

Generalization of the Fierz-Pauli actionClaudia de Rham¹ and Gregory Gabadadze²¹*Département de Physique Théorique, Université de Genève, 24 Quai E. Ansermet, CH-1211, Genève, Switzerland*²*Center for Cosmology and Particle Physics, Department of Physics, New York University, New York, New York, 10003, USA*

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We consider the Lagrangian of gravity covariantly amended by the mass and polynomial interaction terms with arbitrary coefficients and reinvestigate the consistency of such a theory in the decoupling limit, up to the fifth order in the nonlinearities. We calculate explicitly the self-interactions of the helicity-0 mode, as well as the nonlinear mixing between the helicity-0 and -2 modes. We show that ghostlike pathologies in these interactions disappear for special choices of the polynomial interactions and argue that this result remains true to all orders in the decoupling limit. Moreover, we show that the linear and some of the nonlinear mixing terms between the helicity-0 and -2 modes can be absorbed by a local change of variables, which then naturally generates the cubic, quartic, and quintic Galileon interactions, introduced in a different context. We also point out that the mixing between the helicity-0 and -2 modes can be at most quartic in the decoupling limit. Finally, we discuss the implications of our findings for the consistency of the effective field theory away from the decoupling limit, and for the Boulware-Deser problem.

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I. INTRODUCTION AND SUMMARY

In this work we study the covariant polynomial potential of a relativistic and symmetric rank-2 tensor field living in four-dimensional flat space-time.

We start with the mass term in the potential. Poincaré symmetry in four dimensions imposes that any massive spin-2 state has to have five physical degrees of freedom—namely, the helicity- ± 2 , helicity- ± 1 , and helicity-0 modes. The quadratic potential that describes these degrees of freedom is that of Fierz and Pauli (FP), [1]. The latter is known to be the unique ghost-free and tachyon-free mass term for the spin-2 state [2].

No matter how small the graviton mass is in the FP theory, the helicity-0 state couples to the trace of the matter stress-tensor with the same strength as the helicity-2 does [3]. This discontinuity would rule out, on simple observational grounds, the FP mass term for gravity.

As argued first by Vainshtein, the discontinuity problem can be cured by the nonlinear interactions which would become comparable to the linear terms already for very weak fields [4]. Then, the nonlinearities could give rise to the screening of the helicity-0 mode at observable scales, rendering the theory compatible with the known empirical data [4,5].

However, the very same nonlinearities that cure the discontinuity problem typically give rise to a ghost in massive gravity [6]. This ghost, sometimes referred to as the Boulware-Deser (BD) mode, emerges as a sixth degree of freedom, that is infinitely heavy on a flat background, but becomes light on any reasonable nontrivial background (e.g., on a cosmological background [7] or on the weak background of a lump of static matter [8–10]). It is straightforward to see this ghost in the so-called decoupling limit

[8], in which the dynamics of the helicity-0 mode can be made manifest. Then, the sixth degree of freedom ends up being related to the nonlinear interactions of the helicity-0 mode [8–10].¹

The obvious question to ask is then whether there exists a nonlinear model that exhibits the Vainshtein mechanism, but without the ghost mode. This question was raised in Ref. [8] and studied in detail in Ref. [9]. The latter work argued that at the cubic order the ghost can be avoided by tuning the coefficients of the quadratic and cubic order terms. Recently, the cubic terms were calculated in a nonlinear massive spin-2 theory of Refs. [14,15], where it was shown that the necessary tuning is in fact automatic in this model, and the theory is ghost-free to that order [16]!

In the present work we focus instead on addressing this question at higher orders, and in a model-independent framework. We therefore allow for arbitrary nonlinearities in the potential up to the quintic order, but restrict ourselves to considerations in the decoupling limit only.

Our result clashes with one of the conclusions of Ref. [9] which states that the quartic interactions in the decoupling limit ineradicably lead to a ghost. Regrettably, the decoupling limit Lagrangian obtained in Ref. [9] is not reparametrization invariant neither at the cubic nor quartic order, and it gives a tensor equation that does not satisfy the Bianchi identity. The ghost found in the decoupling limit of Ref. [9] is an artifact of these properties. Hence, we

¹Notice also that the discontinuity is absent when a small cosmological constant is included before sending the mass of the graviton to zero [11,12]. Doing so in de Sitter space, however, one passes through the parameter region where helicity-0 becomes a ghost [13], while the anti-de Sitter case is ghost-free [11,12].

reinvestigate this issue in the present work. We find a decoupling limit Lagrangian that is similar to that of Ref. [9] but differs from it in detail, by coefficients of various tensorial structures. In particular, due to those coefficients, our Lagrangian is reparametrization invariant and naturally leads to a tensor equation for which the Bianchi identity is automatically satisfied (as it should be since the helicity-2 mode only mixes linearly in the decoupling limit). Then, not surprisingly, we arrive at a different conclusion, that the quartic theory is also ghost-free in the decoupling limit. Moreover, we go on one step further and investigate the quintic-order theory, which we also show is ghost-free in the decoupling limit. This also allows us to understand the structure of the interactions to all orders and to argue that the decoupling limit can be at most quintic order in interactions (or quartic in the mixing between the helicity-0 and -2 modes) in the ghost-free theory.

Finally, as a corollary, we find that the decoupling limit of the most general consistent theory of massive gravity gives rise to the quadratic, cubic, quartic, and quintic Galileon kinetic interactions introduced in Ref. [17] in a different context (namely, as a generalization of the special cubic term appearing in the decoupling limit of DGP [18] found in Ref. [19]). The Galileon interactions share the important properties of (i) being local, (ii) preserving the shift and Galilean symmetry in the field space of the helicity-0 mode (in particular, in the kinetic and self-interaction terms but not in interactions with matter), and (iii) giving rise to equations of motion with a well-defined Cauchy problem. Since then, the Galileons have developed their own independent and interesting life (see, e.g., [20,21]). We show here that the Galileons naturally arise in the decoupling limit of a general theory of massive gravity. This also helps to prove that upon appropriate choices of the coefficients in the potential, the decoupling limit of massive gravity is stable, at least up to the quintic order in interactions.

We continue this section with a discussion and summary of our main results in more technical terms before turning to the detailed calculations in the subsequent sections.

