One-loop contribution to the neutrino mass matrix in the next-to-minimal supersymmetric standard model with right-handed neutrinos and tribimaximal mixing

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Neutrino mass patterns and mixing have been studied in the context of the next-to-minimal supersymmetric standard model (NMSSM) with three gauge singlet neutrino superfields. We consider the case with the assumption of R-parity conservation. The vacuum expectation value of the singlet scalar field S of NMSSM induces the Majorana masses for the right-handed neutrinos as well as the usual μ term. The contributions to the light neutrino mass matrix at the tree level as well as one-loop level are considered, consistent with the tribimaximal pattern of neutrino mixing. Light neutrino masses arise at the tree level through a TeV-scale seesaw mechanism involving the right-handed neutrinos. Although all the three light neutrinos acquire nonzero masses at the tree level, we show that the one-loop contributions can be comparable in size under certain conditions. Possible signatures to probe this model at the LHC and its distinguishing features compared to other models of neutrino mass generation are briefly discussed.

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I. INTRODUCTION

Several mechanisms of the generation of neutrino masses and mixing in the context of a supersymmetric model have been explored in various works. One of the most popular attempts in this direction is to relax the assumption of R-parity conservation in the minimal supersymmetric standard model (MSSM) by including explicit bilinear and/or trilinear R-parity violating interactions in the superpotential and the scalar potential [1,2]. One can also consider models with spontaneous R-parity violation [3–5] via a singlet sneutrino vacuum expectation value. The low energy limit of such models, where the singlet sneutrino field is decoupled, can be thought of as the bilinear R-parity violating scenario. Thus there are several possibilities within the context of R-parity violation in MSSM. In fact, each of them has been studied in detail in connection with the observed neutrino mass patterns and mixing as provided by the neutrino oscillation experiments. The possible collider signatures of R-parity violating models have also been studied in great detail and correlation between neutrino mixing angles and the decay branching ratios of the lightest supersymmetric particle (LSP) have been obtained [6–15].

Another interesting and well-studied procedure of small neutrino mass generation in a supersymmetric model, with the observed mixing pattern, is the seesaw mechanism [16,17] with the introduction of right-handed neutrino superfields [18–20]. In order to generate small neutrino masses, one introduces $\Delta L = 2$ heavy Majorana mass terms in the superpotential in addition to the trilinear lepton-number conserving Yukawa interactions involving the right-handed neutrino superfields. As long as the neutrino Yukawa couplings are of order one, light neutrino masses $\sim 10^{-2}$ eV require the Majorana masses to be $\sim 10^{15}$ GeV or so. However, such a high seesaw scale is difficult to probe at the LHC or future linear collider experiments. A viable alternative is to look at *TeV-scale* seesaw mechanism where small active neutrino masses are generated with the help of neutrino Yukawa couplings as small as 10^{-6} (same as the electron Yukawa coupling) and this makes the Majorana mass scale of the right-handed neutrino of the order of $\sim$TeV plausible. This gives one an opportunity to test the seesaw models at the LHC. The signatures of TeV-scale supersymmetric seesaw models will be briefly outlined later along with a discussion of the signatures of R-parity violating models.

On the other hand, MSSM is plagued by the so-called "μ problem" which asks the question why the scale of the supersymmetry preserving μ term should be of the same order as the soft supersymmetry breaking terms, which are of the order of TeV. One of the possible solutions to this problem is the next-to-minimal supersymmetric standard model (NMSSM), where a standard model singlet superfield ($\hat{S}$) is introduced to the MSSM superfields with a

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coupling $\lambda\hat{S}\hat{H}_u\hat{H}_d$ in the superpotential (for review and phenomenology, see [21,22]). The scalar component of $\hat{S}$ gets, in general, a nonzero vacuum expectation value (VEV) of the order of ~TeV, as long as the soft mass parameters corresponding to the singlet scalar field are in the same range. This solves the "μ problem" because the μ parameter generated in this way has the right order of magnitude if one considers a coupling $\lambda \sim \mathcal{O}(1)$. In order to generate active neutrino masses and appropriate mixing in the neutrino sector one either includes R-parity violation in the superpotential [23,24] and the scalar potential or introduces gauge-singlet neutrino superfields $\hat{N}_i$ with appropriate couplings with the MSSM superfields and the singlet superfield $\hat{S}$ [25]. In the latter case, the gauge-singlet neutrino superfields $\hat{N}_i$ can have Majorana masses around the TeV scale if there is a coupling of the type $\kappa\hat{N}_i^2\hat{S}$ in the superpotential. When the scalar component of $\hat{S}$ gets a VEV of the order of TeV scale, the right-handed neutrinos also acquire an effective Majorana mass around the TeV values as long as the dimensionless coupling κ is order one [25]. Here it is assumed that the superpotential has a discrete Z_3 symmetry which forbids the appearance of bilinear terms in the superpotential [26].

In this study, within the framework of this TeV-scale seesaw model mentioned above, we calculate the one-loop contributions to the neutrino mass matrix with R-parity conservation and study the effect of these contributions to the neutrino mass patterns and mixing angles. In other words, we consider the case where only the scalar field corresponding to the singlet superfield $\hat{S}$ gets a nonzero VEV along with the neutral Higgs fields. We will show later that these one-loop contributions can be significant and can change the region of parameter space allowed by the three-flavor global neutrino data in comparison to the tree-level results.

The plan of the paper is as follows. In Sec. II we will provide a discussion on the three-flavor neutrino mixing and illustrate the general pattern of the analysis that we are going to follow. Section III describes the model along with the minimization conditions of the neutral scalar potential. One-loop contributions to the neutrino mass matrix in the R-parity conserving scenario and the resulting neutrino mass patterns, which satisfy the three-flavor global neutrino data, are discussed in Sec. IV with numerical results. In Sec. V we outline the possible ways to probe this model at the LHC and present a short critical discussion of the signatures of neutrino mass models involving spontaneous and/or bilinear R-parity violation. We summarize in Sec. VI with possible future directions.

II. NEUTRINO MIXING

The solar, atmospheric, accelerator, and reactor neutrino experiments have shown strong evidence in favor of nonzero neutrino masses and mixing angles [27]. In addition, there is an upper bound on the sum of neutrino mass eigenvalues ~1 eV from cosmological observations [28]. The bound on the 11-element of the neutrino mass matrix resulting from the nonobservation of neutrinoless double beta decay is ≤ 0.3 eV [29]. The global three-flavor fits of various neutrino oscillation experiments point toward the following 3σ ranges of the neutrino oscillation parameters, namely, the two mass-squared differences and three mixing angles [30]:

$$\begin{aligned} &\sin^2\theta_{12} = 0.25 - 0.37, \qquad \sin^2\theta_{23} = 0.36 - 0.67, \\ &\sin^2\theta_{13} \leq 0.056 \qquad \Delta m_{21}^2 = (7.05 - 8.34) \times 10^{-5}\ \mathrm{eV}^2, \\ &|\Delta m_{31}^2| = (2.07 - 2.75) \times 10^{-3}\ \mathrm{eV}^2, \end{aligned} \tag{1}$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$. One can see from these numbers that there are two large mixing angles and one small mixing angle among the three light neutrinos with a mild hierarchy between the mass eigenvalues.

The three-flavor neutrino mixing matrix U can be parametrized as follows, provided that the charged lepton mass matrix is already in the diagonal form and the Dirac as well as Majorana phases are neglected:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}, \tag{2}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, and i, j run from 1 to 3.

The mixing angle data coming from solar, atmospheric and reactor sector indicate that $\theta_{12} \approx 34°$, $\theta_{23} \approx 45°$, and $\theta_{13} \leq 13°$. This is popularly known as the bilarge pattern of neutrino mixing. In order to understand the consequences of such mixing in the *zeroth* order, one considers the tribimaximal structure of the neutrino mixing [31] where $\theta_{23} = \frac{\pi}{4}$, $\theta_{13} = 0$, and $\sin\theta_{12} = \frac{1}{\sqrt{3}}$.

