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Semileptonic and nonleptonic decays of B_c mesons to orbitally excited heavy mesons in the relativistic quark model

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The form factors of weak decays of the B_c meson to orbitally excited charmonium, D, B_s, and B mesons are calculated in the framework of the QCD-motivated relativistic quark model based on the quasipotential approach. Relativistic effects are systematically taken into account. The form factor dependence on the momentum transfer is reliably determined in the whole kinematical range. The form factors are expressed through the overlap integrals of the meson wave functions, which are known from the previous mass spectra calculations within the same model. On this basis, semileptonic and nonleptonic B_c decay rates to orbitally excited heavy mesons are calculated. Predictions for the B_c decays to the orbitally and radially excited $2P$ and $3S$ charmonium states are given, which could be used for clarifying the nature of the recently observed charmoniumlike states above the open charm production threshold.

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I. INTRODUCTION

The investigation of weak decays of mesons composed of a heavy quark and antiquark gives a very important insight in the heavy quark dynamics. The decay properties of the B_c meson are of special interest, since it is the only heavy meson consisting of two heavy quarks with different flavor. This difference of quark flavors forbids annihilation into gluons. As a result, the excited B_c meson states lying below the BD meson threshold undergo pionic or radiative transitions to the pseudoscalar ground state, which is considerably more stable than corresponding charmonium or bottomonium states and decays only weakly. The CDF Collaboration reported the discovery of the B_c ground state in $p\bar{p}$ collisions already more than ten years ago [1]. However, until recently its mass was known with a very large error. Now it is measured with a good precision in the decay channel $B_c \to J/\psi\pi$. The measured value $M_{B_c}^{\rm exp} = 6275.2 \pm 2.9 \pm 2.5$ MeV [2] is in a very good agreement with the prediction of the relativistic quark model $M_{B_c}^{\rm theor} = 6270$ MeV [3]. More experimental data on masses and decays of the B_c mesons are expected to come in the near future from the Tevatron at Fermilab and the Large Hadron Collider (LHC) at CERN.

The characteristic feature of the B_c meson is that both quarks forming it are heavy and thus their weak decays give comparable contributions to the total decay rate. Therefore, it is necessary to consider both the b quark transitions $b \to c$, u with the $\bar{c}$ quark being a spectator and $\bar{c}$ quark transitions $\bar{c} \to \bar{s}$, $\bar{d}$ with the b quark being a spectator. The former transitions lead to weak decays to charmonium and D mesons while the latter lead to decays to B_s and B mesons. The estimates [4] of the B_c decay rates indicate that the c quark transitions give the dominant contribution ($\sim 70\%$) while the b quark transitions and weak annihilation contribute about 20% and 10%, respectively. However, from the experimental point of view, the B_c decays to charmonium are easier to identify. Indeed, CDF and D0 observed the B_c meson and measured its mass analyzing its semileptonic and nonleptonic decays $B_c \to J/\psi l\nu$ and $B_c \to J/\psi\pi$ [1,2,5].

In this paper we extend our previous investigation of B_c properties [3,6,7] to study exclusive weak semileptonic and nonleptonic decay channels to orbitally excited heavy mesons. For the calculations we use the same effective methods [6,7] developed in the framework of the relativistic quark model based on the quasipotential approach for the B_c decays to ground and radially excited states of charmonium, D, B_s, and B mesons. Here weak decays to orbital excitations of these mesons, governed both by the b and c quark decays, are considered. The weak decay matrix elements are parametrized by invariant form factors which are then expressed through the overlap integrals of the meson wave functions. The systematic account for the relativistic effects, including wave function transformations from the rest to the moving frame and contributions from the intermediate negative-energy states, allows us to reliably determine the momentum transfer dependence of the decay form factors in the whole accessible kinematical range. The other important advantage of our approach is that for numerical calculations we use the relativistic wave functions, obtained in the meson mass spectra calculations, and not some *ad hoc* parametrizations, which were widely used in some previous investigations. The calculated form factors are then substituted in the expressions for the differential decay rates.

The important distinction between weak B_c decays, associated with the b and c quark decays, consists of the significant difference of the accessible kinimatical ranges. In the B_c decays to the charmonium and D mesons the kinematical range is considerably broader (by about an order of magnitude) than for decays to B_s and B mesons. As a result, many weak decays that are kinematically allowed in the former case are forbidden in the latter one. The kinematical suppression of semileptonic $B_c \to B_s(B)l\nu$ decays should be more pronounced for the decays to excited states than for the ground ones. The nonleptonic B_c decays to an orbitally excited heavy meson and an energetic light meson can then be considered on the basis of the factorization approximation. The obtained predictions for the decay rates are compared with previous calculations, which are based on different relativistic quark models [8–10], three-point QCD sum rules [11], and light-cone QCD sum rules [12].

We also consider here weak semileptonic and nonleptonic B_c decays to the highly excited $2P$ and $3S$ charmonium states. These states are of special interest since in past years a number of new charmoniumlike states above the open charm production threshold have been observed [13]. They include several unexpectedly narrow states, $X(3872)$, $X(3940)$, $Y(3940)$, $Z(3930)$, $Y(4260)$, $Z(4430)$, for which interpretation is controversial. Some of them could be candidates for excited charmonia. Therefore, experimental observation of such states in B_c decays could help to clarify their real nature.

II. RELATIVISTIC QUARK MODEL

In the quasipotential approach a meson is described as a bound quark-antiquark state with a wave function satisfying the quasipotential equation of the Schrödinger type

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M)\Psi_M(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}, \tag{2}$$

and E_1, E_2 are the center-of-mass energies on mass shell given by

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \qquad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}. \tag{3}$$

Here $M = E_1 + E_2$ is the meson mass, $m_{1,2}$ are the quark masses, and $\mathbf{p}$ is their relative momentum. In the center-of-mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}. \tag{4}$$

The kernel $V(\mathbf{p}, \mathbf{q}; M)$ in Eq. (1) is the quasipotential operator of the quark-antiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. Constructing the quasipotential of the quark-antiquark interaction, we have assumed that the effective interaction is the sum of the usual one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli interaction. The quasipotential is then defined by [3]

$$V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p}, \mathbf{q}; M)u_1(q)u_2(-q), \tag{5}$$

with

$$\mathcal{V}(\mathbf{p}, \mathbf{q}; M) = \tfrac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + V^S_{\rm conf}(\mathbf{k}),$$

where α_s is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge

$$D^{00}(\mathbf{k}) = -\frac{4\pi}{\mathbf{k}^2}, \qquad D^{ij}(\mathbf{k}) = -\frac{4\pi}{k^2}\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right), \qquad D^{0i} = D^{i0} = 0, \tag{6}$$

and $\mathbf{k} = \mathbf{p} - \mathbf{q}$. Here γ_μ and $u(p)$ are the Dirac matrices and spinors

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}}\begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p) + m}\end{pmatrix}\chi^\lambda, \tag{7}$$

where $\boldsymbol{\sigma}$ and χ^λ are Pauli matrices and spinors and $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$. The effective long-range vector vertex is given by

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu, \tag{8}$$

where κ is the Pauli interaction constant characterizing the long-range anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V_V(r) = (1 - \varepsilon)(Ar + B), \qquad V_S(r) = \varepsilon(Ar + B), \tag{9}$$

reproducing

$$V_{\rm conf}(r) = V_S(r) + V_V(r) = Ar + B, \tag{10}$$

where ε is the mixing coefficient.

The expression for the quasipotential of the heavy quarkonia, expanded in v^2/c^2 can be found in Ref. [3]. The quasipotential for the heavy quark interaction with a light antiquark without employing the nonrelativistic (v/c) expansion is given in Ref. [14]. All the parameters of our model like quark masses, parameters of the linear confining potential A and B, mixing coefficient ε and anomalous chromomagnetic quark moment κ are fixed from the analysis of heavy quarkonium masses and radiative decays [3]. The quark masses $m_b = 4.88$ GeV, $m_c = 1.55$ GeV, $m_s = 0.5$ GeV, $m_{u,d} = 0.33$ GeV and the parameters of the linear potential $A = 0.18$ GeV^2 and $B = -0.30$ GeV have values inherent for quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ has been determined from the consideration of the heavy quark expansion for the semileptonic $B \to D$ decays [15] and charmonium radiative decays [3]. Finally, the universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ^3P_J- states [3] and the heavy quark expansion for semileptonic decays of heavy mesons [15] and baryons [16]. Note that the long-range magnetic contribution to the potential in our model is proportional to $(1 + \kappa)$ and thus vanishes for the chosen value of $\kappa = -1$ in accordance with the flux tube model.

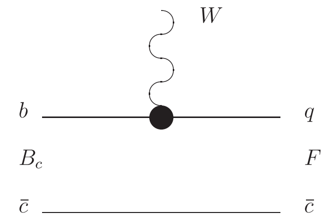

FIG. 1. Lowest order vertex function $\Gamma^{(1)}$ contributing to the current matrix element (11).

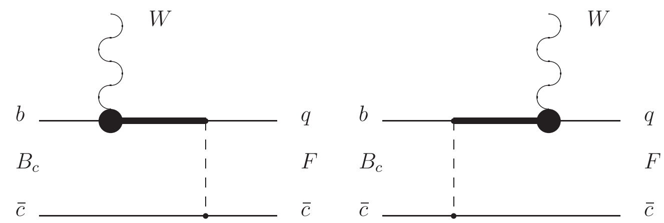

FIG. 2. Vertex function $\Gamma^{(2)}$ taking the quark interaction into account. Dashed lines correspond to the effective potential $\mathcal{V}$ in (5). Bold lines denote the negative-energy part of the quark propagator.

III. MATRIX ELEMENTS OF THE ELECTROWEAK CURRENT FOR $b \to c, u$ AND $c \to s, d$ TRANSITIONS

In order to calculate the exclusive semileptonic decay rate of the B_c meson, it is necessary to determine the corresponding matrix element of the weak current between meson states. First we consider the weak B_c decays governed by the b quark decays. In the quasipotential approach, the matrix element of the weak current $J^W_\mu = \bar{q}\gamma_\mu(1 - \gamma_5)b$, associated with the $b \to q$ ($q = c$ or u) transition, between a B_c meson with mass M_{B_c} and momentum p_{B_c} and a final P-wave meson F [$F = \chi_{cJ}$, h_c or $D_J^{(*)}$] with mass M_F and momentum p_F takes the form [17]

$$\langle F(p_F)|J^W_\mu|B_c(p_{B_c})\rangle = \int \frac{d^3p d^3q}{(2\pi)^6} \bar{\Psi}_{F\mathbf{p}_F}(\mathbf{p})\Gamma_\mu(\mathbf{p}, \mathbf{q}) \times \Psi_{B_c\mathbf{p}_{B_c}}(\mathbf{q}), \tag{11}$$

where $\Gamma_\mu(\mathbf{p}, \mathbf{q})$ is the two-particle vertex function and $\Psi_{M\mathbf{p}_M}$ are the meson ($M = B_c$, F) wave functions projected onto the positive energy states of quarks and boosted to the moving reference frame with momentum $\mathbf{p}_M$.

