidate inflaton in this case. Now, the action for a Euclidean D3-brane wrapping the even four-cycle $\Sigma_L^{(4)}$ is proportional to T_L, so the nonperturbative superpotential depends specifically on the Kähler modulus T_L,[8]

$$W_{ED3} = Ae^{-a_L T_L}. \tag{54}$$

On the other hand, the compactification volume involves a combination (53) of the Kähler modulus and the would-be inflaton b_I. The volume appears in the four-dimensional potential, as usual, through the rescaling to Einstein frame; equivalently, the volume $\mathcal{V}$ appears in the F-term potential via the prefactor e^K. This inflaton-volume mixing is exactly analogous to the problem encountered for D3-brane inflatons in [27]; just as in that case, expansion of the potential around the stabilized value of T_L immediately reveals a Hubble-scale mass for the canonically normalized field ϕ_b corresponding to the axion b. Hence $\eta \sim 1$, preventing prolonged inflation.[9]

We remark that this problem is apparently absent for the case of a perturbatively stabilized volume. Moreover, because the volume depends on b^I but not on c^I, inflaton-volume mixing is also not a problem for a model in which c^I is the inflaton, which we now consider in detail.[10]

C. Instantons and the effective action for RR axions

We are now led to consider a compactification on an orientifold of a Calabi-Yau, in which one or more NS5-branes wrap a curve $\Sigma_I^{(2)}$, and the leading moduli-stabilizing effects from fluxes and Euclidean D3-branes—or gaugino condensation effects—do not contribute to the potential for c^I. The next task is to determine whether there are any further contributions to the inflaton potential which might lead to overly strong dependence on our candidate inflaton direction c^I. In particular, Euclidean D1-branes, when present, introduce sinusoidal contributions. So we must study and control the effects of Euclidean D1-branes.

1. Instanton contributions to the superpotential

A priori, one might expect the superpotential to take the schematic form

$$W = \int (F_3 - \tau H_3)\wedge\Omega + Ae^{-a_L T_L} + Be^{-\tilde{a}(v_+ - G_-/(2\pi)^2)} + Ce^{-a_L T_L - a_L G_-/(2\pi)^2} + \Delta W(T_+), \tag{55}$$

where for simplicity of presentation we have restricted attention to a single pair of cycles freely exchanged by the orientifold, with corresponding fields T_+, G_-, as well as an additional four-cycle $\Sigma_L^{(4)}$ associated with the overall volume; the prefactors A, B, C are constants.

Let us discuss each term in turn. The first two terms represent the moduli-stabilizing contributions of [19,25]; we will discuss additional features arising in the case of the large volume scenario [26] below. The next putative term represents the contribution of Euclidean D1-branes. Here v_+ is the volume of the orientifold-even two-cycle $\Sigma_+^{(2)}$; as explained in [17], v_+ belongs to a linear multiplet, not a chiral multiplet. Holomorphy therefore forbids the superpotential from depending on v_+ (said another way, the proper Kähler coordinates are T_L, T_+, which are four-cycle volumes), but at the same time any Euclidean D1-brane effect must vanish at large volume. So the third term in (55) must be absent [36].

The next term, proportional to C, represents Euclidean D1-brane corrections to the Euclidean D3-brane action (which we will refer to as ED3-ED1 contributions), in the case without wrapped seven-branes on the corresponding four-cycle. (We will discuss the case of strong dynamics on seven-branes further below.) When present, this arises from a Euclidean D1-brane dissolved as flux in a Euclidean D3-brane; see e.g. [37]. Such a contribution

[8] We will soon consider the possibility of axion dependence in the prefactor A; a_L is a constant.

[9] Note that this is *not* an oscillating contribution to η, and hence must be suppressed well below $\mathcal{O}(1)$.

[10] In [35] it was recognized that b^I receives a mass from the leading nonperturbative stabilization effects, whereas c^I does not.

requires that (a supersymmetric representative of) the two-cycle carrying our $C^{(2)}$ axion be embedded in the (supersymmetric) four-cycle wrapped by the original Euclidean D3-brane. When the cycles are configured in this way, the resulting dependence of the superpotential on c^I appears to be unsuppressed compared to the leading moduli-dependence, and in those cases one should worry that the moduli-stabilizing superpotential gives the inflaton a large mass-squared, of order $\mathcal{U}_{\rm mod}/(2\pi f_a)^2 > H^2$.

We could attempt to control this effect using warping. That is, if the two-cycle in question, and all two-cycles in its homology class, are localized in a warped region, then the coefficient C in (55) is suppressed, on dimensional grounds, by three powers of the warp factor $e^{A_{\rm top}}$ at the top of the two-cycle fixed locus in the throat

$$C \sim e^{3A_{\rm top}}. \tag{56}$$

Since this contribution was marginally dangerous to begin with, a modest warp factor satisfying

$$e^{3A_{\rm top}} < \Delta\eta(2\pi f_a/M_P)^2, \tag{57}$$

with $\Delta\eta$ constrained as described in Sec. IV D, suffices to avoid significant contributions to the slow-roll parameters.

