PHYSICAL REVIEW D **82,** 043533 (2010)

Sub-eV scalar dark matter through the super-renormalizable Higgs portal

Federico Piazza[1,2,*] and Maxim Pospelov[1,3,†]

[1]*Perimeter Institute for Theoretical Physics, Waterloo, ON, N2L 2Y5, Canada*
[2]*Canadian Institute for Theoretical Astrophysics (CITA), Toronto, Canada*
[3]*Department of Physics and Astronomy, University of Victoria, Victoria, BC, V8P 1A1, Canada*
(Received 7 April 2010; published 31 August 2010)

The Higgs portal of the standard model provides the opportunity for coupling to a very light scalar field ϕ via the super-renormalizable operator $\phi(H^\dagger H)$. This allows for the existence of a very light scalar dark matter that has coherent interaction with the standard model particles and yet has its mass protected against radiative corrections. We analyze ensuing constraints from the fifth force measurements, along with the cosmological requirements. We find that the detectable level of the fifth force can be achieved in models with low inflationary scales, and by a certain amount of fine-tuning in the initial deviation of ϕ from its minimum.

DOI: 10.1103/PhysRevD.82.043533 PACS numbers: 95.35.+d, 12.10.Dm

I. INTRODUCTION

About 95% of the energy budget of the Universe consists of "dark"—and unknown—components. This is a strong motivation for considering and studying hidden sectors beyond the standard model (SM). Gravitational effects of dark matter cannot reveal the mass of its constituents, and indeed a wide variety of mass ranges, from the inverse galactic size to the super-Planckian scales, is conceivable. While many models that possess stable particles with masses comparable to the SM energy scales have been a subject of incessant theoretical and experimental activity, models with light sub-eV mass scale dark matter received far less attention.

Below the eV mass scale, the dark matter would have to be of integer spin [1] and be produced nonthermally. Thermal candidates in this mass range would only contribute to the "hot dark matter" component, and be subdominant to the "cold dark matter" component and unable to account for the observed patterns of structure formation. The only chance of detecting sub-eV dark matter nongravitationally would occur if such particles are converted into electromagnetic radiation in the external fields or they modify the interaction strength of SM particles. But if light dark matter interacts with the SM, then immediately its lightness comes into question, as the quantum loops with SM particle may easily destabilize the mass scale. A prominent particle in this category is the QCD axion [2] that interacts with the SM currents derivatively, $j_\mu \partial_\mu a$, and has its tiny mass generated by the nonperturbative QCD effects protected at any loop level. Because of the pseudoscalar nature of a and its derivative couplings, it does not generate a long-range attractive force.

A very natural question to ask is whether SM allows for couplings to other types of sub-eV dark matter fields that lead to additional observable effects. For a recent review of the light sector phenomenology, see, e.g., [3]. Real scalar field ϕ and the vector field V_μ provide such opportunities with their couplings to the SM fields via the so-called Higgs and vector portals:

$$\begin{aligned} &(A\phi + \lambda\phi^2)H^\dagger H \qquad \text{Higgs portal} \\ &J_\mu V_\mu; \partial_\mu J_\mu = 0 \qquad \text{Vector portal,} \end{aligned} \tag{1}$$

where H is the Higgs doublet, A and λ are parameters, and J_μ is some locally conserved SM current, such as hypercharge of baryon current. If there is some initial value for ϕ or V_μ fields with respect to their zero-energy configurations, one can source part/all of the Universe's energy density from the coherent oscillations around the minimum.

The perils of low mass scale stabilization are immediately apparent in Eq. (1). Indeed, any loops of the SM fields would tend to induce the correction to the mass of ϕ field $\sim\lambda\Lambda_{\rm UV}^2$, where $\Lambda_{\rm UV}$ is the highest energy scale in the problem, serving as the ultraviolet cutoff. Therefore, λ should be taken to incredibly small values, making this portal irrelevant for the phenomenology of sub-eV dark matter. In contrast, the vector portals and the super-renormalizable Higgs portal, $A\phi H^\dagger H$, allow to avoid problems with technical naturalness. In the latter case, loop corrections scale only as $A^2 \log\Lambda_{\rm UV}$, while the quadratic divergences affect only the term linear in ϕ, which can typically be absorbed in an overall field shift. In this paper, we examine generic consequences of this coupling for the sub-eV scalar dark matter, leaving vector dark matter to future studies.

