# New strategy for  $B_s$  branching ratio measurements and the search for new physics in  $B_s^0 \to \mu^+\mu^-$

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The LHCb experiment at CERN's Large Hadron Collider will soon allow us to enter a new era in the exploration of  $B_s$  decays. A particularly promising channel for the search of "new physics" is  $B_s^0 \rightarrow u^+ u^-$ . The systematic key uncertainty affecting the massurement of this and in fact all B decay  $\mu^+\mu^-$ . The systematic key uncertainty affecting the measurement of this—and in fact all  $B_s$ -decay branching ratios—is the ratio of fragmentation functions  $f_d/f_s$ . As the currently available methods for determining  $f_d/f_s$  are not sufficient to meet the high precision at LHCb, we propose a new strategy using  $\bar{B}_s^0 \to D_s^+ \pi^-$  and  $\bar{B}_d^0 \to D^+ K^-$ . It allows us to obtain a lower experimental bound on BR $(B_s^0 \to \mu^+ \mu^-)$ <br>which offers a powerful probe for new physics. In order to go beyond this bound and to determine f / f which offers a powerful probe for new physics. In order to go beyond this bound and to determine  $f_d/f_s$ with a theoretical precision matching the experimental one it is sufficient to know the  $SU(3)$ -breaking correction to a form-factor ratio from nonperturbative methods at the level of 20%. Thanks to our strategy, we can detect new physics in  $B_s^0 \to \mu^+ \mu^-$  at LHCb with 5 $\sigma$  for a branching ratio as small as twice the standard model value, which represents an improvement of the new physics race by about a factor of 2 standard model value, which represents an improvement of the new-physics reach by about a factor of 2 with respect to the current LHCb expectation.

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#### I. INTRODUCTION

In this decade, we will enter a new round in the precision testing of the flavor sector of the standard model (SM) through B-meson decays. Currently the LHCb experiment at CERN's Large Hardon Collider (LHC) is starting its first physics run. After pioneering results on the  $B_s$  system by the CDF and D0 Collaborations at the Tevatron, LHCb will allow us to explore this still largely unexplored territory of the flavor-physics landscape [[1\]](#page-4-0).

In this respect, one of the most promising channels for detecting signals of ''new physics'' (NP) is the rare decay  $B_s^0 \rightarrow \mu^+ \mu^-$ , which originates in the SM from "penguin"<br>and hox topologies i.e. quantum loop processes. The and box topologies, i.e. quantum loop processes. The corresponding branching ratio is predicted as follows [[2\]](#page-4-1):

<span id="page-0-0"></span>BR 
$$
(B_s^0 \to \mu^+ \mu^-)|_{SM} = (3.6 \pm 0.4) \times 10^{-9}
$$
, (1)

where the error is fully dominated by a nonperturbative "bag parameter" coming from lattice QCD. As is well known, this observable may be significantly enhanced through NP (for a review, see Ref. [\[2](#page-4-1)]). The present upper bounds from the CDF and D0 Collaborations are still about [1](#page-0-0) order of magnitude away from (1) and read as  $4.3 \times 10^{-8}$ [\[3\]](#page-4-2) and  $5.3 \times 10^{-8}$  (95% C.L.) [[4](#page-4-3)], respectively.<br>At LHCb, the extraction of BR( $B_s^0 \rightarrow \mu^+ \mu^-$ ) will rely

At LHCb, the extraction of BR $(B_s^0 \to \mu^+ \mu^-)$  will rely<br>on normalization channels such as  $B_u^+ \to J/\psi K^+$ ,  $B_d^0 \to$ <br> $K^+ \pi^-$  and/or  $B_s^0 \to J/\psi K^{*0}$  in the following way:  $K^+\pi^-$  and/or  $B_d^0 \rightarrow J/\psi K^{*0}$  in the following way:

<span id="page-0-1"></span>BR 
$$
(B_s^0 \to \mu^+ \mu^-)
$$
 = BR $(B_q \to X) \frac{f_q}{f_s} \frac{\epsilon_X}{\epsilon_{\mu \mu}} \frac{N_{\mu \mu}}{N_X}$ , (2)

where the  $\epsilon$  factors are total detector efficiencies and the N factors denote the observed numbers of events. The  $f_q$  are fragmentation functions, which describe the probability that a b quark will fragment in a  $\bar{B}_q$  meson ( $q \in$   $\{u, d, s\}$ ). In ([2\)](#page-0-1),  $f_q/f_s$  is actually the major source of the systematic uncertainty, thereby limiting the ability to detect a 5 $\sigma$  deviation from the SM at LHCb to BR( $B_s^0 \rightarrow u^+ u^-$ ) > 11 × 10<sup>-9</sup> [1]  $\mu^+ \mu^-$ ) > 11  $\times$  10<sup>-9</sup> [\[1](#page-4-0)].

