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Constraints from color and/or charge breaking minima in the supersymmetric standard model with right-handed neutrinos

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We consider a model where right-handed neutrinos and sneutrinos are introduced to the minimal supersymmetric standard model. In the scalar potential of this model, there exist trilinear and quartic terms in scalar potential that are proportional to Yukawa couplings of neutrinos. Because of these trilinear and quartic terms, color and/or charge breaking (CCB) and unbounded-from-below (UFB) directions appear along which sneutrinos have a vacuum expectation value, making the vacuum of the electroweak symmetry breaking unstable. We analyze the scalar potential of this model and derive necessary conditions for color and/or charge breaking and unbounded-from-below directions to vanish.

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I. INTRODUCTION

Neutrino oscillation experiments [1–5] have confirmed that neutrinos have very tiny but nonzero masses. This is a clear evidence of physics beyond the standard model (SM) because neutrinos are massless in the SM. The simplest way to generate their tiny masses is to introduce right-handed neutrinos. There are two scenarios in this regard. One is that right-handed neutrinos are Majorana particles and neutrinos acquire masses via the famous seesaw mechanism [6–10]. The other scenario is that right-handed neutrinos are Dirac particles and neutrinos obtain masses via electroweak symmetry breaking (EWSB).

Combined these right-handed neutrino scenarios with a supersymmetric standard model, which we call νSSM, many works have been done so far. In the seesaw mechanism, it has been investigated recently that Majorana masses are as low as between 100 GeV and 10 TeV. In fact, such low scale Majorana masses can be realized as a consequence of supersymmetry (SUSY) breaking [11–13]. This class of models predicts relatively small Yukawa couplings of neutrinos compared to those of other fermions. In scenarios of Dirac neutrinos, tiny neutrino masses are solely explained by tiny Yukawa couplings. One might think it is unnatural because the Yukawa couplings of neutrinos are too small compared with those of other fermions. However, as was emphasized in [14], it is natural in 't Hooft's sense [15] that a symmetry (i.e. chiral symmetry in the neutrino sector) is recovered in the limit of vanishing neutrino Yukawa coupling constants. In these scenarios, right-handed neutrinos and sneutrinos are light, and therefore the scenarios are testable in astrophysical observations and terrestrial experiments. Studies of these scenarios are, e.g. the dark matter physics [14,16–18], lepton flavor violation searches [19], and collider physics [20,21].

The presence of the scalar partners generally leads color and/or charge breaking (CCB) directions and unbounded-from-below (UFB) directions [22–30]. Along CCB directions, the scalar potential has minima on which color and/or charge symmetries are spontaneously broken. The CCB minimum can be deeper than that of EWSB when the Yukawa coupling of the particle along the CCB direction is small. Along UFB directions, the scalar potential has no global minimum and falls down to negative infinity. These directions make the vacuum of EWSB unstable, and hence must be avoided. In the minimal supersymmetric extension of the SM (MSSM), conditions to avoid the UFB and CCB directions were systematically investigated in [31].[1] Those conditions constrain soft SUSY breaking parameters, mainly trilinear couplings, and exclude a certain region of the parameter space of the MSSM. In the νSSM, due to right-handed sneutrinos, not only new UFB and CCB directions but also false EWSB directions appear. Along false EWSB directions, neither color nor charge symmetry is broken but Higgses and sneutrinos acquire large vacuum expectation values (VEV). Such minima result in too heavy masses of gauge bosons and are excluded by precise electroweak measurements. Since the EWSB vacuum can become unstable along these directions due to small neutrino Yukawa couplings, conditions to avoid those directions should be investigated. In this article, we refer false EWSB directions as CCB directions in view of incorrect vacuum.

In this article, we consider the νSSM where either right-handed Dirac or Majorana (s)neutrinos are introduced to a supersymmetric standard model. We assume that the Majorana masses are below or around the TeV scale so

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[1]See also recent work [32].

that the neutrino Yukawa coupling is small as it is in the Dirac neutrinos case. Then, we analyze the potential along UFB and CCB directions at tree level and derive necessary conditions to avoid dangerous minima and directions. Necessary conditions are generally modified due to radiative corrections [33,34]. The conditions from the tree-level analysis coincide with those from the one-loop analysis when the analysis is performed at a scale in which vacuum expectation values of Higgses with and without radiative corrections coincide [33,34]. We assume that our analysis is performed at this scale.

The outline of this article is as follows. In Sec. II, we briefly review general properties of UFB and CCB directions in the MSSM. Then we analyze the scalar potential and derive necessary conditions in Dirac and Majorana neutrino cases in Secs. III and IV. We show numerical results of constraints on the soft SUSY breaking parameters in Sec. V. Finally we summarize and discuss our analysis in Sec. VI. The scalar potential of the MSSM and notations of fields and couplings are given in Appendix A, and EWSB of the MSSM is summarized in Appendix B. In Appendix C, F terms and soft SUSY breaking terms in the νSSM are shown.

II. GENERAL PROPERTIES FOR UFB DIRECTIONS AND CCB MINIMA IN THE MSSM

We start our discussion with briefly reviewing general properties of UFB and CCB directions in the MSSM [31]. Following the general properties, it is possible to classify all dangerous directions in a field space. As was studied in [31], there are three types of UFB and CCB directions, respectively. Throughout the main part of this paper, we refer H_1 and H_2 to a neutral component of down-type and up-type Higgs scalars, and use the symbol "tilde" to denote scalar partners of the SM fermions. Notations of couplings and fields and the scalar potential of the MSSM are summarized in Appendix A.

A. General properties of UFB directions

In general, UFB directions appear along field configurations such that terms of scalars in a potential are vanishing or kept under control. Along these directions, the potential is unstable and its minimum is driven to negative infinity if quadratic terms of the fields are negative. Two general properties for UFB directions are as follows.

Property 1.—Trilinear scalar terms cannot play a significant role along a UFB direction. This can be understood as follows. If a trilinear term does not vanish, F terms give rise to (positive) quartic terms which lift the potential up for large values of scalar fields. Let us show an example. Suppose that the trilinear term corresponding to the Yukawa couplings of charged sleptons is nonvanishing, and at least one F term of the scalar fields involved in the trilinear term is nonzero, e.g.

$$F_{\tilde{e}_R} = Y_e(H_1\tilde{e}_L - H_1^-\tilde{\nu}_L), \tag{1}$$

where H_1^- is a charged component of the down-type Higgs. It is obvious that a positive quartic F term which is proportional to $|Y_e|^2$ arises from the square of this term in the potential.

Property 2.—Any UFB direction must involve H_2 and perhaps H_1. This is because the terms $|H_2|^2$ and H_1H_2 can have negative soft masses for EWSB to successfully occur, while the other masses must be positive. Furthermore, since these terms are quadratic, all quartic terms coming from F and D terms must be vanishing or kept under control. Thus some additional fields are required except for H_2.

According to these properties, UFB directions are classified into three. A direction along which H_1 and H_2 have an equal VEV and other fields have no VEV's is the so-called UFB-1 direction. Another direction, the so-called UFB-2 direction, is the direction with nonzero VEV's of H_1, H_2, and $\tilde{L}$. Along the last direction called the UFB-3, H_2, $\tilde{L}$ and $\tilde{d}_L$, $\tilde{d}_R$ are nonvanishing. In the following, we show details of the UFB-2 and -3 directions. We will see that the absence of the neutrino Yukawa coupling plays an essential role on these directions.