II. SUPER-RENORMALIZABLE PORTAL TO THE SCALAR DARK MATTER

The specific case of a singlet scalar ϕ coupled via a super-renormalizable term of the type $\phi H^\dagger H$ (see, e.g., [4–9] and references therein) has been mostly studied in

*fpiazza@perimeterinstitute.ca
†mpospelov@perimeterinstitute.ca

connection with electroweak and GeV-scale phenomenology, with a notable exception of [7,10], where a possibility of superweakly interacting Higgs-coupled dark matter was pointed out. The scalar potential in the model of interest reads as

$$V = -\frac{m_h^2}{2}H^\dagger H + \lambda(H^\dagger H)^2 + AH^\dagger H\phi + \frac{m_\varphi^2}{2}\phi^2. \quad (2)$$

This model is explicitly renormalizable and does not require any additional UV completion (if one is willing to tolerate the usual fine-tuning problem with m_h^2 itself). We chose to redefine away possible linear terms in ϕ by shifting the field, and absorbing $A\Delta\phi$ into m_h^2.

After spontaneous symmetry breaking, the two fields acquire a vacuum expectation value, $\langle H^\dagger H\rangle = v^2/2$, $\langle\phi\rangle = \phi_0$, where

$$v^2 = \frac{m_h^2}{2\lambda - A^2/m_\varphi^2}, \qquad \phi_0 = -\frac{Av^2}{2m_\varphi^2} \quad (3)$$

and $v = 246$ GeV. The potential (2) has a stable minimum only if $A^2/m_\varphi^2 < 2\lambda$, which is what we assume in the following; otherwise, it develops a runaway direction in the $(\phi, H^\dagger H)$ plane unless additional nonlinear ϕ^4 terms are introduced. The low energy dynamics are encoded in the two physical fields h and φ, defined as

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h\end{pmatrix}, \qquad \phi = \phi_0 + \varphi, \quad (4)$$

and with Lagrangian

$$\mathcal{L} = \frac{(\partial h)^2}{2} + \frac{(\partial\varphi)^2}{2} - \frac{m_h^2}{2}h^2 - \frac{m_\varphi^2}{2}\varphi^2 - (Av)h\varphi - \frac{A}{2}h^2\varphi + \dots \quad (5)$$

As already noted, Higgs loops give only logarithmically divergent corrections to m_φ, while the radiatively generated quartic terms $\varphi^2 H^\dagger H$ are finite and small, $\sim A^2/v^2$. Therefore, the requirement of technical naturalness bounds the scale of m_φ from below by the coupling A. In summary, by defining the dimensionless ratio $x \equiv A/m_\varphi$, we assume $x < 1$ and $x < \sqrt{2\lambda}$, although values $x \ll 1$ will also be considered. Alternatively, the low-energy Lagrangian (5) could be written in the mass eigenvalues field basis, where the lowest mass eigenvalue is of order $m_\varphi[1 + \mathcal{O}(x)]$.

III. FIFTH FORCE AND EQUIVALENCE PRINCIPLE VIOLATION

The singlet φ couples to SM particles through the mixing with the Higgs field. Depending on the mass m_φ and coupling A, the φ-mediated attractive force can produce testable deviations from $1/r^2$ gravitational force as well as composition dependence, thus violating the equivalence principle (EP). The leading contributions to φ couplings mediated by the φ Higgs propagator is shown in Fig. 1. As a rule of thumb, the φ couplings are suppressed with respect to the Higgs couplings by a factor of Av/m_h^2:

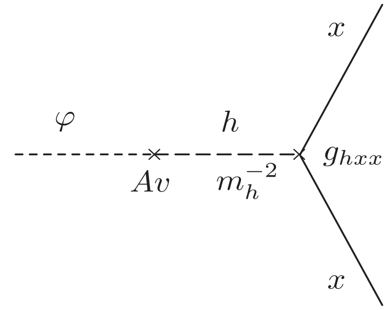

FIG. 1. The mixing with the Higgs Av mediates the coupling of φ to SM particles.

$$g_{\varphi xx} = \frac{Av}{m_h^2}g_{hxx}, \quad (6)$$

where g_{hxx} is the effective dimensionless coupling of the Higgs to x particle at very low momentum transfer. Therefore, the effective Lagrangian describing the interactions with the SM gauge and fermion fields takes the following form:

$$\mathcal{L}_{\rm eff} = \frac{Av}{m_h^2}\left(g_{hff}\bar{f}f + \frac{g_{h\gamma\gamma}}{v}F_{\mu\nu}F^{\mu\nu} + \dots\right)\varphi. \quad (7)$$

In the above, g_{hff} are the Yukawa couplings to fermions. Those can either be fundamental, as the SM couplings to quarks and leptons, $g_{hqq} = m_q/v$, $g_{hll} = m_l/v$ where m_q (m_l) is the mass of the quark (lepton) under consideration, or effective, as in the case of the nucleons. The latter includes the contributions from all heavy quarks contributing to the coupling to gluons g_{hgg} that provide a dominant contribution in the chiral limit [11]. Below the QCD scale, the estimate of the effective Yukawa coupling of the Higgs to nucleons is rather uncertain due to a poorly known strangeness content of the nucleon in the 0^+ channel:

$$g_{hNN} \simeq \frac{200\text{–}500\ \text{MeV}}{v} \sim O(10^{-3}). \quad (8)$$

This is much larger than the naive contribution of up and down quarks.