In this estimate, the current experimental knowledge of  $f_d/f_s$  was assumed, which we summarize in Sec. [II.](#page-0-2) In Sec. [III](#page-1-0), we propose a new strategy to measure  $f_d/f_s$  at LHCb. Its experimental prospects and theoretical limitations are discussed in Secs. [IV](#page-2-0) and [V,](#page-2-1) respectively. In Sec. [VI](#page-4-4), we discuss the implications for the search for NP with the  $B_s \to \mu^+ \mu^-$  branching ratio, while we summarize our conclusions in Sec. [VII.](#page-4-5)

#### II. EXPERIMENTAL STATUS OF  $f_d/f_s$

<span id="page-0-2"></span>The CDF Collaboration has estimated the ratio of fragmentation functions through semi-inclusive  $\bar{B} \to D\ell^- \bar{\nu}_\ell X$ decays [\[5\]](#page-4-6). The reconstructed  $D\ell^-$  signal yields are then related to the number of produced b hadrons by assuming the  $SU(3)$  flavor symmetry and neglecting  $SU(3)$ -breaking corrections (e.g., assuming  $\Gamma(\bar{B}_{d}^{0} \to \ell^{-} \bar{\nu}_{\ell} D^{+}) = \Gamma(\bar{B}_{s}^{0} \to \ell^{-} \bar{\nu}_{s} D^{+})$ ). Together with an earlier result using double  $\ell^- \bar{\nu}_{\ell} D_s^+$ )). Together with an earlier result using double<br>semileptonic decays (containing two muons and either a semileptonic decays (containing two muons and either a  $K^*$  or a  $\phi$  meson) [[6](#page-5-0)] the average value  $f_s/(f_d + f_u)$  =  $0.142 \pm 0.019$  is obtained [\[7](#page-5-1)].<br>An alternative approach use

An alternative approach uses the different mixing probabilities for  $B_d^0$  and  $B_s^0$  mesons. Despite a 1.8 $\sigma$  discrepancy in the time-integrated mixing probability between the LEP and Tevatron data, an average value of  $f_s = 0.119 \pm 0.019$ <br>was determined with this method [8] was determined with this method [\[8\]](#page-5-2).

The CLEO and Belle Collaborations have extracted the fraction  $f_s$  of  $B_s^{(*)}\bar{B}_s^{(*)}$  events among all  $b\bar{b}$  events at the  $\Upsilon(5S)$  resonance from inclusive  $\Upsilon(5S) \rightarrow D_s X$ ,  $\phi X$  decays [\[9](#page-5-3),[10](#page-5-4)]. Here the relation

$$
BR(Y(5S) \to D_s X, \phi X)
$$
  
=  $2f_s BR(B_s^0 \to D_s X, \phi X)$   
+  $(1 - f_s)BR(Y(4S) \to D_s X, \phi X)$  (3)

is assumed with  $BR(B_s^0 \rightarrow D_s X) = (92 \pm 11)\%$ , which<br>relies on a variety of assumptions and vields the modelrelies on a variety of assumptions and yields the modeldependent result  $f_s = 0.194 \pm 0.011(\text{stat}) \pm 0.027(\text{sys})$ [\[8\]](#page-5-2).

It is evident that the fragmentation functions depend on the environment, which becomes apparent when an attempt is made to compare the numerical values for  $f_s$ . At the B factories  $f_d + f_u + f_s = 1$ , whereas at hadron colliders the available energy allows the  $b$  quark to fragment into baryons as well. In addition beam-remnant effects at hadron colliders might affect the b-hadron fractions depending on  $p_T$  and/or pseudorapidity. Consequently, each experiment—and, in particular, LHCb—should calibrate its own value for this quantity. As a result LHCb cannot directly use the value measured at Tevatron or at LEP. The fragmentation function is not only the major limiting parameter for the determination of BR $(B_s^0 \rightarrow \mu^+ \mu^-)$  at LHCb, but in fact for all *R* -decay branching ratio measurements at the fact for all  $B_s$ -decay branching ratio measurements at the LHC, the Tevatron, and an  $e^+e^-$  B factory running at  $\Upsilon(5S)$ . As a result, the usage of  $B_s$  modes as normalization channels, obtained from the KEKB runs at  $\Upsilon(5S)$ , also suffer from an imprecise value of  $f_s$ , in addition to a large statistical uncertainty [\[11\]](#page-5-5).

