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Electroweak symmetry breaking and proton decay in $SO(10)$ supersymmetric GUT with TeV W_R

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In a recent paper, we proposed a new class of supersymmetric $SO(10)$ models for neutrino masses where the TeV-scale electroweak symmetry is $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, making the associated gauge bosons W_R and Z' accessible at the Large Hadron Collider. We showed that there exists a domain of Yukawa coupling parameters and symmetry breaking patterns which give an excellent fit to all fermion masses including neutrinos. In this sequel, we discuss an alternative Yukawa pattern which also gives a good fermion mass fit, and then study the predictions of both models for the proton lifetime. Consistency with current experimental lower limits on the proton lifetime require the squark masses of the first two generations to be larger than ~1.2 TeV. We also discuss how one can have simultaneous breaking of both $SU(2)_R \times U(1)_{B-L}$ and standard electroweak symmetries via radiative corrections.

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I. INTRODUCTION

The nature of TeV-scale new physics beyond the standard model (SM) is a question of enormous interest, as the LHC is poised to collect data in this energy range. Clearly, supersymmetry [especially the minimal supersymmetric extension of the standard model (MSSM)] is one of the prime candidates for this new physics since it not only solves the gauge hierarchy problem, but also has a number of attractive features such as the unification of gauge couplings at a high scale, a potential dark matter candidate, etc. An interesting question along these lines has always been to see if any other new physics can coexist with TeV-scale supersymmetry without conflicting with coupling unification and dark matter, thereby broadening the scope of the LHC physics search.

A particularly appealing possibility is that weak interactions conserve parity asymptotically [1], with the associated gauge group being $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ so that the resulting gauge bosons W_R and Z' are at the TeV scale coexisting with supersymmetry. The case for $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ becomes more compelling when the SM or MSSM are extended to understand small neutrino masses via the seesaw mechanism [2]. As a generic possibility, this scenario is quite consistent with current low-energy observations. Whether a TeV scale $SU(2)_R$ symmetry is compatible with supersymmetric coupling unification has been extensively investigated in the literature [3,4]. With a few exceptions [4], it seems very hard to reconcile this possibility with the observed value of $\sin^2\theta_W$.

In a recent paper [5], we pointed out a new supersymmetric $SO(10)$ scenario where the presence of a vectorlike electroweak singlet and color triplet Higgs multiplet [which is part of the **45** representation in $SO(10)$], in addition to two bidoublets and two right-handed doublets of the left-right electroweak group at the TeV scale, leads to gauge coupling unification with TeV-scale right-handed W_R and Z' bosons. It was shown that the model can reproduce both the observed charged fermion and neutrino masses, making it the first realistic $SO(10)$ supersymmetric-GUT (SUSY-GUT) model with TeV-scale W_R and Z'. This should provide impetus for adding a new search agenda at the LHC in addition to the usual SUSY and extra dimension particles.

Our model is different from other such scenarios considered in the literature [4] in that quark masses and mixing arise in a simple manner. The neutrino masses arise out of an inverse seesaw mechanism [6] and were shown [5] to have interesting phenomenological consequences like leptonic nonunitarity, leptonic CP-violation, lepton flavor violation, etc. which may be testable in the near future. This fit to the fermion masses defines one class of $SO(10)$ models with TeV-scale W_R, which we call model (A).

In this paper, several new results for these $SO(10)$ models are presented: (i) we present an alternative fit to fermion masses, which we call model (B); (ii) we discuss the constraints of proton decay for both fermion mass fits—the one in Ref. [5] and the new one discussed in this paper; (iii) we also show how both $B-L$ and electroweak symmetries can be broken radiatively in these models.

The strength of proton decay has been studied extensively in the context of many SUSY-GUTs (see Ref. [7] for recent reviews). Although there has been no evidence for proton decay so far, current experimental lower bounds on the partial lifetimes of various proton decay modes tend to put severe constraints on these models; e.g. they have now ruled out the simplest versions of SUSY $SU(5)$ and suggest possible modifications of such models [8]. They also constrain the choices of Higgs multiplets that can be used for model building with the $SO(10)$ group [9].

In the models we are discussing here, due to the fact that all the Yukawa couplings responsible for proton decay are constrained by the fermion mass fits, it is possible to estimate the partial lifetimes for the various modes as functions of the squark masses. We get upper bounds on the partial lifetimes of various proton decay channels in model (A) for reasonable squark masses of the first two generations. There are no such bounds in the second case [model (B)]. We find that within a reasonable set of assumptions, all our predicted upper bounds for model (A) are consistent with the current experimental bounds, and some of the modes may be accessible to the next generation proton decay experiments with megaton-size detectors.

We also discuss the constraints imposed by radiative breaking of both $SU(2)_R \times U(1)_{B-L}$ and the SM gauge symmetries via radiative corrections. The idea is to start with positive soft mass squares at the Planck or GUT scale and extrapolate the masses to the weak scale to see if the $SU(2)_R \times U(1)_{B-L}$ symmetry breaks at the TeV scale. We then note that this breaking introduces, via D-terms, a breaking of the SM gauge symmetry to $U(1)_{\rm em}$.

We also discuss the generalization of this model to include R-parity breaking and its implications on proton decay. In addition, we comment on some other interesting aspects of the model such as neutron-antineutron ($n - \bar{n}$) oscillation and TeV-scale resonant leptogenesis.

This paper is organized as follows: In Sec. II, we review the basic structure of our model and the $SO(10)$ symmetry breaking. In Sec. III, we review the fermion mass fit for model (A) already discussed in Ref. [5]. In Sec. IV, we present a new fermion mass fit and define it as model (B). Section V describes the radiative electroweak symmetry breaking (EWSB) in this type of model. In Sec. VI, we discuss the proton decay in both these models. In Sec. VII, we comment on the effect of R-parity breaking terms in the superpotential on proton decay. In Sec. VIII, we make additional comments on some other aspects of the model, namely, $n - \bar{n}$ oscillation and leptogenesis. The results are summarized in Sec. IX. In Appendix A, we present the renormalization group equations (RGEs) for soft SUSY-breaking masses in our supersymmetric left-right (SUSYLR) model. In Appendix B, we derive the anomalous dimensions of the dimension-5 proton decay operators in our model. In Appendix C, we list the hadronic form factors used in our proton decay calculations.

II. A BRIEF OVERVIEW OF THE MODEL

As in the usual $SO(10)$ models, the three generations of quark and lepton fields are assigned to three **16**-dimensional spinor representations. In addition, we add three $SO(10)$ singlet matter fields to implement the inverse seesaw mechanism. The $B - L$ gauge symmetry is broken at the TeV scale by **16**-Higgs fields (denoted by ψ_H), whereas the rest of the gauge symmetry is broken at $\sim 10^{16}$ GeV by **54** and **45** fields (denoted by E and A_a, respectively). We require two **45**-Higgs fields ($a = 1, 2$), one for symmetry breaking and the other to give rise to the vectorlike color triplets at the TeV scale. The SM symmetry is broken by two **10**-Higgs fields (denoted by H_a). We note that the field content of our model is found in many string models after compactification, e.g. fermionic compactification models [10], and it may therefore be easier to embed this GUT model into strings.

The distinguishing feature of our model is that the GUT symmetry breaks down to the left-right symmetric gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ without parity (D parity). The D parity is broken at the GUT scale by the vacuum expectation value (VEV) of the **45**-Higgs field. A consequence of D-parity breaking is that only the right-handed (RH) doublets from **16**-Higgs fields survive below the GUT scale. An interesting feature of this class of models [5] is that if we have two RH Higgs fields $[\chi^c, \bar{\chi}^c(1, 1, 2, \pm 1)]$, two bidoublet fields $[\Phi(1, 2, 2, 0)]$ (all color singlets), and a vectorlike color triplet but $SU(2)_L \times SU(2)_R$ singlet field $[\delta(3, 1, 1, \frac{4}{3}) + \text{c.c.}]$ at the TeV scale, the gauge couplings unify around 10^{16} GeV. The bidoublet fields arise from **10**-Higgs at the GUT scale and the vectorlike color triplet fields arise from the **45**-Higgs field. This is therefore a new class of $SO(10)$ SUSY-GUT theories with TeV-scale W_R and Z' bosons which can be accessible at the LHC.

We consider the symmetry breaking chain

$$SO(10) \xrightarrow{M_G} \mathbf{3}_c \mathbf{2}_L \mathbf{2}_R \mathbf{1}_{B-L} \xrightarrow{M_R} \mathbf{3}_c \mathbf{2}_L \mathbf{1}_Y (\text{MSSM}) \xrightarrow{M_{\rm SUSY}} \mathbf{3}_c \mathbf{2}_L \mathbf{1}_Y (\text{SM}) \xrightarrow{M_Z} \mathbf{3}_c \mathbf{1}_Q, \tag{1}$$

where, as an example of our notation, $\mathbf{3}_c$ means $SU(3)_c$. As shown in Appendix A of Ref. [5], for consistency, we need at least two **45** and one **54** representations of the Higgs fields to break the $SO(10)$ gauge group into the SUSYLR gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ at the scale $M_G \simeq 4 \times 10^{16}$ GeV. Note that to have realistic fermion masses and mixing, we need at least two $SU(2)$ bidoublets of the **10**-Higgs representation to break the $SU(2)_L \times U(1)_Y$ gauge group of the SM to $U(1)_Q$ at the weak scale M_Z. With this minimal set of Higgs fields, we were able to attain not only gauge coupling unification but also the desired fermion masses and mixing at the GUT scale [5]. Incidentally, since our gauge group above the TeV scale is different from the MSSM, we needed to extrapolate fermion masses using the left-right group (see Appendix B of Ref. [5]), which has certain distinguishing features in the running behavior in contrast to the MSSM gauge group.

The superpotential for the model consists of several parts:

$$W = W_{SB} + W_m + W', \tag{2}$$

where W_{SB} is responsible for $SO(10)$ GUT symmetry breaking, doublet triplet splitting, and the remnant sub-GUT-scale multiplets; W_m is the Yukawa superpotential responsible for fermion masses and mixing; W' involves the R-parity violating terms. When we impose an additional matter-parity symmetry under which $\psi_\alpha \to -\psi_\alpha$,

$S_\alpha \to -S_\alpha$, and all other fields even, as was assumed in Ref. [5], we get $W' = 0$; i.e. all R-parity violating terms are absent in the superpotential and the model has a stable dark matter [11]. We discuss the effects of nonzero W' in a subsequent section, where we show that even after including arbitrary R-parity violating terms (i.e. giving up the matter-parity assumption), the model does satisfy proton lifetime bounds since W' conserves baryon number and, after $B-L$ breaking, leads to a highly suppressed amplitude for proton decay. This feature is characteristic only of $SO(10)$ models with low $B-L$ breaking.