The violation of EP is evident from the fact that the electrons and nucleons have couplings to the φ field that do not scale exactly with masses,

$$\frac{g_{hee}}{m_e} \neq \frac{g_{hNN}}{m_{\rm nuc}}. \quad (9)$$

The effective coupling of the Higgs to the electromagnetic field, $g_{h\gamma\gamma}$, is obtained by integrating out heavy charged particles, and the question of which one is "heavy" depends on the characteristic q^2 of (virtual) photons. The coupling $g_{h\gamma\gamma}$ can be written in the following form (see, e.g., [12]):

$$g_{h\gamma\gamma} = \frac{\alpha_{\rm EM}}{6\pi}\left(3\sum_q Q_q^2 + \sum_l Q_l^2 - \frac{21}{4}\right), \tag{10}$$

where summation goes over the quark and lepton fields with charges Q_q and Q_l, and the last term is due to the W bosons. For the purpose of calculating the $\varphi \to \gamma\gamma$ decay, one has to sum over e, μ, τ and c, b, t. Corrections coming from the light quark sector are subdominant, because in the chiral limit they contribute at two loops. In practice, their contribution would amount to at most 10% correction. Including these fermion contributions gives $g_{h\gamma\gamma}(q^2 = m_\varphi^2) \simeq \alpha_{\rm EM}/(8\pi)$. For the purpose of calculating the coupling of φ to nuclei when the EM fraction of energy is taken into account, electrons should not be included in the sum, and muon contribution should include a form factor. We are not going to pursue this calculation, because it turns out that $g_{h\gamma\gamma}$ provides a subleading contribution to the EP violation.

Field φ mediates a fifth force of range $\sim m_\varphi^{-1}$. More precisely, at the Newtonian level of approximation, the total effective gravitational potential between two bodies A and B at relative distance r presents a Yukawa contribution due to the interaction of the long-range field φ,

$$V(r) = -G\frac{m_A m_B}{r}(1 + \alpha_A\alpha_B e^{-m_\varphi r}). \tag{11}$$

The scalar couplings α can be expressed in terms of the log derivative of the masses as

$$\frac{\alpha_A}{\sqrt{2}M_P} = \frac{d\ln m_A(\varphi)}{d\varphi}, \tag{12}$$

where M_P is the reduced Planck mass and $m_A(\varphi)$ includes terms in the Lagrangian that are bilinear in the fields and couple to φ, such as those in Eq. (7). When calculating α_A, one should consider the leading universal contribution from the nucleons and all the corrections that are specific to the element A (See, e.g., [13]). The main, species-independent part of the nuclear mass is given by $m_{\rm nuc}(N_A + Z_A)$, and the universal coupling α is obtained from Eqs. (7), (8), and (12):

$$\alpha = g_{hNN}\frac{\sqrt{2}M_P}{m_{\rm nuc}}\frac{Av}{m_h^2} \simeq 10^{-3}\left(\frac{m_h}{115\ {\rm GeV}}\right)^{-2}\frac{A}{10^{-8}\ {\rm eV}}. \tag{13}$$

In the limit of a very long-range force, the value of α is bounded by post-Newtonian tests of general relativity to $\alpha^2 \lesssim 10^{-5}$ [14]. However, one can easily see that for mass range of m_φ below 10^{-12} eV, the relative strength of the ϕ-induced force drops below 10^{-14} from the gravitational field strength, which would make it extremely challenging for experimental detection and immune to the solar system tests. Thus, it is more interesting to consider intermediate-range forces. Tests of gravitational inverse-square law limit the Yukawa component of the gravitational potential [15,16]. By means of Eq. (13), such tests give a bound on A. This is shown in Fig. 2. The two panels are elaborations of plots taken from Refs. [15,16]. A force with similar values of m_φ and A ($x \simeq 1$) is excluded in the range of masses $m_\varphi \simeq 10^{-8}$ eV–10^{-3} eV.

