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Quartet of spin-3/2 baryons in the chiral multiplet $(1, 1/2) \oplus (1/2, 1)$ with mirror assignment

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We study the possible existence of chiral partners in the spin-$\frac{3}{2}$ sector of the baryon spectrum. We consider a quartet scheme where four spin-$\frac{3}{2}$ baryons, P_{33}, D_{33}, D_{13}, and P_{13}, group into higher-dimensional chiral multiplets $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ with a mirror assignment. With an effective $SU(2)_R \times SU(2)_L$ Lagrangian, we derive constraints imposed by chiral symmetry together with the mirror assignment on the masses and coupling constants of the quartet. Using the effective Lagrangian, we try to find a set of baryons suitable for the chiral quartet. It turns out that two cases reasonably agree with the mass pattern of the quartet: ($\Delta(1600)$, $\Delta(1940)$, $N(1520)$, $N(1720)$) and ($\Delta(1920)$, $\Delta(1940)$, $N(2080)$, $N(1900)$).

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I. INTRODUCTION

Chiral symmetry $SU(N_F)_R \times SU(N_F)_L$ and its spontaneous breaking characterize the QCD vacuum and is a key to understanding the strong interactions. Because of the spontaneous breaking of chiral symmetry (SBCS), the hadron spectrum is classified in terms of the residual symmetry $SU(N_F)_V$, while the role of $SU(N_F)_R \times SU(N_F)_L$ in the hadron spectrum is unclear. Nevertheless, one expects that there exists a set of hadrons reflecting a nature of the original symmetry, which is referred to as chiral partners. Such examples are well known for mesons, e.g. (σ, π) and (ρ, a_1) [1–3], while not well established for baryons. As discussed in the meson's case, finding chiral partners provides us with the understanding of the role of chiral symmetry in the hadron spectrum, and also a clue to study the restoration of chiral symmetry. Recently, the multiplet nature of the chiral group draws renewed attention from an interest in the effective chiral restoration [4–6], which was suggested to be the cause of the observed parity doubling in the high-energy region of the spectrum [7].

In the present work, we address the issue of the multiplet nature of the baryon's chiral partners. We denote a chiral multiplet by (I_R, I_L), where $I_R[I_L]$ is an isospin for $SU(2)_R[SU(2)_L]$. All the members of one chiral multiplet (I_R, I_L) have a fixed spin. The correspondence of the charge algebra between $SU(N_f)_R \times SU(N_f)_L$ and $SU(N_f)_V \times SU(N_f)_A$ leads to a relation $I = I_R \oplus I_L = |I_R + I_L|, \ldots, |I_R - I_L|$. This implies that a chiral multiplet can contain various isospin states. In the presence of the SBCS, the mixing of different chiral representations happens, and a hadron with an isospin I can be described as a superposition of various chiral representations containing I. We are here concerned with the case that a set of hadrons group into one or a few representations even in the presence of the SBCS, or the case where the configuration mixing is small.

In order to find chiral partners, we need to understand the multiplet nature of the chiral group, such as the pattern of the spectrum and coupling constants of the multiplet. Because general relations for masses and axial charges that can be applied to arbitrary chiral representations are not established so far, the properties of the chiral partners are usually studied with focusing on a particular chiral representation. In the meson's case, the properties of chiral partners have been investigated by using e.g. the Nambu Jona-Lasinio (NJL) model [8,9] and Weinberg sum rules [10]. The NJL model was applied to the nucleon [11–15] and $\Delta(1232)$ [16] by solving the Faddeev equation. We applied the NJL model with diquarks to the nucleon [17–19] and the Roper resonance [20], using an auxiliary field method. However, when we apply such microscopic approaches to a baryon with a mass larger than the sum of the masses of the internal degrees of freedom, we encounter the difficulty of the confinement. Because of this difficulty, effective Lagrangian approaches that contain hadrons as degrees of freedom are often employed for the study of baryon's chiral partners [21–28].

In recent papers, we have developed a systematic method to construct an effective $SU(N_f)_R \times SU(N_F)_L$ Lagrangian including higher-dimensional representations [29–33], which we refer to as a projection method. This method is inspired by an NJL model for mesons, and partly extends it to baryons. In Ref. [29], we classified baryon fields consisting of three quarks in terms of chiral multiplets. The Pauli principle implemented by the Fierz transformation plays a crucial role in the classification. The projection method is performed as follows. First we find a chiral invariant operator involving direct products of the quark and diquark fields. This can be achieved by using an analogy between $(\sigma, \vec{\pi})$ and diquarks in chiral transformation property. Then, we project the direct products of the quark and diquark fields onto irreducible parts with the use of the Fierz identities. After the projection, three-

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quark fields are replaced by baryon fields. Thus we can systematically construct chiral invariant Lagrangians including higher-dimensional chiral representations, avoiding problems caused by the lack of the confinement. Although such simple effective Lagrangians have limited validity, they are useful for the present purpose to derive the pattern of the masses and coupling constants of the chiral multiplet.

In Ref. [30], we have applied the projection method to a quartet scheme (QS). The QS was first proposed by Jido *et al.* [34]. They used two kinds of $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ and considered the so-called mirror assignment [22,23,25], where four types of baryons, two with $I = \frac{1}{2}$ and the other two with $I = \frac{3}{2}$, are included in the multiplet. They applied the QS to $J = \frac{1}{2}, \frac{3}{2}$, and $\frac{5}{2}$ and studied the masses and intracoupling constants of the quartet. They did not consider the Dirac structure of the Lagrangian explicitly. Owing to the projection method, we took into account the Dirac structure in the QS Lagrangian, which enables us to include transition terms between $J = \frac{1}{2}$ and $J = \frac{3}{2}$, e.g. N and $\Delta(1232)$. With the QS Lagrangian, we have derived several constraints on the masses and coupling constants, which characterize the multiplet nature of the quartet.

In the present work, we develop the previous study to find a set of baryons suitable for the chiral quartet of spin-$\frac{3}{2}$ baryons. Considering $J = \frac{3}{2}$, the quartet consists of P_{33}, D_{33}, D_{13}, and P_{13}. Among various candidates for this set, we adopted a particular assignment in Ref. [30]: $\Delta(1232)$, $\Delta(1700)$, $N(1520)$, $N(1720)$. It is an important question if there is other assignment suitable for the quartet. One interesting assignment is a set ($\Delta(1920)$, $\Delta(1940)$, $N(2080)$, $N(1900)$). Glozman mentioned the possibility that the approximate degeneracy of these four baryons is a consequence of the effective chiral restoration [6]. If this is the case, there are two possibilities. The first one is that the four baryons form the chiral quartet. The second one is that two Δ's belong to $(\frac{3}{2}, 0) \oplus (0, \frac{3}{2})$ and two N^* belong to $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$. We can study the first case using the QS Lagrangian.

In order to take into account πN interactions in the QS, it is necessary to determine the nucleon's chiral representation. In standard linear σ models of Gell-Mann-Levy type [21] the nucleon belongs to $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$. In the mirror models [22–27], the nucleon is a mixture of two kinds of $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$. The mixing of $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ and $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ was studied in an algebraic approach [35–37] and field theoretical approaches [32,33]. In nonrelativistic quark models the nucleon wave-functions also correspond to the mixing of $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ and $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$. In the present study, we assume the nucleon to be saturated with the fundamental representation $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ due to the following reasons. The linear σ models qualitatively describe the chiral properties of the nucleon. For instance, the linear σ models describe $g_A = 1$ in qualitative agreement with $g_A^{(\mathrm{exp})} = 1.267 \pm 0.004$. Second, the nucleon belongs to $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$, if the nucleon operator has spatially symmetric property [29].

This paper is organized as follows. In Sec. II, we define the baryon fields and derive their $SU(2)_A$ transformation properties. In Sec. III, we construct the $SU(2)_R \times SU(2)_L$ Lagrangian, such as mass terms and πNR interactions with the use of the projection technique. Here R denotes the member of the chiral quartet. Although the QS Lagrangian is not new, we generalize the formulation given in the previous study in an assignment-free manner in order to make it feasible to test various assignments. With the Lagrangian, we derive several constraints on the properties of the quartet. Because the projection method is complicated, we show an alternative derivation of some of the present results, using chiral algebra in Appendix B. Numerical results are shown in Sec. IV. Considering the masses, we find two suitable assignments ($\Delta(1600)$, $\Delta(1940)$, $N(1520)$, $N(1720)$) and ($\Delta(1920)$, $\Delta(1940)$, $N(2080)$, $N(1900)$). We discuss the properties of the quartet for these cases together with the assignment ($\Delta(1232)$, $\Delta(1700)$, $N(1520)$, $N(1720)$). The final section is devoted to a summary.

II. CHIRAL PROPERTIES OF BARYON FIELDS

In this section, we consider baryon fields consisting of three quarks, which serve as a preparation for the projection method. Baryon fields consisting of three quarks in a local form are generally described as

$$B(x) \sim \epsilon_{abc}(q_a^T(x)\Gamma_1 q_b(x))\Gamma_2 q_c(x), \tag{1}$$

where $q(x) = (u(x), d(x))^T$ is an isodoublet quark field at location x, the superscript T represents the transpose, and the indices a, b, and c represent the color. The antisymmetric tensor in color space ϵ_{abc} ensures the baryons being color singlets. From now on, we shall omit the color indices and space-time coordinates. $\Gamma_{1,2}$ describe Dirac and isospin matrices. With a suitable choice of $\Gamma_{1,2}$, a baryon field is defined so that it forms an irreducible representation of the Lorentz and isospin groups.

Concerning $J = \frac{3}{2}$, there are three possible baryon fields with $I = \frac{1}{2}$:

$$N_V^\mu = (\tilde{q}\gamma_\nu q)\Gamma_{3/2}^{\mu\nu}\gamma_5 q, \tag{2a}$$

$$N_A^\mu = (\tilde{q}\gamma_\nu\gamma_5\tau^i q)\Gamma_{3/2}^{\mu\nu}\tau^i q, \tag{2b}$$

$$N_T^\mu = i(\tilde{q}\sigma_{\alpha\beta}\tau^i q)\Gamma_{3/2}^{\mu\alpha}\gamma^\beta\gamma_5\tau^i q, \tag{2c}$$

and two with $I = \frac{3}{2}$:

$$\Delta_A^{\mu i} = (\tilde{q}\gamma_\nu\gamma_5\tau^j q)\Gamma_{3/2}^{\mu\nu}P_{3/2}^{ij} q, \tag{2d}$$

$$\Delta_T^{\mu i} = i(\tilde{q}\sigma_{\alpha\beta}\tau^j q)\Gamma_{3/2}^{\mu\alpha}\gamma^\beta\gamma_5 P_{3/2}^{ij} q, \tag{2e}$$

where $\tilde{q} = q^T C(i\tau_2)\gamma_5$ is a transposed quark field. Here we employ an isospurion formalism [38,39] for an isospin-$\frac{3}{2}$ projection operator $P^{ij}_{3/2}$, which is given by $P^{ij}_{3/2} = \delta^{ij} - \frac{1}{3}\tau^i\tau^j$. Similarly, $\Gamma^{\mu\nu}_{3/2}$ is a local spin-$\frac{3}{2}$ projection operator defined by $\Gamma^{\mu\nu}_{3/2} = g^{\mu\nu} - \frac{1}{4}\gamma^\mu\gamma^\nu$. In the present work, we consider only on-shell spin-$\frac{3}{2}$ states. In order to consider off-shell spin-$\frac{3}{2}$ baryons, we need to employ the nonlocal projector instead of the local one [40–43].

