PHYSICAL REVIEW D **82,** 045015 (2010)

Jackiw-Nohl-Rebbi two-instanton as a source of magnetic monopole loop

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(Received 19 May 2010; published 12 August 2010)

We demonstrate that a Jackiw-Nohl-Rebbi solution, as the most general two-instanton, generates a circular loop of magnetic monopole in four-dimensional Euclidean $SU(2)$ Yang-Mills theory, in contrast to the one-instanton solution in the regular gauge for which no such magnetic monopole loops exist. These results together with our previous result indicate that a two-instanton solution and a two-meron solution with the same asymptotic behavior in the long distance are responsible for quark confinement based on the dual superconductivity picture.

DOI: 10.1103/PhysRevD.82.045015 PACS numbers: 12.38.Aw, 21.65.Qr

I. INTRODUCTION

The quark confinement criterion *à la* Wilson [1] is given by the area decay of the Wilson loop average $\langle W_C[\mathcal{A}]\rangle$, i.e., the vacuum expectation value of the Wilson loop operator $W_C[\mathcal{A}]$ written in terms of the Yang-Mills field [2] $\mathcal{A}_\mu(x)$. In order to clarify the mechanism of quark confinement, it would be efficient to construct ensembles of gauge fields which exhibit confinement. One anticipates that a class of solutions of the classical Yang-Mills equation may give the dominant contribution to quark confinement. Among them, especially, topological (soliton) solutions [3] with nontrivial topological charge Q_P, i.e., the Pontryagin index, are good candidates for a first examination.

The well-known topological solutions are instantons [4–10] and merons [11–13]: The instanton is a solution of the self-duality equation ${}^*F = \pm F$ (first-order differential equation) and has an integer-valued topological charge $Q_P = \pm 1, \pm 2, \ldots$, while the meron is a solution of the original second-order differential equation obtained without imposing the self-duality condition and has a half-integer- or integer-valued topological charge $Q_P = \pm 1/2, \pm 1, \ldots$. Remarkably, the instanton has a finite Euclidean action proportional to the topological charge $S = 8\pi^2|Q_P|/g^2$ due to self-duality, while the meron has the logarithmically divergent Euclidean action due to short distance singularity, since the meron has no collective coordinate corresponding to the size. Therefore, the requirement of finite action inevitably excludes the meron solution from the candidates. However, it has been pointed out in [14] that merons can be responsible for the occurrence of the confinement phase, if we take into account the free energy based on the action-entropy argument, since the logarithmic divergent action is comparable with the entropy associated with the meron configurations calculable from the integration measure in the path-integral formulation.

It is believed that a promising mechanism for quark confinement is a dual superconductivity [15,16]. For this mechanism to work, however, magnetic monopoles must exist and be condensed to cause the dual superconductivity. In this picture, magnetic monopoles are considered to be the most important degrees of freedom relevant to confinement in the dual description. Therefore, we are lead to estimate how magnetic monopoles contribute to the Wilson loop average to fulfill the confinement criterion [17].

On the other hand, we can ask which configuration of the Yang-Mills field can be the source for such magnetic monopoles relevant to confinement in the dual description. The simplest configuration examined first was the one-instanton solution [4]. However, it has been confirmed in [18–21] that magnetic monopole loops are not generated from the one-instanton configuration, as briefly reviewed in the introduction of our previous paper [18]. Note that the magnetic monopole expressed by the current k is a topological object of codimension 3, therefore, it is a one-dimensional object (a closed current due to the topological conservation $\delta k = 0$) in the four-dimensional space, while it is a pointlike object in the three-dimensional space.

In previous papers, we have shown that a two-meron solution with a unit total topological charge $|Q_P| = 1$ leads to circular loops of magnetic monopole joining a pair of merons in an analytical way [18] and a numerical way [22], although a one-instanton and a two-meron have the same total topological charge. This result is in good agreement with the numerical result [23] obtained in the Laplacian Abelian gauge.

In this paper we examine the two-instanton solution of Jackiw-Nohl-Rebbi (JNR) [9] from the viewpoint raised above. In the conventional studies on quark confinement, the multi-instanton solution of 't Hooft type [6] has been used extensively to see the interplay between instantons and magnetic monopoles [20,24–28]. However, the

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't Hooft instanton is not the most general instanton solutions except for the one-instanton case in which the 't Hooft one-instanton agrees with the well-known one-instanton solution in the singular gauge. In contrast, the JNR two-instanton solution is the most general two-instanton solution with the full collective coordinates (moduli parameters), while the 't Hooft two-instanton solution [6] is obtained as a special limit of the JNR solution. We demonstrate in a numerical way that a circular loop of magnetic current k is generated for a JNR two-instanton solution. In addition, we present the configuration of the color field which plays the crucial role in our formulation. The implications of this result for quark confinement will be discussed in the final section.

Incidentally, the JNR two-instanton was used to study the relationship between dyonic instantons as a supertube connecting two parallel D4-branes and the magnetic monopole string loop as the supertube cross section in (4 + 1)-dimensional Yang-Mills-Higgs theory [29], since dyonic instantons of 't Hooft type do not show magnetic string and four-dimensional branes meet on isolated points, instead of some loop. These facts became one of the motivations to study the JNR solution from our point of view.

II. MAGNETIC MONOPOLE CURRENT ON A LATTICE

It is a nontrivial question how to realize the magnetic monopole in the pure Yang-Mills theory without a matter field, while in the Yang-Mills-Higgs system such as the Georgi-Glashow model, the magnetic monopole in a non-Abelian gauge theory has been constructed long ago [30].

In pure Yang-Mills theory in the absence of matter fields, two methods are currently known (to the best of our knowledge) for extracting magnetic monopole degrees of freedom:

(1) Abelian projection due to 't Hooft [31].
(2) Field decomposition due to Cho, Duan and Ge, Faddeev and Niemi, and Shabanov [32–35].

The first method (Abelian projection) has succeeded to exhibit the Abelian dominance [36,37] and magnetic monopole dominance [38] for quark confinement by adopting the maximally Abelian (MA) gauge [39]. See, e.g., [40] for a review. However, the following questions were raised for results obtained in the MA gauge, which are fundamental questions to be answered to establish the gauge-independent dual superconductivity in Yang-Mills theory: (1) how to extract the "Abelian" part responsible for quark confinement from the non-Abelian gauge theory in the gauge-independent way; and (2) how to define the magnetic monopole to be condensed in Yang-Mills theory in the gauge-invariant way even in the absence of any fundamental scalar field, in sharp contrast to the Georgi-Glashow model.

