Vanishing of the *CP* asymmetry in leptogenesis due to form dominance

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We emphasize that the vanishing of the *CP* asymmetry in leptogenesis, previously observed for models with tribimaximal mixing and family symmetry, may be traced to a property of the type I seesaw mechanism satisfied by such models known as form dominance, corresponding to the case of a diagonal Casas-Ibarra *R*-matrix. Form dominance leads to vanishing flavor-dependent *CP* asymmetries irrespective of whether one has tribimaximal mixing or a family symmetry. Successful leptogenesis requires violation of form dominance, but not necessarily violation of tribimaximal mixing. This may be achieved in models where the family symmetry responsible for tribimaximal mixing is implemented indirectly and a strong neutrino mass hierarchy is present with the form dominance broken only softly by the right-handed neutrino responsible for the lightest neutrino mass, as in constrained sequential dominance.

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I. INTRODUCTION

The observation of very small neutrino masses, at least a dozen orders of magnitude lower than the top quark mass, poses a challenge for building a model which accounts for the masses of elementary particles. Since the standard model (SM) of particle physics fails to provide any explanation for the neutrino masses, one is forced to look beyond. A natural explanation for such tiny neutrino masses is provided by postulating an effective five-dimensional operator [1], the only one consistent with the SM, leading to Majorana neutrino masses suppressed by a high mass scale. In the seesaw mechanism [2], such an operator is generated when a heavy particle gets integrated out from the theory, where, under the SM gauge group $SU(2)_L \times U(1)_Y$, the heavy particle can either be a singlet fermion with $Y = 0$, a triplet scalar with $Y = 2$, or a triplet fermion with $Y = 0$. The three cases are known as the type I, type II [3], or type III [4] seesaw mechanisms, respectively.

The seesaw mechanism for generating Majorana masses for the neutrinos opens up another appealing possibility. It allows creation of a lepton asymmetry in the early Universe as a result of *CP* violating out-of-equilibrium decay of the heavy seesaw mediating particle—a phenomenon called leptogenesis [5]. This lepton asymmetry can be subsequently converted to a baryon asymmetry through the $B - L$ conserving and $B + L$ violating sphaleron processes, which are important at temperatures following the epoch of leptogenesis. The seesaw mechanism therefore offers a very natural explanation for baryogenesis through leptogenesis. In this paper, we will discuss only the type I seesaw mechanism, where leptogenesis results from the decay of heavy singlet neutrinos.[1]

Existence of *CP* asymmetry in the heavy right-handed neutrino decays is a prerequisite for leptogenesis within the type I seesaw mechanism. *CP* violation might also be discovered in the upcoming and planned neutrino oscillation experiments. The seesaw mechanism that gives the low energy neutrino mass matrix is also responsible for leptogenesis. Therefore, people have attempted to connect the low energy *CP* violation with the *CP* asymmetry in leptogenesis. It is well known that in the most general framework, there is in general no connection between low and high energy *CP* violation, as there are additional complex parameters involved in the decays of the heavy right-handed neutrinos that are completely independent of the low energy neutrino parameters, as we now discuss.

Neutrino mixing data [6] are well described by the unitary Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix U parametrized by three real mixing angles, with *CP* violation due to one Dirac phase (observable in neutrino oscillations) and two Majorana phases (observable in neutrinoless double beta decay). It is clearly of interest to try to understand the connection between these "low energy" *CP* violating phases in the PMNS matrix and the "high energy" *CP* violation required by leptogenesis. However, in the most general type I seesaw scheme, even zero low energy *CP* violation, corresponding to a real PMNS matrix, does not necessarily preclude the presence of high energy *CP* asymmetry in the heavy right-handed neutrino decays required for leptogenesis. This is because of the presence of additional complex phases at the high scale which are independent of the neutrino parameters accessible to low energy experiments, as a simple parameter

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[1]Our conclusions for the vanishing of leptogenesis are also valid for the type III seesaw scenario.

counting argument shows. This is worth repeating for those unfamiliar with it.

It is well known that, in the flavor basis, the Yukawa coupling matrix of the 3 right-handed neutrinos with the 3 left-handed doublets has 15 physical parameters while the diagonal right-handed neutrino mass matrix has 3 independent real mass parameters. This results in a total of 18 free parameters at the seesaw scale. On the other hand the low energy neutrino mass matrix has only 9 physical parameters (3 neutrino masses, 1 Dirac phase, and 2 Majorana phases). There are therefore 6 additional parameters, plus 3 right-handed neutrino masses, entering physics at the seesaw scale. The most popular way to parametrize these 6 additional parameters at the high scale that is completely independent of the low scale physics is to put them in a complex orthogonal matrix, called the R-matrix [7] involving 3 complex angles. In particular, the R-matrix contains 3 phases which in general are unrelated to low energy CP violation.

It is clear from the above parameter counting that the type I seesaw mechanism introduces three additional phases. These three additional phases could in principle play a role in heavy neutrino decays, perhaps making leptogenesis possible even when there is no low energy CP violation. However, in some models, for example those with texture zeroes or two right-handed neutrinos, the number of extra phases may be reduced. Since the number of parameters or degrees of freedom is reduced at the high scale, in such models it then becomes possible to predict the extent of CP asymmetry at the seesaw scale from the low energy data. In this way one may obtain a one-to-one correspondence between the CP violation at the low and high scales, leading to a link between the PMNS phases and leptogenesis, as many authors have discussed [8].