However, examples generating this contribution to the superpotential are tightly constrained with respect to the basic backreaction condition (42), as follows. A computation of the kinetic term for c shows[11] that the axion decay constant f_c is proportional to the maximal warp factor arising in the homology class of the corresponding two-cycle: $f_c \sim e^{A_{\rm top}}\hat{f}_c$. This implies $N_w \sim 11 e^{-A_{\rm top}}/(\hat{f}_c(2\pi)^2)$. Putting this together with (42), we obtain the constraint $e^{A_{\rm top}} > 11\hat{f}_c/(\pi g_s M_P)$. But the condition (57) is equivalent to the condition $e^{A_{\rm top}} < (2\pi)^2\Delta\eta\hat{f}_c^2/M_P^2$. Together these would require $\Delta\eta > 11M_P/(4\pi^3 g_s^2\hat{f}_c)$.

Because of these issues, we will consider examples where the dangerous ED3-ED1 terms do not arise. One situation in which this occurs naturally is the following. Consider, as in [25], the case that the moduli-stabilizing nonperturbative superpotential arises from gaugino condensation on seven-branes wrapping four-cycles. In that situation, the physics below the KK scale of the four-cycles is given by pure $\mathcal{N} = 1 SU(N_L)$ supersymmetric Yang-Mills theory. In terms of its holomorphic gauge coupling $\tau_{\rm YM} = \theta/(4\pi) + i/g_{\rm YM}^2$, this theory has an exact superpotential of the form [38]

$$W = \Lambda^3 = Ae^{(8\pi^2/N_L)i\tau_{\rm YM}}. \tag{58}$$

In order to determine the dependence of our superpotential on T_L and G_- (and in general on other moduli), we must determine the Yang-Mills gauge coupling, including all significant threshold corrections to it at the KK scale. The Yang-Mills gauge coupling on the seven-branes is classically given by $8\pi^2/g_{\rm YM}^2 = 2\pi\,{\rm Re}T_L$ (in the absence of magnetic two-form flux on the D7-branes [39]), and similarly for other four-cycles in cases with more Kähler moduli. Comparing to (58), we can identify the parameter a_L in (55) as $2\pi/N_L$ for this case.

The holomorphic gauge coupling function $\tau_{\rm YM}$, like the superpotential itself [36], is constrained by holomorphy combined with the condition that in our weakly coupled regime, all nonperturbative corrections decay exponentially as the curvature radii grow. This means that $\tau_{\rm YM}$, like W, cannot develop pure ED1 corrections of order $e^{-2\pi G_-/N_L}$; instead, the leading correction to the gauge coupling function must be exponentially suppressed in T_L. Plugging this into (58), we see that the leading corrections from such threshold effects to the superpotential itself are exponentially suppressed relative to the leading moduli-stabilizing terms in (55). In particular, the coefficient C in (55) is negligible in this setup.

2. Instanton contributions to the Kähler potential

Next, we note that the corrected Kähler potential can be written schematically as follows[12] (with similar terms depending on T_+):

$$\mathcal{K} = -3\log(T_L + \bar{T}_L + \tfrac{3}{2}e^{-\phi}c^{LIJ}b_Ib_J + C_+{\rm Re}e^{-2\pi v^+ - G_-/(2\pi)}) + \ldots \tag{59}$$

In contrast to the holomorphic gauge coupling and superpotential just discussed, the Kähler potential is not protected by holomorphy. The dependence on ϕ_c arising through the appearance of G_- in (59) can naturally be suppressed to the necessary extent by using the exponential suppression in the size v_+ of the two-cycle. The ED1 contribution here yields a shift of η of order

$$\Delta\eta \sim \frac{\mathcal{U}_{\rm mod}}{V_{\rm inflation}}(2\pi)^2\frac{C_+}{g_s}e^{-2\pi v_+}. \tag{60}$$

This is straightforward to suppress with a modest blowup v_+ of the even cycle, as the dependence is exponential. (Alternatively, one may consider the possibility discussed in Sec. II of larger oscillating contributions to η.)

In general, there are several mechanisms one can consider for suppressing instanton effects, including use of local symmetries, warping, and (as just mentioned) exponential suppression of instanton effects with the geometrical sizes of cycles they wrap. Let us elaborate on the latter approach, which is likely to be the generic situation.

In the KKLT mechanism of moduli stabilization, nonperturbative effects are used to stabilize Kähler moduli. Consider using this mechanism to stabilize the Kähler modulus T_+ corresponding to the geometric size of the two-cycle wrapped by our NS5-brane (strictly speaking,

[11] This analysis proceeds as in [24], with c a bulk scalar field in the throat, or alternatively by the method reviewed in Sec. II A.

[12] See e.g. [40] for related work in the type I string.

the Kähler modulus corresponds to the size of the dual four-cycle, and we implicitly use the relation between the T_α's and v_α's.) In doing this, we must keep both T_+ and the overall volume sufficiently small that the barrier heights exceed the GUT scale of our inflaton potential, which takes the form (16):

$$V(c) = \frac{\epsilon}{g_s^2(2\pi)^5\alpha'^{(6-p)/2}}\sqrt{v_+^2 + c^2 g_s^2}. \tag{61}$$

The v_+ dependence in (61) tends to compress the wrapped two-cycle, so a crucial consistency condition, as discussed in Sec. III A for the overall volume and dilaton, is that the modulus T_+ be stabilized strongly enough so as not to shift in such a way as to destabilize inflation. As in Sec. III B, we must therefore compare the tadpole from (61) with the scale of the mass introduced by the moduli stabilization potential $\mathcal{U}_{\rm mod}$. Unlike the overall volume, T_+ need not be exponentially related to its canonical field σ_+ if it makes a subleading contribution to the physical volume $\mathcal{V}$ appearing in the Kähler potential, so we must assess its shift separately.