The MA gauge is reproduced as a special limit of the second method. In fact, the first method is nothing but a gauge-fixed version of the second method. In other words, the second method is a manifestly gauge-covariant reformulation of the first one. Therefore, the second method enables us to answer the above questions. From this viewpoint, the second method has been developed in a series of our papers [41–48], for $SU(2)$ gauge group and [49–51] for $SU(3)$ or $SU(N)$ gauge group. In the second method, a magnetic monopole can be defined in a manifestly gauge-invariant way using new variables obtained from the original Yang-Mills field by change of variables. Moreover, the Wilson loop operator can be exactly rewritten in terms of the magnetic monopole defined in this way through a non-Abelian Stokes theorem [52–54].

In order to define the magnetic monopole on a lattice, we recall the second method based on a nonlinear change of variables in a continuum $SU(2)$ Yang-Mills theory. We introduce a color field $\mathbf{n}(x)$ with a unit length

$$\mathbf{n}(x) = n^A(x)T^A, \qquad (T^A := \sigma_A/2) \tag{1}$$

$$n^A(x)n^A(x) = 1, \tag{2}$$

where σ_A ($A = 1, 2, 3$) are Pauli matrices.

The color field is determined by imposing a condition which we call the reduction condition. A reduction condition is given by minimizing the functional

$$F_{\rm red} := \int d^4x \frac{1}{2} \,{\rm tr}[\{D_\mu[\mathbf{A}]\mathbf{n}(x)\}^2]. \tag{3}$$

The local minima are given by the differential equation which we call the reduction differential equation (RDE) [18]

$$-D_\mu[\mathbf{A}]D_\mu[\mathbf{A}]\mathbf{n}(x) = \lambda(x)\mathbf{n}(x). \tag{4}$$

In this theory, a composite field

$$\mathbf{V}_\mu(x) := c_\mu(x)\mathbf{n}(x) - ig^{-1}[\partial_\mu\mathbf{n}(x), \mathbf{n}(x)] \tag{5}$$

plays an important role where $c_\mu(x) := 2\,{\rm tr}(\mathbf{n}(x)\mathbf{A}_\mu(x))$.

For instance, the field strength of $\mathbf{V}_\mu(x)$ is parallel to $\mathbf{n}(x)$:

$$\begin{aligned}\mathbf{F}_{\mu\nu}[\mathbf{V}] &= \partial_\mu\mathbf{V}_\nu - \partial_\nu\mathbf{V}_\mu - ig[\mathbf{V}_\mu, \mathbf{V}_\nu] \\ &= \{\partial_\mu c_\nu - \partial_\nu c_\mu + 2ig^{-1}\,{\rm tr}(\mathbf{n}[\partial_\mu\mathbf{n}, \partial_\nu\mathbf{n}])\}\mathbf{n} \\ &:= G_{\mu\nu}\mathbf{n}.\end{aligned} \tag{6}$$

Thus, we can define the gauge-invariant field strength $G_{\mu\nu}(x) = 2\,{\rm tr}(\mathbf{n}\mathbf{F}_{\mu\nu}[\mathbf{V}])$ and the gauge-invariant monopole current as

$$k^\mu(x) := \partial_\nu {}^*G^{\mu\nu}(x) = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\partial_\nu G_{\rho\sigma}(x). \tag{7}$$

Once the reduction condition is solved, thus, we can obtain the monopole current k^μ from the original gauge field $\mathbf{A}_\mu$. The gauge-invariant magnetic charge is defined

$$q_m := \int d^3\sigma_\mu k^\mu(x), \tag{8}$$

in a Lorentz (or Euclidean rotation) invariant way [17].

In this paper, we carry out the procedures explained in the above in a numerical way. We use the lattice regularization for numerical calculations where the link variable $U_{x,\mu}$ is related to a gauge field in a continuum theory by

$$U_{x,\mu} = P\exp\left\{ig\int_x^{x+a\hat{\mu}} dy\mathbf{A}_\mu(y)\right\}, \tag{9}$$

where P represents a path-ordered product, a is a lattice spacing, and $\hat{\mu}$ represents the unit vector in the μ direction. The lattice version of the reduction functional in $SU(2)$ Yang-Mills theory is given by

$$F_{\text{red}}[\mathbf{n}, U] = \sum_{x,\mu}\{1\text{–}4\,\text{tr}(U_{x,\mu}\mathbf{n}_{x+a\hat{\mu}}U^\dagger_{x,\mu}\mathbf{n}_x)/\text{tr}(\mathbf{1})\}, \tag{10}$$

where $\mathbf{n}_x$ is a unit color field on a site x,

$$\mathbf{n}_x = n_x^A T^A, \qquad n_x^A n_x^A = 1. \tag{11}$$

We introduce the Lagrange multiplier λ_x to incorporate the constraint of unit length for the color field (11). Then the stationary condition for the reduction functional is given by

$$\frac{\partial}{\partial n_x^A}\left\{F_{\text{red}}[\mathbf{n}, U] - \frac{1}{2}\sum_x \lambda_x(n_x^A n_x^A - 1)\right\} = 0. \tag{12}$$

When F_{red} takes a local minimum for a given and fixed configuration $\{U_{x,\mu}\}$, therefore, a Lagrange multiplier λ_x satisfies

$$W_x^A = \lambda_x n_x^A, \tag{13}$$

and the color field n_x^A satisfies

$$n_x^A n_x^A = 1, \tag{14}$$

where

$$W_x^A = 4\sum_{\mu=1}^{4}\text{tr}(U_{x,\mu}\mathbf{n}_{x+a\hat{\mu}}U^\dagger_{x,\mu}T^A + U_{x-a\hat{\mu},\mu}T^A U^\dagger_{x-a\hat{\mu},\mu}\mathbf{n}_{x-a\hat{\mu}})/\text{tr}(\mathbf{1}). \tag{15}$$

Equation (13) is a lattice version of the reduction differential equation. We are able to eliminate the Lagrange multiplier to rewrite (13) into

$$n_x^A = \frac{W_x^A}{\sqrt{W_x^B W_x^B}}. \tag{16}$$

A derivation of this equation is given in the appendix. The color field configurations $\{\mathbf{n}_x\}$ are obtained by solving (16) in a numerical way.