It is a remarkable observation that global fits to neutrino oscillation data [6] are compatible with the so-called tribimaximal (TB) mixing pattern [9], where the low energy neutrino mixing matrix is given by

$$U_{\rm TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} P, \quad (1)$$

where P is an unspecified diagonal matrix containing two Majorana phases. TB neutrino mixing implies that the neutrino mass matrix has a Klein symmetry which may result either *directly* or *indirectly* from certain classes of discrete family symmetry groups [10]. Many models have been proposed based on various discrete family symmetries to account for the TB mixing [11]. Although TB mixing predicts no CP violation from the Dirac phase, there is no reason why the two Majorana phases or the three extra seesaw phases should not allow the necessary CP violation required for leptogenesis. Nevertheless, it is a curious fact that many models which predict TB mixing also lead to zero leptogenesis as has recently been observed [12–17]. In particular it has been observed that the same family symmetry which predicts TB mixing seems also to predict a vanishing CP asymmetry for leptogenesis.

In this paper we emphasize that the vanishing of the CP asymmetry in leptogenesis, previously observed for models with TB mixing arising from a family symmetry, may be traced to a property of the type I seesaw mechanism known as form dominance (FD) [18]. FD is the requirement that the columns of the Dirac mass matrix in the flavor basis are proportional to the columns of the PMNS matrix, corresponding to the simplest situation when the R-matrix is diagonal. Since the R-matrix is orthogonal, imposing the diagonal condition necessarily makes it also real with its elements being $R = \mathrm{diag}(\pm 1, \pm 1, \pm 1)$, one example of which is the unit matrix $R = I$. It has been pointed out that FD is satisfied by models such as the A_4 seesaw models [18], where tribimaximal mixing is enforced *directly* [10] by a family symmetry. However FD is more general, and leads to vanishing flavor-dependent CP asymmetries independently of the neutrino mass matrix and irrespective of whether one has tribimaximal mixing or a family symmetry.

We remark that it was already known [13] that $R = I$ implies that all the flavor-dependent CP asymmetries vanish exactly. It has also been stated that FD corresponds to $R = I$ and furthermore that A_4 seesaw models leading to TB mixing satisfy FD [18]. However here we shall be more precise and show that a diagonal R-matrix implies and is implied by FD and this is sufficient to lead to vanishing leptogenesis. Moreover the fact that this is the reason why CP asymmetries vanish in such models has apparently not been appreciated in the literature [14–17].

Another purpose of this paper is to discuss a way out of the impasse between family symmetry models of TB mixing and leptogenesis, by emphasizing that successful leptogenesis requires violation of FD, but not necessarily violation of tribimaximal mixing. This may be achieved in models where the family symmetry responsible for tribimaximal mixing is implemented *indirectly* [10] and a strong neutrino mass hierarchy is present with the FD broken only softly by the right-handed neutrino responsible for the lightest neutrino mass, as in constrained sequential dominance (CSD) [19,20]. This was already previously pointed out in [12] but, as before, this observation has been neglected.

The paper is organized as follows. We begin by briefly reviewing the type I seesaw and the R-matrix in Sec. II. We show that the R-matrix is a basis invariant quantity and hence statements made in terms of the R-matrix are true universal. In Sec. III we present the expression for the flavor-dependent and independent CP asymmetries in leptogenesis in terms of the R-matrix and show that these vanish for the case of a unit R-matrix. In Sec. IV we show that the condition that the R-matrix is a diagonal matrix implies and is implied by a Dirac mass matrix of the FD type showing in turn that all models which conform to FD necessarily predict a diagonal R-matrix and hence vanishing leptogenesis. In Sec. V we show that the condition where the Yukawa matrix is unitary (or trivial) is a subclass of models which have FD. We compare this to the situation in models with flavor symmetries and relate our results with some of the previous results in the literature. In Sec. VI we show how violations of FD can lead to successful leptogenesis in models where the family symmetry responsible for tribimaximal mixing is implemented indirectly and a strong neutrino mass hierarchy is present with the form dominance broken only softly by the right-handed neutrino responsible for the lightest neutrino mass, as in CSD. We finally conclude in Sec. VII.

II. THE *R*-MATRIX AND ITS BASIS INVARIANCE

The Yukawa part of the Lagrangian in a SM extension to include three heavy right-handed neutrinos is given by

$$-\mathcal{L}_Y = Y_e \bar{L} H l_R + Y_\nu \bar{L}\,\tilde{H}\,N_R + \frac{1}{2}\bar{N}_R^c M N_R + \text{H.c.}, \quad (2)$$

where L and H are the left-handed lepton doublet and Higgs doublet, respectively, l_R the right-handed charged singlet, and N_R the right-handed neutral singlet. Y_e and Y_ν are the Yukawa couplings and M the right-handed Majorana neutrino mass matrix. In the above equation $\tilde{H} = -i\sigma_2 H^*$. After electroweak symmetry breaking we get the Dirac mass matrix $m_D = Y_\nu v$, where v is the vacuum expectation value of the Higgs doublet. If we consider n generations of heavy right-handed neutrinos N_R, then the Dirac mass matrix m_D is a $3 \times n$ matrix and the Majorana mass matrix M is a $n \times n$ matrix. The $(3+n) \times (3+n)$ neutrino mass matrix turns out to be

$$-\mathcal{L}_m = \begin{pmatrix} \bar{\nu}_L & \bar{N}_R^c \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + \text{H.c.} \quad (3)$$

Once the n heavy right-handed neutrino fields get integrated out from the theory, one obtains the 3×3 light neutrino mass matrix, up to an irrelevant overall sign, as

$$m_\nu \simeq m_D M^{-1} m_D^T, \quad (4)$$

where we have neglected terms higher than $\mathcal{O}(M^{-2})$. The heavy neutrino mass matrix is approximately given by M. This is the celebrated type I seesaw mechanism.