With this tribimaximal pattern, the unitary neutrino mixing matrix turns out to be

$$U_\nu = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{3}$$

Considering m_1, m_2, and m_3 as the three light neutrino mass eigenvalues, we use the matrix U_ν to obtain the neutrino Majorana mass matrix in the flavor basis as

$$m_\nu = U_\nu \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} U_\nu^T = \begin{pmatrix} \frac{1}{3}(2m_1 + m_2) & \frac{1}{3}(-m_1 + m_2) & \frac{1}{3}(m_1 - m_2) \\ \frac{1}{3}(-m_1 + m_2) & \frac{1}{6}(m_1 + 2m_2 + 3m_3) & \frac{1}{6}(-m_1 - 2m_2 + 3m_3) \\ \frac{1}{3}(m_1 - m_2) & \frac{1}{6}(-m_1 - 2m_2 + 3m_3) & \frac{1}{6}(m_1 + 2m_2 + 3m_3) \end{pmatrix}. \tag{4}$$

We can see that a particular structure of neutrino mass matrix emerges from the requirement of tribimaximal mixing, in terms of the neutrino mass eigenvalues. Given a specific model for generating the neutrino mass matrix, one can easily connect the model parameters with the neutrino mass eigenvalues with the help of Eq. (4). This way one can study the normal, inverted, or quasidegenerate mass pattern of the light neutrino mass eigenvalues and try to see the requirement on the model parameters to produce the tribimaximal pattern of neutrino mixing. In this work, we will try to explore the next-to-minimal supersymmetric standard model (NMSSM) where neutrino mass is generated because of the introduction of three right-handed neutrino superfields with the possible interaction terms. Though the assumption of tribimaximal mixing in the neutrino sector is not generic, in the present context it is quite illustrative in studying the role of the soft supersymmetry (SUSY) breaking parameters on the neutrino mass eigenvalues. At the same time, the acceptable domain of the soft parameters consistent with neutrino mass eigenvalues and tribimaximal mixing angles would hardly change with any small shift in θ_{13}.

As mentioned in the introduction, this model was proposed in Ref. [25] where the case with spontaneous violation of R-parity was studied with possible implications on neutrino mass eigenvalues and mixing angles at the tree level. In the present study we shall consider the case when R-parity is conserved and the neutrino mass generation at the tree level is entirely due to the seesaw mechanism involving the TeV-scale right-handed neutrinos. Our aim would be to see if this model can produce the acceptable neutrino mass eigenvalues and mixing angles when the neutrino mass matrix receives contributions at the tree as well as one-loop level. An attractive feature of this model is that the right-handed sneutrino in the form of LSP may become a valid cold dark matter candidate of the universe [32].

This model can also accommodate spontaneous CP and R-parity violation simultaneously. In that case, the neutrino sector is CP violating and the resulting effects on the neutrino masses and mixing angles were studied in Ref. [33]. Similarly, spontaneous R-parity violation motivated by a flavor symmetry may produce tribimaximal mixing pattern in the neutrino sector [34]. However, in the present context we consider the case where the neutrino sector conserves CP symmetry along with R-parity.

There have been some other studies which address the neutrino experimental data in some other extensions of NMSSM. One of these proposals is discussed in Ref. [23], where the effective bilinear R-parity breaking terms are generated through the vacuum expectation value of the scalar component of the singlet superfield $\hat{S}$. In this case, only one neutrino mass is generated at the tree level whereas the other two masses are generated at the one-loop level. In another model [24], nonzero masses for two neutrinos are generated at the tree level by including explicit bilinear R-parity violating terms along with the R-parity breaking term involving $\hat{S}$. It is interesting to note that the R-parity violating NMSSM model may offer a valid dark matter candidate in the form of a gravitino as the R-parity violating decay channels of the gravitino are extremely suppressed because of weak gravitational strength [35].

In another class of models, gauge-singlet neutrino superfields were introduced to solve the μ problem, which can simultaneously address the desired pattern of neutrino masses and mixing [36]. The detailed study of neutrino masses and mixing in this model was presented in Ref. [37] and the correlations of the lightest neutralino decays with neutrino mixing angles were discussed. Subsequently the dominant one-loop contributions toward the tree-level neutrino masses have also been presented [38]. Similar analyses for one and two generations of gauge-singlet neutrinos were presented in Ref. [39] and some other phenomenological implications, in particular the possible signatures at LHC, were addressed. Neutrino masses consistent with different hierarchical scenarios and tribimaximal neutrino mixing can also be generated in an R-parity violating supersymmetric theory with TeV-scale gauge singlet neutrino superfields, where the μ term was not generated by the vacuum expectation values of the singlet sneutrino fields [40]. Another interesting avenue in this direction is to study the role of possible higher dimensional supersymmetry breaking operators in the hidden sector which may render the TeV-scale soft SUSY breaking trilinear and bilinear couplings involving the sneutrinos to produce the observable mass and mixing angles for the neutrinos [41].

III. THE MODEL AND MINIMIZATION CONDITIONS

In this section we review the model along the lines of Ref. [25] and discuss its important characteristics. We introduce the singlet superfield $\hat{S}$ along with three right-handed neutrino superfields $\hat{N}_i$. The superfields $\hat{N}_i$ are odd and the superfield $\hat{S}$ is even under R-parity. The most general superpotential consistent with R-parity conservation is

$$W = W_{\mathrm{NMSSM}} + W_{\mathrm{Singlet}}, \tag{5}$$

where

$$W_{\mathrm{NMSSM}} = f_i^d(\hat{H}_d\hat{Q}_i)\hat{D}_i + f_{ij}^u(\hat{Q}_i\hat{H}_u)\hat{U}_j + f_i^e(\hat{H}_d\hat{L}_i)\hat{E}_i + \lambda_H(\hat{H}_d\hat{H}_u)\hat{S} + \frac{\lambda_s}{3!}\hat{S}^3, \tag{6}$$

$$W_{\mathrm{Singlet}} = f_{ij}^\nu(\hat{L}_i\hat{H}_u)\hat{N}_j + \frac{\lambda_{Ni}}{2}\hat{N}_i^2\hat{S}. \tag{7}$$

Here $\hat{H}_d$ and $\hat{H}_u$ are down-type and up-type Higgs superfields, respectively. The $\hat{Q}_i$ are doublet quark superfields, $\hat{U}_j[\hat{D}_j]$ are singlet up-type [down-type] quark superfields. The $\hat{L}_i$ are the doublet lepton superfields, and the $\hat{E}_j$ are the singlet charged lepton superfields. The indices $i, j = 1, 2, 3$ are generation indices. Note that we have imposed a Z_3 symmetry under which all the superfields have the same charge. This symmetry forbids the appearance of the usual bilinear μ term in the superpotential. The μ term is generated spontaneously through the vacuum expectation value of the singlet scalar $\hat{S}$. In a similar way soft supersymmetry breaking potential can be written as

$$V_{\mathrm{soft}} = V_{\mathrm{soft}}^{\mathrm{NMSSM}} + V_{\mathrm{Singlet}}, \tag{8}$$

where $V_{\mathrm{soft}}^{\mathrm{NMSSM}}$ includes the MSSM soft supersymmetry breaking terms along with a few additional terms as shown below:

$$V_{\mathrm{soft}}^{\mathrm{NMSSM}} = V_{\mathrm{soft}}^{\mathrm{MSSM}} + m_S^2|S|^2 + \left(A^H\lambda_H H_d H_u S + Am\frac{\lambda_s}{3!}S^3 + \mathrm{H.c}\right). \tag{9}$$

The term V_{Singlet} is composed of the soft masses and the trilinear interactions corresponding to the fields $\tilde{N}_i$:

$$V_{\mathrm{Singlet}} = m_{\tilde{N}\tilde{N}^*}^2|\tilde{N}_i|^2 + \left(A^\nu f_{ij}^\nu\tilde{L}_i H_u\tilde{N}_j + Am\frac{\lambda_{Ni}}{2}S\tilde{N}_i^2 + \mathrm{H.c}\right). \tag{10}$$

We have taken a common trilinear coupling A for the singlet fields N_i and S and m is a mass scale. In a supergravity motivated scenario, it is a common practice to choose $m = m_S = m_{\tilde{N}\tilde{N}^*}$ and also a universal trilinear parameter for the fields S, $\tilde{N}_i$. Since these fields are gauge singlets, we assume such universality to hold also at the electroweak scale. Similarly, the mass parameters m_S and $m_{\tilde{N}\tilde{N}^*}$ are very much insensitive to renormalization group equation (RGE) running and their values at the weak scale can be taken to be the same as the values at the high scale. In addition, we have chosen all the parameters f_i^d, f_i^e, λ_{Ni}, λ_H, λ_s, f_{ij}^u, and f_{ij}^ν to be real.

