

**Upper limits on a possible gluon mass**Shmuel Nussinov<sup>1,2</sup> and Robert Shrock<sup>3</sup><sup>1</sup>*School of Physics and Astronomy, Tel Aviv University, Tel Aviv, Israel*<sup>2</sup>*Schmid College of Science, Chapman University, Orange, California 92866, USA*<sup>3</sup>*C. N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794, USA*

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We analyze upper limits on a possible gluon mass,  $m_g$ . We first discuss various ways to modify quantum chromodynamics to include  $m_g \neq 0$ , including a bare mass, a Higgs mechanism, and dynamical breaking of color  $SU(3)_c$ . From an examination of experimental data, we infer an upper limit  $m_g < O(1)$  MeV. As part of our analysis, we show that a claim, hitherto unrefuted in the literature, of a much stronger upper limit on  $m_g$ , is invalid. We discuss subtleties in interpreting gluon mass limits in view of the fact that at scales below  $\Lambda_{\text{QCD}}$ , quantum chromodynamics is strongly coupled, perturbation theory is not reliable, and the physics is not accurately described in terms of the Lagrangian degrees of freedom, including gluons. We also point out a fundamental difference in the behavior of quantum chromodynamics with a nonzero gluon mass and a weakly coupled gauge theory with a gauge boson mass.

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**I. INTRODUCTION**

In quantum chromodynamics (QCD), a mass term for the gluon in the Lagrangian is forbidden by the color  $SU(3)_c$  gauge invariance. Experimental data are consistent with the inference that the gluon mass  $m_g$  is zero. But it is of fundamental importance to inquire how stringent the experimental upper limits are on a gluon mass and what the physical consequences of such a mass would be. Considerable theoretical interest in this question was generated, starting in the late 1970s [1–8], by a report of evidence for free quarks, but later experiments did not confirm this report (some reviews are [9,10]). The literature on gluon mass limits is unsettled; published upper bounds on  $m_g$  range over 10 orders of magnitude, from values of a few MeV [10] to a claim by Yndurain that  $m_g < 2 \times 10^{-10}$  MeV [11]. It is obviously important to clarify this question, and we address it here. From our analysis, we infer an upper bound  $m_g < O(1)$  MeV and thereby resolve this controversy in the literature. In particular, we show (in Sec. III) that the claim by Yndurain [11] is invalid because it is based on the use of a nonrelativistic potential in an ultrarelativistic situation and because it assumes that in a reaction producing a quark-antiquark pair, there is no breaking of the chromoelectric flux tube extending between the quark and antiquark (i.e., no string breaking) out to distances of order  $m_g^{-1}$ , many orders of magnitude beyond the actual distance of  $\sim 1$  fm, where the string does break with the formation of color-singlet hadrons. As will be evident from our discussion, the question of an upper bound on  $m_g$  is interesting partly because it touches on some deeper conceptual issues, such as (i) how one can try to construct a modification of QCD with a gluon mass small compared with the confinement scale of  $\sim (1 \text{ fm})^{-1}$ ; (ii) the question of the extent to which one can get information on the Lagrangian fields in a confining or

quasiconfining theory; and (iii) the related quantum mechanical issue pertaining to the accuracy with which one can measure properties of spatially confined particles.

**II. TYPES OF MODIFICATIONS OF QCD TO INCLUDE A GLUON MASS****A. Bare mass**

It is first necessary to specify which type of modified QCD with nonzero gluon mass or masses one considers. There are several possible approaches to this. First, one can consider a modification of QCD in which the Lagrangian  $\mathcal{L}_{\text{QCD}}$  contains a bare gluon mass term

$$\mathcal{L}_{\text{QCD},m_g} = -\frac{m_g^2}{2} \sum_a A_\mu^a A^{a\mu}, \quad (2.1)$$

where  $a$  is the color index. Here,  $m_g$  is a hard mass [12], which is present for the arbitrarily weak QCD running coupling,  $\alpha_s(\mu) \equiv g_s(\mu)^2/(4\pi)$ . (The scale  $\mu$  will often be left implicit in the notation.) The mass term (2.1) explicitly breaks the  $SU(3)_c$  gauge invariance to a global  $SU(3)_c$  symmetry. One could also consider a more general bare gluon mass term  $-(1/2)\sum_a m_{g,a}^2 A_\mu^a A^{a\mu}$ , but the term in Eq. (2.1) will be sufficient for our purposes here.

Formally, the inclusion of the gluon mass (2.1) in QCD renders the theory perturbatively nonrenormalizable. Thus, for example, if perturbative methods were applicable and one were to compute amplitudes for longitudinally polarized gluon-gluon scattering to multigluon final states, these would have partial wave amplitudes that would involve powers of  $s/m_g^2$  and hence would violate perturbative unitarity when  $\sqrt{s}$  exceeds a value of order  $m_g$ . However, as discussed below, experimental data constrain  $m_g$  to be less than a few MeV, considerably less than the scale,  $\Lambda_{\text{QCD}} \simeq 300$  MeV at which  $\alpha_s$  grows to  $O(1)$  and QCD

exhibits the property of confinement or quasiconfinement. Here, we use the term “quasiconfinement” to mean that free color-nonsinglet states have masses much larger than  $\Lambda_{\text{QCD}}$  and hence are integrated out of the modified QCD, defined as a low-energy effective field theory. An important point is that in the mass region well below  $\Lambda_{\text{QCD}}$ , one cannot use perturbation theory. One consequence of this is that one cannot draw firm conclusions from the apparent violation of perturbative unitarity in the above-mentioned partial wave amplitudes for  $\sqrt{s} \gtrsim m_g$ . Another is that although one can formally insert a nonzero value of  $m_g$  in  $\mathcal{L}_{\text{QCD},m_g}$ , the physical meaning of this is not completely clear, because one does not perform actual physical measurements that are sensitive to this value if it is much less than  $\Lambda_{\text{QCD}}$ .

### B. Higgs mechanism

A second approach to modifying QCD to produce a gluon mass is to try to use a Higgs mechanism, with a Higgs potential arranged so as to produce a vacuum expectation value (VEV) of one or more color-nonsinglet Higgs fields, spontaneously breaking  $\text{SU}(3)_c$ . A scheme with three color triplets of Higgs fields coupled in a manner such as to break the  $\text{SU}(3)_c$  gauge symmetry to global  $\text{SU}(3)$  was studied in Ref. [1]. A second Higgs scheme is based on the observation that the structure of the baryon wave function can be retained if the breaking preserves an  $\text{SO}(3)$  subgroup of  $\text{SU}(3)_c$ , such that the quarks transform as the vector representation,  $\vec{q}$ , of this  $\text{SO}(3)$ ; this involves the equivalence of the wave function  $\epsilon_{abc} q^a q'^b q''^c$  in  $\text{SU}(3)_c$  form with  $\vec{q} \cdot (\vec{q}' \times \vec{q}'')$  in  $\text{SO}(3)$  form [6,13]. In this scheme the five gluons in the coset space  $\text{SU}(3)_c/\text{SO}(3)$  gain masses, while the three gluons corresponding to the generators of  $\text{SO}(3)$  remain massless. These three massless gluons,  $\vec{g}$ , would also naturally form  $\text{SO}(3)$ -singlet bound states,  $\vec{g} \cdot \vec{g}$ . For this scheme, one must thus use a Higgs field that contains a component transforming as a singlet under the  $\text{SO}(3)$  subgroup of  $\text{SU}(3)_c$ . The lowest-dimensional representation that has this property is the 27-dimensional representation of  $\text{SU}(3)_c$  [6,14]. A third type of Higgs model is simpler in that it only uses two Higgs fields, both transforming as fundamental representations of  $\text{SU}(3)_c$ . This breaks  $\text{SU}(3)_c$  in two stages, first to an  $\text{SU}(2)$  subgroup, and then completely, leading to two different scales of masses for gluons, which may be comparable.