By normalizing the  $B_s^0 \rightarrow \mu^+ \mu^-$  decay directly to an-<br>ner B, decay the ratio of the fragmentation functions in other  $B_s$  decay, the ratio of the fragmentation functions in Eq. ([2](#page-0-1)) would trivially disappear. However, at present the best directly measured  $B_s$  branching ratio is  $BR(B_s^0 \rightarrow D_s \pi) = 3.67^{+0.35}_{-0.33}$ (stat) $^{+0.43}_{-0.42} \pm 0.49$ (f<sub>s</sub>) [\[11\]](#page-5-5), determined with 23.6 fb<sup>-1</sup> of data at  $Y(5S)$  Methods are being conwith 23.6 fb<sup>-1</sup> of data at  $\Upsilon(5S)$ . Methods are being con-<br>sidered to improve the present knowledge of f at the B sidered to improve the present knowledge of  $f_s$  at the B factories [\[12\]](#page-5-6). However even considering these possible improvements it is unlikely that they will be sufficient to match the required precision of LHCb. A total uncertainty of about 12% could be expected for a sample of 120 fb<sup>-1</sup> (corresponding to the total available statistics) and assuming these additional improvements in the determination of  $f_s$  [[13](#page-5-7)]. Moreover the decay  $\bar{B}_s^0 \to D_s^+ \pi^-$  poses experimental difficulties when used as a normalization channel mental difficulties when used as a normalization channel for  $B_s^0 \to \mu^+ \mu^-$ , due to the very different decay topology<br>(hadronic final state, number of tracks, flight distance of (hadronic final state, number of tracks, flight distance of the  $D_s$ , etc.). A sizable contribution to the uncertainty in the branching ratio estimation due to the ratio of the efficiencies in Eq. [\(2\)](#page-0-1) must thus be considered. An alternative  $B_{s}$  decay channel for the direct normalization would be  $B_s^0 \rightarrow J/\psi \phi$ , which is, however, affected by a statistical<br>error twice as large compared to  $\bar{B}^0 \rightarrow D^+ \pi^-$ . Assuming error twice as large compared to  $\bar{B}_s^0 \to D_s^+ \pi^-$ . Assuming the full statistics currently available at the *R* factories of the full statistics currently available at the  $B$  factories of  $120$  fb<sup>-1</sup>, in combination with the possible improvements in the determination of  $f_s$ , at best a total relative error of 15% could be expected for this decay.

#### III. A NEW STRATEGY FOR LHCB

<span id="page-1-0"></span>In view of the unsatisfactory situation described in the previous section, we propose a new method for extracting  $f_d/f_s$  at LHCb. The starting point is the following simple expression:

<span id="page-1-2"></span>
$$
\frac{N_s}{N_d} = \frac{f_s}{f_d} \times \frac{\epsilon(B_s \to X_1)}{\epsilon(B_d \to X_2)} \times \frac{\text{BR}(B_s \to X_1)}{\text{BR}(B_d \to X_2)}; \tag{4}
$$

knowing the ratio of the branching ratios, we could obviously extract  $f_d/f_s$  experimentally. In order to implement this feature in practice, the  $B_s \to X_1$  and  $B_s \to X_2$  decays have to satisfy the following three requirements:

- (1) the ratio of their branching ratios must be easy to measure at LHCb;
- (2) the decays must be robust with respect to the impact of NP contributions;
- (3) the ratio of their branching ratios must be theoretically well understood within the SM.

At first sight, an obvious choice seems to use semileptonic decays such as  $\bar{B} \to D^+ \mu^- \nu$ . However, the measurement of such channels at hadron colliders is experimentally challenging since the fully reconstructed B mass is not available and various sources of muons in the background have to be controlled. Therefore, we have to focus at nonleptonic decays, where requirement (1) implies that we look at decays into charged particles and requirement (2) narrows down the search to channels without penguin contributions, which are flavor-changing neutral-current processes which might well be affected by NP contributions. The third requirement finally guides us to the decays  $\bar{B}_s^0 \to D_s^+ \pi^-$  and  $\bar{B}_d^0 \to D^+ K^-$ .<br>As can be seen in Fig. 1, the

As can be seen in Fig. [1](#page-1-1), these channels receive only contributions from color-allowed tree-diagram-like topologies and are related to each other through the interchange of all down and strange quarks, i.e., through the U-spin subgroup of the  $SU(3)$  flavor symmetry. Moreover, the concept of ''factorization'' [\[14\]](#page-5-8) is expected to work well in these transitions. This was expected from ''color transparency'' already two decades ago [[15](#page-5-9),[16](#page-5-10)], while this feature could actually be put on a rigorous theoretical basis in the heavy-quark limit [\[17](#page-5-11)[,18\]](#page-5-12). Consequently, using these decays, we can calculate the corresponding ratio of their branching ratios entering Eq. [\(4](#page-1-2)) up to small, nonfactoriz-

<span id="page-1-1"></span>

FIG. 1. The  $\bar{B}_d^0 \to D^+ K^-$  and  $\bar{B}_s^0 \to D_s^+ \pi^-$  decay topologies.