The calculations of the EP-violating part of the scalar exchange is a far more delicate exercise. One should recognize that the EP is violated already at the level of nucleons, that is, $g_{hnn}/m_n \neq g_{hpp}/m_p$. As is well known, the neutron and proton mass difference comes about because of the unequal quark masses and electromagnetic

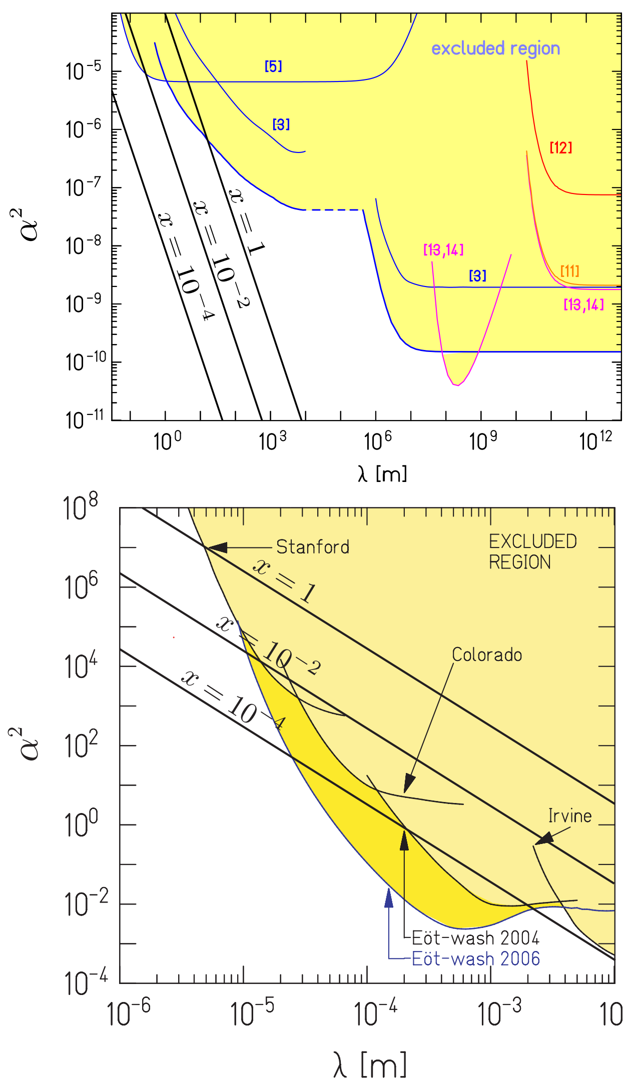

FIG. 2 (color online). We plot the constraints on the mass m_φ and coupling $A = xm_\varphi$ coming from fifth force experiments and taking g_{hNN} to the maximum of its allowed range. The range of the force is just $\lambda = m_\varphi^{-1}$. The coupling α is obtained in Eq. (13) by assuming $m_h \simeq 120$ GeV. For two different mass ranges, the lines corresponding to $x = 1$, $x = 10^{-2}$, and $x = 10^{-4}$ are superimposed on the plots of Refs. [15,16] (upper panel and lower panel, respectively).

contribution to the nucleon mass. One can estimate $(m_n - m_p)|_{m_u \neq m_d} \simeq 2.1$ MeV and $(m_n - m_p)|_{\rm EM} \simeq -0.8$ MeV, so that together both contributions combine to the observable mass difference $\Delta m_{np} = 1.3$ MeV. The φ dependence of both pieces is completely different. Because of the loop smallness of $g_{h\gamma\gamma}$, the electromagnetic fraction of nucleon mass is far less dependent on φ: $\partial(m_n - m_p)|_{\rm EM}/\partial h \ll \partial(m_n - m_p)|_{m_u \neq m_d}/\partial h$. Therefore, when we estimate the mass of an atom, we add to the universal term proportional to the baryon number a correction proportional to the nucleon mass difference:

$$m = (N + Z) m_{\rm nuc}(\varphi) + \frac{N - Z}{2} \Delta_{np} m(\varphi) + \ldots \qquad (14)$$

The first term in (14) produces the universal coupling α calculated in (13). The composition-dependent correction reads

$$\alpha^{\rm EPV} \simeq \alpha \frac{N - Z}{2(N + Z)} \frac{\Delta m_{np}}{m_N} \left(\frac{m_N}{g_{hNN}} \frac{\partial \Delta m_{np}/\partial h}{\Delta m_{np}} - 1 \right)$$
$$\simeq \alpha \frac{N - Z}{2(N + Z)} \times 3 \times 10^{-3}. \qquad (15)$$

This may lead to a sizable variation of acceleration Δa between light atoms with $Z = N$ and heavy atoms with $\frac{N-Z}{2(N+Z)} \simeq 0.1$,

$$\frac{\Delta a}{a} \simeq \alpha^2 \times \mathcal{O}(10^{-3} - 10^{-4}). \qquad (16)$$

Other important effects should be related to the dependence of the nuclear binding energy on φ, that can easily reach a level comparable to (15). More detailed considerations of nuclear mass dependence on φ go outside the scope of the present paper.