The scalar potential of this model can be written as

$$V = V_F + V_D + V_{\mathrm{soft}}, \tag{11}$$

where the neutral part of V_F and V_D can be written as

$$V_F^{\mathrm{neutral}} = \sum_i|f_{ij}^\nu H_u^0\tilde{N}_j|^2 + |\lambda_H H_u^0 S|^2 + |f_{ij}^\nu\tilde{\nu}_i\tilde{N}_j + \lambda_H H_d^0 S|^2 + \sum_j|f_{ij}^\nu(\tilde{\nu}_i H_u^0) + \lambda_{Nj}\tilde{N}_j S|^2 + \left|\lambda_H(H_d^0 H_u^0) + \frac{\lambda_{Ni}}{2}\tilde{N}_i^2 + \frac{\lambda_s}{2}S^2\right|^2, \tag{12}$$

$$V_D^{\mathrm{neutral}} = \frac{g_1^2 + g_2^2}{8}\left(|H_u^0|^2 - |H_d^0|^2 - \sum_i|\tilde{\nu}_i|^2\right)^2. \tag{13}$$

In the above, the repeated indices always mean to sum over the generations. However, the summation sign is used in special cases if required. The VEVs are determined by the minimization of the potential [see Eqs. (11)–(13)]. Here we explore the possibility when only the scalar component of the gauge singlet superfield $\hat{S}$ acquires a VEV along with the doublet Higgs fields. The right chiral sneutrino $\tilde{N}$ can only have a vanishing VEV and thus R-parity is unbroken. On the other hand, when the right-chiral sneutrino $\tilde{N}$ acquires a VEV then R-parity is spontaneously broken and an effective bilinear R-parity violating term of the form $\epsilon_i L_i H_u$ is generated, where $\epsilon_i \equiv f^\nu\langle\tilde{N}_i\rangle$. However, the case of spontaneous R-parity violation will be studied in a separate work [42]. Note that a global continuous symmetry such as lepton number cannot be assigned to the superpotential involving the singlets $\hat{S}$ and $\hat{N}_i$. Thus this model is completely free from the unwanted Nambu-Goldstone boson even if the singlet scalar S and/or $\tilde{N}_i$ acquire VEV. For more details the reader is referred to Refs. [25,43].

Minimization of the scalar potential [see Eq. (11)] leads to the following conditions:

$$
\begin{aligned}
\frac{\partial V}{\partial v_d} &= 2v_d\left(m_{H_d}^2 + \lambda_H^2(v_u^2 + v_s^2) + \frac{g_1^2 + g_2^2}{4}\left(v_d^2 - v_u^2 + \sum_i v_{\tilde{\nu}_i}^2\right) + \tan\beta\left(\frac{1}{2}\lambda_H\lambda_s v_s^2 + \frac{1}{2}\lambda_H\lambda_{N_i}v_{\tilde{N}_i}^2 + A_H\lambda_H v_s\right)\right) \\
&\quad + 2\lambda_H f_\nu^{ij} v_s v_{\tilde{\nu}_i} v_{\tilde{N}_j}, \\
\frac{\partial V}{\partial v_u} &= 2v_u\left(m_{H_u}^2 + \lambda_H^2(v_d^2 + v_s^2) - \frac{g_1^2 + g_2^2}{4}\left(v_d^2 - v_u^2 + \sum_i v_{\tilde{\nu}_i}^2\right) + f_\nu^{ij} f_\nu^{ik} v_{\tilde{N}_j} v_{\tilde{N}_k} + f_\nu^{ji} f_\nu^{ki} v_{\tilde{\nu}_j} v_{\tilde{\nu}_k}\right. \\
&\quad \left. + \cot\beta\left(\frac{1}{2}\lambda_H\lambda_s v_s^2 + \frac{1}{2}\lambda_H\lambda_{N_i}v_{\tilde{N}_i}^2 + A_H\lambda_H v_s\right)\right) + 2A_\nu f_\nu^{ij} v_{\tilde{\nu}_i} v_{\tilde{N}_j} + 2f_\nu^{ji}\lambda_{N_i} v_s v_{\tilde{\nu}_j} v_{\tilde{N}_i}, \\
\frac{\partial V}{\partial v_s} &= 2v_s\left(m_S^2 + \lambda_H^2(v_d^2 + v_u^2) + \lambda_s\lambda_H v_d v_u + \lambda_{N_i}^2 v_{\tilde{N}_i}^2 + \frac{1}{2}Am\lambda_s v_s + \frac{1}{2}\lambda_s^2 v_s^2 + \frac{1}{2}\lambda_s\lambda_{N_i}v_{\tilde{N}_i}^2\right) + 2A_H\lambda_H v_d v_u \\
&\quad + Am\lambda_{N_i}v_{\tilde{N}_i}^2 + 2f_\nu^{ij} v_{\tilde{\nu}_i} v_{\tilde{N}_j}(\lambda_H v_d + \lambda_{N_j} v_u), \\
\frac{\partial V}{\partial v_{\tilde{\nu}_i}} &= 2v_{\tilde{\nu}_i}\left(\tilde{m}_i^2 + \frac{g_1^2 + g_2^2}{4}\left(v_d^2 - v_u^2 + \sum_j v_{\tilde{\nu}_j}^2\right)\right) + 2A_\nu f_\nu^{ij} v_u v_{\tilde{N}_j} + 2\lambda_H f_\nu^{ij} v_d v_s v_{\tilde{N}_j} + 2f_\nu^{ik} f_\nu^{jk} v_u^2 v_{\tilde{\nu}_j} + 2f_\nu^{ij}\lambda_{N_j} v_u v_s v_{\tilde{N}_j} \\
&\quad + 2f_\nu^{ij} f_\nu^{kl} v_{\tilde{N}_j} v_{\tilde{\nu}_k} v_{\tilde{N}_l}, \\
\frac{\partial V}{\partial v_{\tilde{N}_i}} &= 2v_{\tilde{N}_i}\left(m_{\tilde{N}\tilde{N}^*}^2 + Am\lambda_{N_i} v_s + \lambda_{N_i}\lambda_H v_d v_u + \frac{1}{2}\lambda_{N_i}\lambda_s v_s^2 + \lambda_{N_i}^2 v_s^2 + \frac{1}{2}\lambda_{N_i}\lambda_{N_j} v_{\tilde{N}_j}^2\right) + 2f_\nu^{ji} f_\nu^{jk} v_u^2 v_{\tilde{N}_k} + 2A_\nu f_\nu^{ji} v_u v_{\tilde{\nu}_j} \\
&\quad + 2\lambda_H f_\nu^{ji} v_d v_s v_{\tilde{\nu}_j} + 2\lambda_{N_i} f_\nu^{ji} v_u v_s v_{\tilde{\nu}_j} + 2f_\nu^{ji} f_\nu^{kl} v_{\tilde{\nu}_j} v_{\tilde{N}_l} v_{\tilde{\nu}_k}.
\end{aligned}
\tag{14}
$$

Here g_1 and g_2 are the $U(1)$ and $SU(2)$ gauge couplings, respectively, and $\tan\beta = v_u/v_d$. $\tilde{m}_i$ is the soft SUSY breaking mass parameter of the left-chiral sneutrinos. We have assumed that the neutral scalar fields can develop, in general, the following vacuum expectation values:

$$
\begin{aligned}
v_d = \langle H_d^0\rangle; \qquad v_u = \langle H_u^0\rangle; \qquad v_s = \langle S\rangle; \\
v_{\tilde{\nu}_i} = \langle\tilde{\nu}_i\rangle; \qquad v_{\tilde{N}_i} = \langle\tilde{N}_i\rangle.
\end{aligned}
\tag{15}
$$

As has already been mentioned, in the present context we will consider the solutions $v_{\tilde{N}_i} = v_{\tilde{\nu}_i} = 0$ and $v_s \neq 0$ to analyze the neutrino spectra. In our subsequent discussion, we will also ignore the terms in the minimization equations which are bilinear in the neutrino Yukawa couplings. Note that in order to generate very small masses for the active neutrinos ($\lesssim 0.1$ eV) using this TeV-scale seesaw mechanism, the neutrino Yukawa couplings (f^ν) should be below $\mathcal{O}(10^{-6})$, which is around the magnitude of the electron Yukawa coupling.

The VEV v_s comes out as the solution of the following cubic equation (neglecting the Yukawa term):

$$
\begin{aligned}
&\lambda_s^2 v_s^3 + Am\lambda_s v_s^2 + 2v_s(m_s^2 + \lambda_H^2 v_u^2 + \lambda_H^2 v_d^2 + \lambda_H\lambda_s v_d v_u \\
&\quad + \lambda_{N_i}^2 v_{\tilde{N}_i}^2 + \lambda_s\lambda_{N_i} v_{\tilde{N}_i}^2) + 2A_H\lambda_H v_d v_u \\
&\quad + Am\lambda_{N_i} v_{\tilde{N}_i}^2 = 0.
\end{aligned}
\tag{16}
$$

The solutions of the foregoing equation involve soft parameters Am, A_H and m_s^2. In fact these parameters cannot be much away from TeV values to have $v_s \sim$ TeV. In particular, the soft parameters A_H and Am are crucial to produce nonzero VEV for the field S. Any consistent solution that yields $v_s \neq 0$ but $v_{\tilde{N}_i} = 0$ requires $|A| \geq 3$ and also $\lambda_H \leq 1$, $m \geq 100$ GeV, $m_S \geq 100$ GeV [25]. Similarly we also choose the couplings λ_s, λ_{N_i} in such a manner so that the condition for global minima is always satisfied.