A Higgs mechanism for breaking  $\text{SU}(3)_c$  and giving gluons masses has the appeal that it preserves the renormalizability of the theory. If one could analyze the model perturbatively, the gluon mass(es) would be  $\sim g_s |v|$ , where  $v$  denotes a generic VEV of the colored Higgs field(s). To illustrate this, we may consider a very simple case with just one electroweak-singlet Higgs field  $\phi$  transforming as a fundamental representation of  $\text{SU}(3)_c$ , with potential

$$V = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2, \quad (2.2)$$

where  $\mu^2 < 0$ . If one were able to use perturbation theory reliably here, then the minimization of the potential  $V$  would lead to a nonzero VEV for  $\phi$  given by  $v \propto \sqrt{-\mu^2/\lambda}$ . If, indeed, one were able to do this, then, without loss of generality, one could choose the basis for  $\text{SU}(3)_c$  generators such that  $\langle \phi \rangle_0 = (0, 0, v)^T$ . This would break  $\text{SU}(3)_c$  to the  $\text{SU}(2)_c$  subgroup generated by  $T_a$ ,  $a = 1, 2, 3$ . The five gluons in the coset  $\text{SU}(3)_c/\text{SU}(2)_c$  corresponding to the generators  $T_a$ ,  $a = 4, \dots, 8$ , would pick up masses  $m_g \sim g_s |v|$ .

However, there is an important difference between the attempt to use a Higgs mechanism to break  $\text{SU}(3)_c$  and the use of this type of mechanism to break electroweak symmetry in the standard model (SM). Given that, as discussed further below,  $m_g$  must be smaller than a few MeV, considerably below  $\Lambda_{\text{QCD}}$ , the color-nonsinglet Higgs fields interact strongly, and one cannot use perturbation theory to analyze their behavior. In particular, one cannot reliably conclude that setting  $\mu^2$  to a negative value of magnitude small compared with  $\Lambda_{\text{QCD}}^2$  would actually lead to a nonzero VEV for  $\phi$  [3]. This problem, by itself, is sufficiently severe to motivate one to consider a different renormalizable mechanism for giving gluons a mass.

Secondly, although one cannot use perturbation theory reliably to calculate the masses of residual Higgs fields, and hence they might be larger than the perturbative expressions  $m_H \sim \sqrt{\lambda} |v|$ , they could well be sufficiently light as to be excluded by experimental limits. To obtain properties of strongly coupled systems of gauge, Higgs, and fermion fields requires a fully nonperturbative calculational method, and a lattice field theory formulation can provide this. For a lattice theory with a Higgs field transforming according to the fundamental representation of the gauge group the confinement and Higgs phases are analytically connected in the absence of fermions [15], but are separated by a phase boundary when fermions are included [16]. Since the lattice formulation maintains exact local gauge invariance, a Higgs VEV as conventionally defined in the continuum vanishes identically; instead, one measures various gauge-invariant quantities, such as the bilinear fermion condensate and fermion and Higgs masses [17]. One of the issues that these nonperturbative lattice studies confronted was the question of where to take the continuum limit in the space of relevant lattice parameters and the fact that some portions of lattice phase boundaries were first order, with finite correlation lengths, instead of second-order transitions, with the infinite correlation length that is necessary to construct a continuum limit free of lattice artifacts. Notwithstanding this complication, however, these lattice studies tended to find ratios of Higgs to gauge boson masses which did not differ strongly from unity. One could thus use the results from fully nonperturbative studies to support the concern that in a Higgs picture

the spectrum would contain bound states involving the Higgs fields (with themselves or quarks) that are not seen experimentally, disfavoring the Higgs approach to trying to produce nonzero gluon masses.

There are also several other problems with a color-nonsinglet Higgs mechanism. The addition of such Higgs fields to QCD reduces the renormalization-group running of  $\alpha_s(\mu)$  as a function of energy scale  $\mu$ . The increase of  $\alpha_s(\mu)$  as the energy scale decreases from  $\mu = m_Z$  down to the scale of  $b\bar{b}$  quarkonium states is consistent with  $N_f = 5$  dynamical quarks, and the further evolution down to the scale of  $c\bar{c}$  quarkonium states is consistent with  $N_f = 4$  dynamical quarks [10,18]. This agreement would be upset if one added too many additional light color-nonsinglet Higgs to the theory. Equally if not more problematic is the fact that the quartic Higgs coupling is not asymptotically free and hence grows as the energy scale increases, undermining the asymptotic freedom of QCD and leading to a possible Landau singularity. Furthermore, the parameter  $\mu^2$  is quadratically sensitive to ultraviolet physics; i.e., there is a hierarchy problem. Because of all of these problems, it is not clear whether one could, in fact, use a Higgs mechanism to break the  $SU(3)_c$  gauge symmetry (either completely or to a nontrivial subgroup gauge symmetry) and obtain the values of gluon masses of a few MeV. This motivates one to consider alternatives.

### C. Dynamical breaking of $SU(3)_c$ and generation of gluon mass

There is a third way that one might try to break color  $SU(3)_c$  which, to our knowledge, has not received much attention in the literature, namely, via the formation of a color-nonsinglet bilinear fermion condensate produced by another strongly coupled gauge interaction. We will investigate this possibility here using a simple model but will show that this model also has problems. Let us consider an extension of the standard model gauge group  $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$  (where  $Y$  is the weak hypercharge) in which we adjoin another gauge group  $SU(2)_{mc}$ , where mc stands for metacolor (not to be confused with technicolor). Thus, the full gauge group that is operative at scales above a few GeV is taken to be

$$G_{SM'} = SU(2)_{mc} \times G_{SM}. \quad (2.3)$$

In addition to the usual SM fermions, we add left- and right-handed electroweak-singlet chiral fermions (indicated with  $L, R$ ) transforming as

$$\zeta_L^{a\alpha}: (2, 3, 1)_0, \quad (2.4)$$

$$\eta_L^\alpha: (2, 1, 1)_0, \quad (2.5)$$

and

$$\chi_{p,R}^a: 2(1, 3, 1)_0 \quad \text{for } p = 1, 2, \quad (2.6)$$

where  $a$  and  $\alpha$  denote  $SU(3)_c$  and  $SU(2)_{mc}$  gauge indices, respectively, the numbers in parentheses denote the dimensionalities of the representations of  $SU(2)_{mc} \times SU(3)_c \times SU(2)_L$ , the subscript is the value of  $Y$ , and the set includes two copies of the  $(1, 3, 1)_0$  field labeled with a copy number  $p = 1, 2$ .

By analogy with quarks, we assign baryon number  $B = 1/N_c = 1/3$  to the color-triplet fermions  $\zeta_L^{a\alpha}$  and  $\chi_{p,R}^a$ . In the Lagrangian describing the high-scale physics, these fermions are taken to have mass terms whose coefficients are zero or at least negligibly small compared with  $\Lambda_{QCD}$ . The color-singlet fermion  $\eta_L^\alpha$  is included so that there are an even number of left-handed  $SU(2)_{mc}$  doublets, as is required to avoid a global Witten anomaly associated with the homotopy group  $\pi_4(SU(2)) = \mathbb{Z}_2$ . The  $SU(2)_{mc}$  gauge sector thus contains four left-handed chiral fermions, or equivalently, two Dirac fermions, transforming as metacolor doublets. The resultant theory, consisting of these fermions plus those of the standard model, is free of anomalies in all gauged currents. The  $SU(3)_c$  interaction remains vectorial and asymptotically free. Since this model involves the introduction of two additional light flavors of color-triplet Dirac fermions to QCD, it reduces the agreement with  $N_f = 5$  quarks that characterizes the measured dependence of  $\alpha_s(\mu)$  on the scale  $\mu$  between  $\mu = m_Z$  and  $\mu \simeq 10$  GeV, the scale characterizing the  $b\bar{b}$   $Y$  states. But we will show next that the model has even more serious problems.