As long as we adhere to our naturalness condition $A \simeq m_\varphi$, the present bounds on composition-dependent EP violations ($\Delta a/a \lesssim 10^{-13}$) are easily evaded. When the Earth is the common attractor of the two free-falling bodies, the relevant range $m_\phi^{-1} \simeq 10^4$ km turns into extremely tiny values for the coupling A. Still, if we were to consider more fine-tuned scenarios ($m_\varphi \ll A$), it is interesting to note that a fifth force attached to the Higgs portal displays a peculiar relation between composition-independent and composition-dependent effects, as clearly follows from Eq. (15). In principle, this allows us to distinguish between the Higgs portal and, e.g., the string-inspired scenarios [13,17,18].

IV. COSMOLOGICAL CONSTRAINTS

Since all couplings to SM particles are suppressed by a factor Av/m_h^2, the scalar field is sufficiently stable to be a nonthermal relic (for earlier studies of scalar dark matter see, e.g., [19–23]). Its decay rate into photons is smaller than the current Hubble rate as long as the mass is under a keV:

$$\Gamma_\varphi = \frac{m_\varphi^3 A^2 g_{h\gamma\gamma}^2}{4\pi m_h^4} \simeq 10^{-37} \text{ eV} \times x^2 \left(\frac{m_\varphi}{1 \text{ keV}} \right)^5 \left(\frac{100 \text{ GeV}}{m_h} \right)^4. \qquad (17)$$

However, as emphasized in [10], the constraints from the gamma ray background would provide much tighter constraints, and in what follows we will concentrate on the sub-eV range.

The abundance of φ particles today can be estimated [24,25] in terms of the initial misalignment φ_* of the field from its minimum at the time t_* when $m_\varphi \sim 3H$. At that moment, the field starts oscillating around the minimum of its potential and behaves in a nonrelativistic matter. The number of particles in a comoving volume is conserved, so that $n_\varphi/s = \text{const}$, where $s = 0.44 g_* T^3$ is the entropy density, n_φ is the number density of φ particles, and g_* is the number of effective degrees of freedom in equilibrium with the photons. The (average) energy density of φ today is thus given by $\rho_\varphi^0 = m_\varphi n_\varphi s^0/s$, where n_φ and s should be taken at t_*. Using the relations $m_\varphi n_\varphi = m_\varphi^2 \varphi_*^2/2$ and $s^0/s_* = (2/g_*)(T_\gamma^0/T_*)^3$ together with the Hubble rate

$$H \simeq \tilde{g}_*^{1/2} \frac{T^2}{3M_P}, \qquad (18)$$

we express T_* in terms of the parameters of the model to obtain

$$\Omega_\varphi h^2 = 0.4 \frac{\tilde{g}_*^{3/4}}{g_*} \left(\frac{m_\varphi}{10^{-9} \text{ eV}} \right)^{1/2} \left(\frac{\varphi_*}{10^{14} \text{ GeV}} \right)^2, \qquad (19)$$

where $\tilde{g}_*$ is the number of degrees of freedom relevant for estimating H. Since $T_* \sim 10^5 (m_\varphi/\text{eV})^{1/2}$ GeV, in the mass range of interest $\tilde{g}_* = g_*$ and $0.4 \tilde{g}_*^{3/4}/g_* \simeq O(0.1)$.

An important constraint on the model comes from the smallest allowed mass for a dark matter particle. The observations of the smallest halos show their size to be comparable to 1 kpc [26], which means that the Compton wavelength of φ field would have to be comparable to or smaller than this scale. This in turn imposes the constraint on φ_*:

$$m_\varphi > 10^{-26} \text{ eV} \Rightarrow \varphi_* < 2 \times 10^{18} \text{ GeV} \times \left(\frac{\Omega_\varphi h^2}{0.1} \right)^{1/2}. \qquad (20)$$

Notice that at the boundary of the allowed value, $m_\varphi \sim 10^{-26}$ eV, the oscillations start around 10 eV, that is, just before the matter-radiation equality.