The most general Yukawa superpotential of the model is given by

$$W_m = h_{aij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_{H_a} + \frac{f'_{aij}}{M}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_{H_a}\mathbf{45}_H + \frac{f_{aij}}{M^2}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_{H_a}\mathbf{45}_H\mathbf{45}'_H, \tag{3}$$

where the first term is the usual Yukawa coupling term, and the second and third terms are higher-dimensional terms. The second term has two effective contributions (with two different coupling matrices)—one of **10**-Higgs type and another of **120** type. The third has effective fermion couplings of type **10**, **120** as well as **126**, again with different coupling matrices. The effective **10** couplings from the higher-dimensional terms could be absorbed into the first term. Since we are looking to see if we get a fit for quark and lepton masses in the model, we will assume that the effective **120** couplings in the model are zero. We keep only the fully antisymmetric combination in the last term that acts as an effective $\mathbf{126}_H$ operator. This is sufficient for getting a realistic fermion mass spectrum at the GUT scale, as already discussed in Ref. [5]. We define this as our model (A).

The superpotential W_{SB} was discussed in detail in Ref. [5], where it was noted that the following components of the **54**- and **45**-Higgs fields acquire VEVs and leave the left-right subgroup unbroken:

$$\begin{aligned}\langle\mathbf{54}\rangle &= \mathrm{diag}(a,a,a,a,a,a,-\tfrac{3}{2}a,-\tfrac{3}{2}a,-\tfrac{3}{2}a,-\tfrac{3}{2}a);\\ \langle\mathbf{45}\rangle &= \mathrm{diag}(b,b,b,0,0).\end{aligned} \tag{4}$$

III. FERMION MASSES IN MODEL (A)

The model discussed in Ref. [5] is defined by the following VEV pattern of the bidoublets:

$$\langle\Phi_1\rangle = \begin{pmatrix}\kappa_d & 0\\ 0 & 0\end{pmatrix}, \qquad \langle\Phi_2\rangle = \begin{pmatrix}0 & 0\\ 0 & \kappa_u\end{pmatrix}. \tag{5}$$

We define the ratio of the VEVs as $\tan\beta \equiv \frac{\kappa_u}{\kappa_d}$ as in the MSSM. Then the fermion mass matrices at the GUT scale are given by

$$\begin{aligned}M_u &= \tilde{h}_u + \tilde{f}, & M_d &= \tilde{h}_d + \tilde{f},\\ M_e &= \tilde{h}_d - 3\tilde{f}, & M_{\nu_D} &= \tilde{h}_u - 3\tilde{f},\end{aligned} \tag{6}$$

where in the notation of Ref. [5], $\tilde{h}_{u,d} \equiv \kappa_{u,d}h_{u,d}$. The contribution from the effective $\mathbf{126}_H$ operator is assumed to be the same for both up and down sectors, i.e. $\tilde{f} = \kappa_u f_u = \kappa_d f_d$; as a result, we have the relation $f_d = f_u\tan\beta$. Also note that the factor -3 between the quark and lepton sectors is due to the same **126** operator. Using the renormalization group analysis for the fermion masses and mixing in the SUSYLR model (see Appendix B of Ref. [5]), we obtain the GUT-scale fermion masses starting from the experimentally known values at the weak scale. Using these mass values, we obtain a fit for the coupling matrices at the GUT scale defined in Eq. (6). Here we give the results in a down-quark mass diagonal basis for two cases. Case (a) is $\tan\beta_{\mathrm{MSSM}} = 10$. In this case, the GUT-scale values of the charged fermion masses are found to be

$$\begin{aligned}&m_u = 0.0017\ \mathrm{GeV}, \quad m_c = 0.1908\ \mathrm{GeV},\\ &m_t = 77.7\ \mathrm{GeV}, \quad m_d = 0.0013\ \mathrm{GeV},\\ &m_s = 0.0263\ \mathrm{GeV}, \quad m_b = 1.7001\ \mathrm{GeV},\\ &m_e = 0.0004\ \mathrm{GeV}, \quad m_\mu = 0.0910\ \mathrm{GeV},\\ &m_\tau = 1.7061\ \mathrm{GeV},\end{aligned} \tag{7}$$

and $\tan\beta_{\mathrm{GUT}} = 7$. Note that the GUT-scale fermion masses quoted here are slightly different from those given in Ref. [5] because, in this case, we have set the $S\Phi\Phi$ coupling $\mu_\Phi = 0$ (of Ref. [5]) assuming R-parity conservation. With these mass eigenvalues, we find a fit for the GUT-scale couplings of the form

$$f_u = \mathrm{diag}(1.26\times10^{-6}, -0.0001, -9.48\times10^{-6}), \qquad f_d = f_u\tan\beta_{\mathrm{GUT}}, \qquad h_d = \mathrm{diag}(4.86\times10^{-5}, 0.0019, 0.0752),$$

$$h_u = \begin{pmatrix} 7.46\times10^{-5} & 0.0002 - 6.51\times10^{-5}i & 0.0002 - 0.0028i\\ 0.0002 + 6.51\times10^{-5}i & 0.0015 & 0.0118 + 1.26\times10^{-6}i\\ 0.0002 + 0.0028i & 0.0118 - 1.26\times10^{-6}i & 0.4908\end{pmatrix}. \tag{8}$$

Note that, for simplicity, we have chosen the f couplings to be diagonal. Our fit does not allow the off-diagonal components to be too different from zero. Case (b) is for $\tan\beta_{\mathrm{MSSM}} = 30$. In this case, the GUT-scale values of the charged fermion masses are found to be

$$m_u = 0.0121\ \text{GeV}, \quad m_c = 0.3269\ \text{GeV}, \quad m_t = 120.53\ \text{GeV}, \quad m_d = 0.0014\ \text{GeV}, \quad m_s = 0.0277\ \text{GeV},$$
$$m_b = 2.7958\ \text{GeV}, \quad m_e = 0.0006\ \text{GeV}, \quad m_\mu = 0.1266\ \text{GeV}, \quad m_\tau = 2.7737\ \text{GeV}, \tag{9}$$

and $\tan\beta_{\text{GUT}} = 20$. With these mass eigenvalues, we obtain a fit for the couplings of the following form:

$$f_u = \text{diag}(1.5 \times 10^{-6}, -0.0002, 4.2 \times 10^{-5}), \quad f_d = f_u \tan\beta_{\text{GUT}}, \quad h_d = \text{diag}(0.0002, 0.0078, 0.4163),$$
$$h_u = \begin{pmatrix} 0.0002 & 0.0003 - 0.0001i & -0.0008 - 0.0081i \\ 0.0002 + 0.0001i & 0.0029 & 0.0144 + 0.0002i \\ -0.0008 + 0.0081i & 0.0144 - 0.0002i & 0.9145 \end{pmatrix}. \tag{10}$$

We note that in this model, larger values of $\tan\beta(>30)$ are not allowed. This can be seen analytically from the form of the RGEs given in Appendix B of Ref. [5], where it is clear that the up-quark sector masses will increase rapidly at high energies for large $\tan\beta$ and the same effect is induced in the down-quark sector, which makes the Yukawa terms dominant over the gauge terms. This makes all the quark masses run up to unacceptably large values at the GUT scale. We believe this is a general feature of low-scale SUSYLR models, in contrast to the MSSM case [12].

The neutrino mass matrix in this model is given by the inverse seesaw formula [6] and involves, in addition to the Dirac neutrino mass M_{ν_D}, the Majorana mass matrices of the extra singlet S:

$$M_\nu \simeq M_{\nu_D} M_N^{-1} \mu (M_N^{-1})^T M_{\nu_D}^T \equiv F \mu F^T. \tag{11}$$

The Dirac neutrino mass M_{ν_D} is obtained from Eq. (6). As shown in Ref. [5], to satisfy the nonunitary bounds, we require the RH neutrino mass (together with S) to be $\gtrsim$ 1.1 TeV (assuming degenerate eigenvalues for M_N). With these inputs, we can fit the observed neutrino oscillation data by fixing the singlet mass matrix elements of μ in Eq. (11). As an example, for $\tan\beta = 10$ and with the following choice of μ:

$$\mu = \begin{pmatrix} -1.4874 + 0.0267i & 0.2092 - 0.0061i & -0.0041 + 0.0086i \\ 0.2092 - 0.0061i & -0.0300 + 0.0012i & 0.0006 - 0.0012i \\ -0.0041 + 0.0086i & 0.0006 - 0.0012i & (3.8 + 4.8i) \times 10^{-5} \end{pmatrix} \text{GeV}, \tag{12}$$

we find the neutrino masses and mixings to be

$$\begin{gathered} m_1 = 10^{-3}\ \text{eV}, \quad m_2 = 4.88 \times 10^{-2}\ \text{eV}, \quad m_3 = 4.95 \times 10^{-2}\ \text{eV}, \\ \sin^2\theta_{12} = 0.312, \quad \sin^2\theta_{23} = 0.466, \quad \sin^2\theta_{13} = 0, \end{gathered} \tag{13}$$

which satisfy the observed 2σ neutrino oscillation data [13].

IV. A NEW FERMION MASS FIT: MODEL (B)

In this section, we consider an alternative mass fit within the $SO(10)$ models with low-scale $B - L$. It follows from a recent ansatz [14] that in generic $SO(10)$ models which do not use the type I seesaw mechanism to fit neutrino masses, an alternative fit to fermion masses is possible using the idea [14] that one has a rank-one **10**-Higgs Yukawa coupling matrix which dominates the fermion masses while other couplings introduce small corrections; the third generation masses arise from the dominant rank-one coupling matrix with smaller **126** and second **10** couplings generating the Cabibbo-Kobayashi-Maskawa mixing as well as the second and the first generation fermion masses. This idea can be applied to our case, since the neutrino mass is given by the inverse seesaw formula which involves an additional matrix μ. The main difference of model (B) as compared to model (A) resides in the VEV pattern of the two Higgs bidoublets; i.e. in model (B), we have

$$\langle \Phi_1 \rangle = \begin{pmatrix} \kappa_d & 0 \\ 0 & \kappa_u \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} \kappa'_d & 0 \\ 0 & \kappa'_u \end{pmatrix} \tag{14}$$

with $v_{\text{wk}}/\sqrt{2} = \sqrt{\kappa_u^2 + \kappa_d^2 + \kappa_u'^2 + \kappa_d'^2}$. Also we must have $\frac{\kappa_u}{\kappa_d} \neq \frac{\kappa'_u}{\kappa'_d}$ in order to get the right fermion mixing pattern. In the limit $\kappa_u \gg \kappa'_u$, the RG analysis of model (A) can be applied to this case to generate fermion masses at the GUT scale, as well as the symmetry breaking pattern via radiative corrections.