After obtaining the $\{\mathbf{n}_x\}$ configuration for given configurations $\{U_{x,\mu}\}$ in this way, we introduce a new link variable $V_{x,\mu}$ on a lattice corresponding to the gauge potential (5) by

$$V_{x,\mu} = \frac{L_{x,\mu}}{\sqrt{\frac{1}{2}\,\text{tr}[L_{x,\mu}L^\dagger_{x,\mu}]}}, \qquad L_{x,\mu} := U_{x,\mu} + \mathbf{n}_x U_{x,\mu}\mathbf{n}_{x+a\hat{\mu}}. \tag{17}$$

Finally, the monopole current $k_{x,\mu}$ on a lattice is constructed as

$$k_{x,\mu} = \sum_{\nu,\rho,\sigma}\frac{\epsilon_{\mu\nu\rho\sigma}}{4\pi}\frac{\Theta_{x+a\hat{\nu},\rho\sigma}[\mathbf{n}, V] - \Theta_{x,\rho\sigma}[\mathbf{n}, V]}{a}, \tag{18}$$

through the angle variable of the plaquette variable

$$\Theta_{x,\mu\nu}[\mathbf{n}, V] = a^{-2}\arg(\text{tr}\{(\mathbf{1} + \mathbf{n}_x)V_{x,\mu}V_{x+a\hat{\mu},\nu}V^\dagger_{x+a\hat{\nu},\mu}V^\dagger_{x,\nu}\}/\text{tr}(\mathbf{1})). \tag{19}$$

In this definition, $k_{x,\mu}$ takes an integer value [44,45].

To obtain the $\{\mathbf{n}_x\}$ configuration satisfying (16), we recursively apply (16) to $\mathbf{n}_x$ on each site x and update it, keeping $\mathbf{n}_x$ fixed at a boundary ∂V of a finite lattice V until F_{red} converges. Since we calculate the $\{k_{x,\mu}\}$ configuration for the instanton configuration in this paper, we need to decide a boundary condition of the $\{\mathbf{n}_x\}$ configuration in the instanton case. We recall that the instanton configuration approaches a pure gauge at infinity:

$$g\mathbf{A}_\mu(x) \to ih^\dagger(x)\partial_\mu h(x) + O(|x|^{-2}). \tag{20}$$

Then, $\mathbf{n}(x)$ as a solution of the reduction condition is supposed to behave asymptotically

$$\mathbf{n}(x) \to h^\dagger(x)T_3 h(x) + O(|x|^{-\alpha}), \tag{21}$$

for a certain value of $\alpha > 0$. Under this idea, we adopt a boundary condition as

$$\mathbf{n}_x^{\text{bound}} := h^\dagger(x)T_3 h(x), \qquad x \in \partial V. \tag{22}$$

In practice, we start with an initial state of the $\{\mathbf{n}_x\}$ configuration: $\mathbf{n}_x^{\text{init}} = h^\dagger(x)T_3 h(x)$ for $x \in V$. Then, we repeat updating $\mathbf{n}_x$ on each site x according to (16) except for the configuration $\mathbf{n}_x^{\text{bound}}$ on the boundary ∂V.

It should be remarked that these asymptotic forms (20) and (21) satisfy the RDE asymptotically in the sense that

$$D_\mu[\mathbf{A}]\mathbf{n}(x) \to 0 \qquad (|x| \to \infty), \tag{23}$$

together with

$$\lambda(x) \to 0 \qquad (|x| \to \infty), \tag{24}$$

which is necessary to obtain a finite value for the reduction functional [18]

$$F_{\text{red}} = \int d^4x\frac{1}{2}\lambda(x) < \infty. \tag{25}$$

III. ONE-INSTANTON IN THE REGULAR GAUGE

The one-instanton solution in the regular (or nonsingular) gauge is specified by a constant four-vector representing the center $(b^1, b^2, b^3, b^4) \in \mathbb{R}^4$ and a positive real constant representing the size (width) $\rho \geq 0$:

$$g\mathbf{A}_\mu(x) = T^A \eta^{A(+)}_{\mu\nu} \frac{2(x^\nu - b^\nu)}{|x-b|^2 + \rho^2}, \tag{26}$$

where $|x|^2 = x_\mu x_\mu$ is the standard Euclidean norm and $\eta^{A(\pm)}_{\mu\nu}$ is the symbol defined by

$$\eta^{A(\pm)}_{\mu\nu} = \epsilon_{A\mu\nu 4} \pm \delta_{A\mu}\delta_{\nu 4} \mp \delta_{A\nu}\delta_{\mu 4}. \tag{27}$$

In this case, we obtain from (20) and (21)

$$h(x) = \frac{x_\mu}{|x|} e_\mu, \qquad (e_\mu \equiv (-i\sigma_i, \mathbf{1})), \tag{28}$$

where $\mathbf{1}$ is a 2×2 unit matrix and

$$h^\dagger(x) T_3 h(x) = \frac{2(x_1 x_3 - x_2 x_4)}{x^2} T_1 + \frac{2(x_1 x_4 + x_2 x_3)}{x^2} T_2 + \frac{-x_1^2 - x_2^2 + x_3^2 + x_4^2}{x^2} T_3. \tag{29}$$

This exactly agrees with the standard Hopf map [55].

The topological charge density is maximal at the point $x = b$ and decreases algebraically with the distance from this point in such a way that the instanton charge Q_V inside the finite lattice $V = [-aL, aL]^4$ reproduces the total instanton charge $Q_P = 1$. We construct the instanton charge Q_V on a lattice from the configuration of link variables $\{U_{x,\mu}\}$ according to

$$Q_V = a^4 \sum_{x \in \{V - \partial V\}} D_x, \tag{30}$$

$$D_x := \frac{1}{2^4} \frac{a^{-4}}{32\pi^2} \sum_{\mu,\nu,\rho,\sigma=\pm 1}^{\pm 4} \hat{\epsilon}^{\mu\nu\rho\sigma} \operatorname{tr}(\mathbf{1} - U_{x,\mu\nu} U_{x,\rho\sigma}), \tag{31}$$

$$U_{x,\mu\nu} = U_{x,\mu} U_{x+a\hat{\mu},\nu} U^\dagger_{x+a\hat{\nu},\mu} U^\dagger_{x,\nu}, \tag{32}$$

where D_x is a lattice version of the instanton charge density, $V - \partial V$ represents the volume without a boundary, and $\hat{\epsilon}$ is related to the usual ϵ tensor by

$$\hat{\epsilon}^{\mu\nu\rho\sigma} := \operatorname{sgn}(\mu)\operatorname{sgn}(\nu)\operatorname{sgn}(\rho)\operatorname{sgn}(\sigma)\epsilon^{|\mu||\nu||\rho||\sigma|}, \quad \operatorname{sgn}(\mu) := \frac{\mu}{|\mu|}. \tag{33}$$

Our interest is the support of $k_{x,\mu}$, namely, a set of links $\{x, \mu\}$ on which $k_{x,\mu}$ takes nonzero values $k_{x,\mu} \neq 0$. This expresses the location of the magnetic monopole current generated for a given instanton configuration. By definition (18), the number of configurations $\{k_{x,\mu}\}$ are $(2L)^4 \times 4$. [The number of configurations $\{k_{x,\mu}\}$ is not equal to $(2L + 1)^4 \times 4$, because we cannot calculate $k_{x,\mu}$ at positive sides of the boundary ∂V due to the definition of (18) based on the forward lattice derivative.]