IV. NEUTRINO MASSES AND MIXING: *R*-PARITY CONSERVING NMSSM

Let us now discuss in detail the generation of neutrino masses and mixing in this model. Note that this model is different from the models where MSSM is extended with three right-handed singlet neutrino superfields. This is because in those models the right-handed neutrino mass scale is not tied up with the electroweak symmetry breaking scale and is assumed to be very high ($\sim 10^{15}$ GeV or so).

A. Seesaw masses

At the tree level, the (3×3) light neutrino mass matrix, that arises via the seesaw mechanism, has a very well-known structure given by

$$
m_\nu^{\rm tree} = -m_D M_R^{-1} m_D^T,
\tag{17}
$$

where m_D represents the lepton number conserving (3×3) "Dirac" mass matrix and M_R represents the lepton number violating (3×3) "Majorana" mass matrix. Note that, after the electroweak symmetry breaking (EWSB), when the scalar component of $\hat{S}$ gets a VEV, in the effective Lagrangian we can assign a lepton number -1 for the fields N_i^c and $\tilde{N}_i$ (contained in the superfield $\hat{N}_i$). The relevant part of the effective Lagrangian which encompasses both neutrino and sneutrino fields is given by

$$-\mathcal{L}_{\rm eff} = \frac{1}{2}(\lambda_{Ni}v_s)N_i^cN_i^c + f_{ij}^\nu \nu_i v_u N_j^c + \text{H.c.} + m_{\tilde{\nu}_i}^2 \tilde{\nu}_i\tilde{\nu}_i^\star + (m_{\tilde{N}_i\tilde{N}_i^*}^2 + \lambda_{N_i}^2 v_s^2)\tilde{N}_i\tilde{N}_i^\star + (B_{ij}^\nu \tilde{\nu}_i\tilde{N}_j + B_{ij}^{\prime\nu}\tilde{\nu}_i\tilde{N}_j^\star + B_{Ri}\tilde{N}_i\tilde{N}_i + \text{H.c.}), \tag{18}$$

where the coefficients have the following meaning:

$$\begin{aligned} m_{\tilde{\nu}_i}^2 &= \tilde{m}_i^2 + \frac{1}{2}m_Z^2\cos 2\beta,\\ B_{ij}^\nu &= A^\nu f_{ij}^\nu v_u + \lambda_H f_{ij}^\nu v_d v_s,\\ B_{ij}^{\prime\nu} &= f_{ij}^\nu \lambda_{Nj} v_u v_s,\\ B_{Ri} &= \frac{1}{2}\left(\lambda_H\lambda_{N_i}v_dv_u + \frac{\lambda_s\lambda_{N_i}v_s^2}{2} + Am\lambda_{Ni}v_s\right). \end{aligned} \tag{19}$$

It is easy to see from Eq. (18) that $m_{Dij} \equiv f_{ij}^\nu v_u$ and $M_{Ri} = \lambda_{Ni}v_s$, which in turn provide neutrino masses at the tree level through Eq. (17). Note that in Eq. (19) we have neglected a term $\sim m_D^2$ in the expression for $m_{\tilde{\nu}_i}^2$ since it is much smaller compared to the other terms.

The tree-level neutrino masses may receive dominant radiative corrections at the one-loop level. It has been shown in models of MSSM with right-handed neutrino superfields that the loop contributions can be as large as the tree-level value, though the result depends on the soft SUSY breaking parameters [18,20]. In R-parity conserving scenarios the leading contributions to neutrino masses at the one-loop level arise from $\Delta L = 2$ terms in the sneutrino sector. These bilinear interaction terms involving the heavy right-handed sneutrinos fields $\tilde{N}_i$ are $B_{ij}^{\prime\nu}\tilde{\nu}_i\tilde{N}_j^\star$, and $B_{Ri}\tilde{N}_i\tilde{N}_i$ as can be seen from Eq. (18). In association with the $\Delta L = 0$ term, i.e., $B_{ij}^\nu\tilde{\nu}_i\tilde{N}_j$ these $\Delta L = 2$ terms generate lepton number violating *Majorana-like* mass terms ($m_{\tilde{\nu}\tilde{\nu}}^2\tilde{\nu}\,\tilde{\nu}$ +H.c.) for the left-handed sneutrinos. In fact, this can be seen as a scalar seesaw analogue of the usual fermionic seesaw mechanism to generate small masses for the light active neutrinos [20]. This effective Majorana sneutrino mass term in turn induces one-loop radiative corrections to neutrino Majorana masses via the self-energy diagram as shown in Fig. 1. However, rather than computing the one-loop contribution to neutrino masses using the above method, we would choose a different but more general procedure as explained below.

We begin by decomposing the sneutrino fields in terms of real and imaginary components. Thus one has

$$\tilde{\nu}_i = \frac{\tilde{\nu}_{iR} + i\tilde{\nu}_{iI}}{\sqrt{2}}, \qquad \tilde{N}_i = \frac{\tilde{N}_{iR} + i\tilde{N}_{iI}}{\sqrt{2}}, \tag{20}$$

where the components $\tilde{\nu}_{iR}$, $\tilde{N}_{iR}$ are the CP-even and $\tilde{\nu}_{iI}$, $\tilde{N}_{iI}$ are the CP-odd scalar fields. The mass terms of these scalars may be evaluated using the definition

$$M_{R,ij}^2 = \frac{\partial^2 V}{\partial\Phi_{iR}\partial\Phi_{jR}}, \qquad M_{p,ij}^2 = \frac{\partial^2 V}{\partial\Phi_{iI}\partial\Phi_{jI}}, \tag{21}$$

where Φ represents a generic scalar field. Accordingly one obtains the following diagonal mass terms (assuming the right-chiral sneutrino states to be flavor diagonal) for the CP-even and CP-odd right-chiral sneutrinos:

$$\begin{aligned} M_{R,\tilde{N}_i\tilde{N}_i}^2 &= m_{\tilde{N}_i\tilde{N}_i^\star}^2 + \lambda_{Ni}^2 v_s^2 + \left(\lambda_H\lambda_{Ni}v_dv_u + \frac{1}{2}\lambda_{Ni}\lambda_s v_s^2 + Am\lambda_{Ni}v_s\right)\\ M_{P,\tilde{N}_i\tilde{N}_i}^2 &= m_{\tilde{N}_i\tilde{N}_i^\star}^2 + \lambda_{Ni}^2 v_s^2 - \left(\lambda_H\lambda_{Ni}v_dv_u + \frac{1}{2}\lambda_{Ni}\lambda_s v_s^2 + Am\lambda_{Ni}v_s\right). \end{aligned} \tag{22}$$

Similarly, the interactions between $\tilde{N}_{iR,(iI)}$ and $\tilde{\nu}_{iR,(iI)}$ read as

$$\begin{aligned} C_{R,\tilde{\nu}_i\tilde{N}_j} &= f_{ij}^\nu\lambda_H v_d v_s + f_{ij}^\nu\lambda_{Nj}v_uv_s + A^\nu f_{ij}^\nu v_u,\\ C_{P,\tilde{\nu}_i\tilde{N}_j} &= -f_{ij}^\nu\lambda_H v_d v_s + f_{ij}^\nu\lambda_{Nj}v_uv_s - A^\nu f_{ij}^\nu v_u. \end{aligned} \tag{23}$$

The diagonal left-chiral sneutrino mass terms are shown in Eq. (19). As we can see, the off-diagonal terms involving the left-chiral and right-chiral sneutrinos are much smaller compared to the diagonal terms since they are proportional to the small neutrino Yukawa couplings ($f^\nu \sim 10^{-6}$). Hence, we can compute the one-loop correction to the neutrino mass due to the small mixing of the right-chiral sneutrinos with the left-chiral sneutrinos. This is shown in Fig. 2. Note that the right-chiral sneutrino mass matrix

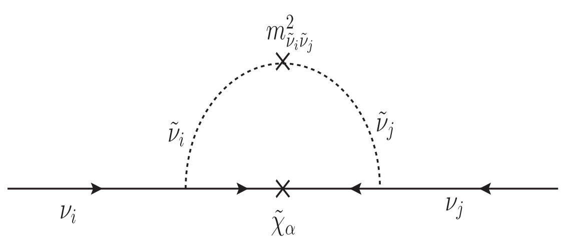

FIG. 1. One-loop contribution to m_ν when $v_{\tilde{N}_i} = v_{\tilde{\nu}_i} = 0$. Here $m_{\tilde{\nu}_i\tilde{\nu}_j}^2$ represents the sneutrino *Majorana* mass term which generates the neutrino mass involving the sneutrino-neutralino loop.

