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Decay of a Yukawa fermion at finite temperature and applications to leptogenesis

Clemens P. Kießig,[1,*] Michael Plümacher,[1,†] and Markus H. Thoma[2,‡]

[1]*Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, D-80805 München, Germany*

[2]*Max-Planck-Institut für Extraterrestrische Physik, Giessenbachstraße, D-85748 Garching, Germany*

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We calculate the decay rate of a Yukawa fermion in a thermal bath using finite-temperature cutting rules and effective Green's functions according to the hard thermal loop resummation technique. We apply this result to the decay of a heavy Majorana neutrino in leptogenesis. Compared to the usual approach where thermal masses are inserted into the kinematics of final states, we find that deviations arise through two different leptonic dispersion relations. The decay rate differs from the usual approach by more than 1 order of magnitude in the temperature range which is interesting for the weak washout regime. We discuss how to arrive at consistent finite-temperature treatments of leptogenesis.

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I. INTRODUCTION

Leptogenesis [1,2] is an extremely successful theory in explaining the baryon asymmetry of the universe by adding three heavy right-handed neutrinos N_i to the standard model,

$$\delta\mathcal{L} = i\bar{N}_i\partial_\mu\gamma^\mu N_i - \lambda_{\nu,i\alpha}\bar{N}_i\phi^\dagger\ell_\alpha - \tfrac{1}{2}M_i\bar{N}_iN_i^c + \text{H.c.} \tag{1}$$

with masses M_i at the scale of grand unified theories (GUTs) and Yukawa couplings $\lambda_{\nu,i\alpha}$ similar to the other fermions. This also solves the problem of the light neutrino masses via the seesaw mechanism without fine-tuning [3–5].

The heavy neutrinos decay into lepton and Higgs boson after inflation; the decay is out of equilibrium since there are no gauge couplings to the standard model. If the CP asymmetry in the Yukawa couplings is large enough, a lepton asymmetry is created by the decays which is then partially converted into a baryon asymmetry by sphaleron processes. As temperatures are high, interaction rates and the CP asymmetry need to be calculated using thermal field theory [6–8] rather than vacuum quantum field theory. However, in the conventional approach [7], thermal masses have been put in by hand without investigating the validity of this approach in detail. We have addressed this issue in [9] and found that corrections arise through the occurrence of two lepton dispersion relations in the thermal bath. In this paper, we calculate the decay rate of the heavy neutrino in a consistent way (Sec. III) which automatically includes the effect of leptonic quasiparticles, compare it to the conventional approach (Sec. IV) and give an outlook of what needs to be done to arrive at consistent descriptions of leptogenesis. Our calculation is general enough to be applied to all decays of a Yukawa fermion at finite temperature, which has interesting implications for other early universe dynamics (Sec. V).

*ckiessig@mpp.mpg.de
†pluemi@mpp.mpg.de
‡mthoma@mpe.mpg.de

II. HARD THERMAL LOOPS AND THERMAL MASSES

If a particle reaction like scattering or decay takes place in the background of a heat bath, e.g. in the hot state of the early Universe, thermal field theory has to be employed to describe this process. There are two different approaches for considering finite temperatures within quantum field theory, the imaginary and real time formalism [10], both yielding the same results. In this work, we will use the imaginary time formalism. Going from zero to finite temperature, ensemble-weighted expectation values of operators have to be used rather than vacuum expectation values. For an operator $\hat{A}$, this reads

$$\langle\hat{A}\rangle_\beta = \mathrm{tr}(\rho\hat{A}), \tag{2}$$

where ρ is the density operator describing the ensemble. In this way it can be shown that the propagator at finite temperature T is given by its usual vacuum expression where the zero component of the momentum is replaced by imaginary discrete Matsubara frequencies $q_0 = 2ni\pi T$ in the case of bosons or $(2n+1)i\pi T$ in the case of fermions with integers n (see e.g. [11]). Perturbation theory at finite temperature then follows from using these propagators and summing over the Matsubara frequencies in loop diagrams.

However, using these bare thermal propagators can lead to inconsistent results, which are not complete to leading order, infrared divergent, and gauge dependent in the case of gauge theories. A famous example is the damping rate of a plasma wave in the quark-gluon plasma, which is different in different gauges. In order to cure this, the hard thermal loop (HTL) resummation has been invented [12,13]. For this purpose, one has to distinguish between hard momenta of the order T or larger and soft momenta of the order gT or smaller, where g is the coupling constant,

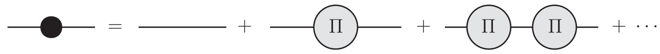

FIG. 1. Resummed propagator.

which is strictly possible only in the weak coupling limit $g \ll 1$. After all, the HTL improved perturbation theory has been successfully applied to thermal QCD for the description of the quark-gluon plasma (see e.g. [14]). The basic idea is that the bare propagators are replaced by resummed propagators, if the external momentum is soft $Q \lesssim gT$.