Note that the baryon fields Eqs. (2) are not independent [44–46]. In addition, they belong to reducible chiral representations, which leads to unphysical mixings of different chiral representations [29]. The cause of the unphysical chiral mixings is the fact that Eqs. (2) are not totally antisymmetric; they are antisymmetric only for the interchange between the first and second quarks. Considering the Fierz transformation as the antisymmetrization of the second and third quarks, we obtain the totally antisymmetric baryon fields

$$N_1^\mu = \frac{\sqrt{3}}{4}N_V^\mu + \frac{1}{4\sqrt{3}}N_A^\mu, \tag{3a}$$

$$\Delta_1^{\mu i} = \tfrac{1}{2}\Delta_A^{\mu i}. \tag{3b}$$

These totally antisymmetric combinations belong to the irreducible chiral multiplet [29]. The derivation of Eq. (3) is shown in Appendix A.

With the baryon fields consisting of the quark fields, it is a straightforward but tedious task to derive their $SU(2)_A$ transformations by using that of the quark field: $\delta_5^{\vec{a}} q = \frac{1}{2} i\boldsymbol{a}\cdot\boldsymbol{\tau}\gamma_5 q$ with $\vec{a}$ being the infinitesimal parameters for $SU(2)_A$. We obtain

$$\delta_5^{\vec{a}} N_1^\mu = \frac{1}{2}\left(\frac{5}{3} i\boldsymbol{a}\cdot\boldsymbol{\tau}\gamma_5 N_1^\mu + \frac{4}{\sqrt{3}} i\gamma_5 \boldsymbol{a}\cdot\Delta_1^\mu\right), \tag{4a}$$

$$\delta_5^{\vec{a}}\Delta_1^{\mu i} = \frac{1}{2}\left(\frac{4}{\sqrt{3}} i\gamma_5 a^j P^{ij}_{3/2} N_1^\mu - \frac{2}{3} i\tau^i\gamma_5 \boldsymbol{a}\cdot\Delta_1^\mu + i\boldsymbol{a}\cdot\boldsymbol{\tau}\gamma_5\Delta_1^{\mu i}\right), \tag{4b}$$

which contain off-diagonal terms $\delta_5^{\vec{a}} N_1^\mu \sim \Delta_1^{\mu i}$ and $\delta_5^{\vec{a}}\Delta_1^{\mu i} \sim N_1^\mu$ as well as the diagonal ones. They restrict possible chiral invariant terms, similar to the case of (σ, π) in the linear sigma model.

For later convenience, we define diquark fields contained in the spin-$\frac{3}{2}$ baryon fields: a Lorentz vector isoscalar diquark V^μ $[I(J)^P = 0(1)^-]$, a Lorentz axial-vector isovector diquark $A^{\mu i}$ $[1(1)^+]$,

$$V^\mu = \tilde{q}\gamma^\mu q, \tag{5a}$$

$$A^{\mu i} = \tilde{q}\gamma^\mu\gamma_5\tau^i q. \tag{5b}$$

It is easy to check that V^μ and $A^{\mu i}$ correspond to σ and $\vec{\pi}$ mesons in chiral transformation properties, which is a key of the projection method.

We introduce the other set of $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$: $(N_2^\mu, \Delta_2^{\mu i})$, where they have the same spin and isospin as the original ones $(N_1^\mu, \Delta_1^{\mu i})$, but the opposite $SU(2)_A$ transformation properties in sign, i.e.,

$$\delta_5^{\vec{a}} N_2^\mu = -\frac{1}{2}\left(\frac{5}{3} i\boldsymbol{a}\cdot\boldsymbol{\tau}\gamma_5 N_2^\mu + \frac{4}{\sqrt{3}} i\gamma_5 \boldsymbol{a}\cdot\Delta_2^\mu\right), \tag{6a}$$

$$\delta_5^{\vec{a}}\Delta_2^{\mu i} = -\frac{1}{2}\left(\frac{4}{\sqrt{3}} i\gamma_5 a^j P^{ij}_{3/2} N_2^\mu - \frac{2}{3} i\tau^i\gamma_5 \boldsymbol{a}\cdot\Delta_2^\mu + i\boldsymbol{a}\cdot\boldsymbol{\tau}\gamma_5\Delta_2^{\mu i}\right). \tag{6b}$$

This property is referred to as the mirror assignment [25], and we refer to $(N_1^\mu, \Delta_1^{\mu i})$ as naïve and to $(N_2^\mu, \Delta_2^{\mu i})$ as mirror. There is a correspondence of the chiral transformation properties between the naive and mirror sets,

$$(N_{1R}^\mu, N_{1L}^\mu, \Delta_{1R}^{\mu i}, \Delta_{1L}^{\mu i}) \leftrightarrow (N_{2L}^\mu, N_{2R}^\mu, \Delta_{2L}^{\mu i}, \Delta_{2R}^{\mu i}), \tag{7}$$

where the indices R and L denote the left- and right-handed projections with the projection operator $P_{R,L} = (1 \pm \gamma_5)/2$. The right-handed parts of N_1^μ and $\Delta_1^{\mu i}$ have the same chiral transformation properties as the left-handed parts of N_2^μ and $\Delta_2^{\mu i}$, and vice versa.

Note that we defined N_2 and Δ_2 by their transformation properties Eqs. (6). It is useful to define the baryon fields for N_2 and Δ_2. It is impossible to describe them in terms of local three-quark fields. Since baryons are composite particles, there are generally various possible expressions for N_2 and Δ_2. For example, we can describe them by using baryon operators having a derivative,

$$N_V'^\mu = \not{D} V_\nu \Gamma^{\mu\nu}_{3/2}\gamma_5 q, \tag{8a}$$

$$N_A'^\mu = \not{D} A^i_\nu \Gamma^{\mu\nu}_{3/2}\tau^i q, \tag{8b}$$

$$\Delta_A'^{\mu i} = \not{D} A^j_\nu \Gamma^{\mu\nu}_{3/2} P^{ij}_{3/2} q, \tag{8c}$$

where D_μ denotes a covariant derivative. The mirror fields N_2^μ and Δ_2^μ are obtained by the same equations as Eqs. (3) with substitution of the primed fields $(N_V'^\mu, N_A'^\mu, \Delta_A'^{\mu i})$ for the original fields $(N_V^\mu, N_A^\mu, \Delta_A^{\mu i})$. Although they would not be a unique possibility for the microscopic description of the mirror fields, Eqs. (8) are enough for the present purpose to construct the chiral invariant Lagrangian.

III. LAGRANGIAN

Now, we proceed to the construction of the $SU(2)_R \times SU(2)_L$ Lagrangian. It is straightforward to show the chiral invariance of the kinetic terms: $\mathcal{L}_K = \bar{N}_{n\mu}(i\not{\partial})N_n^\mu + \bar{\Delta}^i_{n\mu}(i\not{\partial})\Delta_n^{\mu i}$ $(n = 1, 2)$. In order to find interaction terms for higher-dimensional chiral multiplets, it is useful to employ the projection method.

A. Mass terms and πRR terms

The vector and axial-vector diquarks belong to the chiral multiplet $(\frac{1}{2},\frac{1}{2})$, and $V_\mu^2 + A_\mu^2$ is a chiral scalar. The Gell-Mann-Levy–type interaction for the quark $\bar{q}U_5q$ is also a chiral scalar, where $U_5 = \sigma + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}$. Obviously, the following combination of these two terms is also a chiral scalar:

$$\bar{q}(V_\mu^2 + A_\mu^2)U_5 q. \tag{9}$$

This term contains the direct products of the quark and diquark: $V^\mu q$ and $A^{\mu i}q$. They are decomposed into the irreducible parts as

$$\begin{aligned} V^\mu q &= \gamma_5 N_V^\mu + (J = \tfrac{1}{2}\text{terms}),\\ A^{\mu i}q &= \Delta_A^{\mu i} + \tfrac{1}{3}\tau^i N_A^\mu + (J = \tfrac{1}{2}\text{terms}), \end{aligned} \tag{10a}$$

$$\begin{aligned} \bar{q}(V^\mu)^\dagger &= -\bar{N}_V^\mu \gamma_5 + (J = \tfrac{1}{2}\text{terms}),\\ \bar{q}(A^{\mu i})^\dagger &= \bar{\Delta}_A^{\mu i} + \tfrac{1}{3}\bar{N}_A^\mu \tau^i + (J = \tfrac{1}{2}\text{terms}). \end{aligned} \tag{10b}$$

Substituting Eqs. (10) into the chiral invariant term (9), we obtain

$$\begin{aligned} \mathcal{L}_{MRR}^{(1)} = g_1\bigg(&\bar{\Delta}_{1\mu}^i U_5 \Delta_1^{\mu i} - \frac{3}{4}\bar{N}_{1\mu}U_5 N_1^\mu \\ &+ \frac{1}{12}\bar{N}_{1\mu}\tau^i U_5 \tau^i N_1^\mu \\ &+ \frac{\sqrt{3}}{6}(\bar{N}_{1\mu}\tau^i U_5 \Delta_1^{\mu i} + (\text{H.c.}))\bigg) + \bigg(J = \frac{1}{2}\text{terms}\bigg), \end{aligned} \tag{11}$$

where we omit $J = \frac{1}{2}$ terms, which contain the Gell-Mann-Levy–type interaction with local nucleon operators $N_V = V_\mu \gamma^\mu q$ and $N_A = A_\mu^i \gamma^\mu \gamma_5 \tau^i q$. The transition terms between $J = \frac{1}{2}$ and $\frac{3}{2}$ fields vanish due to $\gamma_\mu \Delta_1^{\mu i} = \gamma_\mu N_1^\mu = 0$. The Lagrangian (11) describes several kinds of the interactions; the first three terms describe the diagonal interactions for N_1^μ and $\Delta_1^{\mu i}$ with σ and π, and the fourth term describes a transition between N_1^μ and $\Delta_1^{\mu i}$ with π, where a $\sigma N_1 \Delta_1$ coupling vanishes due to $\tau^i \Delta_1^{\mu i} = 0$.