The contributions to Γ come from Figs. 1 and 2. The contribution $\Gamma^{(2)}$ is the consequence of the projection onto the positive-energy states. Note that the form of the relativistic corrections emerging from the vertex function $\Gamma^{(2)}$ explicitly depend on the Lorentz structure of the quark-antiquark interaction. In the leading order of the v^2/c^2 expansion for B_c and χ_J and in the heavy quark limit $m_c \to \infty$ for D_J only $\Gamma^{(1)}$ contributes, while $\Gamma^{(2)}$ contributes at the subleading order. The vertex functions look like

$$\Gamma^{(1)}_\mu(\mathbf{p}, \mathbf{q}) = \bar{u}_q(p_q)\gamma_\mu(1 - \gamma^5)u_b(q_b)(2\pi)^3\delta(\mathbf{p}_c - \mathbf{q}_c), \tag{12}$$

and

$$\Gamma^{(2)}_\mu(\mathbf{p}, \mathbf{q}) = \bar{u}_q(p_q)\bar{u}_c(p_c)\Big\{\gamma_{1\mu}(1 - \gamma_1^5)\frac{\Lambda_b^{(-)}(k)}{\epsilon_b(k) + \epsilon_b(p_q)} \times \gamma_1^0\mathcal{V}(\mathbf{p}_c - \mathbf{q}_c) + \mathcal{V}(\mathbf{p}_c - \mathbf{q}_c) \times \frac{\Lambda_q^{(-)}(k')}{\epsilon_q(k') + \epsilon_q(q_b)}\gamma_1^0\gamma_{1\mu}(1 - \gamma_1^5)\Big\} \times u_b(q_b)u_c(q_c), \tag{13}$$

where the superscripts "(1)" and "(2)" correspond to Figs. 1 and 2, $\mathbf{k} = \mathbf{p}_q - \mathbf{\Delta}$; $\mathbf{k}' = \mathbf{q}_b + \mathbf{\Delta}$; $\mathbf{\Delta} = \mathbf{p}_F - \mathbf{p}_{B_c}$;

$$\Lambda^{(-)}(p) = \frac{\epsilon(p) - (m\gamma^0 + \gamma^0(\boldsymbol{\gamma}\mathbf{p}))}{2\epsilon(p)}.$$

Here [17]

$$p_{q,c} = \epsilon_{q,c}(p)\frac{p_F}{M_F} \pm \sum_{i=1}^{3} n^{(i)}(p_F)p^i,$$

$$q_{b,c} = \epsilon_{b,c}(q)\frac{p_{B_c}}{M_{B_c}} \pm \sum_{i=1}^{3} n^{(i)}(p_{B_c})q^i,$$

and $n^{(i)}$ are three four-vectors given by

$$n^{(i)\mu}(p) = \left\{\frac{p^i}{M}, \delta_{ij} + \frac{p^i p^j}{M(E+M)}\right\}, \qquad E = \sqrt{\mathbf{p}^2 + M^2}.$$

The wave function of a final P-wave F meson at rest is given by

$$\Psi_F(\mathbf{p}) \equiv \Psi^{J\mathcal{M}}_{F(^{2S+1}P_J)}(\mathbf{p}) = \mathcal{Y}^{J\mathcal{M}}_S \psi_{F(^{2S+1}P_J)}(\mathbf{p}), \tag{14}$$

where J and $\mathcal{M}$ are the total meson angular momentum and its projection, while $S = 0$, 1 is the total spin. $\psi_{F(^{2S+1}P_J)}(\mathbf{p})$ is the radial part of the wave function, which has been determined by the numerical solution of Eq. (1) in [3,14]. The spin-angular momentum part $\mathcal{Y}^{J\mathcal{M}}_S$ has the following form

$$\mathcal{Y}^{J\mathcal{M}}_S = \sum_{\sigma_1\sigma_2} \langle 1\mathcal{M} - \sigma_1 - \sigma_2, S\sigma_1 + \sigma_2 | J\mathcal{M}\rangle \times \left\langle \frac{1}{2}\sigma_1, \frac{1}{2}\sigma_2 \Big| S\sigma_1 + \sigma_2 \right\rangle Y_1^{\mathcal{M}-\sigma_1-\sigma_2} \chi_1(\sigma_1)\chi_2(\sigma_2). \tag{15}$$

Here $\langle j_1 m_1, j_2 m_2 | J\mathcal{M}\rangle$ are the Clebsch-Gordan coefficients, Y_l^m are spherical harmonics, and $\chi(\sigma)$ (where $\sigma = \pm 1/2$) are spin wave functions,

$$\chi(1/2) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \chi(-1/2) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

The heavy-light meson states (such as D_1, D_1' etc.) with $J = L = 1$ are mixtures of spin-triplet $F(^3P_1)$ and spin-singlet $F(^1P_1)$ states:

$$\begin{aligned} \Psi_{F_1} &= \Psi_{F(^1P_1)} \cos\varphi + \Psi_{F(^3P_1)} \sin\varphi, \\ \Psi_{F_1'} &= -\Psi_{F(^1P_1)} \sin\varphi + \Psi_{F(^3P_1)} \cos\varphi, \end{aligned} \tag{16}$$

where φ is the mixing angle and the primed state has the heavier mass. Such mixing occurs due to the nondiagonal spin-orbit and tensor terms in the $Q\bar{q}$ quasipotential. The physical states are obtained by diagonalizing the corresponding mixing terms. The values of the mixing angle φ were determined in the heavy-light meson mass spectra calculations [14] and are given in Table I.

It is important to note that the wave functions entering the weak current matrix element (11) are not in the rest frame in general. For example, in the B_c meson rest frame ($\mathbf{p}_{B_c} = 0$), the final meson is moving with the recoil momentum $\mathbf{\Delta}$. The wave function of the moving meson $\Psi_{F\mathbf{\Delta}}$ is connected with the wave function in the rest frame $\Psi_{F\mathbf{0}} \equiv \Psi_F$ by the transformation [17]

$$\Psi_{F\mathbf{\Delta}}(\mathbf{p}) = D_q^{1/2}(R^W_{L_{\mathbf{\Delta}}}) D_c^{1/2}(R^W_{L_{\mathbf{\Delta}}}) \Psi_{F\mathbf{0}}(\mathbf{p}), \tag{17}$$

where R^W is the Wigner rotation, $L_{\mathbf{\Delta}}$ is the Lorentz boost from the meson rest frame to a moving one, and the rotation matrix $D^{1/2}(R)$ in spinor representation is given by

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} D^{1/2}_{q,c}(R^W_{L_{\mathbf{\Delta}}}) = S^{-1}(\mathbf{p}_{q,c}) S(\mathbf{\Delta}) S(\mathbf{p}), \tag{18}$$

where

$$S(\mathbf{p}) = \sqrt{\frac{\epsilon(p) + m}{2m}} \left(1 + \frac{\boldsymbol{\alpha}\mathbf{p}}{\epsilon(p) + m}\right)$$

is the usual Lorentz transformation matrix of the four-spinor.

The expressions for the matrix elements of the B_c decays to the P-wave B_{sJ} and B_J mesons, governed by the c quark decays, can be obtained from the above expressions by the interchange of the b and c quarks and for the final active quarks $q = s, d$.

IV. FORM FACTORS OF THE SEMILEPTONIC B_c DECAYS TO THE ORBITALLY EXCITED HEAVY MESONS

The matrix elements of the weak current $J^W_\mu = \bar{b}\gamma_\mu(1 - \gamma_5)q$ or $\bar{c}\gamma_\mu(1 - \gamma_5)q$ for B_c decays to orbitally excited scalar light mesons (S) can be parametrized by two invariant form factors

$$\begin{aligned} \langle S(p_F)|\bar{q}\gamma^\mu b|B_c(p_{B_c})\rangle &= 0, \\ \langle S(p_F)|\bar{q}\gamma^\mu\gamma_5 b|B_c(p_{B_c})\rangle &= f_+(q^2)(p^\mu_{B_c} + p^\mu_F) + f_-(q^2)(p^\mu_{B_c} - p^\mu_F), \end{aligned} \tag{19}$$

where $q = p_{B_c} - p_F$, M_S is the scalar meson mass.

The matrix elements of the weak current for B_c decays to axial vector mesons (AV) can be expressed in terms of four invariant form factors

$$\langle A(p_F)|\bar{q}\gamma^\mu b|B_c(p_{B_c})\rangle = (M_{B_c} + M_A) h_{V_1}(q^2)\epsilon^{*\mu} + [h_{V_2}(q^2) p^\mu_{B_c} + h_{V_3}(q^2) p^\mu_F] \times \frac{\epsilon^* \cdot q}{M_{B_c}}, \tag{20}$$

$$\langle A(p_F)|\bar{q}\gamma^\mu\gamma_5 b|B_c(p_{B_c})\rangle = \frac{2ih_A(q^2)}{M_{B_c} + M_A} \epsilon^{\mu\nu\rho\sigma} \epsilon^*_\nu p_{B_c\rho} p_{F\sigma}, \tag{21}$$

TABLE I. Mixing angles φ for heavy-light mesons (in °).

State	D	D_s	B	B_s
$1P$	35.5	34.5	35.0	36.0
$2P$	37.5	37.6	37.3	34.0

where M_A and ϵ^μ are the mass and polarization vector of the axial vector meson.

The matrix elements of the weak current for B_c decays to tensor mesons (T) can be decomposed in four Lorentz-invariant structures

$$\langle T(p_F)|\bar{q}\gamma^\mu b|B_c(p_{B_c})\rangle = \frac{2it_V(q^2)}{M_{B_c}+M_T}\epsilon^{\mu\nu\rho\sigma}\epsilon^*_{\nu\alpha}\frac{p^\alpha_{B_c}}{M_{B_c}} \times p_{B_c\rho}p_{F\sigma}, \tag{22}$$

$$\langle T(p_F)|\bar{q}\gamma^\mu\gamma_5 b|B_c(p_{B_c})\rangle = (M_{B_c}+M_T)t_{A_1}(q^2)\epsilon^{*\mu\alpha}\frac{p_{B\alpha}}{M_{B_c}} + [t_{A_2}(q^2)p^\mu_{B_c} + t_{A_3}(q^2)p^\mu_F]\epsilon^*_{\alpha\beta}\frac{p^\alpha_{B_c}p^\beta_{B_c}}{M^2_{B_c}}, \tag{23}$$

where M_T and $\epsilon^{\mu\nu}$ are the mass and polarization tensor of the tensor meson.

We previously studied the form factors parametrizing the matrix elements of vector and axial vector charged and neutral weak currents for $B_c \to \eta_c(J/\psi)$, $B_c \to D^{(*)}$ [6], $B_c \to B_s^{(*)}(B^{(*)})$ [7], and $B_c \to D_s^{(*)}$ transitions in the framework of our model. Now we apply the same approach, described in detail in Refs. [6,7,18], for the calculation of the form factors for B_c decays to the orbitally excited heavy mesons. Namely, we calculate exactly the contribution of the leading vertex function $\Gamma^{(1)}$ (12) to the transition matrix element of the weak current (11) using the δ-function. For the evaluation of the subleading contribution $\Gamma^{(2)}$ for the $B_c \to \chi_J(h_c)$ and $B_c \to D_J$ transitions, governed by $b \to c$, u transitions, we use expansions in inverse powers of the heavy b-quark mass from the initial B_c meson and large recoil energy of the final heavy meson. Note that the latter contributions turn out to be rather small numerically. Therefore, we obtain reliable expressions for the form factors in the whole accessible kinematical range. It is important to emphasize that in doing these calculations we consistently take into account all relativistic corrections including boosts of the meson wave functions from the rest reference frame to the moving ones, given by Eq. (17). The obtained expressions for the decay form factors are given in the appendix (to simplify these expressions the long-range anomalous chromomagnetic quark moment was explicitly set as $\kappa = -1$). In the limits of infinitely heavy quark mass and large energy of the final meson, the form factors in our model satisfy all heavy quark symmetry relations [19,20].