For a scalar field (Fig. 1), this resummation follows from the Dyson-Schwinger equation as

$$i\Delta^* = i\Delta + i\Delta(-i\Pi)i\Delta + \cdots = \frac{i}{\Delta^{-1} - \Pi} = \frac{i}{Q^2 - m_0^2 - \Pi}. \tag{3}$$

The thermal self-energy Π of the scalar field then acts as a thermal mass $m_{\rm th}^2 = \Pi$ and gives a correction to the zero-temperature mass $m_{\rm tot}^2 = m_0^2 + m_{\rm th}^2$. Since Π is of the order $\sim gT$, the resummation will only affect the propagator when $Q \lesssim \Pi \sim gT$, which is reflected in the prescription to resum only soft momenta. The resummed fermion propagator has a more complicated structure and will be explained in the next section. In general, the self-energy is momentum dependent, e.g. the photon self-energy in QED. In this case, the leading order gauge independent self-energy follows from integrating only over hard momenta in the loop diagram defining the self-energy. This HTL contribution in the resummed propagator leads to a correction of the order gT which cannot be neglected if the momentum of the propagator is soft. The poles of the HTL-resummed propagators then describe the dispersion relations in the medium, e.g. plasma waves following from the resummed photon propagator. In addition to propagators, also HTL effective vertices related to the propagators by Ward identities might have to be used.

III. DECAY AND INVERSE DECAY RATE

In the neutrino decay we want to calculate, the Higgs boson and the lepton acquire thermal masses of the order $m_{\phi,\ell} \sim 0.2$–$0.4T$ via their interactions with other standard model particles. In the regime where the temperature is of the order of the neutrino mass $T \sim M$, one of the momenta of the decay products can be soft and has to be resummed. In the regime where $M \lesssim 0.2$–$0.4T$, both Higgs boson and lepton momentum will be soft and need to be resummed. We are interested in both regimes, therefore we will resum both Higgs boson and lepton propagator. The case of resumming only one propagator is included in this approach, since resumming a hard propagator gives only a negligible correction to the bare propagator. The HTL resummation has been invented for the weak coupling limit $g \ll 1$. This limit does not apply in our case, our more phenomenological approach is rather motivated by the desire to capture effects beyond perturbation theory and justified *a posteriori* by the sizable corrections it reveals, similar to the treatment of meson correlation functions in [15].

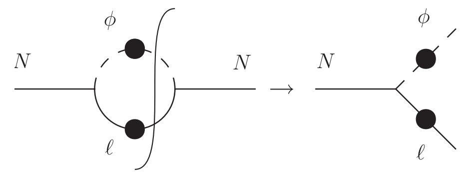

FIG. 2. N decay via the optical theorem with dressed propagators denoted by a blob.

We consider a leptogenesis-inspired model with a massive Majorana fermion N coupling to a massless Dirac fermion ℓ and a massless scalar ϕ. The interaction and mass part of the Lagrangian then reads

$$\mathcal{L}_{\rm int,mass} = g\bar{N}\phi\ell - \tfrac{1}{2}M\bar{N}N^c + \text{H.c.} \tag{4}$$

The HTL resummation technique has been considered in [16] for the case of a Dirac fermion with Yukawa coupling, from which the HTL-resummed propagators for the Lagrangian in Eq. (4) follow directly. We like to calculate the interaction rate Γ of $N \leftrightarrow \ell\phi$.

We cut the N self-energy and use the HTL resummation for the fermion and scalar propagators (Fig. 2).

According to finite-temperature cutting rules [17,18], the interaction rate reads

$$\Gamma(P) = -\frac{1}{2p_0}\,\mathrm{tr}[(\not{P} + M)\mathrm{Im}\Sigma(P)]. \tag{5}$$

At finite temperature, the self-energy reads

$$\Sigma(P) = -g^2T \sum_{k_0 = i(2n+1)\pi T} \int \frac{d^3k}{(2\pi)^3} P_L S^*(K) P_R D^*(Q), \tag{6}$$

where P_L and P_R are the projection operators on left- and right-handed states, $Q = P - K$, and we have summed over Majorana and Dirac spins.