Along the UFB-2 direction, left-handed sleptons have nonvanishing VEV's to cancel quartic terms from D terms. According to property 1, the trilinear term involving left-handed sleptons must be vanishing in order not to give a quartic term proportional to the Yukawa coupling squared. The only possibility for this direction is that the left-handed slepton has a VEV along the sneutrino direction since neutrinos are massless and hence do not have Yukawa couplings. Then, the potential is given

$$V_{\text{UFB-2}} = m_1^2|H_1|^2 + m_2^2|H_2|^2 - 2|m_3^2||H_1||H_2| + m_{\tilde{L}}^2|\tilde{L}|^2 + \frac{g_1^2+g_2^2}{8}(|H_2|^2 - |H_1|^2 - |\tilde{L}|^2)^2. \tag{2}$$

The potential along the UFB-2 direction is obtained by minimizing Eq. (2) with respect to $|\tilde{L}|$ and $|H_1|$,

$$V_{\text{UFB-2}} = \left(m_2^2 + m_{\tilde{L}}^2 - \frac{|m_3^2|^2}{|m_1^2 - m_{\tilde{L}}^2|}\right)|H_2|^2 - \frac{2m_{\tilde{L}}^4}{g_1^2+g_2^2}, \tag{3}$$

where

$$(m_1^2 - m_{\tilde{L}}^2)^2 > |m_3^2|^2, \tag{4}$$

$$|H_2|^2 > \frac{4m_{\tilde{L}}^2}{(g_1^2+g_2^2)(1 - |m_3^2|^2/(m_1^2 - m_{\tilde{L}}^2)^2)}, \tag{5}$$

are assumed. Notice that the condition for the minimum with respect to $|H_2|$, $\partial V/\partial|H_2| = 0$, cannot be satisfied simultaneously; therefore $|H_2|$ is a free parameter in Eq. (3). The potential becomes unbounded from below if the quadratic term of $|H_2|$ is negative. Therefore the condition to avoid the UFB-2 direction is

$$m_2^2 + m_{\tilde{L}}^2 - \frac{|m_3^2|^2}{|m_1^2 - m_{\tilde{L}}^2|} \geq 0. \tag{6}$$

Along the UFB-3 direction, H_1 is vanishing and VEV's of down squarks are chosen to cancel the F term of H_1,

$$F_{H_1} = \mu H_2 + Y_d \tilde{d}_L \tilde{d}_R^* = 0. \tag{7}$$

Then, as we will explain in the next section, VEV's of down squarks are much smaller than those of the Higgs and the sleptons, and can be neglected in the scalar potential. Taking the VEV's along $\tilde{d}_L = \tilde{d}_R^* = \tilde{d}$ so that the $SU(3)$ D term also vanishes, the potential becomes

$$\begin{aligned} V_{\text{UFB-3}} = {} & (m_2^2 - |\mu|^2)|H_2|^2 + (m_{\tilde{Q}}^2 + m_{\tilde{d}_R}^2)|\tilde{d}|^2 + m_{\tilde{L}}^2|\tilde{L}|^2 \\ & + \frac{g_1^2 + g_2^2}{8}(|H_2|^2 + |\tilde{d}|^2 - |\tilde{L}|^2)^2, \end{aligned} \tag{8}$$

where

$$|\tilde{d}|^2 = \frac{|\mu|}{|Y_d|}|H_2|. \tag{9}$$

Repeating the procedure of the UFB-2 direction, we can obtain the constraint preventing the UFB-3 direction

$$m_2^2 - |\mu|^2 + m_{\tilde{L}}^2 \geq 0, \tag{10}$$

assuming

$$|H_2| > \sqrt{\frac{|\mu|^2}{4|Y_d|^2} + \frac{4m_{\tilde{L}}^2}{g_1^2 + g_2^2}} - \frac{|\mu|}{2|Y_d|}. \tag{11}$$

It is important to emphasize here that the quartic terms from F and D terms can vanish simultaneously because the neutrino Yukawa coupling is absent.

B. General properties of CCB minima

CCB minima appear along directions in which a negative trilinear term dominates a potential against quadratic and quartic terms at a certain region of field space. CCB minima become deeper as Yukawa couplings of scalars are smaller. In the following, we show five general properties of CCB minima in the MSSM.

Property 1.—The deepest CCB direction involves only one particular trilinear soft term of one generation. When more than two trilinear terms are nonvanishing, quartic terms arising from F terms are also nonvanishing. Different quartic terms hardly deepen the potential cooperatively, but rather lift up the potential.

Property 2.—It cannot be determined *a priori* which trilinear coupling gives the strongest constraints. Nonvanishing trilinear terms lead quartic terms which are proportional to the square of a Yukawa coupling. Since the quartic terms are more important than the trilinear term for large values of fields, larger Yukawa couplings do not always deepen the potential.

Property 3.—If the trilinear term under consideration has a very small Yukawa coupling, D terms must be vanishing or negligible along the corresponding CCB direction. If D terms are nonvanishing, it lifts up the potential faster than F terms. Then, that direction cannot be the deepest direction.

Property 4.—There are two directions to be explored for CCB. For example, for $A_u Y_u \tilde{Q} \cdot H_2 \tilde{u}_R^*$, one is the direction along which H_2, $\tilde{Q}$, and $\tilde{u}_R$ are nonvanishing, and $|\tilde{d}_L|^2 = |\tilde{d}_R|^2 = |\tilde{d}|^2$ so that $D_{SU(3)}$ and F_{H_1} vanish. This direction is similar to UFB-3 and called direction (a) according to Casas *et al.* [31]. The other direction is along H_1, H_2 and $\tilde{Q}$, $\tilde{u}_R$ are nonzero. Possibly $\tilde{L}$ is also nonzero along this direction. The direction is similar to UFB-2 and called direction (b).

Property 5.—There are two choices of the phases of soft SUSY breaking terms in direction (b). For the same example as the above, the relevant soft terms are

$$\begin{aligned} & 2|A_u Y_u \tilde{Q} H_2 \tilde{u}_R| \cos\varphi_1 + 2|\mu Y_u \tilde{Q} H_1 \tilde{u}_R| \cos\varphi_2 \\ & + 2|B\mu H_1 H_2| \cos\varphi_3, \end{aligned} \tag{12}$$

where φ_1, φ_2, and φ_3 represent phases combined with signs of the couplings and phases of the fields. If $\text{sign}(A_u) = -\text{sign}(B)$, the three phases can be taken as π, so the three terms are negative. On the other hand, if $\text{sign}(A_u) = \text{sign}(B)$, two of them can be taken as π and the other one should be 0. Therefore one of the three terms is positive. For direction (a), only one term with an undetermined phase is $A_u Y_u \tilde{Q} H_2 \tilde{u}_R$. The sign of the term can always be taken negative by rotating the fields involved.

III. CONSTRAINTS FROM CCB MINIMA WITH DIRAC NEUTRINOS

In this section, we analyze the scalar potential of the νSSM with Dirac neutrinos. We consider not only the directions explained in the previous section (MSSM directions) but also new directions along which right-handed sneutrinos have VEV's. In our analysis, we assume that only one sneutrino has a nonvanishing VEV.

A. Constraints from MSSM UFB directions

Let us consider the MSSM UFB-2 direction along which Higgses and left-handed sneutrinos have nonvanishing VEV's. As we emphasized in Sec. II A, absence of the neutrino Yukawa coupling played an important role in the UFB-2 direction. The situation changes when the neutrino Yukawa coupling is introduced. F terms given in Appendix C cannot vanish simultaneously. According to property 2 of the UFB direction, a positive quartic term remains in the scalar potential, $V_{\text{UFB-2}}^{\text{Dirac}}$,

$$V_{\text{UFB-2}}^{\text{Dirac}} = V_{\text{UFB-2}} + |Y_\nu|^2|H_2|^2|\tilde{\nu}_L|^2, \tag{13}$$

where $V_{\text{UFB-2}}$ is given in Eq. (3). The last term lifts up the potential for large values of the fields. Thus, the MSSM UFB-2 direction disappears and turns to a CCB direction. We analyze this CCB direction below.