As a result, we get the following expressions for the B_c decay form factors:

(a) $B_c \to S$ transitions ($S = \chi_{c0}$, D_0)

$$f_\pm(q^2) = f_\pm^{(1)}(q^2) + \varepsilon f_\pm^{S(2)}(q^2) + (1-\varepsilon)f_\pm^{V(2)}(q^2), \tag{24}$$

(b) $B_c \to AV$ transition ($AV = \chi_{c1}$, $D_1(^3P_1)$)

$$h_{V_i}(q^2) = h_{V_i}^{(1)}(q^2) + \varepsilon h_{V_i}^{S(2)}(q^2) + (1-\varepsilon)h_{V_i}^{V(2)}(q^2), \quad (i = 1, 2, 3),$$
$$h_A(q^2) = h_A^{(1)}(q^2) + \varepsilon h_A^{S(2)}(q^2) + (1-\varepsilon)h_A^{V(2)}(q^2), \tag{25}$$

(c) $B_c \to AV'$ transition[1] [$AV' = h_c$, $D_1(^1P_1)$]

$$g_{V_i}(q^2) = g_{V_i}^{(1)}(q^2) + \varepsilon g_{V_i}^{S(2)}(q^2) + (1-\varepsilon)g_{V_i}^{V(2)}(q^2), \quad (i = 1, 2, 3),$$
$$g_A(q^2) = g_A^{(1)}(q^2) + \varepsilon g_A^{S(2)}(q^2) + (1-\varepsilon)g_A^{V(2)}(q^2), \tag{26}$$

(d) $B_c \to T$ transition ($T = \chi_{c2}$, D_2^*)

$$t_V(q^2) = t_V^{(1)}(q^2) + \varepsilon t_V^{S(2)}(q^2) + (1-\varepsilon)t_V^{V(2)}(q^2),$$
$$t_{A_i}(q^2) = t_{A_i}^{(1)}(q^2) + \varepsilon t_{A_i}^{S(2)}(q^2) + (1-\varepsilon)t_{A_i}^{V(2)}(q^2), \quad (i = 1, 2, 3), \tag{27}$$

where $f_\pm^{(1)}$, $f_\pm^{S,V(2)}$, $h_{V_i}^{(1)}$, $h_{V_i}^{S,V(2)}$, $h_A^{(1)}$, $h_A^{S,V(2)}$, $g_{V_i}^{(1)}$, $g_{V_i}^{S,V(2)}$, $g_A^{(1)}$, $g_A^{S,V(2)}$, $t_V^{(1)}$, $t_V^{S,V(2)}$, $t_{A_i}^{(1)}$, and $t_{A_i}^{S,V(2)}$ are given in the appendix. The superscripts "(1)" and "(2)" correspond to Figs. 1 and 2, S and V correspond to the scalar and vector confining potentials of the $q\bar{q}$-interaction. The mixing parameter of scalar and vector confining potentials ε is fixed to be -1 in our model.

In the case of B_c decays to P-wave B_s and B mesons, governed by the $c \to s$, d transitions, the accessible kinematical range is significantly smaller (by almost a factor of 4) than the one for the decays to the S-wave B_s and B mesons. Our previous investigation [7] of the latter decays had shown that intermediate negative-energy states, leading to the subleading term $\Gamma^{(2)}$, give almost negligible contributions to decay form factors (see Fig. 3 of Ref. [7]). Therefore, such contributions can be safely neglected in the present analysis. Thus, for calculations of the form factors of the $B_c \to B_{sJ}$ and $B_c \to B_J$ weak transitions we use the leading order expressions $f_i^{(1)}$, $h_i^{(1)}$, $g_i^{(1)}$, and $t_i^{(1)}$, given in the appendix, where the b and c quarks are interchanged.

For numerical calculations of the form factors we use the quasipotential wave functions of the B_c meson and orbitally excited charmonium and D, B_s, B mesons obtained in their mass spectra calculations [3,14]. Our results for the

[1]The corresponding decay matrix elements are defined by Eqs. (20) and (21).

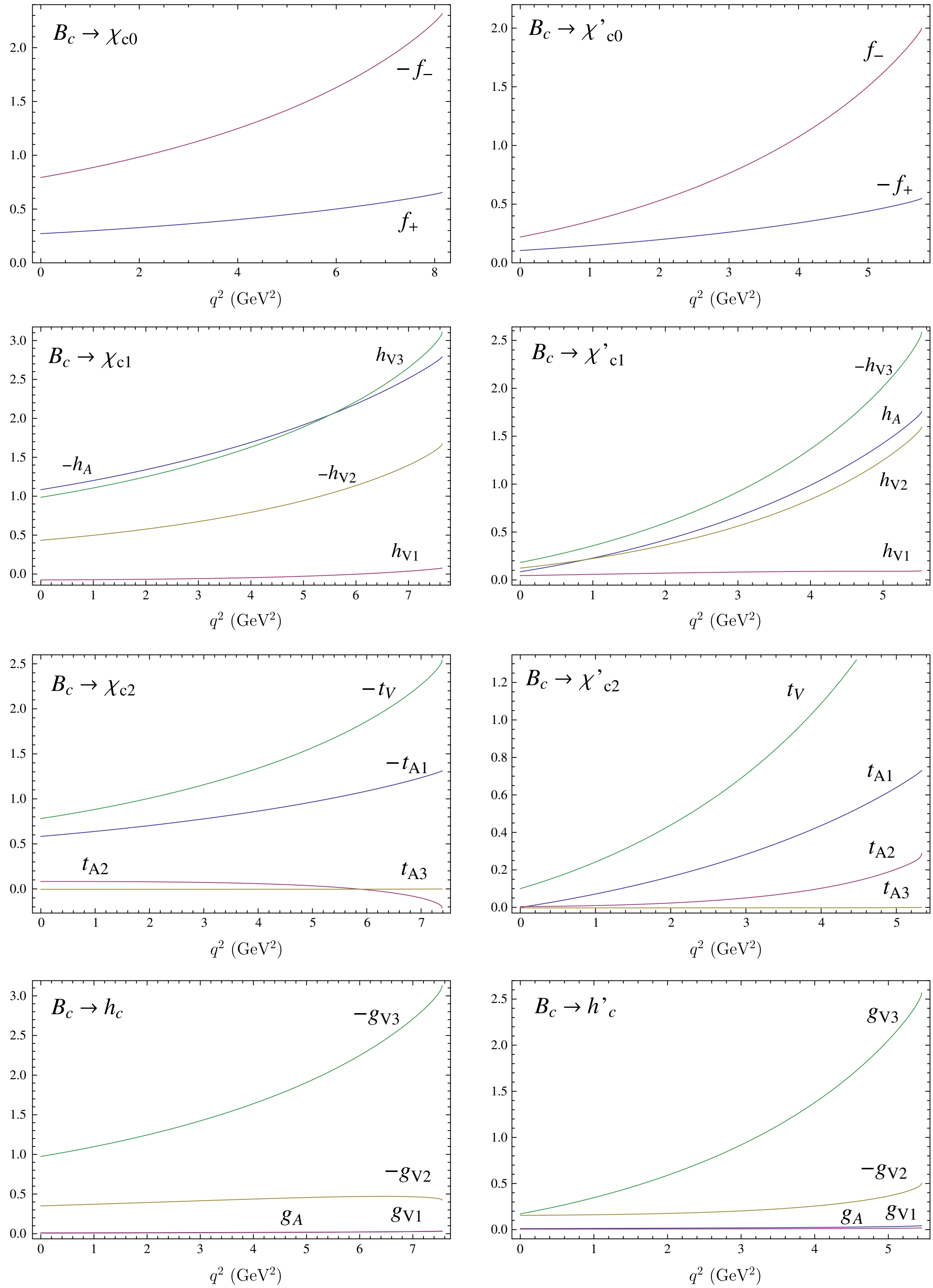

FIG. 3 (color online). Form factors of the B_c decays to the $1P$- and $2P$ -wave charmonium states.

masses of these mesons are in good agreement with available experimental data [21]. Therefore, we use the experimental values for the masses of well-established states and our model predictions for all other masses in the numerical calculations.

In Fig. 3 we plot form factors of the B_c weak transitions to the $1P$ (χ_{cJ}, h_c) and $2P$ (χ'_{cJ}, h'_c) -wave charmonium states as an example. The remaining plots for the B_c weak form factors to the P-wave D_J, B_{sJ}, and B_J mesons have an analogous behavior and are not shown here.

V. SEMILEPTONIC B_c DECAYS TO ORBITALLY EXCITED HEAVY MESONS

The differential decay rate for the B_c meson decay to P-wave heavy mesons reads [8]

$$\frac{d\Gamma(B_c \to F(S, AV, T) l\bar{\nu})}{dq^2} = \frac{G_F^2}{(2\pi)^3}|V_{bf}|^2 \frac{\lambda^{1/2}(q^2 - m_l^2)^2}{24 M_{B_c}^3 q^2} \times \left[HH^\dagger\left(1 + \frac{m_l^2}{2q^2}\right) + \frac{3m_l^2}{2q^2} H_t H_t^\dagger\right], \tag{28}$$

where G_F is the Fermi constant, V_{ij} are the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements, $\lambda \equiv \lambda(M_{B_c}^2, M_F^2, q^2) = M_{B_c}^4 + M_F^4 + q^4 - 2(M_{B_c}^2 M_F^2 + M_F^2 q^2 + M_{B_c}^2 q^2)$, m_l is the lepton mass and

$$HH^\dagger \equiv H_+ H_+^\dagger + H_- H_-^\dagger + H_0 H_0^\dagger. \tag{29}$$

Helicity components of the hadronic tensor are expressed through the invariant form factors.

(a) $B_c \to S(^3P_0)$ transition

$$H_\pm = 0, \qquad H_0 = \frac{\lambda^{1/2}}{\sqrt{q^2}} f_+(q^2),$$
$$H_t = \frac{1}{\sqrt{q^2}}[(M_{B_c}^2 - M_S^2) f_+(q^2) + q^2 f_-(q^2)]. \tag{30}$$

(b) $B_c \to AV(^3P_1)$ transition

$$H_\pm = (M_{B_c} + M_{AV}) h_{V_1}(q^2) \pm \frac{\lambda^{1/2}}{M_{B_c} + M_{AV}} h_A,$$
$$H_0 = \frac{1}{2M_{AV}\sqrt{q^2}}\left\{(M_{B_c} + M_{AV})(M_{B_c}^2 - M_{AV}^2 - q^2) h_{V_1}(q^2) + \frac{\lambda}{2M_{B_c}}[h_{V_2}(q^2) + h_{V_3}(q^2)]\right\},$$
$$H_t = \frac{\lambda^{1/2}}{2M_{AV}\sqrt{q^2}}\left\{(M_{B_c} + M_{AV}) h_{V_1}(q^2) + \frac{M_{B_c}^2 - M_{AV}^2}{2M_{B_c}}[h_{V_2}(q^2) + h_{V_3}(q^2)] + \frac{q^2}{2M_{B_c}}[h_{V_2}(q^2) - h_{V_3}(q^2)]\right\}. \tag{31}$$

(c) $B_c \to AV'(^1P_1)$ transition

H_i are obtained from expressions (31) by replacement of form factors $h_i(q^2)$ by $g_i(q^2)$.

(d) $B_c \to T(^3P_2)$ transition

$$H_\pm = \frac{\lambda^{1/2}}{2\sqrt{2} M_{B_c} M_T}\left[(M_{B_c} + M_T) t_{A_1}(q^2) \pm \frac{\lambda^{1/2}}{M_{B_c} + M_T} t_V\right],$$
$$H_0 = \frac{\lambda^{1/2}}{2\sqrt{6} M_{B_c} M_T^2 \sqrt{q^2}}\left\{(M_{B_c} + M_T)(M_{B_c}^2 - M_T^2 - q^2) t_{A_1}(q^2) + \frac{\lambda}{2M_{B_c}}[t_{A_2}(q^2) + t_{A_3}(q^2)]\right\},$$
$$H_t = \sqrt{\frac{2}{3}} \frac{\lambda}{4M_{B_c} M_T^2 \sqrt{q^2}}\left\{(M_{B_c} + M_T) t_{A_1}(q^2) + \frac{M_{B_c}^2 - M_T^2}{2M_{B_c}}[t_{A_2}(q^2) + t_{A_3}(q^2)] + \frac{q^2}{2M_{B_c}}[t_{A_2}(q^2) - t_{A_3}(q^2)]\right\}. \tag{32}$$

Here the subscripts $\pm$, 0, t denote transverse, longitudinal and time helicity components, respectively.