able, U-spin-breaking corrections. This feature will be discussed in more detail in Sec. [V.](#page-2-1)

Let us note that in contrast to the  $\overline{B}^0 \to D^+\pi^-$  mode<br>ually considered in the literature in the context with usually considered in the literature in the context with factorization, the decays in Fig. [1](#page-1-1) have the advantage of not receiving additional contributions from ''exchange'' topologies, which are expected to be small but are not factorizable. Moreover, thanks to the absence of penguin topologies, the situation concerning factorization is also much more favorable than in  $B \to \pi \pi$ ,  $\pi K$  decays.<br>Applying a notation similar to that of Ref. [17] we

Applying a notation similar to that of Ref. [\[17\]](#page-5-11), we write the branching ratios of the decays at hand as

<span id="page-2-2"></span>BR(
$$
\bar{B}_q^0 \to D_q^+ P^-
$$
)  
\n=
$$
\frac{G_F^2 (m_{B_q}^2 - m_{D_q}^2)^2 |\vec{q}| \tau_{B_q}}{16 \pi m_{B_q}^2} |V_q^* V_{cb}|^2 [f_P F_0^{(q)} (m_P^2)]^2
$$
\n
$$
\times |a_1(D_q P)|^2,
$$
\n(5)

with  $P = K$  and  $\pi$  for  $q = d$  and s, respectively. Here  $G_F$  is Fermi's constant, the *m* factors denote meson masses  $\vec{a}$ is Fermi's constant, the *m* factors denote meson masses,  $\vec{q}$ is the momentum of the final-state  $D_q$  and P mesons in the rest frame of the  $\bar{B}^0_q$  meson,  $\tau_{B_q}$  is the lifetime of the  $\bar{B}^0_q$ ,  $V_q^* V_{cb}$  with  $V_q = V_{us}$  and  $V_{ud}$  for  $q = d$  and s, respectively contains the relevant elements of the Cabibbo tively, contains the relevant elements of the Cabibbo-Kobayashi-Maskawa matrix,  $f<sub>P</sub>$  is the P-meson decay constant, and the form factor  $F_0^{(q)}$  enters the parametrization of the  $\langle D_q^+ | \bar{c} \gamma^\mu b | \bar{B}^0_q \rangle$  matrix element. The quantity  $a_n(D, P)$  describes the deviation from naive factorization.  $a_1(D_aP)$  describes the deviation from naive factorization. As discussed in detail in Ref. [[17](#page-5-11)], this parameter is found in "QCD factorization" as a quasiuniversal quantity  $|a_1| \approx$ 1:05 with very small process-dependent nonfactorizable corrections.

We would like to propose to measure the ratio of the  $\bar{B}_s^0 \to D_s^+ \pi^-$  and  $\bar{B}_d^0 \to D^+ K^-$  branching ratios to deter-<br>mine  $f, f$ . Neglecting, for simplicity kinematical mass mine  $f_d/f_s$ . Neglecting, for simplicity, kinematical mass factors, we have

<span id="page-2-4"></span>
$$
\frac{\text{BR}(\bar{B}_{s}^{0} \to D_{s}^{+}\pi^{-})}{\text{BR}(\bar{B}_{d}^{0} \to D^{+}K^{-})} \sim \frac{\tau_{B_{s}}}{\tau_{B_{d}}} \left| \frac{V_{ud}}{V_{us}} \right|^{2} \left( \frac{f_{\pi}}{f_{K}} \right)^{2} \left[ \frac{F_{0}^{(s)}(m_{\pi}^{2})}{F_{0}^{(d)}(m_{K}^{2})} \right]^{2}
$$

$$
\times \left| \frac{a_{1}(D_{s}\pi)}{a_{1}(D_{d}K)} \right|^{2}.
$$
(6)

On the other hand, the ratio of the corresponding number of signal events observed in the experiment is given by

$$
\frac{N_{D_s \pi}}{N_{D_d K}} = \frac{f_s}{f_d} \frac{\epsilon_{D_s \pi}}{\epsilon_{D_d K}} \frac{\text{BR}(\bar{B}_s^0 \to D_s^+ \pi^-)}{\text{BR}(\bar{B}_d^0 \to D^+ K^-)},\tag{7}
$$

<span id="page-2-3"></span>where the  $\epsilon$  are again total detector efficiencies. Using ([5\)](#page-2-2), we hence obtain

$$
\frac{f_d}{f_s} = 12.88 \times \frac{\tau_{B_s}}{\tau_{B_d}} \times \left[ \mathcal{N}_a \mathcal{N}_F \frac{\epsilon_{D_s \pi}}{\epsilon_{D_d K}} \frac{N_{D_d K}}{N_{D_s \pi}} \right], \quad (8)
$$

with

$$
\mathcal{N}_a = \left| \frac{a_1(D_s \pi)}{a_1(D_d K)} \right|^2, \qquad \mathcal{N}_F = \left[ \frac{F_0^{(s)}(m_\pi^2)}{F_0^{(d)}(m_K^2)} \right]^2. \tag{9}
$$

Let us next first explore the experimental feasibility at LHCb before having a closer look at the theoretical limitations of our new strategy for extracting  $f_d/f_s$ .