In analogy with a massive non-Abelian (Higgs-less) spin-1 [22], the dynamics of the helicity-0 mode, π , can be extracted in a generic theory of gravity with a nonlinear potential by taking the decoupling limit [8]

$$m \rightarrow 0, M_{\text{Pl}} \rightarrow \infty, \quad \text{keeping } \Lambda_5 \equiv (m^4 M_{\text{Pl}})^{1/5} \text{ fixed.} \quad (1)$$

Following [8], in a generic case of the nonlinear potential, the corresponding Lagrangian for the helicity-0 mode reads schematically as follows:

$$\mathcal{L}_\pi = \frac{3}{2} \pi \square \pi + \frac{(\partial^2 \pi)^3}{\Lambda_5^5}. \quad (2)$$

The cubic interaction with six derivatives gives rise to a ghost on locally nontrivial asymptotically-flat backgrounds

(e.g. on the background of a local lump of matter). This could be seen by observing that for $\pi = \pi^{\text{cl}} + \delta\pi$, with π^{cl} denoting the weak field of a local source, and $\delta\pi$ its fluctuation, the cubic term in (2) could generate a four-derivative quadratic term for the fluctuations. This leads to a ghost, which is infinitely heavy on Minkowski space-time, but becomes light enough to be disruptive once a reasonable local background is considered, see Refs. [8–10].

To avoid pathologies such as in (2), the Fierz-Pauli combination in the graviton potential should be pursued further by tuning the coefficients of various higher order terms. This leads to a cancellation of all the terms for π that are suppressed by the scales Λ_5 , $\Lambda_4 = (m^3 M_{\text{Pl}})^{1/4}$, $\Lambda_{11/3} = (m^8 M_{\text{Pl}}^3)^{1/11}$, etc., for any scale $\Lambda < \Lambda_3 = (m^2 M_{\text{Pl}})^{1/3}$, such that only the terms suppressed by the scale Λ_3 survive. Then, Λ_3 is kept fixed in the decoupling limit, and the surviving terms (in addition to the linearized Einstein-Hilbert term) read as follows:

$$\Delta \mathcal{L} = h^{\mu\nu} \left(X_{\mu\nu}^{(1)} + \frac{1}{\Lambda_3^2} X_{\mu\nu}^{(2)} + \frac{1}{\Lambda_3^3} X_{\mu\nu}^{(3)} \right). \quad (3)$$

Here, $h_{\mu\nu}$ denotes the canonically normalized (rescaled by M_{Pl}) tensor field perturbation, while $X_{\mu\nu}^{(1)}$, $X_{\mu\nu}^{(2)}$, and $X_{\mu\nu}^{(3)}$ are, respectively, linear, quadratic and cubic in π . Importantly, they are all transverse (for instance, $X_{\mu\nu}^{(1)} \propto \eta_{\mu\nu} \square \pi - \partial_\mu \partial_\nu \pi$). Not only do these interactions automatically satisfy the Bianchi identity, as they should to preserve diffeomorphism invariance, but they are also at most second order in time derivative. Hence, the interactions (3) are linear in the helicity-2 mode, and unlike the previous results in the literature, present perfectly consistent terms, at least up to the quintic order.

Furthermore, some of the terms in (3) can be absorbed by a local field redefinition. For instance, the quadratic term, $h^{\mu\nu} X_{\mu\nu}^{(1)}$, can be absorbed by a conformal transformation $h_{\mu\nu} \rightarrow h_{\mu\nu} + \eta_{\mu\nu} \pi$. This shift, besides removing the above mixing, generates terms of the form $\pi X^{(2)}$ and $\pi X^{(3)}$, which coincide, up to a total derivative, with the cubic and quartic Galileon terms [17]. Further diagonalization of the cubic mixing term, $h^{\mu\nu} X_{\mu\nu}^{(2)}$, also generates the quintic Galileon, hence exhausting all the possible terms that can arise in the Galileon family at arbitrary order.

Moreover, we also point out that if the decoupling limit happens to pick up the scale Λ_3 (as opposed to another smaller scale such as Λ_5 , Λ_4 , etc.), the mixing between the helicity-0 and -2 modes must stop at the quartic order. Therefore, for appropriate choices of the interaction coefficients, the decoupling limit at this order is exact. It is the subsequent diagonalization of the nonlinear terms in the Lagrangian that generates the quintic Galileon.

Finally, the absence of a ghost in the decoupling limit does not prove the stability of the full theory away from the limit, and the Boulware-Deser ghost is still expected to be

present in general. However, it at least shows that one has a well-defined and consistent effective field theory below the scale Λ_3 . Above this scale, the full theory has to be specified. We discuss related issues in Sec. V. Before that, our work has a two-fold motivation: (i) To establish a consistent effective field theory below Λ_3 (for the full theory to be viable its decoupling limit should be ghost-free as a necessary condition). (ii) All the known examples show that the Boulware-Deser ghost, if present in the full theory, does also show up in the decoupling limit. Therefore, it is encouraging to find no ghosts in this limit.

The paper is organized as follows: In Sec. II we summarize the formalism used to study the decoupling limit of massive gravity with a general potential. We then explicitly compute the decoupling limit Lagrangian to the quartic and quintic orders in Sec. III. We work with a generic nonlinear completion of the FP gravity for which the scale $\Lambda_3^3 = M_{\text{Pl}} m^2$ is fixed. We argue that the π mode does not decouple from the tensor mode, but that the interactions are free of any ghostlike pathologies. In Sec. IV we give a general framework for computing the Lagrangian in the decoupling limit and argue that in theories which are consistent with the fixed scale Λ_3 , at most the quartic order mixing term can be obtained, all the higher order mixing terms being zero. Moreover, we show in Sec. V that upon an appropriate change of variables we recover the standard Galileon interactions. Section VI contains some discussions on open issues and future directions addressing the consistency of massive gravity away from the decoupling limit.