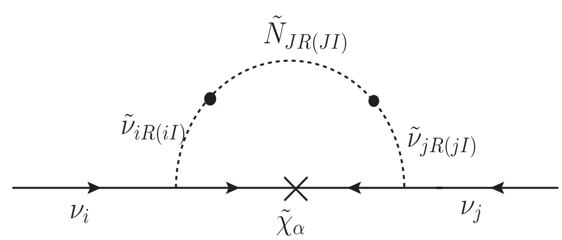

FIG. 2. The same one-loop contribution to m_ν as in Fig. 1 but represented in a different way. Here $\tilde{N}_{JR}$ and $\tilde{N}_{JI}$ are right-handed sneutrino mass eigenstates which couple to $\tilde{\nu}_{i,j}$ to produce the one-loop effective neutrino mass term.

contains bilinear terms like $\lambda_H \lambda_{N_i} v_d v_u$, $\frac{\lambda_s \lambda_{N_i} v_s^2}{2}$ which are originated from the F term contribution in the scalar potential. These are the new contributions to the right sneutrino masses in the present model and thus they are absent in seesaw models of MSSM with only right-handed neutrino superfields. These terms will have important roles to play while calculating the one-loop correction to the neutrino mass matrix, even when the relevant soft breaking trilinear parameters are smaller. The loop contribution can be written as

$$(m_\nu)_{ij}^{\text{loop}} = \frac{g_2^2}{4} \sum_\alpha m_{\tilde{\chi}_\alpha} (N_{\alpha 5} - \tan\theta_w N_{\alpha 4})^2 \times \Big[\sum_{J=1,2,3} C_{R_{iJ}} C_{R_{jJ}} I_4(m_{\tilde{\nu}_{iR}}, m_{\tilde{\nu}_{jR}} m_{\tilde{\chi}_\alpha}, M_{\tilde{N}_{JR}}) - \sum_{J=1,2,3} C_{P_{iJ}} C_{P_{jJ}} I_4(m_{\tilde{\nu}_{iI}}, m_{\tilde{\nu}_{jI}}, m_{\tilde{\chi}_\alpha}, M_{\tilde{N}_{JI}}) \Big], \tag{24}$$

where the integral I_4 is given by

$$I_4(m_{\tilde{\nu}_{iR}}, m_{\tilde{\nu}_{jR}}, m_{\tilde{\chi}_\alpha}, m_{X_J}) = \int \frac{d^4 q}{i(2\pi)^4} \frac{1}{(q^2 - m_{\tilde{\nu}_{iR}}^2)(q^2 - m_{\tilde{\nu}_{jR}}^2)(q^2 - m_{\tilde{\chi}_\alpha}^2)(q^2 - m_{X_J}^2)}. \tag{25}$$

Here X_J denotes right-chiral sneutrino states $\tilde{N}_{JR}$ or $\tilde{N}_{JI}$. One can always evaluate I_4 with the following analytical expressions:

$$\begin{aligned} I_4(m_1, m_2, m_3, m_4) &= \frac{1}{m_3^2 - m_4^2}[I_3(m_1, m_2, m_4) - I_3(m_1, m_2, m_3)], \\ I_3(m_1, m_2, m_3) &= \frac{1}{m_2^2 - m_3^2}[I_2(m_1, m_2) - I_2(m_1, m_3)], \\ I_2(m_1, m_2) &= \frac{1}{(4\pi)^2} \frac{m_2^2}{m_1^2 - m_2^2} \log\frac{m_1^2}{m_2^2}. \end{aligned} \tag{26}$$

Here, $m_{\tilde{\chi}_\alpha}$ represents the eigenvalues of the NMSSM neutralino mass matrix. In the weak interaction basis $(\tilde{S}, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{B}^0, \tilde{W}^0)$, the mass matrix can be written as

$$\mathcal{M} = \begin{pmatrix} \lambda_S v_s & \lambda_H v_u & \lambda_H v_d & 0 & 0 \\ \lambda_H v_u & 0 & \lambda_H v_s & -g_1 v_d/\sqrt{2} & g_2 v_d/\sqrt{2} \\ \lambda_H v_d & \lambda_H v_s & 0 & g_1 v_u/\sqrt{2} & -g_2 v_u/\sqrt{2} \\ 0 & -g_1 v_d/\sqrt{2} & g_1 v_u/\sqrt{2} & M_1 & 0 \\ 0 & g_2 v_d/\sqrt{2} & -g_2 v_u/\sqrt{2} & 0 & M_2 \end{pmatrix}. \tag{27}$$

The mixing matrix elements $N_{\alpha 5}$ and $N_{\alpha 4}$ are the wino and bino component of the neutralino $\tilde{\chi}_\alpha$. The expression [see Eq. (24)] is the most general to compute the one-loop diagram (see Fig. 2). Nevertheless, we would consider a simplified scenario for illustration. In particular, we assume (i) identical values of λ_{Ni} ($\lambda_{Ni} \equiv \lambda_N$) for all three generations and (ii) soft-masses of the sneutrinos (both $\tilde{\nu}_i$ and $\tilde{N}_i$) are flavor blind. This results in identical mass values for all three CP-even right-chiral sneutrinos ($M_{\tilde{N}_{RJ}} \equiv M_{\tilde{N}_R}$) and also for the three CP-odd states ($M_{\tilde{N}_{IJ}} \equiv M_{\tilde{N}_I}$). With these assumptions, it is possible to factor out the flavor structure from Eq. (24) and denote the remaining as the loop factor (LF) which is merely a constant. Then the loop contribution can be cast into a convenient form given by

$$(m_\nu^{\text{loop}})_{ij} = (LF) \sum_{k=1}^{3} f_{iJ}^\nu f_{jJ}^\nu, \tag{28}$$

where

$$LF = \frac{g_2^2}{4} \sum_\alpha m_{\tilde{\chi}_\alpha} (N_{\alpha 5} - \tan\theta_w N_{\alpha 4})^2 \times (I_4(m_{\tilde{\nu}}, m_{\tilde{\nu}}, m_{\tilde{\chi}_\alpha}, M_{\tilde{N}_R}) C_R^2 - I_4(m_{\tilde{\nu}}, m_{\tilde{\nu}}, m_{\tilde{\chi}_\alpha}, M_{\tilde{N}_I}) C_P^2). \tag{29}$$

Here C_R and C_P represent the coefficients of f_{ij}^ν in Eq. (23) and are given as

$$C_R = \lambda_H v_d v_s + \lambda_N v_u v_s + A^\nu v_u, \tag{30}$$

$$C_P = -\lambda_H v_d v_s + \lambda_N v_u v_s - A^\nu v_u. \tag{31}$$

Let us note that the coefficient B_{Ri} can be written as

$$B_{Ri} = B_N M_R, \tag{32}$$

where

$$B_N = \frac{1}{2}\Big(\lambda_H v_d v_u / v_s + \frac{\lambda_s v_s}{2} + Am\Big), \qquad M_R = \lambda_N v_s. \tag{33}$$

Consequently the one-loop contribution can be cast into the well-known form [18,20]

$$m_{\nu ij}^{(\mathrm{loop})} = -\frac{g_2^2 \Delta m_{\tilde{\nu} ij}}{32\pi^2 \cos^2\theta_W} \sum_\alpha f(y_\alpha)|N_{\alpha k}|^2, \qquad f(y_\alpha) = \frac{\sqrt{y_\alpha}[y_\alpha - 1 - \ln(y_\alpha)]}{(1-y_\alpha)^2}, \tag{34}$$

where $y_\alpha \equiv m_{\tilde{\nu}}^2/m_{\tilde{\chi}_\alpha^0}^2$ and $N_{\alpha k} \equiv N_{\alpha 5}\cos\theta_W - N_{\alpha 4}\sin\theta_W$ is the neutralino mixing matrix element and to order in $1/M_R^3$ the left sneutrino mass difference relative to the light neutrino mass is given by

$$\frac{\Delta m_{\tilde{\nu} ij}}{m_{\nu ij}} \simeq \frac{2(A_\nu + \mu\cot\beta - B_N - \frac{B_N(A_\nu + \mu\cot\beta)^2}{M_R^2})}{m_{\tilde{\nu}}}. \tag{35}$$

Here we have used the relation $\Delta m_{\tilde{\nu}}^2 = 2m_{\tilde{\nu}}\Delta m_{\tilde{\nu}}$ and $m_{\tilde{\nu}}$ is an average left-sneutrino mass. In the present case all left-handed sneutrino soft masses are assumed to be identical. The sneutrino Majorana mass $m_{\tilde{\nu}\tilde{\nu}}^2$ shown in Fig. 1 is related to $\Delta m_{\tilde{\nu}}^2$ as $m_{\tilde{\nu}\tilde{\nu}}^2 = \frac{1}{4}\Delta m_{\tilde{\nu}}^2$ [20]. The quantity μ is defined as $\mu = \lambda_H v_s$.

