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Casimir effect for parallel plates involving massless Majorana fermions at finite temperature

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We study the Casimir effect for parallel plates with massless Majorana fermions obeying the bag boundary conditions at finite temperature. The thermal influence will modify the effect. It is found that the sign of the Casimir energy remains negative if the product of the plate distance and the temperature is larger than a special value, otherwise the energy will change to positive. The Casimir energy rises with the stronger thermal influence. We show that the attractive Casimir force between two parallel plates becomes greater with increasing temperature. In the case of the piston system involving the same Majorana fermions with the same boundary conditions, the attractive force on the piston will be weaker in higher-temperature surroundings.

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I. INTRODUCTION

The Casimir effect is essentially a direct consequence of quantum field theory subject to a change in the spectrum of vacuum oscillations when the quantization volume is bounded, or some background field is inserted. Historically, Casimir originated the effect of boundaries [1]. Twenty years later, Boyer found that the Casimir force for a conducting spherical shell is repulsive, showing that the circumstances will determine the nature of the force [2]. Afterwards, more effort has been paid to the problem and related topics and more and more results and methods have been put forward [3–10]. The precision of the measurements has been greatly improved experimentally. Recently, the sign of the Casimir energy and the nature of the Casimir force have been applied in many subjects because the Casimir effect depends on various factors. For example, the Casimir effect can be utilized to study high-dimensional spacetimes. We can research the Casimir effect for parallel plates in the spacetimes with extra compactified dimensions to show that the extra-dimension influence was manifest and distinct while the model was introduced [11–21]. Research on the Casimir effect for parallel plates or piston in the braneworld, such as Randall-Sundrum models, etc., has also made great progress [22–31]. It is interesting that we can estimate the properties of the world with more than four dimensions. The Casimir effect has also been explored in the context of string theory [32].

It is significant to study the Casimir effect for Majorana fermions under various types of boundary conditions. Majorana fermions have been discussed widely in many areas of physics. As a Majorana fermion, the Kaluza-Klein neutrino could be thought as a candidate of weakly interacting massive particle [33]. In the physics of superconductors and superfluids, a Majorana bound state is theoretically predicted in rotating superfluid ^{3}He-A between parallel plates with a gap that is about 10 μm and in triplet superconductors [34]. The Majorana vortex bound state is subject to a singular vortex in chiral p-wave superfluid [35]. Further, the Casimir effect for Majorana fermions in parallel plates with bag or chiral boundary conditions has been studied [36]. It was pointed out that the Casimir energy is singularity free. In addition, the Casimir effect for a massive fermionic field in the geometry of two parallel plates in the Minkowski spacetime with an arbitrary number of toroidally compactified spatial dimensions was also evaluated [37].

The quantum field theory at finite temperature shares many effects. The thermal influence on the Casimir effect cannot be neglected, andits influence certainly modifies the effect. The Casimir energy for a rectangular cavity under a nonzero temperature environment was considered, and the temperature controls the energy sign [38]. The Casimir effect for parallel plates, including thermal corrections in the spacetime with additional compactified dimensions, was explored, and the magnitude of Casimir force as well as the sign of Casimir energy change with the temperature [39–42]. In addition, the Casimir effect for a scalar field within two parallel plates under thermal influence in the bulk region of Randall-Sundrum models was also evaluated [43].

In this paper we are going to consider the thermal corrections to the Casimir effect for parallel plates as well as pistons containing massless Majorana fermions in detail to generalize some of the results of Ref. [36]. The description of the Casimir effect for massless Majorana fermions in the confine region, such as parallel plates, will

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change at different temperatures. At first we derive the frequency of massless Majorana fermions subject to bag boundary conditions with thermal corrections by means of finite-temperature field theories. We regularize the frequency to obtain the Casimir energy density with the help of the zeta function technique. Further, the Casimir force between the parallel plates involving the Majorana fermions can also be gained from the Casimir energy density at finite temperature. Certainly the discussions can be generalized to the case of the piston. It is necessary to explore the nature of Casimir energy and Casimir force for massless Majorana fermions within the parallel-plate system. Our conclusions are given at the end of this paper.