It is convenient—and equivalent—to work out the shift δv_+ of v_+ rather than that of the canonically normalized field σ_+, and then substitute the result back into the potential to determine the size of the resulting corrections to slow-roll parameters in the ϕ_c direction. We obtain from the added exponential terms in $\mathcal{U}_{\rm mod}$ the leading contributions to the mass term for δv_+

$$\begin{aligned}\partial_{v_+}^2\mathcal{U}_{\rm mod} &\sim \left(\frac{\partial T_+}{\partial v_+}\right)^2\mathcal{U}_{\rm mod}\\ &\sim (c^{++L}v_L + 2c^{+++}v_+)^2\mathcal{U}_{\rm mod}.\end{aligned} \tag{62}$$

The tadpole introduced by the expanding the inflaton potential (61) in powers of $v_+^2/(cg_s)^2$—note that this quantity is small in our regime of super-Planckian axion VEV—is of order

$$\partial_{v_+}V \sim V\frac{v_+}{(cg_s)^2}. \tag{63}$$

This leads to a shift in v_+ of order

$$\delta v_+ \sim \frac{V}{\mathcal{U}_{\rm mod}}\frac{(v_+/c^2g_s^2)}{(c^{++L}v_L + 2c^{+++}v_+)^2} \tag{64}$$

and a corresponding correction to the full scalar potential of order

$$\Delta\mathcal{U}_{\rm tot} \sim V\frac{V}{\mathcal{U}_{\rm mod}}\frac{(v_+/c^2g_s^2)^2}{(c^{++L}v_L + 2c^{+++}v_+)^4}. \tag{65}$$

This shift is negligible since $1 \ll v_+ \ll cg_s$.

Once the cycle v_+ is stabilized at a value larger than string scale, the Euclidean D1-brane corrections to the Kähler potential are exponentially suppressed. This provides a natural mechanism for ensuring the conditions (37), (39), and (40) that ϵ_1, ϵ_2, and ϵ_3 are sufficiently small.

3. Effects of enhanced local supersymmetry

In some cases, the instanton corrections might be small without blowing up the cycle $\Sigma_I^{(2)}$. There is ongoing research on stringy instanton effects; a systematic understanding of these effects would substantially improve our ability to build concrete axion inflation models. In particular, in some recent works, desired D-instanton corrections were difficult to obtain because of cancellations arising from extra fermion zero modes [41]. For our purposes this cancellation is advantageous; one situation in which it is particularly likely is when the region near the two-cycle locally preserves extra supersymmetry. A more specific setup of this sort is one in which our two-cycle is locally a $\mathbb{C}^2/\mathbb{Z}_2$ orbifold blowup cycle in a warped throat. In particular, consider such an orbifold singularity, with the fixed point locus extending up the radial direction of the warped throat to a maximal warp factor $e^{A_{\rm top}}$ (see Fig. 2) and along a circle of size $2\pi R$ in the internal X_5 directions.

In this case, in the local six-dimensional system the modulus v_I corresponds to a geometrical blowup of the two-cycle, and is linearly related to the canonically normalized scalar field (as can be seen, for example, by its T-dual relation to relative positions of NS5-branes).

In the case of a Klebanov-Strassler throat, for example, we can orbifold to obtain a fixed point locus which extends radially up the warped throat and along an S^1 within the internal $T^{1,1}$, as follows. In the standard presentation of the deformed conifold,

$$\sum_{i=1}^4 z_i^2 = \varepsilon^2 \tag{66}$$

we obtain this with an orbifold action under which $(z_1, z_2, z_3, z_4) \to (-z_1, -z_2, z_3, z_4)$. This system has $\mathcal{N} = 4$ supersymmetry locally, and the extended nature of the fixed point locus of the orbifold implies the presence of bosonic and fermionic zero modes corresponding to the collective coordinates describing the instanton's position in the radial direction and along the S^1 within the $T^{1,1}$. For this configuration, we note that using (12), the axion decay constant is given by

$$\phi_c \sim \frac{c}{\sqrt{\alpha'}}e^{A_{\rm top}}\left(\frac{R}{\sqrt{\alpha'}}\right) \sim M_P e^{A_{\rm top}}\frac{cg_s}{L^2}, \tag{67}$$

where again we keep track of the maximal warp factor in the region explored by the entire family of homologous blowup two-cycles, a quantity which is determined by the way in which the warped throat is connected to the rest of the compactification. It is worth emphasizing that the simplest methods we outlined above for suppressing corrections to the slow-roll parameters do not require warping of the entire family of two-cycles; one may simply take $e^{A_{\rm top}} \sim 1$ provided that the cycle wrapped by the NS5-brane is stabilized at finite volume and that the moduli-stabilizing nonperturbative effects arise from seven-branes.