The resulting fermion mass formulas in terms of the appropriately redefined Yukawa couplings are given as follows [15]:

$$\begin{aligned} M_u &= \tilde{h} + r_2 \tilde{f} + r_3 \tilde{h}', \quad & M_d &= r_1(\tilde{h} + \tilde{f} + \tilde{h}'), \\ M_l &= r_1(\tilde{h} - 3\tilde{f} + c_e \tilde{h}'), \quad & M_{\nu_D} &= \tilde{h} - 3\tilde{f} + c_\nu \tilde{h}', \end{aligned} \tag{15}$$

where

$$\tilde{h} = \kappa_u h, \qquad \tilde{f} = \frac{\kappa_u \kappa'_d}{\kappa_d} f, \qquad \tilde{h}' = \frac{\kappa_u \kappa'_d}{\kappa_d} h', \qquad r_1 = \frac{\kappa_d}{\kappa_u}, \qquad r_2 = r_3 = \frac{\kappa_d \kappa'_u}{\kappa_u \kappa'_d}. \tag{16}$$

As in the case of model (A), the f coupling above represents the effective **126** coupling arising from the $\psi\psi A_1 A_2 H_2$ term in the superpotential, and h' arises from a coupling of the form $\psi\psi H_2 X$ (with a nonzero VEV for the additional singlet field X). Note that if there is an additional Z_2 symmetry under which H_2, A_2, X are odd and all other fields are even, one can have a superpotential with only the h-, f-, h'-type contributions, as given above, to the fermion mass formulas. In our case with two Higgs bidoublets, $c_e = 1$ and $c_\nu = r_3$. With the GUT-scale mass eigenvalues obtained earlier, we obtain a fit for these couplings as follows:

(i) $\tan\beta_{\mathrm{MSSM}} = 10$:

$$\begin{gathered} \kappa_u = 173.2\ \mathrm{GeV}, \qquad r_1 = 0.0218, \\ r_2 = 0.14, \qquad h = \mathrm{diag}(0, 0, 0.45), \\ f = \begin{pmatrix} 0 & -0.0006 & 0.0019 \\ -0.0006 & 0.0115 & 0.0101 \\ 0.0019 & 0.0101 & 0.0001 \end{pmatrix}, \\ h' = i\begin{pmatrix} 0 & -0.0022 & 0.0005 \\ 0.0022 & 0 & 0.0181 \\ -0.0005 & -0.0181 & 0 \end{pmatrix}. \end{gathered} \tag{17}$$

(ii) $\tan\beta_{\mathrm{MSSM}} = 30$:

$$\begin{gathered} \kappa_u = 172.4\ \mathrm{GeV}, \qquad r_1 = 0.0231, \\ r_2 = 0.21, \qquad h = \mathrm{diag}(0, 0, 0.70), \\ f = \begin{pmatrix} 0 & -0.0016 & 0.0062 \\ -0.0016 & 0.0140 & 0.0111 \\ 0.0062 & 0.0111 & 0.0019 \end{pmatrix}, \end{gathered} \tag{18}$$

and h' is the same as in case (i). It may be noted here that in both cases, all the fermion mass values predicted using the couplings above agree with those obtained from the RGEs within the experimental uncertainty, the only exception being the up-quark mass in case (i), where our predicted value is about 4 times larger. Note, however, that in our discussion, we have not included contributions from threshold corrections or higher-dimensional operators. Those contributions can generally be of order MeVs when their couplings are chosen appropriately, in which case, they will not affect the second and third generation masses but could easily bring the up-quark mass into agreement with RGE predictions.

As in model (A), the observed neutrino oscillation data can be fitted with the following singlet Majorana mass matrix μ:

$$\mu = \begin{pmatrix} -1.5213 + 0.2016i & -0.0798 - 0.0883i & -0.0019 - 0.0008i \\ -0.0798 - 0.0883i & -0.0021 - 0.0089i & -3.5\times 10^{-5} - 0.0002i \\ -0.0019 - 0.0008i & -3.5\times 10^{-5} - 0.0002i & (-1.7 - 2.2i)\times 10^{-6} \end{pmatrix} \mathrm{GeV}. \tag{19}$$

With the Yukawa couplings completely fixed in our model, we can analyze the predictions for the proton decay rate. But before doing so, we discuss the details of the electroweak symmetry breaking in this model, which was not done in the original paper [5]. This discussion applies to both models (A) and (B).

V. SYMMETRY BREAKING BY RADIATIVE CORRECTIONS

In this section, we propose a way to break both the $SU(2)_R \times U(1)_{B-L}$ and the SM symmetry via radiative corrections from renormalization group extrapolation of the scalar Higgs masses from the GUT to the TeV scale. As is well known, the large top quark coupling enables us to achieve a similar goal i.e. radiative EWSB in the case of MSSM [16]. The simple generalization of that procedure cannot work in our model since the bidoublet Higgs of LR models contains both the $H_{u,d}$ components of the MSSM; as a result, large top quark coupling will necessarily turn both their masses negative, and this is known not to give a stable vacuum.

Our proposal is that we use a domain of parameter space for the soft SUSY-breaking mass squares for the RH Higgs doublets χ^c and $\bar{\chi}^c$, where the mass square of one of them turns negative by RG running to the TeV scale due to the $L^c\bar{\chi}^c S$ Yukawa coupling being large. This leads to a breaking of the $SU(2)_R$ and $B-L$ symmetry. The mass square of the χ^c remains positive throughout but it acquires an induced VEV. The differences in their VEVs, via the D-term, can make the mass square of the H_u field negative while keeping the mass square of H_d positive, as in the case of the MSSM, thereby also giving rise to the EWSB. The main point is that both symmetry breakings owe their origin to one radiative correction.

In order to show that it is indeed possible to achieve negative mass square for one of the RH Higgs doublets while keeping all other soft mass squares positive, we need to examine the RG running of all the soft mass parameters from the GUT to the TeV scale. In this regime, the model is

SUSYLR, for which the superpotential and soft SUSY-breaking Lagrangian are given by [17]

$$W = ih_a Q^T\tau_2\Phi_a Q^c + ih'_a L^T\tau_2\Phi_a L^c + i\mu^\alpha_{\chi^c_{pq}} S^\alpha \chi^{c^T}_p \tau_2\bar{\chi}^c_q + i\mu^\alpha_{L^c_p} S^\alpha L^{c^T}\tau_2\bar{\chi}^c_p + iM_{\chi^c}\chi^{c^T}\tau_2\bar{\chi}^c + \mu^\alpha_{\Phi_{ab}} S^\alpha \mathrm{Tr}(\Phi^T_a\tau_2\Phi_b\tau_2) + M_{\Phi_{ab}}\mathrm{Tr}(\Phi^T_a\tau_2\Phi_b\tau_2) + \tfrac{1}{6}Y^{\alpha\beta\gamma}S^\alpha S^\beta S^\gamma + \tfrac{1}{2}M^{\alpha\beta}_S S^\alpha S^\beta, \tag{20}$$

$$\begin{aligned}\mathcal{L}_{\rm soft} = &-\tfrac{1}{2}(M_3\tilde{g}\,\tilde{g} + M_{2L}\tilde{W}_L\tilde{W}_L + M_{2R}\tilde{W}_R\tilde{W}_R + M_1\tilde{B}\,\tilde{B} + \text{H.c.}) - [iA_{Q_a}\tilde{Q}^T\tau_2\Phi_a\tilde{Q}^c + iA_{L_a}\tilde{L}^T\tau_2\Phi_a\tilde{L}^c \\ &+ iA^\alpha_{\chi^c_{pq}} S^\alpha\chi^{c^T}_p\tau_2\bar{\chi}^c_q + iA^\alpha_{L^c_p}S^\alpha\tilde{L}^{c^T}\tau_2\bar{\chi}^c_p + \tfrac{1}{6}A^{\alpha\beta\gamma}_S S^\alpha S^\beta S^\gamma + A^\alpha_{\Phi_{ab}}S^\alpha\mathrm{Tr}(\Phi^T_a\tau_2\Phi_b\tau_2) + \text{H.c.}] \\ &- [iB_{\chi^c_{pq}}\chi^{c^T}_p\tau_2\bar{\chi}^c_q + B_{ab}\mathrm{Tr}(\Phi^T_a\tau_2\Phi_b\tau_2) + \tfrac{1}{2}B^{\alpha\beta}_S S^\alpha S^\beta] - [m^2_Q\tilde{Q}^T\tilde{Q}^* + m^2_{Q^c}\tilde{Q}^{c^\dagger}\tilde{Q}^c + m^2_L\tilde{L}^T\tilde{L}^* \\ &+ m^2_{L^c}\tilde{L}^{c^\dagger}\tilde{L}^c + m^2_{\chi^c}\chi^{c^\dagger}_p\chi^c_p + m^2_{\bar{\chi}^c}\bar{\chi}^{c^\dagger}_p\bar{\chi}^c_p + m^2_{\Phi_{ab}}\mathrm{Tr}(\Phi^\dagger_a\Phi_b) + m^2_{S_{\alpha\beta}}S^{\alpha^*}S^\beta],\end{aligned} \tag{21}$$

where we have suppressed the generational and $SU(2)$ indices, and a, $b = 1, 2$ (for two bidoublets), p, $q = 1, 2$ [for two $SU(2)_R$ doublets], and α, β, $\gamma = 1, 2, 3$ (for three gauge singlets). Note that we do not have any χ-term in these expressions as there is no $SU(2)_L$ Higgs doublet in our model. Also, we have an additional term in the superpotential (the $SL^c\chi^c$ term) and a corresponding trilinear term in the soft-breaking Lagrangian (the $S\tilde{L}^c\bar{\chi}^c$ term) as compared to the expressions given in Ref. [17]; this additional term in the superpotential is required for the inverse seesaw mechanism to work. Moreover, if we assume R-parity conservation, then the $S\chi^c\bar{\chi}^c$ and $S\Phi\Phi$ terms are not allowed in the superpotential and also in the soft-breaking Lagrangian; i.e. the couplings μ_{χ^c} and μ_Φ as well as Y_{abc} in Eq. (20) and the corresponding terms in Eq. (21) are set to zero and μ_{L^c} is the only nonzero coupling in Eq. (20) which can be fixed by requiring $b-\tau$ unification at the GUT scale. In this section, we work with this assumption; the effects of R-parity breaking will be discussed later.