The light and heavy neutrino mass matrices can be diagonalized by unitary matrices U and U_M, respectively. Hence we have the relations $U^\dagger m_\nu U^* = D_k$ and $U_M^\dagger M U_M^* = D_M$, where D_k and D_M are diagonal matrices containing the light and heavy neutrino mass eigenvalues. In the basis where Y_e is diagonal we identify U as the PMNS matrix. From above, one obtains

$$U^\dagger m_D M^{-1} m_D^T U^* = D_k. \quad (5)$$

Substituting $U_M^\dagger M U_M^* = D_M$ in the above equation we get

$$U^\dagger m_D U_M^* D_M^{-1} U_M^\dagger m_D^T U^* = D_k. \quad (6)$$

The R-matrix is defined as [7]

$$R = D_{\sqrt{M}}^{-1} U_M^\dagger m_D^T U^* D_{\sqrt{k}}^{-1}, \quad (7)$$

where R is clearly a complex orthogonal matrix $R^T R = I$. Equation (7) parametrizes the freedom in the Dirac matrix m_D, for fixed values of U, D_k, and D_M, in terms of a complex orthogonal matrix R.

Following the discussion in [13], we show that the R-matrix is invariant under any kind of basis transformation of the heavy Majorana neutrinos as well as the well-known invariance under charged lepton basis transformations [7,13]. This is realized by the fact that $U_M^\dagger m_D^T$ and U are invariant under the heavy Majorana basis transformation. To show this explicitly, let us consider two bases (m_D, M) and $(\hat{m}_D, \hat{M})$ which are related by a unitary basis transformation of the heavy Majorana neutrinos as

$$\hat{M} = S^T M S, \quad (8)$$

and

$$\hat{m}_D = m_D S. \quad (9)$$

The matrix S is unitary and hence satisfies the relation $S^\dagger S = I$. In the old nonhatted basis, the diagonalizing relation for the heavy Majorana mass matrix is

$$U_M^\dagger M U_M^* = D_M. \quad (10)$$

Plugging $\hat{M} = S^T M S$ back into the above equation one will get

$$U_M^\dagger (S^T)^{-1} \hat{M} S^{-1} U_M^* = D_M. \quad (11)$$

For S to be a unitary matrix this above equation represents the diagonalizing relation in the new hatted basis. Hence one can define the new eigenvectors as $\hat{U}_M^* = S^\dagger U_M^*$. So the $U_M^\dagger m_D^T$ transforms as

$$U_M^\dagger m_D^T = \hat{U}_M^\dagger S^T S^* \hat{m}_D^T = \hat{U}_M^\dagger \hat{m}_D^T. \quad (12)$$

Using Eqs. (8) and (9) one can very easily prove that the low energy neutrino mass matrix is also invariant under this unitary basis transformation,

$$M_\nu = m_D M^{-1} m_D^T = \hat{m}_D \hat{M}^{-1} \hat{m}_D^T. \quad (13)$$

Hence the low energy neutrino mixing matrix U will remain unaffected under this heavy Majorana neutrino basis transformation. We have already defined the R-matrix in Eq. (7). Because the heavy Majorana masses D_M and the low energy neutrino masses D_k are physical observables and are basis independent and also the neutrino mixing matrix U is basis independent, hence the statement "$U_M^\dagger m_D^T$ is invariant under heavy Majorana basis transformation" is sufficient to prove that the R-matrix is invariant under the heavy Majorana basis transformation.

Similarly one can also prove that the quantity $m_D^T U^*$ is invariant under the leptonic basis transformation. In this case for $\bar{L} \to \bar{L}W$, we get $\hat{m}_D = W m_D$ and the neutrino mixing matrix would change to $\hat{U}^* = W^* U^*$. Hence $m_D^T U^* = \hat{m}_D^T \hat{U}^*$. Therefore, here also the R-matrix would be invariant. Hence the R-matrix is invariant under any kind of basis transformation [2] as well as nonunitary transformation of the heavy Majorana fields [13].

III. *CP* ASYMMETRY IN LEPTOGENESIS AND *R*-MATRIX

As discussed before, *CP* asymmetric out-of-equilibrium heavy singlet Majorana neutrino decay could lead to leptogenesis. The *CP* asymmetry generated by N_i decays into a lepton doublet L (written as l_α with a flavor index $\alpha = e, \mu, \tau$) and a Higgs doublet H (written as ϕ) is given by [5,22]

$$\varepsilon_i^\alpha = \frac{\Gamma(N_i \to \phi \bar{l}_\alpha) - \Gamma(N_i \to \phi^\dagger l_\alpha)}{\sum_\beta [\Gamma(N_i \to \phi \bar{l}_\beta) + \Gamma(N_i \to \phi^\dagger l_\beta)]} = \frac{1}{8\pi v^2} \frac{1}{(m_D^\dagger m_D)_{ii}} \sum_{j \neq i} \left(\mathcal{I}_{ij}^\alpha f(M_j^2/M_i^2) + \mathcal{J}_{ij}^\alpha \frac{1}{1 - M_j^2/M_i^2} \right), \tag{14}$$

where we have written

$$\mathcal{I}_{ij}^\alpha = \mathrm{Im}[(m_D^\dagger)_{i\alpha}(m_D)_{\alpha j}(m_D^\dagger m_D)_{ij}], \quad \mathcal{J}_{ij}^\alpha = \mathrm{Im}[(m_D^\dagger)_{i\alpha}(m_D)_{\alpha j}(m_D^\dagger m_D)_{ji}]. \tag{15}$$