The diagonal interactions with σ generate the masses of N_1^μ and $\Delta_1^{\mu i}$ in the presence of the SBCS $\sigma \to \langle\sigma\rangle = f_\pi = 92.4$ (MeV). We obtain a mass relation $|m_{\Delta_1}|:|m_{N_1}| = 2{:}1$. If we assign N_1^μ with $N(1520)$, which is the lowest lying state for $I(J) = \frac{1}{2}(\frac{3}{2})$, its partner $\Delta_1^{\mu i}$ has the mass of $2\times 1520 \sim 3000$ MeV. We do not find a baryon suitable for this mass relation in the experimental data [47].

There are several directions to solve this mass problem: the inclusion of higher-order terms in the Lagrangian and of higher-order diagrams, the extension of the chiral basis such as $(\frac{3}{2},0)\oplus(0,\frac{3}{2})$ and of the mirror assignment. It was shown [34] that the inclusion of the mirror assignment reasonably reproduces the masses and some properties of observed baryons. Using Eq. (7), we find a chiral invariant interaction term

$$\begin{aligned} \mathcal{L}_{MRR}^{(2)} = g_2\bigg(&\bar{\Delta}_{2\mu}^i U_5^\dagger \Delta_2^{\mu i} - \frac{3}{4}\bar{N}_{2\mu}U_5^\dagger N_2^\mu \\ &+ \frac{1}{12}\bar{N}_{2\mu}\tau^i U_5^\dagger \tau^i N_2^\mu + \frac{\sqrt{3}}{6}(\bar{N}_{2\mu}\tau^i U_5^\dagger \Delta_2^{\mu i} + \text{H.c.})\bigg), \end{aligned} \tag{12}$$

which is almost the same as Eq. (11). The difference appears in the signs of the terms accompanying π ($U_5 \to U_5^\dagger$), which is a feature of the mirror assignment [25].

Considering Eqs. (4), (6), and (7), $\bar{\Delta}_{1R}\Delta_{2L} + \bar{N}_{1R}N_{2L}$ is chiral invariant, which leads to the following term:

$$\mathcal{L}_{RR} = -m_0(\bar{\Delta}_{1\mu}^i \Delta_2^{\mu i} + \bar{N}_{1\mu}N_2^\mu + \text{H.c.}), \tag{13}$$

which describes off-diagonal mass terms between N_1^μ and N_2^μ and between $\Delta_1^{\mu i}$ and $\Delta_2^{\mu i}$. The parameter m_0 describes a chiral scalar, so-called mirror mass [25].

The mass terms included in $\mathcal{L}_{MRR}^{(1)} + \mathcal{L}_{MRR}^{(2)} + \mathcal{L}_{RR}$ are rewritten in the following matrix forms:

$$\begin{aligned} \mathcal{L}_M = &-(\bar{\Delta}_{1\mu}^i, \bar{\Delta}_{2\mu}^i)\begin{pmatrix} -g_1 f_\pi & m_0 \\ m_0 & -g_2 f_\pi \end{pmatrix}\begin{pmatrix} \Delta_1^{\mu i} \\ \Delta_2^{\mu i} \end{pmatrix} \\ &- (\bar{N}_{1\mu}, \bar{N}_{2\mu})\begin{pmatrix} \frac{1}{2}g_1 f_\pi & m_0 \\ m_0 & \frac{1}{2}g_2 f_\pi \end{pmatrix}\begin{pmatrix} N_1^\mu \\ N_2^\mu \end{pmatrix}. \end{aligned} \tag{14}$$

Because of the off-diagonal terms in these mass matrices, physical states and their masses are obtained through the diagonalization of the mass matrices. Note that the mass eigenvalues can take both positive and negative values. A state with a negative eigenvalue can be transformed into a state with a positive mass, but has opposite parity to the original state. It is carried out by multiplying a state having negative mass by γ_5 [25]. In the present paper, we consider the case that two states form a pair of positive and negative parity states both in Δ and N^* sectors.

For the Δ part in Eq. (14), we obtain the mass eigenvalues of two Δ states

$$m_{\Delta^\pm} = \tfrac{1}{2}\left[\sqrt{(g_1 - g_2)^2 f_\pi^2 + 4m_0^2} \mp (g_1 + g_2)f_\pi\right], \tag{15}$$

and the eigenstates

$$\Delta_+^{\mu i} = \cos\theta_\Delta \Delta_1^{\mu i} + \sin\theta_\Delta \Delta_2^{\mu i}, \tag{16a}$$

$$\Delta_-^{\mu i} = \gamma_5(-\sin\theta_\Delta \Delta_1^{\mu i} + \cos\theta_\Delta \Delta_2^{\mu i}), \tag{16b}$$

$$\tan 2\theta_\Delta = \frac{2m_0}{(g_2 - g_1)f_\pi}. \tag{16c}$$

Here we define $\Delta_+^{\mu i}$ and $\Delta_-^{\mu i}$ as positive and negative parity states, respectively, where the indices $\pm$ denote the parity. Hence $\Delta_+^{\mu i}$ and $\Delta_-^{\mu i}$ are identified with $\Delta(P_{33})$ and $\Delta(D_{33})$, respectively. Note that γ_5 in Eq. (16b) appears due to the parity redefinition. Similarly, for the N^* part, we obtain the mass eigenvalues

$$m_{N^\pm} = \frac{1}{2}\left[\sqrt{\frac{1}{4}(g_1 - g_2)^2 f_\pi^2 + 4m_0^2} \pm \frac{(g_1 + g_2)f_\pi}{2}\right], \tag{17}$$

and the eigenstates

$$N_+^\mu = \cos\theta_N N_1^\mu + \sin\theta_N N_2^\mu, \tag{18a}$$

$$N_-^\mu = \gamma_5(-\sin\theta_N N_1^\mu + \cos\theta_N N_2^\mu), \tag{18b}$$

$$\tan 2\theta_N = \frac{4m_0}{(g_1 - g_2)f_\pi}. \tag{18c}$$

N_+^μ and N_-^μ are identified with $N(D_{13})$ and $N(P_{13})$, respectively. Again, γ_5 in Eq. (18b) appears due to the parity redefinition. The four masses $m_{\Delta^\pm}$ and $m_{N^\pm}$ are given by the three parameters g_1, g_2, and m_0, which offer constraints on the four masses [34],

$$(m_{\Delta^+} + m_{\Delta^-}) \geq (m_{N^+} + m_{N^-}), \tag{19a}$$

$$m_{\Delta^-} - m_{\Delta^+} = 2(m_{N^+} - m_{N^-}). \tag{19b}$$

The inequality in the first line of Eq. (19) is controlled by m_0. Thus, the mass splittings and average masses are determined by chiral symmetry and the mirror mass m_0.

It is worthwhile considering the correspondence between the basis states and the physical states. Obviously, the mixing angles vanish in the absence of the mirror mass; $\theta_N, \theta_\Delta \to 0$ for $m_0 \to 0$. In this limit, the naive and mirror sectors decouple, and the physical states correspond to the basis states: $(\Delta_+^{\mu i}, N_+^\mu) \to (\Delta_1^{\mu i}, N_1^\mu)$ and $(\Delta_-^{\mu i}, N_-^\mu) \to (\Delta_2^{\mu i}, N_2^\mu)$. It should be noted that the decoupling of the two sectors does not violate chiral invariance. Contrarily, the two sectors are maximally mixed in the m_0 dominant case: $\theta_N, \theta_\Delta = \pi/4$.

The Lagrangians (11) and (12) contain the one-pion interaction terms between the spin-$\frac{3}{2}$ baryons (πRR) as well as the mass terms. Having the four spin-$\frac{3}{2}$ baryons, there are ten coupling constants $g_{\pi RR}$: four diagonal and six off-diagonal terms. All the ten coupling constants are functions of g_1, g_2, and m_0, which are determined by the masses. It is straightforward to derive the πRR coupling constants, $g_{\pi RR}$ from Eqs. (11) and (12). For the Δ part, we obtain

$$\Delta - \Delta \begin{cases} g_{\pi\Delta^+\Delta^+} = -(g_1\cos^2\theta_\Delta - g_2\sin^2\theta_\Delta) \\ g_{\pi\Delta^-\Delta^-} = (g_1\sin^2\theta_\Delta - g_2\cos^2\theta_\Delta) \\ g_{\pi\Delta^+\Delta^-} = (g_1 + g_2)\cos\theta_\Delta \sin\theta_\Delta \end{cases} \tag{20a}$$

which are defined by $\mathcal{L} = -g_{\pi\Delta_P\Delta_{P'}}\bar{\Delta}_{P\mu i}(i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})\Gamma_5\Delta_{P'}^{\mu i}$. Here P and P' denote parity, i.e., $P, P' = +$ or $-$, and $\Gamma_5 = 1$ for $P = P'$ and γ_5 for $P \neq P'$. For the N^* part, we obtain

$$N^* - N^* \begin{cases} g_{\pi N^+N^+} = \frac{5}{6}(g_1\cos^2\theta_N - g_2\sin^2\theta_N) \\ g_{\pi N^-N^-} = -\frac{5}{6}(g_1\sin^2\theta_N - g_2\cos^2\theta_N) \\ g_{\pi N^+N^-} = -\frac{5}{6}(g_1 + g_2)\cos\theta_N \sin\theta_N \end{cases} \tag{20b}$$

which are defined by $\mathcal{L} = -g_{\pi N_P N_{P'}}\bar{N}_{P\mu}(i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})\Gamma_5 N_{P'}^{\mu i}$. For N^*-Δ transition terms,

$$N^* - \Delta \begin{cases} g_{\pi N^+\Delta^+} = -\frac{\sqrt{3}}{3}(g_1\cos\theta_\Delta\cos\theta_N - g_2\sin\theta_\Delta\sin\theta_N) \\ g_{\pi N^+\Delta^-} = \frac{\sqrt{3}}{3}(g_2\cos\theta_\Delta\sin\theta_N + g_1\cos\theta_N\sin\theta_\Delta) \\ g_{\pi N^-\Delta^+} = -\frac{\sqrt{3}}{3}(g_1\cos\theta_\Delta\sin\theta_N + g_2\cos\theta_N\sin\theta_\Delta) \\ g_{\pi N^-\Delta^-} = \frac{\sqrt{3}}{3}(g_1\sin\theta_\Delta\sin\theta_N - g_2\cos\theta_N\cos\theta_\Delta) \end{cases} \tag{20c}$$

which are defined by $\mathcal{L} = -g_{\pi N_P\Delta_{P'}}\bar{N}_{P\mu}(i\gamma_5\Gamma_5)\pi^i\Delta_{P'}^{\mu i}$. In order to understand the features of $g_{\pi RR}$, it is useful to consider the axial charges, which are obtained by the Noether theorem

$$\Delta - \Delta \begin{cases} g_A^{\Delta^\pm\Delta^\pm} = \pm\cos 2\theta_\Delta, \\ g_A^{\Delta^+\Delta^-} = -\sin 2\theta_\Delta, \end{cases} \quad N^* - N^* \begin{cases} g_A^{N^{*\pm}N^{*\pm}} = \pm\frac{5}{3}\cos 2\theta_N, \\ g_A^{N^{*+}N^{*-}} = -\frac{5}{3}\sin 2\theta_N, \end{cases} \quad N^* - \Delta \begin{cases} g_A^{N^{*\pm}\Delta^\pm} = \pm\frac{4}{\sqrt{3}}\cos(\theta_N + \theta_\Delta), \\ g_A^{N^{*\pm}\Delta^\mp} = \pm\frac{4}{\sqrt{3}}\sin(\theta_N + \theta_\Delta). \end{cases} \tag{21}$$