Now we analyze the RG evolution of the gaugino and soft mass parameters from the GUT to the TeV scale. It is well known that in SUSY-GUTs, the β function for the gaugino mass is proportional to the β function for the corresponding gauge coupling. Explicitly, the RGEs for the gaugino mass parameters are given by

$$\frac{dM_i}{dt} = \frac{2b_i}{16\pi^2}M_i g_i^2, \tag{22}$$

where the β-function coefficients in our SUSYLR model are [5] $b_i = (13, 2, 4, -2)$, corresponding to $i = \mathbf{1}_{B-L}$, $\mathbf{2}_L$, $\mathbf{2}_R$, $\mathbf{3}_c$, respectively. This implies that the three gaugino masses, like the three gauge couplings, must unify at $\mu = M_{\rm GUT}$. In order to solve Eq. (22), we adopt the universality hypothesis at the GUT scale [as in typical minimal supergravity (mSUGRA)-type models],

$$M_1 = M_{2L} = M_{2R} = M_3 \equiv m_{1/2}, \tag{23}$$

together with the initial condition

$$g_1^2 = g_{2L}^2 = g_{2R}^2 = g_3^2 \equiv 4\pi\alpha_{\rm GUT}, \tag{24}$$

where $M_{\rm GUT} \simeq 4\times 10^{16}$ GeV and $\alpha^{-1}_{\rm GUT} \simeq 20.3$ in our model [5]. Using these initial conditions, we can obtain the running masses for the gauginos at the TeV scale, starting with a given value $m_{1/2}$ at the GUT scale, as shown in Fig. 1 for a typical value of $m_{1/2} = 200$ GeV. The value of M_3 increases, since it has a negative β function, while the other gaugino masses decrease as we go down the energy scale. Thus the gluino is much heavier than other gauginos at the weak scale.

The one-loop RGEs for the soft SUSY-breaking mass parameters are given in Appendix A. As initial conditions, we assume universality and reality of the soft fermion and Higgs masses at the GUT scale, i.e.

$$\begin{aligned}&(m^2_Q)_{ij} = (m^2_{Q^c})_{ij} = (m^2_L)_{ij} = (m^2_{L^c})_{ij} \equiv m_0^2\delta_{ij}, \\ &m^2_{\chi^c} = m^2_{\bar{\chi}^c} = m_0^2, \qquad (m^2_\Phi)_{ab} = m_0^2\delta_{ab},\end{aligned} \tag{25}$$

whereas a different scale is assumed for the soft singlet scalar mass:

$$(m^2_S)_{\alpha\beta} = m_0'^2 \quad \forall\ \alpha, \beta = 1, 2, 3. \tag{26}$$

In principle, we can choose a different mass scale for the Higgs bidoublets and even for different generations of fermions. The only constraint due to the $SO(10)$ symmetry requires us to have the same mass for each generation of

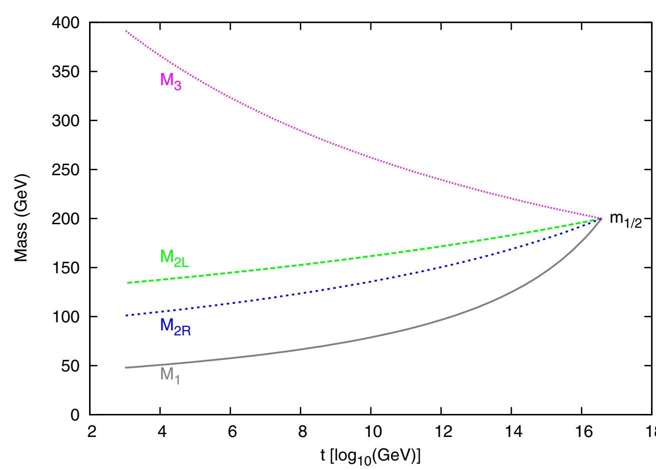

FIG. 1 (color online). RG evolution of gaugino masses from GUT to TeV scale for $m_{1/2} = 200$ GeV.

fermions. Note that all the off-diagonal soft SUSY-breaking scalar masses have been set to zero. The inter-generational mixing at the low-energy scale then occurs only via the superpotential Yukawa couplings. With these initial conditions, we solve the coupled RGEs for the soft masses given in Appendix A, along with the Yukawa RGEs given in Ref. [5], to get the running soft masses at the low scale. We find that it is indeed possible to find a parameter space such that $m^2_{\bar{\chi}^c} < 0$ [for $SU(2)_R$ breaking] and $m^2_{\Phi_1} < 0$ (for EWSB) while keeping all other mass squares positive. Figure 2 illustrates such a scenario for the choices $m_{1/2} = 200$ GeV, $m_0 = 1.2$ TeV, and $m'_0 = 1.27$ TeV. We have chosen the $SL^c\bar{\chi}^c$ coupling $\mu_{L^c} = 0.7$ to achieve a realistic fermion mass spectrum, and in particular, the $b - \tau$ unification at the GUT scale. Note that the RH slepton masses evolve much more rapidly than their LH counterparts due to this large coupling μ_{L^c}. The value of m'_0 is chosen such that all the other eigenvalues (especially $m^2_{L^c_3}$ and m^2_S) remain positive at the TeV scale. Note that the low-energy values of $m^2_{L^c_3}$ and m^2_S are of order $(10\text{ GeV})^2$. However the physical masses of these particles also receive a contribution from the $\langle\bar{\chi}^c\rangle$, which pushes the masses up to a TeV scale. As far as the squark masses are concerned, they evolve more than the slepton masses due to the strong interaction loop contributions to their RGEs. The small intragenerational mass splitting is due to the differences in their electroweak interactions. We can see clearly that at the weak scale, the values of $m^2_{\bar{\chi}^c}$ and $m^2_{\Phi_1}$ are negative, thus triggering the $SU(2)_R$ and electroweak symmetry breaking, respectively. Note that both bidoublet mass squares need not be negative, as one negative value will induce the symmetry breaking via the cross terms of the type $\Phi_1\Phi_2$ in the Lagrangian.

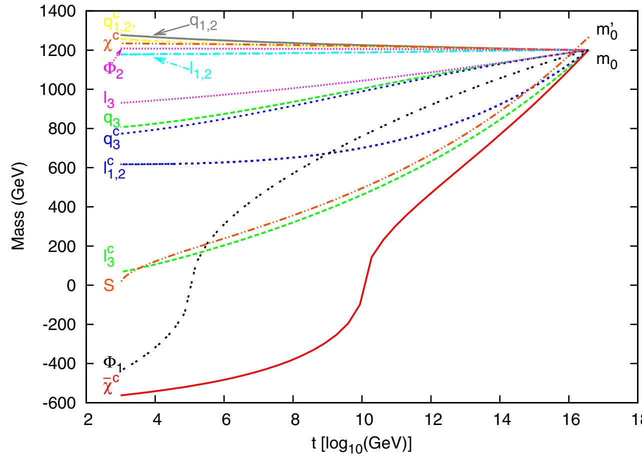

FIG. 2 (color online). Evolution of the scalar mass parameters for $m_{1/2} = 200$ GeV, $m_0 = 1.20$ TeV, and $m'_0 = 1.27$ TeV. For the scalar masses, we actually plot the sign $(m^2) \cdot \sqrt{|m^2|}$, so that the negative values on the curves correspond to negative values of m^2.

We also verify that the low-energy values of the sfermion mass square matrices satisfy all the flavor changing neutral current (FCNC) constraints [18], due to the smallness of the off-diagonal entries. As an example, we give the values here for the parameter values shown in Fig. 2:

$$m^2_Q = \begin{pmatrix} 1.63\times10^6 & -1.45\times10^1 + 8.64\times10^1 i & -4.79\times10^2 + 3.57\times10^3 i \\ -1.45\times10^1 - 8.64\times10^1 i & 1.63\times10^6 & -2.31\times10^4 + 1.68i \\ -4.79\times10^2 - 3.57\times10^3 i & -2.31\times10^4 - 1.68i & 6.51\times10^5 \end{pmatrix}\text{GeV}^2,$$

$$m^2_{Q^c} = \begin{pmatrix} 1.58\times10^6 & -1.45\times10^1 + 8.64\times10^1 i & -4.79\times10^2 + 3.57\times10^3 i \\ -1.45\times10^1 - 8.64\times10^1 i & 1.58\times10^6 & -2.31\times10^4 + 1.68i \\ -4.79\times10^2 - 3.57\times10^3 i & -2.31\times10^4 - 1.68i & 5.99\times10^5 \end{pmatrix}\text{GeV}^2,$$

$$m^2_L = \begin{pmatrix} 1.39\times10^6 & -7.28 + 8.39\times10^1 i & -2.59\times10^2 + 3.45\times10^3 i \\ -7.28 - 8.39\times10^1 i & 1.39\times10^6 & -1.25\times10^4 + 7.45\times10^{-1} i \\ -2.59\times10^2 - 3.45\times10^3 i & -1.25\times10^4 - 7.45\times10^{-1} i & 8.66\times10^5 \end{pmatrix}\text{GeV}^2,$$

$$m^2_{L^c} = \begin{pmatrix} 3.81\times10^5 & -7.18 + 8.24\times10^1 i & -2.57\times10^2 + 3.41\times10^3 i \\ -7.18 - 8.24\times10^1 i & 3.81\times10^5 & -1.24\times10^4 + 7.75\times10^{-1} i \\ -2.57\times10^2 - 3.42\times10^3 i & -1.24\times10^4 - 7.75\times10^{-1} i & 5.00\times10^3 \end{pmatrix}\text{GeV}^2.$$

VI. PROTON DECAY

In this section, we discuss the partial lifetimes of various proton decay channels.

A. Proton decay operators

In generic SUSY-GUTs, there exist three sources for proton decay:

(i) D-type (dimension-6) operators that arise from the exchange of gauge bosons:

$$\frac{1}{M_G^2}\int d^2\theta d^2\bar{\theta}\Phi^\dagger\Phi\Phi^\dagger\Phi, \qquad (27)$$

which may be generated both by heavy gauge boson exchange and by heavy chiral (Higgs) superfield exchange. For a unification scale $\gtrsim 10^{16}$ GeV, these contributions to proton decay are sufficiently small and well beyond the range of current experiments.

