# Radiative decay into $\gamma$ -baryon of dynamically generated resonances from the vector-baryon interaction

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We study the radiative decay into  $\gamma$  and a baryon of the SU(3) octet and decuplet of nine and ten resonances that are dynamically generated from the interaction of vector mesons with baryons of the octet and the decuplet, respectively. We obtain quite different partial decay widths for the various resonances, and for different charge states of the same resonance, suggesting that the experimental investigation of these radiative decays should bring much information on the nature of these resonances. For the case of baryons of the octet we determine the helicity amplitudes and compare them with experimental data when available.

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## **I. INTRODUCTION**

In a recent paper [1], the  $\rho\Delta$  interaction was studied within the local hidden gauge formalism for the interaction of vector mesons. The results of the interaction gave a natural interpretation for the  $\Delta(1930)(5/2^{-})$  as a  $\rho\Delta$ bound state, which otherwise is extremely problematic in quark models since it involves a  $3\hbar\omega$  excitation and appears with much higher mass. At the same time two states with  $J^P = 1/2^-$ ,  $3/2^-$  were obtained, degenerate with the  $5/2^-$ , which could be accommodated with two known  $\Delta$ states in that energy range. Also, three degenerate  $N^*$  states with  $1/2^{-}$ ,  $3/2^{-}$ ,  $5/2^{-}$  were obtained, which were more difficult to identify with known resonances since that sector is not so well established. The work of [1] was extended to the SU(3) sector in [2] to account for the interaction of vectors of the octet with baryons of the decuplet. In this case ten resonances, all of them also degenerate in the three spin states, were obtained, many of which could be identified with existing resonances, while there were predictions for a few more. At the same time in [2] the poles and residues at the poles of the resonances were evaluated, providing the coupling of the resonances to the different vector-baryon of the decuplet components.

One of the straightforward tests of these theoretical predictions is the radiative decay of these resonances into photon and the member of the baryon decuplet to which it couples. Radiative decay of resonances into  $\gamma N$  is one of the observables traditionally calculated in hadronic models. Work in quark models on this issue is abundant [3–18]. For resonances that appear as dynamically generated in chiral unitary theories there is also much work done on the radiative decay into  $\gamma N$  [19–23]. Experimental work in this topic is also of current interest [24–26].

In the present work we address the novel aspect of radiative decay into a photon and a baryon of the decuplet of the  $\Delta$ , since the underlying dynamics of the resonances that we study provides this as the dominant mode of radiative decay into photon baryon. This is so, because the underlying theory of the studies of [1,2] is the local hidden gauge formalism for the interaction of vector mesons developed in [27-30], which has the peculiar feature, inherent to vector meson dominance, that the photons couple to the hadrons through the conversion into a vector meson. In this case a photon in the final state comes from either a  $\rho^0$ ,  $\omega$ ,  $\phi$ . Thus, the radiative decay of the resonances into  $\gamma B$  is readily obtained from the theory by taking the terms with  $\rho^0 B$ ,  $\omega B$ ,  $\phi B$  in the final state and coupling the  $\gamma$  to any of the final  $\rho^0$ ,  $\omega$ ,  $\phi$  vector mesons. This procedure was used in [31] and provided good results for the radiative decay into  $\gamma\gamma$  of the  $f_0(1370)$  and  $f_2(1270)$  mesons which were dynamically generated from the  $\rho\rho$  interaction within the same framework [32]. This latter work was also extended to the interaction of vectors with themselves within SU(3), where many other states are obtained which can be also associated with known resonances [33]. The radiative decay of the latter resonances into  $\gamma\gamma$  or a  $\gamma$  and a vector has been studied in [34], with good agreement with experiment when available. Given the success of the theory in its predictions and the good results obtained for the  $\gamma\gamma$  decay of the  $f_0(1370)$ ,  $f_2(1270)$  and  $f'_2(1525)$  mesons, the theoretical framework stands on good foot and the predictions made should be solid enough to constitute a test of the theory by contrasting with experimental data.

The extension of the work of [1,2] to the interaction of vector mesons with baryons of the octet of the proton has also been successful [35] and nine resonances, degenerated in spin-parity  $1/2^-$  and  $3/2^-$ , appear dynamically generated in the approach, many of which can be naturally associated to known resonances in the PDG [36]. We also extend the present work to study the radiative decay of these resonances into a photon and a baryon of the octet.

In this case we can also evaluate helicity amplitudes and compare them with experimental results when available.

The experimental situation in that region of energies is still poor. The PDG [36] quotes many radiative decays of  $N^*$  resonances, and of the  $A_{1/2}, A_{3/2}$  helicity amplitudes for decay of resonances into  $\gamma N$ , with N either proton or neutron. However, there are no data to our knowledge for radiative decay into  $\gamma B$ , with B a baryon of the decuplet. The reason for it might be the difficulty in the measurement, or the lack of motivation, since there are also no theoretical works devoted to the subject. With the present work we hope to reverse the situation offering a clear motivation for these experiments since they bear close connection with the nature invoked for these resonances, very different to the ordinary three quark structure of the baryons.

The numbers obtained for the radiative widths are well within the measurable range, of the order of 1 MeV, and the predictions are interesting, with striking differences of 1 order of magnitude between decay widths for different charges of the same resonance.

The work will proceed as follows. In the next two sections we present the framework for the evaluation of amplitudes of radiative decay. In Sec. IV, we show the results obtained for the different resonances generated with the baryon decuplet. Section V introduces the equations for the baryon octet, which are used in Sec. VI to obtain results for the decay width of the resonances dynamically generated with a vector and the baryon octet. In Sec. VII, we present the results for the helicity amplitudes of some resonances used in the previous section, and finish with some conclusions.