II. THE CASIMIR EFFECT OF MASSLESS MAJORANA FERMIONS IN THE PARALLEL-PLATE SYSTEM WITH BAG BOUNDARY CONDITIONS AT FINITE TEMPERATURE

In finite-temperature field theories the imaginary time formalism can be utilized to describe the fermion field in thermal equilibrium. We introduce a partition function for a system containing Dirac fields,

$$Z = N \int_{\text{antiperiodic}} D\bar{\psi} D\psi \exp\left[\int_0^\beta \int d^3x \mathcal{L}(\psi, \partial_E \psi)\right], \tag{1}$$

where $\mathcal{L}$ is the Lagrangian density for the system under consideration. N is a constant and ''antiperiodic'' means,

$$\psi(0, \mathbf{x}) = -\psi(\tau = \beta, \mathbf{x}), \tag{2}$$

where $\beta = \frac{1}{T}$ is the inverse of the temperature and $\tau = it$. Here the massless Majorana fermions can be described through the Dirac equation as follows:

$$i\gamma^\mu \partial_\mu \psi = 0, \tag{3}$$

where $\mu = 0, 1, 2, 3$. These massless fields satisfying the MIT bag boundary conditions, which are expressed as

$$in^\mu \gamma_\mu \psi = \psi, \tag{4}$$

are confined to the interior of parallel-plate system. In Eq. (4) $n^\mu = (0, \mathbf{n})$ and $\mathbf{n}$ the vector normal to the surface of plates and directing to the interior of the slab configuration. According to the solutions to the equations of motion (3) and the boundary conditions (4), the generalized zeta function can be written as

$$\begin{aligned}\zeta(s, -\partial_E) &= \text{Tr}(-\partial_E)^{-s} \\ &= -\int \frac{d^2k}{(2\pi)^2} \sum_{n=0}^{\infty} \sum_{l=-\infty}^{\infty} \left[k^2 + \frac{\pi^2(n+\frac{1}{2})^2}{R^2} + \frac{4\pi^2(l+\frac{1}{2})^2}{\beta^2}\right]^{-s},\end{aligned} \tag{5}$$

where

$$\partial_E = \frac{\partial^2}{\partial \tau^2} + \nabla^2 \tag{6}$$

and k denote the transverse components of the momentum. R is the distance of the plates. We obtain the total energy density of the system with thermal corrections by virtue of $\varepsilon = -\frac{\partial}{\partial\beta}(\frac{\partial\zeta(s;-\partial_E)}{\partial s}|_{s=0})$ and regularize the expression by means of the zeta function technique to obtain the Casimir energy per unit area for parallel plates with massless Majorana fermions at finite temperature as follows:

$$\begin{aligned}\varepsilon_C = &-\frac{7}{8}\frac{\pi^2}{720R^3} + \frac{1}{\sqrt{2}}\frac{1}{\beta^{3/2}R^{3/2}} \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} n_1^{-(3/2)} \\ &\times \left(n_2 + \frac{1}{2}\right)^{3/2} K_{3/2}\left[\frac{\pi\beta}{R} n_1 \left(n_2 + \frac{1}{2}\right)\right] \\ &+ \frac{\pi}{\sqrt{2}}\frac{1}{\beta^{1/2}R^{5/2}} \sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty} n_1^{-(1/2)}\left(n_2+\frac{1}{2}\right)^{5/2} \\ &\times \left[K_{1/2}\left(\frac{\pi\beta}{R} n_1\left(n_2+\frac{1}{2}\right)\right) \right. \\ &\left. + K_{5/2}\left(\frac{\pi\beta}{R}n_1\left(n_2+\frac{1}{2}\right)\right)\right],\end{aligned} \tag{7}$$

where $K_\nu(z)$ is the modified Bessel function of the second kind. Here we use the zeta function regularization. In Eq. (7) the terms with series converge very quickly and only the first several summands need to be taken into account for numerical calculation to further discussion. If the temperature approaches zero, the Casimir energy density will turn to be the results of Ref. [36], so will the Casimir force. When the temperature becomes extremely high, the asymptotic behavior of the Casimir energy per unit area for massless Majorana fermions within the parallel plates is