The HTL-resummed scalar propagator is

$$D^*(Q) = \frac{1}{Q^2 - m_\phi^2}, \tag{7}$$

where $m_\phi^2 = g^2T^2/12$ is the thermal mass of the scalar, created by the interaction with fermions. Because of the reduced Majorana degrees of freedom, m_ϕ differs from the Dirac-Dirac case by a factor $1/2$ [16].

The effective fermion propagator in the helicity-eigenstate representation is given by [19–21]

$$S^*(K) = \frac{1}{2}\Delta_+(K)(\gamma_0 - \hat{\mathbf{k}}\cdot\gamma) + \frac{1}{2}\Delta_-(K)(\gamma_0 + \hat{\mathbf{k}}\cdot\gamma), \tag{8}$$

where

$$\Delta_\pm(K) = \left[-k_0 \pm k + \frac{m_\ell^2}{k}\left(\pm 1 - \frac{\pm k_0 - k}{2k}\ln\frac{k_0+k}{k_0-k}\right)\right]^{-1} \tag{9}$$

and

$$m_\ell^2 = \frac{1}{32}g^2T^2. \tag{10}$$

This again differs from the Dirac case by a factor 1/2 [16].

The trace can be evaluated as

$$\mathrm{tr}[(\not{P} + M)P_L S^*(K)P_R] = \Delta_+(p_0 - p\eta) + \Delta_-(p_0 + p\eta), \tag{11}$$

where $\eta = \cos\theta$ is the angle between $\mathbf{p}$ and $\mathbf{k}$. We evaluate the sum over Matsubara frequencies by using the Saclay method [22]. For the scalar propagator, the Saclay representation reads

$$D^*(Q) = -\int_0^\beta d\tau e^{q_0\tau}\frac{1}{2\omega_q} \times \{[1 + n_B(\omega_q)]e^{-\omega_q\tau} + n_B(\omega_q)e^{\omega_q\tau}\}, \tag{12}$$

where $\beta = 1/T$, $n_B(\omega_q) = 1/(e^{\omega_q\beta} - 1)$ is the Bose-Einstein distribution, and $\omega_q^2 = q^2 + m_\phi^2$. For the fermion propagator it is convenient to use the spectral representation [23],

$$\Delta_\pm(K) = -\int_0^\beta d\tau' e^{k_0\tau'}\int_{-\infty}^{\infty} d\omega\rho_\pm(\omega, k) \times [1 - n_F(\omega)]e^{-\omega\tau'}, \tag{13}$$

where $n_F(\omega) = 1/(e^{\omega\beta} + 1)$ is the Fermi-Dirac distribution and $\rho_\pm$ the spectral density [19,20].

The fermion propagator in Eq. (8) has two different poles for $1/\Delta_\pm = 0$, which correspond to two leptonic quasiparticles with a positive (Δ_+) or negative (Δ_-) ratio of helicity over chirality [24–27]. The spectral density $\rho_\pm$ has two contributions, one from the poles and one discontinuous part. Since the quasiparticles are our final states, we will set K such that $1/\Delta_\pm(K) = 0$.

Thus we are only interested in the pole contribution,

$$\rho_\pm^{\mathrm{pole}}(\omega, k) = \frac{\omega^2 - k^2}{2m_\ell^2}(\delta(\omega - \omega_\pm) + \delta(\omega + \omega_\mp)), \tag{14}$$

where $\omega_\pm$ are the dispersion relations for the two quasiparticles, i.e. the solutions for k_0 such that $1/\Delta \pm (\omega_\pm, \mathbf{k}) = 0$, shown in Fig. 3. There exists an analytical solution for $\omega_\pm$ making use of the Lambert W function which has not yet been reported in the literature. The analytical solution is explained in detail in the Appendix. One can assign a momentum-dependent thermal mass $m_\pm(k)^2 = \omega_\pm(k)^2 - k^2$ to the two modes as shown in Fig. 4 and for very large momenta the heavy mode m_+ approaches $\sqrt{2}m_\ell$, while the light mode becomes massless.