Since the  $SU(2)_{mc}$  gauge interaction is asymptotically free, as the reference energy scale  $\mu$  decreases from large values, its coupling  $\alpha_{mc}(\mu) \equiv g_{mc}(\mu)^2/(4\pi)$  increases. Let us first consider the case where the value of  $\alpha_{mc}(\mu)$  is sufficiently large at a high scale  $\mu \gg \Lambda_{QCD}$  so that this coupling grows to values of order unity at a scale  $\Lambda_{mc} > \Lambda_{QCD}$ . (The reason for this assumption will be explained below.) Both general arguments and a study of the Dyson-Schwinger equation for the fermion propagator in an asymptotically free vectorial  $SU(N)$  gauge theory (at zero temperature) with  $N_f$  copies of massless fermions transforming according to the fundamental representation of this gauge group suggest that if  $N_f < N_{f,cr}$ , then as the theory evolves into the infrared, it produces a bilinear fermion condensate that spontaneously breaks the global chiral symmetry, whereas if  $N_f > N_{f,cr}$ , such a condensate is not formed and instead the chiral symmetry remains exact, where  $N_{f,cr}$  is a certain critical number [19]. For the case  $N = 2$  relevant here, a solution of the Dyson-Schwinger equation in the one-gluon exchange approximation yields the value  $N_{f,cr} \simeq 8$ . Since this is well above the number  $N_f = 2$  that we have in the  $SU(2)_{mc}$  model, we can confidently infer, given our assumption that, as the scale  $\mu$  decreases, the metacolor coupling gets large before the color coupling does, that the  $SU(2)_{mc}$  interaction produces bilinear fermion condensates. The most attractive channel for these is  $2 \times 2 \rightarrow 1$ , where the numbers refer to



the dimensionalities of fermion  $SU(2)_{\text{mc}}$  representations. These include a condensate  $\langle \epsilon_{\alpha\beta} \zeta_L^{a\alpha} T C \zeta_L^{b\beta} \rangle$ , where  $\epsilon_{\alpha\beta}$  is the antisymmetric tensor density for  $SU(2)_{\text{mc}}$ . This is automatically antisymmetrized in the color indices  $a, b$  and hence is proportional to

$$\langle \epsilon_{abc} \epsilon_{\alpha\beta} \zeta_L^{a\alpha} T C \zeta_L^{b\beta} \rangle, \quad (2.7)$$

where  $\epsilon_{abc}$  is the antisymmetric tensor density for  $SU(3)_c$ . The condensate (2.7) transforms as a color  $\bar{3}$  and hence dynamically breaks  $SU(3)_c$  to an  $SU(2)$  subgroup. A second condensate formed by the  $SU(2)_{\text{mc}}$  interaction is

$$\langle \epsilon_{\alpha\beta} \zeta_L^{a\alpha} T C \eta_L^\beta \rangle. \quad (2.8)$$

This transforms as a  $\underline{3}$  under  $SU(3)_c$  and hence also breaks it to an  $SU(2)$  subgroup. One can use vacuum alignment arguments to infer that these  $SU(2)$  subgroups are identical. Then, without loss of generality, one may choose the index  $c = 3$  in the condensate (2.7) and  $a = 3$  in the condensate (2.8), so that the residual  $SU(2)$  subgroup of  $SU(3)_c$  preserved by these condensates, which we will denote as  $SU(2)_c$ , is the one for which the corresponding Lie algebra is generated by  $T_a$  with  $a = 1, 2, 3$ . The five gluons in the coset  $SU(3)_c/SU(2)_c$ , i.e., those corresponding to  $T_a$ ,  $4 \leq a \leq 8$ , gain masses of order

$$m_g \sim g_s(\Lambda_{\text{mc}}) \Lambda_{\text{mc}} \sim \Lambda_{\text{mc}}. \quad (2.9)$$

The fermions involved in these condensates,  $\zeta_L^{a\alpha}$  with  $a = 1, 2, 3$  and  $\eta_L^\alpha$  for  $\alpha = 1, 2$ , also gain dynamical masses of order  $\Lambda_{\text{mc}}$ .

This, then, is a renormalizable, dynamical way to break  $SU(3)_c$  [to  $SU(2)_c$ ]. In contrast to a Higgs mechanism, it is technically natural and does not suffer from any hierarchy problem. This model shows that the property that a gauge symmetry is vectorial is not sufficient, in itself, to ensure that it remains unbroken. Indeed, since the weak isospin  $SU(2)_L$  gauge interaction is asymptotically free, if it had not been broken at the electroweak scale but instead had been able to grow in strength to a sufficient level, it would have broken  $SU(3)_c$  to  $SU(2)_c$  in a somewhat analogous manner [20]. One could presumably add additional fields and/or interactions to this metacolor model so that  $SU(3)_c$  would be broken completely. However, although this dynamical approach avoids some of the problems with the other approaches to breaking  $SU(3)_c$  that we have described above, the gluon masses that it produces, given in Eq. (2.9), are too large to be allowed by experiment. This is clear from an example. Consider, say, the value  $\Lambda_{\text{mc}} = 10$  GeV. Experimental data determine  $\alpha_s(10 \text{ GeV}) = 0.18$  (see, e.g., Fig. 6 of Ref. [18]), i.e.,  $g_s(10 \text{ GeV}) = 1.5$ . Then from Eq. (2.9), we would obtain  $m_g \sim 15$  GeV, which is much too large to agree with experiment. A second illustrative choice is  $\Lambda_{\text{mc}} = 1$  GeV. For this choice, one has  $\alpha_s(1 \text{ GeV}) \approx 0.5$  [18]; from Eq. (2.9) one obtains  $m_g \sim 2.5$  GeV, which is again much too large. One cannot

improve this situation by selecting initial conditions for  $\alpha_{\text{mc}}(\mu)$  at a high scale  $\mu$  so that  $\Lambda_{\text{mc}}$  is smaller than  $\Lambda_{\text{QCD}}$ , because if the  $SU(3)_c$  interaction becomes strongly coupled, with  $\alpha_s \sim O(1)$ , at a scale where the  $SU(2)_{\text{mc}}$  interaction is still weakly coupled, then among the bilinear fermion condensates produced by the QCD interaction, in addition to  $\langle \bar{q}_L q_R \rangle + \text{H.c.}$  for  $q = u, d, s$ , there would be

$$\langle \bar{\zeta}_{a,\alpha,L} \chi_{p,R}^a \rangle, \quad p = 1, 2, \quad (2.10)$$

which would break  $SU(2)_{\text{mc}}$ . This is analogous to the fact that the QCD quark condensates  $\langle \bar{q}_L q_R \rangle + \text{H.c.}$  break  $SU(2)_L$  (which, however, is already broken at the much higher scale 250 GeV). Since the  $SU(2)_{\text{mc}}$  symmetry would not be active in the low-energy effective theory applicable at energy scales below  $\Lambda_{\text{QCD}}$ , its coupling  $\alpha_{\text{mc}}$  would be frozen at this scale and hence would not become large enough to break color. Thus, although this model for dynamically breaking  $SU(3)_c$  is renormalizable and does not have a hierarchy problem, it is excluded by the fact that it would yield excessively large values for the gluon masses. It also would have the problem that it would predict new hadronic states at experimentally accessible masses, and these have not been observed.

One could also consider other mechanisms such as attempting to formulate QCD in five or more dimensions and choosing boundary conditions in the higher-dimensional space that break  $SU(3)_c$  and give rise to a gluon mass in the usual  $(3 + 1)$ -dimensional Minkowski space. The fact that the higher-dimensional theory is not renormalizable leads to a number of complications, and we do not pursue this direction here.