#### <span id="page-2-0"></span>IV. EXPERIMENTAL PROSPECTS AT LHCB

At LHCb, both the  $\bar{B}_s^0 \rightarrow D_s^+ \pi^-$  and  $\bar{B}_d^0 \rightarrow D^+ K^-$  de-<br>v channels can be exclusively reconstructed using the cay channels can be exclusively reconstructed using the  $D^+ \to K^- \pi^+ \pi^+$  and  $D_s^+ \to K^+ K^- \pi^+$  final states. An expected *R*-mass resolution of 18 MeV and excellent parexpected B-mass resolution of 18 MeV and excellent particle identification capabilities will allow LHCb to select and reconstruct a clean sample of these decays. Since both channels are selected with an identical flavor final state containing the four charged hadrons  $KK\pi\pi$ , the uncertainty on  $\epsilon_{D_s\pi}/\epsilon_{D_dK}$  is expected to be small.

We estimated the corresponding statistical uncertainty on  $r = \epsilon_{D_s} \pi N_{D_dK}/(\epsilon_{D_dK}N_{D_s}\pi)$  with a toy Monte Carlo, generating a sample equivalent to  $0.2$  fb<sup>-1</sup>. This is the expected integrated luminosity at the end of 2010, taking a lower bb cross section of 250  $\mu$ b due to the reduced LHC beam energy of 3.5 TeV into account. Following the estimates from full simulation [[19](#page-5-13)], and assuming a total trigger efficiency of 30% [\[20\]](#page-5-14), we expect to select 5500  $\overline{B_8^0} \to D_s^+ \pi^-$  and 1100  $\overline{B_8^0} \to D^+ K^-$  events, with a back-<br>ground of approximately 6600  $\overline{R^0} \to D^+ \pi^-$  events, where ground of approximately  $\ddot{6}600 \bar{B}_d^0 \rightarrow D^+\pi^-$  events, where<br>one of the three pions is misidentified as a kaon (assuming one of the three pions is misidentified as a kaon (assuming a 5% probability to misidentify a pion as a kaon). Combinatorial background from inclusive bb events is expected to yield 6000 events inside a mass window  $5220 < m < 5420$  MeV around the B mass. We expect a precision of  $7.5\%$  on r, where the dominant uncertainty originates from  $BR(D_s \to K^+ K^- \pi) = (5.50 \pm 0.28)\%$ .<br>With an integrated luminosity of 1 fb<sup>-1</sup> as expected at With an integrated luminosity of  $1$  fb<sup>-1</sup> as expected at the end of 2011, the statistical uncertainty becomes negligible, thereby reducing the total uncertainty to  $\sim$  5.6%.

The ratio  $f_d/f_s$  is not only crucial for the precise determination of  $\overline{BR(B_9^0 \rightarrow \mu^+ \mu^-)}$  but actually for the mea-<br>surement of any B pranching ratio. Similarly the general surement of any  $B_s$  branching ratio. Similarly, the general purpose LHC experiments ATLAS and CMS rely on a precise value of  $f_d/f_s$  for the determination of BR $(B_s^0 \rightarrow u^+u^-)$ . Unfortunately our proposed hadronic decays are  $\mu^+\mu^-$ ). Unfortunately our proposed hadronic decays are not ideal for these experiments due to trigger and particle identification requirements. However, we advocate to apply the value of  $f_d/f_s$  as determined by LHCb also at ATLAS and CMS, once the dependence of  $f_d/f_s$  on  $p_T$ and/or rapidity is measured to be small.

## V. THEORETICAL LIMITATIONS

<span id="page-2-1"></span>In the extraction of  $f_d/f_s$  through ([8\)](#page-2-3), we have theoretical uncertainties related to U-spin–breaking effects in  $\mathcal{N}_a$  <span id="page-3-0"></span>and  $\mathcal{N}_F$ . In the case of the first factor, we can write

$$
\mathcal{N}_a \approx 1 + 2\Re(a_1^{\text{NF}}(D_s\pi) - a_1^{\text{NF}}(D_dK)),\tag{10}
$$

where the  $a_1^{\text{NF}}$  describe the nonuniversal, i.e., processdependent, nonfactorizable contributions to the decays at hand. These contributions cannot be calculated reliably. However, they arise as power corrections to the heavyquark limit, i.e., they are suppressed by at least one power of  $\Lambda_{\text{OCD}}/m_b$ , and are—in the decays at hand—numerically expected at the few percent level [[17](#page-5-11)]. Let us note that within QCD factorization, also final-state interaction effects such as  $\bar{B}_d^0 \to [D^0 \bar{K}^0] \to D^+ K^-$  arise only as non-<br>factorizable  $\Lambda_{\text{con}}/m$ , corrections factorizable  $\Lambda_{\text{QCD}}/m_b$  corrections.