By Eq. (19), it is clear that the abundance of φ particles depends on the VEV of the field at the moment when the Hubble parameter becomes of the order of its mass. This is ultimately a matter of initial conditions. However, it is interesting to study the preceding evolution of φ up to electroweak symmetry breaking. During inflation, while the field would classically stay constant, its vacuum expectation value gets random kicks of order $H/2\pi$ every Hubble time due to quantum fluctuations (see, e.g., [27]). Its behavior can be described formally with a Langevin-type equation [17]:

$$\frac{d\phi}{dp} = \frac{H(p)}{2\pi}\zeta(p). \tag{21}$$

In the above, $p = \ln a$ is the number of e folds and ζ is a Gaussian random variable. Its p averages are $\langle\zeta(p)\rangle = 0$, $\langle\zeta(p)\zeta(p')\rangle = \delta(p - p')$. It is straightforward to estimate the expected shift in the field during inflation:

$$|\Delta\phi_{\rm inf}| \equiv \sqrt{\langle(\phi_{\rm end} - \phi_{\rm in})^2\rangle} = \frac{1}{2\pi}\left(\int_{p_{\rm in}}^{p_{\rm end}} dp' H^2(p')\right)^{1/2}. \tag{22}$$

As a rough order of magnitude, this gives $\Delta\phi_{\rm inf} \gtrsim 10H_{\rm CMB}$, where $H_{\rm CMB}$ is the Hubble parameter at the epoch where the scales relevant for the CMB left the horizon. However, in scenarios with a long epoch of self-regenerating inflation, $\Delta\phi_{\rm inf}$ can be much larger.

At the onset of radiation domination, quantum fluctuations become irrelevant and the field is governed by the classical equation

$$\ddot{\phi} + 3H\dot{\phi} + m_\varphi^2\phi + A\langle H^\dagger H\rangle = 0. \tag{23}$$

The behavior of ϕ up to electroweak phase transition is obtained by neglecting the second derivative and the mass term from the above equation. While the Higgs field is in thermal equilibrium we have, with good approximation [25], $\langle H^\dagger H\rangle = 3T^2$. By using (18) we thus get

$$\dot{\phi}\tilde{g}_*^{1/2} = -3AM_P. \tag{24}$$

To a good approximation, the field has a constant velocity, which justifies neglecting the second derivative term in (23). Every time a relativistic species leaves the thermal bath, $\tilde{g}_*$ decreases, giving a little "kick" to the field's velocity. Thus, the details of this mechanism depend on the physics beyond the SM. By making the minimal assumption $\tilde{g}_* \simeq 100$, we count only for the SM degrees of freedom and obtain a shift in field space.

$$\frac{\phi_{\rm EW} - \phi_{\rm end}}{M_P} = -0.4At_{\rm EW} = -2x \times 10^{-6}\frac{m_\varphi}{10^{-10}\ {\rm eV}}, \tag{25}$$

where the subscript EW indicates quantities at electroweak phase transition.

Finally, at the onset of EW phase transition, the field finds itself displaced from its true minimum by an amount $\varphi_{\rm EW} \equiv \phi_{\rm EW} - \phi_0$, where ϕ_0 is given in Eq. (3),

$$\phi_0 \simeq -3x \times 10^{23}\left(\frac{m_\varphi}{10^{-10}\ {\rm eV}}\right)^{-1}\ {\rm GeV}. \tag{26}$$

A potential disparity between (20) and (26) signifies a possible fine-tuning problem. The starting point for the φ field at the end of inflation would have to be reasonably close to ϕ_0. Therefore, if we start, say, with $\phi_{\rm end} = 0$ at the end of inflation, the field starts running toward its true minimum, ϕ_0, thanks to the coupling to the Higgs (25). However, for masses $m_\varphi \lesssim 10^{-5}$ eV, the shift during radiation domination (25) is irrelevant with respect to the scale set by (26). If φ mediates long-/intermediate-range ($\lambda \gtrsim$ cm) forces, its initial value φ_* has to be fine-tuned ($\varphi_* \ll |\phi_0|$), or otherwise φ particles are overproduced.

Any supercold dark matter, such as axion or φ field discussed in this paper, is prone to the CMB constraints on the amount of isocurvature perturbations (For the recent discussions of the axion isocurvature perturbations see, e.g., [28,29]). Before going to implications of these constraints for the model, we would like to comment that the scalar field with the quadratic potential is less susceptible to the isocurvature constraints than axions. In the case of the quadratic potential, the increase in the homogeneous displacement from the minimum, φ_*, over the fluctuating value $\delta\phi$ leads to the $\delta\varphi/\varphi_*$ suppression of the isocurvature perturbations, that in principle can be made arbitrarily small by the increase of φ_*. In contrast, the increase in the homogenous value of the axion field due to the periodicity of the potential $V_a(a) = V(a + 2\pi f_a)$ can lead to, at most, $\delta a/f_a$ suppression of the isocurvature perturbations.