In order to reproduce the result in Eq. (34), we assumed that B_N, $m_{\tilde{N}\tilde{N}*} < M_R$ and $A_\nu > B_N$. Now, in addition, if we assume $M_R > A_\nu$, the last term becomes negligible compared to the other terms in the expression Eq. (35) and this keeps only the terms to leading order in $1/M_R$. However, this is not always true as all soft SUSY breaking mass parameters as well as the right-handed neutrino masses may have similar magnitudes as in the present scenario. Hence, rather than using Eq. (34), we evaluate the neutrino mass terms corrected up to one-loop order, from

$$(m_\nu^{\mathrm{total}})_{ij} = \left(\frac{-v_u^2}{M_R} + LF\right)\sum_{k=1}^{3} f_{iJ}^\nu f_{jJ}^\nu. \tag{36}$$

Clearly, the coefficient of the loop contribution shifts the tree-level neutrino masses by a constant amount. This coefficient involves the soft SUSY breaking parameters and in this work we explore the effect of these parameters on the neutrino mass matrix.

This simple structure of the neutrino mass matrix [see Eq. (36)] can indeed be very helpful to examine the neutrino mixing pattern. In particular, we are interested in exploring the conditions which could yield the mixing matrix into a tribimaximal structure. Thus we compare Eq. (36) with Eq. (4), where the latter provides with the neutrino mass matrix consistent with the tribimaximal mixing pattern. Then, with a symmetric neutrino Yukawa matrix, neutrino masses can be evaluated using the following expressions:

$$\begin{aligned}
\frac{2}{3}m_1 + \frac{1}{3}m_2 &= C[(f_{11}^\nu)^2 + (f_{12}^\nu)^2 + (f_{13}^\nu)^2],\\
\frac{1}{6}(m_1 + 2m_2 + 3m_3) &= C[(f_{12}^\nu)^2 + (f_{22}^\nu)^2 + (f_{23}^\nu)^2], = C[(f_{13}^\nu)^2 + (f_{23}^\nu)^2 + (f_{33}^\nu)^2],\\
\frac{1}{3}(-m_1 + m_2) &= C[f_{11}^\nu f_{12}^\nu + f_{12}^\nu f_{22}^\nu + f_{13}^\nu f_{23}^\nu],\\
\frac{1}{3}(m_1 - m_2) &= C[f_{11}^\nu f_{13}^\nu + f_{12}^\nu f_{23}^\nu + f_{13}^\nu f_{33}^\nu],\\
\frac{1}{6}(-m_1 - 2m_2 + 3m_3) &= C[f_{12}^\nu f_{13}^\nu + f_{22}^\nu f_{23}^\nu + f_{23}^\nu f_{33}^\nu].
\end{aligned} \tag{37}$$

Here the constant C is defined as $(\frac{-v_u^2}{M_R} + LF)$. As a simple choice we consider $f_{22}^\nu = f_{33}^\nu$ and also $f_{12}^\nu = f_{13}^\nu = 0$ to obtain the solutions. This choice, coupled with the consistency condition $f_{11}^\nu = f_{22}^\nu - f_{23}^\nu$, leads to the following solutions of the neutrino spectra:

$$m_1 = m_2 = \left(\frac{-v_u^2}{M_R} + LF\right)(f_{11}^\nu)^2, \qquad m_3 = \left(\frac{-v_u^2}{M_R} + LF\right)(2f_{22}^\nu - f_{11}^\nu)^2. \tag{38}$$

It is obvious that the mass pattern as depicted above satisfies the desired tribimaximal structure of the neutrino mixing. The mass terms, as expected, contain tree-level contributions which are always negative. On the other hand, the loop contribution can go both ways depending on the sign of the soft SUSY breaking parameters. For a large B_R, which primarily depends on Am, the radiative correction to the neutrino masses could be enhanced to supersede the tree-level results [18].

Before presenting the numerical results a few comments regarding the lepton flavor violating (LFV) processes are in order. Recall that we assume flavor diagonal mass terms for the left and right-chiral sneutrinos. The loop induced processes like $\mu \to e\gamma$, $\tau \to e\gamma$, or $\tau \to \mu\gamma$ can get contributions primarily via the couplings B_{ij}^ν or $B_{ij}^{\prime\nu}$ [see Eqs. (18) and (19)]. Clearly, any such contribution at the leading order would involve a product of two small neutrino Yukawa couplings f_{ij}^ν and are expected to be very suppressed. Moreover, our assumption $f_{12}^\nu = f_{13}^\nu = 0$ would lead to vanishing contributions for the processes $\mu \to e\gamma$ and $\tau \to e\gamma$ in this model.

We now explore whether the obtained mass pattern could fit with the different hierarchical structure that we know so far. In particular, we show our numerical results to identify the regions in the parameter space consistent with the normal, inverted, and quasidegenerate neutrino mass pattern. In the numerical computation we choose different soft parameters and couplings in such a way, that the proper minima condition of the scalar potential is always satisfied [25].

The choices of various parameters are listed below. The value of $\tan\beta$ is taken to be equal to 10. In addition to that, other parameter choices are (i) superpotential parameters $\lambda_h = -0.3$, $\lambda_s = 0.6$, $\lambda_{N1} = \lambda_{N2} = \lambda_{N3} = \lambda_N = 0.2$; and (ii) soft SUSY breaking parameters $m_S = 100$ GeV, $m_{\tilde{N}_i\tilde{N}_i^*} = 300$ GeV, $m_{\tilde{\nu}_i} = 100$ GeV, $A_H = 100$ GeV, $A_\nu = 1000$ GeV.

Apart from the above parameters which are fixed to the quoted values, we have also varied the parameter Am in the calculation. This would cause changes in v_s [see Eq. (16)], which in turn produces variation in the neutrino spectrum. We list the values of Am and v_s in Table I.

B. Different neutrino spectra

The two mass-squared differences shown in Eq. (1) indicate three possible neutrino mass hierarchies [44], namely

(1) Normal hierarchy: this neutrino mass pattern can be established if m_1, m_2, and m_3 are related with the observables $\sqrt{\Delta m_{21}^2}$ and $\sqrt{|\Delta m_{32}^2|}$ as

$$m_1 \approx m_2 \sim \sqrt{\Delta m_{21}^2}, \qquad m_3 \sim \sqrt{|\Delta m_{32}^2|}. \tag{39}$$

However, in principle m_1 can also be much smaller than m_2 or even be zero. Since in this case m_3 is much greater than both m_1 and m_2, we can approximately use the relation shown in Eq. (39) for illustration.

(2) Inverted hierarchy: this hierarchical scenario can be achieved if one chooses

$$m_1 \approx m_2 \sim \sqrt{|\Delta m_{32}^2|}, \qquad m_3 \ll \sqrt{|\Delta m_{32}^2|}. \tag{40}$$

We assume the maximum possible value for m_3 to be ~ 0.01 eV while the minimum value could be vanishing. Obviously, the solar mass-squared difference Δm_{21}^2 will come from the small mass splitting between m_2 and m_1, where $\Delta m_{21}^2 \ll m_2, m_1$. Hence, for a simple analysis we can assume that $m_2 = m_1$.

(3) Degenerate masses: finally, this scenario is defined by

$$m_1 \approx m_2 \approx m_3 \gg \sqrt{|\Delta m_{32}^2|}. \tag{41}$$

Here we assume that the upper bound of the neutrino masses could be 0.33 eV, which comes from the cosmological observations. The lower bound is chosen to be 0.1 eV.

In Fig. 3, three neutrino mass eigenvalues m_1, m_2, m_3, consistent with the normal hierarchical pattern, are plotted as functions of neutrino Yukawa couplings. The difference in the contours manifests how the neutrino masses depend on the soft bilinear coupling parameter (Am). The variation occurs, as v_s depends on (Am), thereby acquiring a differ-