The nonfactorizable terms can actually be probed [\[15\]](#page-5-9) through the differential rate of the semileptonic decay  $\bar{B}_q^0 \rightarrow D_q^+ \ell^- \bar{\nu}_\ell$  which yields the following expression [\[17\]](#page-5-11):

$$
BR(\bar{B}_q^0 \to D_q^+ P^-) \tau_{B_q}
$$
  
\n
$$
\overline{d\Gamma(\bar{B}_q^0 \to D_q^+ \ell^- \bar{\nu}_{\ell})/dq^2|_{q^2 = m_p^2}}
$$
  
\n
$$
= 6\pi^2 |V_q|^2 f_P^2 |a_1(D_q P)|^2 X_P,
$$
\n(11)

where  $X_P$  deviates from 1 below the percent level. Replacing the pseudoscalar mesons P by their vectormeson counterparts, i.e.  $K^- \to K^{*-}$  and  $\pi^- \to \rho^-$ , the corresponding  $X_t$ , would be exactly given by 1. However corresponding  $X_V$  would be exactly given by 1. However, these modes are more challenging for LHCb. The current experimental value  $BR(\bar{B}_d^0 \to D^+ K^-) = (2.0 \pm 0.6) \times 10^{-4}$  [7] agrees well with the number in Ref. [17] although  $10^{-4}$  [\[7\]](#page-5-1) agrees well with the number in Ref. [[17](#page-5-11)], although the uncertainty is still too large to probe the nonfactorizable effects. This will be feasible at LHCb by combining the measurement of the  $\bar{B}_d^0 \to D^+ K^-$  branching ratio de-<br>scribed above with measurements of the differential semiscribed above with measurements of the differential semileptonic  $\bar{B}^0 \to D^+ \ell^- \bar{\nu}_{\ell}$  rate at  $q^2 = M_K^2$  by the *B*-factory experiments *BABAR* and Belle experiments BABAR and Belle.

It is interesting to note that factorization was already tested in a similar setting at the  $B$  factories, measuring the branching ratios and the  $D_{(s)}^{*-}$  polarization in the decays<br> $P_{(s)}^{0} \rightarrow D^{*-} \circ \pi^{+}$  and  $P_{(s)}^{0} \rightarrow D^{*-} \circ \pi^{+}$ . Good agreement week  $B_s^0 \to D_s^*^- \rho^+$  and  $B^0 \to D^{*-} \omega \pi^+$ . Good agreement was<br>found between factorization predictions and the experifound between factorization predictions and the experimental results within the current errors [\[21](#page-5-15)[,22\]](#page-5-16).

The deviation of [\(10](#page-3-0)) from 1 is actually not only suppressed by  $\Lambda_{\text{OCD}}/m_b$  but also through the feature that this is a U-spin–breaking difference. In this context, it should again be emphasized that any decay topology contributing to  $\bar{B}_d^0 \to D^+ K^-$ , even the most complicated rescattering<br>topology, has a counterpart in  $\bar{B}^0 \to D^+ \pi^-$ , which is topology, has a counterpart in  $\bar{B}_s^0 \to D_s^+ \pi^-$ , which is<br>related to the  $B_s$  case through the interchange of all related to the  $B_d$  case through the interchange of all down and strange quarks. Consequently, taking all these considerations into account, we eventually conclude that  $1 - \mathcal{N}_a$  is conservatively expected to be at most a few percent.

The major uncertainty affecting ([8\)](#page-2-3) is hence the formfactor ratio  $\mathcal{N}_F$ , where U-spin–breaking corrections arise from d and s spectator-quark effects, which were neglected in previous determinations of  $f<sub>s</sub>$  [\[5](#page-4-6)]. Making the same approximation, we would simply have  $\mathcal{N}_F = 1$ . Unfortunately, the  $B_s \rightarrow D_s$  form factors have so far received only small theoretical attention. In Ref. [[23](#page-5-17)], such effects were explored using heavy-meson chiral perturbation theory, while QCD sum-rule techniques were applied in Ref. [\[24\]](#page-5-18). The numerical value given in the latter paper yields  $\mathcal{N}_F = 1.3 \pm 0.1$ .<br>Interestingly we can g