It is evident that $\mathcal{I}_{ij}^\alpha = -\mathcal{I}_{ji}^\alpha$ and $\mathcal{J}_{ij}^\alpha = -\mathcal{J}_{ji}^\alpha$. In the minimal supersymmetric standard model, the function $f(x)$ has the form [23]

$$f(x) = \sqrt{x}\left[\frac{2}{1-x} - \ln\left(\frac{1+x}{x}\right)\right]. \tag{16}$$

In the above equations we have considered the *CP* asymmetry generated in each flavor and thus have taken into account the so-called flavor effects in leptogenesis [24–26]. We have also considered the decay asymmetry created from the decay of all the three right-handed neutrinos, including the case of N_2 dominated leptogenesis [27].

In many cases only the term proportional to $\mathcal{I}_{ij}^\alpha$ in Eq. (14) is relevant, since the second term proportional to $\mathcal{J}_{ij}^\alpha$ is often suppressed by ratios of right-handed neutrino masses M_i/M_j. Furthermore, the second term in Eq. (14) vanishes when one sums over flavors to obtain the flavor-independent decay asymmetry:

$$\varepsilon_i = \sum_\alpha \varepsilon_i^\alpha \equiv \frac{\sum_\alpha [\Gamma(N_i \to \phi \bar{l}_\alpha) - \Gamma(N_i \to \phi^\dagger l_\alpha)]}{\sum_\beta [\Gamma(N_i \to \phi \bar{l}_\beta) + \Gamma(N_i \to \phi^\dagger l_\beta)]}, = \frac{1}{8\pi v^2} \frac{1}{(m_D^\dagger m_D)_{ii}} \sum_{j \neq i} \mathrm{Im}[(m_D^\dagger m_D)_{ij}^2] f(M_j^2/M_i^2). \tag{17}$$

Note that the flavor-independent *CP* asymmetry given by Eq. (17) depends on the imaginary part of the combination $(m_D^\dagger m_D)_{ij}$, where $i \neq j$.

Since ε_i^α depend on the Dirac mass matrix, we can express them also in terms of the R-matrix as

$$\varepsilon_i^\alpha = -\frac{3M_i}{16\pi v^2} \frac{\mathrm{Im}[\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{\alpha j}^* U_{\alpha k} R_{ij}^* R_{ik}^*]}{\sum_j m_j |R_{ij}|^2}. \tag{18}$$

For the case where flavor effects are inconsequential, the corresponding *CP* asymmetry is given by summing over the flavors as

$$\varepsilon_i = -\frac{3M_i}{16\pi v^2} \frac{\mathrm{Im}[\sum_j m_j^2 (R_{ij}^*)^2]}{\sum_j m_j |R_{ij}|^2}, \tag{19}$$

where m_j are the eigenvalues of the light neutrino mass matrix and we have assumed hierarchical masses for the right-handed neutrinos. It is interesting to compare the flavor-dependent asymmetry in Eq. (18) to the flavor-independent case in Eq. (19). Equation (19) shows that the flavor-independent *CP* asymmetry is directly proportional to the imaginary components of the R-matrix. Therefore, for models where the R-matrix is real, the flavor-independent *CP* asymmetry becomes identically zero and one has no leptogenesis. By contrast, from Eq. (18) it is clear that a real R-matrix allows the flavor-dependent asymmetries to be nonzero [25,26] due to the PMNS phases, allowing a link between low energy *CP* violation and leptogenesis for the case of a real R-matrix [28]. Such a link was first observed for flavor-dependent leptogenesis, independently of the R-matrix parametrization, in [12].

In [13] it was pointed out that a real R-matrix is an automatic consequence of CSD since in this case R is equal to the unit matrix. It was also pointed out [13] that $R = I$ implies the stronger result that flavor-dependent *CP* asymmetries in Eq. (18) vanish identically due to the unitarity of U. We point out that, since the R-matrix enters Eq. (18) quadratically, a diagonal R-matrix with diagonal elements being ± 1 is sufficient to lead to vanishing flavor-dependent *CP* asymmetries. The same conclusion applies to both $\mathcal{I}_{ij}^\alpha$ and $\mathcal{J}_{ij}^\alpha$ even though for simplicity we have only considered the terms arising from $\mathcal{I}_{ij}^\alpha$ above. (In Sec. VI we

[2] A simple physical reason why the R-matrix has to be basis invariant can be understood from the fact that the R-matrix encodes the three right-handed neutrino decay rates as well as the three leptogenesis *CP* asymmetry observables. Therefore since the R-matrix is fixed by six physical observables it must be basis invariant .

consider both terms.) In the next section we shall show that a diagonal R-matrix with diagonal elements being ± 1 corresponds to FD [18]. Since models of TB mixing enforced directly by a family symmetry satisfy FD, this is the reason why the leptonic flavor-dependent *CP* asymmetries vanish for these models.