D. Backreaction condition

Let us next address the question of backreaction of our wrapped brane inside the warped Calabi-Yau. The basic condition (42) becomes

$$N_w \ll \frac{\pi^3}{4}\,\mathrm{Re}(T_L), \tag{68}$$

where we used the relation $\mathrm{Re}(T_L) = \frac{\mathrm{Vol}_4}{(2\pi)^4 g_s}$ between the chiral field T_L and the size $\mathrm{Vol}_4\alpha'^2$ of the corresponding four-cycle in the Calabi-Yau (which we then identified with $L^4 \sim (2R_\perp)^4/\alpha'^2$). Now let us combine this with the condition that the moduli potential barriers exceed the scale of our inflation potential. The scale of the moduli-stabilizing barriers is given in terms of the rank of the gauge group N_L on the seven-branes as

$$\mathcal{U}_{\mathrm{mod}} \simeq \frac{|A|^2}{T_L^3 M_P^2} e^{-4\pi T_L/N_L}. \tag{69}$$

Setting $\mathcal{U}_{\mathrm{mod}} \geq V_{\mathrm{inf}} \simeq 2.4\times 10^{-9} M_P^4$ yields the constraint

$$T_L \leq -\frac{N_L}{4\pi}\log\left(2.4\times 10^{-9} T_L^3 \frac{M_P^6}{|A|^2}\right)$$
$$\Rightarrow N_w \ll -\frac{\pi^2}{16} N_L \log\left(2.4\times 10^{-9} T_L^3 \frac{M_P^6}{|A|^2}\right). \tag{70}$$

The coefficient A in the superpotential may depend holomorphically on complex structure moduli. As in much of the previous literature on KKLT moduli stabilization, we will take $A \approx M_P^3$. However, we should note a standard subtlety with loop corrections to the gauge coupling and how it affects our considerations. In our system, the four-dimensional $\mathcal{N} = 1$ supersymmetric Yang-Mills theory on the seven-branes crosses over at the KK scale $M_{\mathrm{KK}} \sim (2\pi) g_s M_P/L^4$ to an eight-dimensional maximally supersymmetric gauge theory. The effective cutoff scale in the quantum field theoretic analysis of the $\mathcal{N} = 1$ supersymmetric Yang-Mills theory [38,42] is therefore M_{KK}. This might naively suggest $A \sim M_{\mathrm{KK}}^3$, but this would not be holomorphic. The holomorphy of the superpotential (58) may be maintained by an appropriate redefinition of the chiral superfield T_α appearing in the gauge coupling function for the seven-brane stack wrapping the four-cycle $\Sigma_\alpha^{(4)}$. This in turn introduces a shift proportional to $-\frac{N_\alpha}{2\pi} \times \log(T_\alpha + \bar{T}_\alpha)$ (plus a constant) in the argument of the logarithm appearing in the Kähler potential (53). Since we require relatively high moduli-stabilizing barriers, our system lies close to the interior of Kähler moduli space, and this shift can become significant depending on the details of the example. A preliminary investigation suggests that with reasonable numbers the effect is (barely) "neglibigle" in the stabilization of T_α, and that more generally it may in fact push T_α to larger values at fixed, high, barrier heights. Overall, we find that with modest choices of N_L, our system can tolerate hundreds of circuits of the basic axion period.

The large volume scenario

In the large volume mechanism [26] for stabilizing Calabi-Yau flux compactifications, both power-law and nonperturbative terms play a role in stabilizing the Kähler moduli. In this setting, one can increase the volume, maintaining the required high barrier heights, by increasing W_0. This provides another method for ensuring satisfaction of the backreaction constraint.

E. Numerical toy examples

Our analysis of the conditions for inflation suggests that they are reasonably straightforward to satisfy. Because the full potential is somewhat complicated, it is worth checking numerically how the scales work out in a four-dimensional supersymmetric effective action which encodes the essence of our mechanism, including the basic structures required for moduli stabilization and inflation. We will therefore consider the four-dimensional action descending from a compactification with the minimal possible content—$h_+^{1,1} = 2$ and $h_-^{1,1} = 1$—required for the mechanism described above, including the effects of the orientifold action.

We will consider setups of the sort described in the previous subsection, with two size moduli denoted by an index $\alpha = L$ for the overall volume mode and $\alpha = +$ for the even combination of cycles under an orientifold action. (The odd combination of cycles, as described above, supports our $C^{(2)}$ inflaton field, $c_- = \mathrm{Re}[G_-]$.)

To mock this up, guided by the structure of orientifolds of Calabi-Yau manifolds such as $\mathbb{P}^4_{11169}$ and $\mathbb{P}^4_{13335}$, we define a class of toy models by a classical Kähler potential of the form

$$\begin{aligned} K &= -2\ln\mathcal{V}_E \\ &= -2\ln\{(T_L + \bar{T}_L)^{3/2} \\ &\quad - [T_+ + \bar{T}_+ + \tfrac{3}{8} g_s c_{+--}(G_- + \bar{G}_-)^2]^{3/2}\}. \end{aligned} \tag{71}$$

plus contributions depending on the dilaton and complex structure moduli, where we defined

$$\mathcal{V}_E = \frac{L^6}{g_s^{3/2}(2\pi)^6} = \frac{\mathcal{V}}{g_s^{3/2}(2\pi)^6}. \tag{72}$$

From the requirement of getting a positive-definite kinetic term for G_- we deduce $c_{+--} > 0$ and for the following examples we choose for convenience $c_{+--} = +1$. We also include corrections to K of the form given in (59).

The superpotential we take to be of a generalized KKLT-type structure [25,28]

$$W = W_0 + A_+ e^{-a_+ T_+} + \begin{cases} A_L e^{-a_L T_L} & \text{“KKLT”} \\ A_L^{(1)} e^{-a_L^{(1)} T_L} + A_L^{(2)} e^{-a_L^{(2)} T_L} & \text{“KL”} \end{cases}. \quad (73)$$

For simplicity in this section, we will work in units of M_P.