Interestingly, we can obtain a lower bound on the  $B_s^0 \rightarrow \pm \mu$  - branching ratio from our strategy Ising (2) and (8)  $\mu^+ \mu^-$  branching ratio from our strategy. Using [\(2\)](#page-0-1) and [\(8\)](#page-2-3) and assuming  $\mathcal{N}_a = 1$  yields

$$
BR(B_s^0 \to \mu^+ \mu^-) = \mathcal{N}_F BR(B_s^0 \to \mu^+ \mu^-)_0, \quad (12)
$$

where  $BR(B_s^0 \to \mu^+ \mu^-)_0$  follows from the analysis described above by assuming vanishing *U*-spin-breaking scribed above by assuming vanishing U-spin–breaking corrections. Since the radius of the  $B_s^0$  is smaller than that of the  $B_d^0$ , we expect  $\mathcal{N}_F > 1$  [[23](#page-5-17)]. This behavior is actually reproduced in the calculation of the chiral logarithms in Ref. [\[23\]](#page-5-17), as well as in the QCD sum-rule calculation in Ref. [[24](#page-5-18)]. Moreover, the sign of the chiral logarithmic correction to the  $SU(3)$ -breaking ratio of the decay constants of  $D_{(s)}$  and  $B_{(s)}$  mesons agrees with experimental [for  $D_{(s)}$ ] and lattice results (and also the numerical values are found of similar size). The inequality  $\mathcal{N}_F > 1$  implies then the following bound:

<span id="page-3-1"></span>BR 
$$
(B_s^0 \to \mu^+ \mu^-) > BR(B_s^0 \to \mu^+ \mu^-)_0
$$
, (13)

which offers an interesting tool for the detection of possible NP contributions to  $B_s^0 \to \mu^+ \mu^-$  at LHCb.<br>Assuming that we will measure  $BR(B_0^0 \to \mu^+ \mu^-)$  to be Assuming that we will measure  $\overline{BR}(B_s^0 \to \mu^+ \mu^-)_0$  to be  $5\sigma$  above the SM prediction (1)  $U_s$ spin–breaking effects  $5\sigma$  above the SM prediction [\(1\)](#page-0-0), U-spin–breaking effects could only enhance the measured branching ratio and could not move it down towards the SM value.

In the long run, we would of course like to measure  $BR(B_s^0 \to \mu^+ \mu^-)$  as accurately as possible. In order to match the experimental precision for r of about 5% dismatch the experimental precision for  $r$  of about 5% discussed above, it is sufficient to know the U-spin–breaking corrections to the form-factor ratio  $F_0^{(s)}(m_\pi^2)/F_0^{(d)}(m_K^2)$ <br>from nonperturbative calculations, such as lattice OCD from nonperturbative calculations, such as lattice QCD, at the level of 20%. This looks feasible to us, in particular, in view of the tremendous amount of work that was invested to study  $B \rightarrow D$  form factors on the lattice for the extraction of  $|V_{cb}|$  from semileptonic  $B \to D\ell\bar{\nu}_\ell$  decays. We are not aware of any lattice calculation of the  $SU(3)$ -breaking corrections to the form-factor ratio entering our strategy, which is due to the fact that such analyses did so far not appear phenomenologically interesting.

Finally, we would like to note that the  $SU(3)$ -breaking effects in Eq. [\(6\)](#page-2-4) coming from the ratios of decay constants and form factors tend to cancel each other. Assuming  $\mathcal{N}_F = 1.3$  with  $f_\pi/f_K = 0.8$ , we get an overall  $SU(3)$ -breaking correction to the ratio of branching ratios  $SU(3)$ -breaking correction to the ratio of branching ratios of only about 10%, which is surprisingly small and suggests that also the  $SU(3)$  suppression of  $1 - \mathcal{N}_a$  is very efficient.

<span id="page-4-7"></span>

FIG. 2 (color online). Illustration of the LHCb NP discovery potential in  $B_s^0 \to \mu^+ \mu^-$  resulting from our strategy  $(\mathcal{N}_F \in [1 \ 2 \ 1 \ 4])$ . We show the smallest value of  $\text{BR}(R^0 \to \mu^+ \mu^-)$ . [1.2, 1.4]). We show the smallest value of  $BR(B_3^0 \rightarrow \mu^+ \mu^-)$ <br>allowing the detection of a 5  $\sigma$  deviation from the SM as a allowing the detection of a  $5\sigma$  deviation from the SM as a function of the luminosity at LHCb (at the nominal beam energy of 14 TeV). The figure on the left-hand side shows the lowluminosity regime, whereas the one on the right-hand side illustrates the asymptotic behavior (curves extrapolated from Ref. [\[1\]](#page-4-0)).