II. FORMALISM

A. Gauge invariant potential for gravity

Below we consider in detail the decoupling limit of a general Lagrangian of a massive spin-2 field endowed with a potential on Minkowski space-time. We use the technique developed in Ref. [8]. The covariant Lagrangian with the potential reads as follows:

$$\mathcal{L} = M_{\text{Pl}}^2 \sqrt{-g} R - \frac{M_{\text{Pl}}^2 m^2}{4} \sqrt{-g} (U_2(g, H) + U_3(g, H) + U_4(g, H) + U_5(g, H) \dots), \quad (4)$$

where U_i denotes the interaction term at i th order in $H_{\mu\nu}$,

$$U_2(g, H) = H_{\mu\nu}^2 - H^2, \quad (5)$$

$$U_3(g, H) = c_1 H_{\mu\nu}^3 + c_2 H H_{\mu\nu}^2 + c_3 H^3, \quad (6)$$

$$U_4(g, H) = d_1 H_{\mu\nu}^4 + d_2 H H_{\mu\nu}^3 + d_3 H_{\mu\nu}^2 H_{\alpha\beta}^2 + d_4 H^2 H_{\mu\nu}^2 + d_5 H^4, \quad (7)$$

$$U_5(g, H) = f_1 H_{\mu\nu}^5 + f_2 H H_{\mu\nu}^4 + f_3 H^2 H_{\mu\nu}^3 + f_4 H_{\alpha\beta}^2 H_{\mu\nu}^3 + f_5 H (H_{\mu\nu}^2)^2 + f_6 H^3 H_{\mu\nu}^2 + f_7 H^5. \quad (8)$$

Here the index contractions are performed using the inverse metric, so that $H = g^{\mu\nu} H_{\mu\nu}$, $H_{\mu\nu}^2 = g^{\mu\nu} g^{\alpha\beta} H_{\mu\alpha} H_{\nu\beta}$, etc. The coefficients c_i , d_i , and f_i are *a priori* arbitrary but will be determined by demanding that no ghosts are present at least up to the quintic order in the decoupling limit.

Finally, the tensor $H_{\mu\nu}$ is related to the metric tensor as follows:

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_{\text{Pl}}} = H_{\mu\nu} + \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b, \quad (9)$$

where $a, b = 0, 1, 2, 3$, $\eta_{ab} = \text{diag}(-1, 1, 1, 1)$, and $H_{\mu\nu}$ is a covariant tensor as long as the four fields ϕ^a transform as scalars under a change of coordinates. Furthermore, expressing ϕ^a in terms of the coordinates x^α , and the field π^α as $\phi^a = (x^\alpha - \pi^\alpha) \delta_\alpha^a$, we obtain

$$H_{\mu\nu} = \frac{h_{\mu\nu}}{M_{\text{Pl}}} + \partial_\mu \pi_\nu + \partial_\nu \pi_\mu - \eta_{\alpha\beta} \partial_\mu \pi^\alpha \partial_\nu \pi^\beta. \quad (10)$$

In (10), and in what follows, we adopt the convention that the indices on π_μ are raised and lowered with respect to the Minkowski metric $\eta_{\mu\nu}$. Crucially, the expression for the tensor $H_{\mu\nu}$ in (10) differs by a minus sign in front of the last term from the analogous expression in Eq. (5) used in Ref. [9]. To emphasize the importance of this sign, we derive in the Appendix the decoupling limit using the opposite sign in (10), recover the results of Ref. [9], and show that the Bianchi identity is then not automatically satisfied, since the reparametrization invariance is not retained in the resulting Lagrangian.

From (4) it is not immediately clear what is the scale of the effective field theory represented by this Lagrangian, i.e., what is the energy/momentum scale by which the higher polynomial interactions would be suppressed as compared with the leading ones. This will become clear by studying the decoupling limit of the theory.

In what follows, we focus on the helicity-2 and helicity-0 modes but ignore the vector mode. The latter enters only quadratically in the decoupling limit (since the vector does not couple to a conserved stress-tensor in the linearized order) and can be set to zero self-consistently. This does not prove that the vector sector is ghost-free; however, the findings of Ref. [16] that the cubic nonlinearities for the vector are completely harmless due to the $U(1)$ gauge invariance of the resulting terms suggest that the vector sector is not going to reintroduce the BD ghost. Therefore, we use the substitution $\pi_\alpha = \partial_\alpha \pi / \Lambda_3^3$ so that

$$H_{\mu\nu} = \frac{h_{\mu\nu}}{M_{\text{Pl}}} + \frac{2}{M_{\text{Pl}} m^2} \Pi_{\mu\nu} - \frac{1}{M_{\text{Pl}}^2 m^4} \Pi_{\mu\nu}^2, \quad (11)$$

where we use the same notation as in [9], $\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$

and $\Pi_{\mu\nu}^2 = \eta^{\alpha\beta} \Pi_{\mu\alpha} \Pi_{\beta\nu}$. Moreover, in what follows the square brackets $[\dots]$ will represent the trace of a tensor contracted using the Minkowski metric, e.g. $[\Pi^2] = \Pi^{\mu\nu} \Pi_{\mu\nu}$ and $[\Pi]^2 = \Pi_{\mu}^{\mu} \Pi_{\nu}^{\nu}$.

B. Decoupling scale

As mentioned in the introduction, the interactions U_2 and U_3 typically lead to terms of the form $(\partial^2 \pi)^3 / (M_{\text{Pl}} m^4)$, and the decoupling limit should be taken keeping the scale $\Lambda_5^5 = M_{\text{Pl}} m^4$ fixed, while $M_{\text{Pl}} \rightarrow \infty$ and $m \rightarrow 0$. However, we will show in what follows (see also [16]) that for some special values of the coefficients c_i , such interactions cancel (up to a total derivative), generalizing the FP term to the cubic order. This procedure can be extended further to an arbitrary order:

At a given order the leading contributions are of the form

$$\mathcal{L}_n \sim \frac{(\partial \partial \pi)^n}{M_{\text{Pl}}^{n-2} m^{2(n-1)}}, \quad (12)$$

then, one chooses the interactions $U_n(H) \sim H^n$ so that the above terms combine into a total derivative. At each order, there exists a unique total derivative combination $\mathcal{L}_{\text{der}}^{(n)}$ that can be written as follows:

$$\mathcal{L}_{\text{der}}^{(n)} = - \sum_{m=1}^n (-1)^m \frac{(n-1)!}{(n-m)!} [\Pi^m] \mathcal{L}_{\text{der}}^{(n-m)}, \quad (13)$$

with $\mathcal{L}_{\text{der}}^{(0)} = 1$ and $\mathcal{L}_{\text{der}}^{(1)} = [\Pi]$. Up to the quartic order, the total derivatives are

$$\mathcal{L}_{\text{der}}^{(2)} = [\Pi]^2 - [\Pi^2], \quad (14)$$

$$\mathcal{L}_{\text{der}}^{(3)} = [\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3], \quad (15)$$

$$\begin{aligned} \mathcal{L}_{\text{der}}^{(4)} &= [\Pi]^4 - 6[\Pi^2][\Pi]^2 + 8[\Pi^3][\Pi] + 3[\Pi^2]^2 \\ &\quad - 6[\Pi^4]. \end{aligned} \quad (16)$$