Our analysis of various approaches to producing a gluon mass that is small compared with  $\Lambda_{\text{QCD}}$  has thus shown the difficulties that one encounters with each of these approaches. Although our analysis is not exhaustive, it does show how challenging it is to construct a self-consistent calculable model that could explain a gluon mass that is small compared with the scale where QCD becomes strongly coupled and confines. It is also worth noting that the property  $m_g = 0$  is protected by the color gauge invariance, and once this condition is removed, i.e., once one considers  $m_g \neq 0$ , breaking  $SU(3)_c$  gauge invariance, then there is no obvious symmetry that could naturally keep  $m_g$  small compared with other relevant scales, in particular,  $\Lambda_{\text{QCD}}$ .

### III. AN UPPER BOUND ON $m_g$ FROM EXPERIMENTAL DATA

Here we step back from the construction of a model that could account naturally for a small gluon mass and, in a more phenomenological framework, analyze how large a value of  $m_g$  might be allowed by experimental data. To the extent that we need a theoretical framework, we will use that given by the hard bare mass term in Eq. (2.1),

recognizing that it would require an ultraviolet completion to answer the question of the origin of the gluon mass. There are many pieces of experimental evidence showing that  $m_g$  must be considerably smaller than  $\Lambda_{\text{QCD}}$ ; the question is how much smaller.

By standard Bohr-Oppenheimer and effective field theory arguments, if a particle has a mass  $m$ , then it does not play a dynamical role in the low-energy effective theory that is operative at scales well below  $m$ . It follows quite generally that  $m_g$  must be smaller than  $\Lambda_{\text{QCD}}$  because if it were not, then as the reference energy scale  $\mu$  decreased, gluons would be integrated out before  $\mu$  decreased to  $\Lambda_{\text{QCD}}$ , and hence  $\alpha_s(\mu)$  would never grow to values of  $O(1)$ . A weakly coupled QCD with a nonzero  $m_g$  would not confine, so that there would be color-nonsinglet physical states in nature. (This would be analogous to the fact that since weak-isospin  $SU(2)_L$  is a broken gauge symmetry, it does not confine neutrinos or charged leptons.) But, in fact, there are no confirmed observations of such states, in particular, free quarks, and there are quite stringent upper limits on them, both from searches in matter and in collider experiments [9,10]. Furthermore, analyses of the Dyson-Schwinger equation for the quark propagator [21,22] have shown that if one starts with a zero-mass quark, then, if  $C_2(R)\alpha_s$  is greater than roughly unity [where  $C_2(R)$  is the quadratic Casimir invariant for the representation  $R$ , equal to  $4/3$  for the fundamental representation of  $SU(3)_c$ ], this equation yields a solution with a nonzero value of the quark mass. This constitutes dynamical generation of a constituent quark mass, the result of spontaneous chiral symmetry breaking in QCD. That this is associated with confinement can be understood by a simply physical argument [23]: as a quark is headed outward from the center of a hadron and is reflected back inward at the boundary, there is a flip of chirality, which amounts to the presence of a  $\bar{q}q$  term in the effective Lagrangian. Since this dynamical mass is the coefficient of  $\bar{q}q$  in the effective QCD Lagrangian, one may also associate this with the dynamical generation of a condensate  $\langle \bar{q}q \rangle$ . This spontaneous chiral symmetry breaking gives rise to the approximate Nambu-Goldstone bosons of QCD, the  $\pi$ ,  $K$ , and  $\eta$  mesons [24,25]. However, if the gluons were integrated out of the theory so that  $\alpha_s$  never grew to values of  $O(1)$ , then this spontaneous chiral symmetry breaking would not occur.

In the presence of a nonzero gluon mass, even one much smaller than  $\Lambda_{\text{QCD}}$ , the  $SU(3)_c$  color gauge invariance is broken. A consequence of this is that color is, in fact, not completely confined. This is obvious in the conventional Higgs picture, given the inferred nonzero VEV of the color-nonsinglet Higgs. It also holds in the model presented above in which there is dynamical breaking of  $SU(3)_c$ . Consequently, an isolated quark or gluon does not have infinite energy, and the Wilson-Polyakov line for such a particle does not vanish identically. If the limit  $m_g/\Lambda_{\text{QCD}} \rightarrow 0$  is smooth, one expects that the mass of a

free quark or gluon would diverge. Thus, in a low-energy effective theory, these color-nonsinglet states would disappear from the spectrum as  $m_g \rightarrow 0$ , and the resultant theory would exhibit the property of being quasiconfining, in the sense that we defined above. Thus, for  $m_g \ll \Lambda_{\text{QCD}}$ , QCD, considered as a low-energy effective theory, is quasiconfining.

It is worth noting that this expectation for the dependence of the mass of a free quark or gluon on  $m_g$  is borne out by a specific calculation within the context of the Massachusetts Institute of Technology (MIT) bag model. Let us denote these states, dressed with their cloud of glue, as  $q_{\text{dr}}$  and  $g_{\text{dr}}$ . The MIT bag model yields the masses [1]

$$m_{q_{\text{dr}}} = \frac{\sigma}{m_g} [1 + O((m_g/\sqrt{\sigma})^{1/3})] = \frac{0.18 \text{ GeV}^2}{m_g}, \quad (3.1)$$

and

$$m_{g_{\text{dr}}} = \frac{3}{2} m_{q_{\text{dr}}}, \quad (3.2)$$

where

$$\sigma = \frac{1}{2\pi\alpha'} = (420 \text{ MeV})^2 = 0.90 \text{ GeV/fm} \quad (3.3)$$

is the QCD string tension and  $\alpha' = 0.9 \text{ GeV}^{-2}$  is the Regge slope. Because the string tension  $\sigma \propto \Lambda_{\text{QCD}}^2$ , these masses can also be written in the form

$$m_{q_{\text{dr}}} = \frac{2}{3} m_{g_{\text{dr}}} = \text{const} \frac{\Lambda_{\text{QCD}}^2}{m_g} [1 + O((m_g/\sqrt{\sigma})^{1/3})]. \quad (3.4)$$

These estimates show how the limit  $m_g/\Lambda_{\text{QCD}} \rightarrow 0$  can be smooth, in the sense of low-energy effective field theory, since as  $m_g \rightarrow 0$ , the masses of a free quark or gluon diverge and they are integrated out of the low-energy theory. The physical, finite-mass states in this low-energy theory are color singlets.

Searches for free quarks in collider experiments depend on assumptions about their electroweak transformation properties and decays [10,26]; current lower limits from collider searches on a quark of charge  $2/3$  or  $-1/3$  vary between about 200 and 340 GeV. Taking the lower bound of 300 GeV as a representative illustrative value and inserting this into Eq. (3.4), one obtains the nominal upper bound  $m_g \lesssim 0.5 \text{ MeV}$ . Since there are model-dependent aspects to the MIT bag model estimates of the masses of a free quark or gluon, it is appropriate to allow a factor of a few to represent the theoretical uncertainty, and also a similar factor to represent the effect of model-dependent assumptions in the limits obtained from experimental searches. Including these, we infer that a reasonable upper bound on a possible gluon mass is

$$m_g < O(1) \text{ MeV}. \quad (3.5)$$

It is also useful to estimate the dependence of the size of the  $q_{\text{dr}}$  and  $g_{\text{dr}}$  states on  $m_g$  for  $m_g \ll \Lambda_{\text{QCD}}$ ; one sets the masses in Eq. (3.4) equal to the approximate volume  $(4\pi/3)r^3$  times the energy density, set by  $\Lambda_{\text{QCD}}^4$ , and hence obtains

$$\begin{aligned} r &\sim \left( \frac{1}{m_g \Lambda_{\text{QCD}}^2} \right)^{1/3}, \\ &\sim \frac{1}{\Lambda_{\text{QCD}}} \left( \frac{\Lambda_{\text{QCD}}}{m_g} \right)^{1/3}, \end{aligned} \quad (3.6)$$

i.e.,  $r \sim 1 \text{ fm} (\Lambda_{\text{QCD}}/m_g)^{1/3}$ . Hence, for  $m_g$  small compared with  $\Lambda_{\text{QCD}}$ , the sizes of deconfined, dressed quarks and gluons would be substantially larger than the typical 1 fm size of a usual hadron.