We parametrize the VEV's for convenience,

$$|\tilde{\nu}_L| = \alpha|H_2|, \qquad |H_1| = \gamma|H_2|, \tag{14}$$

where α and γ are real numbers. The potential is written using this parametrization,

$$V^{\text{Dirac}}_{\text{UFB-2}} = |Y_\nu|^2 F(\alpha, \gamma)\alpha^2\gamma^2|H_2|^4 + \hat{m}^2(\alpha, \gamma)|H_2|^2, \tag{15}$$

where

$$F(\alpha, \gamma) = \frac{1}{\gamma^2} + \frac{1}{\alpha^2\gamma^2} f(\alpha, \gamma), \tag{16a}$$

$$f(\alpha, \gamma) = \frac{1}{8}\frac{g_1^2 + g_2^2}{|Y_\nu|^2}(\alpha^2 + \gamma^2 - 1)^2, \tag{16b}$$

$$\hat{m}^2(\alpha, \gamma) = m_1^2\gamma^2 - 2|m_3^2|\gamma + m_2^2 + m_{\tilde{L}}^2\alpha^2. \tag{16c}$$

The minimum of the potential is obtained by differentiating Eq. (15) with respect to $|H_2|$,

$$|H_2|^2_{\text{ext}} = -\frac{1}{2}\frac{\hat{m}^2(\alpha, \gamma)}{|Y_\nu|^2 F(\alpha, \gamma)\alpha^2\gamma^2}, \tag{17}$$

where $|H_2|_{\text{ext}}$ is the VEV of H_2 at extremal. Here we assumed that $\hat{m}^2(\alpha, \gamma)$ is negative. According to property 3 of the CCB direction, we set $\alpha^2 = 1 - \gamma^2$ to cancel the D term or $f(\alpha, \gamma)$. Inserting Eq. (17) into the potential, the minimum is expressed

$$V^{\text{Dirac}}_{\text{UFB-2 min}} = -\frac{1}{4}\frac{(\hat{m}^2(\gamma))^2}{|Y_\nu|^2(1 - \gamma^2)}, \tag{18}$$

where

$$\hat{m}^{\,2}(\gamma) = (m_1^2 - m_{\tilde{L}}^2)\gamma^2 - 2|m_3^2|\gamma + m_2^2 + m_{\tilde{L}}^2. \tag{19}$$

The minimum would be much deeper than that of the EWSB, (B9), because the neutrino Yukawa coupling is very small. A necessary condition to avoid the dangerous minimum is that $\hat{m}^2$ is positive for any γ. It imposes a constraint on the soft masses as

$$0 \le |m_3^2|^2 - m_1^2 m_2^2 \le m_{\tilde{L}}^2(m_1^2 - m_2^2 + m_{\tilde{L}}^2), \tag{20}$$

where the left inequality is imposed by Eq. (B6). The constraint forbids a small soft mass for the left-handed sleptons unless $|m_3^2|^2$ is close to $m_1^2 m_2^2$.

For the MSSM UFB-3 direction, the same quartic term remains in the potential,

$$V^{\text{Dirac}}_{\text{UFB-3}} = V_{\text{UFB-3}} + |Y_\nu|^2|H_2|^2|\tilde{\nu}_L|^2, \tag{21}$$

and alters the MSSM UFB-3 direction to a CCB direction. $V_{\text{UFB-3}}$ is given in (8). As shown below, the VEV's of the Higgses and sneutrinos are of order m_{soft}/Y_ν, where m_{soft} is a typical scale of the soft SUSY breaking masses. These VEV's are much larger than those of the down squarks. Therefore, we can neglect down squarks in the following discussion. Similar to the UFB-2 direction, the potential is expressed using a parametrization, $|\tilde{\nu}_L| = \alpha|H_2|$,

$$V^{\text{Dirac}}_{\text{UFB-3}} = |Y_\nu|^2 F(\alpha)\alpha^2|H_2|^4 + \hat{m}^2(\alpha)|H_2|^2, \tag{22}$$

where

$$F(\alpha) = 1 + \frac{1}{\alpha^2} f(\alpha), \tag{23a}$$

$$f(\alpha) = \frac{1}{8}\frac{(g_1^2 + g_2^2)}{|Y_\nu|^2}(\alpha^2 - 1)^2, \tag{23b}$$

$$\hat{m}^2(\alpha) = m_2^2 - |\mu|^2 + m_{\tilde{L}}^2\alpha^2. \tag{23c}$$

Minimizing the potential with respect to $|H_2|$, the value of the $|H_2|$ at extremal, $|H_2|_{\text{ext}}$, is obtained,

$$|H_2|^2_{\text{ext}} = -\frac{1}{2}\frac{\hat{m}^2(\alpha)}{|Y_\nu|^2 F(\alpha)\alpha^2}, \tag{24}$$

and the minimum of the potential is given by

$$V^{\text{Dirac}}_{\text{UFB-3 min}} = -\frac{1}{4}\frac{(\hat{m}^2)^2}{|Y_\nu|^2}, \tag{25}$$

where $\alpha^2 = 1$ is used and $\hat{m}^2 = \hat{m}^2(\alpha^2 = 1)$. Again, the minimum is much deeper than that of the EWSB, (B9). A necessary condition to avoid the CCB minimum is

$$m_2^2 - |\mu|^2 + m_{\tilde{L}}^2 \ge 0. \tag{26}$$

B. Constraint from CCB-1 minimum

In the following, we analyze the scalar potential along CCB directions. Along the CCB directions of the MSSM, there are no important modifications on the constraints given in [31] since the trilinear term involving right-handed sneutrinos is vanishing and the quartic term proportional to the neutrino Yukawa coupling is very small. Once we consider directions that right-handed sneutrinos are nonvanishing, there appear new directions along which the minimum can become much deeper than that of EWSB. We focus our analysis on new CCB directions and derive constraints to evade such CCB minima.

First, we consider a direction similar to the MSSM CCB direction (a). From properties 1 and 3 of the CCB direction, we assume

$$H_2, \quad \tilde{\nu}_L, \quad \tilde{\nu}_R \neq 0, \tag{27}$$

$$|\tilde{d}_L|^2 = |\tilde{d}_R|^2 = |d|^2, \tag{28}$$

and $\text{sign}(A_\nu) = -\text{sign}(B)$ for simplicity. Other fields are vanishing. The assumption $|\tilde{d}_L|^2 = |\tilde{d}_R|^2$ is made to cancel the $SU(3)$ D term. Furthermore $\tilde{d}_L\tilde{d}_R^*$ is chosen to cancel F_{H_1}. Analogous to the MSSM UFB-3 direction, the VEV's of the Higgses and the sneutrinos are inversely proportional to the Yukawa coupling of neutrinos and are much larger than those of the down squarks. Hence we neglect down squarks in the potential.

The scalar potential from F, D terms and the soft SUSY breaking terms is given in Appendix C and also Appendix A. Following the procedure of the MSSM UFB-3 direction, we parametrize VEV's as

$$|\tilde{\nu}_L| = \alpha|H_2|, \qquad |\tilde{\nu}_R^*| = \beta|H_2|, \tag{29}$$

where α and β are real numbers. Then, the scalar potential is written

$$V_{\text{CCB-1}}^{\text{Dirac}} = |Y_\nu|^2 F(\alpha,\beta)\alpha^2\beta^2|H_2|^4 - 2|Y_\nu|\hat{A}\alpha\beta|H_2|^3 + \hat{m}^2(\alpha,\beta)|H_2|^2, \tag{30}$$

where

$$F(\alpha,\beta) = 1 + \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\alpha^2\beta^2} f(\alpha), \tag{31a}$$

$$f(\alpha) = \frac{1}{8}\frac{g_1^2 + g_2^2}{|Y_\nu|^2}(\alpha^2 - 1)^2, \tag{31b}$$

$$\hat{A} = |A_\nu|, \tag{31c}$$

$$\hat{m}^2(\alpha,\beta) = m_{H_2}^2 + m_{\tilde{L}}^2\alpha^2 + m_{\tilde{\nu}_R}^2\beta^2. \tag{31d}$$

Here, $|H_2|_{\text{ext}}$ is obtained by minimizing the right-hand side of Eq. (30) with respect to $|H_2|$ for fixed values of α and β,

$$|H_2|_{\text{ext}} = \frac{3\hat{A}}{4|Y_\nu|F(\alpha,\beta)\alpha\beta}\left(1 + \sqrt{1 - \frac{8\hat{m}^2(\alpha,\beta)F(\alpha,\beta)}{9\hat{A}^2}}\right). \tag{32}$$