Moduli stabilization then proceeds from the F-term scalar potential for the fields T_L, T_+, G_- which is determined by

$$V_F(T_L, T_+, G_-) = e^K (K^{I\bar{J}} D_I W \overline{D_J W} - 3|W|^2), \quad (74)$$

where $K^{I\bar{J}}$ is understood to be the inverse Kähler metric derived by keeping the dilaton dependence in K (and thus for its determination the tree-level dilaton Kähler potential $K_\tau = -\ln[-i(\tau + \bar{\tau})]$ has to be included in K). The dilaton is assumed to be fixed by three-form fluxes at $D_\tau W = 0$, and we will take $g_s \sim 1/2$ for concreteness. Thus, here I, J run over the values $L, +, -$, corresponding to the fields T_L, T_+, G_-.

With a choice of parameters in e.g. the KKLT case of

$$A_L = -1, \qquad A_+ = 1, \qquad a_L = \frac{2\pi}{25}, \qquad a_+ = \frac{2\pi}{3}, \qquad W_0 = 3 \times 10^{-2} \text{“KKLT”} \quad (75)$$

this setup stabilizes $T_L \sim 20$, $T_+ \sim 4$ and $b \sim 0$ in a way consistent with the most basic conditions for inflation. In particular, the moduli potential barriers exceed V_{inf}, and the moduli suffer practically negligible shifts in their VEVs during inflation driven by an NS5-brane wrapped on the blown-up two-cycle.[13]

In terms of the supersymmetric multiplets, the NS5-brane potential is given by

$$V_{NS5} = M_P^4 e^{4A_{\text{bottom}}} \frac{1}{(2\pi)^3 g_s \mathcal{V}_E^2} \times \sqrt{\underbrace{g_s \frac{T_+ + \bar{T}_+ + \frac{3}{8} g_s c_{+--} (G_- + \bar{G}_-)^2}{2}}_{\text{this is} v_+^2} + g_s^2 c^2} \quad (76)$$

with $e^{A_{\text{bottom}}}$ denoting the warp factor at the bottom of the throat. We obtain a GUT-scale inflaton potential for $e^{A_{\text{bottom}}} \sim 0.04$. Inputting the axion decay constant for this case (67), with $e^{A_{\text{top}}} \sim 1$, we find of order $N_w \sim 70$ cycles during inflation, easily satisfying the backreaction constraint. Increasing a_+ to π introduces oscillating corrections to η of amplitude ~ 0.04.

[13]Note that given a fully explicit model, knowledge of the intersection numbers in $T_+ = c_{+\alpha\beta} v^\alpha v^\beta$ may allow for having smaller values of T_+, while still yielding $v_+ \sim 2$, as necessary for sufficiently suppressing the ED1 contributions to the Kähler potential.

In all cases discussed above, the uplifting contribution of an anti-D3-brane

$$\delta V_{\overline{D3}-\text{uplift}} = \frac{\delta_{\overline{D3}}}{\mathcal{V}_E^{4/3}} \quad (77)$$

is assumed present and is fine-tuned as in [25] so as to provide the post-inflationary minimum at $c = 0$ with small positive cosmological constant.

F. Gravity waves and low-energy supersymmetry

It is interesting to consider the possibility of combining low-energy supersymmetry with high-scale inflation. The present work moves a step closer to an understanding of this question by implementing large-field inflation in string compactifications which have a four-dimensional effective theory with spontaneously broken $\mathcal{N} = 1$ supersymmetry.

In the particular case of KKLT moduli stabilization—with an uplift of a SUSY-breaking AdS minimum—the scale of the moduli barriers decreases with decreasing scale of supersymmetry breaking. Kallosh and Linde [28] explained how—with extra fine-tuning via an additional racetrack in the superpotential—one may decouple these scales (see also the recent work [43]).

In our setup, we may also apply this mechanism, with the following caveat. Our wrapped five-brane action, at its post-inflationary minimum, itself constitutes a supersymmetry-breaking ‘‘uplifting’’ contribution to the potential energy for nonzero v_+. This contribution would need to be very small in order to obtain a low scale of supersymmetry breaking. Such a suppression might be possible by (i) blowing down v_+, which may lead to a larger, but still viable, oscillating contribution to η (modulo suppressions coming from the enhanced local super-

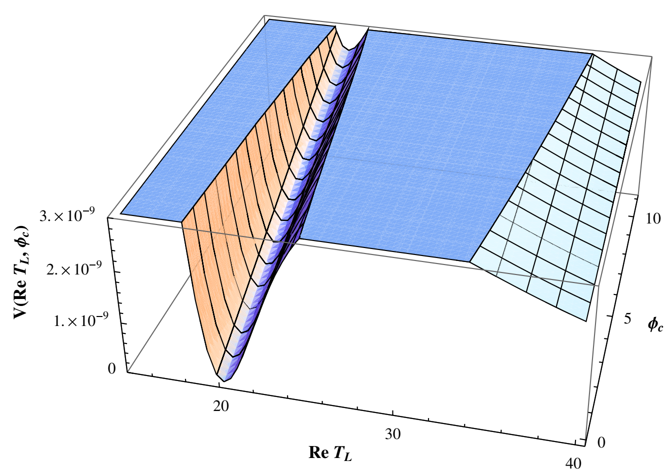

FIG. 3 (color online). Linear axion inflaton potential $V(\mathrm{Re}T_L, \phi_c)$ with KKLT Kähler moduli stabilization scenario. The linear inflaton valley is clearly visible. The potential looks very similar (but for the second AdS minimum at larger volume) for the KL case. The cutoff surfaces at the top of the plotted box denote the further rise of the scalar potential in the barriers.

symmetry near the cycle in some examples), or (ii) warping the NS5-brane further down, as long as this is consistent with the backreaction constraints.