Now we substitute the weak decay form factors calculated in the previous section in the above expressions for decay rates. The resulting differential distributions for B_c decays to the $1P$ (χ_J, h_c) and $2P$ (χ'_J, h'_c) charmonium states are plotted in Fig. 4. The difference of the plot shapes for the corresponding $1P$ and $2P$ charmonium states is the consequence of their different nodal structure. We calculate the total rates of the semileptonic B_c decays to the

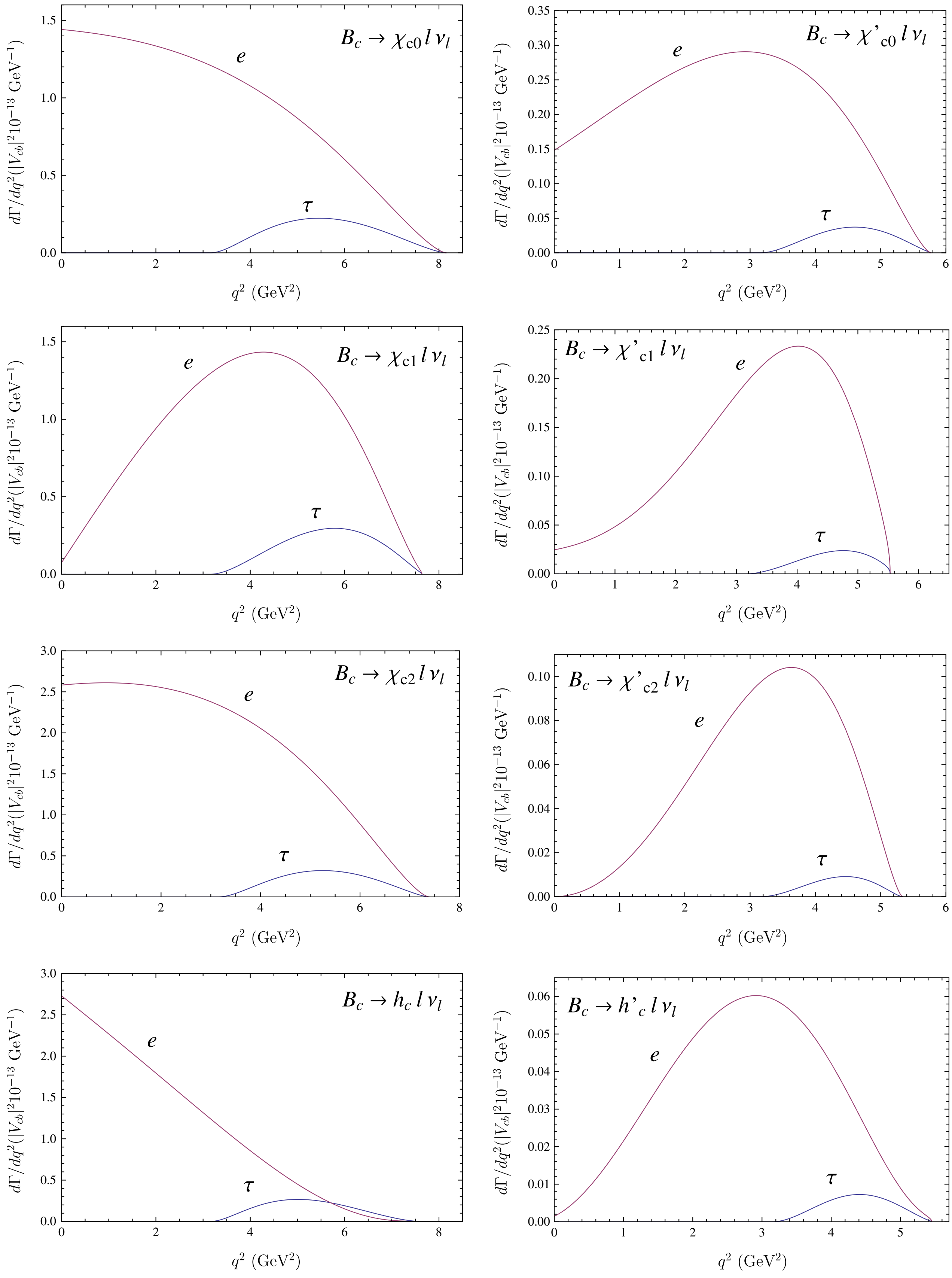

FIG. 4 (color online). Predictions for the differential decay rates of the B_c semileptonic decays to the $1P$- and $2P$-wave charmonium states.

TABLE II. Comparison of our predictions for the rates of the semileptonic B_c decays to the P-wave charmonium states with previous calculations (in 10^{-15} GeV).

Decay	our	[8]	[9]	[10]	[11]	[12]
$B_c \to \chi_{c0} e\nu$	1.27	2.52	1.55	1.69	2.60 ± 0.73	
$B_c \to \chi_{c0} \tau\nu$	0.11	0.26	0.19	0.25	0.70 ± 0.23	
$B_c \to \chi_{c1} e\nu$	1.18	1.40	0.94	2.21	2.09 ± 0.60	
$B_c \to \chi_{c1} \tau\nu$	0.13	0.17	0.10	0.35	0.21 ± 0.06	
$B_c \to \chi_{c2} e\nu$	2.27	2.92	1.89	2.73		
$B_c \to \chi_{c2} \tau\nu$	0.13	0.20	0.13	0.42		
$B_c \to h_c e\nu$	1.38	4.42	2.40	2.51	2.03 ± 0.57	4.2 ± 2.1
$B_c \to h_c \tau\nu$	0.11	0.38	0.21	0.36	0.20 ± 0.05	0.53 ± 0.26
$B_c \to \chi'_{c0} e\nu$	0.19					10 ± 6
$B_c \to \chi'_{c0} \tau\nu$	0.0089					0.39 ± 0.20
$B_c \to \chi'_{c1} e\nu$	0.12					8.6 ± 4.8
$B_c \to \chi'_{c1} \tau\nu$	0.0056					0.31 ± 0.18
$B_c \to \chi'_{c2} e\nu$	0.048					
$B_c \to \chi'_{c2} \tau\nu$	0.0019					
$B_c \to h'_c e\nu$	0.031					0.76 ± 0.33
$B_c \to h'_c \tau\nu$	0.0016					0.028 ± 0.014

TABLE III. Predictions for the rates of the semileptonic B_c decays to the P-wave D mesons (in 10^{-15} GeV).

Decay	Γ	Decay	Γ
$B_c \to D_0 e\nu$	0.016	$B_c \to D_0 \tau\nu$	0.0067
$B_c \to D_1 e\nu$	0.016	$B_c \to D_1 \tau\nu$	0.0056
$B_c \to D'_1 e\nu$	0.027	$B_c \to D'_1 \tau\nu$	0.016
$B_c \to D_2 e\nu$	0.052	$B_c \to D_2 \tau\nu$	0.019

P-wave heavy mesons by integrating the corresponding differential decay rates over q^2. For calculations we use the following values of the CKM matrix elements: $|V_{cb}| = 0.041$, $|V_{ub}| = 0.0038$, $|V_{cs}| = 0.974$, $|V_{cd}| = 0.223$. Our predictions for the rates of the semileptonic B_c decays to the P-wave charmonium states are compared with the previous calculations [8–12] in Table II. The authors of Refs. [8–10] use different types of relativistic quark models. Calculations in Ref. [11] are based on the three-point QCD sum rules, while Ref. [12] employs the light-cone QCD sum rules. We find that significantly different theoretical approaches give values for the $B_c \to \chi_J(h_c)l\nu$ decay rates consistent in the order of magnitude, while for the B_c decays to first radial excitations of the P-wave charmonium ($B_c \to \chi'_J(h'_c)l\nu$) our results are almost an order of magnitude lower than the predictions of the light-cone QCD sum rules [12], which are the only available ones at present. The latter decays can play an important role in studying charmonium states above the open charm production threshold. Their observation at Tevatron and LHC can help to clarify the nature of the new charmoniumlike states. Our results for the rates of the CKM suppressed semileptonic B_c decays to the P-wave D mesons, governed by the weak $b \to u$ transitions, are given in Table III.

In Fig. 5 we plot predicted differential semileptonic decay rates of the B_c to P-wave B_s meson states, governed by the $c \to s$ weak transitions. The allowed kinematical range for these transitions is rather narrow. Therefore semileptonic decays involving the τ lepton are forbidden. From these plots we see that even the account of the rather small muon mass significantly modifies the differential decay rates. The corresponding plots for $B_c \to B_J l\nu$ decays have similar shape and are not shown here. The predicted values for the rates of the semileptonic B_c decays to the P-wave B_s and B mesons are given in Tables IV and V. Note that, notwithstanding the fact that these decay rates have significantly larger values of the CKM matrix elements than the rates of B_c decays to charmonium, they have the same order of magnitude. This is the result of the above-mentioned strong phase space suppression of the $B_c \to B_{sJ} l\nu$ decays.

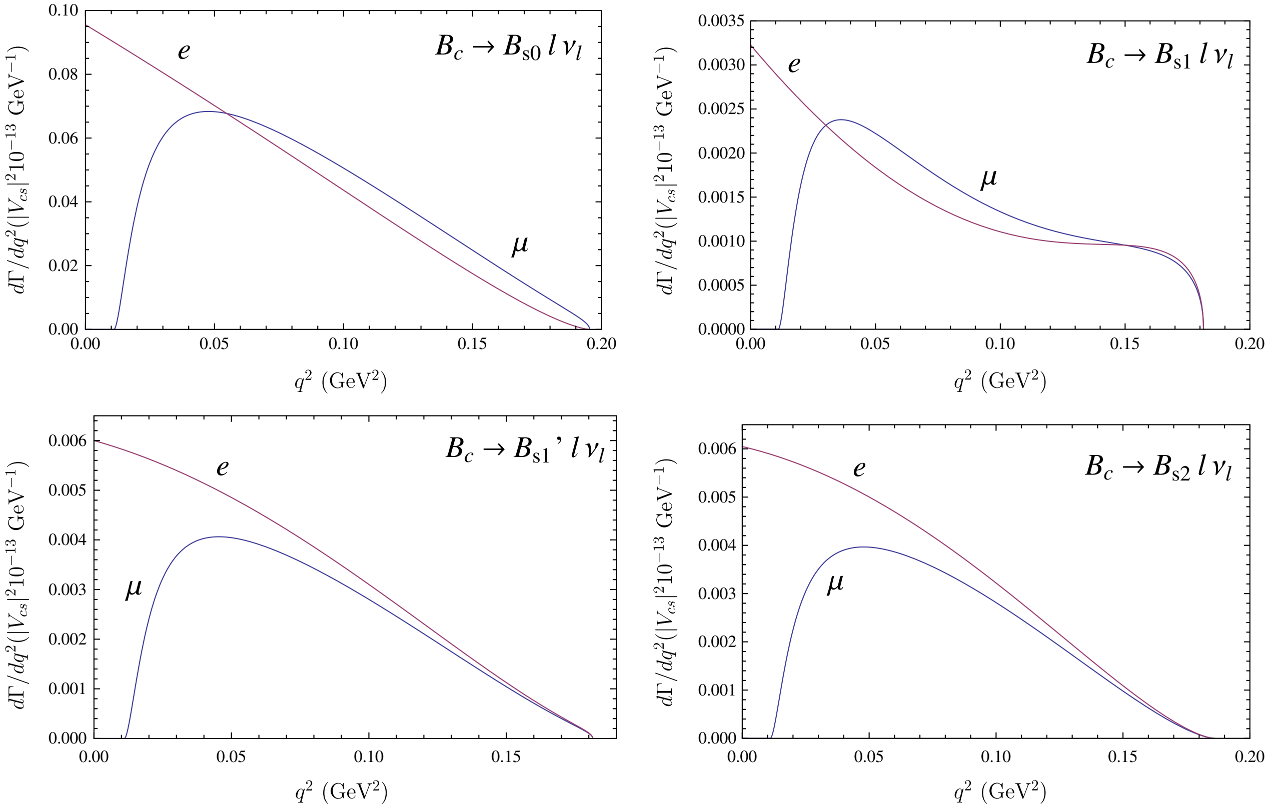

FIG. 5 (color online). Predictions for the differential decay rates of the B_c semileptonic decays to the P-wave B_s meson states.

TABLE IV. Predictions for the rates of the semileptonic B_c decays to the P-wave B_s mesons (in 10^{-15} GeV).

Decay	Γ	Decay	Γ
$B_c \to B_{s0} e\nu$	0.96	$B_c \to B_{s0} \mu\nu$	0.82
$B_c \to B_{s1} e\nu$	0.029	$B_c \to B_{s1} \mu\nu$	0.026
$B_c \to B'_{s1} e\nu$	0.065	$B_c \to B'_{s1} \mu\nu$	0.044
$B_c \to B_{s2} e\nu$	0.066	$B_c \to B_{s2} \mu\nu$	0.031

TABLE V. Predictions for the rates of the semileptonic B_c decays to the P-wave B mesons (in 10^{-15} GeV).