$$\lim_{T\to\infty} \varepsilon_C = \frac{7}{240}\frac{\pi}{\Gamma(\frac{3}{2})}\frac{R}{\beta^4}, \tag{8}$$

meaning that the Casimir energy rises as the temperature increases. Having performed the burden calculation, we find that the sign of the Casimir energy will be positive if the plates separation and temperature satisfy as $RT > 0.37$ in natural unit. The special value has something to do with the systems and boundary conditions.

The Casimir force on the plates is given by the derivative of the Casimir energy with respect to the plate distance. Here the Casimir force per unit area on the plates enclosing the massless Majorana fermions with bag boundary conditions can be written as

$$\begin{aligned} f_C &= -\frac{\partial \varepsilon_C}{\partial R} \\ &= -\frac{7\pi^2}{1920}\frac{1}{R^4} + \frac{3}{2^{3/2}}\frac{1}{\beta^{3/2}R^{5/2}}\sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty} n_1^{-(3/2)}\left(n_2+\frac{1}{2}\right)^{3/2} K_{3/2}\left[\frac{\pi\beta}{R}n_1\left(n_2+\frac{1}{2}\right)\right] \\ &\quad - \frac{\pi}{2^{3/2}}\frac{1}{\beta^{1/2}R^{7/2}}\sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty} n_1^{-(1/2)}\left(n_2+\frac{1}{2}\right)^{5/2}\left[K_{1/2}\left(\frac{\pi\beta}{R}n_1\left(n_2+\frac{1}{2}\right)\right) + K_{5/2}\left(\frac{\pi\beta}{R}n_1\left(n_2+\frac{1}{2}\right)\right)\right] \\ &\quad + \frac{5}{2^{3/2}}\frac{1}{\beta^{1/2}R^{7/2}}\sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty} n_1^{-(1/2)}\left(n_2+\frac{1}{2}\right)^{5/2}\left[K_{1/2}\left(\frac{\pi\beta}{R}n_1\left(n_2+\frac{1}{2}\right)\right) + K_{5/2}\left(\frac{\pi\beta}{R}n_1\left(n_2+\frac{1}{2}\right)\right)\right] \\ &\quad - \frac{\pi^2}{2^{3/2}}\frac{\beta^{1/2}}{R^{9/2}}\sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty} n_1^{1/2}\left(n_2+\frac{1}{2}\right)^{7/2}\left[K_{1/2}\left(\frac{\pi\beta}{R}n_1\left(n_2+\frac{1}{2}\right)\right) + 2K_{3/2}\left(\frac{\pi\beta}{R}n_1\left(n_2+\frac{1}{2}\right)\right) + K_{7/2}\left(\frac{\pi\beta}{R}n_1\left(n_2+\frac{1}{2}\right)\right)\right]. \end{aligned} \tag{9}$$

Similarly, if the temperature vanishes, the Casimir pressure shown in Eq. (9) keeps the first term that is consistent with that of Ref. [36]. When the two plates part from each other, the Casimir force per unit area does not vanish because of the temperature and is expressed as

$$\lim_{R\to\infty} f_C = -\frac{7\pi^2}{120}T^4. \tag{10}$$

It is evident that the Casimir force between the two plates where the Majorana fermions satisfy the bag boundary conditions becomes greater during the process where the temperature increases, while the force nature remains attractive. As a function of the plate distance for a definite temperature, the Casimir force per unit area with thermal influence is plotted in Fig. 1. The curves of the Casimir pressure associated with the separation of the plates for different magnitudes of temperature are similar. As the temperature becomes higher, the entire Casimir pressure curve will become lower, which means that the stronger thermal influence leads the attractive Casimir force for two parallel plates containing massless Majorana fermions to be greater. It is interesting that here the asymptotic value of the Casimir force with thermal corrections does not vanish unless the temperature is equal to zero. It should also be pointed out that the asymptotic behavior of the Casimir force shown in Eq. (10) is independent of the plates position.