Moreover, at higher orders these total derivatives vanish identically, i.e. $\mathcal{L}_{\text{der}}^{(n)} \equiv 0$, for any $n \geq 5$. By ensuring that all the leading terms (12) take the form of a total derivative (13), all the interactions that arise at an energy scale lower than Λ_3 disappear. Keeping this in mind we will therefore consider below the following decoupling limit (first considered in [19] in the context of the DGP model)

$$m \rightarrow 0, \quad M_{\text{Pl}} \rightarrow \infty, \quad \text{keeping } \Lambda_3 \equiv (m^2 M_{\text{Pl}})^{1/3} \text{ fixed.} \quad (17)$$

Note that the procedure of taking the limit in the present case is well defined for fields that decay fast enough at spatial infinity. For these we introduce an infrared regulator of the theory, say a large sphere of radius $L \gg 1/m$, and take the radius to infinity, $L \rightarrow \infty$, before taking the limit (17). This hierarchy of scales enables us to put all the surface terms to zero before taking the decoupling limit.

Furthermore, as it should be becoming clear from the above discussions, the scale Λ_3 will end up being the effective field theory scale. The higher interaction terms, both written or implied in (4), will be subdominant to the leading ones for energy/momentum scales below Λ_3 .

III. DECOUPLING LIMIT OF MASSIVE GRAVITY

A. Cubic order

We now explicitly compute the decoupling limit for the interactions considered in (5)–(9), and thus generalize the Fierz-Pauli term to higher orders. In terms of the ‘‘Einstein operator’’ $\hat{\mathcal{E}}$ defined for an arbitrary symmetric field $Z_{\mu\nu}$ as

$$\begin{aligned} \hat{\mathcal{E}}_{\mu\nu}^{\alpha\beta} Z_{\alpha\beta} &= -\frac{1}{2}(\square Z_{\mu\nu} - \partial_{\mu} \partial_{\alpha} Z_{\nu}^{\alpha} - \partial_{\nu} \partial_{\alpha} Z_{\mu}^{\alpha} + \partial_{\mu} \partial_{\nu} Z_{\alpha}^{\alpha} \\ &\quad - \eta_{\mu\nu} \square Z_{\alpha}^{\alpha} + \eta_{\mu\nu} \partial_{\alpha} \partial_{\beta} Z^{\alpha\beta}), \end{aligned} \quad (18)$$

the decoupling limit Lagrangian of massive gravity up to the cubic order reads as follows:

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2} h^{\mu\nu} \hat{\mathcal{E}}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} + h^{\mu\nu} X_{\mu\nu}^{(1)} - \frac{1}{4\Lambda_5^5} ((8c_1 - 4)[\Pi^3] \\ &\quad + (8c_2 + 4)[\Pi][\Pi^2] + 8c_3[\Pi]^3) + \frac{1}{\Lambda_3} h^{\mu\nu} X_{\mu\nu}^{(2)}, \end{aligned} \quad (19)$$

with

$$X_{\mu\nu}^{(1)} = [\Pi] \eta_{\mu\nu} - \Pi_{\mu\nu}, \quad (20)$$

and $X_{\mu\nu}^{(2)}$ quadratic in Π . Using the total derivative combination (15), the interactions arising at the scale Λ_5 can be removed by setting

$$c_1 = 2c_3 + \frac{1}{2} \quad \text{and} \quad c_2 = -3c_3 - \frac{1}{2}. \quad (21)$$

As a result, we find the following expression for the tensor $X_{\mu\nu}^{(2)}$:

$$\begin{aligned} X_{\mu\nu}^{(2)} &= -(6c_3 - 1) \{ (\Pi_{\mu\nu}^2 - [\Pi] \Pi_{\mu\nu}) - \frac{1}{2} ([\Pi^2] \\ &\quad - [\Pi]^2) \eta_{\mu\nu} \}. \end{aligned} \quad (22)$$

Notice that both $X_{\mu\nu}^{(1)}$ and $X_{\mu\nu}^{(2)}$ are automatically conserved, as they should for the reparametrization invariance to be retained and the Bianchi identity to be satisfied.

Moreover, it is straightforward to check that these cubic interactions bear at most two time derivatives, and are therefore free of any ghostlike pathologies. One should also check that the lapse (which coincides with h_{00} in the decoupling limit) still propagates a constraint, which is indeed the case here as neither $X_{00}^{(1)}$ nor $X_{00}^{(2)}$ contain any time derivatives. Furthermore, these cubic interactions with the specific coefficient $c_3 = 1/4$ have already been discussed in detail in Ref. [16].

We now apply the same formalism to quartic interactions for which ghostlike pathologies have been argued to arise inexorably in Ref. [9].

B. Quartic order

At the quartic order, we find the following interactions in the decoupling limit:

$$\begin{aligned} \mathcal{L}^{(4)} = & \frac{1}{\Lambda_3^6} h^{\mu\nu} X_{\mu\nu}^{(3)} + \frac{1}{\Lambda_4^8} \left\{ \left(3c_1 - 4d_1 - \frac{1}{4} \right) [\Pi^4] \right. \\ & + \left(c_2 - 4d_3 + \frac{1}{4} \right) [\Pi^2]^2 + (2c_2 - 4d_2) [\Pi][\Pi^3] \\ & \left. + (3c_3 - 4d_4) [\Pi^2][\Pi]^2 - 4d_5 [\Pi]^4 \right\}, \end{aligned} \quad (23)$$

with $\Lambda_4 = (M_{\text{Pl}} m^3)^{1/4}$ and $X_{\mu\nu}^{(3)}$ cubic in Π . Here again the pathological terms arising at the scale Λ_4 can be removed by using the total derivative combination (16) and by setting c_1 and c_2 as in (21), as well as

$$d_1 = -6d_5 + \frac{1}{16}(24c_3 + 5), \quad (24)$$

$$d_2 = 8d_5 - \frac{1}{4}(6c_3 + 1), \quad (25)$$

$$d_3 = 3d_5 - \frac{1}{16}(12c_3 + 1), \quad (26)$$

$$d_4 = -6d_5 + \frac{3}{4}c_3. \quad (27)$$

Substituting these coefficients in $X_{\mu\nu}^{(3)}$ we obtain the mixing term between the helicity-0 and -2 modes determined by

$$\begin{aligned} X_{\mu\nu}^{(3)} = & (c_3 + 8d_5) \{ 6\Pi^3_{\mu\nu} - 6[\Pi]\Pi^2_{\mu\nu} + 3([\Pi]^2 \\ & - [\Pi^2])\Pi_{\mu\nu} - ([\Pi]^3 - 3[\Pi][\Pi^2]) \\ & + 2[\Pi^3]\eta_{\mu\nu} \}. \end{aligned} \quad (28)$$