Decay	Γ	Decay	Γ
$B_c \to B_0 e\nu$	0.089	$B_c \to B_0 \mu\nu$	0.082
$B_c \to B_1 e\nu$	0.0048	$B_c \to B_1 \mu\nu$	0.0043
$B_c \to B'_1 e\nu$	0.010	$B_c \to B'_1 \mu\nu$	0.0082
$B_c \to B_2 e\nu$	0.012	$B_c \to B_2 \mu\nu$	0.0067

VI. NONLEPTONIC DECAYS

In the standard model, nonleptonic B_c decays are described by the effective Hamiltonian, obtained by integrating out the heavy W-boson and top quark.

(a) For the case of the $b \to c$, u transitions, one gets

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb}[c_1(\mu) O_1^{cb} + c_2(\mu) O_2^{cb}] + \frac{G_F}{\sqrt{2}} V_{ub}[c_1(\mu) O_1^{ub} + c_2(\mu) O_2^{ub}] + \ldots. \tag{33}$$

(b) For the case of the $c \to s$, d transitions, we have

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cs}[c_1(\mu) O_1^{cs} + c_2(\mu) O_2^{cs}] + \frac{G_F}{\sqrt{2}} V_{cd}[c_1(\mu) O_1^{cd} + c_2(\mu) O_2^{cd}] + \ldots. \tag{34}$$

The Wilson coefficients $c_{1,2}(\mu)$ are evaluated perturbatively at the W scale and then are evolved down to the renormalization scale $\mu \approx m_b$ by the renormalization-group equations. The ellipsis denote the penguin operators, the Wilson coefficients of which are numerically much smaller than $c_{1,2}$. The local four-quark operators O_1 and O_2 are given by

$$O_1^{qb} = [(\tilde{d}u)_{V-A} + (\tilde{s}c)_{V-A}](\bar{q}b)_{V-A},$$
$$O_2^{qb} = (\bar{q}u)_{V-A}(\tilde{d}b)_{V-A} + (\bar{q}c)_{V-A}(\tilde{s}b)_{V-A}, \quad (35)$$
$$q = (u, c),$$

and

$$O_1^{cq} = (\tilde{d}u)_{V-A}(\bar{c}q)_{V-A},$$
$$O_2^{cq} = (\bar{c}u)_{V-A}(\tilde{d}q)_{V-A}, \quad (36)$$
$$q = (s, d),$$

where the rotated antiquark fields are

$$\tilde{d} = V_{ud}\bar{d} + V_{us}\bar{s}, \qquad \tilde{s} = V_{cd}\bar{d} + V_{cs}\bar{s}, \quad (37)$$

and for the hadronic current the following notation is used

$$(\bar{q}q')_{V-A} = \bar{q}\gamma_\mu(1-\gamma_5)q' \equiv J^W_\mu.$$

The factorization approach, which is extensively used for the calculation of two-body nonleptonic decays, such as $B_c \to FM$, assumes that the nonleptonic decay amplitude reduces to the product of a meson transition matrix element and a decay constant [22]. This assumption in general cannot be exact. However, it is expected that factorization can hold for energetic decays, where the final F meson is heavy and the M meson is light [23]. A justification of this assumption is usually based on the issue of color transparency [24]. In these decays the final hadrons are produced in the form of almost pointlike color-singlet objects with a large relative momentum. And thus the hadronization of the decay products occurs after they are too far separated for strongly interacting with each other. That provides the possibility to avoid the final state interaction. A more general treatment of factorization is given in Refs. [25,26].

Here we first analyze the B_c^+ nonleptonic decays to the P-wave charmonium and the light π^+, ρ^+ or $K^{(*)+}$ mesons, governed by the weak $b \to c, u$ transitions. The corresponding diagram is shown in Fig. 6(a), where $q_1 = d, s$ and $q_2 = u$. Then the decay amplitude can be approximated by the product of one-particle matrix elements

$$\langle F^0 M^+|H_{\rm eff}|B_c^+\rangle = \frac{G_F}{\sqrt{2}} V_{cb}V_{q_1q_2}a_1\langle F|(\bar{b}c)_{V-A}|B_c\rangle \times \langle M|(\bar{q}_1q_2)_{V-A}|0\rangle, \quad (38)$$

where

$$a_1 = c_1(\mu) + \frac{1}{N_c}c_2(\mu) \quad (39)$$

and N_c is the number of colors.

Next we consider nonleptonic decays of the B_c meson to the P-wave B_s or B mesons and the final light M^+ meson, governed by the weak $c \to s, d$ transitions. Only the pion is kinematically allowed. The corresponding diagram is shown in Fig. 6(b). Then in the factorization approximation the decay amplitude can be expressed through the product of one-particle matrix elements

$$\langle F^0 M^+|H_{\rm eff}|B_c^+\rangle = \frac{G_F}{\sqrt{2}} V_{cq}V_{q_1q_2}a_1\langle F|(\bar{c}q)_{V-A}|B_c\rangle \times \langle M|(\bar{q}_1q_2)_{V-A}|0\rangle. \quad (40)$$

The matrix element of the weak current J^W_μ between vacuum and a final pseudoscalar (P) or vector (V) meson is parametrized by the decay constants $f_{P,V}$

$$\langle P|\bar{q}_1\gamma^\mu\gamma_5 q_2|0\rangle = if_P p_P^\mu,$$
$$\langle V|\bar{q}_1\gamma_\mu q_2|0\rangle = \epsilon_\mu M_V f_V. \quad (41)$$

The pseudoscalar f_P and vector f_V decay constants were calculated within our model in Ref. [27]. It was shown that the complete account of relativistic effects is necessary to get agreement with experiment for decay constants especially of light mesons. We use the following values of the decay constants: $f_\pi = 0.131$ GeV, $f_\rho = 0.208$ GeV, $f_K = 0.160$ GeV, and $f_{K^*} = 0.214$ GeV. The relevant CKM matrix elements are $|V_{ud}| = 0.975$, $|V_{us}| = 0.222$.

The matrix elements of the weak current between the B_c meson and the final heavy meson F entering the factorized nonleptonic decay amplitude (38) are parametrized by the set of decay form factors defined in Eqs. (24)–(27). Using the form factors obtained in Sec. IV, we get predictions for the nonleptonic $B_c^+ \to \chi_J(h_c)^0 M^+$ decay rates and give them in Table VI in comparison with other calculations [8–10,28], which are available for the decays to the 1P

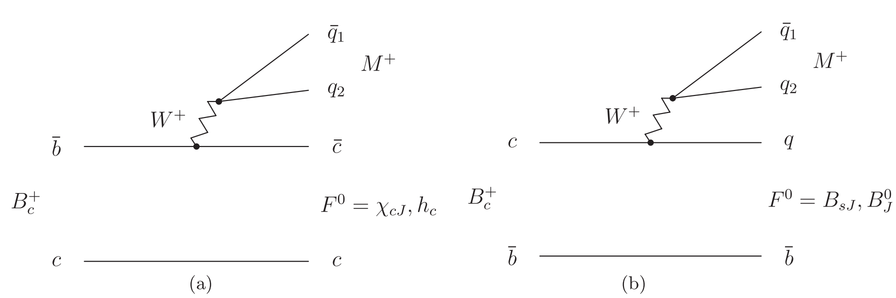

FIG. 6. Quark diagram for the nonleptonic $B_c^+ \to F^0 M^+$ decay.

TABLE VI. The rates of the nonleptonic B_c decays to the P-wave charmonium and light mesons (in 10^{-15} GeV).

Decay	our	[8]	[9]	[10]	[28]
$B_c^+ \to \chi_{c0}\pi^+$	$0.23a_1^2$	$0.622a_1^2$	$0.28a_1^2$	$0.317a_1^2$	$11a_1^2$
$B_c^+ \to \chi_{c0}\rho^+$	$0.64a_1^2$	$1.47a_1^2$	$0.73a_1^2$	$0.806a_1^2$	$37a_1^2$
$B_c^+ \to \chi_{c0}K^+$	$0.018a_1^2$	$0.0472a_1^2$	$0.022a_1^2$	$0.00235a_1^2$	
$B_c^+ \to \chi_{c0}K^{*+}$	$0.045a_1^2$	$0.0787a_1^2$	$0.041a_1^2$	$0.00443a_1^2$	
$B_c^+ \to \chi_{c1}\pi^+$	$0.22a_1^2$	$0.0768a_1^2$	$0.0015a_1^2$	$0.0815a_1^2$	$0.10a_1^2$
$B_c^+ \to \chi_{c1}\rho^+$	$0.16a_1^2$	$0.326a_1^2$	$0.11a_1^2$	$0.331a_1^2$	$5.2a_1^2$
$B_c^+ \to \chi_{c1}K^+$	$0.016a_1^2$	$0.0057a_1^2$	$0.00012a_1^2$	$0.0058a_1^2$	
$B_c^+ \to \chi_{c1}K^{*+}$	$0.010a_1^2$	$0.0201a_1^2$	$0.0080a_1^2$	$0.00205a_1^2$	
$B_c^+ \to \chi_{c2}\pi^+$	$0.41a_1^2$	$0.518a_1^2$	$0.24a_1^2$	$0.277a_1^2$	$8.9a_1^2$
$B_c^+ \to \chi_{c2}\rho^+$	$1.18a_1^2$	$1.33a_1^2$	$0.71a_1^2$	$0.579a_1^2$	$36a_1^2$
$B_c^+ \to \chi_{c2}K^+$	$0.031a_1^2$	$0.0384a_1^2$	$0.018a_1^2$	$0.00199a_1^2$	
$B_c^+ \to \chi_{c2}K^{*+}$	$0.082a_1^2$	$0.0732a_1^2$	$0.041a_1^2$	$0.00348a_1^2$	
$B_c^+ \to h_c\pi^+$	$0.51a_1^2$	$1.24a_1^2$	$0.58a_1^2$	$0.569a_1^2$	$18a_1^2$
$B_c^+ \to h_c\rho^+$	$1.11a_1^2$	$2.78a_1^2$	$1.41a_1^2$	$1.40a_1^2$	$60a_1^2$
$B_c^+ \to h_cK^+$	$0.039a_1^2$	$0.0939a_1^2$	$0.045a_1^2$	$0.0043a_1^2$	
$B_c^+ \to h_cK^{*+}$	$0.077a_1^2$	$0.146a_1^2$	$0.078a_1^2$	$0.0076a_1^2$	
$B_c^+ \to \chi'_{c0}\pi^+$	$0.023a_1^2$				
$B_c^+ \to \chi'_{c0}\rho^+$	$0.080a_1^2$				
$B_c^+ \to \chi'_{c0}K^+$	$0.0019a_1^2$				
$B_c^+ \to \chi'_{c0}K^{*+}$	$0.0055a_1^2$				
$B_c^+ \to \chi'_{c1}\pi^+$	$0.011a_1^2$				
$B_c^+ \to \chi'_{c1}\rho^+$	$0.016a_1^2$				
$B_c^+ \to \chi'_{c1}K^+$	$0.0095a_1^2$				
$B_c^+ \to \chi_{c1}K^{*+}$	$0.0011a_1^2$				
$B_c^+ \to \chi'_{c2}\pi^+$	$8.5\times10^{-7}a_1^2$				
$B_c^+ \to \chi'_{c2}\rho^+$	$0.0022a_1^2$				
$B_c^+ \to \chi'_{c2}K^+$	$8.5\times10^{-6}a_1^2$				
$B_c^+ \to \chi'_{c2}K^{*+}$	$0.00015a_1^2$				
$B_c^+ \to h'_c\pi^+$	$1.0\times10^{-5}a_1^2$				
$B_c^+ \to h'_c\rho^+$	$0.0051a_1^2$				
$B_c^+ \to h'_cK^+$	$3.4\times10^{-6}a_1^2$				
$B_c^+ \to h_cK^{*+}$	$0.00035a_1^2$				

charmonium states only. Predictions for the energetic nonleptonic decays to the $2P$ charmonium states are made for the first time and their measurement could be important for the identification of these states. Our results for the nonleptonic $B_c^+ \to B_{sJ}M^+$ and $B_c^+ \to B_JM^+$ decay rates are presented in Table VII. Note that in the latter case only decays involving pions are kinematically allowed.