### **II. FRAMEWORK**

In Refs. [1,2], the *s*-wave scattering amplitudes for vector-decuplet baryon  $VB \rightarrow V'B'$  are given by

$$t_{VB \to V'B'} = t\vec{\epsilon} \cdot \vec{\epsilon}' \delta_{m_s, m'_s},\tag{1}$$

where  $\vec{\epsilon}$ ,  $\vec{\epsilon}'$  refer to the initial and final vector polarization and the matrix is diagonal in the third component of the baryons of the decuplet. The transition is diagonal in the spin of the baryon and the spin of the vector, and as a consequence in the total spin. To make this property more explicit, we write the total spin state as

$$|S, M\rangle = \sum_{m_s} C(3/2, 1, S; m_s, M - m_s, M) |3/2, m_s\rangle |\vec{\epsilon}_{M - m_s}\rangle$$
(2)

and

$$\langle S, M | = \sum_{m'_s} C(3/2, 1, S; m'_s, M - m'_s, M) \\ \times \langle 3/2, m'_s | \langle \vec{\epsilon}^*_{M - m'_s} |, \qquad (3)$$

where  $C(3/2, 1, S; m_s, M - m_s, M)$  are the Clebsch-

Gordan coefficients and  $\epsilon_{\mu}$  the polarization vectors in the spherical basis in the Coulomb Gauge and with the photon in the *z* direction

$$\vec{\epsilon}_{+} = -\frac{1}{\sqrt{2}}(\vec{\epsilon}_{1} + i\vec{\epsilon}_{2}), \qquad \vec{\epsilon}_{-} = \frac{1}{\sqrt{2}}(\vec{\epsilon}_{1} - i\vec{\epsilon}_{2}), \qquad (4)$$
$$\vec{\epsilon}_{0} = \vec{\epsilon}_{3}.$$

We can write Eq. (1) in terms of the projectors  $|S, M\rangle\langle S, M|$  as

$$t_{VB\to V'B'} = t\langle \vec{\epsilon}' | \langle 3/2, m'_s | \sum_{S,M} | S, M \rangle \langle S, M | 3/2, m_s \rangle | \vec{\epsilon} \rangle.$$
(5)

Since the Clebsch-Gordan coefficients satisfy the normalization condition

$$\sum_{S} C(3/2, 1, S; m_{s}, M - m_{s}, M) \times C(3/2, 1, S; m'_{s}, M' - m'_{s}, M') = \delta_{m_{s}m'_{s}} \delta_{MM'}, \quad (6)$$

we then have

$$\sum_{S,M} |S, M\rangle \langle S, M| = \sum_{M} \sum_{m_s} |3/2, m_s\rangle \langle 3/2, m_s| |\vec{\epsilon}_{M-m_s}\rangle \langle \vec{\epsilon}_{M-m_s}^* |$$
$$= \sum_{M'} \sum_{m_s} |3/2, m_s\rangle \langle 3/2, m_s| |\vec{\epsilon}_{M'}\rangle \langle \vec{\epsilon}_{M'}^* | \equiv 1.$$
(7)

We can depict the contribution of a specific resonant state of spin S to the amplitude described by means of Fig. 1. Then the amplitude for the transition of the resonance to a final vector-baryon state is depicted by means of Fig. 2. As shown in Refs. [1,2], the  $VB \rightarrow V'B'$  scattering amplitudes develop poles corresponding to resonances and a resonant



FIG. 1. Diagram contributing to the vector-baryon interaction via the exchange of a resonance.



FIG. 2. Diagram on the decay of the resonance in a decuplet baryon and a vector meson.

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amplitude is written as Eq. (1) with t given by

$$t_{ij} = \frac{g_i g_j}{\sqrt{s} - M + i\Gamma/2},\tag{8}$$

with  $g_i$  and  $g_j$  the couplings to the initial and final states. Accordingly, the amplitude for the transition from the resonance to a final state of vector-baryon is given by

$$t_{SM \to V'B'} = g_i \langle \vec{\epsilon} | \langle 3/2, m_s | S, M \rangle$$
  
=  $g_i C(3/2, 1, S; m_s, M - m_s, M) \langle \vec{\epsilon} | \vec{\epsilon}_{M - m_s} \rangle.$   
(9)

The generalization of Eq. (9) for the octet is rather obvious, the spin 3/2 becomes now 1/2 and then we have

$$t_{SM \to V'B'} = g_i C(1/2, 1, S; m_s, M - m_s, M) \langle \vec{\epsilon} | \vec{\epsilon}_{M - m_s} \rangle,$$
(10)

and the equations that determine the radiative decay width will be identical with this trivial change.

When calculating the decay width of the resonance into VB, we will sum  $|t|^2$  over the vector and baryon polarization, and average over the resonance polarization M. Thus, we have

$$\frac{1}{2S+1} \sum_{M,m_s,\vec{\epsilon}} |t_{SM\to V'B'}|^2 = |g_i|^2 \frac{1}{2S+1} \sum_{M,m_s,\vec{\epsilon}} C(3/2, 1, S; m_s, M - m_s, M)^2 \langle \vec{\epsilon}_{M-m_s}^* | \vec{\epsilon} \rangle \langle \vec{\epsilon} | \vec{\epsilon}_{M-m_s} \rangle$$

$$= |g_i|^2 \frac{1}{2S+1} \sum_{M'} \sum_{m_s} \frac{2S+1}{3} C(3/2, S, 1; m_s, -m_s - M', -M')^2 \langle \vec{\epsilon}_{M'}^* | \vec{\epsilon}_{M'} \rangle$$

$$= |g_i|^2 \frac{1}{3} \sum_{M'} \delta_{M'M'} = |g_i|^2, \tag{11}$$

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where in the first step we have permuted the two last spins in the Clebsch-Gordan coefficients and in the second we applied their orthogonality condition.

We observe that the normalization of the amplitudes is done in a way such that the sum and average of  $|t|^2$  is simply the modulus squared of the coupling of the resonance to the final state. The width of the resonance for decay into *VB* is given in the Mandl and Shaw normalization [37] by

$$\Gamma = \frac{M_B}{2\pi M_R} q|g_i|^2, \qquad (12)$$

where q is the momentum of the vector in the resonance rest frame and  $M_B$ ,  $M_R$  the masses of the baryon and the resonance. We should note already that later on when the vector polarizations are substituted by the photon polarizations in the sum over M' in Eq. (11), we will get a factor 2 rather than 3, because we only have two transverse polarizations, and then Eq. (12) must be multiplied by the factor 2/3.

#### **III. RADIATIVE DECAY**

Next we study the radiative decay into  $B\gamma$  of the resonances dynamically generated in Ref. [2] with *B* a baryon of the decuplet. Recalling the results of [2], we obtained ten resonances there, dynamically generated, each of them degenerated in three states of spin,  $1/2^-$ ,  $3/2^-$ ,  $5/2^-$ . As we have discussed in the preceding section, the radiative width will not depend on the spin of the resonance, but only on the coupling which is the same for all three spin states due to the degeneracy. This would be of course an interesting experimental test of the nature of these resonances.