During inflation, the field φ undergoes fluctuations of order $\delta\varphi = H/2\pi$ as any other light field, H being the Hubble rate at the time when the fluctuation exits the horizon. The produced perturbations are of isocurvature type. Following the standard treatment that also applies to axions [24,30], we can estimate the power spectrum of entropy perturbations $\mathcal{P}_\mathcal{S}(k)$ and compare it to that of curvature perturbations $\mathcal{P}_\mathcal{R}(k)$. The ratio of the two defines a parameter

$$\frac{\alpha(k)}{1 - \alpha(k)} \equiv \frac{\mathcal{P}_\mathcal{S}(k)}{\mathcal{P}_\mathcal{R}(k)} = 8\epsilon c_s\frac{\Omega_\varphi^2}{\Omega_c^2}\frac{M_P^2}{\varphi_*^2}, \tag{27}$$

where Ω_c is proportional to the total energy density in dark matter, ϵ is the usual inflationary slow-roll parameter, and c_s is the speed of sound of the adiabatic fluctuations during inflation [31].

By using (19) and $\Omega_c h^2 \simeq 0.1$, we get, for small $\alpha(k_0)$,

$$\alpha(k) = 4.7 \times 10^9\epsilon c_s\frac{\Omega_\varphi}{\Omega_c}\left(\frac{m_\phi}{10^{-9}\ {\rm eV}}\right)^{1/2}. \tag{28}$$

The above result can nicely be reexpressed in terms of the tensor to scalar ratio $r = 16\epsilon c_s$. At the pivot wave number $k_0 = 0.002\ {\rm Mpc}^{-1}$, the limit set by WMAP + BAO + SN is $\alpha(k_0) < 0.067$ [30]. This gives the rather strict constraint

$$r\frac{\Omega_\varphi}{\Omega_c}\left(\frac{m_\phi}{10^{-9}\ {\rm eV}}\right)^{1/2} \lesssim 2.3 \times 10^{-10}, \tag{29}$$

which is very similar to the conclusions reached for the axion cosmology [28,29]. If we insist on Ω_φ making most of the cold dark matter density, this result shows that the detectable level of inflationary gravitational waves ($r > 10^{-2}$) implies a very light scalar close to the bound (20), which would make φ-mediated fifth force totally negligible. Conversely, a detectable level of the fifth force ($m_\varphi > A > 10^{-9}$ eV) would imply tiny r on the order of 10^{-10} favoring some intermediate-scale inflationary scenarios, $H \sim O(r^{1/2} \times 10^{14})$ GeV. Given that in some models of inflation (see, e.g., [32]) the Hubble parameter can be as low as $H \sim$ GeV, producing a tensor to scalar ratio $r \simeq 10^{-28}$, constructing an inflationary model with the Hubble parameter at some intermediate scale does not pause any model-building challenge.

V. DISCUSSION

The model we considered in this work is very similar to the linearized version of the Brans-Dicke (BD) theory when the scalar field is supplied with the mass term. Indeed, the transformation from the Jordan to the Einstein frame puts the BD scalar in front of any dimensionful parameter. Therefore, the A parameter from the model considered here can be identified with $A \sim m_h^2/(\omega^{1/2} M_P)$, where ω is the BD parameter. It is very important to keep in mind, however, one crucial difference. In the BD theory, the ϕ field also couples to all massive states that may exist beyond the SM states, and therefore, even at the electroweak scale one should expect the extension of Eq. (2) by additional higher-dimensional operators. Such terms alter the couplings of BD scalar to matter, and make couplings to gauge bosons, e.g., $g_{\phi\gamma\gamma}$, different from the values in the model considered here. Moreover, the BD theory requires explicit UV completion, while the model with coupling via the super-renormalizable portal assumes that higher-dimensional operators are absent from the beginning and generated only via the SM loops with the φ-independent UV cutoff.

The key feature of the model considered here is its technical naturalness. It allows for a relatively light scalar dark matter that generates medium-range attractive force without extra fine-tuning of the parameters in the Lagrangian. A detectable level of the fifth force would have to be combined with inflationary scenarios with low r and would face the potential fine-tuning problem in the initial condition of the scalar field value. One of the most interesting (albeit fine-tuned) scenarios that can have particle physics implications not considered in this paper is the ϕ dependence of the electroweak phase transition. If m_φ is taken comparable to the Hubble rate at $T = 100$ GeV, $A > m_\varphi$ can lead to $|A\phi_*| \sim 10^4$ GeV^2, thus altering the properties of the electorweak sector close to the phase tansition point. This way, one could change the order of the phase transition and make it first-order if the effective Higgs mass is pushed below 50 GeV.

The model considered here falls into the class of the ‘‘supercool’’ dark matter models, such as axion dark matter. Another example worthy of investigation is the vector dark matter. There, the coupling of vector fields to the SM and the mass of the vector fields do not have to follow the strength $\times$ range $=$ const constraint of the scalar case. This could open more room for the fifth force mediated by the vectorlike sub-eV dark matter.