In order to execute the sum over Matsubara frequencies, we write $k_0 = i\omega_n$ with $\omega_n = (2n + 1)\pi T$ and remember that when evaluating frequency sums, also $p_0 = i\omega_m = i(2m + 1)\pi T$ can be written as a Matsubara frequency and later on be continued analytically to real values of p_0 [10,28,29]. In particular $e^{p_0\beta} = e^{i\omega_m\beta} = -1$. We can write

$$T\sum_n e^{i\omega_n\tau} = \sum_{n'=-\infty}^{\infty}\delta(\tau - n'\beta), \tag{15}$$

then

$$T\sum_n e^{(p_0 - k_0)\tau}e^{k_0\tau'} = e^{p_0\tau}\delta(\tau' - \tau), \tag{16}$$

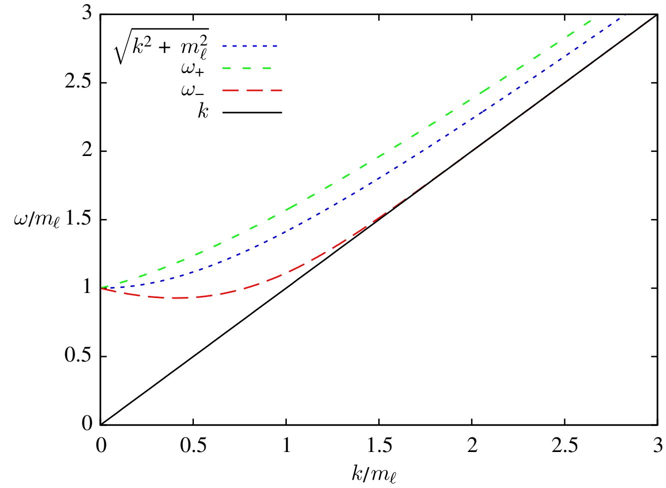

FIG. 3 (color online). The two leptonic dispersion relations compared with the standard dispersion relation $\omega^2 = k^2 + m_\ell^2$ in blue are shown.

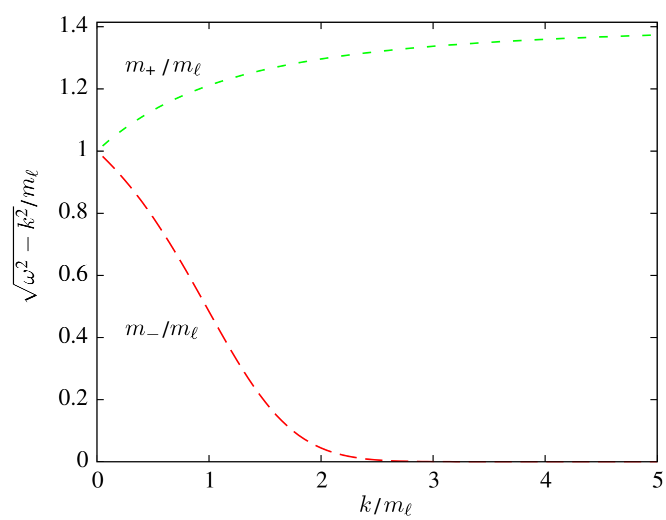

FIG. 4 (color online). The momentum-dependent quasiparticle masses $m_\pm^2 = \omega_\pm^2 - k^2$ are shown.

since $-\beta \leq \tau' - \tau \leq \beta$. After evaluating the sum over k_0 and carrying out the integrations over τ and τ', we get

$$T\sum_{k_0} D^*(Q)\Delta_\pm(K) = -\int_{-\infty}^{\infty} d\omega \rho_\pm(\omega, k)\frac{1}{2\omega_q} \times \left[\frac{1 + n_B(\omega_q) - n_F(\omega)}{p_0 - \omega - \omega_q} + \frac{n_B(\omega_q) + n_F(\omega)}{p_0 - \omega + \omega_q}\right]. \tag{17}$$

Integrating ω over the pole part of $\rho_\pm$ in Eq. (14), we get

$$T\sum_{k_0} D^*\Delta_\pm = -\frac{1}{2\omega_q}\left\{\frac{\omega_\pm^2 - k^2}{2m_\ell^2}\left[\frac{1 + n_B - n_F}{p_0 - \omega_\pm - \omega_q} + \frac{n_B + n_F}{p_0 - \omega_\pm + \omega_q}\right] + \frac{\omega_\mp^2 - k^2}{2m_\ell^2} \times \left[\frac{n_B + n_F}{p_0 + \omega_\mp - \omega_q} + \frac{1 + n_B - n_F}{p_0 + \omega_\mp + \omega_q}\right]\right\}, \tag{18}$$

where $n_B = n_B(\omega_q)$ and $n_F = n_F(\omega_\pm)$ or $n_F(\omega_\mp)$, respectively.