In order to proceed further, we use the same formalism of the hidden gauge local symmetry for the vector mesons of [27–30]. The peculiarity of this theory concerning photons is that they couple to hadrons by converting first into a vector meson,  $\rho^0$ ,  $\omega$ ,  $\phi$ . Diagrammatically this is depicted in Fig. 3. This idea has already been applied with success to obtain the radiative decay of the  $f_2(1270)$ ,  $f_0(1370)$ ,  $f'_2(1525)$  and  $f_0(1710)$  resonances into  $\gamma\gamma$  in Refs. [31,34]. In Ref. [31], the question of gauge invariance was addressed, and it was shown that the theory fulfills it. In Ref. [38], it is also proved in the case of radiative decay of axial vector resonances.

The amplitude of Fig. 3 requires the  $\gamma V$  conversion Lagrangian, which comes from Refs. [27–29] and is given by (*see Ref.* [38] *for practical details*)

$$\mathcal{L}_{V\gamma} = -M_V^2 \frac{e}{\tilde{g}} A_\mu \langle V^\mu Q \rangle, \qquad (13)$$

with  $A_{\mu}$  the photon field,  $V_{\mu}$  the SU(3) matrix of vector fields



FIG. 3. Diagram on the radiative decay of the resonance in a baryon and a photon.

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$$V_{\mu} \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^{0} + \frac{1}{\sqrt{2}}\omega & \rho^{+} & K^{*+} \\ \rho^{-} & -\frac{1}{\sqrt{2}}\rho^{0} + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_{\mu}, \quad (14)$$

and Q the charge matrix

$$Q \equiv \begin{pmatrix} 2/3 & 0 & 0\\ 0 & -1/3 & 0\\ 0 & 0 & -1/3 \end{pmatrix}.$$
 (15)

In Eq. (13),  $M_V$  is the vector meson mass, for which we take an average value  $M_V = 800$  MeV, *e* the electron charge,  $e^2 = 4\pi\alpha$ , and

$$\tilde{g} = \frac{M_V}{2f}; \qquad f = 93 \text{ MeV}$$

The sum over polarizations in the intermediate vector meson for a fixed final photon polarization, in the product of the  $R \rightarrow BV$  transition amplitude of Eq. (9) and the  $V\gamma$  Lagrangian of Eq. (13) converts the polarization vector of the vector meson of the  $R \rightarrow BV$  amplitude into the photon polarization of the  $R \rightarrow B\gamma$  amplitude, and leads to the equation

$$-it_{\gamma V}D_V = -iM_V^2 \frac{e}{\tilde{g}} \frac{i}{-M_V^2} F_j$$
(16)

with

$$F_{j} = \begin{cases} \frac{1}{\sqrt{2}} & \text{for } \rho^{0}, \\ \frac{1}{3\sqrt{2}} & \text{for } \omega, \\ -\frac{1}{3} & \text{for } \phi, \end{cases}$$
(17)

and  $D_V$  the vector propagator for  $p^2 = 0$ . Thus, finally our amplitude for the  $R \to B\gamma$  transition, omitting the spin matrix element of Eq. (9),  $C(3/2, 1, S; m_s, M - m_s, M) \times \langle \vec{\epsilon} | \vec{\epsilon}_{M-m_s} \rangle$ , is given by

$$t_{\gamma} = -\frac{e}{\tilde{g}} \sum_{j=\rho^{0},\omega,\phi} g_{j} F_{j}.$$
 (18)

As discussed in the preceding section, the radiative decay width will then be given by

$$\Gamma_{\gamma} = \frac{1}{2\pi} \frac{2}{3} \frac{M_B}{M_R} q |t_{\gamma}|^2.$$
(19)

The couplings  $g_j$  for different resonance and *VB* with  $V = \rho^0$ ,  $\omega$ ,  $\phi$ , and *B* different baryon of the decuplet can be found in Ref. [2], and we use them here for the evaluation of  $\Gamma_{\gamma}$ . The factor 2/3 in Eq. (19) additional to Eq. (12) appears because now we have only two photon polarizations and the sum over M' in Eq. (11) gives 2 instead of 3 for the case of vector mesons.

# IV. RESULTS FOR RADIATIVE DECAYS INTO THE $\gamma$ AND BARYON DECUPLET

The couplings of the resonances to the different *VB* channels are given in Ref. [2] in the isospin basis. For the case of  $\omega B$  and  $\phi B$ , there is no change to be made, but for the case of  $\rho B$ , one must project over the  $\rho^0 B$  component. Since this depends on the charge of the resonance *R*, the radiative decays will depend on this charge, as we will see. We recall that in our phase convention  $|\rho^+\rangle = -|1, 1\rangle$  of isospin. The information on the resonances and their couplings to different baryons of decuplet and vector mesons  $\rho$ ,  $\omega$ ,  $\phi$  for different channels is listed in Table I. We detail the results below.

# A. S = 0, I = 1/2 channel

A resonance is obtained at  $z_R = 1850 + i5$  MeV, which couples to  $\Delta \rho$ . We have in this case

$$\left|\Delta\rho, \frac{1}{2}, \frac{1}{2}\right\rangle = \sqrt{\frac{1}{2}} |\Delta^{++}\rho^{-}\rangle - \sqrt{\frac{1}{3}} |\Delta^{+}\rho^{0}\rangle - \sqrt{\frac{1}{6}} |\Delta^{0}\rho^{+}\rangle$$
(20)

and

$$\left| \Delta \rho, \frac{1}{2}, -\frac{1}{2} \right\rangle = \sqrt{\frac{1}{6}} |\Delta^{+} \rho^{-} \rangle - \sqrt{\frac{1}{3}} |\Delta^{0} \rho^{0} \rangle - \sqrt{\frac{1}{2}} |\Delta^{-} \rho^{+} \rangle.$$
(21)

The coupling of the resonance to  $\rho^0$  is obtained multiplying the coupling of Table I by the corresponding Clebsch-Gordan coefficient for  $\Delta \rho^0$  of Eqs. (20) and (21). Then by means of Eqs. (18) and (19), one obtains the decay width. In this case since the  $\Delta \rho^0$  component is the same for  $I_3 = 1/2$  and  $I_3 = -1/2$ , one obtains the same radiative width for the two channels, which is  $\Gamma = 0.722$  MeV.