Another approach to the question of an upper limit on a gluon mass is to study the effects of a nonzero  $m_g$  on the static quark potential between a very heavy quark  $Q$  and antiquark  $\bar{Q}$ . The short-distance part of this potential for  $r \ll \Lambda_{\text{QCD}}^{-1}$  has the Coulombic form

$$V_{Q\bar{Q},\text{sd}}(r) = \frac{(4/3)\alpha_s(r)}{r}, \quad (3.7)$$

and because short distances are equivalent to large  $\mu$ , a nonzero  $m_g$  that is small compared with  $\Lambda_{\text{QCD}}$  would not affect this significantly. In regular QCD,

$$V_{Q\bar{Q}} = \sigma r \quad \text{for } r \gtrsim \Lambda_{\text{QCD}}^{-1} \sim 1 \text{ fm}. \quad (3.8)$$

This linear growth in  $V_{Q\bar{Q}}$  corresponds to the property that a chromoelectric flux tube with energy per unit length  $\sigma$  stretches between the  $Q$  and  $\bar{Q}$ . Making  $m_g$  nonzero changes this so that for  $r \gtrsim m_g^{-1}$ ,  $V_{Q\bar{Q}}(r)$  is damped by a factor  $e^{-m_g r}$  and hence decreases to zero for large  $r$  rather than increasing without bound. In turn, this implies that  $V_{Q\bar{Q}}(r)$  reaches a maximum at some value of  $r \sim m_g^{-1}$ , where the force between the  $Q$  and  $\bar{Q}$ ,  $\vec{F} = -\vec{\nabla} V_{Q\bar{Q}}(r)$  vanishes. This is another indication that once  $m_g$  is nonzero, QCD no longer precisely confines, since a quark can tunnel through this potential barrier. If QCD only had very heavy quarks, then an analysis of this static quark potential could provide a useful guide to an upper limit on  $m_g$ .

However, real QCD has light quarks. This has two effects: first, one cannot use nonrelativistic quantities such as a potential energy associated with a  $q\bar{q}$  state reliably, because the physics is relativistic, and second, the chromoelectric flux tube forming the string breaks in the process of hadronization. That is, when an actual  $q\bar{q}$  pair is produced in a reaction like  $e^+e^- \rightarrow q\bar{q}$ , as the  $q$  and  $\bar{q}$  separate to a distance  $r \sim \Lambda_{\text{QCD}}^{-1} \sim 1 \text{ fm}$ , and hence the energy in the chromoelectric flux tube (string) is sufficient to produce hadronic final states, such as  $2\pi$ ,  $4\pi$ , etc. it is energetically favorable for the string to break with production of additional light  $q\bar{q}$  pairs and gluons, followed by

hadronization. The presence of a string extending to a few fm can be interpreted as being responsible for short-lived hadronic resonances lying on Regge trajectories up to masses of several GeV. But the string (chromoelectric flux tube) does not stretch beyond a few fm; instead, it is divided into smaller string bits as the  $q\bar{q}$  pairs are created. For the relevant range of  $m_g$  of a few MeV, recalling that  $(1 \text{ MeV})^{-1} = 200 \text{ fm}$ , the string breaking and  $q\bar{q}$  pair creation and resultant hadronization occur before the  $e^{-m_g r}$  factor becomes relevant. If, nevertheless, one were to attempt to apply a static quark potential assuming no string breaking out to distances of order  $m_g^{-1}$ , one would obtain apparently quite stringent apparent upper bounds on  $m_g$  [11]. We consider that both the use of a nonrelativistic potential in an ultrarelativistic situation and this assumption of no string breaking out to distances of order  $m_g^{-1}$  are unjustified and hence that the very stringent bound  $m_g < 2 \times 10^{-10} \text{ MeV}$  claimed by Yndurain [11] is invalid. Indeed, if  $m_g$  were really  $2 \times 10^{-10} \text{ MeV}$ , then the string persistence length assumed in Ref. [11] would be a factor  $10^{12}$  longer than the actual hadronization distance of  $\sim 1 \text{ fm}$ .

The hadronization process can be modeled approximately via a non-Abelian extension of the Schwinger mechanism [27]. Although this has not been calculated for  $m_g \neq 0$ , a rough estimate of the effect of a gluon mass can be obtained from the result for  $q\bar{q}$  production by the Schwinger mechanism [27],

$$\frac{dW}{d^4x} \simeq \frac{\xi^2}{4\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(\frac{-n\pi m_q^2}{\xi}\right), \quad (3.9)$$

where  $\xi \equiv (g_s/2)\mathcal{E}$ , with  $\mathcal{E}$  serving as a measure of the magnitude of the chromoelectric field in the flux tube (a general expression in terms of quantities that are manifestly gauge invariant and Lorentz invariant is given in [28]), and  $m_q$  is the quark mass. We denote the area of a cross section of the flux tube by  $A$ . The string tension, given by  $\sigma \sim (\mathcal{E}^2/2)A$ , and Gauss's law implies that  $\mathcal{E}A = g/2$ ; combining these to eliminate  $A$  and using the fact that  $\sigma = 1/(2\pi\alpha')$ , one obtains

$$\xi = (g_s/2)\mathcal{E} \simeq 2\sigma = 0.35 \text{ GeV}^2. \quad (3.10)$$

It is plausible that the kinematic dependence of  $dW/d^4x$  on  $m_g$  would be somewhat similar to the dependence on  $m_q$ . We shall assume this and require that  $dW/d^4x$  not change by more than a small fractional amount  $\epsilon$ . For a rough bound, we retain just the first term in the sum (3.9), which is the dominant term, and we require that the fractional change in this term be less than  $\epsilon$ , i.e.,  $1 - \exp(-\pi m_g^2/\xi) < \epsilon$ . This yields the upper limit

$$m_g < \left[ \frac{\xi}{\pi} \ln\left(\frac{1}{1-\epsilon}\right) \right]^{1/2}. \quad (3.11)$$



With the above estimate for  $\xi$  and the illustrative value  $\epsilon = 0.01$ , this yields the upper bound  $m_g \lesssim 35$  MeV, a somewhat less stringent bound than was obtained in (3.5).

There is currently no evidence for the proton decay or decays of neutrons that are stably bound in nuclei, with typical partial lifetime limits  $\tau/B > 10^{33}-10^{34}$  yr, where  $B$  denotes the branching ratio for the given mode [10]. Different types of  $SU(3)_c$  breaking and gluon mass generation yield different predictions for how this would change. Thus, the binding of protons would disappear if all gluons got masses of order  $\Lambda_{\text{QCD}}$ , or if  $SU(3)_c$  were broken to an  $SU(2)$  subgroup, but would remain if  $SU(3)_c$  were broken to a gauged  $SO(3)$  subgroup. Thus, although in principle one could use limits on proton and bound neutron instability to constrain  $m_g$ , the results would depend strongly on the assumed type of  $SU(3)_c$  color breaking and gluon mass generation. Let us consider the case where  $SU(3)_c$  is either broken completely or broken to an  $SU(2)$ , rather than a  $SO(3)$  subgroup. Then a proton could decay via a tunneling process in which a quark tunneled out. However, this tunneling process would be very different from the tunneling mechanism responsible for  $\alpha$  decays of heavy nuclei. In the  $\alpha$  decays of heavy nuclei, the emitted  $\alpha$  particle has essentially the same mass that it has inside the parent nucleus. In contrast, for the relevant range of  $m_g \lesssim O(1)$  MeV given by (3.5), a  $u$  or  $d$  quark with a current-quark mass of about 5 or 10 MeV and a constituent mass of about 330 MeV inside a proton would have a mass of order hundreds of GeV outside of the proton. Clearly, not only would there be suppression of the tunneling process that might give rise to this emission of a quark, but also it would be energetically forbidden.