The four terms in Eq. (18) correspond to the processes with the energy relations indicated in the denominator, i.e. the decay $N \to \phi\ell$, the production $N\phi \to \ell$, the production $N\ell \to \phi$, and the production of $N\ell\phi$ from the vacuum, as well as the four inverse reactions [17]. We are only interested in the process $N \leftrightarrow \phi\ell$, where the decay and inverse decay are illustrated by the statistical factors,

$$1 + n_B - n_F = (1 + n_B)(1 - n_F) + n_B n_F. \tag{19}$$

Our term thus reads

$$T\sum_{k_0} D^*\Delta_\pm = -\frac{1}{2\omega_q}\frac{\omega_\pm^2 - k^2}{2m_\ell^2}\frac{1 + n_B - n_F}{p_0 - \omega_\pm - \omega_q}. \tag{20}$$

For carrying out the integration over the angle η, we use

$$\mathrm{Im}\frac{1}{p_0 - \omega_\pm - \omega_q} = -\pi\delta(p_0 - \omega_\pm - \omega_q) = -\pi\frac{\omega_q}{kp}\delta(\eta - \eta_\pm), \tag{21}$$

where

$$\eta_\pm = \frac{1}{2kp}[2p_0\omega_\pm - M^2 - (\omega_\pm^2 - k^2) + m_\phi^2] \tag{22}$$

denotes the angle for which the energy conservation $p_0 = \omega + \omega_q$ holds. The integration over η then yields

$$\int_{-1}^{1} d\eta\, \mathrm{Im}\left(T\sum_{k_0} D^*\Delta_\pm\right) = \frac{\pi}{2kp}\frac{\omega_\pm^2 - k^2}{2m_\ell^2} \times [1 + n_B(\omega_{q\pm}) - n_F(\omega_\pm)], \tag{23}$$

where $\omega_{q\pm} = p_0 - \omega_\pm$. It follows that

$$\Gamma(P) = -\frac{1}{2p_0}\mathrm{tr}[(\not{P} + M)\mathrm{Im}\Sigma(P)] = \frac{1}{2p_0}\mathrm{Im}\left\{g^2 T\sum_{k_0}\int\frac{d^3k}{(2\pi)^3}\mathrm{tr}[(\not{P} + M)P_L S^* P_R]D^*\right\}$$
$$= \frac{g^2}{8\pi^2 p_0}\mathrm{Im}\left\{T\sum_{k_0}\int dk d\eta k^2 D^*[\Delta_+(p_0 - p\eta) + \Delta_-(p_0 + p\eta)]\right\}$$
$$= \frac{g^2}{32\pi p_0 p}\sum_\pm\int_{-1\leq\eta_\pm\leq 1} dk\frac{\omega_\pm^2 - k^2}{2m_\ell^2}[1 + n_B(\omega_{q\pm}) - n_F(\omega_\pm)][2p_0(k \mp \omega_\pm) \pm M^2 \pm (\omega_\pm^2 - k^2) \mp m_\phi^2], \tag{24}$$

where we only integrate over regions with $-1 \leq \eta \leq 1$.

Using finite-temperature cutting rules, one can also write the interaction rates for the two modes in a way which resembles the zero-temperature case [17]:

$$\Gamma_\pm(P) = \frac{1}{2p_0}\int d\tilde{k}d\tilde{q}(2\pi)^4\delta^4(P - K - Q)|\mathcal{M}_\pm(P, K)|^2 \times [1 + n_B(\omega_q) - n_F(\omega_\pm)], \tag{25}$$

where

$$d\tilde{k} = \frac{d^3k}{(2\pi)^3 2k_0} \tag{26}$$

and $d\tilde{q}$ analogously and the matrix elements are

$$|\mathcal{M}_\pm(P, K)|^2 = g^2\frac{\omega_\pm^2 - k^2}{2m_\ell^2}\omega_\pm(p_0 \mp p\eta_\pm). \tag{27}$$