IV. DIAGONAL *R*-MATRIX AND FORM DOMINANCE

In this section we first review the argument that $R = I$ implies and is implied by FD [18] and then extend it to the case of a diagonal R-matrix with diagonal elements being ± 1.

Let us first consider the case that the R-matrix is the unit matrix, i.e.,

$$R = I. \tag{20}$$

From Eq. (7) this implies that

$$D^{-1}_{\sqrt{M}} m_D^T U^* D^{-1}_{\sqrt{k}} = I, \tag{21}$$

where for simplicity we choose to work in the basis where the heavy Majorana mass matrix is real and diagonal. However, since the R-matrix is basis invariant, the physical results are basis invariant. Equation (21) yields the condition on the Dirac mass matrix as

$$m_D = U.(D_{\sqrt{k}} D_{\sqrt{M}}) = U.D, \tag{22}$$

where D is a diagonal matrix of the form $D = \mathrm{diag}(a_1, a_2, a_3)$. They are given as $a_1 = \sqrt{m_1}\sqrt{M_1}$, $a_2 = \sqrt{m_2}\sqrt{M_2}$, and $a_3 = \sqrt{m_3}\sqrt{M_3}$, where m_i and M_i ($i = 1, 2, 3$) are the eigenvalues of the light and heavy Majorana neutrinos, respectively. We work in the convention where all mass eigenvalues are taken as real and positive. Therefore, the parameters a_1, a_2, a_3 are real. Since we are working in a basis where the right-handed Majorana mass is $M = \mathrm{diag}(M_1, M_2, M_3)$, inserting the FD $m_D = U.D$ in the seesaw formula yields

$$m_\nu = m_D M^{-1} m_D^T = U.D_k.U^T, \tag{23}$$

which serves as a consistency check. Also, it is trivial to see that the diagonal matrix containing the light neutrino mass eigenvalues is given by

$$D_k = \mathrm{diag}\left(\frac{a_1^2}{M_1}, \frac{a_2^2}{M_2}, \frac{a_3^2}{M_3}\right), \tag{24}$$

which is obviously consistent with $D = \mathrm{diag}(a_1, a_2, a_3) = D_{\sqrt{k}} D_{\sqrt{M}}$.

The above discussion shows that any model which produces a Dirac mass matrix that is of the form given by Eq. (22) will give $R = I$ and hence zero leptogenesis. In fact, this form for the Dirac mass matrix has been discussed in detail before in the literature and has been called FD [18]. Hence we confirm that $R = I$ implies FD [18].

Let us now assume that R is diagonal, i.e., $R = R_d$ with diagonal elements being ± 1,

$$R = R_d. \tag{25}$$

Assuming $R = R_d$ gives

$$m_D = U.D_{\sqrt{k}}.R_d.D_{\sqrt{M}} = U.D', \tag{26}$$

where D' is a real and diagonal matrix $D' = \mathrm{diag}(\pm\sqrt{m_1}\sqrt{M_1}, \pm\sqrt{m_2}\sqrt{M_2}, \pm\sqrt{m_3}\sqrt{M_3})$. Since FD [18] is a criterion whereby the columns of the Dirac matrix m_D are proportional to the respective columns of the neutrino mixing matrix while working in a basis where the charged lepton and heavy Majorana mass matrix are diagonal, it is clear that Eq. (26) implies FD. Therefore the condition that R is a diagonal matrix $R = R_d$ with diagonal elements being ± 1 leads to FD, wherein $m_D = U.D'$, where D' is a real diagonal matrix. We can turn the argument around to state that, for any real diagonal matrix D', FD leads to R that is real and diagonal. Hence, FD necessarily predicts zero *CP* asymmetry for leptogenesis.

Finally, we stress that the condition that the R-matrix is diagonal is independent of the low energy neutrino parameters. This is because it only demands that the Dirac mass matrix should obey FD. In particular it does not restrict the PMNS mixing matrix U, which could have any form.

V. FORM DOMINANCE AND UNITARITY OF m_D

It was shown in [15] that if the right-handed neutrinos belong to the irreducible representation of a family symmetry group G_F, then one gets

$$m_D^\dagger m_D \propto I \tag{27}$$

from the invariance of the Lagrangian under G_F. Hence the Dirac mass matrix in these flavor models is predicted to be unitary. Unitary m_D occurs in several of the models with A_4 and S_4 flavor symmetry in [11]. In this section we show that the case of unitary Dirac matrices corresponds to an interesting subclass of FD cases. Since a unitary m_D is only a subclass of the class of models which conform to FD, one concludes that the set of flavor models which give unitary m_D, and hence vanishing leptogenesis, is only a subclass of a more general class of models with vanishing leptogenesis characterized by $R = \mathrm{diag}(\pm 1, \pm 1, \pm 1)$.

In the approach here, the unitary Dirac matrices emerge from the condition for FD in Eq. (26), generalized to any arbitrary right-handed neutrino mass basis, for any real diagonal matrix D,

$$m_D = U.D.U_M^T, \tag{28}$$

which leads to the FD condition

$$m_D m_D^\dagger = U.D^2.U^\dagger. \tag{29}$$

From this equation it is clear that if $D^2 = I$ then m_D is unitary,

$$m_D m_D^\dagger = I, \tag{30}$$

which is satisfied trivially by the special case $m_D = I$.[3] Conversely, if m_D is unitary then this implies FD, since one can always go to a basis where a general unitary m_D can be expressed as $m_D = U.D.U_M^T$.