This expression bears two expected but important features:

- (i) It is conserved $\partial^\mu X_{\mu\nu}^{(3)} = 0$, as it should be for the reparametrization invariance to be present and the Bianchi identity to be automatically satisfied.
- (ii) For i, j spacelike indices and 0 timelike index:

$$X_{ij}^{(3)} \quad \text{has at most two time derivatives,}$$

$$X_{0i}^{(3)} \quad \text{has at most one time derivative,}$$

$$X_{00}^{(3)} \quad \text{has no time derivatives.}$$

These properties ensure that no ghostlike pathology arise at the quartic level in the decoupling limit as long as the interactions come in with the generalized FP structure set by the coefficients (21) and (24)–(27).

C. Quintic order

At the fifth order in the decoupling limit, we consider interactions as given in (9). The pathological terms that scale as

$$\mathcal{L}_{\Pi^5} \sim \frac{1}{M_{\text{Pl}}^3 m^8} (\partial \partial \pi)^5 \quad (29)$$

can be canceled with an appropriate choice of the coefficients f_1 to f_6 :

$$\begin{aligned} f_1 &= \frac{7}{32} + \frac{9}{8}c_3 - 6d_5 + 24f_7, \\ f_2 &= -\frac{5}{32} - \frac{15}{16}c_3 + 6d_5 - 30f_7, \\ f_3 &= \frac{3}{8}c_3 - 5d_5 + 20f_7, \\ f_4 &= -\frac{1}{16} - \frac{3}{4}c_3 + 5d_5 - 20f_7, \\ f_5 &= \frac{3}{16}c_3 - 3d_5 + 15f_7, \\ f_6 &= d_5 - 10f_7. \end{aligned} \quad (30)$$

As a result, the quintic interactions in π arrange themselves to form the expression for $\mathcal{L}_{\text{der}}^{(5)}$, as derived from (13)

$$\begin{aligned} \mathcal{L}_{\text{der}}^{(5)} = & 24[\Pi^5] - 30[\Pi][\Pi^4] + 20[\Pi^3][\Pi^2] - [\Pi^2] \\ & + 15[\Pi][\Pi^2]^2 - 10[\Pi^2][\Pi]^3 + [\Pi]^5 \equiv 0. \end{aligned} \quad (31)$$

Notice that $\mathcal{L}_{\text{der}}^{(5)}$ is not simply a total derivative as for the previous orders but instead vanishes identically. This implies, in particular, that any limiting Lagrangian of the form $\mathcal{L}^{(n)} \sim f(\Pi)\mathcal{L}_{\text{der}}^{(5)}$, where f is an analytic function, gives no dangerous π interactions and can be used at higher orders. Beyond the quintic order the degrees of freedom in the coefficients to be tuned should therefore increase, and make it easier to remove any ghostlike interactions.

With the above choice of coefficient (30), the only quintic interaction in the decoupling limit then is

$$\mathcal{L}^{(5)} = \frac{1}{\Lambda_3^9} h^{\mu\nu} X_{\mu\nu}^{(4)}, \quad (32)$$

with

$$\begin{aligned} X_{\mu\nu}^{(4)} \sim & 24(\Pi^4_{\mu\nu} - \Pi\Pi^3_{\mu\nu}) + 12\mathcal{L}_{\text{der}}^{(2)}\Pi^2_{\mu\nu} \\ & - 4\mathcal{L}_{\text{der}}^{(3)}\Pi_{\mu\nu} + \mathcal{L}_{\text{der}}^{(4)}\eta_{\mu\nu} \equiv 0, \end{aligned} \quad (33)$$

with $\mathcal{L}_{\text{der}}^{(2,3,4)}$ given, respectively, in (14)–(16). The decoupling limit is therefore well behaved up to the quintic order, and the number of free parameters at higher orders suggests that one can always make appropriate choices to avoid any ghost mode from appearing in the entire decoupling limit. To be certain, one should, however, analyze a fully nonlinear theory, such as the one proposed in [14, 15].

Motivated by the above obtained results, we set up in the next section a general formalism for obtaining the interactions to all orders.

Before we do so, some important comments are in order. We might of course argue that the absence of the ghost up to the quintic order represents no proof of the stability of the theory even in the decoupling limit, since the ghost could be pushed to the next order in interactions. It is also not a proof of the consistency of the full theory, as was discussed in Sec. I, since the ghost may appear away from

the decoupling limit. The arguments concerning these two points, respectively, are as follows:

- (1) Beyond the quintic order, the number of free coefficients in the interactions seems sufficient to eliminate pathological contributions of the form $(\partial\partial\pi)^n$. Furthermore, beyond the quartic order all conserved tensors of the form $X_{\mu\nu}^{(n)} \sim (\partial\partial\pi)_{\mu\nu}^n$ vanish identically, and cannot lead to any ghostlike pathologies in the mixing $h^{\mu\nu}X_{\mu\nu}^{(n)}$ between the helicity-0 and -2 modes.
- (2) The ghost may exist in a given order away from the decoupling limit (say at the quartic or higher order) but disappear in the decoupling limit. If so, then, the ghost should come with a mass greater than Λ_3 . Then, the theory would be acceptable as an effective theory below the Λ_3 scale. However, at scales above Λ_3 , one would need to specify an infinite number of terms in the full nonlinear theory in order to conclude whether or not the ghost is removed by the resummation of these terms. This will be made more precise in the last section.