#### B. S = 0, I = 3/2 channel

One resonance is obtained at  $z_R = 1972 + i49$  MeV, which couples to  $\Delta \rho$ ,  $\Delta \omega$ , and  $\Delta \phi$ . The isospin states for  $\Delta \rho$  can be written as

$$\left|\Delta\rho, \frac{3}{2}, \frac{3}{2}\right\rangle = \sqrt{\frac{3}{5}} |\Delta^{++}\rho^{0}\rangle + \sqrt{\frac{2}{5}} |\Delta^{+}\rho^{+}\rangle, \qquad (22)$$

$$\left| \Delta \rho, \frac{3}{2}, \frac{1}{2} \right\rangle = \sqrt{\frac{2}{5}} |\Delta^{++} \rho^{-}\rangle + \sqrt{\frac{1}{15}} |\Delta^{+} \rho^{0}\rangle + \sqrt{\frac{8}{15}} |\Delta^{0} \rho^{+}\rangle,$$
(23)

$$\left|\Delta\rho, \frac{3}{2}, -\frac{1}{2}\right\rangle = \sqrt{\frac{8}{15}} |\Delta^+\rho^-\rangle - \sqrt{\frac{1}{15}} |\Delta^0\rho^0\rangle + \sqrt{\frac{2}{5}} |\Delta^-\rho^+\rangle, \qquad (24)$$

<i>S</i> , <i>I</i>	Channel			
		$z_R = 1850 + i5$		
0, 1/2	$\Delta  ho$	4.9 + i0.1		
		$z_R = 1972 + i49$		
0, 3/2	$\Delta  ho$	5.0 + i0.2		
	$\Delta \omega$	-0.1 + i0.2		
	$\Delta \phi$	0.2 - i0.4		
		$z_R = 2052 + i10$		
-1, 0	$\Sigma^* ho$	4.2 + i0.1		
		$z_R = 1987 + i1$	$z_R = 2145 + i58$	$z_R = 2383 + i73$
-1, 1	$\Sigma^* ho$	1.4 + i0.0	-4.3 - i0.7	0.4 + i1.1
	$\Sigma^* \omega$	1.4 + i0.0	1.3 - i0.4	-1.4 - i0.4
	$\Sigma^* \phi$	-2.1 - i0.0	-1.9 + i0.6	2.1 + i0.6
		$z_R = 2214 + i4$	$z_R = 2305 + i66$	$z_R = 2522 + i38$
-2, 1/2	$\Xi^* ho$	1.8 - i0.1	-3.5 - i1.7	0.2 + i1.0
	$\Xi^*\omega$	1.7 + i0.1	2.0 - i0.7	-0.6 - i0.3
	$\Xi^*\phi$	-2.5 - i0.1	-3.0 + i1.0	0.9 + i0.4
		$z_R = 2449 + i7$		
-3, 0	$\Omega \omega$	1.6 - i0.2		
	$\Omega \phi$	-2.4 + i0.3		

TABLE I. The coupling  $g_i$  of the resonance obtained dynamically to the  $\rho B$ ,  $\omega B$ , and  $\phi B$  channels.

$$\left| \Delta \rho, \frac{3}{2}, -\frac{3}{2} \right\rangle = \sqrt{\frac{2}{5}} |\Delta^0 \rho^-\rangle - \sqrt{\frac{3}{5}} |\Delta^- \rho^0\rangle.$$
 (25)

Since all the Clebsch-Gordan coefficients to  $\Delta \rho^0$  are now different, we obtain different radiative decay width for each charge of the state. The results are  $\Gamma = 1.402$  MeV for  $I_3 = 3/2$ ,  $\Gamma = 0.143$  MeV for  $I_3 = 1/2$ ,  $\Gamma =$ 0.203 MeV for  $I_3 = -1/2$  and  $\Gamma = 1.582$  MeV for  $I_3 =$ -3/2. It is quite interesting to see that there is an order of magnitude difference between for I = 3/2 and I = 1/2, and it is a clear prediction that could be tested experimentally.

### C. S = -1, I = 0 channel

We get a resonance at  $z_R = 2052 + i10$  MeV, which couples to  $\Sigma^* \rho$ . In this case,

$$|\Sigma^*\rho, 0, 0\rangle = \sqrt{\frac{1}{3}} |\Sigma^{*+}\rho^-\rangle - \sqrt{\frac{1}{3}} |\Sigma^{*0}\rho^0\rangle - \sqrt{\frac{1}{3}} |\Sigma^{*-}\rho^+\rangle,$$
(26)

and the radiative decay obtained is  $\Gamma = 0.583$  MeV.

## D. S = -1, I = 1 channel

Here we find three resonances at  $z_R = 1987 + i1$  MeV, 2145 + *i*58 MeV, and 2383 + *i*73 MeV, which couple to  $\Sigma^* \rho$ ,  $\Sigma^* \omega$ , and  $\Sigma^* \phi$ . The relevant isospin states are

$$|\Sigma^* \rho, 1, 1\rangle = \sqrt{\frac{1}{2}} |\Sigma^{*+} \rho^0\rangle + \sqrt{\frac{1}{2}} |\Sigma^{*0} \rho^+\rangle,$$
 (27)

$$|\Sigma^*\rho, 1, 0\rangle = \sqrt{\frac{1}{2}} |\Sigma^{*+}\rho^-\rangle + \sqrt{\frac{1}{2}} |\Sigma^{*-}\rho^+\rangle, \qquad (28)$$

and

$$|\Sigma^* \rho, 1, -1\rangle = \sqrt{\frac{1}{2}} |\Sigma^{*0} \rho^-\rangle - \sqrt{\frac{1}{2}} |\Sigma^{*-} \rho^0\rangle.$$
 (29)

The results obtained in this case are summarized in Table II.