TABLE I. The distribution of $k_{x,\mu}$ and the instanton charge Q_V for a given r.

ρ	$\lvert k_{x,\mu}\rvert > 1$	$k_{x,\mu} = -1$	$k_{x,\mu} = 0$	$k_{x,\mu} = 1$	Q_V
5	0	4	59 105 336	4	0.9674
10	0	8	59 105 328	8	0.9804
15	0	12	59 105 320	12	0.9490
20	0	12	59 105 320	12	0.8836

The results are summarized in Table I and Fig. 1. In our calculations of the monopole current configuration, we fix the center on the origin

$$(b^1, b^2, b^3, b^4) = (0, 0, 0, 0), \tag{34}$$

and change the value of ρ.

For a choice of $L = 31$, the total number of configurations $\{k_{x,\mu}\}$ are $62^4 \times 4 = 59\,105\,344$. Although the current $k_{x,\mu}$ is zero on almost all the links (x, μ), it has a nonzero value $|k_{x,\mu}| = 1$ on a small number of links, e.g., $4 + 4$ links for $\rho = 5a$. The number of links with $k_{x,\mu} = +1$ is equal to one with $k_{x,\mu} = -1$, which reflects the fact that the current $k_{x,\mu}$ draws a closed path of links. It turns out that there are no configurations such that $|k_{x,\mu}| > 1$.

We see how Q_V reproduces the total instanton charge $Q_P = 1$ for the choice of ρ. The instanton charge density is equal to zero when $\mathbf{A}_\mu(x)$ is a pure gauge, i.e., $\mathbf{F}_{\mu\nu}(x) = 0$. Therefore, the more rapidly $\mathbf{A}_\mu(x)$ converge to a pure gauge, the more precisely Q_V reproduces the proper value Q_P for a given L. For one-instanton in the regular gauge, clearly, $\mathbf{A}_\mu(x)$ more rapidly converge to a pure gauge for smaller ρ. Indeed, Q_V for $\rho = 10a$ reproduces $Q_P = 1$ more precisely than $\rho = 15a$ and $\rho = 20a$. On the contrary, Q_V for $\rho = 5a$ reproduces $Q_P = 1$ poorly compared with $\rho = 10a$. This is because the lattice is too coarse (a lattice spacing is not sufficiently small) to estimate properly the rapid change of the instanton charge distribution concentrated to the neighborhood of the origin for small ρ.

If $\mathbf{A}_\mu(x)$ converge rapidly to a pure gauge $ih^\dagger(x)\partial_\mu h(x)$ so that Q_V gives a good approximation for an expected value Q_P, then the color field is expected to behave as $\mathbf{n}(x) \to h^\dagger(x) T_3 h(x)$ asymptotically and our choice of the boundary condition $\mathbf{n}^{\text{bound}}_x := h^\dagger(x) T_3 h(x)$ at the boundary $x \in \partial V$ is well motivated.

In Fig. 1, the support of $k_{x,\mu}$ is drawn by projecting the four-dimensional space on the $x_4 = 0$ hyperplane (three-dimensional space) for the choice of $\rho = 5a$, $10a$, $15a$, and $20a$. This figure shows that the nonzero monopole current forms a small loop. The size of the magnetic monopole loop hardly changes while ρ increases. This is an indication that the magnetic monopole loop for the one-instanton