It is worth emphasizing that despite much progress in recent years, specific models arise very much under a lamppost, and it is difficult—if not impossible—to determine generic patterns without a systematic analysis of string compactifications.[14] Thus, although there is no known natural construction combining high-scale inflation with low scale supersymmetry, neither is there is a compelling "no go" theorem. The answer to this question must await further development of the subject.

V. SPECIFIC MODELS II: PERTURBATIVELY STABILIZED COMPACTIFICATIONS

Let us next briefly outline some potential examples of our mechanism in the context of perturbative stabilization of moduli. This class of examples includes compactifications on more generic—Ricci-curved—manifolds, and a correspondingly higher scale of supersymmetry breaking. The conditions that the flux-induced axion masses not lift b and c, which were automatically satisfied in the no-scale type IIB Calabi-Yau compactifications discussed above, will need to be assessed separately in these cases. The perturbative models, on the other hand, enjoy some complementary simplifications of their own, such as the fact that one need not balance classical effects against nonperturbative effects to stabilize moduli. The moduli-stabilizing barriers, being power law in the volume as well as in the dilaton, may be naturally higher, and the η problem for b derived in the previous section does not directly apply when the volume is perturbatively stabilized.

As with Calabi-Yau compactifications, only a small subset of models in this class have been analyzed in any detail. The simplest examples of this sort involve known classical compactification geometries and a relatively small set of additional ingredients, and are therefore accessible to more detailed analysis than the typical Calabi-Yau compactification (as in [10,32]). The most specific, tractable examples, however, do not incorporate the warping effects one expects to arise in a typical compactification (whether low-energy supersymmetric or not). Clearly the implementation of our mechanism for linear inflation from axion monodromies will benefit from further developments in string compactification.

A. Compactifications on nilmanifolds

First, consider compactifications of type IIA string theory on a product of two nilmanifolds

$$ds^2_{\mathrm{Nil}\times\mathrm{Nil}} = \frac{L_u^2}{\beta} du_1^2 + \beta L_u^2 du_2^2 + L_x^2(dx + M u_1 du_2)^2 + \frac{L_u^2}{\beta} d\tilde{u}_1^2 + \beta L_u^2 d\tilde{u}_2^2 + L_x^2(d\tilde{x} + M\tilde{u}_1 d\tilde{u}_2)^2, \tag{78}$$

compactified via projection by a discrete set of isometries, and stabilized, for example, with the ingredients described in [32], including an orientifold action exchanging the tilded and untilded coordinates. In the presence of D4-branes, these manifolds yield monodromy-driven large field inflation with a $\phi^{2/3}$ potential [10]. It is interesting to consider the angular closed string moduli in [10], to see if monodromy from wrapped branes might yield linear inflation in axion directions also in these models.

To begin, we note that the flux couplings in Sec. III C prevent inflation in the Neveu-Schwarz axion (b) directions in this model, because of the zero-form flux m_0 which plays a leading role in moduli stabilization. Ramond-Ramond axions come from those components of $C^{(1)}$, $C^{(3)}$, and $C^{(5)}$ that are invariant under the orientifold projection. With the NS-NS H_3 flux configuration of the specific example analyzed in [32], $C^{(1)} \wedge H_3$ is always nonzero.

Many components of $C^{(3)}$ consistent with the orientifold projection satisfy $C^{(3)} \wedge H_3 = 0$. The next question is whether any ingredients which fit into the compactification introduce monodromy in one or more of these directions. Consider (16) for the case of an NS5-brane in the presence of a $C^{(2)}$ axion (i.e. $p = 2$). T-duality in a direction $y_\perp$ transverse to the NS5-brane yields a KK5-brane—a Kaluza-Klein monopole with fiber direction $y_\perp$. The T-duality transforms the $C^{(2)}$ field to a $C^{(3)}$ field with two legs along the KK5-brane world volume and one along $y_\perp$. Hence a KK5-brane thus oriented with respect to a $C^{(3)}$ axion c_3 introduces a linear potential for c_3.

The setup [10,32] includes of order $1/\beta$ sets of M KK5-branes wrapped along a linear combination of the u_2 and $\tilde{u}_2$ directions times a combination of the x and $\tilde{x}$ directions, with its fiber circle in the transverse combination of x, $\tilde{x}$ directions. The components of $C^{(3)}$ with legs along these three directions are lifted by the nilmanifold's "metric flux"—that is, the fiber circle is a torsion cycle. Thus, in order to implement c_3 axion inflation we need to add additional wrapped branes.