TABLE VII. The rates of the nonleptonic B_c decays to the P-wave B_s or B mesons and π meson (in 10^{-15} GeV).

Decay	Γ	Decay	Γ
$B_c^+ \to B_{s0}\pi^+$	$5.82a_1^2$	$B_c^+ \to B_0^0\pi^+$	$0.46a_1^2$
$B_c^+ \to B_{s1}\pi^+$	$0.30a_1^2$	$B_c^+ \to B_1^0\pi^+$	$0.041a_1^2$
$B_c^+ \to B'_{s1}\pi^+$	$0.31a_1^2$	$B_c^+ \to B_1'^0\pi^+$	$0.061a_1^2$
$B_c^+ \to B_{s2}\pi^+$	$0.26a_1^2$	$B_c^+ \to B_2^0\pi^+$	$0.047a_1^2$

VII. CONCLUSIONS

We calculated the form factors of the weak B_c decays to orbitally excited heavy mesons, governed both by the $b \to c, u$ and $c \to s, d$ transitions, in the framework of the QCD-motivated relativistic quark model based on the quasipotential approach. The momentum dependence of the weak decay form factors was reliably determined in the whole accessible kinematical range. This is particularly important for B_c decays to the P-wave charmonium and D mesons since they have a rather broad kinematically allowed range ($q^2_{\rm max} \sim 6$–15 GeV^2). All essential relativistic effects were taken into account including transformations of the meson

TABLE VIII. Branching fractions (in %) of exclusive B_c decays calculated for the fixed values of the B_c lifetime $\tau_{B_c} = 0.46$ ps and $a_1 = 1.14$ for the $b \to c$ transitions and $a_1 = 1.20$ for the $c \to s, d$ transitions.

Decay	Br	Decay	Br	Decay	Br
$B_c \to \chi_{c0} e\nu$	0.087	$B_c^+ \to \chi_{c0}\pi^+$	0.021	$B_c^+ \to \chi'_{c0}\pi^+$	0.0020
$B_c \to \chi_{c0} \tau\nu$	0.0075	$B_c^+ \to \chi_{c0}\rho^+$	0.058	$B_c^+ \to \chi'_{c0}\rho^+$	0.0071
$B_c \to \chi_{c1} e\nu$	0.082	$B_c^+ \to \chi_{c0}K^+$	0.0016	$B_c^+ \to \chi'_{c0}K^+$	0.000 17
$B_c \to \chi_{c1} \tau\nu$	0.0092	$B_c^+ \to \chi_{c0}K^{*+}$	0.0040	$B_c^+ \to \chi'_{c0}K^{*+}$	0.000 49
$B_c \to \chi_{c2} e\nu$	0.16	$B_c^+ \to \chi_{c1}\pi^+$	0.020	$B_c^+ \to \chi'_{c1}\pi^+$	0.0010
$B_c \to \chi_{c2} \tau\nu$	0.0093	$B_c^+ \to \chi_{c1}\rho^+$	0.015	$B_c^+ \to \chi_{c1}\rho^+$	0.0014
$B_c \to h_c e\nu$	0.096	$B_c^+ \to \chi_{c1}K^+$	0.0015	$B_c^+ \to \chi'_{c1}K^+$	0.000 086
$B_c \to h_c \tau\nu$	0.0077	$B_c^+ \to \chi_{c1}K^{*+}$	0.0010	$B_c^+ \to \chi'_{c1}K^{*+}$	0.000 10
$B_c \to \chi'_{c0} e\nu$	0.014	$B_c^+ \to \chi_{c2}\pi^+$	0.038	$B_c^+ \to \chi'_{c2}\pi^+$	7.7×10^{-8}
$B_c \to \chi'_{c0} \tau\nu$	0.000 63	$B_c^+ \to \chi_{c2}\rho^+$	0.11	$B_c^+ \to \chi'_{c2}\rho^+$	0.000 20
$B_c \to \chi'_{c1} e\nu$	0.0085	$B_c^+ \to \chi_{c2}K^+$	0.0028	$B_c^+ \to \chi'_{c2}K^+$	7.8×10^{-7}
$B_c \to \chi'_{c1} \tau\nu$	0.000 39	$B_c^+ \to \chi_{c2}K^{*+}$	0.0074	$B_c^+ \to \chi'_{c2}K^{*+}$	0.000 014
$B_c \to \chi'_{c2} e\nu$	0.0033	$B_c^+ \to h_c\pi^+$	0.046	$B_c^+ \to h'_c\pi^+$	9.4×10^{-7}
$B_c \to \chi'_{c2} \tau\nu$	0.000 13	$B_c^+ \to h_c\rho^+$	0.10	$B_c^+ \to h'_c\rho^+$	0.000 46
$B_c \to h'_c e\nu$	0.0021	$B_c^+ \to h_cK^+$	0.0035	$B_c^+ \to h'_cK^+$	3.1×10^{-7}
$B_c \to h'_c \tau\nu$	0.000 11	$B_c^+ \to h_cK^{*+}$	0.0070	$B_c^+ \to h'_cK^{*+}$	0.000 032
$B_c \to D_0 e\nu$	0.0011	$B_c \to B_{s0} e\nu$	0.0066	$B_c \to B_0 e\nu$	0.0061
$B_c \to D_0 \tau\nu$	0.000 46	$B_c \to B_{s0} \mu\nu$	0.0057	$B_c \to B_0 \mu\nu$	0.0056
$B_c \to D_1 e\nu$	0.0011	$B_c \to B_{s1} e\nu$	0.0020	$B_c \to B_1 e\nu$	0.000 33
$B_c \to D_1 \tau\nu$	0.000 39	$B_c \to B_{s1} \mu\nu$	0.0018	$B_c \to B_1 \mu\nu$	0.000 30
$B_c \to D'_1 e\nu$	0.0019	$B_c \to B'_{s1} e\nu$	0.0045	$B_c \to B'_1 e\nu$	0.000 72
$B_c \to D'_1 \tau\nu$	0.0011	$B_c \to B'_{s1} \mu\nu$	0.0031	$B_c \to B'_1 \mu\nu$	0.000 57
$B_c \to D_2 e\nu$	0.0036	$B_c \to B_{s2} e\nu$	0.0046	$B_c \to B_2 e\nu$	0.000 84
$B_c \to D_2 \tau\nu$	0.0013	$B_c \to B_{s2} \mu\nu$	0.0022	$B_c \to B_2 \mu\nu$	0.000 47
$B_c \to \eta''_c e\nu$	0.000 55	$B_c^+ \to B_{s0}\pi^+$	0.55	$B_c^+ \to B_0\pi^+$	0.043
$B_c \to \eta''_c \tau\nu$	5.0×10^{-7}	$B_c^+ \to B_{s1}\pi^+$	0.028	$B_c^+ \to B_1\pi^+$	0.0039
$B_c \to \psi'' e\nu$	0.000 57	$B_c^+ \to B'_{s1}\pi^+$	0.029	$B_c^+ \to B'_0\pi^+$	0.0058
$B_c \to \psi'' \tau\nu$	3.6×10^{-6}	$B_c^+ \to B_{s2}\pi^+$	0.024	$B_c^+ \to B_2\pi^+$	0.0044

wave functions from the rest to the moving reference frame and contributions from the intermediate negative-energy states. The resulting form factors are expressed through the overlap integrals of the meson wave functions. These wave functions were obtained previously in the meson mass spectra calculations and are used in the present numerical evaluations. The influence of mixing effects on the P-wave heavy-light meson wave functions due to the nondiagonal spin-orbit and tensor terms in the $Q\bar{q}$ quasipotential was explicitly considered. The reliable determination of the q^2 dependence of the from factors in the whole kinematical range is an important achievement, since in many previous calculations form factors were determined only at the single point of either zero ($q^2 = q^2_{\max}$) or maximum ($q^2 = 0$) recoil of the final meson, and then different *ad hoc* extrapolations were employed.

The obtained weak form factors were used for the calculation of the semileptonic and nonleptonic B_c decays to corresponding orbitally excited heavy mesons. For the nonleptonic decays the factorization approximation was used. The calculated branching fractions are summarized in Table VIII. In this table we give our predictions not only for B_c decays to the first 1P-wave charmonium states (χ_J, h_c), but also for their radial excitations (2P-wave charmonium χ'_J, h'_c). For completeness, we also present there our predictions for the semileptonic B_c decays to the 3S charmonium states (ψ'', η''_c) which were not given in our previous study [6]. These decays to highly (both radially and orbitally) excited charmonium are of special interest, since their observation could help to reveal the nature of the newly observed charmoniumlike states above the open charm production threshold.

Summing the corresponding branching fractions in Table VIII we find that the semileptonic[2] and the considered energetic nonleptonic decays to the 1P charmonium states contribute about 0.88% and 0.44% of the total rate,

[2]We also take into account semileptonic decays involving the muon, for which rates are almost equal to the ones with the electron.

respectively. The corresponding decays to the $2P$ charmonium states are significantly suppressed (by an order of magnitude) mainly due to the presence of the node in the $2P$ wave function and give about 0.057% and 0.013% of the total rate. The same pattern was previously observed in B_c decays to the S-wave charmonia [6], where the rates of decays to the $2S$ states were also suppressed by an order of magnitude compared to decays to the $1S$ states. For decays to higher charmonium excitations such suppression should be even more pronounced. The CKM suppressed semileptonic decays to the D_J mesons contribute about 0.019%. Thus the total contribution of the considered B_c decays, governed by the $b \to c, u$ weak transitions, is about 1.41%.

The B_c semileptonic decays to orbitally excited B_{sJ} and B_J mesons, governed by the $c \to s, d$ weak transitions, turn out to have smaller branching fractions than B_c decays to orbitally excited charmonium, notwithstanding the significantly larger values of the CKM matrix elements, due to the substantial phase space suppression. Such decays involving the τ are kinematically forbidden, while the muon mass starts to play an important role, reducing branching fractions by more than 10%. In total, such semileptonic decays give about 0.045% of the B_c decay rate. On the other hand, there is no kinematical suppression in the corresponding nonleptonic decays and they contribute about 0.69%. Thus, the total contribution of the considered B_c decays, governed by the $c \to s, d$ weak transitions, is about 0.73%.

The semileptonic and nonleptonic B_c decays to excited heavy mesons can be investigated at Tevatron and LHC, especially in the LHCb experiment, where the B_c mesons are expected to be copiously produced.