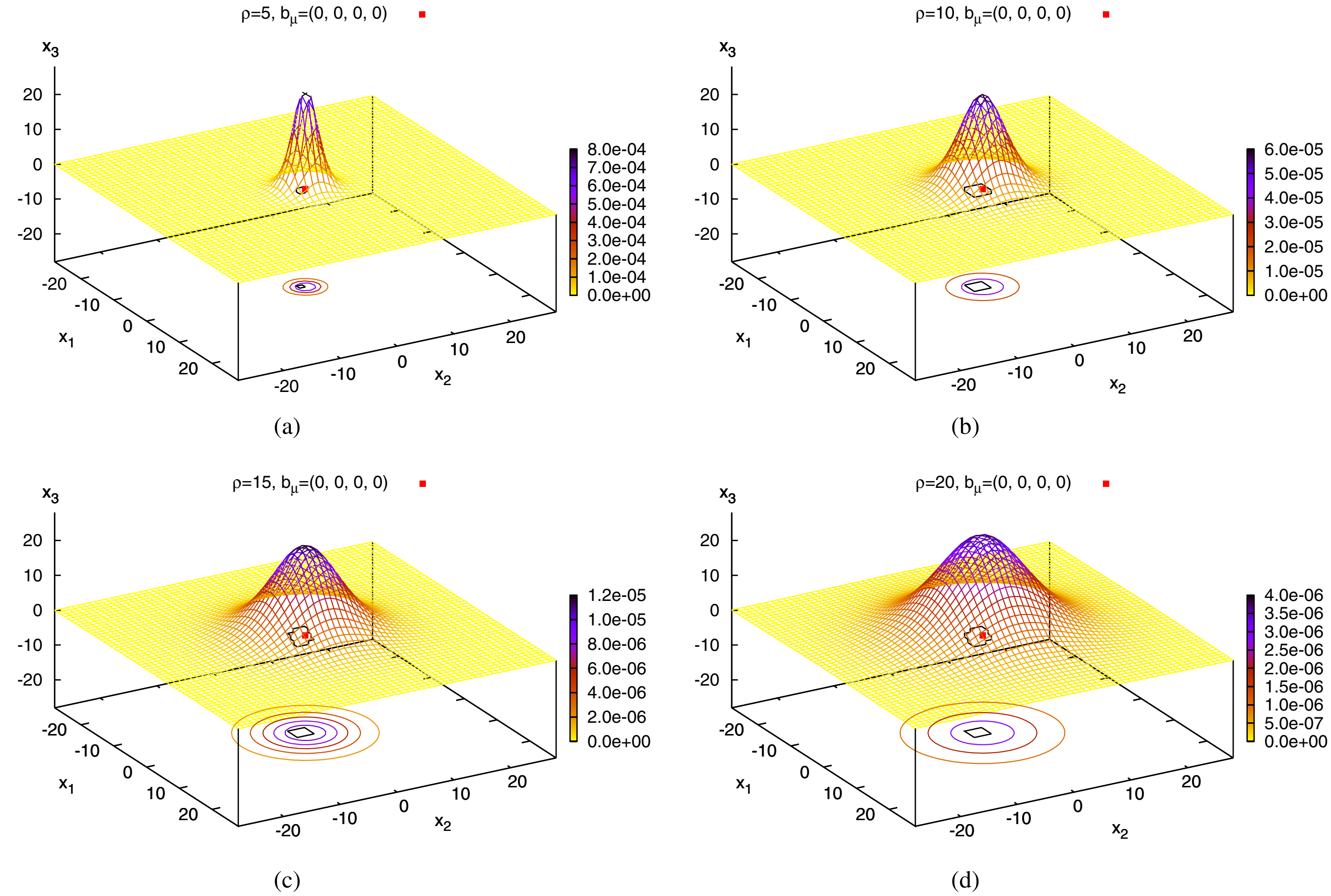

FIG. 1 (color online). One-instanton in the regular gauge and the associated magnetic-monopole current $k_{x,\mu}$ for various choices of size parameter ρ: (a) $\rho = 5a$, (b) $\rho = 10a$, (c) $\rho = 15a$, and (d) $\rho = 20a$. The grid shows an instanton charge density D_x on the x_1-x_2 ($x_3 = x_4 = 0$) plane. The black (thick) line on the base shows the magnetic monopole loop projected and the colored (thin) lines show a contour plot of the instanton charge density. Figures are drawn in units of a.

solution disappears in the continuum limit of the lattice spacing a going to zero.

IV. TWO-INSTANTON OF THE JACKIW-NOHL-REBBI TYPE

It is known by the Atiyah-Hitchin-Drinfeld-Manin construction [10] that the N-instanton moduli space has dimension $8N$. For $N = 1$, the 8 moduli parameters are interpreted as $4 + 1 + 3$ degrees of freedom for the position, size, and $SU(2)$ orientation (global gauge rotations), respectively.

For $N = 2$, the JNR instanton [9] is the most general two-instanton as explained below. The explicit form of the JNR two-instanton solution is

$$gA_\mu(x) = -T^A \eta^{A(-)}_{\mu\nu} \partial_\nu \ln\phi_{\mathrm{JNR}}, \tag{35}$$

$$= T^A \eta^{A(-)}_{\mu\nu} \phi^{-1}_{\mathrm{JNR}} \sum_{r=0}^{2} \frac{2\rho_r^2 (x^\nu - b_r^\nu)}{(|x - b_r|^2)^2}, \tag{36}$$

$$\phi_{\mathrm{JNR}} := \sum_{r=0}^{2} \frac{\rho_r^2}{|x - b_r|^2}. \tag{37}$$

The JNR two-instanton has $4 \times 3 + 3 + 3 = 18$ parameters, which consist of three pole positions $(b_0^1, b_0^2, b_0^3, b_0^4)$, $(b_1^1, b_1^2, b_1^3, b_1^4)$, $(b_2^1, b_2^2, b_2^3, b_2^4)$ and three scale parameters ρ_0, ρ_1, ρ_2 including the overall $SU(2)$ orientation. Note that the number of poles is one greater than the number of the instanton charge. Although the parameter count of the JNR two-instanton appears to exceed the 16 dimensions of the $N = 2$ moduli space, the JNR two-instanton has precisely the required number of parameters for the $N = 2$ general solution. In fact, one parameter is reduced by noting that the multiplication of the scale parameter by a constant does not alter the solution, so only the ratios ρ_r/ρ_0 ($r = 1, 2$) are relevant. Moreover, one of the degrees of freedom corresponds to a gauge transformation [9].

The 't Hooft two-instanton, which is more popular and has been used extensively in the preceding investigations, is given by

$$\phi_{\rm tHooft} := 1 + \sum_{r=1}^{2} \frac{\rho_r^2}{|x - b_r|^2}. \tag{38}$$

The 't Hooft two-instanton has only $4 \times 2 + 2 + 3 = 13$ parameters, which consist of two pole positions $(b_1^1, b_1^2, b_1^3, b_1^4)$, $(b_2^1, b_2^2, b_2^3, b_2^4)$ and two scale parameters ρ_1, ρ_2 including the overall $SU(2)$ orientation. The 't Hooft solution is reproduced from the JNR solution in the limit $\rho_0 = |b_0| \to \infty$, namely, the location of the first pole b_0 is sent to infinity keeping the relation $\rho_0 = |b_0|$.

The crucial difference between the JNR and the 't Hooft solutions is the asymptotic behavior. The JNR solution goes to zero slowly, while the gauge potential produced by the 't Hooft ansatz tends rapidly to zero at spatial infinity $|x| \to \infty$. In fact, for $N = 1$ the 't Hooft ansatz gives the one-instanton in the singular gauge with the asymptotic behavior

$$\mathbf{A}_\mu(x) \sim O(|x|^{-3}) \qquad |x| \to \infty, \tag{39}$$

while the one-instanton in the regular gauge exhibits the asymptotic behavior

$$\mathbf{A}_\mu(x) \sim O(|x|^{-1}) \qquad |x| \to \infty. \tag{40}$$

In the case of the JNR two-instanton, we obtain

$$h(x) = \frac{x_\mu}{|x|} \bar{e}_\mu, \qquad (\bar{e}_\mu \equiv e_\mu^\dagger = (i\sigma_i, \mathbf{1})) \tag{41}$$

and

$$h^\dagger(x) T_3 h(x) = \frac{2(x_1x_3 + x_2x_4)}{x^2} T_1 + \frac{2(-x_1x_4 + x_2x_3)}{x^2} T_2 + \frac{-x_1^2 - x_2^2 + x_3^2 + x_4^2}{x^2} T_3. \tag{42}$$

This is another form of the standard Hopf map [55].

We equate three size parameters and put three pole positions $(b_0^1, b_0^2, b_0^3, b_0^4)$, $(b_1^1, b_1^2, b_1^3, b_1^4)$, $(b_2^1, b_2^2, b_2^3, b_2^4)$ on the $x_3 = x_4 = 0$ plane, so that the three poles are located at the vertices of an equilateral triangle:

$$\rho_0 = \rho_1 = \rho_2 \equiv \rho, \tag{43}$$

$$(b_0^1, b_0^2, b_0^3, b_0^4) = (r, 0, 0, 0) + \Delta, \tag{44}$$

$$(b_1^1, b_1^2, b_1^3, b_1^4) = \left(-\frac{r}{2}, \frac{\sqrt{3}}{2} r, 0, 0\right) + \Delta, \tag{45}$$

$$(b_2^1, b_2^2, b_2^3, b_2^4) = \left(-\frac{r}{2}, -\frac{\sqrt{3}}{2} r, 0, 0\right) + \Delta, \tag{46}$$

where Δ is a small parameter introduced to avoid the pole singularities at $x = b_r$.