Now that we have arrived at an expression for the full HTL decay rate of a Yukawa fermion, we would like to compare it to the conventional approximation adopted by [7]. To this end, we do the same calculation for an approximated fermion propagator,

$$S^*_{\mathrm{approx}}(K) = \frac{1}{\not{K} - m_\ell}. \tag{28}$$

This yields the following approximated interaction rate:

$$\Gamma_{\rm approx}(P) = \frac{g^2}{32\pi p_0 p}\int_{k_1}^{k_2} dk \frac{k}{\omega}[1 + n_B(\omega_q) - n_F(\omega)] \times [M^2 + m_\ell^2 - m_\phi^2] = \frac{1}{2p_0}\int d\tilde{k} d\tilde{q} (2\pi)^4 \delta^4(P - K - Q)|\mathcal{M}|^2 \times [1 + n_B(\omega_q) - n_F(\omega)], \tag{29}$$

where $\omega^2 = k^2 + m_\ell^2$, $\omega_q = p_0 - \omega$, and the integration boundaries

$$k_{1,2} = \frac{1}{2M^2}\left[p_0\sqrt{(M^2 + m_\ell^2 - m_\phi^2)^2 - (2Mm_\ell)^2} \mp p(M^2 + m_\ell^2 - m_\phi^2)\right] \tag{30}$$

ensure $-1 \le \eta \le 1$, where

$$\eta = \frac{1}{2kp}[2p_0\omega - M^2 - m_\ell^2 + m_\phi^2]. \tag{31}$$

We see that the matrix element is

$$|\mathcal{M}|^2 = \frac{g^2}{2}(M^2 + m_\ell^2 - m_\phi^2). \tag{32}$$

This result resembles the zero-temperature result

$$\Gamma_{T=0}(P) = \frac{g^2}{32\pi p_0 p}\int_{k_1}^{k_2} dk \frac{k}{\omega}[M^2 + m_\ell^2 - m_\phi^2] \tag{33}$$

with zero-temperature masses m_ℓ, m_ϕ. The missing factor

$$1 + n_B - n_F = (1 + n_B)(1 - n_F) + n_B n_F \tag{34}$$

accounts for the statistical distribution of the initial or final particles. As pointed out in more detail in [9], we have shown that the approach to treat thermal masses like zero-temperature masses in the final state [7] is justified since it equals the HTL treatment with an approximate fermion propagator. However, this approach does not equal the full HTL result.

Concluding this calculation, a caveat has to be added: The external Majorana fermion will also acquire a thermal mass of order gT. Thus, if its zero-temperature mass is smaller than that, the external fermion also needs to be described by leptonic quasiparticles to be consistent. However, in our leptogenesis application, the Yukawa coupling giving rise to the Majorana neutrino decay is much smaller than the couplings giving rise to the thermal masses of the Higgs boson (scalar) and the lepton (Dirac fermion) and thus the thermal mass of the heavy neutrino can be neglected.

We have calculated the decay rate assuming a Majorana particle, but the result can be very easily generalized to the case of two Dirac fermions by inserting the appropriate factors of 2 in the decay rate and the thermal masses.

IV. NEUTRINO DECAY IN LEPTOGENESIS

When turning to leptogenesis with

$$\delta\mathcal{L} = i\bar{N}_i\partial_\mu\gamma^\mu N_i - \lambda_{\nu,i\alpha}\bar{N}_i\phi^\dagger\ell_\alpha - \tfrac{1}{2}M_i\bar{N}_i N_i^c + \text{H.c.}, \tag{35}$$

we sum over the two components of the doublets, particles, and antiparticles and the three lepton flavors. Thus, we need to replace $g^2 \to 4(\lambda_\nu^\dagger\lambda_\nu)_{11}$. Integrating over all neutrino momenta, the decay density in equilibrium is

$$\gamma_D^{\rm eq} = \int\frac{d^3p}{(2\pi)^3} f_N^{\rm eq}(E)\Gamma_D = \frac{1}{2\pi^2}\int_M^\infty dE E p f_N^{\rm eq}\Gamma_D, \tag{36}$$

where $E = p_0$, $f_N^{\rm eq}(E) = [\exp(E\beta) - 1]^{-1}$ is the equilibrium distribution of the neutrinos, and $\Gamma_D = [1 - f_N^{\rm eq}(E)]\Gamma$.

Since $\lambda_{\nu,i\alpha} \ll 1$, the thermal masses are

$$m_\phi^2(T) = \left(\frac{3}{16}g_2^2 + \frac{1}{16}g_Y^2 + \frac{1}{4}y_t^2 + \frac{1}{2}\lambda\right)T^2 \tag{37}$$

and

$$m_\ell^2(T) = \left(\frac{3}{32}g_2^2 + \frac{1}{32}g_Y^2\right)T^2. \tag{38}$$

The couplings denote the SU(2) coupling g_2, the U(1) coupling g_Y, the top Yukawa coupling y_t, and the Higgs self-coupling λ, where we assume a Higgs mass of 115 GeV. The other Yukawa couplings can be neglected since they are much smaller than unity and the remaining couplings are renormalized at the first Matsubara mode $2\pi T$ as explained in [7].