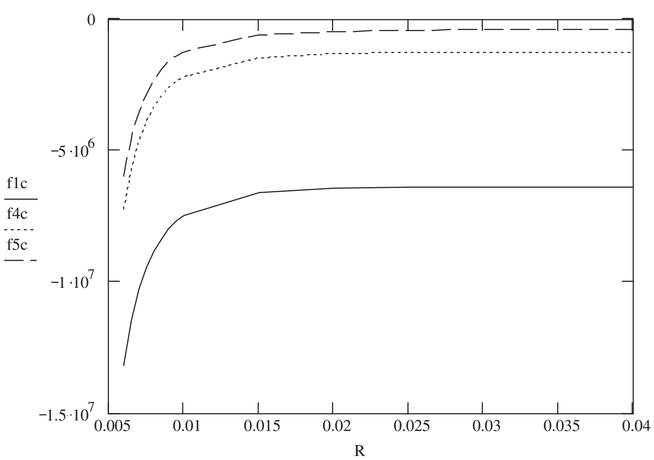

FIG. 1. The solid, dotted, and dashed curves of the Casimir force per unit area between two parallel plates as functions of plate separation R for $\beta = 0.02, 0.03, 0.04$, respectively.

III. THE CASIMIR PISTON OF MASSLESS MAJORANA FERMIONS WITH BAG BOUNDARY CONDITIONS AT FINITE TEMPERATURE

It is significant to explore the Casimir effect for massless Majorana within the device of the piston at finite temperature. More effort has been contributed to the topics related to the piston [15–22,36]. Under the environment with nonzero temperature, we discuss the massless Majorana fermions in the system consisting of three parallel plates and the bag boundary conditions are imposed on the plates. Having found and regularized the total vacuum energy per unit area of the three-parallel-plate system, we obtain the Casimir energy per unit area $\varepsilon_C = \varepsilon_C^1(R,T) + \varepsilon_C^2(L-R,T) + \varepsilon_C^{\mathrm{out}}(T)$, where $\varepsilon_C^1(R,T)$, $\varepsilon_C^2(L-R,T)$, and $\varepsilon_C^{\mathrm{out}}(T)$ represent the Casimir energy per unit area of two inner parts and the Casimir energy per unit area outside the system, respectively. Here $\varepsilon_C^1(R,T) = \varepsilon_C$ presented in Eq. (7) and $\varepsilon_C^2(L-R,T) = \varepsilon_C|_{R\to L-R}$. The Casimir energy per unit area outside the three-parallel-plate system has nothing to do with the distance of the plates. Further, the Casimir force per unit area on the piston is given with the help of a derivative of the Casimir energy per unit area with respect to the plates separation like $f'_{PC} = -\frac{\partial \varepsilon_C}{\partial R} = -\frac{\partial}{\partial R}\varepsilon^1_C(R,T) - \frac{\partial}{\partial R}\varepsilon^2_C(L-R,T) - \frac{\partial}{\partial R}\varepsilon^{\text{out}}_C(T)$. As one outer plate is moved to the extremely distant place or equivalently $L \to \infty$, the Casimir pressure between the remaining two plates with Majorana fermions is

$$\begin{aligned} f_{\text{PC}} &= \lim_{L\to\infty} f'_{\text{PC}} \\ &= \frac{3}{\sqrt{2}}\frac{1}{R^{3/2}\beta^{5/2}}\sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty} n_1^{-(3/2)}\left(n_2+\frac{1}{2}\right)^{3/2} K_{3/2}\left(\frac{4\pi R}{\beta}n_1\left(n_2+\frac{1}{2}\right)\right) \\ &\quad + \frac{(6+2\pi)\sqrt{2}}{R^{1/2}\beta^{7/2}}\sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty} n_1^{-(1/2)}\left(n_2+\frac{1}{2}\right)^{5/2}\left[K_{1/2}\left(\frac{4\pi R}{\beta}n_1\left(n_2+\frac{1}{2}\right)\right) + K_{5/2}\left(\frac{4\pi R}{\beta}n_1\left(n_2+\frac{1}{2}\right)\right)\right] \\ &\quad - 2^{7/2}\pi^2\frac{R^{1/2}}{\beta^{9/2}}\sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty} n_1^{1/2}\left(n_2+\frac{1}{2}\right)^{7/2}\left[K_{1/2}\left(\frac{4\pi R}{\beta}n_1\left(n_2+\frac{1}{2}\right)\right) + 2K_{3/2}\left(\frac{4\pi R}{\beta}n_1\left(n_2+\frac{1}{2}\right)\right)\right. \\ &\quad \left. + K_{7/2}\left(\frac{4\pi R}{\beta}n_1\left(n_2+\frac{1}{2}\right)\right)\right]. \end{aligned} \tag{11}$$