# E. S = -2, I = 1/2 channel

Here we also find three states at  $z_R = 2214 + i4$  MeV, 2305 + *i*66 MeV and 2522 + *i*38 MeV, which couple to  $\Xi^*\rho$ ,  $\Xi^*\omega$  and  $\Xi^*\phi$ . The isospin states for  $\Xi^*\rho$  are written as

$$\left|\Xi^{*}\rho,\frac{1}{2},\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}|\Xi^{*-}\rho^{+}\rangle + \sqrt{\frac{1}{3}}|\Xi^{*0}\rho^{0}\rangle, \qquad (30)$$

TABLE II. The radiative decay widths in units of MeV for the S = -1, I = 1 resonances with different isospin projection  $I_3$ .

$\overline{I_3}$	(1987)	(2145)	(2383)
1	0.561	0.399	0.182
0	0.199	0.206	0.277
-1	0.020	2.029	0.537

TABLE III. The radiative decay widths in units of MeV for the S = -2, I = 1/2 resonances with the different isospin projection  $I_3$ .

I <sub>3</sub>	(2214)	(2305)	(2522)
1/2	0.815	0.320	0.044
-1/2	0.054	1.902	0.165

$$\left|\Xi^*\rho, \frac{1}{2}, -\frac{1}{2}\right\rangle = -\sqrt{\frac{1}{3}}|\Xi^{*-}\rho^0\rangle + \sqrt{\frac{2}{3}}|\Xi^{*0}\rho^-\rangle. \quad (31)$$

The radiative decay widths in this case are shown in Table III.

F. 
$$S = -3$$
,  $I = 0$  channel

Here we have only one state at  $z_R = 2449 + i7$  MeV, which couples to  $\Omega \omega$  and  $\Omega \phi$ . The radiative decay width obtained in this case is  $\Gamma = 0.330$  MeV.

As one can see, there is a large variation in the radiative width of the different states, which should constitute a good test for the model when these widths are measured.

In Table IV we summarize all the results obtained making an association of our states to some resonances found in the PDG [36].

## V. RESULTS FOR RADIATIVE DECAYS INTO $\gamma$ AND BARYON OCTET

The couplings of the resonances to the different *VB* channels are given in Ref. [2] in the isospin basis. For the case of  $\omega B$  and  $\phi B$ , there is no change to be done, but for the case of  $\rho B$ , one must project over the  $\rho^0 B$  component. Since this depends on the charge of the resonance *R*, the radiative decays will depend on this charge, as we will

see. We recall that in our phase convention  $|\rho^+\rangle = -|1,1\rangle$  of isospin. The information on the resonances and their couplings to different baryons of octet and vector mesons  $\rho, \omega, \phi$  for different channels is listed in Table V. We detail the results below and compile them in Table VI.

## A. S = 0, I = 1/2 channel

Two resonances are obtained at  $z_R = 1696$  MeV and  $z_R = 1977 + i53$  MeV which couple to  $\rho N$ ,  $\omega N$  and  $\phi N$ . We have in this case

$$\left|\rho N, \frac{1}{2}, \frac{1}{2}\right\rangle = -\sqrt{\frac{1}{3}} \left|\rho^0 p\right\rangle - \sqrt{\frac{2}{3}} \left|\rho^+ n\right\rangle \qquad (32)$$

and

$$\left| \rho N, \frac{1}{2}, -\frac{1}{2} \right\rangle = \sqrt{\frac{1}{3}} \left| \rho^0 n \right\rangle - \sqrt{\frac{2}{3}} \left| \rho^- p \right\rangle.$$
 (33)

The coupling of the resonance to  $\rho^0$  is obtained multiplying the coupling of Table V by the corresponding Clebsch-Gordan coefficient for  $\rho^0 N$  of Eqs. (32) and (33). Then by means of Eqs. (18) and (19), one obtains the decay width.

## B. S = -1, I = 0 channel

We get three resonances at  $z_R = 1784 + i4$  MeV,  $z_R = 1906 + i70$  MeV and  $z_R = 2158 + i13$  MeV respectively, which couple to  $\rho \Sigma$ ,  $\omega \Lambda$  and  $\phi \Lambda$ . In this case,

$$|\rho\Sigma, 0, 0\rangle = \sqrt{\frac{1}{3}}|\rho^{-}\Sigma^{+}\rangle - \sqrt{\frac{1}{3}}|\rho^{0}\Sigma^{0}\rangle - \sqrt{\frac{1}{3}}|\rho^{+}\Sigma^{-}\rangle.$$
(34)

TABLE IV. The predicted radiative decay widths of the 10 dynamically generated resonances for different isospin projection  $I_3$ . Their possible PDG counterparts are also listed. Note that the  $\Sigma(2000)$  could be the spin parter of the  $\Sigma(1940)$ , in which case the radiative decay widths would be those of the  $\Sigma(1940)$ .

S, I	Theory	PDG o	data	Predicted width (KeV) for $I_3$						
,	Pole position (MeV)	Name	$J^P$	-3/2	-1	-1/2	0	1/2	1	3/2
0, 1/2	1850 + i5	N(2090)	$1/2^{-}$			722		722		
		N(2080)	$3/2^{-}$							
0, 3/2	1972 + i49	$\Delta(1900)$	$1/2^{-}$	1582		203		143		1402
		$\Delta(1940)$	$3/2^{-}$							
		$\Delta(1930)$	$5/2^{-}$							
-1, 0	2052 + i10	$\Lambda(2000)$	??				583			
-1, 1	1987 + i1	$\Sigma(1940)$	$3/2^{-}$		20		199		561	
		$\Sigma(2000)$	$1/2^{-}$							
	2145 + i58	$\Sigma(2250)$	??		2029		206		399	
	2383 + i73	$\Sigma(2455)$	$?^{?}$		537		277		182	
-2, 1/2	2214 + i4	$\Xi(2250)$	$?^{?}$			54		815		
	2305 + i66	$\Xi(2370)$	$?^{?}$			1902		320		
	2522 + i38	$\Xi(2500)$	$?^{?}$			165		44		
-3, 1	2449 + i7	Ω(2470)	??				330			