In the limit $\theta_{N,\Delta} \to 0$ ($m_0 \to 0$), the absolute values of the parity-nonchanging interactions reach the maximum values: $|g_A^{\Delta^\pm\Delta^\pm}| \to 1$, $|g_A^{N^{*\pm}N^{*\pm}}| \to \frac{5}{3}$, and $|g_A^{N^{*\pm}\Delta^\pm}| \to \frac{4}{\sqrt{3}}$, while the parity-changing terms vanish $g_A^{\Delta^+\Delta^-} = g_A^{N^+N^-} = g_A^{N^\pm\Delta^\mp} = 0$. The mixing angles become larger, as m_0 becomes larger. Since the naive and mirror sectors have the opposite axial charges, the mixing of the two sectors suppresses the parity-nonchanging interactions and enhances the parity-changing interactions. In the m_0 dominance, the parity-nonchanging interactions vanish $g_A^{\Delta^\pm\Delta^\pm} = g_A^{N^{*\pm}N^{*\pm}} = g_A^{N^{*\pm}\Delta^\pm} \to 0$, while the parity-changing terms reach the maximum values $|g_A^{\Delta^+\Delta^-}| = 1$, $|g_A^{N^+N^-}| = \frac{5}{3}$, and $|g_A^{N^\pm\Delta^\mp}| = \frac{4}{\sqrt{3}}$. Of course, $g_{\pi RR}$ have the same features as the axial charges due to the Goldberger-Treiman relations:

$$
\begin{aligned}
&\Delta-\Delta\begin{cases} f_\pi g_{\pi\Delta^+\Delta^+} = \cos 2\theta_\Delta m_{\Delta^+}, \\ f_\pi g_{\pi\Delta^-\Delta^-} = -\cos 2\theta_\Delta m_{\Delta^-}, \\ f_\pi g_{\pi\Delta^+\Delta^-} = -\frac{1}{2}\sin 2\theta_\Delta (m_{\Delta^+}-m_{\Delta^-}), \end{cases} \\
&N^*-N^*\begin{cases} f_\pi g_{\pi N^+N^+} = \frac{5}{3}\cos 2\theta_N m_{N^+}, \\ f_\pi g_{\pi N^-N^-} = -\frac{5}{3}\cos 2\theta_N m_{N^-}, \\ f_\pi g_{\pi N^+N^-} = -\frac{5}{6}\sin 2\theta_N (m_{N^+}-m_{N^-}), \end{cases} \\
&N^*-\Delta\begin{cases} f_\pi g_{\pi N^+\Delta^+} = \frac{2}{\sqrt{3}}\cos(\theta_N+\theta_\Delta)(m_{N^+}+m_{\Delta^+}), \\ f_\pi g_{\pi N^+\Delta^-} = -\frac{2}{\sqrt{3}}\sin(\theta_N+\theta_\Delta)(m_{N^+}-m_{\Delta^-}), \\ f_\pi g_{\pi N^-\Delta^+} = -\frac{2}{\sqrt{3}}\sin(\theta_N+\theta_\Delta)(m_{N^-}-m_{\Delta^+}), \\ f_\pi g_{\pi N^-\Delta^-} = -\frac{2}{\sqrt{3}}\cos(\theta_N+\theta_\Delta)(m_{N^-}+m_{\Delta^-}). \end{cases}
\end{aligned} \tag{22}
$$

B. Interaction with the nucleon

Next, we construct the interactions between the nucleon (N) and the chiral quartet. As we have discussed in the introduction, we assume that the nucleon belongs to $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$. With the nucleon's chiral multiplet, we can classify the products of the chiral properties of $N\otimes\Delta$:

$$
\begin{aligned}
N\otimes\Delta &= \left[\left(\frac{1}{2},1\right)\oplus\left(1,\frac{1}{2}\right)\right]\otimes\left[\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)\right] \\
&= \begin{cases} (1,0)\oplus(0,1) & \text{for} \quad (N_1^\mu, \Delta_1^{\mu i}), \\ (\frac{1}{2},\frac{1}{2}) & \text{for} \quad (N_2^\mu, \Delta_2^{\mu i}), \end{cases}
\end{aligned} \tag{23}
$$

where we omit four-meson terms $(1,1)$ and $[(\frac{3}{2},\frac{1}{2})\oplus(\frac{1}{2},\frac{3}{2})]$. In the derivation of Eq. (23), it is important to take into account the chirality conservation. This classification implies that chiral invariant interactions between N and $(N_1^\mu, \Delta_1^{\mu i})$ accompany two-meson fields, while those between N and $(N_2^\mu, \Delta_2^{\mu i})$ accompany one-meson fields.

We find two chiral scalars $\sigma V_\mu + i\boldsymbol{\pi}\cdot\boldsymbol{A}_\mu$ and $\bar{N}U_5 q$. Multiplying them, we find two chiral invariant terms: $(-i)\bar{N}U_5[(\partial^\mu\sigma)V_\mu + i(\partial^\mu\boldsymbol{\pi})\cdot\boldsymbol{A}_\mu]q$ and $(-i)\bar{N}(\partial^\mu U_5)\times(\sigma V_\mu + i\boldsymbol{\pi}\cdot\boldsymbol{A}_\mu)q$. Using Eqs. (10), we obtain the chiral invariant interaction terms between N and $(N_1^\mu, \Delta_1^{\mu i})$,

$$
\mathcal{L}_{MNR}^{(1)} = \frac{g_3}{\Lambda^2}[\bar{N}O_{1\mu}^i\Delta_1^{\mu i} + \bar{N}O_{2\mu}N_1^\mu] + (\text{H.c.}), \tag{24a}
$$

$$
\mathcal{L}_{MNR}^{(2)} = \frac{g_4}{\Lambda^2}[\bar{N}O_{3\mu}^i\Delta_1^{\mu i} + \bar{N}O_{4\mu}N_1^\mu] + (\text{H.c.}), \tag{24b}
$$

where the dimensional parameter Λ (mass) is introduced to keep the coupling constants g_3 and g_4 dimensionless. We also introduce shorthand notations O_n $(n=1,\ldots,4)$ for mesonic operators

$$
O_1^{\mu i} = U_5(\partial^\mu\pi^i), \tag{24c}
$$

$$
O_2^\mu = -\frac{\sqrt{3}}{2}U_5\left((\partial^\mu\sigma)\gamma_5 + \frac{1}{3}(i\partial^\mu\boldsymbol{\pi}\cdot\boldsymbol{\tau})\right), \tag{24d}
$$

$$
O_3^{\mu i} = (\partial^\mu U_5)(\pi^i), \tag{24e}
$$

$$
O_4^\mu = -\frac{\sqrt{3}}{2}(i\partial^\mu U_5)\left(\sigma\gamma_5 + \frac{1}{3}i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\right). \tag{24f}
$$

One may think it possible to construct similar interaction terms for the mirror fields by the replacement Eq. (7). However, such terms are forbidden by chirality conservation, as is shown in Eq. (23).[1] The mirror fields have one-meson interactions with the nucleon. It can be constructed by using the chiral invariant operators $(-i)(\sigma V_\mu + i\boldsymbol{\pi}\cdot\boldsymbol{A}_\mu)$ and $\bar{N}\not{D}q$. We obtain

$$
\mathcal{L}_{MNR}^{(3)} = \frac{g_5}{\Lambda}[\bar{N}O_{5\mu}^i\Delta_2^{\mu i} + \bar{N}O_{6\mu}N_2^\mu], \tag{25a}
$$

where O_5 and O_6 are also mesonic operators,

$$
O_5^{\mu i} = (\partial^\mu\pi^i), \tag{25b}
$$

$$
O_6^\mu = -\frac{\sqrt{3}}{2}(i\partial^\mu)\left(\sigma\gamma_5 + \frac{1}{3}i\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right). \tag{25c}
$$

In the mass basis, $\mathcal{L}_{MNR} = \mathcal{L}_{MNR}^{(1)} + \mathcal{L}_{MNR}^{(2)} + \mathcal{L}_{MNR}^{(3)}$ is rewritten as

$$
\begin{aligned}
\mathcal{L}_{MNR} = {}& \bar{N}[(O_{1\mu}^i + O_{3\mu}^i)\cos\theta_\Delta + O_{5\mu}^i\sin\theta_\Delta]\Delta_+^{\mu i} \\
&+ \bar{N}[-(O_{1\mu}^i + O_{3\mu}^i)\sin\theta_\Delta + O_{5\mu}^i\cos\theta_\Delta]\gamma_5\Delta_-^{\mu i} \\
&+ \bar{N}[(O_{2\mu} + O_{4\mu})\cos\theta_N + O_{6\mu}\sin\theta_N]N_+^{\mu i} \\
&+ \bar{N}[-(O_{2\mu} + O_{4\mu})\sin\theta_N + O_{6\mu}\cos\theta_N]\gamma_5 N_-^{\mu i},
\end{aligned} \tag{26}
$$