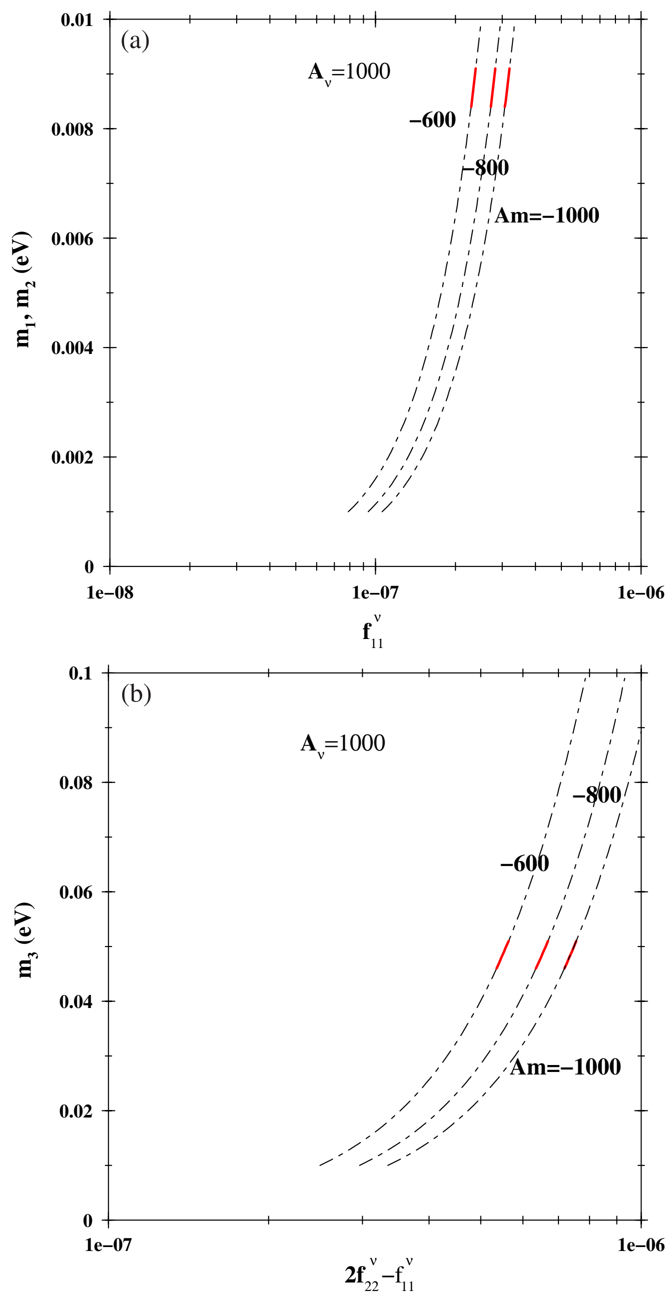

FIG. 3 (color online). Normal hierarchy: variation of m_ν with the Yukawa parameters. The red (solid) segments denote the range of the Yukawa parameters that satisfy neutrino data. Each contour represents a separate set of v_s and Am, as given in Table I. All mass parameters are in GeV.

TABLE I. Different values of v_s corresponding to the different values of the coupling parameter Am.

Am (GeV)	−600.0	−800.0	−1000.0	−1200.0
v_s (GeV)	927.56	1280.76	1625.18	1965.67

ent value at the global minima which has already been mentioned in Table I. In particular $|v_s|$ always increases as we increase the $|Am|$ parameter, which in turn increases the right-handed neutrino masses. This results in a smaller value for m_ν^{tree}. On the other hand, loop correction does not increase appreciably by this small variation of Am if A_ν is around TeV scale, as we will discuss later. We should note here that neutrino loop correction is always an order of magnitude smaller compared to the tree-level value for the parameters we have chosen. Thus with increase in Am parameter, one requires large values of Yukawa couplings to satisfy the neutrino data. The red zone in each contour (see Fig. 3) represents the range of the Yukawa couplings that can satisfy the neutrino data.

In case of inverted hierarchy, we have shown the variation of m_3 with the respective Yukawa couplings in Fig. 4(a). The other mass parameters m_1, m_2 depend on the Yukawa coupling f_{11}^ν, but that can be estimated from the Fig. 3(b) if in that plot we replace m_3 in the y axis by m_1/m_2 and $2f_{22}^\nu - f_{11}^\nu$ in the x axis by f_{11}^ν [see Eq. (38)]. In fact knowing the value of the Yukawa coupling f_{11}^ν would allow us to determine the coupling f_{22}^ν.

The Fig. 4(b) depicts the variation of m_3 with the Yukawa coupling f_{22}^ν for the quasidegenerate mass scenario. In this scenario, the neutrino spectrum is approximately degenerate, i.e., m_1, m_2, and m_3 turn out to be almost identical if one chooses f_{23}^ν much smaller compared to the diagonal Yukawa coupling f_{22}^ν, which essentially means that $f_{22}^\nu \approx f_{11}^\nu$.

Finally a few comments on the dependence of the one-loop contribution to the neutrino mass on the soft SUSY breaking parameters Am and A_ν. The loop contribution is always suppressed unless the parameter A_ν is sufficiently large, as can be seen from Fig. 5. As for illustration, the Yukawa couplings are chosen as $f_{11}^\nu = 1.75 \times 10^{-7}$ and $f_{22}^\nu = f_{33}^\nu = 3.95 \times 10^{-7}$. Similarly, we choose

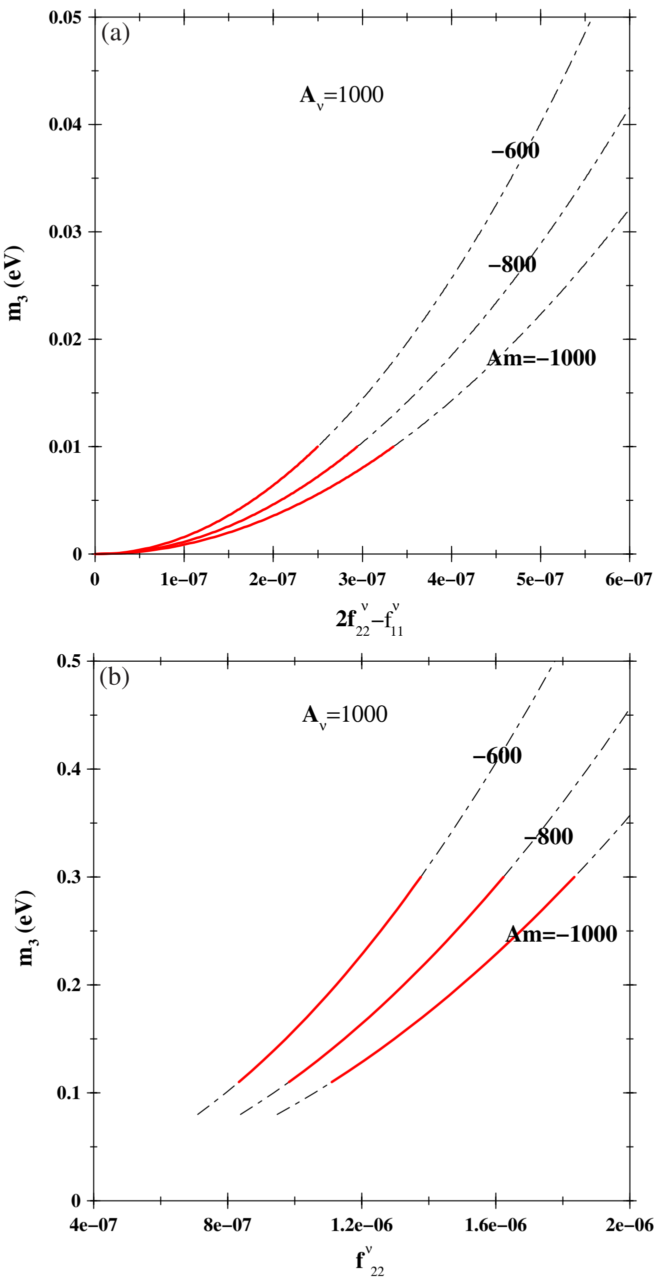

FIG. 4 (color online). Variation of m_3 with the corresponding Yukawa parameter is shown for (a) inverted hierarchical mass pattern and also for (b) the degenerate spectrum. All mass parameters are in GeV.

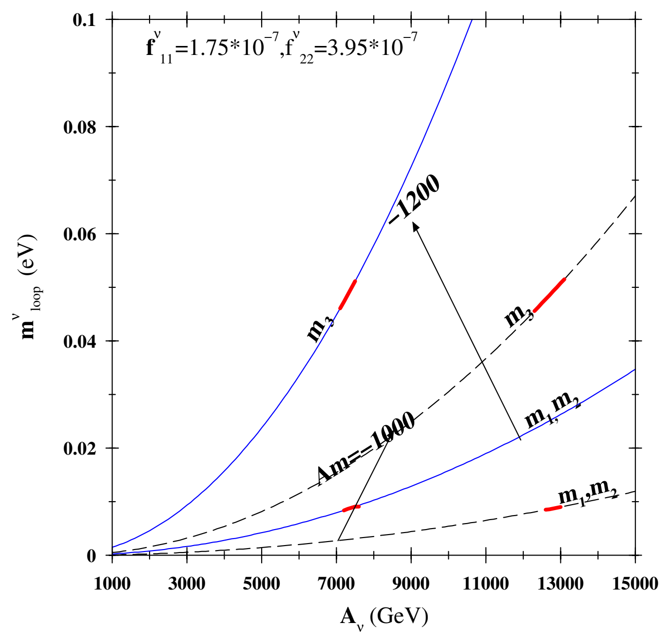

FIG. 5 (color online). Variation of the m_ν^{loop} with A^ν in the normal hierarchical scenario. All mass parameters are in GeV.