S, I	Channel			
		$z_R = 1696$	$z_R = 1977 + i53$	
0, 1/2	$\rho N$	3.2 + i0	-0.3 - i0.5	
	$\omega N$	0.1 + i0	-1.1 - i0.4	
	$\phi N$	-0.2 + i0	1.5 + i0.6	
		$z_R = 1784 + i4$	$z_R = 1906 + i70$	$z_R = 2158 + i13$
-1, 0	$\omega\Lambda$	1.4 + i0.03	0.4 + i0.2	-0.3 - i0.2
	$\rho\Sigma$	-1.5 + i0.03	3.1 + i0.7	0.01 - i0.08
	$\phi\Lambda$	-1.9 - i0.04	-0.6 - i0.3	0.5 + i0.3
		$z_R = 1830 + i40$	$z_R = 1987 + i240$	
-1, 1	$ ho\Lambda$	-1.6 + i0.2	-0.3 + i0.9	
	$\rho\Sigma$	-1.6 + i0.07	2.6 + i0.0	
	ωΣ	-0.9 + i0.1	-0.2 + i0.5	
	$\phi\Sigma$	1.2 - i0.2	0.2 - i0.7	
		$z_R = 2039 + i67$	$z_R = 2082 + i31$	
-2, 1/2	$ ho\Xi$	2.4 + i0.7	0.4 + i0.3	
	$\omega \Xi$	0.6 - i0.08	1.1 + i0.3	
	$\phi\Xi$	-0.8 + i0.1	-1.6 - i0.4	

TABLE V. The coupling  $g_i$  of the resonance obtained dynamically to the  $\rho B$ ,  $\omega B$ , and  $\phi B$  channels.

# C. S = -1, I = 1 channel

Here we find two resonances at 1830 + i40 MeV and 1987 + i240 MeV, which couple to  $\rho\Lambda$ ,  $\rho\Sigma$ ,  $\omega\Sigma$  and  $\phi\Sigma$ . The relevant isospin states are

$$|\rho\Sigma, 1, 1\rangle = -\sqrt{\frac{1}{2}}|\rho^0\Sigma^+\rangle - \sqrt{\frac{1}{2}}|\rho^+\Sigma^0\rangle, \qquad (35)$$

$$|\rho\Sigma, 1, 0\rangle = -\sqrt{\frac{1}{2}}|\rho^{+}\Sigma^{-}\rangle - \sqrt{\frac{1}{2}}|\rho^{-}\Sigma^{+}\rangle, \qquad (36)$$

and

$$|\rho\Sigma, 1, -1\rangle = -\sqrt{\frac{1}{2}}|\rho^{-}\Sigma^{0}\rangle + \sqrt{\frac{1}{2}}|\rho^{0}\Sigma^{-}\rangle.$$
(37)

# D. S = -2, I = 1/2 channel

Here we also find two states at  $z_R = 2039 + i67$  MeV and 2082 + i31 MeV, which couple to  $\rho \Xi$ ,  $\omega \Xi$ , and  $\phi \Xi$ . The isospin states for  $\rho \Xi$  are written as

$$\left|\rho\Xi,\frac{1}{2},\frac{1}{2}\right\rangle = -\sqrt{\frac{2}{3}}|\rho^{+}\Xi^{-}\rangle - \sqrt{\frac{1}{3}}|\rho^{0}\Xi^{0}\rangle, \qquad (38)$$

TABLE VI. The predicted radiative decay widths of the nine dynamically generated resonances for different isospin projection  $I_3$ . Their possible PDG counterparts are also listed. The values in the bracket for  $I_3 = 0$  denote widths for the radiative decay into  $\Lambda \gamma$ , while the values outside the bracket denote widths for  $\Sigma \gamma$ .

S, I	Theory	PDG o	lata	Predicted width (KeV) for $I_3$					
	Pole position (MeV)	Name	$J^P$	-1	-1/2	0	1/2	1	
0, 1/2	1696	N(1650)	1/2-		334		253		
		N(1700)	$3/2^{-}$						
	1977 + i53	N(2080)	$3/2^{-}$		196		79		
		N(2090)	$1/2^{-}$						
-1, 0	1784 + i4	$\Lambda(1690)$	$3/2^{-}$			65 (166)			
		$\Lambda(1800)$	$1/2^{-}$						
	1907 + i70	$\Lambda(2000)$	??			321 (21)			
	2158 + i13					0 (17)			
-1, 1	1830 + i40	$\Sigma(1750)$	$1/2^{-}$	363		69 (240)		7	
	1987 + i240	$\Sigma(1940)$	$3/2^{-}$	307		27 (90)		426	
		$\Sigma(2000)$	$1/2^{-}$						
-2, 1/2	2039 + i67	$\Xi(1950)$	??		400		89		
. ,	2082 + i31	Ξ(2120)	$?^{?}$		212		84		

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$$\left|\rho\Xi,\frac{1}{2},-\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}|\rho^{0}\Xi^{-}\rangle - \sqrt{\frac{2}{3}}|\rho^{-}\Xi^{0}\rangle.$$
(39)

In Table VI, we summarize all the results obtained, associating our states to some resonances found in the PDG [36].

As one can see, there is a large variation in the radiative width of the different states, which should constitute a good test for the model. For the case of the vector-baryon octet states, which decay into  $\gamma$  and a baryon of the octet, it is customary to express the experimental information in terms of helicity amplitudes  $A_{1/2}$  and  $A_{3/2}$ . We evaluate these amplitudes below to facilitate the comparison with experiment.

## VI. HELICITY AMPLITUDES

Recalling Eq. (10) for the dynamically generated states from a vector and a baryon of the octet, we have the two cases  $J^P = 1/2^-$  and  $J^P = 3/2^-$ . The helicity amplitudes with the choice of polarization vectors of Eq. (4), which imply the use of the Coulomb gauge with the photon momentum in the z direction, are defined as

$$A_{1/2}^{N^*} = \sqrt{\frac{2\pi\alpha}{k}} \frac{1}{e} \langle N^*, J_z = 1/2 | \boldsymbol{\epsilon}_{\mu}^{(+)} J^{\mu} | N, J_z = -1/2 \rangle,$$
(40)

$$A_{3/2}^{N^*} = \sqrt{\frac{2\pi\alpha}{k}} \frac{1}{e} \langle N^*, J_z = 3/2 | \boldsymbol{\epsilon}_{\mu}^{(+)} J^{\mu} | N, J_z = 1/2 \rangle,$$
(41)

where  $\alpha = 1/137$ , k is the center of mass photon momentum and  $e^2 = 4\pi\alpha$ . To accommodate these amplitudes to our