Consider adding a second set of M KK5-branes wrapped along the u_2 and $\tilde{u}_2$ directions, with their fiber circle in a linear combination of the x and $\tilde{x}$ directions. Let us denote this set by KK5$'$. They carry a linear potential in the c_3 direction. The ratio of the KK5$'$ potential energy to the original KK5 potential energy is

$$\frac{\beta^{1/2}}{L_x L_u}\sqrt{\beta^2 L_u^4 + c_3^2 g_s^2/L_x^2}. \tag{79}$$

[14]Moreover, generic compactifications with this much supersymmetry involve many further ingredients, such as generalized fluxes, which significantly affect questions of genericity.

We now observe that the decay constant of an axion arising from a potential that threads a product space of the form $\Sigma^{(p)} \times \Sigma^{(6-p)}$ is given by

$$\phi_c^2 \sim \frac{L^6}{(2\pi)^7 \ell_{(p)}^{2p} \alpha'} c^2 \sim M_P^2 \frac{g_s^2 c^2}{2\ell_{(p)}^{2p}} \qquad \text{(product space)}. \tag{80}$$

Using (80) and (21) for $p = 3$, we find that $g_s N_w \sim c g_s/(2\pi)^2 \sim 11(2\beta L^3)/(2\pi)^2$. In order for our added KK5$'$ branes to be subdominant to the moduli potential all along the inflation trajectory, we need to tune the anisotropy β such that the ratio (79) is less than unity.

Next let us assess systematically the rest of the consistency conditions delineated in Sec. III. First, consider the KK5$'$ branes before the effect of the axion VEV c_3. The core size of a KK monopole is its fiber size, here L_x; in the present case we obtain $r_{\rm core}^{\rm KK'} \sim M L_x$. This fits well within the transverse u_1, $\tilde{u}_1$ directions, and is marginal for $M \sim 1$ within the transverse linear combination of x, $\tilde{x}$ directions.

We now consider the effect of c_3 on the core size of the object. In the present case where our manifold is locally a product space, the c_3 term in the brane action contributes to its effective tension. In our regime of interest, the tension is of order $\frac{\phi_c}{M_P} \sim 11$ times what its tension would be at $c_3 = 0$. In other words, it behaves like 11 sets of KK5$'$ branes. This increases the core size by a factor of 11.

Locally in the u_1 directions, the KK5$'$ branes are BPS objects, and hence the corresponding formula for their tension applies to good approximation. Moreover, as discussed in [32], there are more elaborate methods which might be used to warp down the tensions of KK5-branes in this space to separate such marginal ratios of scales, bringing NS5-branes wrapped on the x and $\tilde{x}$ directions close to the positions of the KK5$'$ branes.

Finally, we note that instanton effects which depend on c_3 arise from Euclidean D2-branes. These are safely suppressed by a factor of order $\exp[-\beta L^3/g_s]$.

B. Compactifications on hyperbolic spaces (Riemann surfaces)

Generic compact manifolds are negatively curved, and moduli stabilization has been outlined for a very special case of this—type IIB string theory on a product of three Riemann surfaces [30]. Let us therefore sketch the possibilities for linear inflation from axion monodromies in this class of compactifications.

Because the volume is directly stabilized, these models do not suffer from the η problem discussed in Sec. IVA in the Neveu-Schwarz axion (b) directions. Moreover, in contrast to the massive IIA models discussed in the previous subsection, the flux couplings of Sec. III C do not immediately lift all the b type axions. It is therefore possible that inflation with b type axions as well as c type axions might arise in this example. The main potential obstruction to this is the rich set of intersecting (p, q) seven-branes prescribed in [30]. Some of these—in particular those which combine to form O7-planes—impose boundary conditions that components of B_{MN} vanish which are fully transverse or fully parallel to the O7. The negative term in the moduli-fixing potential in [30] arises from triple intersections of (p, q) seven-branes (which contribute anomalous O3 tension as in [19]).

Finally, we note an intriguing feature of more general supercritical compactifications (of which a special case was studied in [29])—compactifications of D-dimensional type II string theory contain exponentially many RR axions, of order 2^D with D the total spacetime dimension. On the other hand, there are many *a priori* possible flux-induced masses for these axions. Again in this setting, a systematic analysis of axion monodromy inflation awaits further progress in the study of string compactification.

VI. OBSERVATIONAL PREDICTIONS

We have seen that the monodromy produced by wrapped branes yields a linear potential, over a super-Planckian distance, for the canonically normalized axion field. The leading corrections to this structure are periodic modulations induced by instantons.

Because of the natural exponential suppression of instanton effects, it is reasonably straightforward to arrange that these modulations are negligible, as we argued in our examples above. When this is the case, the linear inflaton potential gives for the tensor to scalar ratio r and tilt n_s of the power spectrum

$$r \approx 0.07 \qquad n_s \approx 0.975. \tag{81}$$

The uncertainty comes only from the usual fact that the number of e-folds is not known precisely in the absence of a specification of reheating. The resulting predictions are indicated in Fig. 4, which exhibits their consistency with current exclusion contours. We note that several authors have exhibited a preference in the data [44] for potentials with $V'' \leq 0$, which arises naturally in the case of monodromy-driven inflation. Upcoming CMB experiments promise to reduce these contours to $\mathcal{O}(10^{-2})$ in both directions, which will go a long way toward discriminating different inflationary mechanisms.