(ii) F-type (dimension-5) operators that arise from the exchange of color triplet Higgsino fields in **10**-Higgs fields as shown in Fig. 3(a):

$$\frac{1}{M_G}\int d^2\theta\Phi\Phi\Phi\Phi, \tag{28}$$

where Φ's are used to denote quark and lepton doublets. In the component language, they give rise to dimension-5 operators of the form $(QQ)(\tilde{Q}\tilde{L})$ and $(QL)(\tilde{Q}\tilde{Q})$. As these operators involve squark and slepton fields, they cannot induce proton decay at the lowest order. Proton decay occurs by converting the squark and slepton legs into quarks and leptons by exchanging a gaugino, as shown in the box diagram of Fig. 3(b).

(iii) Another class of dimension-5 operators can arise from R-parity breaking Planck suppressed operators, which are absent when we assume R parity. We discuss them in Sec. VI and show that their effects are very small due to the low $B-L$ breaking scale. These are absent in models where **126** Higgs fields break $B-L$, but are present in our model.

There are two effective dimension-5 operators of $LLLL$ type that involve only left-handed quark and lepton fields, given by Eq. (28), and a corresponding $RRRR$ type, both invariant under the MSSM [19]. In superspace notation, these are explicitly given by

$$\mathcal{O}_L = \int d^2\theta\epsilon^{\alpha\beta\gamma}\epsilon^{ab}\epsilon^{cd}Q_{\alpha ai}Q_{\beta bj}Q_{\gamma ck}L_{dl}, \tag{29}$$

$$\mathcal{O}_R = \int d^2\theta\epsilon^{\alpha\beta\gamma}(Q^c)_{\alpha i}(Q^c)_{\beta j}(Q^c)_{\gamma k}(L^c)_l, \tag{30}$$

where α, β, $\gamma = 1, 2, 3$ are $SU(3)_c$ color indices; a, b, c, $d = 1, 2$ are $SU(2)_L$ isospin indices; and i, j, k, $l = 1, 2, 3$ are generation indices. It is clear from the form of these operators that they break baryon number by one unit, but preserve the $B-L$ symmetry, leading to the proton decay to a pseudoscalar and an antilepton. As argued in Ref. [20] for kinematical reasons and explicitly shown in Ref. [21] for a small to moderate $\tan\beta$ region of the SUSY parameter space, the $RRRR$ contributions are at least an order of magnitude smaller than the $LLLL$ contributions. We also verify this explicitly in our model, as shown later; for the time being therefore, we concentrate only on the $LLLL$ operator.

In component form, the effective superpotential due to the $LLLL$ operator is explicitly given by [22]

$$\begin{aligned}\mathcal{W}_{\Delta B=1} = \frac{1}{M_T}\epsilon^{\alpha\beta\gamma}[&(C_{ijkl}-C_{kjil})u_{\alpha i}d_{\beta j}u_{\gamma k}e_l \\ &- (C_{ijkl}-C_{ikjl})u_{\alpha i}d_{\beta j}d_{\gamma k}\nu_l],\end{aligned} \tag{31}$$

where M_T is the effective mass of the color triplet Higgs field belonging to the $\mathbf{10}_H$ representation and, in our model, is of the order of the unification scale M_G (see Appendix A of Ref. [5]). This superpotential leads to the effective dimension-5 operators involving two fermions and two sfermions, as shown in Fig. 3(b), which lead to proton decay by four-Fermi interactions when "dressed" via the exchange of gauginos, namely, gluinos, binos, and winos. A typical diagram for the effective four-Fermi interaction induced by this dressing is shown in Fig. 4.

The coefficients C_{ijkl} associated with the superpotential given by Eq. (31) can be expressed in terms of the products of the GUT-scale Yukawa couplings. For model (A), this is given by

$$\begin{aligned}C_{ijkl} = {}& h_{u_{ij}}h_{u_{kl}} + x_1h_{d_{ij}}h_{d_{kl}} + x_2h_{u_{ij}}h_{d_{kl}} + x_3h_{d_{ij}}h_{u_{kl}} \\ &+ \tfrac{1}{2}[h_{u_{ij}}f_{u_{kl}} + f_{u_{ij}}h_{u_{kl}} + x_1(h_{d_{ij}}f_{d_{kl}} + f_{d_{ij}}h_{d_{kl}}) \\ &+ x_2(f_{u_{ij}}h_{d_{kl}} + h_{u_{ij}}f_{d_{kl}}) + x_3(h_{d_{ij}}f_{u_{kl}} + f_{d_{ij}}h_{u_{kl}})] \\ &+ \tfrac{1}{4}(f_{u_{ij}}f_{u_{kl}} + x_1f_{d_{ij}}f_{d_{kl}} + x_2f_{u_{ij}}f_{d_{kl}} + x_3f_{d_{ij}}f_{u_{kl}}),\end{aligned} \tag{32}$$

while for model (B) this becomes

$$\begin{aligned}C_{ijkl} = {}& h_{ij}h_{kl} + x_1h'_{ij}h'_{kl} + x_2h_{ij}h'_{kl} + x_3h'_{ij}h_{kl} \\ &+ \tfrac{1}{2}[x_1(h'_{ij}f_{kl} + f_{ij}h'_{kl}) + x_2h_{ij}f_{kl} + x_3f_{ij}h_{kl}] \\ &+ \tfrac{1}{4}x_1f_{ij}f_{kl},\end{aligned} \tag{33}$$

where x_i's are the ratios of the $\mathbf{10}_H$ color triplet Higgs masses and mixings and the factor $\frac{1}{2}$ is the Clebsch-Gordan coefficient for the $\mathbf{10}\cdot\mathbf{10}\cdot\mathbf{126}$ coupling. Note that there are only three mixing parameters, as there are only four

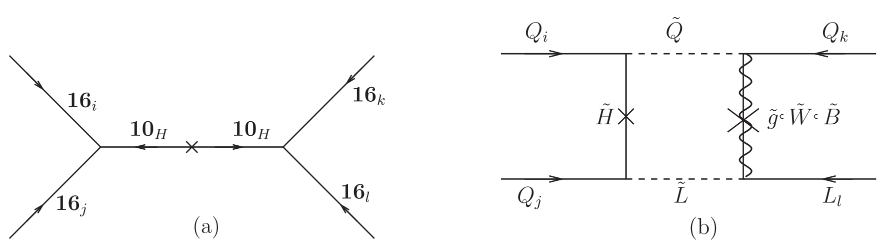

FIG. 3. (a) Supergraph giving rise to effective dimension-5 proton decay operators, and (b) box diagram involving gaugino exchange that converts the dimension-5 operator of panel (a) into an effective four-Fermi operator that induces proton decay.

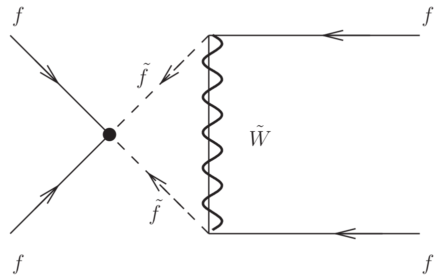

FIG. 4. The effective four-Fermi interaction diagram induced by the gaugino dressing of the effective dimension-5 operator given by Fig. 3(b).

color triplet Higgses in the MSSM gauge group, corresponding to the two $\mathbf{10}_H$ fields in our model. As we are interested only in the upper bound for the partial lifetimes of various proton decay channels, we do not need to know the detailed form for the x_i parameters in terms of these masses and mixings. We just vary these parameters numerically to get the maximum value for the partial lifetimes.

It can be shown that [23] in the limit of all squark masses being degenerate as in typical mSUGRA-type models, the gluino and bino contributions to the dressing of the dimension-5 operators vanish. This basically follows from the use of the Fierz identity for the chiral two component spinors representing quarks and leptons. In realistic models, the FCNC constraints allow only very small deviations from universality of squark masses. Hence, these gluino and bino contributions are expected to be small compared to the wino contributions, and can be ignored altogether. The charged wino dressing diagrams have been evaluated earlier [24], and in the limit of degenerate squark masses, this leads to the effective Lagrangian [22]

$$\mathcal{L}_{\Delta B=1} = 2I\epsilon^{\alpha\beta\gamma}(C_{kjil} - C_{ijkl})[u^T_{\alpha k}Cd_{\beta j}d^T_{\gamma i}C\nu_l + u^T_{\beta j}Cd_{\gamma k}u^T_{\alpha i}Ce_l], \tag{34}$$

where C denotes the charge-conjugation matrix and I is given by

$$I = \frac{\alpha_2}{4\pi}\frac{m_{\tilde{W}}}{M^2_{\tilde{f}}}, \tag{35}$$

$m_{\tilde{W}}$ being the wino mass and $M_{\tilde{f}}$ the sfermion mass. Using this expression and adding a similar contribution from the neutral wino exchange diagram, we can write down the total contribution to various proton decay channels. This is summarized in Table I. We note that the proton decay operators with the s quark lead to K-meson final states, whereas the ones without s lead to π final states. As shown in Table I, the amplitude for nonstrange quark final states will be Cabibbo suppressed compared to the strange quark final states. It is also important to mention here that the total amplitude for final states involving neutrinos is the incoherent sum of the rates for all three neutrino states.

TABLE I. The coefficients for various $\Delta B = 1$ dimension-5 operators obtained from the effective Lagrangian to leading order. Here θ_C is the Cabibbo angle (with $\sin\theta_C \sim 0.22$) and the C_{ijkl}'s are products of the Yukawa couplings, as defined in Eqs. (32) and (33).

Decay channel	$\mathcal{C}$ coefficient
$p \to K^+\bar{\nu}_l$	$(C_{112l} - C_{121l})$
$p \to K^0e^+$	$(C_{1121} - C_{1211})$
$p \to K^0\mu^+$	$(C_{1122} - C_{1212})$
$p \to \pi^+\bar{\nu}_l$	$\sin\theta_C(C_{211l} - C_{112l})$
$p \to \pi^0e^+$	$\sin\theta_C(C_{2111} - C_{1121})$
$p \to \pi^0\mu^+$	$\sin\theta_C(C_{2112} - C_{1122})$

This leads to large decay rates for $p \to K^+\bar{\nu}$ and $p \to \pi^+\bar{\nu}$ channels compared to the other decay channels due to the large Yukawa couplings of the third generation.