Eq. (10), we rewrite them taking  $\epsilon_{\mu}^{(+)}J^{\mu} = -\vec{\epsilon}^{(+)}\vec{J}$  as

$$A_{1/2}^{J=1/2} = -t_{\gamma} \frac{1}{\sqrt{2k}} C(1/2, 1, 1/2; m_s, M - m_s, M)$$
$$\times \langle \vec{\epsilon}_{M-m_s} | \vec{\epsilon}^* \rangle^*, \qquad (42)$$

where  $t_{\gamma}$  is given by Eq. (18), with  $m_s = -1/2$ ,  $\vec{\epsilon} = \vec{\epsilon}^{(+)}$ , which fixes  $M - m_s = 1$ , and similarly for the other amplitudes. Hence, we obtain

$$A_{1/2}^{J=1/2} = -t_{\gamma} \frac{1}{\sqrt{2k}} C(1/2, 1, 1/2; -1/2, 1, 1/2)$$
$$= \frac{1}{\sqrt{2k}} \sqrt{\frac{2}{3}} t_{\gamma}$$
(43)

$$A_{1/2}^{J=3/2} = -t_{\gamma} \frac{1}{\sqrt{2k}} C(1/2, 1, 3/2; -1/2, 1, 1/2)$$
$$= -\frac{1}{\sqrt{2k}} \sqrt{\frac{1}{3}} t_{\gamma}, \qquad (44)$$

$$A_{3/2}^{J=3/2} = -t_{\gamma} \frac{1}{\sqrt{2k}} C(1/2, 1, 3/2; 1/2, 1, 1/2)$$
$$= -\frac{1}{\sqrt{2k}} t_{\gamma}.$$
 (45)

The ordinary formula to get the radiative decay width in terms of  $A_{1/2}$  and  $A_{3/2}$  is given in the PDG [36] as

$$\Gamma_{\gamma} = \frac{k^2}{\pi} \frac{2M_B}{(2J_R + 1)M_R} [(A_{1/2})^2 + (A_{3/2})^2] \quad (46)$$

One can see that using in Eq. (46), the values of the helicity amplitudes obtained in Eqs. (43)–(45) one obtains the same result of Eq. (19) for both spins of the resonances.

It is interesting to note that the values of  $A_{1/2}$  for J = 1/2, 3/2 and  $A_{3/2}$  for J = 3/2 are all related by the simple relations of Eqs. (43)–(45) for these dynamically generated states, and the ratio of  $1/\sqrt{3}$  between  $A_{1/2}^{J=3/2}$  and  $A_{3/2}^{J=3/2}$  is something that could be contrasted with experiment. In Table VII, we compile all the results obtained for the resonances that are likely to be associated to states in the PDG for which there are data. The theoretical errors have been obtained by assuming a 10% uncertainty in the largest coupling of the resonance to the different channels and 15% in the other ones. This is only a rough estimate, and the uncertainties can easily be double this amount.

We should comment on the signs of the helicity amplitudes. For us they are determined by means of Eqs. (43)– (45) in terms of  $t_{\gamma}$  given in Eq. (18). We should mention that the  $g_j$  couplings appearing there are all determined with their relative sign, but one of them has an arbitrary sign. Thus, we have an arbitrary sign in the helicity amplitudes, but the relative sign between the  $A_{1/2}^{J=3/2}$  and  $A_{3/2}^{J=3/2}$  is well determined. Within our convention of SU(2) phases also the *n* and *p* helicity amplitudes are correlated.

Experimentally, the phases of the helicity amplitudes are a subject of convention since there is one phase arbitrary in the non diagonal transition of baryon states. In practice when dealing with physical processes to determine the helicity amplitudes, one has a strong vertex and an electromagnetic one, like  $\gamma N \rightarrow N^*$  and  $N^* \rightarrow N \pi$  [44–47], and one could determine the sign of the product of these two vertices of some resonances with respect to other ones, with some global phase undetermined. Theoretically one could do a similar thing, but this would require to study the transition amplitude of the VN resonances to  $\pi N$ , something that has not been done yet, though work in this direction is underway [48].

PDG data					Helicity amplitudes	$10^{-3} (\text{GeV}^{-1/2})$			
Name	$J^P$	Decay	Theory	Exp. [36] PDG	Exp. [39] Barbour	Exp. [40] Devenish	Th. [41]	Th. [42]	Th. [43]
N(1650)	$1/2^{-}$	$A_{1/2}^{p}$	64 ± 7	$53 \pm 16$			5	46	54
		$A_{1/2}^{n}$	$-74\pm7$	$-15 \pm 4$			-16	-58	-35
N(1700)	$3/2^{-}$	$A_{1/2}^{p}$	$-46\pm5$	$-18 \pm 13$	$-33 \pm 21$		-13	-3	-33
		$A_{3/2}^{p}$	$-79 \pm 9$	$-2 \pm 24$	$-14 \pm 25$		-10	15	18
		$A_{1/2}^{n}$	$52\pm5$	$0 \pm 50$	$50 \pm 42$		16	14	-3
		$A_{3/2}^{n}$	$91\pm9$	$-3 \pm 44$	$35 \pm 30$		-42	-23	-30
N(2080)	3/2-	$A_{1/2}^{p}$	21 ± 5	$-20 \pm 8$		$26 \pm 52$			
		$A_{3/2}^{p}$	$36 \pm 8$	$17 \pm 11$		$128 \pm 57$			
		$A_{1/2}^{n}$	$29 \pm 5$	$7 \pm 13$		$53 \pm 83$			
		$A_{3/2}^{n}$	$50\pm8$	$-53 \pm 34$		$100 \pm 141$			
N(2090)	$1/2^{-}$	$A_{1/2}^{p}$	$-30\pm 6$						
		$A_{1/2}^{n'}$	$-41 \pm 6$						

TABLE VII. Comparison with experiments and other theories

In view of this, we have chosen two phases to compare with experiment, the one of  $A_{1/2}^p$  for N(1700) and that of  $A_{1/2}^p$  for N(2080). The choice has been made to agree with the experiments of Barbour and Devenish, respectively.