The minimum is given by inserting Eq. (32) into Eq. (30),

$$V_{\text{CCB-1 min}}^{\text{Dirac}} = -\frac{1}{2}\alpha\beta|H_2|_{\text{ext}}^2\left(Y_\nu\hat{A}|H_2|_{\text{ext}} - \frac{\hat{m}^2(\alpha,\beta)}{\alpha\beta}\right). \tag{33}$$

The CCB-1 minimum would be much deeper than the EWSB minimum, (B9), because it is inversely proportional to $|Y_\nu|^2$. A necessary condition to avoid the minimum is that $V_{\text{CCB-1 min}}$ becomes positive, which reads

$$|A_\nu|^2 \le \frac{1 + 2\beta^2}{\beta^2}(m_{H_2}^2 + m_{\tilde{L}}^2 + m_{\tilde{\nu}_R}^2\beta^2), \tag{34}$$

where $\alpha^2 = 1$ is set to cancel the D term, according to property 3 of the CCB direction. We can further simplify the condition by minimizing the right-hand side of Eq. (34). Differentiating the right-hand side with respect to β^2, β^2 for extremal is obtained,

$$\beta_{\text{ext}}^4 = \frac{m_{H_2}^2 + m_{\tilde{L}}^2}{2m_{\tilde{\nu}_R}^2}, \tag{35}$$

and inserting Eq. (35), the condition becomes

$$|A_\nu| \le \sqrt{2(m_{H_2}^2 + m_{\tilde{L}}^2)} + m_{\tilde{\nu}_R}, \tag{36}$$

and the trilinear term is bounded from above. It is important to notice that condition (26) appears in the right-hand side. Therefore one can avoid both dangerous CCB minima once the constraint (36) is satisfied.

C. Constraint from CCB-2 minimum

Next, we analyze a direction similar to the MSSM CCB direction (b). We assume

$$H_1, \quad H_2, \quad \tilde{\nu}_L, \quad \tilde{\nu}_R \neq 0, \tag{37}$$

and other fields are zero. It is also assumed that $\text{sign}(A_\nu) = -\text{sign}(B)$. Note that neither color nor charge symmetry is broken along this direction. Instead VEV's of Higgses and sneutrinos are so large that weak gauge bosons are too heavy, and therefore EWSB does not occur correctly. As we mentioned in the Introduction, we call this direction the CCB direction. Then, we parametrize VEV's as

$$|\tilde{\nu}_L| = \alpha|H_2|, \qquad |\tilde{\nu}_R| = \beta|H_2|, \qquad |H_1| = \gamma|H_2|, \tag{38}$$

where α, β, and γ are real numbers. Then, the scalar potential is written

$$V_{\text{CCB-2}}^{\text{Dirac}} = Y_\nu^2 F(\alpha,\beta,\gamma)\alpha^2\beta^2|H_2|^4 - 2Y_\nu\hat{A}(\gamma)\alpha\beta|H_2|^3 + \hat{m}^2(\alpha,\beta,\gamma)|H_2|^2, \tag{39}$$

where

$$F(\alpha,\beta,\gamma) = 1 + \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\alpha^2\beta^2} f(\alpha,\gamma), \tag{40a}$$

$$f(\alpha,\gamma) = \frac{1}{8}\frac{g_1^2 + g_2^2}{|Y_\nu|^2}(\alpha^2 + \gamma^2 - 1)^2, \tag{40b}$$

$$\hat{A}(\gamma) = |A_\nu| + \gamma|\mu|, \tag{40c}$$

$$\hat{m}^2(\alpha,\beta,\gamma) = m_1^2\gamma^2 + m_2^2 + m_{\tilde{L}}^2\alpha^2 + m_{\tilde{\nu}_R}^2\beta^2 - 2|m_3^2|\gamma, \tag{40d}$$

and $\text{sign}(A_\nu) = -\text{sign}B$ is assumed. The constraint from the CCB-2 direction is obtained by iterating the same procedure of the CCB-1 or the MSSM UFB-2 direction,

$$|A_\nu| \le -|\mu|\gamma + \left(1 + \frac{2 - \gamma^2}{1 - \gamma^2}\beta_{\text{ext}}^2(\gamma)\right)m_{\tilde{\nu}_R}, \tag{41}$$

where $\beta_{\text{ext}}(\gamma)$ is

$$\beta_{\text{ext}}^4(\gamma) = \frac{1 - \gamma^2}{2 - \gamma^2}\frac{(m_1^2 - m_{\tilde{L}}^2)\gamma^2 - 2|m_3^2|\gamma + m_2^2 + m_{\tilde{L}}^2}{m_{\tilde{\nu}_R}^2}, \tag{42}$$

and $\alpha^2 = 1 - \gamma^2$ is used. It is seen that the constraint (20) is satisfied; hence the MSSM UFB-2 can be evaded if $\beta_{\text{ext}}^4(\gamma)$ is positive for any γ.

The stringent constraint on $|A_\nu|$ is given by minimizing the right-hand side of Eq. (41) with respect to γ, but it is not easy to obtain γ_{ext} analytically because of complications. Therefore we just give an equation that $\gamma_{\rm ext}$ must be satisfied,

$$-|\mu|\beta_{\rm ext}^2(\gamma_{\rm ext}) + m_{\tilde{\nu}_R}\left[\frac{\gamma_{\rm ext}}{(1-\gamma_{\rm ext}^2)^2}\beta_{\rm ext}^4(\gamma_{\rm ext}) + \frac{(m_1^2 - m_{\tilde{L}}^2)\gamma_{\rm ext} - |m_3^2|}{m_{\tilde{\nu}_R}^2}\right] = 0. \quad (43)$$

Equation (43) should be solved numerically. If $\gamma_{\rm ext}$ is negative, $\gamma_{\rm ext} = 0$ is chosen and the condition from the CCB-1 direction is obtained by replacing $m_{H_2}^2$ with m_2^2. If $\gamma_{\rm ext}$ is larger than unity, $\gamma_{\rm ext} = 1$ and $\alpha = 0$ are chosen. Then, the potential becomes

$$V_{\rm CCB\text{-}2}^{\rm Dirac} = Y_\nu^2\beta^2|H_2|^4 + \hat{m}^2(0,\beta,1)|H_2|^2, \quad (44)$$

where

$$\hat{m}^2(0,\beta,1) = m_1^2 + m_2^2 - 2|m_3^2| + m_{\tilde{\nu}_R}^2\beta \geq m_{\tilde{\nu}_R}^2 > 0, \quad (45)$$

and we used Eq. (B6). Thus, the potential has a global minimum at $H_1 = H_2 = \tilde{\nu}_L = \tilde{\nu}_R = 0$.

D. Constraint from CCB-3 minimum

The CCB-3 direction is defined as the CCB-2 with $\text{sign}(A_\nu) = \text{sign}(B)$. Along this direction, one of the signs among $|A_\nu|$, $|\mu|$, and $|m_3^2|$ is flipped according to property 5.

When the sign of $|A_\nu|$ or $|\mu|$ is flipped, the condition to avoid the CCB minimum is given as

$$|A_\nu| \leq |\mu|\gamma + \left(1 + \frac{2-\gamma^2}{1-\gamma^2}\beta_{\rm ext}^2(\gamma)\right)m_{\tilde{\nu}_R}, \quad (46)$$

where $\beta_{\rm ext}^4$ is the same as Eq. (42).

When the sign of $|m_3^2|$ is flipped, the constraint becomes

$$|A_\nu| \leq |\mu|\gamma + \left(1 + \frac{2-\gamma^2}{1-\gamma^2}\tilde{\beta}_{\rm ext}^2(\gamma)\right)m_{\tilde{\nu}_R}, \quad (47)$$

where

$$\tilde{\beta}_{\rm ext}^4(\gamma) = \frac{1-\gamma^2}{2-\gamma^2}\frac{(m_1^2 - m_{\tilde{L}}^2)\gamma^2 + 2|m_3^2|\gamma + m_2^2 + m_{\tilde{L}}^2}{m_{\tilde{\nu}_R}^2}. \quad (48)$$

The corresponding sign of $|\mu|$ or $|m_3^2|$ in Eq. (43) for $\gamma_{\rm ext}$ should also be flipped appropriately.