[1]It can be shown explicitly. For example, the first term in Eq. (24a) is rewritten in terms of left- and right-handed parts of the fields as $\bar{N}U_5(\partial_\mu\pi^i)\Delta_1^{\mu i} = \bar{N}_L U_5(\partial_\mu\pi^i)\Delta_{1R} + (l\leftrightarrow r)$. Replacing $\Delta_{1R}\to\Delta_{2L}$, $\bar{N}_L U_5(\partial_\mu\pi^i)\Delta_{1R}\to\bar{N}_L U_5(\partial_\mu\pi^i)\Delta_{2L}$, which vanishes due to $P_L P_R = 0$ $[P_{R,L} = (1\pm\gamma_5)/2]$.

which contains several kinds of the interaction terms, πNR, $\pi\pi NR$, σNR, and $\sigma\sigma NR$. Among them, we consider πNR and $\pi\pi NR$ interaction terms in order for the comparison with experiments. The πN interactions of the chiral quartet are given by

$$\mathcal{L}_{\pi NR} = \frac{g_{\pi N\Delta^+}}{\Lambda}\bar{N}(\partial_\mu \pi^i)\Delta^{+\mu i} + \frac{g_{\pi N\Delta^-}}{\Lambda}\bar{N}(\partial_\mu \pi^i)\gamma_5\Delta^{-\mu i} + \frac{g_{\pi NN^{*-}}}{\Lambda}\bar{N}(\partial_\mu \boldsymbol{\pi}\cdot\boldsymbol{\tau})\gamma_5 N^{-\mu} + \frac{g_{\pi NN^{*+}}}{\Lambda}\bar{N}(\partial_\mu \boldsymbol{\pi}\cdot\boldsymbol{\tau})N^{+\mu}, \tag{27a}$$

where the coupling constants $g_{\pi NN^{*\pm}}$ and $g_{\pi N\Delta^\pm}$ are given by

$$g_{\pi N\Delta^+} = \frac{1}{\Lambda}(g_5\Lambda\sin\theta_\Delta + g_3 f_\pi\cos\theta_\Delta), \tag{27b}$$

$$g_{\pi N\Delta^-} = \frac{1}{\Lambda}(g_5\Lambda\cos\theta_\Delta - g_3 f_\pi\sin\theta_\Delta), \tag{27c}$$

$$g_{\pi NN^{*+}} = \frac{\sqrt{3}}{6\Lambda}(g_5\Lambda\sin\theta_N + (g_3 + 3g_4)f_\pi\cos\theta_N), \tag{27d}$$

$$g_{\pi NN^{*-}} = \frac{\sqrt{3}}{6\Lambda}(g_5\Lambda\cos\theta_N - (g_3 + 3g_4)f_\pi\sin\theta_N). \tag{27e}$$

Four $g_{\pi NR}$ are expressed in terms of three parameters g_3, g_4, and g_5, which leads to one identity

$$(\sin\theta_\Delta g_{\pi N\Delta^+} + \cos\theta_\Delta g_{\pi N\Delta^-}) = 2\sqrt{3}(\sin\theta_N g_{\pi NN^{*+}} + \cos\theta_N g_{\pi NN^{*-}}). \tag{28}$$

Here it must be noted that the derivation of the πN interactions is based on the assumption of the nucleon's chiral multiplet. If the nucleon together with the negative parity resonance group into $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ with the mirror assignment, we can include three additional interactions, which spoils the constraint Eq. (28). Another possibility is that the nucleon contains $(1,\frac{1}{2})\oplus(\frac{1}{2},1)$ as well as $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$. In this case, we can include one additional interaction that has a similar form to Eq. (11). With the new term, Eq. (28) becomes a loose constraint and gives the ordering of the coupling constants. So, Eq. (28) is one of the strictest constraints. The point is that it is possible to improve this result without changing the masses and πRR interactions of the quartet.

We obtain two-pion interaction terms

$$\mathcal{L}_{\pi\pi N\Delta} = \frac{g^{(v)}_{\pi\pi N\Delta_+}}{\Lambda}\bar{N}(\epsilon^{abc}\pi^a\pi^b_{,\mu}\gamma_5)\Delta^{\mu c}_+ + \frac{g^{(t)}_{\pi\pi N\Delta^+}}{\Lambda}\bar{N}(\pi^a\pi^b_{,\mu} + \pi^a_{,\mu}\pi^b)(i\gamma_5\tau^a)\Delta^{\mu b}_+ + \frac{g^{(v)}_{\pi\pi N\Delta_-}}{\Lambda}\bar{N}(\epsilon^{abc}\pi^a\pi^b_{,\mu})\Delta^{\mu c}_- + \frac{g^{(t)}_{\pi\pi N\Delta_-}}{\Lambda}\bar{N}(\pi^a\pi^b_{,\mu} + \pi^a_{,\mu}\pi^b)(i\tau^a)\Delta^{\mu b}_-, \tag{29}$$

$$\mathcal{L}_{\pi\pi NN^*} = \frac{g^{(s)}_{\pi\pi NN^*_+}}{\Lambda}\bar{N}(i\gamma_5\boldsymbol{\pi}\cdot\boldsymbol{\pi}_{,\mu})N^\mu_+ + \frac{g^{(v)}_{\pi\pi NN^*_+}}{\Lambda}\bar{N}(\epsilon^{abc}\pi^a\pi^b_{,\mu}\tau^c)\gamma_5 N^\mu_+ + \frac{g^{(s)}_{\pi\pi NN^*_-}}{\Lambda}\bar{N}(i\boldsymbol{\pi}\cdot\boldsymbol{\pi}_{,\mu})N^\mu_- + \frac{g^{(v)}_{\pi\pi NN^*_-}}{\Lambda}\bar{N}(\epsilon^{abc}\pi^a\pi^b_{,\mu}\tau^c)N^\mu_-, \tag{30}$$

with

$$\Delta\text{-sector}\begin{cases} g^{(v)}_{\pi\pi N\Delta_+} &= \frac{\cos\theta_\Delta}{2\Lambda}(g_3 - g_4),\\ g^{(t)}_{\pi\pi N\Delta^+} &= \frac{\cos\theta_\Delta}{2\Lambda}(g_3 + g_4),\\ g^{(v)}_{\pi\pi N\Delta_-} &= -\frac{\sin\theta_\Delta}{2\Lambda}(g_3 - g_4),\\ g^{(t)}_{\pi\pi N\Delta_-} &= -\frac{\sin\theta_\Delta}{2\Lambda}(g_3 + g_4),\end{cases}$$
$$N^*\text{-sector}\begin{cases} g^{(s)}_{\pi\pi NN^*_+} &= +\frac{\sqrt{3}\cos\theta_N}{6\Lambda}(g_3 + g_4),\\ g^{(v)}_{\pi\pi NN^*_+} &= -\frac{\sqrt{3}\cos\theta_N}{6\Lambda}(g_3 - g_4),\\ g^{(s)}_{\pi\pi NN^*_-} &= -\frac{\sqrt{3}\sin\theta_N}{6\Lambda}(g_3 + g_4),\\ g^{(v)}_{\pi\pi NN^*_-} &= \frac{\sqrt{3}\sin\theta_N}{6\Lambda}(g_3 - g_4),\end{cases} \tag{31}$$

where they are classified into three types: the symmetric $(\boldsymbol{\pi}\cdot\boldsymbol{\pi}_{,\mu})$, antisymmetric $(i\epsilon^{abc}\pi^a\pi^b_{,\mu})$, and symmetric $(\pi^a\pi^b_{,\mu} + \pi^a_{,\mu}\pi^b)$ types. They correspond to an isoscalar $(\boldsymbol{\pi}\cdot\boldsymbol{\pi}_{,\mu})$, isovector $(i\epsilon^{abc}\pi^a\pi^b_{,\mu})$, and isotensor $(\pi^a\pi^b_{,\mu} + \pi^a_{,\mu}\pi^b)$. Since the two-pion coupling constants $g_{\pi\pi NR}$ contain only g_3 and g_4, their strengths are determined by the πN coupling constants through $g_3 = (\Lambda/f_\pi)\times((g_{\pi N\Delta^+} - g_{\pi N\Delta^-})/(\cos\theta_\Delta + \sin\theta_\Delta))$ and $g_4 = (2\Lambda/\sqrt{3}f_\pi)((g_{\pi NN^*_+} - g_{\pi NN^*_-})/(\cos\theta_N + \sin\theta_N))$. Furthermore, $g_{\pi\pi NR}$ are proportional to either $(g_3 + g_4)$ or $(g_3 - g_4)$, which provides a selection rule; either $\pi\pi$ isoscalar or isovector interaction is suppressed each for $N^*_\pm$, and either the isovector or isotensor interaction is suppressed each for $\Delta_\pm$.

Using the $SU(2)_R\times SU(2)_L$ Lagrangian, we have derived several constraints on the properties of the chiral quartet. We concentrate on the construction of the lowest-order terms and the derivation of the chiral constraints at tree level. In general, it is possible to insert chiral invariant operators such as $(\sigma^2 + \pi^2)^n$ into the chiral Lagrangians we derived. However, those terms do not change the above constraints and can be absorbed into the parameters. Regarding the πRR interactions, it is possible to include an additional interaction term with a derivative [25]. The constraint for the πNR interactions rely on the assumption of the saturation of $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ in the nucleon. The inclusion of the $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ component in the nucleon causes one additional chiral invariant πN interaction term similar to Eq. (11). In this case, four $g_{\pi NR}$ are given by four parameters. It must be noted that the inclusion of $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ for the nucleon does not affect the multiplet nature of the quartet.