Note that, for the special case $m_D = I$, the generalized FD condition $m_D = U.D.U_M^T$, implies that

$$U = U_M^* D^{-1}. \tag{31}$$

However, for $m_D = I$ the seesaw formula gives

$$m_\nu = M^{-1}, \tag{32}$$

and hence one gets

$$U = U_M^*. \tag{33}$$

Therefore for the case where $m_D = I$, one has $D = I$.

The main results in this paper so far can then be summarized as follows:

(1) FD implies and is implied by a diagonal R-matrix $R = R_d$ with diagonal elements being ± 1.

(2) FD may be expressed as the generalized condition in Eq. (28) where D is a real and diagonal matrix, and U and U_M are the matrices which diagonalize the low and heavy neutrino mass matrices, respectively, which can be arbitrary.

(3) Models which have unitary Dirac mass matrix are a subclass of FD, corresponding to the real diagonal matrix D having elements ± 1.

(4) A special case of unitary m_D is the case where the Dirac Yukawa matrix is proportional to I, where $m_D = I$ implies $D = I$.

(5) Models which respect FD with $R = R_d$ have vanishing flavor-dependent CP asymmetries for leptogenesis. A subclass of such models has a unitary or unit Dirac mass matrix.

VI. VIOLATIONS OF FORM DOMINANCE

In order to explore violations of FD, we shall introduce a more explicit notation for the Dirac neutrino mass matrix m_D, the right-handed neutrino mass matrix M, the type I seesaw effective light Majorana mass matrix m_ν in Eq. (4), and the PMNS matrix U, as well as the R-matrix. We shall write the (not necessarily TB) PMNS matrix U in terms of three column vectors Φ_i:

$$U = (\Phi_1, \Phi_2, \Phi_3), \tag{34}$$

where the complex Φ_i include the respective Majorana phase associated with that particular column of U as well as the Dirac phase in U. The columns of U obey the unitarity relations

$$\Phi_i^\dagger \Phi_j = \delta_{ij}. \tag{35}$$

According to FD, in the diagonal right-handed neutrino mass matrix basis M,

$$M = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}, \tag{36}$$

the columns of the Dirac neutrino mass matrix m_D (in the left-right convention for m_D) are proportional to columns of the PMNS matrix,

$$m_D = (a_1\Phi_1, a_2\Phi_2, a_3\Phi_3), \tag{37}$$

where a_i are the real parameters introduced previously. Then the type I seesaw mechanism implies

$$m_\nu \simeq m_D M^{-1} m_D^T = m_1 \Phi_1 \Phi_1^T + m_2 \Phi_2 \Phi_2^T + m_3 \Phi_3 \Phi_3^T, \tag{38}$$

where $m_i = a_i^2/M_i$. Using this notation, it is clear that the effective light Majorana neutrino mass matrix m_ν is diagonalized by the PMNS matrix,

$$U^\dagger m_\nu U^* = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \tag{39}$$

using $U = (\Phi_1, \Phi_2, \Phi_3)$ with Eq. (38) and the unitarity relations in Eq. (35). This notation makes the essential feature of FD, that the PMNS matrix U is unrelated to the seesaw parameters which determine the neutrino masses m_i, completely manifest, since U is given by Φ_i and the neutrino masses are given by the combinations $m_i = a_i^2/M_i$, with Φ_i independent of a_i, M_i.

From Eq. (7), the R-matrix may be defined as the matrix which parametrizes m_D in the basis where M and Y_e are diagonal as

$$m_D D_{\frac{-1}{\sqrt{M}}} = U D_{\sqrt{k}} R^T. \tag{40}$$

It is instructive to expand this equation in terms of the columns of m_D and U,

$$\begin{aligned} &((m_D)_{i1} M_1^{-1/2}, (m_D)_{i2} M_2^{-1/2}, (m_D)_{i3} M_3^{-1/2}) \\ &\quad = (U_{i1} m_1^{1/2}, U_{i2} m_2^{1/2}, U_{i3} m_3^{1/2}) R^T. \end{aligned} \tag{41}$$

Since $m_D = (a_1\Phi_1, a_2\Phi_2, a_3\Phi_3)$ and $U = (\Phi_1, \Phi_2, \Phi_3)$, it is apparent that R is equal to the unit matrix in the case of FD with $m_i^{1/2} = a_i M_i^{-1/2}$. Note that, in our convention, assuming FD, we have assumed that $m_i = a_i^2/M_i$. In other words we have defined M_1 to be the mass of the right-handed neutrino which is responsible for the physical neutrino mass m_1, M_2 to be the mass of the right-handed neutrino which is responsible for the physical neutrino mass m_2, and M_3 to be the mass of the right-handed neutrino which is responsible for the light neutrino mass

[3] Similarly if $D \propto I$ then $m_D m_D^\dagger \propto I$, which is satisfied trivially by the special case $m_D \propto I$.

m_3. This differs from the usual convention where $M_1 < M_2 < M_3$ and the right-handed neutrinos are ordered such that the lightest one with mass M_1 appears in the first column, the second lightest with mass M_2 appears in the second column, and the heaviest with mass M_3 appears in the third column of the matrices m_D and M. While, in the usual convention, there is an ambiguity in the R-matrix due to the reordering of the right-handed neutrino masses, in our convention there is no such ambiguity, and the R-matrix for FD is thus equal to the unit matrix, with no reordering ambiguity. In our convention the mass ordering of the right-handed neutrino masses M_i remains general and for us it is *not* generally true that $M_1 < M_2 < M_3$ (although this possibility is not excluded) and other mass orderings such as $M_3 < M_2 < M_1$ are permitted. Similarly the mass orderings of the physical neutrino masses is also left general with $m_1 < m_2 < m_3$ being the normal mass ordering and $m_3 < m_1 < m_2$ being the inverted one (all mass eigenvalues taken to be positive).