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APPENDIX: FORM FACTORS OF WEAK B_c DECAYS TO ORBITALLY EXCITED HEAVY MESONS

(a) $B_c \to S(^3P_0)$ transition ($S = \chi_{c0}, D_0^*$)

$$\begin{aligned} f_\pm^{(1)}(q^2) &= \sqrt{\frac{E_S}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\bar\psi_S\Big(\mathbf{p}+\frac{2m_c}{E_S+M_S}\mathbf{\Delta}\Big)\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}}\\ &\times\Big\{\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\Big[\frac{\mathbf{\Delta}^2}{\epsilon_q(p+\Delta)+m_q}\pm(M_{B_c}\mp E_S)\Big(1+\frac{\mathbf{p}^2}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}\Big)\Big]\\ &+\frac{2}{3}\frac{p}{E_S+M_S}\Big(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\Big)\\ &\times\Big[\frac{\mathbf{\Delta}^2}{\epsilon_q(p+\Delta)+m_q}\pm(M_{B_c}\mp E_S)\Big(1-\frac{\mathbf{p}^2}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}\Big)\Big]\\ &+p\Big(\frac{1}{\epsilon_q(p+\Delta)+m_q}+\frac{1}{\epsilon_b(p)+m_b}\pm\frac{M_{B_c}\mp E_S}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}\Big)\Big\}\psi_{B_c}(\mathbf{p}), \end{aligned} \tag{A1}$$

$$\begin{aligned} f_\pm^{S(2)}(q^2) &= \sqrt{\frac{E_S}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\bar\psi_S\Big(\mathbf{p}+\frac{2m_c}{E_S+M_S}\mathbf{\Delta}\Big)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\\ &\times\Big\{-\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\frac{\mathbf{\Delta}^2}{\epsilon_q(\Delta)[\epsilon_q(p+\Delta)]}\Big(1\mp\frac{M_{B_c}\mp E_S}{\epsilon_q(\Delta)+mq}\Big)\Big[M_S-\epsilon_q\Big(p+\frac{2m_c}{E_S+M_S}\Delta\Big)-\epsilon_c\Big(p+\frac{2m_c}{E_S+M_S}\Delta\Big)\Big]\\ &-p\Big(\frac{1}{4m_b^2}+\frac{1}{2\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}\Big)\Big[M_{B_c}+M_S-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\Big(p+\frac{2m_c}{E_S+M_S}\Delta\Big)\\ &-\epsilon_c\Big(p+\frac{2m_c}{E_S+M_S}\Delta\Big)\Big]+p\frac{\epsilon_q(\Delta)-m_q}{2m_b\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}\\ &\times\Big[M_S-\epsilon_q\Big(p+\frac{2m_c}{E_S+M_S}\Delta\Big)-\epsilon_c\Big(p+\frac{2m_c}{E_S+M_S}\Delta\Big)\Big]\Big\}\psi_{B_c}(\mathbf{p}), \end{aligned} \tag{A2}$$

$$f_{\pm}^{V(2)}(q^2) = \sqrt{\frac{E_S}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_S\left(\mathbf{p}+\frac{2m_c}{E_S+M_S}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\frac{p}{2m_c}$$
$$\times\left\{\frac{1}{\epsilon_q(p+\Delta)+m_q}\left(1\mp\frac{M_{B_c}\mp E_S}{\epsilon_q(\Delta)+m_q}\right)+\frac{1}{2m_b}\left(1\pm\frac{M_{B_c}\mp E_S}{\epsilon_q(\Delta)+m_q}\right)\right.$$
$$\left.-\frac{\epsilon_q(\Delta)-m_q}{3\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}\left(1\mp\frac{M_{B_c}\mp E_S}{\epsilon_q(\Delta)+m_q}\right)\right\}\left[M_{B_c}+M_S-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_S+M_S}\Delta\right)\right.$$
$$\left.-\epsilon_c\left(p+\frac{2m_c}{E_S+M_S}\Delta\right)\right]\psi_{B_c}(\mathbf{p}), \qquad \text{(A3)}$$

(b) $B_c \to AV(^3P_1)$ transition ($AV = \chi_{c1}, D_1(^3P_1)$)

$$h_{V_1}^{(1)}(q^2) = \frac{2\sqrt{E_{AV}M_{B_c}}}{M_{B_c}+M_{AV}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}}\left\{\frac{(\mathbf{p\Delta})}{p}\frac{1}{\epsilon_q(p+\Delta)+m_q}\right.$$
$$\left.+p\left[\frac{E_{AV}-M_{AV}}{\epsilon_q(p+\Delta)+m_q}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right)+\frac{2}{3}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_b(p)+m_b}\right)\right]\right\}\psi_{B_c}(\mathbf{p}), \qquad \text{(A4)}$$

$$h_{V_1}^{S(2)}(q^2) = \frac{2\sqrt{E_{AV}M_{B_c}}}{M_{B_c}+M_{AV}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}$$
$$\times\left\{-\frac{(\mathbf{p\Delta})}{p}\frac{1}{\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}[M_{B_c}-\epsilon_b(p)-\epsilon_c(p)]\right.$$
$$-\frac{p}{3}\left[\left(\frac{1}{\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}-\frac{1}{2m_b^2}\right)\left[M_{B_c}+M_{AV}-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right.\right.$$
$$\left.-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]+\frac{\epsilon_q(\Delta)-m_q}{2m_b\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}$$
$$\left.\left.\times\left[M_{AV}-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]\right]\right\}\psi_{B_c}(\mathbf{p}), \qquad \text{(A5)}$$

$$h_{V_1}^{V(2)}(q^2) = \frac{2\sqrt{E_{AV}M_{B_c}}}{M_{B_c}+M_{AV}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\frac{p}{3m_c}$$
$$\times\left\{\frac{1}{\epsilon_q(\Delta)+m_q}\left[M_{AV}-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]\right.$$
$$-\frac{m_c}{\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}[M_{B_c}-\epsilon_b(p)-\epsilon_c(p)]$$
$$\left.-\frac{1}{2m_b}\left[M_{B_c}+M_{AV}-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]\right\}\psi_{B_c}(\mathbf{p}), \qquad \text{(A6)}$$

$$h_{V_2}^{(1)}(q^2) = 2E_{AV}\sqrt{\frac{E_{AV}}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\,\bar\psi_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}}$$
$$\times\Bigg\{\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\frac{E_{AV}}{\epsilon_q(p+\Delta)+m_q}\left[\frac{M_{AV}^2}{E_{AV}^2}-\frac{2}{3}\frac{\mathbf{p}^2}{E_{AV}+M_{AV}}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right)\right]$$
$$-\frac{2}{3}\frac{p}{E_{AV}+M_{AV}}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right)$$
$$\times\left(1-\frac{E_{AV}}{2[\epsilon_q(p+\Delta)+m_q]}+\frac{\mathbf{p}^2}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}+\frac{3}{2}\frac{\mathbf{\Delta}^2}{E_{AV}[\epsilon_q(p+\Delta)+m_q]}\right)$$
$$+\frac{2}{3}p\left[\frac{1}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}+\frac{1}{E_{AV}}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_b(p)+m_b}\right)\right]\Bigg\}\psi_{B_c}(\mathbf{p}), \tag{A7}$$

$$h_{V_2}^{S(2)}(q^2) = 2E_{AV}\sqrt{\frac{E_{AV}}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\,\bar\psi_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}$$
$$\times\Bigg\{-\left(\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\frac{M_{AV}^2}{E_{AV}}+\frac{2p}{3[\epsilon_q(\Delta)+m_q]}\right)\frac{1}{\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}[M_{B_c}-\epsilon_b(p)-\epsilon_c(p)]$$
$$+\frac{p}{3}\left(\frac{1}{E_{AV}}-\frac{1}{\epsilon_q(\Delta)+m_q}\right)\Bigg[\left(\frac{1}{\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}-\frac{1}{2m_b^2}\right)$$
$$\times\left[M_{B_c}+M_{AV}-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]$$
$$+\frac{1}{m_b\epsilon_q(\Delta)}\left[M_{AV}-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]\Bigg]\Bigg\}\psi_{B_c}(\mathbf{p}), \tag{A8}$$

$$h_{V_2}^{V(2)}(q^2) = 2E_{AV}\sqrt{\frac{E_{AV}}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\,\bar\psi_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\frac{p}{3m_c}$$
$$\times\Bigg\{\left[\frac{1}{[\epsilon_q(\Delta)+m_q]^2}\left(1-\frac{\epsilon_q(\Delta)+m_q}{E_{AV}}-\frac{E_{AV}}{2\epsilon_q(\Delta)}\right)+\frac{1}{2m_b}\left(\frac{1}{\epsilon_q(\Delta)+m_q}+\frac{1}{E_{AV}}\right)\right]$$
$$\times\left[M_{B_c}+M_{AV}-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]$$
$$+\frac{1}{E_{AV}\epsilon_q(\Delta)}[M_{B_c}-\epsilon_b(p)-\epsilon_c(p)]\Bigg\}\psi_{B_c}(\mathbf{p}), \tag{A9}$$

$$h_{V_3}^{(1)}(q^2) = 2E_{AV}\sqrt{E_{AV}M_{B_c}}\int\frac{d^3p}{(2\pi)^3}\,\bar\psi_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}}$$
$$\times\Bigg\{-\frac{1}{\epsilon_q(p+\Delta)+m_q}\left(\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\left[1-\frac{2}{3}\frac{\mathbf{q}^2}{E_{AV}+M_{AV}}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right)\right]\right.$$
$$\left.+\frac{q}{3}\frac{1}{E_{AV}+M_{AV}}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right)\right)\Bigg\}\psi_{B_c}(\mathbf{p}), \tag{A10}$$

$$h_{V_3}^{S(2)}(q^2) = 2E_{AV}\sqrt{E_{AV}M_{B_c}}\int\frac{d^3p}{(2\pi)^3}\,\bar\psi_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\frac{1}{\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}$$
$$\times[M_{B_c}-\epsilon_b(p)-\epsilon_c(p)]\psi_{B_c}(\mathbf{p}), \tag{A11}$$

$$h_{V_3}^{V(2)}(q^2) = 2E_{AV}\sqrt{E_{AV}M_{B_c}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\frac{p}{6m_c\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]^2}$$
$$\times\left[M_{B_c}+M_{AV}-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]\psi_{B_c}(\mathbf{p}), \quad (A12)$$

$$h_A^{(1)}(q^2) = (M_{B_c}+M_{AV})\sqrt{\frac{E_{AV}}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}}$$
$$\times\left\{\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}+\frac{p}{3}\left[\frac{1}{E_{AV}+M_{AV}}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right)+\frac{2}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}\right]\right\}\psi_{B_c}(\mathbf{p}), \quad (A13)$$

$$h_A^{S(2)}(q^2) = (M_{B_c}+M_{AV})\sqrt{\frac{E_{AV}}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}$$
$$\times\left\{\left([\epsilon_q(\Delta)-m_q]\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}+\frac{p}{3[\epsilon_q(\Delta)+m_q]}\right)\frac{1}{\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}[M_{B_c}-\epsilon_b(p)-\epsilon_c(p)]\right.$$
$$+\frac{p}{3m_b[\epsilon_q(\Delta)+m_q]}\left(\frac{1}{4m_b}\left[M_{B_c}+M_{AV}-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right.\right.$$
$$\left.\left.\left.-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]+\frac{1}{\epsilon_q(\Delta)}\left[M_{AV}-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]\right)\right\}\psi_{B_c}(\mathbf{p}), \quad (A14)$$

$$h_A^{V(2)}(q^2) = (M_{B_c}+M_{AV})\sqrt{\frac{E_{AV}}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\left(-\frac{p}{6m_c[\epsilon_q(\Delta)+m_q]}\right)\left(\frac{1}{\epsilon_q(\Delta)}+\frac{1}{m_b}\right)$$
$$\times\left[M_{B_c}+M_{AV}-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]\psi_{B_c}(\mathbf{p}), \quad (A15)$$