IV. CONSTRAINTS FROM UFB AND CCB MINIMA WITH MAJORANA NEUTRINOS

We consider the νSSM with Majorana neutrinos and analyze its potential given in Appendix C. Differences from the Dirac case are the Majorana mass term in the superpotential and the corresponding soft SUSY breaking mass. These additional terms result in linear and quadratic terms of the right-handed sneutrinos in the scalar potential.

It is immediately understood that the constraints from the MSSM UFB directions are the same as those of the Dirac case, (20) and (26), because the right-handed sneutrinos do not have VEV's. There appears a new UFB direction along

$$\tilde{\nu}_R \neq 0, \qquad \text{other fields} = 0, \quad (49)$$

and $\text{sign}(B_\nu M_R) = -1$. The potential along this direction is given as

$$V_{\rm UFB}^{\rm Majorana} = (m_{\tilde{\nu}_R}^2 - |B_\nu M_R| + |M_R|^2)|\tilde{\nu}_R|^2, \quad (50)$$

and it is unbounded from below unless

$$m_{\tilde{\nu}_R}^2 - |B_\nu M_R| + |M_R|^2 \geq 0. \quad (51)$$

Along the CCB directions, we simply show results because the procedure to find the conditions is the same as in the Dirac case. The conditions are obtained by making replacements,

$$|A_\nu| \to |A_\nu| + |M_R|, \quad (52)$$

$$m_{\tilde{\nu}_R} \to m_{\tilde{\nu}_R} + |B_\nu M_R| + |M_R|^2, \quad (53)$$

where $\text{sign}(B_\nu M_R) = 1$ is assumed. From the CCB-1 minimum, it is given from Eq. (36),

$$|A_\nu| \leq -|M_R| + \sqrt{2(M_{H_2}^2 + M_{\tilde{L}}^2)} + \sqrt{m_{\tilde{\nu}_R}^2 + |B_\nu M_R| + |M_R|^2}. \quad (54)$$

From the CCB-2 minimum, the condition is obtained from Eq. (41),

$$|A_\nu| \leq -(|\mu|\gamma_{\rm ext} + |M_R|) + \left(1 + \frac{2-\gamma^2}{1-\gamma^2}\beta_{\rm ext}^2(\gamma_{\rm ext})\right) \times \sqrt{m_{\tilde{\nu}_R}^2 + |B_\nu M_R| + |M_R|^2}, \quad (55)$$

where

$$\beta_{\rm ext}^4(\gamma) = \frac{1-\gamma^2}{2-\gamma^2}\frac{(m_1^2 - M_{\tilde{L}}^2)\gamma^2 - 2|m_3^2|\gamma + m_2^2 + M_{\tilde{L}}^2}{m_{\tilde{\nu}_R}^2 + |B_\nu M_R| + |M_R|^2}. \quad (56)$$

Here $\gamma_{\rm ext}$ is determined from

$$-|\mu|\beta_{\rm ext}^2(\gamma_{\rm ext}) + m_{\tilde{\nu}_R}\left[\frac{\gamma_{\rm ext}}{(1-\gamma_{\rm ext}^2)^2}\beta_{\rm ext}^4(\gamma_{\rm ext}) + \frac{(m_1^2 - m_{\tilde{L}}^2)\gamma_{\rm ext} - |m_3^2|}{m_{\tilde{\nu}_R}^2 + |B_\nu M_R| + |M_R|^2}\right] = 0. \quad (57)$$

Along the CCB-3 direction, the same replacement should be done. For the case of $\text{sign}(B_\nu M_R) = -1$, the sign of $|B_\nu M_R|$ is flipped.

V. NUMERICAL ANALYSIS

We show numerical results for the Dirac neutrino case to demonstrate a strategy to constrain the soft SUSY parameters with the conditions from UFB and CCB-1, -2, i.e. (20), (36), and (41). The conditions are important for relatively light sneutrinos, and therefore we vary masses of sneutrinos fixing the Higgs masses.

We calculate the Higgs soft masses using the SPS1a point [35] as an example. The parameters we use are

$$\mu = 3.57 \times 10^2, \qquad B = 47.2, \tag{58}$$

$$m_{H_1}^2 = 3.24 \times 10^4, \qquad m_{H_2}^2 = -1.28 \times 10^5, \tag{59}$$

in the unit of GeV, and $m_{\tilde{L}}$ is taken as 360 and 560 GeV so that $\beta_{\rm ext}^4$ along the CCB-1 direction is positive. It is assumed that $\mathrm{sign}(A_\nu) = -\mathrm{sign}(B)$. The EWSB occurs correctly and the lighter Higgs mass is above 114 GeV with these parameters.

We start by checking that Eq. (19) is positive between $0 \le \gamma \le 1$ for a given set of the parameters. Figure 1 shows Eq. (19) with respect to γ. Figure 1(a) is for $m_{\tilde{L}} = 360$ GeV and 1(b) is for $m_{\tilde{L}} = 560$ GeV. It is seen that $\hat{m}^2$ is positive in both cases, and hence Eq. (20) is satisfied.

Second, we calculate $\gamma_{\rm ext}$ using Eq. (43). Figure 2 shows the left-hand side of Eq. (43) normalized by $m_{\tilde{\nu}_R}$ in terms of γ. $m_{\tilde{\nu}_R}$ is varied from 100 to 500 GeV. The mass of the left-handed slepton for each curve is shown in the figures. $m_{\tilde{L}}$ is 360 GeV in Fig. 2(a) and 560 GeV in Fig. 2(b). The crossing point of each curve to zero corresponds to $\gamma_{\rm ext}$. It is seen that $\gamma_{\rm ext}$ is independent of $m_{\tilde{\nu}_R}$. This is because $m_{\tilde{\nu}_R}$ can be factored out by inserting the concrete form of $\beta_{\rm ext}$. From the figures, we can obtain $\gamma_{\rm ext} = 0.62$ and 0.73, respectively. It is also seen that $\gamma_{\rm ext}$ for $m_{\tilde{L}} = 560$ GeV is larger than that for 360 GeV. Generally $\gamma_{\rm ext}$ becomes larger as $m_{\tilde{L}}$ increases for fixed values of the other parameters, although the dependence of $\gamma_{\rm ext}$ on the other parameters is so complicated that it cannot be understood easily.

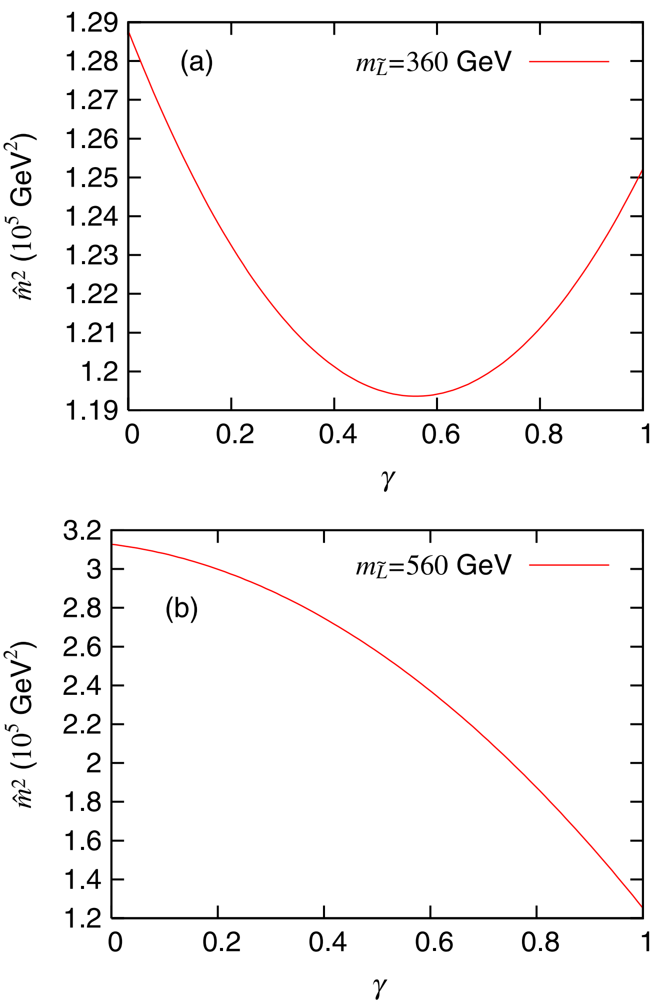

FIG. 1 (color online). $\hat{m}^2(\gamma)$ in terms of γ. $m_{\tilde{L}}$ is taken as 360 GeV in (a), and 560 GeV in (b), respectively.