To sum up, the sub-eV dark matter considered in this paper represents a very different possibility compared to the often discussed models of dark matter. The direct or indirect detection of weakly interacting massive particle-type dark matter is not possible in the sub-eV case, because it carries only a minuscule amount of energy. Instead, the presence of sub-eV dark matter could be discovered via an extra contribution to the gravitational force. Should the scalar field discussed in this paper have an additional (derivative) pseudoscalar coupling, there would be more options with respect to the potential signatures of dark matter. Those can include the direct conversion of dark matter scalars to electromagnetic radiation in the magnetic field, as well as the parity-odd rotations of the polarization of light traveling over cosmological distances.

ACKNOWLEDGMENTS

We would like to thank N. Afshordi, N. Barnaby, J. Bond, C. Burgess, A. Erickcek, and A. Nicolis for useful discussions. Research at the Perimeter Institute was supported in part by the Government of Canada through NSERC and by the Province of Ontario through MEDT.

[1] S. Tremaine and J.E. Gunn, Phys. Rev. Lett. **42**, 407 (1979).
[2] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).
[3] J. Jaeckel and A. Ringwald, arXiv:1002.0329.
[4] B. Patt and F. Wilczek, arXiv:hep-ph/0605188.
[5] D. O'Connell, M.J. Ramsey-Musolf, and M.B. Wise, Phys. Rev. D **75**, 037701 (2007).
[6] V. Barger, P. Langacker, M. McCaskey, M.J. Ramsey-Musolf, and G. Shaughnessy, Phys. Rev. D **77**, 035005 (2008).
[7] O. Bertolami and R. Rosenfeld, Int. J. Mod. Phys. A **23**, 4817 (2008).
[8] B. Batell, M. Pospelov, and A. Ritz, arXiv:0911.4938.
[9] M. Ahlers, J. Jaeckel, J. Redondo, and A. Ringwald, Phys. Rev. D **78**, 075005 (2008).

[10] M. Pospelov, A. Ritz, and M. B. Voloshin, Phys. Rev. D **78**, 115012 (2008).
[11] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Lett. **78B**, 443 (1978).
[12] C. P. Burgess, J. Matias, and M. Pospelov, Int. J. Mod. Phys. A **17**, 1841 (2002).
[13] T. Damour and A. M. Polyakov, Nucl. Phys. **B423**, 532 (1994); Gen. Relativ. Gravit. **26**, 1171 (1994).
[14] B. Bertotti, L. Iess, and P. Tortora, Nature (London) **425**, 374 (2003).
[15] S. Schlamminger, K. Y. Choi, T. A. Wagner, J. H. Gundlach, and E. G. Adelberger, Phys. Rev. Lett. **100**, 041101 (2008).
[16] D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle, and H. E. Swanson, Phys. Rev. Lett. **98**, 021101 (2007).
[17] T. Damour, F. Piazza, and G. Veneziano, Phys. Rev. D **66**, 046007 (2002).
[18] T. Damour, F. Piazza, and G. Veneziano, Phys. Rev. Lett. **89**, 081601 (2002).
[19] J. McDonald, Phys. Rev. D **50**, 3637 (1994).
[20] J. W. Lee and I. G. Koh, Phys. Rev. D **53**, 2236 (1996).
[21] C. P. Burgess, M. Pospelov, and T. ter Veldhuis, Nucl. Phys. **B619**, 709 (2001).
[22] M. C. Bento, O. Bertolami, and R. Rosenfeld, Phys. Lett. B **518**, 276 (2001).
[23] T. Matos, J. A. Vázquez-González, and J. Magaña, Mon. Not. R. Astron. Soc. **393** 4, 1359 (2009);
[24] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, New York, 1990).
[25] V. Mukhanov, *Physical Foundations of Cosmology* (Cambridge University Press, Cambridge, England, 2005).
[26] B. Willman, Adv. Astron. Astrophys. (to be published).
[27] A. D. Linde, *Particle Physics and Inflationary Cosmology* (CRC, Boca Raton, FL, 1990), p. 1.
[28] M. P. Hertzberg, M. Tegmark, and F. Wilczek, Phys. Rev. D **78**, 083507 (2008).
[29] J. Hamann, S. Hannestad, G. G. Raffelt, and Y. Y. Y. Wong, J. Cosmol. Astropart. Phys. 06 (2009) 022.
[30] E. Komatsu *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **180**, 330 (2009).
[31] J. Khoury and F. Piazza, J. Cosmol. Astropart. Phys. 07 (2009) 026.
[32] R. Allahverdi, K. Enqvist, J. Garcia-Bellido, and A. Mazumdar, Phys. Rev. Lett. **97**, 191304 (2006).