(c) $B_c \to AV'(^1P_1)$ transition ($AV' = h_c, D_1(^1P_1)$)

$$g_{V_1}^{(1)}(q^2) = \frac{2\sqrt{E_{AV}M_{B_c}}}{M_{B_c}+M_{AV}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}}\frac{p}{3}$$
$$\times\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}+\frac{1}{\epsilon_b(p)+m_b}\right)\psi_{B_c}(\mathbf{p}), \quad (A16)$$

$$g_{V_1}^{S(2)}(q^2) = \frac{2\sqrt{E_{AV}M_{B_c}}}{M_{B_c}+M_{AV}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}$$
$$\times\left\{-\frac{p}{6}\left(\frac{1}{\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}+\frac{1}{2m_b^2}\right)\left[M_{B_c}+M_{AV}-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right.\right.$$
$$\left.\left.-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]\right\}\psi_{B_c}(\mathbf{p}), \quad (A17)$$

$$g_{V_1}^{V(2)}(q^2) = \frac{2\sqrt{E_{AV}M_{B_c}}}{M_{B_c}+M_{AV}} \int \frac{d^3p}{(2\pi)^3} \bar{\psi}_{AV}\left(\mathbf{p} + \frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\frac{p}{6m_c}\left(\frac{1}{\epsilon_q(\Delta)+m_q}+\frac{1}{2m_b}\right) \times \left[M_{B_c}+M_{AV}-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]\psi_{B_c}(\mathbf{p}), \quad \text{(A18)}$$

$$g_{V_2}^{(1)}(q^2) = 2E_{AV}\sqrt{\frac{E_{AV}}{M_{B_c}}}\int \frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}} \times \left\{\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\left(1+\frac{\mathbf{p}^2}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}-\frac{E_{AV}}{\epsilon_q(p+\Delta)+m_q} + \frac{2}{3}\frac{\mathbf{p}^2}{E_{AV}+M_{AV}}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right) \times \left[\frac{\mathbf{\Delta}^2}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}+E_{AV}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_b(p)+m_b}\right)\right]\right) + \frac{p}{3}\left[\frac{1}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}-\frac{1}{E_{AV}}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}+\frac{1}{\epsilon_b(p)+m_b}\right)\right]\right\}\psi_{B_c}(\mathbf{p}), \quad \text{(A19)}$$

$$g_{V_2}^{S(2)}(q^2) = 2E_{AV}\sqrt{\frac{E_{AV}}{M_{B_c}}}\int \frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}} \times \left\{\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\frac{E_{AV}+\epsilon_q(\Delta)-m_q}{\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}[M_{B_c}-\epsilon_b(p)-\epsilon_c(p)] + \frac{p}{3}\left[\frac{1}{2\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}\left(\frac{1}{E_{AV}}+\frac{1}{\epsilon_q(\Delta)+m_q}\right) + \frac{1}{4m_b^2}\left(\frac{1}{E_{AV}}-\frac{1}{\epsilon_q(\Delta)+m_q}\right)\right]\left[M_{B_c}+M_{AV}-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right) - \epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]-\frac{1}{2m_c\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]} \times \left[M_{AV}-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]\right\}\psi_{B_c}(\mathbf{p}), \quad \text{(A20)}$$

$$g_{V_2}^{V(2)}(q^2) = 2E_{AV}\sqrt{\frac{E_{AV}}{M_{B_c}}}\int \frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\frac{p}{6m_c} \times \left\{\frac{1}{[\epsilon_q(\Delta)+m_q]^2}\left(\frac{E_{AV}}{\epsilon_q(\Delta)}+2\right)[M_{B_c}-\epsilon_b(p)-\epsilon_c(p)]-\left(\frac{1}{\epsilon_q(\Delta)+m_q}+\frac{1}{E_{AV}}\right)\left(\frac{1}{\epsilon_q(\Delta)+m_q}+\frac{1}{2m_b}\right) \times \left[M_{B_c}+M_{AV}-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]\right\}\psi_{B_c}(\mathbf{p}), \quad \text{(A21)}$$

$$g_{V_3}^{(1)}(q^2) = 2E_{AV}\sqrt{E_{AV}M_{B_c}}\int \frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}}\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2} \times \left[\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{2}{3}\frac{\mathbf{q}^2}{E_{AV}+M_{AV}}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right)\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_b(p)+m_b}\right)\right] \times \psi_{B_c}(\mathbf{p}), \quad \text{(A22)}$$

$$g_{V_3}^{S(2)}(q^2) = 2E_{AV}\sqrt{E_{AV}M_{B_c}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\left(-\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\right)\frac{1}{\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]} \times [M_{B_c}-\epsilon_b(p)-\epsilon_c(p)]\psi_{B_c}(\mathbf{p}), \tag{A23}$$

$$g_{V_3}^{V(2)}(q^2) = 2E_{AV}\sqrt{E_{AV}M_{B_c}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\left(-\frac{p}{6m_c\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]^2}\right) \times\left[M_{B_c}+M_{AV}-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]\psi_{B_c}(\mathbf{p}), \tag{A24}$$

$$g_A^{(1)}(q^2) = (M_{B_c}+M_{AV})\sqrt{\frac{E_{AV}}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}}\frac{p}{3} \times\left[\frac{1}{E_{AV}+M_{AV}}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right)\left(1-\frac{\mathbf{q}^2}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}\right) + \frac{1}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}\right]\psi_{B_c}(\mathbf{p}), \tag{A25}$$

$$g_A^{S(2)}(q^2) = (M_{B_c}+M_{AV})\sqrt{\frac{E_{AV}}{M_{B_c}}}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\frac{p}{3[\epsilon_q(\Delta)+m_q]} \times\left\{-\left(\frac{1}{\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}+\frac{1}{2m_b^2}\right)\left[M_{B_c}+M_{AV}-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right) -\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]-\frac{1}{m_b\epsilon_q(\Delta)}\left[M_{AV}-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]\right\}\psi_{B_c}(\mathbf{p}), \tag{A26}$$

$$g_A^{V(2)}(q^2) = (M_{B_c}+M_{AV})\sqrt{\frac{E_{AV}}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{AV}\left(\mathbf{p}+\frac{2m_c}{E_{AV}+M_{AV}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\frac{p}{6m_c[\epsilon_q(\Delta)+m_q]} \times\left(-\frac{1}{\epsilon_q(\Delta)+m_q}+\frac{1}{2m_b}\right)\left[M_{B_c}+M_{AV}-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right) -\epsilon_c\left(p+\frac{2m_c}{E_{AV}+M_{AV}}\Delta\right)\right]\psi_{B_c}(\mathbf{p}), \tag{A27}$$

(d) $B_c \to T(^3P_2)$ transition ($T = \chi_{c2}, D_2^*$)

$$t_V^{(1)}(q^2) = (M_{B_c}+M_T)E_T\sqrt{\frac{E_T}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_T\left(\mathbf{p}+\frac{2m_c}{E_T+M_T}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}} \times\left\{\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\left[\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{3}\frac{\mathbf{p}^2}{E_T+M_T}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right)\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_b(p)+m_b}\right)\right] -\frac{p}{3(E_T+M_T)[\epsilon_q(p+\Delta)+m_q]}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right)\right\}\psi_{B_c}(\mathbf{p}), \tag{A28}$$

$$t_V^{S(2)}(q^2) = (M_{B_c} + M_T)E_T\sqrt{\frac{E_T}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_T\left(\mathbf{p}+\frac{2m_c}{E_T+M_T}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\left(-\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\right)\frac{1}{\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}\times[M_{B_c}-\epsilon_b(p)-\epsilon_c(p)]\psi_{B_c}(\mathbf{p}), \tag{A29}$$

$$t_V^{V(2)}(q^2) = 0, \tag{A30}$$

$$t_{A_1}^{(1)}(q^2) = 2\sqrt{E_TM_{B_c}}\frac{E_T}{M_{B_c}+M_T}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_T\left(\mathbf{p}+\frac{2m_c}{E_T+M_T}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}}\times\left\{\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\left[1-\frac{\mathbf{p}^2}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}\right]-\frac{1}{3}\frac{\mathbf{p}^2}{E_T+M_T}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right)\times\left[1+\frac{\mathbf{p}^2}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}\right]\right\}\psi_{B_c}(\mathbf{p}), \tag{A31}$$

$$t_{A_1}^{S(2)}(q^2) = 2\sqrt{E_TM_{B_c}}\frac{E_T}{M_{B_c}+M_T}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_T\left(\mathbf{p}+\frac{2m_c}{E_T+M_T}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\times\left\{\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\frac{\epsilon_q(\Delta)-m_q}{\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}[M_{B_c}-\epsilon_b(p)-\epsilon_c(p)]+\frac{p}{6[\epsilon_q(\Delta)+m_q]}\left[\left(\frac{1}{\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}+\frac{1}{2m_b^2}\right)\left[M_{B_c}+M_T-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_T+M_T}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_T+M_T}\Delta\right)\right]+\frac{1}{m_b\epsilon_q(\Delta)}\left[M_T-\epsilon_q\left(p+\frac{2m_c}{E_T+M_T}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_T+M_T}\Delta\right)\right]\right]\right\}\psi_{B_c}(\mathbf{p}), \tag{A32}$$

$$t_{A_1}^{V(2)}(q^2) = 2\sqrt{E_TM_{B_c}}\frac{E_T}{M_{B_c}+M_T}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_T\left(\mathbf{p}+\frac{2m_c}{E_T+M_T}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\frac{p}{3m_c[\epsilon_q(\Delta)+m_q]^2}\times\left[M_T-\epsilon_q\left(p+\frac{2m_c}{E_T+M_T}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_T+M_T}\Delta\right)\right]\psi_{B_c}(\mathbf{p}), \tag{A33}$$

$$t_{A_2}^{(1)}(q^2) = 2E_T^2\sqrt{\frac{E_T}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_T\left(\mathbf{p}+\frac{2m_c}{E_T+M_T}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}}\times\left\{\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\left[\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{E_T}\left(1-\frac{\mathbf{p}^2}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}\right)-\frac{\mathbf{p}^2}{E_T+M_T}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right)\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}+\frac{1}{\epsilon_b(p)+m_b}-\frac{E_T}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}\right)\right]-\frac{p}{3(E_T+M_T)}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right)\times\left[\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{E_T}\left(1+\frac{\mathbf{p}^2}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}\right)\right]\right\}\psi_{B_c}(\mathbf{p}), \tag{A34}$$

$$t_{A_2}^{S(2)}(q^2) = 2E_T^2\sqrt{\frac{E_T}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_T\left(\mathbf{p}+\frac{2m_c}{E_T+M_T}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}$$
$$\times\left\{-\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\frac{1}{\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}\left(1-\frac{\epsilon_q(\Delta)+m_q}{E_T}\right)[M_{B_c}-\epsilon_b(p)-\epsilon_c(p)]\right.$$
$$-\frac{p}{3E_T[\epsilon_q(\Delta)+m_q]}\left[\left(\frac{1}{2\epsilon_q(\Delta)[\epsilon_q(\Delta)+m_q]}+\frac{1}{4m_b^2}\right)\left[M_{B_c}+M_T-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_T+M_T}\Delta\right)\right.\right.$$
$$\left.\left.\left.-\epsilon_c\left(p+\frac{2m_c}{E_T+M_T}\Delta\right)\right]+\frac{1}{2m_b\epsilon_q(\Delta)}\left[M_T-\epsilon_q\left(p+\frac{2m_c}{E_T+M_T}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_T+M_T}\Delta\right)\right]\right]\right\}\psi_{B_c}(\mathbf{p}), \tag{A35}$$

$$t_{A_2}^{V(2)}(q^2) = 2E_T^2\sqrt{\frac{E_T}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_T\left(\mathbf{p}+\frac{2m_c}{E_T+M_T}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\frac{p}{3m_c[\epsilon_q(\Delta)+m_q]^2}$$
$$\times\left\{\frac{1}{2\epsilon_q(\Delta)}\left[M_{B_c}+M_T-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_c}{E_T+M_T}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_T+M_T}\Delta\right)\right]\right.$$
$$\left.-\frac{1}{E_T}\left[M_T-\epsilon_q\left(p+\frac{2m_c}{E_T+M_T}\Delta\right)-\epsilon_c\left(p+\frac{2m_c}{E_T+M_T}\Delta\right)\right]\right\}\psi_{B_c}(\mathbf{p}), \tag{A36}$$

$$t_{A_3}^{(1)}(q^2) = 2\sqrt{E_TM_{B_c}}E_T^2\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_T\left(\mathbf{p}+\frac{2m_c}{E_T+M_T}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}}$$
$$\times\left\{-\frac{(\mathbf{p\Delta})}{p\mathbf{\Delta}^2}\frac{\mathbf{p}^2}{E_T+M_T}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right)\frac{1}{[\epsilon_q(p+\Delta)+m_q][\epsilon_b(p)+m_b]}\right\}\psi_{B_c}(\mathbf{p}), \tag{A37}$$

$$t_{A_3}^{S(2)}(q^2) = 0, \tag{A38}$$

$$t_{A_3}^{V(2)}(q^2) = 0, \tag{A39}$$

where the subscript $q = c, u$ refers to the final active quark, the superscripts "(1)" and "(2)" correspond to Figs. 1 and 2, S and V correspond to the scalar and vector potentials of the $q\bar{q}$-interaction, and

$$\Delta \equiv |\mathbf{\Delta}| = \sqrt{\frac{(M_{B_c}^2+M_F^2-q^2)^2}{4M_{B_c}^2}-M_F^2}, \qquad (F = S, AV, AV', T) \qquad E_F = \sqrt{M_F^2+\mathbf{\Delta}^2},$$
$$\epsilon_Q(p+a\Delta) = \sqrt{m_Q^2+(\mathbf{p}+a\mathbf{\Delta})^2} \qquad (Q = b, c, s, u, d).$$

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