In the numerical calculation, we choose

$$\Delta = (0.1a, 0.1a, 0.1a, 0.1a). \tag{47}$$

Then we have searched for monopole currents by changing r. We have adopted $L = 31$. In this paper we focus on the special case $\rho_0 = \rho_1 = \rho_2 \equiv \rho$ of the JNR solution. Then the ratio is uniquely fixed: $\rho_1/\rho_0 = \rho_2/\rho_0 = \rho/\rho_0 \equiv 1$, and hence ρ can be set to an arbitrary value without loss of generality in this class. In the following, we put $\rho = \rho_0 = 3a$.

TABLE II. The distribution of $k_{x,\mu}$ and the instanton charge Q_V for a given r.

r	$\lvert k_{x,\mu}\rvert > 1$	$k_{x,\mu} = -1$	$k_{x,\mu} = 0$	$k_{x,\mu} = 1$	l/r	Q_V
5	0	14	59 105 316	14	5.6	1.903
10	0	26	59 105 292	26	5.2	1.969
15	0	40	59 105 264	40	5.3	1.950
20	0	51	59 105 242	51	5.1	1.862

The results are summarized in Table II and Figs. 2 and 3. In Fig. 2, as in the case of one-instanton in the regular gauge, we draw the distribution of the instanton charge density D_x and the support of the magnetic monopole current $k_{x,\mu}$ projected on the $x_4 = 0$ hyperspace for $r = 5a, 10a, 15a, 20a$.

The instanton charge density D_x of the JNR two-instanton takes the maximal value at a circle with radius R_I, rather than on the origin, on the x_1-x_2 plane. This is not the case for the 't Hooft two-instanton in which the instanton charge distribution concentrates near the two pole positions, as is well-known.

We have found that nonvanishing monopole currents originating from the JNR two-instanton forms a circular loop. The circular loops of the magnetic monopole current are located on the same plane as that specified by three poles b_0, b_1, b_2. The size of the circular loop, e.g., the radius R, increases proportionally as r increases and the circular loops constitute concentric circles with the center at the origin, within the accuracy of our numerical calculations.

To reproduce the correct distribution of the instanton charge density D_x in the numerical calculations, we need to further approximate the link variable $U_{x,\mu}$ defined by (9) using a discretization with a width $\epsilon = a/n$ for the line integral on a link:

$$U_{x,\mu} \approx \exp\left\{ i\epsilon \sum_{k=1}^{n} \frac{1}{2}[g\mathbf{A}_\mu(x + (k-1)\epsilon\hat{\mu}) + g\mathbf{A}_\mu(x + k\epsilon\hat{\mu})]\right\}. \tag{48}$$

We have chosen the lattice spacing $a = 1$ and the total number of partition points $n = 20$. If n is relatively small, the numerical result for the instanton charge density D_x gives a wrong distribution with remnant of three poles.

In Table II, we find that the ratio between the length $\ell := \sum_{x,\mu} |k_{x,\mu}|$ of the magnetic monopole current $k_{x,\mu}$ and the size r of the equilateral triangle of JNR is nearly constant,