In fact, the piston device can help us to cancel the terms that are independent of the plate gap. According to Eq. (11), we discover that

$$\lim_{R\to\infty} f_{\text{PC}} = 0, \tag{12}$$

which is favored by the observational results and

$$\lim_{T\to\infty} f_{\text{PC}} = 0, \tag{13}$$

which means that the Casimir force is weaker with stronger thermal influence. The dependence of the Casimir force per unit area for the piston on the distance between the piston and the other plate is shown in Fig. 2. It is manifest that the attractive Casimir force for Majorana fermion piston will be weaker as the temperature increases because the parts in the Casimir pressure like Eq. (10) are plate-position independent and are certainly cancelled in the piston device, although their magnitude is an increasing function of temperature. The curves of the force with different temperature are similar.

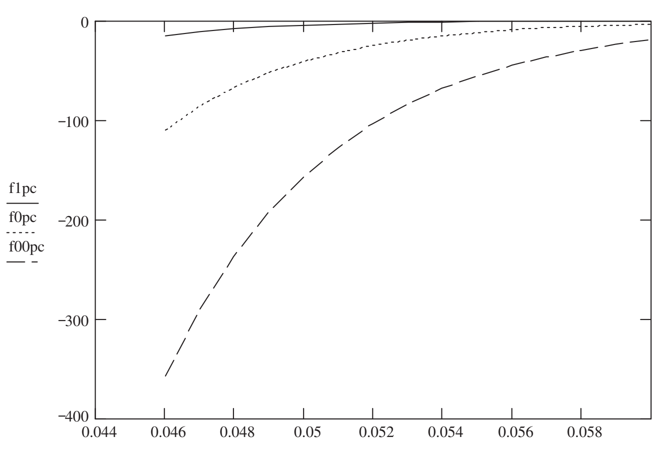

FIG. 2. The solid, dotted, and dashed curves of the Casimir force per unit area for piston as functions of plate separation R for $\beta = 0.02, 0.025, 0.03$, respectively.

IV. CONCLUSION

In this work we investigate the Casimir effect with thermal modification for parallel plates and piston filled with massless Majorana fermions while there are bag boundary conditions imposed at the plates. At first we discover that the Casimir energy for two parallel plates rises with the increase in the temperature. When the product of the plates separation and the temperature is larger than a special value like $RT > 0.37$, the sign of the Casimir energy will change to positive. The special value will be different in a different system with different boundary conditions. It is also found that the magnitude of the Casimir force between two parallel plates with massless Majorana fermions satisfying the bag boundary conditions at the plates become greater as the thermal influence is stronger while the Casimir force remains attractive. In the case of the piston system, which contains the massless Majorana fermions with the same bag boundary conditions at the plates, the attractive Casimir force on the piston will be weaker when the surroundings arehotter. As the temperature is extremely high, the Casimir force on the piston will vanish. When the distance of the platesis extremely great, the asymptotic value of the Casimir force between two parallel plates depends on the temperature and does not disappear. In the context of the piston model, the Casimir force between the piston and the other plate will approach zero when the two plates part from each other much farther. Our results can return to be those of Ref. [36] when the temperature is equal to zero. Here we explain how the Casimir effect for massless Majorana fermions within the parallel system subject to the bag boundary conditions depends on the temperature.

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