IV. GENERAL FORMULATION FOR AN ARBITRARY ORDER

All our findings up to the quintic order presented in the previous section can be formulated in a unified way, which may also suggest how things could work at higher orders. For this, in the N th order expansion (so far $N \leq 5$), we introduce the notations

$$\bar{U}_N(g, H) \equiv -\frac{M_{\text{Pl}}^2 m^2}{4} \sum_{i=2}^N \sqrt{-g} U_i(g, H), \quad (34)$$

where the tensor $H_{\mu\nu}$ is defined as in Sec. II. If the N th order expression for the function $\bar{U}_N(g, H)$ satisfies

$$\bar{U}_N(g, H)|_{h_{\mu\nu}=0, A_\mu=0} = \text{total derivative}, \quad (35)$$

(where A_μ denotes the helicity-1 field) then the decoupling limit Lagrangian for the helicity-0 and -2 interactions, up to a total derivative, takes the form

$$\mathcal{L}_{\Lambda_3}^{\text{lim}} = -\frac{1}{2} h^{\mu\nu} \mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} + h^{\mu\nu} \bar{X}_{\mu\nu}^{(N)}(\pi), \quad (36)$$

with the conserved tensor $\bar{X}_{\mu\nu}^{(N)}$:

$$\bar{X}^{(N)\mu\nu}(\pi) = \frac{\delta \bar{U}_N(g, H)}{\delta h_{\mu\nu}} \Big|_{h_{\mu\nu}=0, A_\mu=0}. \quad (37)$$

We have checked that the above Lagrangian gives rise to equations of motion with no more than two time derivatives and appropriate constraints for $N \leq 5$. It seems reasonable to conjecture that this will also be the case for $N > 5$. Furthermore, in four dimensions $\bar{X}_{\mu\nu}^{(N)}$ can only contain a finite number of terms if it is local and conserved.

It is therefore likely that this formalism leads to a finite number of interactions in the decoupling limit.

At a given order n in the expansion, there should be enough freedom to set the polynomial $U_n(g, H)$ appropriately, so as to ensure that the leading interactions (12) enter as a total derivative of the form (13), or as $f(\Pi)\mathcal{L}_{\text{der}}^{(m)}$ for $m \geq 5$ and f being an arbitrary function of $\Pi_{\mu\nu}$. The resulting leading contribution is then of the form

$$\mathcal{L}^{(n)} = \frac{\beta}{\Lambda_3^{n-1}} h^{\mu\nu} X_{\mu\nu}^{(n)}, \quad (38)$$

where β depends on the coefficient c 's, d 's, etc. and $X_{\mu\nu}^{(n)} \sim \Pi_{\mu\nu}^n$ must be conserved as a straightforward consequence of reparametrization invariance in the decoupling limit (since higher interactions in h are then suppressed). At each order n , there is a unique combination of $\Pi_{\mu\nu}^n$'s which is conserved. This combination is of the form

$$X_{\mu\nu}^{(n)} \propto \frac{\delta \mathcal{L}_{\text{der}}^{(n+1)}}{\delta \Pi_{\mu\nu}}. \quad (39)$$

In four dimensions, however, $\mathcal{L}_{\text{der}}^{(5)} \equiv 0$ as pointed out earlier, and the same remains true at higher orders. This further implies that there is a limit on the number of possible interactions in the decoupling limit: $X_{\mu\nu}^{(n)} \equiv 0$ for any $n \geq 4$. This suggests that all theories of massive gravity (with the scale Λ_3) can only have at most quartic couplings between the helicity-0 and -2 modes in the decoupling limit.

V. MASSIVE GRAVITY AND THE GALILEON

When making the generalized FP choice for the coefficients (21), (24)–(27), and (30), the higher interactions in the decoupling limit only arise as a coupling between the tensor mode and the helicity-0 mode of the form

$$\mathcal{L}_{\text{int}} = h^{\mu\nu} \bar{X}_{\mu\nu}^{(N)} = h^{\mu\nu} \left(X_{\mu\nu}^{(1)} + \frac{1}{\Lambda_3} X_{\mu\nu}^{(2)} + \frac{1}{\Lambda_3^2} X_{\mu\nu}^{(3)} \right), \quad (40)$$

where $X^{(1)}$ is given by (20), $X^{(2)}$ by (22), and $X^{(3)}$ by (28). Moreover, as emphasized before, $\partial^\mu X_{\mu\nu}^{(i)} = 0$. We proceed further by noticing that

$$X_{\mu\nu}^{(1,2)} = \hat{\mathcal{E}}_{\mu\nu}^{\alpha\beta} Z_{\alpha\beta}^{(1,2)}, \quad (41)$$

with

$$Z_{\mu\nu}^{(1)} = \pi \eta_{\mu\nu}, \quad (42)$$

$$Z_{\mu\nu}^{(2)} = (6c_3 - 1) \partial_\mu \pi \partial_\nu \pi. \quad (43)$$

We can therefore diagonalize the action up to the cubic order by performing a local but nonlinear change of the variable

$$h_{\mu\nu} = \hat{h}_{\mu\nu} + Z_{\mu\nu}^{(1)} + \frac{1}{\Lambda_3^2} Z_{\mu\nu}^{(2)}, \quad (44)$$

such that, up to total derivatives, the Lagrangian is

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \hat{h}_{\mu\nu} \hat{\mathcal{E}}^{\mu\nu\alpha\beta} \hat{h}_{\alpha\beta} + \frac{3}{2} \pi \square \pi + \frac{3}{2} \frac{(6c_3 - 1)}{\Lambda_3^3} (\partial\pi)^2 \square \pi \\ & + \frac{1}{\Lambda_3^6} \left(\frac{1}{2} (6c_3 - 1)^2 - 2(c_3 + 8d_5) \right) (\partial\pi)^2 \\ & \times ([\Pi^2] - [\Pi]^2) + \frac{1}{\Lambda_3^6} \hat{h}^{\mu\nu} X_{\mu\nu}^{(3)} - \frac{5}{2\Lambda_3^9} (6c_3 - 1) \\ & \times (c_3 + 8d_5) (\partial\pi)^2 ([\Pi]^3 - 3[\Pi][\Pi]^2 + 2[\Pi^3]). \quad (45) \end{aligned}$$

In the first line we see appearing the quadratic and cubic Galileon terms, [17] (the usual kinetic term for π , as well as the interaction present in DGP). Next we notice the quartic Galileon interaction and finally the quintic, last interaction of the Galileon family, appears in the last line.

By setting $c_3 = -8d_5$ we precisely recover the Galileon family of terms up to quartic order, and all the remaining couplings with the tensor mode disappear at the quintic order. Since there is still a lot of freedom in the coefficients at higher orders, it is only natural to expect this result to be maintained to all orders.