Before proceeding to calculate the rate of proton decay induced by these $LLLL$-type operators, let us estimate the contribution from the $RRRR$-type operators in our model. The gluino dressing graphs do not contribute in the limit of universal sfermion masses by the same Fierz arguments as for the $LLLL$ case. Moreover, since all superfields in the $RRRR$ operator are $SU(2)_L$ singlets, there is no wino contribution to the leading order. Also the bino dressing generates an effective four-Fermi operator of the type $\epsilon^{\alpha\beta\gamma}\epsilon^{ij}\epsilon^{kl}u^{cT}_{\beta j}Cd^c_{\gamma k}u^{cT}_{\alpha i}Ce^c_l$ which, in the flavor basis, is antisymmetric in the flavor indices i and j and hence, in the mass basis, must involve a charm quark. Thus to leading order, the bino contribution also vanishes due to phase space constraints. Thus the only dominant contribution comes from the Higgsino exchange, and the largest amplitude in this case, which comes from stop intermediate states, is estimated to be [22] (using the C_{ijkl} values calculated later in our model)

$$C_{1323}\frac{m_t m_\tau V_{ub}}{16\pi^2 v^2_{\mathrm{wk}}\sin\beta\cos\beta} \sim 4.0 \times 10^{-10} \tag{36}$$

for $\tan\beta = 30$, as compared to the $LLLL$ contribution which is typically of order

$$C_{1123}\frac{\alpha_2}{4\pi} \sim 4.5 \times 10^{-9}. \tag{37}$$

As the $RRRR$ contribution is proportional to $\frac{1}{\sin\beta\cos\beta}$ which is $\sim \tan\beta$ for large β, for smaller $\tan\beta$, this contribution is further suppressed. This justifies why we can ignore the $RRRR$ contributions in the following calculation of the proton decay rate.

B. Proton decay rate

In order to calculate the proton decay rate, we must extrapolate these dimension-5 operators defined at the GUT scale to the scale of $m_p = 1$ GeV. In our model, we can divide this whole energy range into three parts, following the breaking chain given by Eq. (1):

(i) from the GUT scale M_G to the $B-L$ breaking scale M_R (SUSYLR),
(ii) from M_R to the SUSY-breaking scale M_S (MSSM), and
(iii) from M_S to 1 GeV (SM).

The values of these extrapolation factors are given in the literature [20,25–27] for both the SM and the MSSM, but not for the SUSYLR model. In this section, we derive these factors using the anomalous dimensions for the dimension-5 operators in our model calculated in Appendix B. We denote the overall extrapolation factor by A_e. We noted some discrepancies in the values of the anomalous dimensions quoted in different papers, but found that our results for the SM and MSSM cases agree with those given in Refs. [20,25] and quoted in Appendix E of Ref. [7].

We also need to include the QCD effects in going from three quarks to a proton. As the low-energy hadrons are involved in the decay, this is a highly nonperturbative process, and it is difficult to calculate the exact form of the hadronic mixing matrix element for the process. Even though various QCD models have been constructed for this purpose, the estimates vary by a factor of $\mathcal{O}(10)$ between the smallest and the largest [28]. As the partial width of the decay is proportional to the matrix element squared, the variation in the estimate of the proton lifetime in different models will be $\mathcal{O}(100)$. A different approach using lattice QCD techniques gives more consistent results [29]. We use these recent results to estimate the chiral symmetry breaking effects which can be parametrized by two hadronic parameters, D and F. Then the hadronic mixing matrix for the proton decay can be written as $\frac{\beta}{f_\pi} f(F,D)$, where $f_\pi = (130.4 \pm 0.04 \pm 0.2)$ MeV [30] is the pion decay constant and $|\beta| = 0.0120(26)\ \mathrm{GeV}^3$ [29] is a low-energy parameter of the $SU(3)_f$ baryon chiral Lagrangian with the baryon number violating interaction. The factors $f(F,D)$ for different final states are listed in Appendix C.

Finally, combining all the factors discussed above, the proton decay rate for a given decay mode $p \to Ml$ (M denotes the meson and l the lepton) is given by [22]

$$\Gamma_p(Ml) \simeq \frac{m_p}{32\pi M_T^2}\frac{|\beta|^2}{f_\pi^2}\left(\frac{\alpha_2}{4\pi}\right)^2\left(\frac{m_{\tilde W}}{M_{\tilde f}^2}\right)^2 4|\mathcal{C}|^2|A_e|^2|f(F,D)|^2$$
$$\simeq (1.6\times 10^{-49}\ \mathrm{GeV})\left(\frac{2\times 10^{16}\ \mathrm{GeV}}{M_T}\right)^2 \times \left(\frac{m_{\tilde W}}{200\ \mathrm{GeV}}\right)^2\left(\frac{1\ \mathrm{TeV}}{M_{\tilde f}}\right)^4 |\mathcal{C}|^2|A_e|^2|f(F,D)|^2, \tag{38}$$

where the coefficients $\mathcal{C}$ are given in Table I, the hadronic factors $f(F,D)$ are listed in Appendix C, and the extrapolation factors A_e are derived below.

C. The extrapolation factors for the dimension-5 operator

As noted in the previous section, we need to extrapolate the dimension-5 operators defined at the GUT scale to the scale of 1 GeV. In our model, this whole energy range is divided into three parts, with different running behavior for the gauge couplings. First, we have the SM sector from 1 GeV to the SUSY-breaking scale M_S, in which we have the usual non-SUSY enhancement factor [25] for the $LLLL$ operator:

$$A_e^{\mathrm{NS}} = \left[\frac{\alpha_3(1\ \mathrm{GeV})}{\alpha_3(M_S)}\right]^{2/(11-(2/3)n_f)}, \tag{39}$$

where n_f is the number of quark flavors below the energy scale of interest. Here we have neglected the effects of $SU(2)_L$ and $U(1)_Y$ couplings as they are much smaller compared to that of $SU(3)_c$. In our model, as $M_S = 300\ \mathrm{GeV} > m_t$, the enhancement factor explicitly becomes

$$A_e^{\mathrm{NS}} = \left[\frac{\alpha_3(1\ \mathrm{GeV})}{\alpha_3(m_c)}\right]^{2/9}\left[\frac{\alpha_3(m_c)}{\alpha_3(m_b)}\right]^{6/25}\left[\frac{\alpha_3(m_b)}{\alpha_3(m_t)}\right]^{6/23} \times\left[\frac{\alpha_3(m_t)}{\alpha_3(M_S)}\right]^{2/7} = 1.49 \tag{40}$$

using the values of $\alpha_3(\mu)$ at $\mu = 1$ GeV, m_c and m_b obtained by interpolating the renormalization group equation for the effective QCD coupling [31], and at $\mu = m_t$ by the SM running from $\mu = m_Z$.

Now above M_S, we have the usual MSSM till the $B-L$ breaking scale M_R, and then the SUSYLR model till the GUT scale M_G. The extrapolation factor in this case is given by

$$A_e^{\mathrm{S}} = A_e^{\mathrm{MSSM}} A_e^{\mathrm{SUSYLR}}, \tag{41}$$

where the corresponding factors in the two sectors are given by

$$A_e^{\mathrm{MSSM}} = \prod_{i=1}^{3}\left[\frac{\alpha_i(M_S)}{\alpha_i(M_R)}\right]^{\gamma_i/b_i} \quad \text{and}$$
$$A_e^{\mathrm{SUSYLR}} = \prod_{j=1}^{4}\left[\frac{\alpha_j(M_R)}{\alpha_j(M_G)}\right]^{\gamma_j/b_j}. \tag{42}$$

Here $b_i = (\frac{33}{5}, 1, -3)$ for $i = \mathbf{1}_Y, \mathbf{2}_L, \mathbf{3}_c$ are the well-known MSSM β-function coefficients; $b_j = (13, 2, 4, -2)$ for $j = \mathbf{1}_{B-L}, \mathbf{2}_L, \mathbf{2}_R, \mathbf{3}_c$ are the β-function coefficients for the SUSYLR model [5]; and γ_i's are the anomalous dimensions for the $LLLL$ operator, calculated in Appendix B. From these results, we obtain

$$A_e^{\mathrm{MSSM}} = \left[\frac{\alpha_3(M_S)}{\alpha_3(M_R)}\right]^{-4/3}\left[\frac{\alpha_{2_L}(M_S)}{\alpha_{2_L}(M_R)}\right]^{3}\left[\frac{\alpha_{1_Y}(M_S)}{\alpha_{1_Y}(M_R)}\right]^{1/33} = 0.91 \tag{43}$$

using the MSSM running of the gauge couplings, and similarly,

$$A_e^{\mathrm{SUSYLR}} = \left[\frac{\alpha_3(M_R)}{\alpha_3(M_G)}\right]^{-2}\left[\frac{\alpha_{2_L}(M_R)}{\alpha_{2_L}(M_G)}\right]^{3/2}\left[\frac{\alpha_{2_R}(M_R)}{\alpha_{2_R}(M_G)}\right]^{3/4} \times \left[\frac{\alpha_{1_{B-L}}(M_R)}{\alpha_{1_{B-L}}(M_G)}\right]^{1/26} = 0.08 \qquad (44)$$

using the SUSYLR running of the gauge couplings [5]. Combining all these results, we get the overall extrapolation factor in bringing the operators from the GUT scale down to 1 GeV:

$$A_e = A_e^{\mathrm{NS}} A_e^{\mathrm{MSSM}} A_e^{\mathrm{SUSYLR}} = 0.11. \qquad (45)$$

D. Predictions for partial lifetimes

Substituting the extrapolation factor obtained in Eq. (45) in the expression for the partial decay width given by Eq. (38) and using $M_T \simeq M_U \simeq 4 \times 10^{16}$ GeV in our model, we obtain the partial lifetimes of different decay modes:

$$\tau_p(Ml) = \frac{\hbar}{\Gamma_p} \simeq \frac{(4.42 \times 10^{33}\ \mathrm{yr})}{|f(F,D)|^2}\left(\frac{10^{-14}}{|\mathcal{C}|^2}\right)\left(\frac{200\ \mathrm{GeV}}{m_{\tilde{W}}}\right)^2 \times \left(\frac{M_{\tilde{f}}}{1\ \mathrm{TeV}}\right)^4. \qquad (46)$$

The wino mass $m_{\tilde{W}}$ has been constrained at the CERN LEP to be larger than $\sim$100 GeV [32], essentially independent of any specific model. As a typical value, we choose the universal gaugino mass, $m_{1/2} = 200$ GeV, which when extrapolated to the weak scale gives $m_{\tilde{W}} \simeq 134$ GeV for the wino mass.