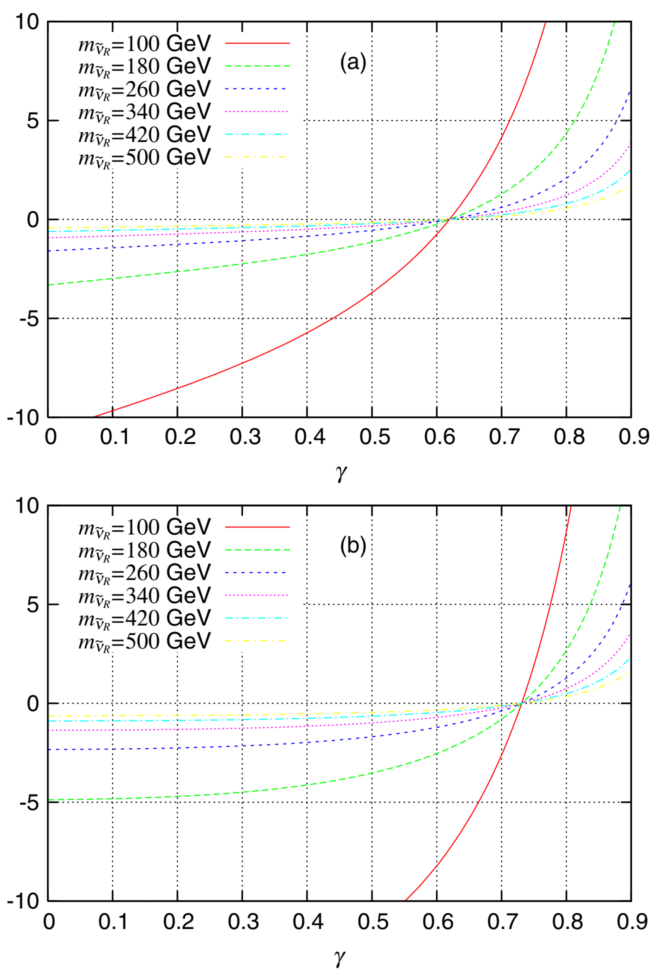

FIG. 2 (color online). The left-hand side of Eq. (43) normalized by $m_{\tilde{\nu}_R}$ in terms of γ for various $m_{\tilde{\nu}_R}$. The values of $m_{\tilde{\nu}_R}$ are shown in the figures. The left-handed slepton soft mass is 360 GeV in (a), and 560 GeV in (b), respectively.

Third, the constraints from CCB-1 and CCB-2 are calculated. In Fig. 3, we plot the constraints normalized with $m_{\tilde{\nu}_R}$ by varying the right-handed slepton mass from 100 to 1000 GeV. The mass of the left-handed slepton is taken as 360 GeV in Fig. 3(a), and 560 GeV in Fig. 3(b), respectively. The solid (red) curve represents Eq. (36) and the dashed (green) curve represents Eq. (41). It is seen from Fig. 3 that the constraint of CCB-1 is stronger than that of CCB-2 for $m_{\tilde{L}} = 360$ GeV, while the constraint of CCB-2 is stronger for 560 GeV. The dependence of Eq. (36) on $m_{\tilde{\nu}_R}$ is trivial, and that of Eq. (41) can be understood as follows. As we explained in Fig. 2, $\gamma_{\rm ext}$ becomes large as $m_{\tilde{L}}$ increases. Then, the right-hand side of Eq. (41) increases due to a factor of $1 - \gamma^2$ in the denominator. This result is nontrivial, and therefore we always have to check both constraints. The CCB-1 and CCB-2 constraint curves approach $|A_\nu|/m_{\tilde{\nu}_R} = 1$ as $m_{\tilde{\nu}_R}$ becomes large. In the large $m_{\tilde{\nu}_R}$ limit, the right-hand side of Eq. (36) is dominated by $m_{\tilde{\nu}_R}$, and $\beta_{\rm ext}$ goes to zero since $m_{\tilde{\nu}_R}$ appears in the denominator in Eq. (35).

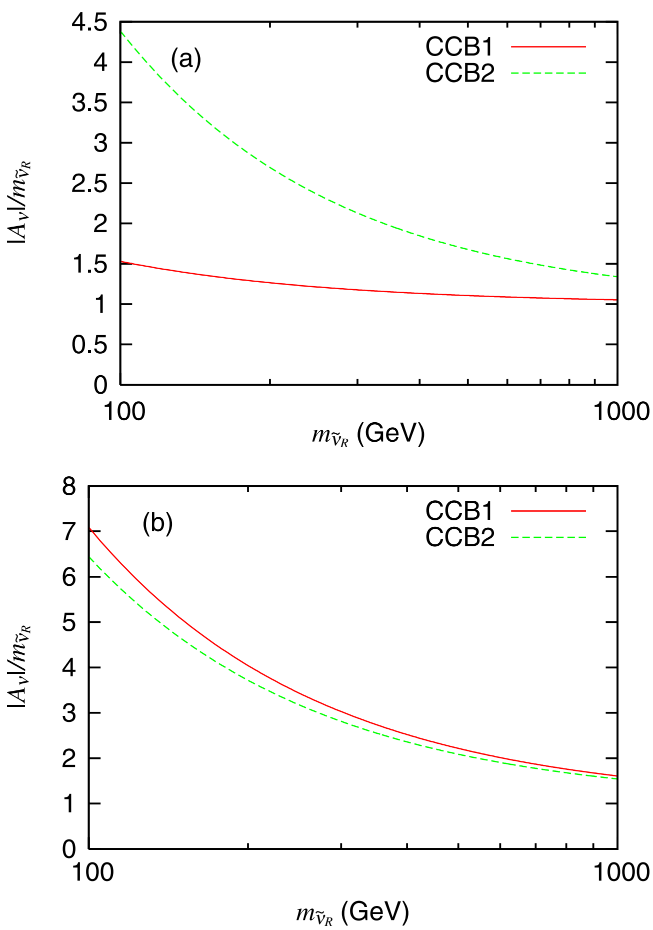

FIG. 3 (color online). The constraints from CCB minimum normalized with $m_{\tilde{\nu}_R}$ in terms of $m_{\tilde{\nu}_R}$. The left-handed slepton soft mass is 360 GeV in (a), and 560 GeV in (b), respectively. The solid (red) curve represents Eq. (36) and the dashed (green) curve represents Eq. (41).