On the other hand, if $c_3 \neq -8d_5$, then the last mixing term $h^{\mu\nu} X_{\mu\nu}^{(3)}$ does not seem to be removable via any *local* field redefinition. This mixing term may be crucial to address the issue of superluminality of the massive theory, as the Galileon without the mixing terms does exhibit superluminal behavior [17].

In a more general case, as soon as the cubic Galileon is present in (45), we are also bound to have either the quartic Galileon and no other terms (for $c_3 = -8d_5$), or a quartic mixing and the quintic Galileon [for $4(c_3 + 8d_5) = (6c_3 - 1)^2 \neq 0$], or all of the above terms together.

If, however, the cubic Galileon is absent (for $c_3 = 1/6$), one in general is left with the quartic Galileon and the quartic mixing term.

Finally, notice also that for the specific choice $c_3 = 1/6$ and $d_5 = -1/48$, all the interactions at the scale Λ_3 disappear. This may be an example of a theory for which the decoupling limit picks up a higher scale $\Lambda_* > \Lambda_3$, if such a theory exists. Alternatively, this may also be a theory in which all the nonlinear terms disappear in the decoupling limit. This would suggest that the theory has no strongly coupled behavior (i.e., no Vainshtein mechanism) and would be ruled out observationally.

VI. OUTLOOK

The previous analysis shows that for appropriate choices of interactions that generalize the Fierz-Pauli term to higher orders, one can construct a consistent and local theory of massive gravity where no ghostlike instabilities are present, at least up to the quintic order in the decou-

pling limit, and positive prospects can be foreseen for higher orders. In particular, the connection with the Galileon generalization of the cubic term appearing in the DGP decoupling limit provides a natural framework for studying ghost-free theories of gravity [17,21].

Furthermore, the decoupling limit considerations of this paper suggest that the higher nonlinear terms in (5)–(9) become equally important at the scale Λ_3 . Since the scale $\Lambda_3 = (M_{\text{Pl}} m^2)^{1/3}$ is very low (typically $\Lambda_3 \sim 10^{-9}$ eV), the effective theory below Λ_3 can only be used for large scale cosmological studies.² To extend the scope of applicability of massive gravity to shorter length scales, however, one would need to go above Λ_3 , and, hence, the higher interactions should be taken into account. For a viable model, it will therefore be necessary to consider all the higher polynomial interactions, $U_n(g, H)$, and not only the ones up to the quintic order as presented here (even though the decoupling limit may only have a finite number of interactions).

A theory that provides such a resummation is the model of Refs. [14,15]. In particular, by integrating out the auxiliary dimension in that model, one gets an infinite series of interactions of the form (5)–(8) and beyond, with certain specific coefficients. In [16], it has been checked that the coefficients of the quadratic and cubic terms were equal to those used in Sec. III for the specific choice $c_3 = 1/4$. Thus, in the decoupling limit, the theory is ghost-free up to the cubic order. Furthermore, the theory in the cubic order preserves the Hamiltonian constraint even away from the decoupling limit [16], and the BD term cancels out in the exact all-order Hamiltonian [14]. Moreover, it was shown in Ref. [15] that the nonlinear terms giving rise to a ghost at a scale $\Lambda < \Lambda_3$ cancel out in that specific theory. These findings constitute an important evidence (but not a proof yet) that the theory of [14,15] may be consistent, at least classically, to all orders.

How about other possible theories of massive gravity that would yield the terms discussed here with the coefficients still consistent with the absence of the ghost but not coinciding with the ones obtained in [16]? Is there any hope for these theories away from the decoupling limit and above the scale Λ_3 ? Naively, the answer seems to be a negative one: As was shown in [9], in the order-by-order expansion, and beginning with the quartic order, one cannot avoid higher powers of the lapse function in the Hamiltonian, and hence, the emergence of the sixth degree of freedom (which typically is a ghost) seems to be unavoidable in massive gravity [9].

However, there may be a way to circumvent this problem in the full theory if its Hamiltonian, due to a resummation of perturbative terms, ends up having a very special

²Once external classical sources, such as planets, stars, galaxies, etc., are present, the energy scale of nonlinearities—the Vainshtein scale—depends on the mass/energy of the source and is significantly lower [5].

dependence on the lapse and shift functions. Here we demonstrate this in a toy example, that is motivated by the Hamiltonian of the theory [14,15] discussed in [14].

Consider the toy Hamiltonian:

$$H = N(R^0 + m^2 f(\gamma)) + N_j(R^j + m^2 Q^j(\gamma)) + m^2 P(\gamma) \frac{N_j N^j}{2N}, \quad (46)$$

where N , N_j , γ_{ij} , and R^0 , R^j are the standard Arnowitt-Deser-Misner canonical formalism variables and functions, respectively [23]; $f(\gamma)$, $Q_j(\gamma)$, and $P(\gamma)$ are some functions that modify the general relativity constraints by the mass terms. The shift function N_j is not a Lagrange multiplier but is algebraically determined, as should be the case for a massive theory with 5 degrees of freedom. However, the lapse function also enters in the last term in a way that seems to prevent it from being a Lagrange multiplier, and if so, it would give rise to the sixth degree of freedom. This is not the case, however: One can introduce a new variable $n_j \equiv N_j/N$ in terms of which the Hamiltonian reads

$$H = N(R^0 + m^2 f(\gamma)) + N n_j(R^j + m^2 Q^j(\gamma)) + N m^2 P(\gamma) \frac{n_j n^j}{2}. \quad (47)$$

The shift n_j still has no conjugate momentum, hence $\delta H/\delta n_j = 0$. This determines the new shift variable, $n^j = -(R^j + m^2 Q^j(\gamma))/(m^2 P(\gamma))$, and yields the following Hamiltonian

$$H|_{n_j} = N \left(R^0 + m^2 f(\gamma) - \frac{(R_j + m^2 Q_j(\gamma))^2}{2m^2 P(\gamma)} \right). \quad (48)$$

Here, the lapse does certainly appear as the Lagrange multiplier. Hence, the BD term does not arise, and the theory does not propagate the sixth degree of freedom.³

On the other hand, a direct perturbative expansion of the last term in (46) in powers of $\delta N = N - 1$ with subsequent truncation of this series at any finite nonlinear order necessarily yields higher powers of δN in the Hamiltonian.⁴ Naively, this truncated theory would give rise to the poten-

³In general, it could still be propagating “5.5” modes even if the Hamiltonian constraint is maintained. For instance, since the toy model described by (46) is not Lorentz invariant for general functions f , Q_j , and P , there may exist nonpropagating instantaneous modes in this model. For discussions of related issues, see [24]. In contrast, the model of Refs. [14,15] is 4D Lorentz invariant and the instantaneous mode in 4D is not expected. For a rigorous proof that there are only 5 degrees of freedom, and not “5.5,” however, a detailed study of the algebra of the Hamiltonian constraint should be performed. The fact that the decoupling limit gives only 5 degrees of freedom is an important hint that the full theory is not likely to have the extra “0.5” degree of freedom.