However, it is very interesting to consider the more general case in which the instanton-induced[15] modulations of the linear potential are non-negligible; one example like this might involve a vanishing v_+ in the models described in Sec. IV. In this case we must incorporate *oscillating*

[15]Shifts of the moduli during inflation may be contrived to give small corrections to the potential, but this requires inflationary energy that is marginally sufficient to destabilize the compactification. We expect that most successful models of inflation based on our mechanism have negligible corrections from this effect.

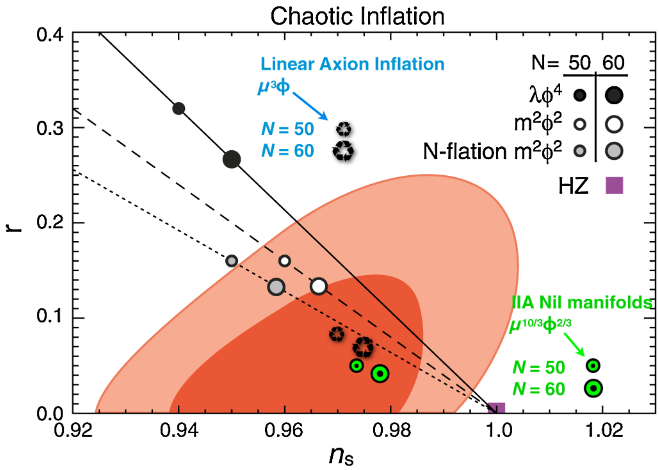

FIG. 4 (color online). Red: 5-year WMAP + BAO + SN [13] combined joint 68% and 95% error contours on (n_s, r). Recycling symbol: general prediction of the linear axion inflation potential $V(\phi_c) = \mu_c^3 \phi_c$, for $N = 50, 60$ e-folds before the end of inflation.

corrections to the slow-roll parameters, and correspondingly to the power spectrum. We leave a complete study of this case for the future. For now we note only that modulations of sufficiently high frequency but non-negligible amplitude may not affect the *average* tilt, but could conceivably lead to signatures in the more detailed structure of the power spectrum.

Let us also remark that the prospect of recurrent modulations of the perturbation spectrum is quite general in systems making use of the monodromy mechanism [10]: as the system moves repeatedly around the monodromy direction, it may interact periodically with localized degrees of freedom, including, for example, degrees of freedom into which the system reheats.

VII. DISCUSSION

Monodromy is a generic phenomenon in string compactifications. We have shown that the axion monodromy introduced by space-filling wrapped five-branes leads to a linear potential, over a super-Planckian distance, for the canonically normalized axion field. Axion monodromy therefore provides a mechanism for realizing chaotic inflation, with a linear potential, in string theory. We have shown that this mechanism is compatible with various methods of moduli stabilization, including nonperturbative stabilization of type IIB string theory on warped Calabi-Yau manifolds, as well as perturbative stabilization on more general Ricci-curved spaces. This produces a clear signature in the CMB of $r \approx 0.07$, with model-dependent opportunities for further, novel, signatures arising from oscillating corrections to the slow-roll parameters.

Our mechanism is reasonably robust and natural because of the presence of perturbative axionic shift symmetries. In our examples, the inflaton potential itself is the leading effect that breaks the shift symmetry, with instanton corrections naturally exponentially suppressed. We would like to remark that a related symmetry structure is plausibly also present in configurations with more general monodromies not involving axions. Monodromy in the potential energy arises when a would-be circle in a direction γ in the approximate moduli space is lifted by an additional ingredient whose potential energy grows as one moves in the γ direction. This unwraps the circle direction and extends the kinematic range of the corresponding field. Then, symmetries translating around the original circle do much to control the structure of the potential along the eventually unwrapped direction. Thus, monodromy-extended directions are not just long; they also generically profit from approximate symmetries. Monodromy-extended directions can be used for large-field inflation if the underlying moduli potential depends sufficiently weakly on γ and if all corrections to the slow-roll parameters are sufficiently suppressed; the classes of compactifications analyzed here and in [10] provide two particular realizations of this effect.[16]

There is much more to be done at the level of model building. The examples we have provided in this work are useful as proofs of principle, and to that end we have focused on demonstrating parametric suppression of corrections to the inflaton potential, and, in particular, on enumerating a wide array of mechanisms, such as warping, axionic symmetries, extended local supersymmetry, etc., that serve to control such contributions. We have not yet attempted to construct a minimal realization of linear axion inflation that uses the smallest possible subset of these control mechanisms. This is an interesting problem for future work, as methods for analyzing string compactifications and string-theoretic instantons improve.

A further lesson of this work, as of [10], is that in large-field models based on monodromy, a degree of suppression of otherwise problematic contributions to the potential that suffices for inflation is also sufficient to make firm predictions for the tilt of the scalar power spectrum. This is in sharp contrast to typical small-field models, where fine-tuning the inflaton potential to be flat enough for inflation is not a strong enough restriction to be predictive: slight variations in the fine-tuned contributions can noticeably change the tilt. The difference in our case is that the problematic terms arise as periodic modulations of the potential; requiring that inflation occurs at all implies that the amplitude of these modulations is small compared to the scale of changes in the inflaton potential itself. In turn, this implies that the *average* tilt is not affected at a detectable level by these modulations. On the other hand, it would be very interesting if the oscillations in the detailed power spectrum produced by a modulated linear potential had characteristic features accessible to future observa-

[16] Recently, monodromy has been used as a method to model chain inflation [45] in string theory [46].

tions. In any case, this class of models is falsifiable on the basis of its gravity wave signature.

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