By looking at Table VII, we can see that the agreement with the data of  $A_{1/2}^p$  for the PDG average of  $N^*(1650)$  is good. For the case of  $A_{1/2}^n$  the results obtained are larger than experiment but the sign is good. In the case of the  $N^*(1700)$ ,  $A_{1/2}^p$  can be considered qualitatively fine within theoretical and experimental errors,  $A_{3/2}^p$  seems to be larger than experiment but one can see that individual measurements, such as the one of Barbour [39], diverge appreciably from the PDG average values. Similarly  $A_{1/2}^n$  would be compatible with experiment within errors, and  $A_{3/2}^n$  seems also a bit larger, but not qualitatively too much, taking into account the large experimental uncertainties. This last magnitude is very relevant since the predictions of the dynamically generated model have an opposite sign to all the quark model calculations mentioned in the table. Since a global sign is these nondiagonal transitions can always appear in different models, more relevant than the absolute sign is the relative one to  $A_{1/2}^n$  which is the same in our case and the opposite in [41,42]. In Ref. [43], one has the same signs but there is 1 order of magnitude difference between the two helicity amplitudes, while in our model the ratio is  $\sqrt{3}$ . It is clear that precise measurements of these magnitudes are very useful to discriminate among models and help us better understand the structure of these resonances.

The case of the  $N^*(2080)$  and  $N^*(2090)$  is more unclear. The agreement with the PDG average does not seem too good, although the uncertainties are very large, but we find

PDG data		$t_{\gamma} 10^{-3} (A_{1/2}^{3/2} / A_{1/2}^{1/2})$								
Name J <sup>P</sup>		Determined from	Theory		Exp. $[39]$ Barbour		Exp. [40] Devenish			
N(1650)	$1/2^{-}$	$A_{1/2}^{p}$	$85 \pm 9$							
		$A_{1/2}^{n}$	$-98\pm10$							
N(1700)	$3/2^{-}$	$A_{1/2}^{p}$	$85 \pm 9$		$60 \pm 38$					
		$A_{3/2}^{p}$	$85 \pm 9$	$(1.7 \pm 0.3)$	$15 \pm 26$	$(0.4 \pm 0.8)$				
		$A_{1/2}^{n}$	$-98\pm10$		$-94 \pm 79$					
		$A_{3/2}^{n}$	$-98\pm10$	$(\textbf{1.7}\pm\textbf{0.2})$	$-35\pm32$	$(\textbf{0.7}\pm\textbf{0.8})$				
N(2080)	3/2-	$A_{1/2}^{p}$	$-45 \pm 10$				$-57 \pm 116$			
		$A_{3/2}^{p}$	$-45 \pm 10$	$(1.7 \pm 0.6)$			$-164 \pm 73$	$(4.9 \pm 10.1)$		
		$A_{1/2}^{n}$	$-62\pm10$				$-119\pm186$			
		$A_{3/2}^{n}$	$-62\pm10$	$(\textbf{1.7} \pm \textbf{0.4})$			$-129\pm182$	$(\textbf{1.9} \pm \textbf{4.0})$		
N(2090)	$1/2^{-}$	$A_{1/2}^{p'}$	$-45 \pm 10$							
		$\mathbf{A_{1/2}^{n}}$	$-62\pm10$							

TABLE VIII. Comparison of  $t_{\gamma}$  obtained from the theoretical and experimental helicity amplitudes. The numbers in bold characters refer to neutron and the normal ones to the proton.

it more appropriate to compare with the results of a single experiment. For this purpose, we also show the experimental results of Devenish [40] for the resonances to show that individual measurements are very different from the PDG averages. The agreement with signs of all amplitudes is now good (recall that one theoretical sign is chosen to agree) and also the sizes are similar, taking into account the large experimental uncertainties. Since under the umbrella of the  $N^*(2080)$  and  $N^*(2090)$  there are apparently different states compiled, it would be possible that the averages of the PDG were not done for different measurements on the same state but for measurements on different states. The experimental situation is hence unclear but the results obtained here should be a motivation for further reanalysis.

A different way to make the comparison with experiment is to take the relationship of Eqs. (43)–(45), tied to the nature of these resonances as dynamically generated from the vector-baryon interaction, and extract the only independent amplitude,  $t_{\gamma}$ , from the experimental data, hence comparing  $t_{\gamma}$  directly. We have done this and take the data of Barbour and Devenish, for the  $N^*(1700)$  and  $N^*(2080)$ , respectively, to guarantee that they come from the same state. The results are found in Table VIII. We observe consistency in the sizes and signs (only one sign is chosen theoretically for one amplitude of each resonance), although the experimental errors are still very large. Needless to say that improvements on these measurements would be most welcome.

#### **VII. CONCLUSIONS**

We have studied the radiative decay into  $\gamma B$ , with *B* a baryon of the octet and decuplet of SU(3), of the dynamically generated resonances obtained within the framework of the local hidden gauge mechanism for vector interactions. The framework is particularly rewarding for the study of such observable, since the photon in the final state appears coupling directly to the vector  $V = \rho^0$ ,  $\omega$ ,  $\phi$  in the  $R \rightarrow BV$  amplitudes which are studied in previous works. The rates obtained are large and the radiative widths are of the order of 1 MeV. On the other hand, one of the appealing features of the results is the large difference, of about 1

order of magnitude, that one finds between the widths for different charge states of the same particle. Although, such differences are also found in quark models, the precise values are tied to the details of the theory, concretely the coupling of the resonances to VB, which sometimes produce large interferences between the different contributions of the three vector mesons to which the photon couples. As a consequence, the radiative decay widths that we have evaluated bear much information on the nature of those resonances, which should justify efforts for a systematic measurement of these observables.

We have studied the decay into  $\gamma$ -baryon octet and  $\gamma$ -baryon decuplet of the states dynamically generated from the vector-baryon octet and vector-baryon decuplet interaction. In the first case, one can define the helicity amplitude  $A_{1/2}$  and  $A_{3/2}$  for the *n* and *p* type states of the  $N^*$ , which makes the comparison with data more useful. We have found good agreement with data in some cases and rough in others, but we have warned about the large experimental uncertainties and the possibility that the PDG averages are done over different states. Perhaps the most practical way to test the nature of the resonances that we have discussed would be to check experimentally the relationships of Eqs. (43)–(45), which relate the spin 1/2and 3/2 helicity amplitudes for the approximately degenerated spin partners, as we have done. What stands clear from the work and the discussion is that these observables are very useful in helping us better understand the nature of the resonances discussed here. Further experimental work is most desirable. We hope the present work stimulates work in this direction.

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