⁴Note that away from the decoupling limit, and at a nonlinear order, δN is the right variable and not $h_{00} = 1 - N^2 + N_j^2$, which was used before as the lapse in the decoupling limit.

tially false impression that the lapse is not a Lagrange multiplier, and that there is a sixth degree of freedom in the model.

Noticing that the higher powers of δN at any finite nonlinear order emerge from the expansion of the theory (46) is trivial, in this toy model. However, a similar, albeit more complicated structure, emerges in the Hamiltonian of the model of [14,15] (see [14]), and the fact that the terms in the expansion come up from a single term in the exact Hamiltonian is not as simple to observe.

Last but not least, in this work we discussed the classical theory. Generic quantum loop corrections are expected to renormalize and detune the coefficients of the polynomial terms needed to avoid the ghost. One way to be protected against this problem is to have a theory in which the tuned coefficients automatically emerge as a consequence of a symmetry that would be respected by the loop corrections. In this respect, the recent findings of [16] that the cubic terms with the automatically tuned coefficients emerge as an expansion of the theory, which by itself exhibits an evidence for a hidden nonlinearly realized symmetry, makes us hopeful for the existence of a quantum-mechanically stable effective field theory of massive gravity.

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APPENDIX: DECOUPLING LIMIT WITH THE OPPOSITE SIGN IN $H_{\mu\nu}$

As mentioned in Sec. II A, the expression (10) for $H_{\mu\nu}$ differs by a minus sign in front of the third term on the right-hand side from its counterpart considered in Eq. (5) of [9]:

$$H_{\mu\nu} = \frac{h_{\mu\nu}}{M_{\text{Pl}}} + \partial_\mu \pi_\nu + \partial_\nu \pi_\mu + \eta_{\alpha\beta} \partial_\mu \pi^\alpha \partial_\nu \pi^\beta. \quad (\text{A1})$$

To emphasize the importance of this sign difference, we show that we recover the results of Ref. [9] when deriving the decoupling limit using (A1), but stress that the Bianchi identity is then not satisfied, as a consequence of the fact that $H_{\mu\nu}$ is then not a covariant tensor if $g_{\mu\nu}$ and $h_{\mu\nu}$ are conventionally defined.

Up to the cubic order, the Lagrangian in the decoupling limit is then

$$\tilde{\mathcal{L}} = -\frac{1}{2} h^{\mu\nu} \hat{\mathcal{E}}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} + h^{\mu\nu} \tilde{X}_{\mu\nu}^{(1)} - \frac{1}{4\Lambda_5^2} ((8c_1 + 4)[\Pi^3] + (8c_2 - 4)[\Pi][\Pi^2] + 8c_3[\Pi]^3) + \frac{1}{\Lambda_3^3} h^{\mu\nu} \tilde{X}_{\mu\nu}^{(2)}, \quad (\text{A2})$$

with $\tilde{X}_{\mu\nu}^{(1)} = X_{\mu\nu}^{(1)}$, since both approaches only differ at quadratic order in π , and

$$\begin{aligned} \tilde{X}_{\mu\nu}^{(2)} = & -(3c_1 - \frac{3}{2})\Pi_{\mu\nu}^2 - 2(1 + c_2)[\Pi]\Pi_{\mu\nu} \\ & + (\frac{1}{2} - 3c_3)[\Pi]^2\eta_{\mu\nu} - c_2[\Pi^2]\eta_{\mu\nu}. \end{aligned} \quad (\text{A3})$$

Setting $c_1 = 2c_3 - \frac{1}{2}$ and $c_2 = -3c_3 + \frac{1}{2}$ to obtain the total derivative combination (15), we get

$$\begin{aligned} \tilde{X}_{\mu\nu}^{(2)} = & -6(c_3 - \frac{1}{2})(\Pi_{\mu\nu}^2 - [\Pi]\Pi_{\mu\nu}) \\ & - (3c_3 - \frac{1}{2})([\Pi]^2 - [\Pi^2])\eta_{\mu\nu}, \end{aligned} \quad (\text{A4})$$

which is not conserved for any choice of c_3 since the reparametrization invariance is not present with this choice of $H_{\mu\nu}$, and the Bianchi identity has no reason to be satisfied.

Similarly at the quartic order, we would need to impose the relation between the coefficients $d_1 = -6d_5 - \frac{1}{16} \times (24c_3 - 5)$, $d_2 = 8d_5 + \frac{1}{4}(6c_3 - 1)$, $d_3 = 3d_5 + \frac{1}{16} \times$

$(12c_3 - 1)$, and $d_4 = -6d_5 - \frac{3}{4}c_3$ to cancel the terms of the form $\Lambda_4^{-8}(\partial\partial\pi)^4$. The mixing with the helicity-2 mode will then enter with the quantity $\tilde{X}_{\mu\nu}^{(3)}$ as derived in [9]:

$$\begin{aligned} \tilde{X}_{\mu\nu}^{(3)} = & (-1 + 9c_3 + 24d_5)(\Pi_{\mu\nu}^3 - [\Pi]\Pi_{\mu\nu}^2) \\ & - (9c_3 + 24d_5)\Pi_{\mu\nu}([\Pi^2] - [\Pi]^2) \\ & - (c_3 + 8d_5)\eta_{\mu\nu}([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]). \end{aligned} \quad (\text{A5})$$

As noticed in [9], not only would there then be no choice of c_3 and d_5 for which this interaction disappears, but also it would always lead to higher derivative equations of motion, suggesting a ghostlike instability. However, the fact that $\tilde{X}_{\mu\nu}^{(3)}$ is not conserved is an artifact of the sign choice in the expression for $H_{\mu\nu}$ that does not lead to reparametrization invariant results.

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