E. Model (A)

As we are interested in obtaining an upper bound on the partial lifetimes of various proton decay modes, we adopt the strategy of varying the mixing parameters x_i defined by Eq. (32) to maximize the expression (46), and simultaneously satisfying the present experimental lower bounds [33]. We find that the most stringent constraint comes from the $p \to K^+\bar{\nu}$ decay mode, and for this decay rate to be consistent with the present experimental bound, we must have the sfermion mass $M_{\tilde{f}} \geq 1.2(2.1)$ TeV for the MSSM $\tan\beta = 10(30)$. This value of $M_{\tilde{f}}$, when extrapolated to the GUT scale, puts a lower limit on the universal squark mass m_0 for a given value of $m_{1/2}$. The allowed region in the $m_0 - m_{1/2}$ plane satisfying the proton decay constraints and also satisfying the EWSB constraints is shown in Fig. 5. It is clear that this model favors low values of $\tan\beta$.

The model predictions for the upper bound on the partial lifetime of various proton decay modes are given in

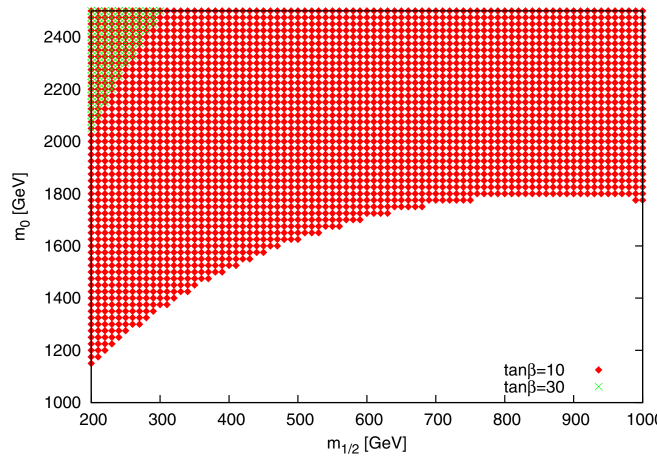

FIG. 5 (color online). Allowed region for model (A) in the $m_0 - m_{1/2}$ plane satisfying the proton decay and EWSB constraints for $\tan\beta = 10$ (red, larger area) and $\tan\beta = 30$ (green, smaller area).

Table II. We also list the present experimental lower bounds for comparison. As noted above, the most stringent constraint on the parameter space comes from the $p \to K^+\bar{\nu}$ decay mode; this is due to the fact that the neutrino final states add incoherently for the three generations, and hence, the decay rate for the neutrino final states will be much larger compared to the rates of other decay modes due to the third generation Yukawa coupling dominance. This also explains why the $p \to \pi^+\bar{\nu}$ decay rate is so large, even though it is Cabibbo suppressed. The predicted upper bounds for these neutrino final states may be testable in the future proton decay searches, as in the next round of Super-Kamiokande [33] or megaton-type detector searches.

F. Model (B)

As in model (A), we maximize the function $|C|^{-2}$ given by Eq. (33) with respect to the x_i parameters to find an upper bound on the proton decay lifetime. However, due to the particular structure of the Yukawa matrices in this model, as given by Eqs. (17) and (18), the parameters x_2 and x_3 have no effect on the amplitude and the only effective mixing parameter is x_1. The experimental lower bounds on the lifetime of various proton decay modes will then put a lower bound on the ratio $\frac{M_{\tilde{f}}^2}{x_1 m_{\tilde{W}}}$. It turns out that the most stringent bound is $p \to K^+\bar{\nu}(\pi^0\mu^+)$ for $\tan\beta = 10(30)$, and we must have

$$\frac{M_{\tilde{f}}^2}{x_1 m_{\tilde{W}}} \geq 1.44(1.06) \times 10^5\ \mathrm{GeV}. \qquad (47)$$

As an example, for $m_{1/2} = 200$ GeV and $x_1 = 0.1$, it puts a lower bound on the first and second generation squark masses to be $M_{\tilde{f}} \geq 1.4(1.2)$ TeV for $\tan\beta = 10(30)$. The model predictions for $x_1 = 0.1$ for various decay modes are given in Table III. We note that the observation of one

TABLE II. Model (A) predictions for the upper limits on the partial lifetimes of various proton decay modes in $SO(10)$ with low-scale SUSYLR for $\tan\beta = 10$ and 30 for $m_{1/2} = 200$ GeV. We have chosen the value of the universal scalar mass m_0 to be 1.2 (2.1) TeV for $\tan\beta = 10(30)$ so that the $p \to K^+\bar{\nu}$ constraint is just satisfied. The present experimental lower limits are also given for comparison.

Decay mode	Experimental lower limit ($\times 10^{33}$ yr)	Predicted upper limit ($\times 10^{33}$ yr)	
		$\tan\beta = 10$	$\tan\beta = 30$
$p \to K^+\bar{\nu}$	2.3	2.3	2.3
$p \to K^0\mu^+$	1.3	399.3	738.8
$p \to K^0 e^+$	1.0	1.3×10^3	49.7
$p \to \pi^0 e^+$	10.1	5.8×10^3	230.0
$p \to \pi^0\mu^+$	6.6	2.4×10^4	1.3×10^4
$p \to \pi^+\bar{\nu}$	0.025	1.5	0.8

TABLE III. The predictions for the upper limits on the partial lifetimes of various proton decay modes for the new mass fit in our model for $m_{1/2} = 200$ GeV and $x_1 = 0.1$. The most stringent constraint is from the $p \to K^+\bar{\nu}(\pi^0\mu^+)$ mode for $\tan\beta = 10(30)$, and hence, the squark mass has been chosen to be 1.4 (1.2) TeV so as to just satisfy the most stringent bound. Note that in this case, the model does not have any predictions for the decay modes $p \to K^0 e^+$ and $p \to \pi^0 e^+$. This is because the C coefficients for both these modes involve products of (1,1) elements of the Yukawa coupling matrices, and by construction, these elements are zero for all three coupling matrices; hence these modes have vanishing decay rates.

Decay mode	Experimental lower limit ($\times 10^{33}$ yr)	Predicted upper limit ($\times 10^{33}$ yr)	
		$\tan\beta = 10$	$\tan\beta = 30$
$p \to K^+\bar{\nu}$	2.3	2.3	3.5
$p \to K^0\mu^+$	1.3	2.3	1.6
$p \to K^0 e^+$	1.0	*	*
$p \to \pi^0 e^+$	10.1	*	*
$p \to \pi^0\mu^+$	6.6	9.8	6.6
$p \to \pi^+\bar{\nu}$	0.025	1.7	2.7

of the decay modes in the last two columns of Table III at a given rate will fix x_1, and the rates for the remaining modes (the ones without stars) are then predicted and should provide a test of this model. It should also be noted here that within the mSUGRA framework at low $\tan\beta$, the Tevatron has put a lower limit of 375 GeV for the squark mass based on an integrated luminosity of 1 fb^{-1}. We expect our predicted lower bound on the squark mass, which is of order 1 TeV, to be testable at higher luminosities within the reach of the LHC.

VII. COMMENT ON *R*-PARITY BREAKING

So far we have assumed matter parity so that there is no R-parity violating terms in the superpotential (i.e. $W' = 0$). In this section we discuss the implications for relaxing this assumption on the proton lifetime. This is an interesting exercise in view of the fact that in the MSSM embedding into $SU(5)$, relaxing R-parity (or matter-parity) conservation leads to new contributions to baryon number violation with arbitrary strength, so that, in principle, such models are not viable without the matter-parity assumption. We would like to study in this section the situation in the case of our $SO(10)$ model.

The most general R-parity violating interactions up to dimension-5 operators in our model are the following:

$$W' = M'_a \psi_a \bar{\psi}_H + \lambda'_a \psi_a \psi_H H + \frac{\lambda''_{abc}}{M_{\rm Pl}} \psi_a \psi_b \psi_c \psi_H + \frac{\mu'_{abc}}{M_{\rm Pl}} S_a S_b S_c + \mu''_{ab} S_a S_b, \tag{48}$$

where $\psi_{a,b,c}$ denote matter spinors and ψ_H and $\bar{\psi}_H$ are Higgs spinor fields. Before proceeding to discuss their implications, note that M'_a must be of order TeV; otherwise the right-handed neutrino field would decouple from the low-energy sector and break the gauge multiplet required to implement the inverse seesaw mechanism. The following classes of R-parity violating operators follow from this in conjunction with the $W_m + W_{SB}$ at the TeV scale:

$$W'(\text{TeV}) = M'_a L^c_a \bar{\chi}^c + \lambda' L \Phi \chi^c + \frac{\lambda''_{abc}}{M_{\rm Pl}} \chi^c [Q^c_a Q^c_b Q^c_c + L_a Q_b Q^c_c + L^c_a L_b L_c + \cdots]. \tag{49}$$

Note that the first three terms within the square brackets, after $B-L$ breaking, give rise to the familiar MSSM R-parity breaking terms with, however, couplings determined to be of order $\frac{v_{BL}}{M_{\rm Pl}}$ which is of order 10^{-15}. Hence their contribution to proton decay is negligible. Note that this would not be the case with $SO(10)$ models, where $B-L$ symmetry is broken at the GUT scale.

VIII. ADDITIONAL COMMENTS

In this section, we make brief comments on some other aspects of the model:

(i) Even though the model has low-scale $B-L$ violation, it does not lead to neutron-antineutron ($n-\bar{n}$) oscillation. The reason is that $n-\bar{n}$ oscillation requires color sextet Higgs bosons [34]. These fields are part of **126**-dimensional multiplets, which are not used in this paper.

(ii) An interesting question in this model is to explore the origin of matter via leptogenesis. This requires degenerate right-handed neutrinos with TeV-scale mass, which are a feature of this model. Thus the model has all the necessary ingredients for understanding the origin of matter in the Universe. There are some detailed issues such as the amount of washout and the magnitude of the lepton asymmetry which need to be investigated. This interesting feature of the model is currently under investigation.