$M_1 = 60$ GeV and $M_2 = 120$ GeV, where M_1 and M_2 are the $U(1)$ and $SU(2)$ gaugino mass parameters, respectively. For larger values of electroweak gaugino masses, the one-loop contribution would be reduced further. For $Am = -1$ TeV, higher A_ν values ~ 13 TeV can satisfy the current neutrino data. However, even if A_ν is ~ 13 TeV, the quantity $A_\nu f^\nu$ is very small, i.e., $\sim 10^{-2}$ GeV. Note that for such a choice of the parameter space, the tree-level values of the neutrino masses are not sufficient to accommodate the three-flavor global neutrino data. Increasing the value of Am requires a relatively smaller value of A_ν (~ 7 TeV) to reproduce the neutrino data. It is very important to point out that, for a fixed λ_s and λ_{Ni}, one cannot increase the trilinear coupling parameter Am to an arbitrary high value as the right-chiral sneutrinos may turn out to be tachyonic. Thus, a relatively larger soft trilinear parameter A_ν is required to enhance the one-loop contribution to neutrino masses.

The requirement of a large A_ν can be understood from the following discussion.

(i) The one-loop contribution to the neutrino mass originating from the mass splitting in the left-handed sneutrinos depends on the parameters μ, A_ν, and B_N as can be seen from Eqs. (34) and (35). It has been argued in Ref. [18] that, in order to have the one-loop contribution to the neutrino mass comparable to its tree-level value, the ratio $\Delta m_{\tilde{\nu}_{ij}}/m_\nu$ should be $\sim 10^3$.

(ii) Substituting $\mu = \lambda_H v_s$ and the expression for B_N from Eq. (33), we may write

$$\begin{aligned}\Delta m_{\tilde{\nu}ij}/m_\nu \simeq 2(A_\nu &+ \lambda_H v_s \cot\beta \\ &- \left(\frac{1}{4}\lambda_s v_s + Am/2 + \lambda_H v_d v_u/2v_s\right) \\ &\times (1 + (A_\nu + \lambda_H v_s \cot\beta)^2)/M_R^2)/m_{\tilde{\nu}}.\end{aligned}$$

We can see from the above expression that one may increase either the Am or A^ν parameter to enhance the one-loop contribution to make it countable. But in the present context, raising the soft parameter Am alone would not serve the purpose. This is because the VEV v_s increases significantly with $|Am|$ (see Table I). Thus there is always a partial cancellation between different terms in the above expression for the left sneutrino mass splitting. In particular, the effective bilinear coupling B_N is reduced because of this partial cancellation. In addition, we choose the sign of the coupling λ_H as negative in order to determine the correct global minima. This also causes a partial cancellation between various terms, but to a lesser extent. With this cancellation effect in mind, it is easy to check that the ratio $\Delta m_{\tilde{\nu}ij}/m_\nu$ always resides near the value ~ 10 with the soft parameters A^ν and B_N around the TeV scale.

(iii) Now, as mentioned above, the trilinear coupling parameter Am is restricted if one does not want the right-chiral sneutrinos to become tachyonic. Of course this depends on the choice of the soft Dirac mass term $m_{\tilde{N}\tilde{N}^*}$ of the $\tilde{N}$s, which we have chosen to have a quite moderate value (300 GeV) in this case. However, the parameter A^ν can be pushed to a reasonably high value without affecting any other results. This explains why a large A^ν parameter is required to make the one-loop contribution to the neutrino mass comparable to its tree-level value.

V. SIGNATURES AT LHC

It is extremely important to investigate the possible signatures of this TeV-scale seesaw mechanism at the LHC. One of the search strategies could be to produce the right-handed neutrino N (or the corresponding right-handed sneutrino $\tilde{N}$) with a large enough cross section and then look at the decay branching ratios in different available modes. However, in this type of model the production of TeV-scale right-handed neutrinos (or sneutrinos) at the LHC is suppressed[1] by the light neutrino mass [46]. Nevertheless, it is possible to construct models where the production mechanism of the right-handed neutrino (sneutrino) can be decoupled from the neutrino mass generation. For example, extended gauge symmetries such as $U(1)_{B-L}$ or $SU(2)_R$ may offer extra gauge bosons near the TeV scale whose couplings to quarks and the right-handed neutrino (sneutrino) are unsuppressed [47]. In such models a single or a pair of right-handed neutrinos can be produced with large cross sections leading to dilepton signals (same sign) with no missing energy [17(a),48–51], trilepton signals [52], or four-lepton signals [53–55].

In the context of the present model, the left-sneutrino Majorana mass term can lead to oscillation between the left-chiral sneutrino and the corresponding antisneutrino [18,56,57]. This can be interpreted as the observation of a sneutrino decaying into a final state with a ‘‘wrong-sign’’ charged lepton. In order to have a large oscillation probability the total decay width Γ of the sneutrino/antisneutrino and the mass splitting Δm must be of the same order. Since Δm is constrained by the neutrino data, one needs a very small total decay width of the sneutrino/antisneutrino. It has been shown in [18] that this can be achieved in a scenario where the lighter stau is long-lived and the left-chiral sneutrino can only have three-body decay modes involving the lighter stau in the final states. This can lead to signals such as like-sign dileptons, single charged lepton plus like-sign distaus (leading to heavily ionizing charged tracks), or like-sign distau charged tracks at future linear colliders [18,58,59] or at the LHC [60]. The resulting charge asymmetry of the final states can be measured to

[1] A very recent analysis along with the discovery potential at the LHC is presented in Ref. [45].

get an estimate of the sneutrino-antisneutrino oscillation probability [60]. In addition, for a very small sneutrino decay width one can also observe a displaced vertex in the detector. However, a detailed study of such signals in the context of the present model is beyond the scope of the present paper.

In comparison, now we discuss briefly the signatures of R-parity violating models in general. In models with spontaneous violation of R-parity, the singlet sneutrino vacuum expectation value leads to the existence of a Majoron which is an additional source of missing energy. This can change the decay pattern of the lightest Higgs and the lightest neutralino with the corresponding signatures at the LHC. For more details and the relevant references the reader is referred to Ref. [46]. In the case of bilinear R-parity violation, the ratios of certain decay branching ratios of the LSP show very nice correlation with the neutrino mixing angles. This can lead to very interesting signatures at the LHC where comparable numbers of events with muons and taus, respectively, can be observed in the final state [9–15].

From the above discussion we see that the canonical type-I supersymmetric seesaw case that we have considered in this paper has characteristic signatures which can be tested at the LHC. At the same time one can also distinguish the predictions of this model with those of the models with spontaneous or bilinear R-parity violating scenarios.

VI. CONCLUSIONS

We have studied the neutrino masses and mixing in an R-parity conserving supersymmetric standard model with three right-handed neutrino superfields $\hat{N}_i$ and another gauge singlet superfield $\hat{S}$. This model is similar to the next-to-minimal supersymmetric standard model (NMSSM), where the scalar component of $\hat{S}$ gets a VEV to generate a μ term of correct order of magnitude. In addition, the same VEV also generates TeV-scale Majorana masses for the right-handed neutrinos. The small neutrino masses are generated at the tree level by the usual seesaw mechanism at the TeV scale. We also calculate the one-loop contribution to the neutrino mass matrix and investigate the constraints on the model parameters to produce the tribimaximal pattern of neutrino mixing for three different neutrino mass hierarchies. The neutrino mass matrix gets a contribution at the one-loop level controlled by the sneutrino Majorana mass terms. We show that the one-loop contribution can be important for certain choices of the soft SUSY breaking parameters. This we have demonstrated by evaluating the one-loop contribution in two different ways. In particular, we observe that the one-loop contributions can be significant when the soft SUSY breaking trilinear parameter $A_\nu f^\nu$ is $\sim\mathcal{O}(10^{-3}\ \text{GeV})$ with $A_\nu \sim 10$ TeV. This observation is quite robust and does not change much if one introduces a small θ_{13} in the neutrino sector. Our choice of neutrino Yukawa couplings also predicts vanishing contributions to the lepton flavor violating processes $\mu \to e\gamma$ and $\tau \to e\gamma$ as well as an extremely suppressed contribution to $\tau \to \mu\gamma$.

As has been stated earlier, it is also possible to have nonzero vacuum expectation values for the left- and right-chiral sneutrinos. In that case, R-parity is violated spontaneously. The neutrino mass matrix can have contributions from two different sources, namely, the effective bilinear R-parity violating interactions and the TeV-scale seesaw mechanism. One-loop contributions to the neutrino mass matrix can be very important in this case too. However, the tree-level and one-loop calculations are rather involved and require a separate discussion altogether. We plan to present these results in a subsequent paper [42].

The characteristic signatures of this model at the LHC include like-sign dilepton (without missing energy), trilepton, or four-lepton final states as well as single lepton plus two heavily ionizing charged tracks or only two heavily ionizing charged tracks stemming from long-lived staus. By looking at these signals one can possibly distinguish this model from the models of spontaneous or bilinear R-parity violation.

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