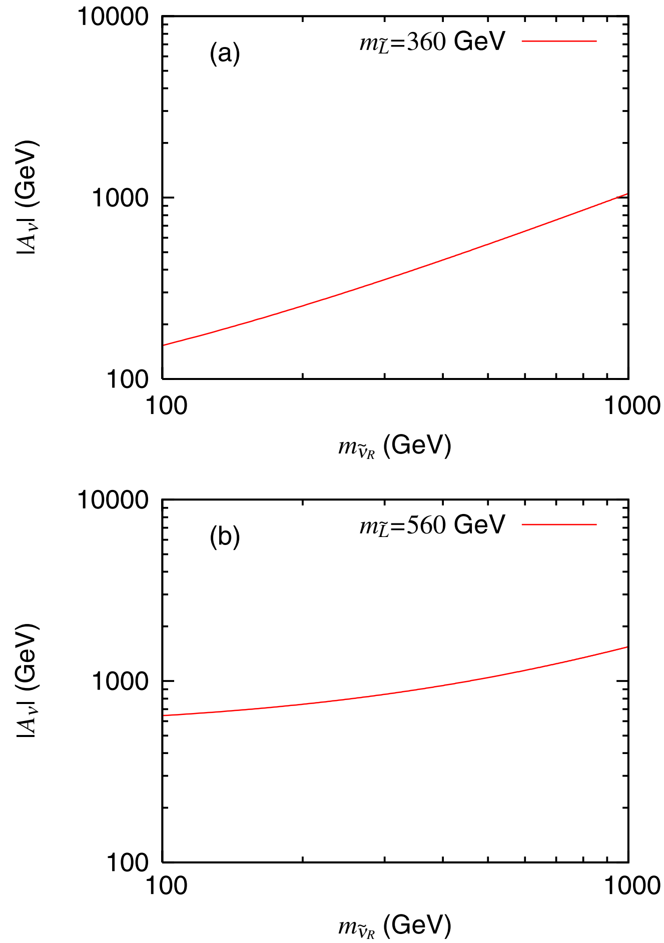

FIG. 4 (color online). The upper bound on A_ν in terms of $m_{\tilde{\nu}_R}$. The left-handed slepton soft mass is 360 GeV in (a), and 560 GeV in (b), respectively.

Figure 4 shows the upper bound on $|A_\nu|$ in terms of $m_{\tilde{\nu}_R}$. The value of the left-handed slepton mass is indicated in the figures. The upper bound is more strict as $m_{\tilde{\nu}_R}$ is smaller and $m_{\tilde{L}}$ is smaller. In our example, $|A_\nu|$ must be smaller than 153 GeV for $m_{\tilde{\nu}_R} = 100$ GeV and 1550 GeV for $m_{\tilde{\nu}_R} = 1000$ GeV in Fig. 4(a) and 640 and 2020 GeV in Fig. 4(b), respectively. From the numerical analysis, if the mass of the right-handed slepton is between several 100 GeV, the A_ν term must be smaller than 1 TeV.

VI. SUMMARY AND DISCUSSION

We have considered the νSSM where either Dirac or Majorana (s)neutrinos are introduced to the MSSM, and analyzed its scalar potential along the MSSM UFB/CCB directions as well as new CCB directions which appear due to nonvanishing VEV's of right-handed sneutrinos.

We have found that the MSSM UFB directions disappear and turn to CCB directions because the quartic term proportional to the square of the neutrino Yukawa coupling lifts up the potential for large values of fields. We have shown that the depth of the minima along these directions is inversely proportional to the square of the neutrino Yukawa coupling; therefore it would be much deeper than that of EWSB. We derived necessary conditions to avoid the CCB minima along the MSSM UFB directions. The conditions impose constraints among the soft SUSY breaking masses of the Higgses and the left-handed sneutrinos.

Then we have analyzed the potential along which the right-handed sneutrinos have nonvanishing VEV's. We showed that CCB and incorrect EWSB minima exist along these directions. The minima are inversely proportional to the square of the neutrino Yukawa coupling and hence very deep in the cases of the Dirac neutrinos and Majorana neutrinos in the TeV scale seesaw. Necessary conditions to evade these minima are derived for both Dirac and Majorana neutrinos. The conditions constrain the trilinear coupling of the sneutrino with respect to the soft masses. In the Majorana neutrino case, we found that one UFB direction appears due to the presence of the soft SUSY breaking mass terms of sneutrinos. A necessary condition to avoid the potential unbounded from below was also found.

In Sec. V, we have performed a numerical analysis of the conditions to demonstrate a strategy to avoid the UFB and CCB minima. The strategy is that for a given set of the parameters consistent with the EWSB, first we check the condition from the MSSM UFB directions, (20) and (26). Next, we calculate $\gamma_{\rm ext}$ using Eq. (43) for the CCB-2 direction. Finally we check the conditions from CCB minima, (36) and (41). In Fig. 3, we have shown that the condition (36) is more severe for $m_{\tilde{L}} = 360$ GeV and (41) is for $m_{\tilde{L}} = 560$ GeV. We have also shown in Fig. 4 that the trilinear coupling is strictly constrained for smaller sneutrino masses. In the case that the right-handed sneutrinos are the lightest SUSY particles, this constraint is important to calculate their lifetime.

The conditions we found in this article are necessary conditions but not sufficient conditions. With these conditions satisfied, one can avoid dangerous UFB and CCB directions when radiative corrections are small compared with tree-level potential. As we mentioned in the Introduction, it would be needed to include radiative corrections to obtain viable conditions at the electroweak scale. Since finite temperature effects would lift up the potential, it would also be important to consider finite temperature effects. We leave these for our future work.

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APPENDIX A: SCALAR POTENTIAL OF THE MSSM

In this Appendix, we give notations of the scalars and the full scalar potential of the MSSM. The down-type and the up-type Higgs scalars are denoted as

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix}, \qquad H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix}, \tag{A1}$$

where H_1^1 and H_2^2 are electrically neutral. The left-handed squarks and the right-handed squarks are denoted as

$$\tilde{Q} = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}, \qquad \tilde{u}_R, \quad \tilde{d}_R, \tag{A2}$$

and the left-handed sleptons and the right-handed sleptons are denoted as

$$\tilde{L} = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}, \quad \tilde{e}_R. \tag{A3}$$

The scalar potential is divided into three parts which consist of F terms, D terms, and soft SUSY breaking terms,

$$V = V_F + V_D + V_{\rm soft}. \tag{A4}$$

The F term potential, V_F, is given by a sum of absolute square of all matter auxiliary fields,

$$V_F = \sum_{i=\rm matter} |F_i|^2, \tag{A5}$$

where

$$F^*_{H_1^1} = \mu H_2^2 + Y_e \tilde{e}_L \tilde{e}_R^* + Y_d \tilde{d}_L \tilde{d}_R^*, \tag{A6a}$$

$$F^*_{H_1^2} = -\mu H_2^1 - Y_e \tilde{\nu}_L \tilde{e}_R^* - Y_d \tilde{u}_L \tilde{d}_R^*, \tag{A6b}$$

$$F^*_{H_2^1} = -\mu H_1^2 + Y_u \tilde{d}_L \tilde{u}_R^*, \tag{A6c}$$

$$F^*_{H_2^2} = \mu H_1^1 - Y_u \tilde{u}_L \tilde{u}_R^*, \tag{A6d}$$

$$F_{\tilde{e}_R} = Y_e (H_1^1 \tilde{e}_L - H_1^2 \tilde{\nu}_L), \tag{A6e}$$

$$F^*_{\tilde{e}_L} = Y_e H_1^1 \tilde{e}_R^*, \tag{A6f}$$

$$F^*_{\tilde{\nu}_L} = -Y_e H_1^2 \tilde{e}_R^*, \tag{A6g}$$

$$F_{\tilde{d}_R} = Y_d (H_1^1 \tilde{d}_L - H_1^2 \tilde{u}_L), \tag{A6h}$$

$$F^*_{\tilde{d}_L} = Y_d H_1^1 \tilde{d}_R^* + Y_u H_2^1 \tilde{u}_R^*, \tag{A6i}$$

$$F_{\tilde{u}_R} = Y_u (H_2^1 \tilde{d}_L - H_2^2 \tilde{u}_L), \tag{A6j}$$

$$F^*_{\tilde{u}_L} = -Y_d H_1^2 \tilde{d}_R^* - Y_u H_2^2 \tilde{u}_R^*. \tag{A6k}$$

Here μ is a supersymmetric Higgs mass and Y_i ($i = u$, d, and e) are Yukawa couplings.