IV. RESULTS

In this section, we proceed to numerical discussions and look for a set of baryons suitable for the QS. Possible candidates for the members of the quartet are shown in Table I. There are six parameters in our model: m_0, g_1, g_2, g_3, g_4, and g_5. The dimensional parameter Λ does not play any role in the present study, so then we do not need to determine it. Since the masses $m_{\Delta_\pm}$ and $m_{N^*_\pm}$ are the functions of m_0, g_1, and g_2, we can determine them by minimizing $\chi^2_{\rm mass} = \sum_R (m_R - m_R^{(\rm exp)})^2/(\delta m_R^{(\rm exp)})^2$ ($R = \Delta_\pm$ and $N^*_\pm$). Here $m_R^{(\rm exp)}$ and $\delta m_R^{(\rm exp)}$ are the central values and errors of the observed masses, which are shown in Tables II and III. Considering the states listed in Table I, there are 36 possible assignments. Among them, we discuss four cases *[Case (1)]* ($\Delta(1232)$, $\Delta(1700)$, $N(1520)$, $N(1720)$), *[Case (2)]* ($\Delta(1600)$, $\Delta(1940)$, $N(1520)$, $N(1720)$), *[Case (3-1)]* and *[Case (3-2)]* ($\Delta(1920)$, $\Delta(1940)$, $N(2080)$, $N(1900)$). Although case (1) was studied in Refs. [30,34], we reanalyze this case with the use of the different methods for the determination of the parameters. As we will show, case (2) agrees with the mass pattern of the QS with the smallest $\chi^2_{\rm mass}$. We also discuss ($\Delta(1920)$, $\Delta(1940)$, $N(2080)$, $N(1900)$). Because of a variety in the data, we consider two cases, (3-1) and (3-2), for this assignment, using two different data sets shown in Table III. There are three other assignments that reproduce the masses with $\chi^2_{\rm mass}$ less than one: ($\Delta(1600)$, $\Delta(1700)$, $N(1700)$, $N(1720)$), ($\Delta(1600)$, $\Delta(1940)$, $N(1700)$, $N(1900)$), and ($\Delta(1920)$, $\Delta(1940)$, $N(1700)$, $N(1720)$). We concentrate on the above four cases in the present work. Instead of discussing all of them, we discuss the general behaviors of the QS later. Results for the masses are shown in Table IV. For case (1), the present result differs from the previous study [30], which is due to the difference of the method to determine the mass parameters. In Ref. [30], we adopted the minimization of a standard deviation $\sigma^2 = \Sigma_R (m_R - m_R^{(\rm exp)})^2$, while we employ the χ^2-minimum method in the present work. These two methods differ in how $\Delta(1232)$ is included in the fitting procedure, because the error of the observed $\Delta(1232)$'s mass is much smaller than those of the other three states. We found $\chi^2_{\rm mass}$ amounts to 60, which is significantly large. It is favorable for the QS that the masses of the $\Delta_\pm$ are larger than those of $N^*_\pm$, as shown in Eqs. (19). The mass of $\Delta(1232)$ is much smaller compared with other spin-$\frac{3}{2}$ baryons. This causes the significantly large discrepancy. We also found that $\chi^2_{\rm mass}$ becomes larger if assignments include $\Delta(1232)$ as a member of the quartet, which implies that the mass of $\Delta(1232)$ is too small for the QS.

Cases (2), (3-1), and (3-2) are new in this work. Case (2) is the best assignment for the quartet with $\chi^2_{\rm mass} = 0.0025$, which is the smallest value among $\chi^2_{\rm mass}$ for 36 possible assignments. For $\Delta(1940)$ in this case, we use the data by Horn *et al.* [48]. We confirmed that the result for (2) is

TABLE I. Observed states listed in Particle Data Group (PDG) [47] corresponding to the quantum numbers of the members of the quartet. The number of the stars denotes PDG ratings of the states.

L_{2I2J}	Observed states
P_{33}	$\Delta(1232)^{****}$, $\Delta(1600)^{***}$, $\Delta(1920)^{***}$
D_{33}	$\Delta(1700)^{***}$, $\Delta(1940)^{**}$
D_{13}	$N(1520)^{****}$, $N(1700)^{***}$, $N(2080)^{**}$
P_{13}	$N(1720)^{****}$, $N(1900)^{*}$

TABLE II. Data for masses, πN decay widths, and πN coupling constants of the observed states used in cases (1) and (2). The data are taken from PDG [47]. The values in the bracket for $m_R^{(\rm exp)}$ are central values of the observed masses, while those for $\Gamma_{\pi N}^{(\rm exp)}$ are the average values between minimum and maximum values. The definition of $g_{\pi N}^{(\rm exp)}$ is given in the main text. For $\Delta(1940)$ in case (2), we use the data in Ref. [48].

States R	$m_R^{(\rm exp)}$ (MeV)	$\Gamma_{\pi N}^{(\rm exp)}$ (MeV)	$g_{\pi N}^{(\rm exp)}/\Lambda$ (GeV^{-1})
$\Delta(1232)[P_{33}]$	1231–1233 (1232)	116–120 (118)	15.7–16.0 (15.8)
$\Delta(1600)[P_{33}]$	1550–1700 (1600)	25.0–113 (68.8)	2.37–5.04 (3.70)
$\Delta(1700)[D_{33}]$	1670–1750 (1700)	20.0–80.0 (50.0)	6.34–12.7 (9.51)
$\Delta(1940)[D_{33}]$	1950–2030 (1990)	17.0–62.4 (39.7)	3.23–6.20 (4.72)
$N(1520)[D_{13}]$	1515–1525 (1520)	55.0–81.3 (68.1)	7.64–9.30 (8.46)
$N(1720)[P_{13}]$	1700–1750 (1720)	15.0–60.0 (37.5)	1.72–3.44 (2.58)

TABLE III. Data for masses, πN decay widths, and πN coupling constants of the observed states used in cases (3-1) and (3-2). See also the caption of Table II.

		Case (3-1)		
States R	$m_R^{(\mathrm{exp})}$ (MeV)	$\Gamma_{\pi N}^{(\mathrm{exp})}$ (MeV)	$g_{\pi N}^{(\mathrm{exp})}/\Lambda$ (GeV^{-1})	Reference
$\Delta(1920)[P_{33}]$	1900–1970 (1920)	7.50–60.0 (33.8)	0.825–2.33(1.58)	PDG average [47]
$\Delta(1940)[D_{33}]$	1950–2030 (1990)	17.0–62.4 (39.7)	3.23–6.20(4.72)	Horn *et al.* [48]
$N(2080)[D_{13}]$	1945–1947 (1946)	85.2–121 (103)	4.63–5.23(5.08)	Penner *et al.* [49]
$N(1900)[P_{13}]$	1855–1975 (1915)	2.80–19.8 (11.3)	0.574–1.53(1.05)	Nikonov *et al.* [50]
		Case (3-2)		
States R	$m_R^{(\mathrm{exp})}$ (MeV)	$\Gamma_{\pi N}^{(\mathrm{exp})}$ (MeV)	$g_{\pi N}^{(\mathrm{exp})}/\Lambda$ (GeV^{-1})	Reference
$\Delta(1920)[P_{33}]$	1900–1970 (1920)	7.50–60.0 (33.8)	0.825–2.33 (1.58)	PDG average [47]
$\Delta(1940)[D_{33}]$	1947–2167 (2057)	8.40–234 (121)	2.04–10.8 (6.40)	Manley *et al.* [51]
$N(2080)[D_{13}]$	1749–1859 (1804)	53.0–165 (109)	4.45–7.84 (6.15)	Manley *et al.* [51]
$N(1900)[P_{13}]$	1855–1975 (1915)	2.8.0–19.8 (11.3)	0.574–1.53 (1.05)	Nikonov *et al.* [50]

TABLE IV. Result for the masses and parameters. For the experimental data, see Tables II and III.

		Masses (MeV) [Assigned states]		
State	Case (1)	Case (2)	Case (3-1)	Case (3-2)
Δ^+ $[P_{33}]$	1233 [$\Delta(1232)$]	1594 [$\Delta(1600)$]	1935 [$\Delta(1920)$]	1917 [$\Delta(1920)$]
Δ^- $[D_{33}]$	2190 [$\Delta(1700)$]	1992 [$\Delta(1940)$]	1980 [$\Delta(1940)$]	2083 [$\Delta(1940)$]
N^- $[D_{13}]$	1473 [$N(1520)$]	1520 [$N(1520)$]	1946 [$N(2080)$]	1817 [$N(2080)$]
N^+ $[P_{13}]$	1951 [$N(1720)$]	1719 [$N(1720)$]	1969 [$N(1900)$]	1899 [$N(1900)$]
χ^2_{mass}	68	0.0025	0.26	0.045
		Parameters and angles		
State	Case (1)	Case (2)	Case (3-1)	Case (3-2)
g_1	5.2	12	0.25	10
g_2	5.2	−7.5	0.25	−8.3
m_0 (MeV)	1712	1557	1957	1809
θ_N (degree)	45	37	45	38
θ_Δ (degree)	45	60	45	58

insensitive to the choice of the data for $\Delta(1940)$. Cases (3-1) and (3-2) also reproduce the masses of the quartet with $\chi^2_{\mathrm{mass}} = 0.26$ and 0.045, respectively.

Once the masses are determined, we obtain the one-pion coupling constants between two members of the quartet, which are shown in Table V. First, we consider qualitative features of the one-pion coupling constants. It was found [34] that in case (1) the parity-nonchanging interactions vanish, while the parity-changing interactions remain to be finite. However, even for the parity-changing interactions, their strengths are smaller than a typical order of one-pion interactions, e.g. $g_{\pi NN} \sim 13$ [18]. On the other hand, $g_{\pi RR}$ behaves in an opposite way in case (2). All of the coupling constants survive in the case, where the parity-changing interactions are suppressed compared to the parity-nonchanging ones. In addition, diagonal coupling constants are comparable to $g_{\pi NN}$, e.g. $g_{\pi\Delta_-\Delta_-} = 11$. Interestingly, cases (3-1) and (3-2) show different results, although they are the same assignment. This is caused by the difference of the ordering of the masses of the quartet, especially that of $\Delta(1920)$ and $N(2080)$. We turn back to this point later.

Among various coupling constants, $g_{\pi\Delta(1232)\Delta(1232)}$ are investigated in several approaches. Quark models [52] and large N_c [53] predict large values, especially, $g_A^{\pi\Delta\Delta} = (9/5)g_A$ in large N_c, which gives $g_{\pi\Delta(1232)\Delta(1232)} \sim 30$. A light-cone QCD sum rule reported half of the quark model

TABLE V. The one-pion coupling constants between the members of the quartet, $g_{\pi RR}$. The values of the parameters are shown in Table IV.

$g_{\pi RR}$	Case (1)	Case (2)	Case (3-1)	Case (3-2)
$g_{\pi\Delta^+\Delta^+}$	0	−8.6	0	−8.9
$g_{\pi\Delta^-\Delta^-}$	0	11	0	9.6
$g_{\pi\Delta^+\Delta^-}$	5.2	1.9	0.25	0.81
$g_{\pi N^+N^+}$	0	8.5	0	7.9
$g_{\pi N^-N^-}$	0	−7.5	0	−7.5
$g_{\pi N^+N^-}$	−4.3	−1.7	−0.21	−0.73
$g_{\pi N^+\Delta^+}$	0	−5.0	0	−5.0
$g_{\pi N^+\Delta^-}$	3.0	3.4	0.14	2.3
$g_{\pi\Delta^+N^-}$	−3.0	0.92	−0.14	1.2
$g_{\pi N^-\Delta^-}$	0	5.3	0	5.1

prediction [54] but still large values compared to our result. The $g_{\pi\Delta(1232)\Delta(1232)}$ were also determined in coupled channel analysis. Krehl *et al.* obtained $g_{\pi\Delta\Delta} = 31$ [55], while Schneider *et al.* obtained $g_{\pi\Delta\Delta} = 12.5$ [56]. In case (1), $g_{\pi\Delta(1232)\Delta(1232)}$ vanishes, which is inconsistent with these studies. Krehl *et al.* and Schneider *et al.* also investigated $g_{\pi\Delta(1232)N(1520)}$ and obtained $g_{\pi N(1520)\Delta(1232)} = 0.95$ and 1.3, respectively. The present result $|g_{\pi\Delta(1232)N(1520)}| = 3.0$ is qualitatively consistent with these values.