Other effects of  $SU(3)_c$  color breaking and nonzero gluon masses would occur in the early universe. Here, however, the temperature is finite rather than zero, so that, strictly speaking, one would not be dealing with a Lorentz-invariant gluon mass, but rather a gluonic screening mass. For the relevant range of allowed values of  $m_g$  given by Eq. (3.5), which are considerably below  $\Lambda_{\text{QCD}}$ , it follows that the finite-temperature phase transition in the early universe would occur at a temperature  $T_c \simeq 200$  MeV, where  $SU(3)_c$ -breaking effects were negligible. Hence, the formation of free quarks in the early universe would mostly occur starting from color-singlet states. As in our discussion above, this formation process depends on assumptions about how far the string between  $q$  and  $\bar{q}$  stretches before it breaks. Owing to this and other model-dependent features of the calculation, there is, for a given  $m_g$ , a wide range of possible predictions for the resultant ratio in the number density of free quarks to baryons,  $n_q/n_B$ . For  $m_g \sim \text{few MeV}$ , using results from Ref. [27], Ref. [7] found that  $n_q/n_B$  could be in accord with experimental bounds of order  $10^{-22}$  [7]. (See Ref. [10] for current upper limits on  $n_q/n_B$ .) We are in agreement with Ref. [7] but note that Ref. [11], assuming considerably longer string

persistence lengths, claimed the much more stringent limit  $m_g < 2 \times 10^{-10}$  MeV.

#### IV. ON THE MEASURABILITY OF A SMALL GLUON MASS

The rough upper limit (3.5) shows that  $m_g$  must be small compared with  $\Lambda_{\text{QCD}}$ . To what extent can one make this more precise? In this section, we address this question and stress that there is a basic problem that one encounters in trying to do this. Our starting point is the property that QCD with  $m_g = 0$  (at zero temperature) confines. It may be recalled that in addition to the experimental evidence, a convincing theoretical understanding of this has come from the lattice gauge theory formulation of the theory. Because the measure of the Euclidean QCD path integral on the lattice is compact, one avoids inserting a Faddeev-Popov determinant in this measure and maintains exact gauge invariance at all stages of the calculation. One can then rigorously define an order parameter for confinement, namely, the Wilson loop. The area-law behavior of the Wilson loop at strong bare coupling, i.e., small  $\beta = 2N_c/g_0^2$ , in conjunction with numerical simulations that suggest that one can analytically continue from this limit to the continuum limit at  $\beta \rightarrow \infty$  constitute strong evidence that the continuum QCD defined in this limit confines color. The physical picture for this is the chromoelectric flux tube that extends between an infinitely heavy, static quark and antiquark, producing a static quark potential (3.8), which grows without bound as  $r \rightarrow \infty$ . We have seen how, for  $m_g \ll \Lambda_{\text{QCD}}$ , although the modified QCD does not precisely confine, the deconfined quarks and gluons have masses that are much larger than  $\Lambda_{\text{QCD}}$ . Hence, insofar as one deals with QCD as an effective low-energy theory, the physical states in this theory with masses that are of order  $\Lambda_{\text{QCD}}$  or at least not many orders of magnitude larger than this scale are color singlets. But this means that in this effective low-energy theory, the physics is well described at realizable energies by a confining theory and not by the Lagrangian fields, the quarks and gluons. This statement becomes progressively more accurate as  $m_g/\Lambda_{\text{QCD}}$  decreases toward zero. This suggests that it would be futile to try to set an upper limit on a gluon mass that is many orders of magnitude smaller than  $\Lambda_{\text{QCD}}$  because there is no well-defined gluon in the effective QCD theory that is applicable in this energy range.

Indeed, it follows that because gluons are quasiconfined, basic quantum mechanics places a limit on how precisely one can probe for a nonzero but small  $m_g$  and prevents one from setting an upper limit on  $m_g$  that is many orders of magnitude less than  $\Lambda_{\text{QCD}}$ . Since the color-singlet hadrons have sizes of order  $1/\Lambda_{\text{QCD}} \sim 1$  fm and the gluons are effectively confined within a distance of this order, the Heisenberg uncertainty principle dictates that one cannot, even in principle, measure the gluon momentum or energy

to a better accuracy than  $\Delta|\vec{k}_g| \sim \Lambda_{\text{QCD}}$  and  $\Delta E_g \sim \Lambda_{\text{QCD}}$ . Hence, from such a measurement, one cannot, in principle, distinguish between the case where  $E_g = \sqrt{|\vec{k}_g|^2 + m_g^2}$  and the case where  $E_g = |\vec{k}_g|$  for  $m_g \ll \Lambda_{\text{QCD}}$ . Confinement in the effective QCD theory also implies a minimum bound-state gluon momentum  $\vec{k}_g$  of order  $\Lambda_{\text{QCD}}$  [29]. Indeed, the confined gluon propagator does not have a pole, and hence the gluon does not have a well-defined mass.

In principle, one might attempt to calculate glueball masses as a function of  $m_g$ , then compute how their mixing with  $q\bar{q}$  mesons to form mass eigenstates changes as a result of varying  $m_g$ , and then compare the results with experimental data to derive an upper bound on  $m_g$ . For many calculations of QCD properties, the lattice formulation is the appropriate tool. As noted above, the most natural formulation of lattice QCD maintains exact local gauge invariance, and one would have to give up this advantage if one were to try to use the lattice to study glueballs in the case of a nonzero gluon mass, since this mass breaks the color gauge invariance. Lattice QCD calculations of glueball masses have been done for pure glue or glue with quenched, but not light dynamical fermions [30]. Ideally, one would do this calculation with light dynamical fermions, compute the mixing matrix that maps the (isosinglet)  $J^{PC} = 0^{++}$  states of  $(|u\bar{u}\rangle + |d\bar{d}\rangle)/\sqrt{2}$ ,  $|s\bar{s}\rangle$ , and  $|\text{glue}\rangle$  to the  $f_0$  mesons  $f_0(1370)$ ,  $f(1500)$ , and  $f(1710)$  (among others) and then compare with experiment. However, there is no consensus what this mixing matrix is experimentally, even for regular QCD with  $m_g = 0$  [31]. Another idea would be to try to look for some kinematic signature of a small nonzero  $m_g$  in hadron decays, similar to a test for quark masses in  $D_s$  decays [32], using helicity suppression arguments. But the situation is not analogous because of the presence of gluonic self-coupling and resultant  $g \rightarrow gg$  transitions.