Adopting the above FD conventions, summarized by $m_i = a_i^2/M_i$, where the i-th right-handed neutrino mass is associated with i-th physical neutrino mass, we now consider the CP asymmetry parameters associated with the decay of such an i-th right-handed neutrino. We emphasize that the i-th right-handed neutrino could be the lightest, second lightest, or heaviest right-handed neutrino (e.g., $i = 3$ could be the lightest right-handed neutrino in our convention). We shall write Eq. (15) as follows:

$$\begin{aligned} \mathcal{I}_{ij}^{\alpha} &= \mathrm{Im}[(m_D^\dagger)_{i\alpha}(m_D^T)_{j\alpha}(m_D^\dagger m_D)_{ij}], \\ \mathcal{J}_{ij}^{\alpha} &= \mathrm{Im}[(m_D^\dagger)_{i\alpha}(m_D^T)_{j\alpha}(m_D^\dagger m_D)_{ji}]. \end{aligned} \tag{42}$$

In the case of FD we may use Eq. (37) to we express Eq. (15) as

$$\begin{aligned} \mathcal{I}_{ij}^{\alpha} &= \mathrm{Im}[a_i^2 a_j^2 \Phi_{i\alpha}^* \Phi_{j\alpha}(\Phi_i^\dagger \Phi_j)], \\ \mathcal{J}_{ij}^{\alpha} &= \mathrm{Im}[a_i^2 a_j^2 \Phi_{i\alpha}^* \Phi_{j\alpha}(\Phi_j^\dagger \Phi_i)]. \end{aligned} \tag{43}$$

From Eq. (43), which assumes FD, it is clear that both flavor-dependent leptonic CP asymmetry parameters $\mathcal{I}_{ij}^{\alpha}$ and $\mathcal{J}_{ij}^{\alpha}$ vanish exactly due to the unitarity condition in Eq. (35). The vanishing of $\mathcal{I}_{ij}^{\alpha}$ and $\mathcal{J}_{ij}^{\alpha}$ for all values of i, j, α means that all types of leptogenesis vanish, including flavor (α) dependent leptogenesis and so-called N_1 and N_2 leptogenesis arising from the lightest and second lightest right-handed neutrino, including thermal and nonthermal leptogenesis—all these types of leptogenesis vanish identically as a result of FD. It is clear that this vanishing of CP asymmetry in leptogenesis arises from FD in a very simple way, independently of the PMNS matrix, and hence the vanishing is not directly related to TB mixing or family symmetry. However, as discussed in [18], many models that describe TB mixing via family symmetry do satisfy FD, and that is the reason for vanishing CP asymmetry in these cases.

We have seen that exact FD leads to exactly zero leptogenesis. Therefore in order to achieve successful leptogenesis we must consider violations of FD. In the remainder of this section we show how FD may be violated softly, without perturbing the PMNS matrix U, in the case of a hierarchical neutrino mass spectrum in the limit that the lightest physical neutrino mass $m_1 \to 0$. In this limit, assuming FD, the neutrino masses and mixing parameters are insensitive to the coefficient a_1 of the first column of the Dirac mass matrix $a_1\Phi_1$ and the first right-handed neutrino mass eigenvalue M_1 since they are responsible for $m_1 = a_1^2/M_1$ and by assumption m_1 is negligible. Moreover, in this limit, we can replace the first column of the Dirac mass matrix $a_1\Phi_1$ by any other column vector

$$a_1\Phi_1 \to a_1\tilde{\Phi}_1, \tag{44}$$

so that the Dirac neutrino mass matrix becomes

$$\tilde{m}_D = (a_1\tilde{\Phi}_1, a_2\Phi_2, a_3\Phi_3), \tag{45}$$

leaving the PMNS matrix approximately unchanged,

$$U \approx (\Phi_1, \Phi_2, \Phi_3). \tag{46}$$

We call this a soft violation of FD since Eq. (46) becomes exact in the limit that $m_1 \to 0$. We emphasize again that, in our convention, M_1 need not be the lightest right-handed neutrino mass eigenvalue, even though in this example m_1 is the lightest physical neutrino mass eigenvalue. Making the replacement in Eq. (44) it is clear that we will now obtain nonzero CP asymmetries for $\mathcal{I}_{ij}^{\alpha}$ and $\mathcal{J}_{ij}^{\alpha}$ with either $i = 1$ or $j = 1$.