With regard to the πN coupling constants $g_{\pi NR}$, we need to determine three parameters g_3, g_4, and g_5. Since $g_{\pi NR}$ are the functions of g_3, g_4, and g_5, we can determine them by the χ^2-minimum method with $\chi^2_{\pi NR} = \Sigma_R(g_{\pi NR} - g^{(\mathrm{exp})}_{\pi NR})^2/(\delta g^{(\mathrm{exp})}_{\pi NR})^2$. Here $g^{(\mathrm{exp})}_{\pi NR}$ and $\delta g^{(\mathrm{exp})}_{\pi NR}$ are the average and errors of the coupling constants determined from the experimental πN decay widths. We obtain them by using a relation $g^{(\mathrm{exp})}_{\pi NR}/\Lambda = \sqrt{\Gamma^{(\mathrm{exp})}_{\pi N}/\tilde{\Gamma}_{\pi N}}$, where $\tilde{\Gamma}$ is πN decay widths obtained by setting the coupling constant to be one, and $\Gamma^{(\mathrm{exp})}_{\pi N}$ are the experimental values of the πN decay widths shown in Tables II and III. The dimensional parameter Λ does not play any role in the determination of the coupling constants because of the cancellation between the numerator and denominator in $\chi^2_{\pi NR}$. We obtain $\tilde{\Gamma}_{\pi N}$ by calculating the simplest tree diagram. Note that we can determine only absolute values of the coupling constants from the πN decay widths. Hence, the positive sign of $g^{(\mathrm{exp})}_{\pi NR}$ in Tables II and III is our assumption. The result is shown in Table VI.

Case (1) reproduces the reasonable values for the four $g_{\pi NR}$ with small $\chi^2_{\pi NR}$, which are almost within the ranges of the experimental values. In case (2), the $\chi^2_{\pi NR}$ value is significantly large. The discrepancy is mostly caused by the small values of the πN decay width of $\Delta(1600)$ and $\Delta(1940)$. In the QS, it is favored that the average values of $g_{\pi NR}$ between $\Delta_\pm$ is larger than that between $N^*_\pm$, as is shown in Eq. (28). Because of the same reason, $\chi^2_{\pi NR}$ is large for case (3-1). We obtain reasonable results for case (3-2) with small $\chi^2_{\pi NR}$. Our result underestimates the value of $g_{\pi NR}$ for $R = N(2080)(N^*_-)$, which gives πN decay widths half of the minimum of the experimental values.

Mass pattern and one-pion coupling constant

The quartet scheme shows two different behaviors for the one-pion coupling constants, as shown in Table V. Especially, the assignment ($\Delta(1920)$, $\Delta(1940)$, $N(2080)$, $N(1900)$) shows two different behaviors, depending on the choice of the experimental data. Equations (21) show that the one-pion coupling constants are controlled by the mixing angles. Cases (1) and (3-1) correspond to the maximally mixing with the angles $\theta_{N,\Delta} = 45°$, while cases (2) and (3-2) correspond to moderate mixing. Since the mixing angles are the functions of m_0 and $(g_1 - g_2)f_\pi$ as shown in Eqs. (16) and (18), we can understand the behavior of the one-pion coupling constants, comparing m_0 with $(g_1 - g_2)f_\pi$. These parameters also determine the masses of the quartet. Therefore, we can relate the masses to the one-pion constants.

In order to understand their relation, we approximate the masses in two ways. In the small m_0 case, the masses are, up to $\mathcal{O}(m_0^2)$, given by

$$m_{\Delta^\pm} = 2X \mp 2Y + Z, \qquad m_{N^\pm} = X \pm Y + 2Z,$$

where $X = f_\pi|g_1 - g_2|/4$, $Y = (g_1 + g_2)f_\pi/4$, and $Z = 4m_0^2/(f_\pi|g_1 - g_2|)$. In the m_0 dominant case, they are, up to $\mathcal{O}((f_\pi/m_0))$, given by

$$m_{\Delta^\pm} = m_0 \mp 2a, \qquad m_{N^{*\pm}} = m_0 \pm a,$$

where $a = (g_1 + g_2)f_\pi/4$. The mass patterns for these cases are shown in Fig. 1. The two cases are different in the ordering of Δ^+ and N^{*-}. In the $m_0 \to 0$ limit, they have mass ratio 2:1 and Δ^+ is heavier than N^{*-}. Small values of m_0 do not change this ordering, which corresponds to the left panel in Fig. 1. When m_0 becomes much larger, the ordering is changed and Δ^+ becomes the lowest state. Cases (1) and (3-1) correspond to the mass pattern shown in the right panel in Fig. 1, while cases (2) and (3-2)

TABLE VI. Result for the πN coupling constants and parameters. For the experimental data, see Tables II and III.

	Case (1)	Case (2)	Case (3-1)	Case (3-2)
	πN coupling constants Theo (Expt) (GeV^{-1})			
$\frac{g_{\pi N\Delta^+}}{\Lambda}$	16 (15.7–16.0)	7.2 (2.37–5.04)	2.7 (0.825–2.33)	1.8 (0.825–2.33)
$\frac{g_{\pi N\Delta^-}}{\Lambda}$	14 (6.34–12.7)	7.2 (3.23–6.20)	8.9 (3.23–6.20)	12 (2.04–10.8)
$\frac{g_{\pi NN^{*-}}}{\Lambda}$	7.3 (7.64–9.30)	4.2 (7.64–9.30)	3.8 (4.63–5.23)	2.2 (4.45–7.84)
$\frac{g_{\pi NN^{*+}}}{\Lambda}$	1.3 (1.72–3.44)	−0.89 (1.72–3.44)	−0.44 (0.574–1.53)	0.81(0.574–1.53)
$\chi^2_{\pi NR}$	1.5	13	7.1	1.8
	Parameters (GeV^{-1})			
	Case (1)	Case (2)	Case (3-1)	Case (3-2)
$\frac{g_3 f_\pi}{\Lambda^2}$	1.1	−2.6	−4.4	−8.8
$\frac{g_4 f_\pi}{\Lambda^2}$	−5.2	−2.9	−2.0	2.1
$\frac{g_5}{\Lambda}$	21	9.8	8.2	7.7

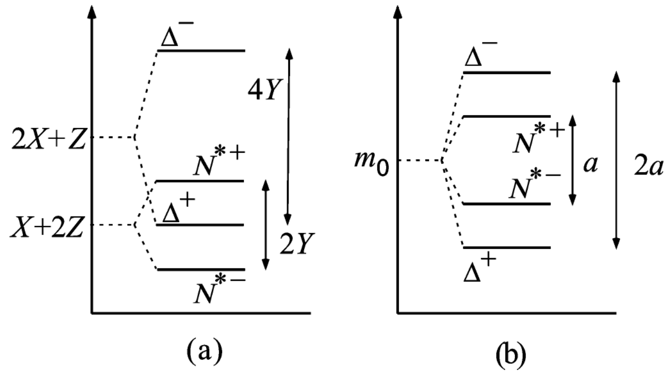

FIG. 1. Schematic figures for the mass pattern of the QS. (a) Small m_0 case. (b) m_0-dominant case.

correspond to the left panel. Actually, m_0 is not small in cases (2) and (3-2), but comparable to $(g_1 - g_2)f_\pi$. However, the left panel in Fig. 1 well described the mass pattern of these cases. Using Eqs. (16) and (18), mixing angles in the small m_0 case takes moderate values and all the one-pion coupling constants survive. On the other hand, in the m_0-dominant case, mixing angles are $\theta_{N,\Delta} \sim \pi/4$ and the parity-nonchanging interactions vanish. Thus, the behavior of the one-pion coupling constants is related to the mass pattern of the quartet. According to this discussion, cases (3-1) and (3-2) are different due to the ordering of $\Delta(1920)$ and $N(2080)$, although they describe the same assignments. This is the reason why the assignment ($\Delta(1920)$, $\Delta(1940)$, $N(2080)$, $N(1900)$) is sensitive to the choice of the experimental data. This discussion can be applied to other assignments we do not take into account. As we have mentioned, the other three assignments reproduce the masses of the quartet with χ^2_{mass} less than one: ($\Delta(1600)$, $\Delta(1700)$, $N(1700)$, $N(1720)$), ($\Delta(1600)$, $\Delta(1940)$, $N(1700)$, $N(1900)$), and ($\Delta(1920)$, $\Delta(1940)$, $N(1700)$, $N(1720)$). According to the above discussions, the first and second cases correspond to maximal mixing with the vanishing of the parity-nonchanging interactions, while all the coupling constants survive in the third case.

V. SUMMARY

We have investigated the possibility that chiral partners exist in the spin-$\frac{3}{2}$ baryon sector by considering the quartet scheme, where four spin-$\frac{3}{2}$ baryons, P_{33}, D_{33}, D_{13}, and P_{13}, form the chiral multiplets $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ with the mirror assignment. Using the $SU(2)_R \times SU(2)_L$ Lagrangian, we tried to find a set of four baryons suitable for the chiral quartet. We discussed three assignments: (1) ($\Delta(1232)$, $\Delta(1700)$, $N(1520)$, $N(1720)$), (2) ($\Delta(1600)$, $\Delta(1940)$, $N(1520)$, $N(1720)$), and (3-1) and (3-2) ($\Delta(1920)$, $\Delta(1940)$, $N(2080)$, $N(1900)$). Here we investigated ($\Delta(1920)$, $\Delta(1940)$, $N(2080)$, $N(1900)$) using two data sets.

For case (1) we found that there is significant discrepancy for the masses, which implies the mass of $\Delta(1232)$ is too small for the quartet scheme. In addition, the vanishing of $g_{\pi\Delta(1232)\Delta(1232)}$ is inconsistent with other theories. Considering the discrepancy for the masses and the inconsistencies of $g_{\pi\Delta(1232)\Delta(1232)}$, it seems that this case is less suitable for the quartet.

For case (2), the masses of the observed baryons agree well with the mass pattern of the QS. Among all the possible assignments, the χ^2 value becomes the smallest in this case. Considering the masses, this case is most suitable for the quartet. Regarding the πN interactions, this case does not reproduce reasonable results.