In Fig. 5, we compare our consistent HTL calculation to the approximation adopted by [7], while we add quantum statistical distribution functions to their calculation which then equals the approach of using an approximated lepton

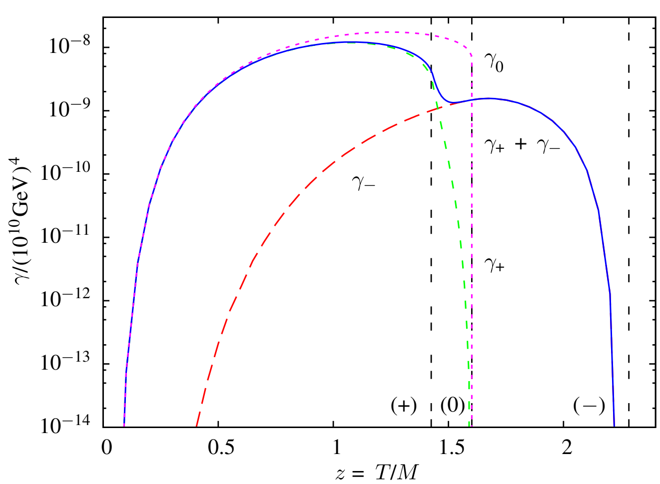

FIG. 5 (color online). The neutrino decay density with the one lepton mode approach γ_0 and the two-mode treatment $\gamma_\pm$ for $M_1 = 10^{10}$ GeV and $\tilde{m}_1 = 0.06$ eV. The thresholds for the two modes (+), (−) and one mode (0) are indicated.

propagator $1/(\not{K} - m_\ell)$ as in Eq. (28) [9]. We assume the heavy neutrino masses to be hierarchical and evaluate the decay rate for the typical value $M_1 = 10^{10}$ GeV, which is inspired by putting M_3 to the GUT scale (10^{15} GeV) and assuming $M_1/M_3 \sim 10^{-5}$ analogous to the quark sector. The combination of Yukawa couplings $(\lambda_\nu \lambda_\nu^\dagger)_{11}$ which governs the decay rate is often parametrized by the so-called "effective" neutrino mass $\tilde{m}_1 = (\lambda_\nu \lambda_\nu^\dagger)_{11} v^2/M_1$, where $v = 174$ GeV is the vacuum expectation value of the Higgs field. We take $\tilde{m}_1 = 0.06$ eV, inspired by the mass scale of the atmospheric mass splitting. However, our results can be generalized to all regions of parameter space.

In the one-mode approach, the decay is forbidden when the thermal masses of Higgs boson and lepton become larger than the neutrino mass, $M < m_\ell + m_\phi$. Considering two modes, the kinematics exhibit a more interesting behavior. For the positive mode, there are two critical temperatures. Below z_+, where $M > m_+(\infty) + m_\phi$, the decay is possible for all final lepton momenta k. However, the momentum-dependent masses of the final state leptons are larger than for the one-mode approach, $m_+(\infty) > m_+(k) > m_\ell$, so the phase space of the two body decay is reduced. Contrary to this, the matrix element increases with increasing lepton energy; however, the effect is small and the suppression of the phase space dominates, so the decay rate is reduced by up to 1 order of magnitude. At $z_+ < z < z_0$, the neutrino can still decay into positive-mode leptons; however, their momentum k has to be small enough so that the condition $m_+(k) + m_\phi \leq M$ is satisfied. Thus, the decay rate drops drastically until z_0, where the decay is strictly forbidden. The decay into the negative, quasimassless mode is suppressed since its residue is much smaller than the one of the positive mode. However, the decay is possible up to $M = m_\phi$. Because of the various effects, the two-mode rate differs from the one-mode approach by more than 1 order of magnitude in the interesting temperature regime of $z = T/M \gtrsim 1$.

It is extremely tempting to put this result in a Boltzmann solver and obtain an effect for the produced baryon asymmetry. However, in the quest for consistent treatments which capture effects of the same origin and size, other effects need to be included as well. At higher temperatures, when $m_\phi > M + m_\pm(k)$, the Higgs can decay into neutrino and lepton modes and this process acts as a production mechanism for neutrinos [7]. Moreover, the CP asymmetry needs to be calculated taking into account the two lepton modes in order to have a consistent treatment.