If $i = 1$, then

$$\begin{aligned} \mathcal{I}_{1j}^{\alpha} &= \mathrm{Im}[a_1^2 a_j^2 \tilde{\Phi}_{1\alpha}^* \Phi_{j\alpha}(\tilde{\Phi}_1^\dagger \Phi_j)], \\ \mathcal{J}_{1j}^{\alpha} &= \mathrm{Im}[a_1^2 a_j^2 \tilde{\Phi}_{1\alpha}^* \Phi_{j\alpha}(\Phi_j^\dagger \Phi_1)]. \end{aligned} \tag{47}$$

If $j = 1$, then

$$\begin{aligned} \mathcal{I}_{i1}^{\alpha} &= \mathrm{Im}[a_i^2 a_1^2 \Phi_{i\alpha}^* \tilde{\Phi}_{1\alpha}(\Phi_i^\dagger \tilde{\Phi}_1)], \\ \mathcal{J}_{i1}^{\alpha} &= \mathrm{Im}[a_i^2 a_1^2 \Phi_{i\alpha}^* \tilde{\Phi}_{1\alpha}(\tilde{\Phi}_1^\dagger \Phi_i)]. \end{aligned} \tag{48}$$

It is clear that $\mathcal{I}_{ij}^{\alpha}$ and $\mathcal{J}_{ij}^{\alpha}$ with either $i = 1$ or $j = 1$ are nonzero since in general both $\Phi_i^\dagger \tilde{\Phi}_1 \neq 0$ and $\tilde{\Phi}_1^\dagger \Phi_i \neq 0$. An example of such a soft violation of FD is provided by constrained sequential dominance [19] which just corresponds to FD for the case of a strong neutrino mass hierarchy $m_1 \to 0$ together with the assumption of TB mixing U_{TB}. CSD is in turn a special case of sequential dominance which corresponds to the case of a general PMNS matrix U [20]. In [12] it was first pointed out that (flavor-dependent) CP asymmetries vanish in the limit $m_1 \to 0$ for the case of CSD and TB mixing where leptogenesis is dominated by the CP asymmetry of the lightest right-handed neutrino which is associated with either the m_2 or m_3 due to $\Phi_2^\dagger \Phi_3 = \Phi_3^\dagger \Phi_2 = 0$. Furthermore it was realized [12] that, under the similar assumptions, but with the lightest right-handed neutrino being associated with m_1, then, since $\Phi_i^\dagger \tilde{\Phi}_1 \neq 0$ and $\tilde{\Phi}_1^\dagger \Phi_i \neq 0$, the CP asymmetries would no longer be zero for small $m_1 \sim 10^{-3}$ eV but could in fact be rather large or optimal.

A full numerical estimate of leptogenesis for this case with M_1 being the lightest right-handed neutrino was performed [12] where it was shown that realistic values of baryon asymmetry could result for $m_1 \sim 10^{-3}$ eV and with approximate TB mixing arising from the dominant and subdominant right-handed neutrinos of mass M_3 and M_2. In that analysis a zero initial abundance of right-handed neutrinos was assumed with $m_1 \sim 10^{-3}$ eV leading to optimal washout. If instead a thermal initial abundance of right-handed neutrinos were assumed then m_1 could become arbitrarily small with zero washout in the soft FD limit $m_1 \to 0$ where TB mixing becomes exact, which is the limit considered here. This example shows that the vanishing of the CP asymmetry in leptogenesis has nothing to do with TB mixing but instead is a consequence of FD.

We remark that the conditions required for TB mixing suggest the presence of a family symmetry. However, as previously observed, the family symmetry may lead to TB mixing in two ways, either directly or indirectly [10]. In the direct implementation of the family symmetry, where some of the generators of the family symmetry are preserved as symmetries of the TB neutrino mass matrix, it is rather unnatural to achieve a strong neutrino mass hierarchy. On the other hand, if the family symmetry is achieved in an indirect way, with the family symmetry being responsible for the alignments along the directions of the TB mixing matrix columns Φ_2 and Φ_3, then a strong hierarchy with $m_1 \to 0$ is completely natural [10]. In [18] this was called natural FD, but really it is just an example of CSD. The presence of a strong neutrino mass hierarchy, together with TB mixing resulting from a family symmetry, can therefore lead to successful leptogenesis if the family symmetry is implemented in the indirect way as in CSD.

VII. CONCLUSIONS

In this paper we have emphasized that the vanishing of the CP asymmetry in leptogenesis, previously observed for models with tribimaximal mixing and family symmetries such as A_4 or S_4 may be traced to a property of the type I seesaw mechanism satisfied by such models known as FD, corresponding to the case of an R-matrix characterized by $R = \mathrm{diag}(\pm 1, \pm 1, \pm 1)$. FD with such a diagonal R-matrix leads to vanishing flavor-dependent CP asymmetries irrespective of whether one has tribimaximal mixing or a family symmetry. In particular, one could have a non-TB mixing matrix at the low scale and yet have vanishing leptogenesis, if the Dirac mass matrix conforms to FD. On the other hand one may have exact TB mixing and nonvanishing leptogenesis if FD is violated. The only significance of the family symmetry seems to be that it can give rise to models with FD.

The other main results of the paper are summarized in Sec. V. Many models where the right-handed neutrinos are in an irreducible representation of the flavor group have been observed to give a Dirac mass matrix which is unitary. We have shown that such cases are a subclass of FD models having both a diagonal R-matrix and a diagonal D-matrix with elements ± 1, where the D-matrix is the one appearing in Eq. (28). A special case is where the Dirac matrix is proportional to the unit matrix. Clearly FD is again responsible for the vanishing leptogenesis in all these cases.

Finally we showed that successful leptogenesis requires violation of FD, but not necessarily violation of TB mixing. Violation of FD but not TB mixing can be achieved in models based on constrained sequential dominance where a strong neutrino mass hierarchy is present. In this case the FD is violated only softly by the right-handed neutrino responsible for the lightest neutrino mass. This seems to be possible in models where the neutrino flavor symmetry responsible for TB mixing emerges from the family symmetry *indirectly* rather than *directly.*

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