## V. CONTRAST WITH ESTIMATES OF QUARK MASSES

It is of interest to contrast the situation concerning an upper bound on a possible gluon mass with estimates of what are denoted the current or ‘‘hard’’ masses of the light quarks  $u$  and  $d$  and the intermediate-mass quark  $s$  [33]. Here, these hard masses are to be distinguished from the ‘‘soft’’ constituent masses of order  $\Lambda_{\text{QCD}}$  that are generated dynamically for the light quarks by the formation of the quark condensates  $\langle \bar{q}q \rangle$  that spontaneously break chiral symmetry. The key point here is that in the limit in which one turns off color gauge interactions, quarks still have weak and electromagnetic interactions, but there are no gluons, since the gluons only enter as the gauge bosons of QCD. The current or hard masses of the quarks are, indeed, defined as the masses that these particles would have in

the hypothetical limit in which QCD is turned off [33]. It has been challenging to determine the current-quark masses  $m_u$  and  $m_d$  of the light quarks  $u$  and  $d$ . Two further differences with respect to the gluons have enabled one to obtain approximate values for these. First, from the nucleon masses  $m_p$  and  $m_n$ , one can infer that  $m_u$  and  $m_d$  differ only by a few MeV and that  $m_d > m_u$ . Second, because of spontaneous chiral symmetry breaking, one has Gell-Mann–Oakes–Renner (GMOR) relations such as [34]

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} \langle \bar{q}q \rangle. \quad (5.1)$$

The measured values of  $m_\pi$  and  $f_\pi$ , together with a determination of  $\langle \bar{q}q \rangle$  from, e.g., the lattice, then yield the value of  $m_u + m_d$ . From the corresponding GMOR relations for  $m_{K^+}^2$  and  $m_{K^0}^2$ , assuming flavor independence of  $\langle \bar{q}q \rangle$  for  $q = u, d, s$ , one can obtain approximate values for the ratios  $m_d/m_u \simeq 2$  and  $m_s/m_d \simeq 20$  (e.g., [35]). The fact that, even with these methods and modern refinements [35], there is still non-negligible uncertainty in  $m_u$  and  $m_d$  shows the difficulty of extracting (hard) masses of light confined particles.

## VI. HIGH-ENERGY BEHAVIOR OF CROSS SECTIONS IN QCD WITH $m_g \neq 0$

We elaborate here on an interesting point that we noted at the beginning of this paper. Let us consider a modified QCD theory with the nonzero  $m_g$  in Eq. (2.1) satisfying the bound (3.5). We treat this theory as an effective field theory, valid up to some UV cutoff  $\Lambda_{\text{UV}}$ . In order for it to be a useful theory and to match experimental data, it is necessary that  $\Lambda_{\text{UV}} \gg \Lambda_{\text{QCD}}$ . This condition should hold if  $m_g/\Lambda_{\text{QCD}} \ll 1$ . A very interesting aspect of this construction is its contrast with the Higgs mechanism in a weakly interacting theory, such as the standard model. Taking the limit of large Higgs mass in the SM, one obtains an estimate of the highest energy to which the resultant theory (denoted SM') can be used as a perturbatively calculable effective field theory, namely  $\Lambda_{\text{UV,SM}'} \simeq 4\sqrt{\pi}v_{\text{EW}} = 8\sqrt{\pi}m_W/g = 1.7 \text{ TeV}$  [36], where  $g$  is the weak  $\text{SU}(2)_L$  gauge coupling and  $v_{\text{EW}} \equiv 2^{-1/4}G_F^{-1/2} = 246 \text{ GeV}$ . For  $\sqrt{s} \gtrsim \Lambda_{\text{UV,SM}'}$ , the  $J = 0$  partial wave amplitude for longitudinal vector boson scattering violates perturbative unitarity, indicating the onset of strongly coupled physics. However, the analogous procedure is not applicable in our present case of QCD with  $m_g \neq 0$ , because for the relevant range given by Eq. (3.5), longitudinal gluon-gluon scattering is not perturbatively calculable, as a result of the strong coupling  $\alpha_s \sim O(1)$ . The nonapplicability of the perturbative partial wave amplitude analysis to QCD with  $m_g \simeq O(1) \text{ MeV}$  is clear, because if one were able to apply it, then, in terms of the color-nonsinglet Higgs VEV  $|v|$ ,



one would get  $\Lambda_{UV} \sim |v| \sim m_g/g_s$ . But this result would not make physical sense, since it would imply that, for example, QCD with  $m_g = 1$  eV would break down at a scale of order 1 eV. In QCD with  $m_g \neq 0$ , the expression for  $\Lambda_{UV}$  has a form that is fundamentally different from the form  $\Lambda_{UV} \sim m_g/g_s$  that one would obtain for a perturbative theory. The property that QCD with  $m_g \neq 0$  matches onto the theory with  $m_g = 0$  as  $m_g \rightarrow 0$  implies that QCD with  $m_g \neq 0$  should be a good effective theory up to a scale  $\Lambda_{UV}$  of the form

$$\Lambda_{UV} \sim \Lambda_{QCD} \left( \frac{\Lambda_{QCD}}{m_g} \right)^\nu, \quad (6.1)$$

where the exponent  $\nu > 0$ . The strong-coupling nature of the theory in the region of energies  $\sqrt{s} \sim \Lambda_{QCD}$  makes it difficult to obtain a precise value of the exponent  $\nu$ , but it is plausible that  $\nu \sim O(1)$ . This value is in accord with the MIT bag model estimate of the masses of free quarks and gluons in Eqs. (3.1) and (3.2).

## VII. CONTRAST WITH LIMITS ON A PHOTON MASS

There is a striking contrast in the modest upper limit of a few MeV that one can obtain for  $m_g$  and the very stringent upper limit on the photon mass,  $m_\gamma \lesssim 10^{-19}$  eV [37]. The fact that the upper bound on  $m_\gamma$  is so much smaller than the upper bound on  $m_g$  can be traced to the property that the photon is not confined, together with the property that matter is electrically neutral on a macroscopic scale, and the ability to observe electromagnetic fields, such as those associated with planetary dipole fields and the solar wind, that have quite large spatial extent. Note that conditions other than confinement could limit one's ability to set a bound on the photon mass. For example, consider the hypothetical situation in which one were restricted to making observations in the interior of a metal, where, instead of freely propagating photons, there are plasmons with plasma frequency  $\omega_p$  given by  $\omega_p^2 = 4\pi n e^2 / m_e$  ( $n$  = number density of electrons). Then one would only be

able to obtain an upper bound on  $m_\gamma$  that was a small fraction of  $\omega_p$  (where  $\hbar\omega_p \sim$  few eV in typical metals). A similar comment would apply if one were restricted to making observations in a medium where there is Debye screening.

## VIII. CONCLUSION

In this paper we have revisited the question of upper limits on a possible gluon mass. We have discussed various ways of modifying QCD to produce gluon masses. From an analysis of experimental constraints, we have concluded that a reasonably robust upper bound is  $m_g < O(1)$  MeV, given in Eq. (3.5). We believe that our analysis settles an existing disagreement (by 10 orders of magnitude) in the literature concerning an upper bound on  $m_g$ . We have discussed some of the subtle conceptual issues that one must confront in trying to set an upper bound on  $m_g$  that would be very far below the scale,  $\Lambda_{QCD} \approx 300$  MeV. These include the fact that in this mass range one cannot use perturbation theory reliably and the physics is not accurately described in terms of the Lagrangian degrees of freedom, including gluons. Since the inapplicability of perturbation theory makes it difficult to use a Higgs mechanism reliably to produce a small gluon mass, we have explored how one might do this with a nonperturbative dynamical mechanism and have shown how this attempt would yield excessively large values of  $m_g$ . We have shown how quasiconfinement in QCD with a small gluon mass, in conjunction with the Heisenberg uncertainty principle, renders it difficult to set an upper limit on  $m_g$  that is very small compared with  $\Lambda_{QCD}$ . As part of our analysis, we have also shown that the ultraviolet cutoff  $\Lambda_{UV}$  on QCD with  $m_g \neq 0$ , considered as an effective field theory, has a very different form from the ultraviolet cutoff in the electroweak theory with a heavy Higgs.