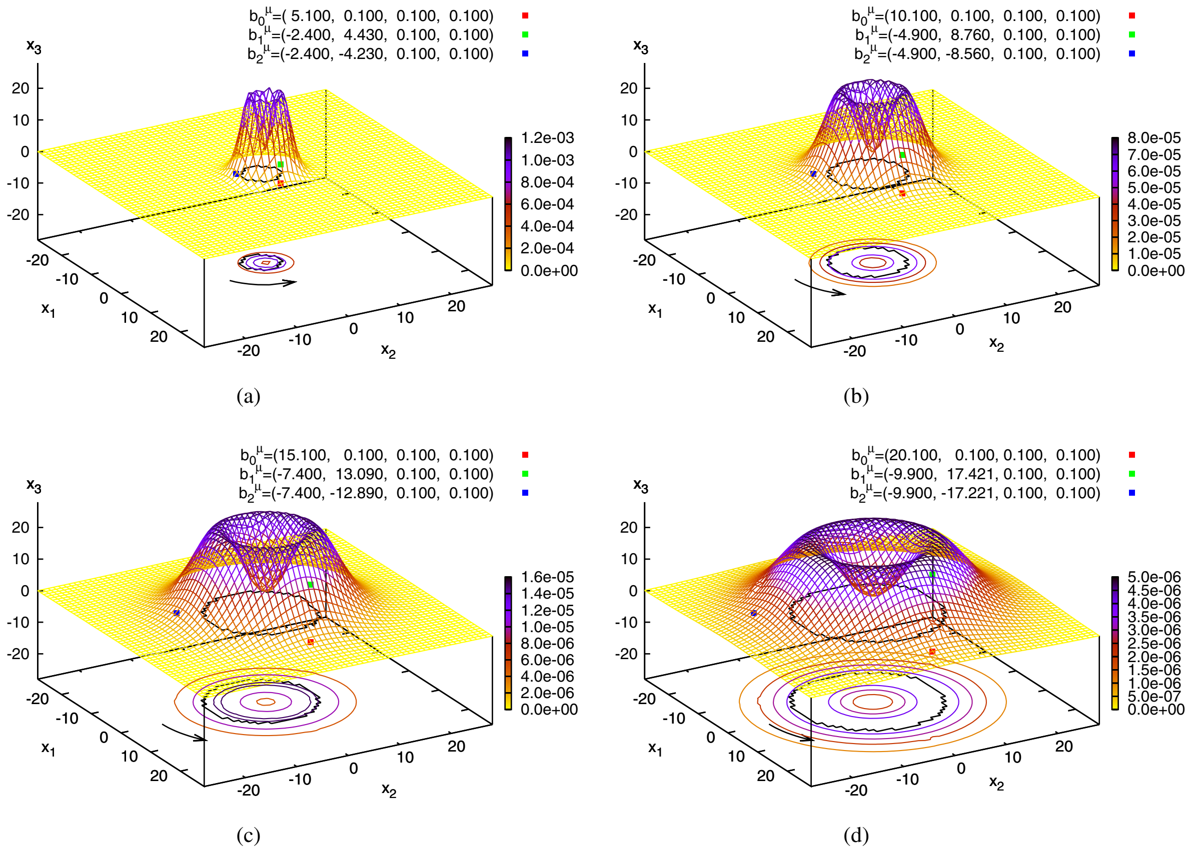

FIG. 2 (color online). JNR two-instanton and the associated circular loop of the magnetic monopole current $k_{x,\mu}$. The JNR two-instanton is defined by fixing three scales $\rho_0 = \rho_1 = \rho_2 = 3a$ and three pole positions b_0^μ, b_1^μ, b_2^μ which are arranged to be three vertices of an equilateral triangle specified by r: (a) $r = 5a$, (b) $r = 10a$, (c) $r = 15a$, and (d) $r = 20a$. The grid shows an instanton charge density D_x on the x_1-x_2 ($x_3 = x_4 = 0$) plane. The associated circular loop of the magnetic monopole current is located on the same plane as that specified by three poles. The black (thick) line on the base shows the magnetic monopole loop projected on the x_1-x_2 plane and the arrow indicates the direction of the monopole current, while colored (thin) lines on the base show the contour plot for the equi-D_x lines. Figures are drawn in units of a.

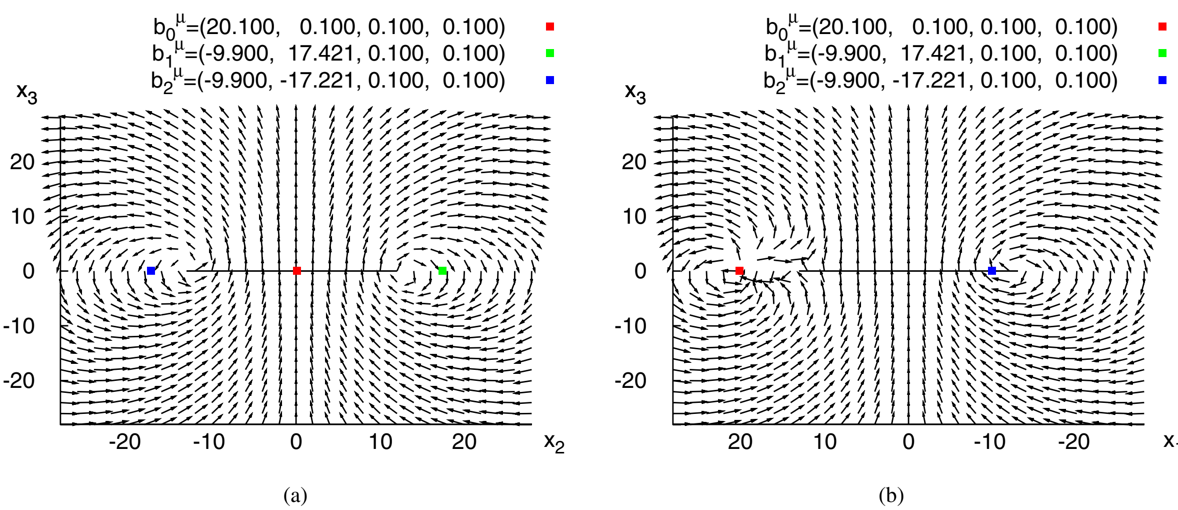

FIG. 3 (color online). The configuration of the color field $\mathbf{n}_x = (n_x^1, n_x^2, n_x^3)$ and a circular loop of the magnetic monopole current $k_{x,\mu}$ obtained from the JNR two-instanton solution ($r = 20a$), viewed in (a) the x_2-x_3 ($x_1 = x_4 = 0$) plane which is off three poles, and (b) the x_1-x_3 ($x_2 = x_4 = 0$) plane which goes through a pole b_0^μ. The magnetic monopole current $k_{x,\mu}$ and the three poles of the JNR solution are projected onto the same plane. Here the $SU(2)$ color field (n_x^1, n_x^2, n_x^3) is identified with a unit vector in the three-dimensional space (x_1, x_2, x_3). Figures are drawn in units of a.

$$\ell/r \simeq 5.2, \tag{49}$$

which implies

$$R_m/r \simeq 0.65, \tag{50}$$

where we have used a relation $\ell \simeq 8R_m$ for large ℓ since a closed current $k_{x,\mu}$ consists of links on a lattice and the relation $\ell \simeq 2\pi R_m$ in the continuum does not hold. Moreover, the magnetic monopole loop passes along the neighborhood of contour giving the absolute maxima of the instanton charge density

$$R_m/R_I \simeq 0.65/0.54 \simeq 1.2, \tag{51}$$

where $R_I \simeq 0.54r$.

Figure 3 shows the relationship between the magnetic monopole loop $k_{x,\mu}$ and the color field $\mathbf{n}_x$ configuration. The vector field $\{\mathbf{n}_x\}$ is winding around the loop, and it is indeterminate at points where the loop pass. This is the first time that an explicit configuration of a color field leading to the magnetic monopole loop has been obtained from an instanton solution based on the new reformulation of Yang-Mills theory.

V. CONCLUSION AND DISCUSSION

For given instanton solutions of the classical Yang-Mills equation in the four-dimensional Euclidean space, we have solved in a numerical way the reduction condition to obtain the color field which plays the key role to define a gauge-invariant magnetic monopole in our reformulation of the Yang-Mills theory written in terms of new variables. Here we have used a lattice regularization [44] for performing numerical calculations. Then we have constructed the magnetic monopole current k_μ on a dual lattice where the resulting magnetic charge is gauge invariant and quantized according to the quantization condition of the Dirac type.

For the two-instanton solution of the Jackiw-Nohl-Rebbi type, we have discovered that the magnetic monopole current k_μ has the support on a circular loop which is located near the maxima of the instanton charge density. Thus, we have shown that the two-instanton solution of the Jackiw-Nohl-Rebbi type generates the magnetic monopole loop in four-dimensional $SU(2)$ Yang-Mills theory. In the same setting, we have found that the magnetic current has the support only on a plaquette around the center of the one-instanton. This result confirms that no magnetic monopole loop is generated for a one-instanton solution in the continuum limit.