# <span id="page-4-4"></span>VI. IMPLICATIONS FOR THE NP REACH IN THE MEASUREMENT OF BR $(B_s \to \mu^+ \mu^-)$

In Fig. [2,](#page-4-7) we illustrate the NP discovery potential in  $B_s^0 \rightarrow \mu^+ \mu^-$  at LHCb through our method. We show<br>contours corresponding to a 5 $\sigma$  NP signal with respect to contours corresponding to a  $5\sigma$  NP signal with respect to [\(1\)](#page-0-0) for the bound in ([13](#page-3-1)) and the extracted value of the branching ratio. Here we have assumed that the uncertainty on  $BR(D_s \to K^+ K^- \pi)$  in the determination of  $f_d/f_s$  is<br>distributed Gaussian, and likewise for the uncertainty on distributed Gaussian, and likewise for the uncertainty on  $BR(B_0^0 \to J/\psi K^*)$  in the extraction of  $BR(B_0^0 \to \mu^+ \mu^-)$ .<br>We conservatively varied  $\mathcal{N}_- \subseteq [1, 2, 1, 4]$  resulting in a We conservatively varied  $\mathcal{N}_F \in [1.2, 1.4]$ , resulting in a negligible change to the predicted sensitivity. Similarly, a variation of  $\mathcal{N}_a \in [0.97, 1.03]$  does essentially not affect the contour.

As can be seen in the plot on the right-hand side in Fig. [2,](#page-4-7) the resulting NP discovery potential is about twice as large as the present LHCb expectation [\[1](#page-4-0)] (upper horizontal line) enabling a possible discovery of NP down to  $BR(B_s^0 \to \mu^+ \mu^-) > 6 \times 10^{-9}$  (lower horizontal line). In addition to the increased sensitivity in the regime of low addition to the increased sensitivity in the regime of low branching ratios, even for large values close to the current CDF exclusion limit the significance of a possible NP discovery would be increased. Thanks to the decrease of the systematical uncertainty, LHCb will be able to fully exploit the statistical improvement, taking full advantage of the accumulated LHCb data up to 10 fb<sup>-1</sup>, which corresponds to five years of nominal LHCb data taking.

At a future LHCb upgrade, even the  $B_d^0 \rightarrow \mu^+ \mu^-$  decay will become accessible, and the proposed determination of  $f_d/f_s$  will be an important tool for the measurement of  $BR(B_d^0 \to \mu^+ \mu^-)/BR(B_s^0 \to \mu^+ \mu^-)$ , which provides an even stronger test of the SM [2] even stronger test of the SM [[2\]](#page-4-1).

Let us finally emphasize that a future super- $B$  factory running at  $\Upsilon(5S)$  would allow us to check the calculations<br>of the  $SU(3)$ -breaking effects in the form factor through of the  $SU(3)$ -breaking effects in the form factor through<br>the measurement of  $\bar{B}_s^0 \to D_s^+ \ell \bar{\nu}_\ell$  decays. The possible<br>discovery of NP in  $B_0^0 \to \mu^+ \mu^-$  at LHCb does not rely discovery of NP in  $B_3^0 \rightarrow \mu^+ \mu^-$  at LHCb does not rely<br>on this input but constraining—and even extracting on this input, but constraining—and even extracting—  $SU(3)$ -breaking form-factor ratios would lead to a more precise determination of BR( $B_s^0 \rightarrow \mu^+ \mu^-$ ).

## VII. CONCLUSIONS

<span id="page-4-5"></span>The current experimental knowledge of the ratio  $f_d/f_s$ of fragmentation functions is unsatisfactory and affects all absolute  $B<sub>s</sub>$ -decay branching ratio measurements at hadron colliders. In particular, this quantity is also the major uncertainty for the extraction of the  $B_s^0 \to \mu^+ \mu^-$  branch-<br>ing ratio from the I HCb data. In view of this situation, we ing ratio from the LHCb data. In view of this situation, we have proposed a new strategy for determining  $f_d/f_s$  at LHCb. It uses the pair of the color-allowed, U-spin–related tree decays  $\bar{B}_{s}^{0} \rightarrow D_{s}^{+} \pi^{-}$  and  $\bar{B}_{d}^{0} \rightarrow D^{+} K^{-}$ , which are very favorable from an experimental point of view for I HCb favorable from an experimental point of view for LHCb, robust with respect to NP contributions, and theoretically well understood, thereby offering a precise measurement of  $f_d/f_s$  at LHCb. The resulting decrease of the total systematic uncertainty on  $f_d/f_s$  allows us to detect a  $5\sigma$ NP signal in the measured  $\overline{B_8^0} \to \mu^+ \mu^-$  branching ratio<br>for values as small as twice the SM value. This corresponds for values as small as twice the SM value. This corresponds to an improvement of the corresponding NP reach by a factor of 2 with respect to the present LHCb expectation. Once the dependence of  $f_d/f_s$  on  $p_T$  and/or rapidity is measured to be small, the value of  $f_d/f_s$  as determined at LHCb by means of our strategy can also be applied at ATLAS and CMS.

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