For the assignment ($\Delta(1920)$, $\Delta(1940)$, $N(2080)$, $N(1900)$), we consider two cases (3-1) and (3-2) with the use of different data sets because of the variety of the experimental data. Both cases reproduce the masses of the quartet with χ^2 less than one. The one-pion coupling constants for this assignment are quite sensitive to the ordering of the masses of $\Delta(1920)$ and $N(2080)$. If the mass of $\Delta(1920)$ is smaller than that of $N(2080)$, only the parity-changing one-pion interactions survive. On the other hand, if the mass of $N(2080)$ is smaller, all the coupling constants are finite and the parity-nonchanging interactions are larger than the parity-changing ones. Regarding the πN interactions, we obtained reasonable results for case (3-2).

For further confirmation, experiments or lattice calculations for the one-pion coupling constants are needed. For instance, we can test the validity of case (2) using coupling constants such as $g_{\pi N(1520)N(1520)}$, $g_{\pi N(1720)N(1720)}$, and $g_{\pi N(1520)N(1720)}$. For the further study of the assignment ($\Delta(1920)$, $\Delta(1940)$, $N(2080)$, $N(1900)$), we need information about the masses because of a variety of the data. Especially, detailed information of the masses of $\Delta(1920)$ and $N(2080)$ are needed, because the one-pion coupling constants are sensitive to the ordering of the masses of them. If the mass ordering is determined, we can test this assignment using one-pion coupling constants such as $g_{\pi\Delta(1920)\Delta(1920)}$.

It is important to extend the present framework with the inclusion of higher-dimensional chiral representations for the nucleon. For the πN interactions with the quartet, we adopted the assumption that the nucleon belongs to the fundamental chiral representation. There are other possibilities for the nucleon's chiral representation. Hence, the disagreements for the πN interactions may come from this assumption and can be resolved by including higher-dimensional chiral representations for the nucleon. Furthermore, it may be possible to test the nucleon's chiral representations through the πN interactions with the quartet, if we can confirm the QS by using the one-pion interactions for the quartet.

In the present study, we employed the effective Lagrangian approach, where we truncated higher-order terms in the Lagrangian and we neglected quantum effects. With the high-lying baryons in the multiplet, we need to include various resonances in order to evaluate the quantum effects properly, which would cause additional difficulties. Rather, it is desired to reproduce and confirm the present result using different methods. For instance, an algebraic method proposed by Weinberg is one of the useful methods to study chiral partners. This method is based on the commutation relations derived from the superconvergence property of pion-nucleon scattering amplitudes and can be applied to baryons [35–37]. We have already started a study along this line in Ref. [32].

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APPENDIX A: FIERZ TRANSFORMATION

We show the derivation of Eqs. (3). We define totally antisymmetric fields as linear combinations of Eqs. (2):

$$B_N = \boldsymbol{a}_N \cdot \boldsymbol{\phi}_N, \tag{A1a}$$

$$B_\Delta = \boldsymbol{a}_\Delta \cdot \boldsymbol{\phi}_\Delta, \tag{A1b}$$

where

$$\vec{\phi}_N = (N_V^\mu, N_A^\mu, N_T^\mu), \tag{A1c}$$

$$\vec{\phi}_\Delta = (\Delta_A^{\mu i}, \Delta_T^{\mu i}), \tag{A1d}$$

$$\vec{a}_N = (a_1^N, a_2^N, a_3^N), \tag{A1e}$$

$$\vec{a}_\Delta = (a_1^\Delta, a_2^\Delta). \tag{A1f}$$

The coefficients $\vec{a}_N$ and $\vec{a}_\Delta$ are determined by the totally antisymmetric condition, which is implemented by the antisymmetric condition under the interchange between the second and third quark and is given by

$$\mathcal{F}[B_n] = -[B_n], \qquad (n = N, \Delta), \tag{A2}$$

where $\mathcal{F}[B]$ denotes a baryon field obtained from the Fierz transformation of B. The Fierz transformation formula is given in Ref. [29]. This equation can be read as two kinds of the eigenvalue problems: (a) for the vector space $\vec{B}_{N,\Delta}$, and (b) for the vector space $\vec{a}_{N,\Delta}$. The eigenvalue problem (a) gives identities between the baryon operators

$$N_V^\mu = N_A^\mu, 2N_A^\mu = N_T^\mu, \tag{A3a}$$

$$\Delta_A^{\mu i} = -\Delta_T^{\mu i}, \tag{A3b}$$

which reduce the number of the independent fields [29,44–46]. The eigenvalue problem (b) determines the values of the coefficients $\vec{a}_N$ and $\vec{a}_\Delta$

$$\vec{a}_N = (3, 1, 1), \tag{A4a}$$

$$\vec{a}_\Delta = (-2, 1), \tag{A4b}$$

with which B_N and B_Δ are totally antisymmetric. This determines the ratio between N_V^μ and N_A^μ in N_1^μ. It is convenient to replace N_T^μ by N_V^μ and N_A^μ and $\Delta_T^{\mu i}$ by $\Delta_A^{\mu i}$ with the use of Eqs. (A3), which can be done without the change of chiral transformation properties of B_N and B_Δ. Finally, we obtain Eqs. (3).

APPENDIX B: ALTERNATIVE DERIVATION OF CHIRAL PROPERTIES

We show an alternative derivation of the chiral transformation properties of $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ and the mass relation. The starting point is a standard definition of the transformation in terms of the chiral algebra between charges and fields. In general, the $SU(2)_A$ transformation is given by $\psi' = \psi + ia^i[Q_A^i, \psi]$ with generators Q_A^i $(i = 1, 2, 3)$ and infinitesimal parameters a^i for the $SU(2)_A$ transformation. We describe $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ by the product of the isovector and isospinor $\psi^i = (\psi^i)_a$ $(a = 1, 2)$. For simplicity, we suppress the Lorentz indices in this section.

In the left- and right-handed representations, they correspond to $\psi_R^i = (1, \frac{1}{2})$ and $\psi_L^i = (\frac{1}{2}, 1)$: $\psi_R^i = (1, \frac{1}{2})$ transforms as $I = 1$ under $SU(2)_R$ and $I = \frac{1}{2}$ under $SU(2)_L$, while $\psi_L^i = (\frac{1}{2}, 1)$ transforms $I = \frac{1}{2}$ under $SU(2)_R$ and $I = 1$ under $SU(2)_L$. Note that this field ψ^i corresponds to Δ_T^i and N_T in Eq. (2). It is easy to check that N_A, N_V, and Δ_A consist of $(RL)R$, $(RL)L$, $(LR)R$, and $(LR)L$, while N_T and Δ_T contain $(RR)L$ and $(LL)R$. Jido *et al.* employed $(RR)L$ and $(LL)R$ for the description of $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ [34]. The chiral transformations of these fields are given by

$$\begin{aligned} \delta_R^a \psi_{Ri}^b &= \epsilon^{abc}(\psi_r)_i^c, & \delta_R^a \psi_{Li}^b &= it^a \psi_l^b, \\ \delta_L^a \psi_{Ri}^b &= \epsilon^{abc}(\psi_r)_i^c, & \delta_L^a \psi_{Li}^b &= it^a \psi_r^b, \end{aligned} \tag{B1}$$

where we have defined $\delta^a \psi^b = -i[Q^a, \psi^b]$. Using $Q_V^a = Q_R^a + Q_L^a$ and $Q_A^a = Q_R^a - Q_L^a$, we obtain $SU(2)_V$ and $SU(2)_A$ transformation properties

$$\delta_V^a \psi_i^b = [(\epsilon^{abc} + it^a \delta^{bc})]\psi^c, \tag{B2}$$

$$\delta_A^a \psi^b = \gamma_5(\epsilon^{abc} - it^a \delta^{bc})\psi_i^c. \tag{B3}$$

Employing an isospurion formalism, $I = \frac{1}{2}$ and $I = \frac{3}{2}$ components are obtained by $\psi_{1/2} = \tau^i \psi^i$ and $\psi_{3/2}^i = P_{3/2}^{ij}\psi^j$. After the irreducible decomposition, we obtain

$$\delta_A^a \psi_{1/2} = \tfrac{1}{2} i\gamma_5[\tfrac{5}{3}\tau^a \psi_{1/2} - 4\psi_{3/2}^a], \tag{B4a}$$

$$\delta_A^a \psi_{3/2}^b = \tfrac{1}{2} i\gamma_5[\tau^a \psi_{3/2}^b - \tfrac{2}{3}\tau^b \psi_{3/2}^a - \tfrac{4}{3}P_{3/2}^{ba}\psi_{1/2}]. \tag{B4b}$$

Here note that the coefficients differ from Eqs. (4). This is because $\psi_{1/2}$ and $\psi_{3/2}^a$ describe N_T and Δ_T^i, respectively. Using Eqs. (3) and (A3), we obtain $\psi_{1/2} = N_T = 2\sqrt{3}N_1$

and $\psi_{3/2} = \Delta_T = -2\Delta_1$. Substituting these relations into Eqs. (B4), we reproduce Eqs. (4).

Considering the $I_z = \frac{1}{2}$ components, it is easy to show that the $SU(2)_A$ transformations of the $I = \frac{1}{2}$ and $\frac{3}{2}$ fields

$$\delta_A^a \begin{pmatrix} \psi_{1/2}^{I_z=(1/2)} \\ \psi_{3/2}^{I_z=(1/2)} \end{pmatrix} = T \begin{pmatrix} \psi_{1/2}^{I_z=(1/2)} \\ \psi_{3/2}^{I_z=(1/2)} \end{pmatrix}, \qquad T = \frac{1}{2}\begin{pmatrix} \frac{5}{3} & \frac{4\sqrt{2}}{3} \\ \frac{4\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}, \tag{B5}$$

where T is the axial-transformation matrix Eq. (B4) for $I_z = \frac{1}{2}$ components. We introduce the mass matrix for $(\psi_{1/2}^{I_z=(1/2)}, \psi_{3/2}^{I_z=(1/2)})^T$ as $M = \mathrm{diag}(a, b)$ with a and b being the masses of $\psi_{1/2}$ and $\psi_{3/2}$. We also introduce the pion interaction matrix M_π for their pseudoscalar couplings. With chiral invariance, the matrices T, M, and M_π must obey

$$M = \{T, M_\pi\}, \qquad M_\pi = \{T, M\},$$

which leads to a double-commutation relation

$$M = \{T, \{T, M\}\}. \tag{B6}$$

This double-commutation relation gives $a = -2b$, which reproduces the mass relation between N_1^μ and $\Delta_1^{\mu i}$. Note that the double commutator Eq. (B6) is the necessity condition of chiral invariance.

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