Combining the result in this paper with the previous one [18], we have found that both the JNR two-instanton solution and two-meron solution with the same asymptotic behavior at spatial infinity

$$\mathbf{A}_\mu(x) \sim O(|x|^{-1}) \qquad |x| \to \infty \tag{52}$$

generate circular loops of magnetic monopole, which should be compared with the 't Hooft multi-instanton with the asymptotic behavior at spatial infinity

$$\mathbf{A}_\mu(x) \sim O(|x|^{-3}) \qquad |x| \to \infty. \tag{53}$$

We expect that these loops of magnetic monopole are responsible for confinement in the dual superconductivity picture. This result seems to be consistent with the claim made in [56,57]. However, the correspondence between the instanton and magnetic monopole loop is not one-to-one. To draw the final conclusion, we need to collect more data for supporting this claim. In particular, it is not yet clear which relationship between instanton charge and the magnetic charge holds in our case, as studied in [58–60].

Moreover, it is known at finite temperature that there exist self-dual solutions with nontrivial holonomy (calorons) [61] which exhibit a nontrivial monopole content by construction. It will be interesting to study how magnetic monopoles, to be obtained in our approach from calorons, are related to dyons inherent in calorons. These issues will be investigated in future works.

ACKNOWLEDGMENTS

The authors would like to thank Koji Hashimoto for very helpful discussions on instantons in the early stage of this work. This work is financially supported by Grant-in-Aid for Scientific Research (C) 21540256 from the Japan Society for the Promotion of Science (JSPS).

APPENDIX: DERIVING THE REDUCTION EQUATION ON A LATTICE

Taking the square root of both sides of (13) yields

$$\lambda_x = \pm\sqrt{W_x^A W_x^A}, \tag{A1}$$

which is substituted into (13) to obtain

$$n_x^A = \pm \frac{W_x^A}{\sqrt{W_x^B W_x^B}}. \tag{A2}$$

We must choose a correct sign in the right-hand side of Eq. (A2) so that a solution of this equation gives a solution of the original Eq. (13).

In what follows, we adopt a direct method of choosing a correct sign of the right-hand side by examining whether Eq. (A2) converges to the RDE in the continuum limit $a \to 0$. By substituting $U_{x,\mu} = e^{-iag\mathbf{A}_\mu(x)}$ into (15) and expanding it in powers of a, W_x^A is written as

$$\begin{aligned}
W_x^A &= 4\sum_{\mu=1}^{4} \mathrm{tr}[\{e^{-iag\mathbf{A}_\mu(x)}\mathbf{n}(x+a\hat{\mu})e^{iag\mathbf{A}_\mu(x)} + e^{iag\mathbf{A}_\mu(x-a\hat{\mu})}\mathbf{n}(x-a\hat{\mu})e^{-iag\mathbf{A}_\mu(x-a\hat{\mu})}\}T^A]/\mathrm{tr}(\mathbf{1}) \\
&= 4\sum_{\mu=1}^{4} \mathrm{tr}\Bigg[\Bigg\{\mathbf{n}(x+a\hat{\mu}) - ia[g\mathbf{A}_\mu(x), \mathbf{n}(x+a\hat{\mu})] - \frac{a^2}{2}[g\mathbf{A}_\mu(x), [g\mathbf{A}_\mu(x), \mathbf{n}(x+a\hat{\mu})]] + \mathbf{n}(x-a\hat{\mu}) \\
&\quad + ia[g\mathbf{A}_\mu(x-a\hat{\mu}), \mathbf{n}(x-a\hat{\mu})] - \frac{a^2}{2}[g\mathbf{A}_\mu(x-a\hat{\mu}), [g\mathbf{A}_\mu(x-a\hat{\mu}), \mathbf{n}(x-a\hat{\mu})]]\Bigg\}T^A\Bigg]/\mathrm{tr}(\mathbf{1}) + O(a^3) \\
&= 4\sum_{\mu=1}^{4} \mathrm{tr}[\{2\mathbf{n}(x) + a^2\partial_\mu\partial_\mu\mathbf{n}(x) - 2ia^2[g\mathbf{A}_\mu(x), \partial_\mu\mathbf{n}(x)] - ia^2[g\partial_\mu\mathbf{A}_\mu(x), \mathbf{n}(x)] \\
&\quad - a^2[g\mathbf{A}_\mu(x), [g\mathbf{A}_\mu(x), \mathbf{n}(x)]]\}T^A]/\mathrm{tr}(\mathbf{1}) + O(a^3) \\
&= 4\sum_{\mu=1}^{4} \mathrm{tr}[\{2\mathbf{n}(x) + a^2 D_\mu[\mathbf{A}]D_\mu[\mathbf{A}]\mathbf{n}(x)\}T^A]/\mathrm{tr}(\mathbf{1}) + O(a^3) = 8n^A(x) + a^2\sum_{\mu=1}^{4}(D_\mu[\mathbf{A}]D_\mu[\mathbf{A}]\mathbf{n}(x))^A + O(a^3). \qquad \text{(A3)}
\end{aligned}$$

Then, the magnitude of W_x^A is given by

$$\sqrt{W_x^A W_x^A} = \sqrt{64 + 16a^2\sum_\mu n^A(x)(D_\mu[\mathbf{A}]D_\mu[\mathbf{A}]\mathbf{n}(x))^A + O(a^3)} = 8 + a^2\sum_\mu n^A(x)(D_\mu[\mathbf{A}]D_\mu[\mathbf{A}]\mathbf{n}(x))^A + O(a^3). \qquad \text{(A4)}$$

Substituting (A3) and (A4) into (A2), therefore, we obtain

$$8(-n^A(x) \pm n^A(x)) + a^2\left(-\sum_{\mu=1}^{4}(D_\mu[\mathbf{A}]D_\mu[\mathbf{A}]\mathbf{n}(x))^A \pm n^A(x)\sum_\mu n^B(x)(D_\mu[\mathbf{A}]D_\mu[\mathbf{A}]\mathbf{n}(x))^B\right) + O(a^3) = 0, \qquad \text{(A5)}$$

where the double-signs $\pm$ in (A2) and (A5) correspond to each other. If we choose the minus sign, we have an unrealistic result $n^A(x) = 0$. Thus, by choosing the plus sign in (16), the leading term vanishes and the next-to-leading term leads to

$$\sum_{\mu=1}^{4}(D_\mu[\mathbf{A}]D_\mu[\mathbf{A}]\mathbf{n}(x))^A = -\lambda(x)n^A(x), \qquad \text{(A6)}$$

$$\lambda(x) \equiv -\sum_\mu n^B(x)(D_\mu[\mathbf{A}]D_\mu[\mathbf{A}]\mathbf{n}(x))^B, \qquad \text{(A7)}$$

which is nothing but the RDE in the continuum theory:

$$-D_\mu[\mathbf{A}]D_\mu[\mathbf{A}]\mathbf{n}(x) = \lambda(x)\mathbf{n}(x). \qquad \text{(A8)}$$

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