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- [1] A. De Rújula, R. C. Giles, and R. L. Jaffe, *Phys. Rev. D* **17**, 285 (1978).
  - [2] J. D. Bjorken, in *Proceedings of the EPS International Conference on High Energy Physics, Geneva, 1979* (CERN, Geneva, 1979).
  - [3] H. Georgi, *Phys. Rev. D* **22**, 225 (1980).
  - [4] A. De Rújula, R. C. Giles, and R. L. Jaffe, *Phys. Rev. D* **22**, 227 (1980).
  - [5] L. B. Okun and M. A. Shifman, *Z. Phys. C* **8**, 17 (1981).
  - [6] R. Slansky, T. Goldman, and G. L. Shaw, *Phys. Rev. Lett.* **47**, 887 (1981); G. Shaw and R. Slansky, *Phys. Rev. Lett.* **50**, 1967 (1983).
  - [7] E. W. Kolb, G. Steigman, and M. S. Turner, *Phys. Rev. Lett.* **47**, 1357 (1981).
  - [8] M. Glück and E. Reya, *Phys. Rev. Lett.* **48**, 662 (1982).
  - [9] M. Marinelli and G. Morpurgo, *Phys. Rep.* **85**, 161 (1982); L. Lyons, *Phys. Rep.* **129**, 225 (1985); M. L. Perl, E. R. Lee, and D. Lomba, *Mod. Phys. Lett. A* **19**, 2595 (2004).

- [10] Particle Data Group, *Phys. Lett. B* **667**, 1 (2008); <http://pdg.lbl.gov>.
- [11] F.J. Yndurain, *Phys. Lett. B* **345**, 524 (1995).
- [12] The various modifications of QCD with nonzero gluon masses associated with broken color  $SU(3)_c$  considered here are to be contrasted with treatments of usual QCD without any gluon mass in  $\mathcal{L}_{\text{QCD}}$  that discuss notions of confinement-induced modifications of the effective gluon propagator without any breaking of color; see, e.g., J. Cornwall, *Phys. Rev. D* **26**, 1453 (1982); J. E. Mandula, *Phys. Rep.* **315**, 273 (1999); R. Alkofer and L. von Smekal, *Phys. Rep.* **353**, 281 (2001).
- [13] It should be noted, however, that the  $SO(3)$  gauge symmetry would also allow a  $SO(3)$  singlet but fractionally charged diquark bound states  $\tilde{q} \cdot \tilde{q}'$  and quark-gluon bound states  $\tilde{q} \cdot \tilde{g}$  that are forbidden by  $SU(3)_c$  invariance.
- [14] We note that it would be very difficult to construct a dynamical model in which the condensates would break  $SU(3)_c$  in the same manner as a Higgs field in the  $\underline{27}$  representation of  $SU(3)_c$ . The reason for this is that these condensates would form in the most attractive channel. A criterion for the attractiveness of a channel involving fermions in representations  $R_1$  and  $R_2$  of a given gauge group producing a condensate in the representation  $R_{\text{cond}}$  is  $\Delta C_2 = C_2(R_1) + C_2(R_2) - C_2(R_{\text{cond}})$ , where  $C_2(R)$  is the quadratic Casimir invariant for  $R$ . Thus, for a given  $R_1$  and  $R_2$ , this favors the smallest possible  $R_{\text{cond}}$ . To get  $R_{\text{cond}} = \underline{27}$  of  $SU(3)_c$ , one would have to start with  $R_1$  and/or  $R_2$  in very large representations, which would be disfavored by the resultant large effect on the QCD beta function.
- [15] E. H. Fradkin and S. H. Shenker, *Phys. Rev. D* **19**, 3682 (1979).
- [16] I-H. Lee and R. Shrock, *Phys. Rev. Lett.* **59**, 14 (1987); *Phys. Lett. B* **201**, 497 (1988).
- [17] Some reviews of lattice models with gauge, Higgs, and fermion fields include R. Shrock, *Nucl. Phys. B, Proc. Suppl.* **4**, 373 (1988); J. Kuti, L. Lin, and Y. Shen, *ibid.* **4**, 397 (1988); I. Montvay, *ibid.* **4**, 443 (1988); J. Shigemitsu, *Nucl. Phys. B, Proc. Suppl.* **20**, 515 (1991); A. K. De and J. Jersak, *Adv. Ser. Dir. High Energy Phys.* **10**, 732 (1992); A. Hasenfratz, in *Quantum Fields on the Lattice*, edited by M. Creutz (World Scientific, Singapore, 1992), p. 125; R. Shrock, *ibid.*, p. 150.
- [18] S. Bethke, *Eur. Phys. J. C* **64**, 689 (2009).
- [19] T. Banks and A. Zaks, *Nucl. Phys.* **B196**, 189 (1982); T. Appelquist, J. Terning, and L. C. R. Wijewardhana, *Phys. Rev. Lett.* **77**, 1214 (1996); **79**, 2767 (1997).
- [20] C. Quigg and R. Shrock, *Phys. Rev. D* **79**, 096002 (2009).
- [21] K. Lane, *Phys. Rev. D* **10**, 1353 (1974); **10**, 2605 (1974).
- [22] H. D. Politzer, *Nucl. Phys.* **B117**, 397 (1976).
- [23] A. Casher, *Phys. Lett.* **83B**, 395 (1979).
- [24] Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961); **124**, 246 (1961).
- [25] A recent discussion on modeling these pseudoscalar mesons as  $q\bar{q}$  bound states and also as approximate Nambu-Goldstone bosons and references to the literature on this are S. Nussinov and R. Shrock, *Phys. Rev. D* **79**, 016005 (2009).
- [26] T. Aaltonen *et al.* (CDF Collaboration), *Phys. Rev. Lett.* **104**, 091801 (2010).
- [27] A. Casher, H. Neuberger, and S. Nussinov, *Phys. Rev. D* **20**, 179 (1979); **21**, 1966 (1980).
- [28] G. C. Nayak, *Phys. Rev. D* **72**, 125010 (2005); G. C. Nayak and R. Shrock, *Phys. Rev. D* **77**, 045008 (2008).
- [29] S. J. Brodsky and R. Shrock, *Phys. Lett. B* **666**, 95 (2008).
- [30] Recent lattice calculations of glueball masses include W. Lee and D. Weingarten, *Phys. Rev. D* **61**, 014015 (1999); C. J. Morningstar and M. Peardon, *Phys. Rev. D* **60**, 034509 (1999); A. Hart and M. J. Teper, *Phys. Rev. D* **65**, 034502 (2002); Y. Chen *et al.*, *Phys. Rev. D* **73**, 014516 (2006).
- [31] See, e.g., W. Lee and D. Weingarten [30]; C. Amsler, in Ref. [10], *op cit.*; P. Minkowski and W. Ochs, *Eur. Phys. J. C* **9**, 283 (1999); H.-Y. Cheng, C.-K. Chua, and K.-F. Liu, *Phys. Rev. D* **74**, 094005 (2006); F. E. Close and Q. Zhao, *Phys. Rev. D* **71**, 094022 (2005); S. Nussinov and R. Shrock, *Phys. Rev. D* **80**, 054003 (2009), and references therein.
- [32] J. Milana and S. Nussinov, *Phys. Rev. D* **52**, 6612 (1995).
- [33] Note that if a “hard” current-quark mass is dynamically generated, then the corresponding running mass is actually soft at the high scale where it is generated [N. C. Christensen and R. Shrock, *Phys. Rev. Lett.* **94**, 241801 (2005)].
- [34] M. Gell-Mann, R. J. Oakes, and B. Renner, *Phys. Rev.* **175**, 2195 (1968).
- [35] S. Weinberg, *Trans. N.Y. Acad. Sci.* **38**, 185 (1977); see, e.g., J. Gasser and H. Leutwyler, *Nucl. Phys.* **B250**, 465 (1985); **B250**, 517 (1985); U. Meissner, *Rep. Prog. Phys.* **56**, 903 (1993); M. Harada and K. Yamawaki, *Phys. Rep.* **381**, 1 (2003); H. Leutwyler, *Proc. Sci.*, CD09 (2009) 005, and references therein.
- [36] B. W. Lee, C. Quigg, and H. B. Thacker, *Phys. Rev. D* **16**, 1519 (1977); M. S. Chanowitz and M. K. Gaillard, *Nucl. Phys.* **B261**, 379 (1985).
- [37] A. S. Goldhaber and M. M. Nieto, *Rev. Mod. Phys.* **82**, 939 (2010).