The D term potential, V_D, is given by a sum of square of all gauge auxiliary fields,

$$V_D = \tfrac{1}{2}((D^a_{SU(3)})^2 + (D^a_{SU(2)})^2 + (D_{U(1)})^2), \tag{A7}$$

where a runs from 1 to 8 (3) for $SU(3)$ [$SU(2)$] and the summation should be understood. The auxiliary fields, $D^a_{SU(3)}$, $D^a_{SU(2)}$, and $D_{U(1)}$, are given by

$$D^a_{SU(3)} = g_3\left(\tilde{Q}^\dagger \frac{\lambda^a}{2}\tilde{Q} - \tilde{u}^*_R \frac{\lambda^a}{2}\tilde{u}_R - \tilde{d}^*_R \frac{\lambda^a}{2}\tilde{d}_R\right), \tag{A8a}$$

$$D^a_{SU(2)} = g_2(\tilde{Q}^\dagger T^a \tilde{Q} + \tilde{L}^\dagger T^a \tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2), \tag{A8b}$$

$$D_{U(1)} = g_1(\tfrac{1}{6}\tilde{Q}^\dagger \tilde{Q} - \tfrac{2}{3}\tilde{u}^*_R \tilde{u}_R + \tfrac{1}{3}\tilde{d}^*_R\tilde{d}_R - \tfrac{1}{2}\tilde{L}^\dagger \tilde{L} + \tilde{e}^*_R\tilde{e}_R - \tfrac{1}{2}H_1^\dagger H_1 + \tfrac{1}{2}H_2^\dagger H_2), \tag{A8c}$$

where g_i ($i = 1, 2, 3$) is a gauge coupling constant, and λ^a and T^a are Gell-Mann and Pauli matrices, respectively.

The soft SUSY breaking terms, $V_{\rm soft}$, are

$$\begin{aligned} V_{\rm soft} = {}& m^2_{H_1} H_1^\dagger H_1 + m^2_{H_2} H_2^\dagger H_2 + (B\mu H_1 \cdot H_2 + \text{H.c.}) \\ & + m^2_{\tilde{Q}}\tilde{Q}^\dagger\tilde{Q} + m^2_{\tilde{u}_R}\tilde{u}^*_R\tilde{u}_R + m^2_{\tilde{d}_R}\tilde{d}^*_R\tilde{d}_R + m^2_{\tilde{L}}\tilde{L}^\dagger\tilde{L} \\ & + m^2_{\tilde{e}_R}\tilde{e}^*_R\tilde{e}_R + (A_d Y_d H_1 \cdot \tilde{Q}\tilde{d}^*_R + A_u Y_u H_2 \cdot \tilde{Q}\tilde{u}^*_R \\ & + A_e Y_e H_1 \cdot \tilde{L}\tilde{e}^*_R + \text{H.c.}), \end{aligned} \tag{A9}$$

where m_i ($i = H_1$, H_2, Q, u, d, L, and E) are soft masses and $B\mu$ is a soft term for Higgses. A "dot" symbol represents an inner product for $SU(2)$ doublets, $A \cdot B = A^1B^2 - A^2B^1$. The trilinear terms, A_i ($i = u$, d, and e), are defined to be proportional to the corresponding Yukawa coupling.

APPENDIX B: ELECTROWEAK SYMMETRY BREAKING OF THE MSSM

We review the Higgs potential and the constraint from EWSB of the MSSM.

The Higgs potential of the MSSM is given by

$$\begin{aligned} V = {}& m_1^2 H_1^2 + m_2^2 H_2^2 - (m_3^2 H_1 H_2 + \text{H.c.}) \\ & + \tfrac{1}{8}(g_1^2 + g_2^2)(|H_1|^2 - |H_2|^2)^2, \end{aligned} \tag{B1}$$

where

$$m_1^2 = m^2_{H_1} + |\mu|^2, \tag{B2a}$$

$$m_2^2 = m^2_{H_2} + |\mu|^2, \tag{B2b}$$

$$m_3^2 = -B\mu. \tag{B2c}$$

A UFB direction is found along the D flat direction, namely,

$$|H_1|^2 = |H_2|^2, \tag{B3}$$

and the potential becomes

$$V = (m_1^2 + m_2^2 - 2|m_3^2|)|H_1|^2. \tag{B4}$$

The potential is unbounded from below if the quadratic term is negative. Thus the constraint from the UFB direction is given as

$$m_1^2 + m_2^2 - 2|m_3^2| \geq 0. \tag{B5}$$

This is the so-called UFB-1 condition in [31]. For the EWSB to occur correctly, the potential must be a saddle point at the origin. The condition for such a saddle point is

$$\left(\frac{\partial^2 V}{\partial|H_1|\partial|H_2|}\right)^2 - \frac{\partial^2 V}{\partial|H_1|^2}\frac{\partial^2 V}{\partial|H_2|^2} = |m_3^2|^2 - m_1^2 m_2^2 > 0. \tag{B6}$$

The EWSB vacuum is found by minimizing the potential, (B1), with respect to the Higgses under the conditions, Eqs. (B5) and (B6).

The EW symmetry is successfully broken at $|H_1| = v\cos\beta/\sqrt{2}$ and $|H_2| = v\sin\beta/\sqrt{2}$ if the following relations are satisfied:

$$m_1^2 + m_2^2 = -\frac{2m_3^2}{\sin 2\beta}, \tag{B7}$$

$$m_1^2 - m_2^2 = -\cos 2\beta(m_Z^2 + m_1^2 + m_2^2), \tag{B8}$$

where $m_Z^2 = \frac{1}{4}(g_1^2 + g_2^2)v^2$. The minimum of the potential is obtained using Eqs. (B7) and (B8),

$$V_{\rm real\ min} = -\frac{1}{2}\frac{m_Z^4}{g_1^2 + g_2^2}\cos^2 2\beta. \tag{B9}$$

APPENDIX C: SCALAR POTENTIAL OF THE νSSM

We list up here modifications of the MSSM scalar potential to the νSSM for Dirac neutrino and Majorana neutrino cases. The gauge auxiliary fields are the same as that of the MSSM because the right-handed (s)neutrinos are gauge singlets.

First, we list modifications in the Dirac neutrino case. Three of the matter auxiliary fields are replaced with

$$F^*_{H_2^1} = -\mu H_1^2 + Y_\nu \tilde{e}_L\tilde{\nu}^*_R + Y_u\tilde{d}_L\tilde{u}^*_R, \tag{C1a}$$

$$F^*_{H_2^2} = \mu H_1^1 - Y_\nu\tilde{\nu}_L\tilde{\nu}^*_R - Y_u\tilde{u}_L\tilde{u}^*_R, \tag{C1b}$$

$$F^*_{\tilde{\nu}_L} = -Y_e H_1^2\tilde{e}^*_R - Y_\nu H_2^2\tilde{\nu}^*_R, \tag{C1c}$$

$$F^*_{\tilde{e}_L} = Y_e H_1^1\tilde{e}^*_R + Y_\nu H_2^1\tilde{\nu}^*_R, \tag{C1d}$$

where $\tilde{\nu}_R$ is the right-handed sneutrinos and Y_ν is the Yukawa couplings of neutrinos. The auxiliary fields of the right-handed neutrinos are added,

$$F_{\tilde{\nu}_R} = Y_\nu(H_2^1\tilde{e}_L - H_2^2\tilde{\nu}_L). \tag{C2}$$

For the soft SUSY breaking term, a trilinear term A_ν and a soft mass $m_{\tilde{\nu}_R}$ for sneutrinos are added,

$$m^2_{\tilde{\nu}_R}\tilde{\nu}^*_R\tilde{\nu}_R + (A_\nu Y_\nu\tilde{L}\cdot H_2\tilde{\nu}^*_R + \text{H.c.}). \tag{C3}$$

Here $m_{\tilde{\nu}_R}$ is a soft SUSY breaking mass of the right-handed sneutrinos.

Next, we show two modifications in the Majorana neutrino case. One is on $F_{\tilde{\nu}_R}$ such as

$$F_{\tilde{\nu}_R} = Y_\nu (H_2^1 \tilde{e}_L - H_2^2 \tilde{\nu}_L) + M_R \tilde{\nu}_R^*, \tag{C4}$$

where M_R is the masses of the right-handed neutrinos. The other one is on the soft SUSY breaking term, that is, the following soft SUSY breaking mass is added,

$$\frac{1}{2} B_\nu M_R \tilde{\nu}_R^* \tilde{\nu}_R^* + \text{H.c.}, \tag{C5}$$

where $B_\nu M_R$ is a soft mass for the right-handed sneutrinos.

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