V. CONCLUSIONS

As discussed in detail in [9], we have, by employing HTL resummation and finite-temperature cutting rules, confirmed that treating thermal masses as kinematic masses as in [7] is a reasonable approximation. However, quantum statistical functions need to be included as they always appear in thermal field theory. Moreover, the full HTL lepton propagator shows a nontrivial two-mode behavior which is not accounted for by the conventional approach. We have calculated the effect of the two modes in a general way which is applicable to any decay and inverse decay rates involving fermions at high temperature. Thus, this calculation is a valuable tool for other particle processes in the early universe, as other leptogenesis processes, the thermal production of gravitinos, or the like.

The behavior of the decay density of the two lepton modes can be explained by considering the dispersion relations $\omega_\pm$ of the modes and assigning momentum-dependent quasimasses to them. The thresholds for neutrino decay reported in [7] are shifted and the decay density shows deviations of more than an order of magnitude in the interesting temperature regime $T/M \sim 1$. Thus, we expect these effects to have a sizable impact on the final baryon asymmetry. However, in order to arrive at a minimal consistent treatment, also the decay $\phi \to N\ell$ at high temperatures needs to be included as well as a CP asymmetry that is corrected for lepton modes. In a further step, it will be interesting to include the effect of thermal widths in the calculations.

As for all effects arising from thermal field theory, the effects are only important in the weak washout regime, where leptogenesis takes place at high temperatures. We are aware of the progress that is currently being made in approaching the effects of quantum statistics [30–33], quantum transport equations [34–39], or other collective phenomena as, e.g., the Landau-Pomeranchuk effect [8]. These efforts contribute to getting an idea of the size and impact of various thermal effects by approaching the extremely complex situation from different angles.

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APPENDIX: ANALYTICAL SOLUTION FOR HTL LEPTON DISPERSION RELATIONS

The dispersion relations of the two lepton modes are given by the poles of the corresponding propagator. Hence, we seek the zeros of

$$\begin{aligned} D_\pm(K) &= \Delta_\pm(K)^{-1} \\ &= \left[-k_0 \pm k + \frac{m_\ell^2}{k}\left(\pm 1 - \frac{\pm k_0 - k}{2k} \ln\frac{k_0 + k}{k_0 - k} \right) \right]^{-1}. \end{aligned} \tag{A1}$$

The equations $D_\pm = 0$ can be transformed by the substitutions

$$x_+ := \frac{k_0 + k}{k_0 - k} \tag{A2}$$

$$x_- := \frac{k_0 - k}{k_0 + k} = \frac{1}{x_+} \tag{A3}$$

$$c := \frac{k^2}{m_\ell^2}. \tag{A4}$$

This yields

$$D_\pm = \pm\frac{k}{c}\frac{1}{x_\pm - 1}(-2c - 1 + x_\pm - \ln x_\pm). \tag{A5}$$

Further introducing

$$s := -\exp(-2c - 1) \tag{A6}$$

leads to

$$D_\pm = \frac{\mp 2k}{1 + \ln(-s)}\frac{1}{x_\pm - 1}[x_\pm + \ln(-s) - \ln x_\pm]. \tag{A7}$$

Since the prefactor does not have poles for the values of K we are looking at, solving $D_\pm = 0$ amounts to solving

$$x_\pm + \ln(-s) - \ln x_\pm = 0, \tag{A8}$$

which in turn means

$$s = -x_\pm e^{-x_\pm}. \tag{A9}$$

This is the defining equation of the Lambert W function [40,41], thus the solution reads

$$x_\pm = -W(s). \tag{A10}$$

According to the definition in Eq. (A6),

$$-1/e \leq s \leq 0, \tag{A11}$$

thus the two real branches of the Lambert function, W_0 and W_{-1}, correspond to the two solutions we seek. In the range given by Eq. (A11) $W_0 \geq -1$ and $W_{-1} \leq -1$. For $k_0 \geq k$ we have $x_+ \geq 1$ and $x_- \leq 1$. Hence, the physical solutions for $x_\pm$ read

$$x_+ = -W_{-1}(s) \quad \text{and} \quad x_- = -W_0(s). \tag{A12}$$

The corresponding results for $\omega_\pm$ are then given by

$$\omega_+ = k\frac{W_{-1}(s) - 1}{W_{-1}(s) + 1} \tag{A13}$$

$$\omega_- = -k\frac{W_0(s) - 1}{W_0(s) + 1}. \tag{A14}$$

Making use of the relations [42]

$$W_{0,-1}(z) + \ln(W_{0,-1}(z)) = \ln z, \tag{A15}$$

one can directly prove the result by plugging Eqs. (A13) and (A14) into Eq. (A1).

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