IX. CONCLUSION

In summary, we have discussed proton decay as well as electroweak symmetry breaking in a new class of recently proposed $SO(10)$ models with TeV-scale W_R. We showed in an earlier paper that the model explains small neutrino masses via the inverse seesaw mechanism and has the feature of gauge coupling unification. The right-handed neutrinos in this model are almost Dirac type (pseudo-Dirac) with masses also in the TeV range, making them (as well as the W_R and Z' bosons) accessible at the LHC. This result is exciting since this brings in a new class of particles within the grand unification framework, which can be searched at the LHC. The signals are different from the case with Majorana right-handed neutrinos of the conventional type I seesaw mechanism, which do not lead to a grand unified theory.

We have explored two classes of fermion mass fits in these models. In both cases, all the Yukawa couplings entering the dimension-5 proton decay operators are fixed within certain assumptions by charged fermion mass fits, thereby leading to definite expectations for the partial lifetimes of various proton decay modes. We find that it is possible to satisfy the current experimental lower limits on the lifetimes with a wino mass of 100–200 GeV and squark and slepton masses of order TeV. More specifically, to satisfy the most stringent bound coming from the $p\to K^+\bar{\nu}$ decay mode, we need to have a lower limit of 1.2 (2.1) TeV on the squark masses in the case of model (A) for $\tan\beta=10(30)$ and similar lower bounds for model (B) for a given **10**-Higgs mixing, assuming the universality of squark and slepton masses, as in a typical mSUGRA-type scenario. Thus, discovery of squarks at the LHC can throw light on the validity of these models. It is also worth pointing out that the choice of $SO(10)$ multiplets in this class of models is derivable from fermionic string compactification.

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APPENDIX A: RGES FOR SOFT SUSY-BREAKING MASSES IN THE SUSYLR MODEL

Assuming R-parity conservation and the trilinear couplings A and Y in the superpotential and the soft-breaking Lagrangian given by Eqs. (20) and (21) to be zero, the soft-breaking mass RGEs at one-loop level are given by [17]

$$16\pi^2\frac{d}{dt}m_Q^2 = 2m_Q^2 h_a h_a^\dagger + h_a(2h_a^\dagger m_Q^2 + 4m_{Q^c}^2 h_a^\dagger + 4m_{\Phi_{ab}}^2 h_b^\dagger) - \frac{1}{3}M_1M_1^\dagger g_1^2 - 6M_{2L}M_{2L}^\dagger g_{2L}^2 - \frac{32}{3}M_3M_3^\dagger g_3^2 + \frac{1}{8}g_1^2 S_2, \tag{A1}$$

$$16\pi^2\frac{d}{dt}m_{Q^c}^2 = 2m_{Q^c}^2 h_a^\dagger h_a + h_a^\dagger(2h_a m_{Q^c}^2 + 4m_Q^2 h_a + 4h_b m_{\Phi_{ba}}^2) - \frac{1}{3}M_1M_1^\dagger g_1^2 - 6M_{2R}M_{2R}^\dagger g_{2R}^2 - \frac{32}{3}M_3M_3^\dagger g_3^2 - \frac{1}{8}g_1^2 S_2, \tag{A2}$$

$$16\pi^2\frac{d}{dt}m_L^2 = 2m_L^2 h'_a h'^\dagger_a + h'_a(2h'^\dagger_a m_L^2 + 4m_{L^c}^2 h'^\dagger_a + 4m_{\Phi_{ab}}^2 h'^\dagger_b) - 3M_1M_1^\dagger g_1^2 - 6M_{2L}M_{2L}^\dagger g_{2L}^2 - \frac{3}{8}g_1^2 S_2, \tag{A3}$$

$$16\pi^2\frac{d}{dt}m_{L^c}^2 = 2m_{L^c}^2 h'^\dagger_a h'_a + h'^\dagger_a(2h'_a m_{L^c}^2 + 4m_L^2 h'_a + 4h'_b m_{\Phi_{ba}}^2) + 2\mu_{L^c}^{\alpha^*}[m_{L^c}^2\mu_{L^c}^\alpha + m_{\bar{\chi}^c}^2\mu_{L^c}^\alpha + \mu_{L^c}^\beta(m_{\bar{S}}^2)_{\beta\alpha}] - 3M_1M_1^\dagger g_1^2 - 6M_{2R}M_{2R}^\dagger g_{2R}^2 + \frac{3}{8}g_1^2 S_2, \tag{A4}$$

$$16\pi^2 \frac{d}{dt} m^2_{\bar{\chi}^c} = 2\mu^{\alpha*}_{L^c}[m^2_{L^c}\mu^\alpha_{L^c} + m^2_{\bar{\chi}^c}\mu^\alpha_{L^c} + \mu^\beta_{L^c}(m^2_S)_{\beta\alpha}] - 3M_1M_1^\dagger g_1^2 - 6M_{2R}M_{2R}^\dagger g_{2R}^2 - \frac{3}{8}g_1^2 S_2, \tag{A5}$$

$$16\pi^2 \frac{d}{dt} m^2_{\chi^c} = -3M_1M_1^\dagger g_1^2 - 6M_{2R}M_{2R}^\dagger g_{2R}^2 + \frac{3}{8}g_1^2 S_2, \tag{A6}$$

$$16\pi^2 \frac{d}{dt}(m^2_S)^{\alpha\beta} = 4\mu^{\alpha*}_{L^c}\mu^\beta_{L^c}(m^2_{\bar{\chi}^c} + m^2_{L^c}) + 2\mu^{\alpha*}_{L^c}\mu^\rho(m^2_S)^\beta_\rho, \tag{A7}$$

$$16\pi^2 \frac{d}{dt} m^2_{\Phi_{ab}} = m^2_{\Phi_{ac}}\mathrm{Tr}(3h_c^\dagger h_b + h_c'^\dagger h_b') + \mathrm{Tr}(3h_a^\dagger h_c + h_a'^\dagger h_c')m^2_{\Phi_{cb}} + \mathrm{Tr}(6h_a^\dagger h_b m^2_{Q^c} + 6h_a^\dagger m_Q^2 h_b + 2h_a'^\dagger h_b' m^2_{L^c} + 2h_a'^\dagger m_L^2 h_b') + (-6M_{2L}M_{2L}^\dagger g_{2L}^2 - 6M_{2R}M_{2R}^\dagger g_{2R}^2)\delta_{ab}, \tag{A8}$$

where

$$S_2 \equiv 4[\mathrm{Tr}(m_Q^2 - m^2_{Q^c} - m_L^2 + m^2_{L^c}) + (m^2_{\chi^c} - m^2_{\bar{\chi}^c})]. \tag{A9}$$

We have ignored the RG running of the coupling $\mu^\alpha_{L^c}$, as this is a higher order effect.

APPENDIX B: ANOMALOUS DIMENSIONS OF THE DIMENSION-5 OPERATOR

Here we present the derivation of the anomalous dimensions of the dimension-5 operators of the $LLLL$ type given by Eq. (29). The calculation is straightforward in a supersymmetric gauge due to the fact that the operator $\mathcal{O}_L$ is purely chiral (it is an F-term), and hence, it follows from nonrenormalization theorems that, in a supersymmetric gauge, it will only have wave function renormalization. Then it is easy to show that the anomalous dimensions of any purely chiral operator are given by

$$\gamma_{\mathcal{O}} = \sum_r C_2(r), \tag{B1}$$

where $C_2(r)$ is the eigenvalue of the quadratic Casimir operator in the representation r, and the sum runs over all the chiral superfields occurring in the chiral coupling. As the gauge bosons belong to the adjoint representation, we have

$$C_2(r) = \begin{cases} \frac{N^2-1}{2N} & \text{for } SU(N) \\ \frac{1}{4}X^2 & \text{for } U(1)_X. \end{cases} \tag{B2}$$

Thus we have for $SU(3)_c$,

$$\gamma_{\mathbf{3}_c} = 3 \times \tfrac{4}{3} = 4, \tag{B3}$$

as there are three $SU(3)_c$ fields in the $LLLL$ operator [e.g. $(qq)(\tilde{q}\tilde{l})$]. Similarly, we have

$$\gamma_{\mathbf{2}_{L,R}} = 4 \times \tfrac{3}{4} = 3, \tag{B4}$$

$$\gamma_{\mathbf{1}_Y} = \tfrac{1}{4}[3(\tfrac{1}{3})^2 + 1]\tfrac{3}{5} = \tfrac{1}{5}, \tag{B5}$$

$$\gamma_{\mathbf{1}_{B-L}} = \tfrac{1}{4}[3(\tfrac{1}{3})^2 + 1]\tfrac{3}{2} = \tfrac{1}{2}. \tag{B6}$$

Here the factors $\frac{3}{5}$ and $\frac{3}{2}$ are the GUT normalization factors for $U(1)_Y$ and $U(1)_{B-L}$, respectively.

We note that the same results would have been obtained in a nonsupersymmetric gauge, though the calculation is much more involved. For instance, the same results were obtained for the MSSM case in a Wess-Zumino gauge in Ref. [20].

APPENDIX C: THE HADRONIC FACTORS $f(F, D)$

As noted in Sec. IV, the hadronic factor $f(F, D)$ estimates the chiral symmetry breaking effects on different final states. The low-energy parameters D and F are usually chosen to be the same as the analogous parameters in weak semileptonic decays [35]. Then $D + F = g_A^{(np)} = 1.27$ is the nucleon axial charge, while $D - F = g_A^{(\Sigma^- n)} = 0.33$–$0.34$ [30]. This gives $D = 0.8$ and $F = 0.47$. Using these constants and the approximations $m_{u,d} \ll m_s \ll m_p$ as well as $-q^2 \ll m_p^2$, where q_μ is the momentum transfer (the momentum of the antilepton for physical decays), all the hadronic matrix elements can be obtained [29]. In Table IV, we list the results for different decay channels.

TABLE IV. The hadronic factors $f(F, D)$ for different proton decay modes. Here we have used $m_N = 0.94$ GeV for the mass of the nucleon and $m_B = 1.15$ GeV for the average baryon mass ($m_B \simeq m_\Sigma \simeq m_\Lambda$).

Decay mode	$f(F, D)$	$\lvert f(F, D)\rvert^2$
$p \to \pi^0 l^+$	$\frac{1}{\sqrt{2}}(1 + D + F)$	2.58
$p \to \pi^+ \bar{\nu}_l$	$1 + D + F$	5.15
$p \to K^0 l^+$	$1 - \frac{m_N}{m_B}(D - F)$	0.53
$p \to K^+ \bar{\nu}_l$	$\frac{m_N}{m_B}\frac{2D}{3}$	0.19

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