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No-go theorem prohibiting inflation in the entropic force scenario

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We show that to accommodate inflation in the entropic force scenario of Verlinde, it is necessary to introduce a negative temperature on a holographic screen. This will introduce several puzzles such as energy nonconservation. If one tries to modify the derivation of the Einstein equations to avoid a negative temperature, we prove that it is impossible to find a proper new definition of temperature to derive the Einstein equations.

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Verlinde recently proposed that gravity is actually a thermodynamic phenomenon emerging from the holographic principle [1], based on an earlier observation of Jacobson [2] (some other speculations on emergent gravity can be found in [3]). One of the important applications of the entropic force scenario is to cosmology [4], and, in particular, to the dark energy problem [5] if the new paradigm has anything new to say about gravity. For other studies following Verlinde, we refer to [6,7] for an incomplete list.

One of the authors of the present paper and Wang observed in [5] that in order to explain the current acceleration of the universe, it is necessary to introduce a global holographic screen in addition to those studied by Verlinde. One then naturally wonders whether this is also the case for inflation. In this paper we show that it is impossible to accommodate inflation with a single holographic screen for inflation.

First, we present the derivation of the Einstein equations, following Verlinde [1]. Choosing a local timelike Killing vector ξ^a and defining the generalized Newtonian potential $\phi = 1/2\ln(-\xi^a\xi_a)$, the temperature on a surface S is

$$T = \frac{\hbar}{2\pi} e^{\phi} N^a \partial_a \phi, \tag{1}$$

where N^a is a unit vector normal to S as well as to ξ^a. Assuming the equipartition theorem, the total mass on the holographic screen is

$$M = \frac{1}{2}\int T dN = \frac{1}{4\pi G}\int e^{\phi}\nabla\phi\cdot dS. \tag{2}$$

Before proceeding we pause to note that the physics principle requires that the temperature defined in (1) must be positive.

Utilizing the definition of Newtonian potential and applying the Stokes theorem, we arrive at

$$M = \frac{1}{4\pi G}\int_{\Sigma} R_{ab} n^a \xi^b dV, \tag{3}$$

where Σ is the volume enclosed by S, and n^a is the unit future vector normal to Σ. To derive the Einstein equations, we have to assume that the mass measured against the Killing vector ξ^a is the one given by the so-called Tolman-Komar mass, thus

$$M = \frac{1}{4\pi G}\int_{\Sigma} R_{ab} n^a \xi^b dV = 2\int_{\Sigma}\left(T_{ab} - \frac{1}{2}g_{ab}T\right)n^a\xi^b dV. \tag{4}$$

Choosing an arbitrary Killing vector as well as an arbitrary Σ, we deduce the Einstein equations $R_{ab} = 8\pi G(T_{ab} - \frac{1}{2}g_{ab}T)$ or $R_{ab} - \frac{1}{2}g_{ab}R = 8\pi G T_{ab}$.

Now, let us examine some details of the above derivation when it comes to inflation. For simplicity, let us consider the case that inflation as well as quantum fluctuations against the background of inflation can be described by the form of an ideal fluid; the stress tensor is

$$T_{ab} = (\rho + p)u_a u_b + p g_{ab}. \tag{5}$$

The Tolman-Komar stress tensor reads

$$T_{ab} - \frac{1}{2}g_{ab}T = (\rho + p)u_a u_b + \frac{1}{2}(\rho - p)g_{ab}. \tag{6}$$

Since Σ is arbitrary, one particular choice is the one comoving with the fluid in which $u\cdot n = -1$, and one particular choice of the local Killing vector also satisfies $u\cdot\xi = -1$. We have in this case $2(T_{ab} - \frac{1}{2}g_{ab}T)n^a\xi^b = \rho + 3p$. We know that for the universe to be inflating this quantity is negative; namely, the Tolman-Komar mass is negative.

Thus, we are facing the problem that when the holographic screen is comoving with the inflation fluid, the energy is negative; thus either the temperature is negative or the number of bits is negative, and the latter is unattainable so we have to assume the temperature is negative. A negative temperature is not unfamiliar; for instance, some 50 years ago physicists had to consider this possibility in

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nuclear physics (we thank Wang for pointing this out to us). There, one simply takes a system of finite energy levels, then the Boltzmann partition function is well defined for a negative temperature. However, when we introduce such a system into the holographic screen, we encounter the following problems:

(i) We need to consider the holographic screen to consist of two systems, one with a negative temperature and another with a positive temperature, since when inflation ends radiation and matter starts to dominate.

(ii) A test particle outside of the holographic screen must experience a repulsive force; according to the formula $F = T\frac{\Delta S}{\Delta x}$, entropy increases when the test particle approaches the screen. Even in a system with negative temperature, entropy increases with an increase of the number of bits. Or, put another way, after the test particle crosses into the screen, we need more bits to describe the system. However, according to the equipartition theorem, the energy of this test particle on the screen is $m = \frac{1}{2}Tn$, where n is the number of bits describing the test particle. Thus, m is negative and energy is not conserved.

(iii) There is also a contradiction between the increase of entropy when the test particle crosses into the screen and the usual wisdom that after more matter is dumped into a horizon of an accelerating universe, the area of the horizon actually decreases.

All of the above puzzles can be resolved at once if, rather than choosing an arbitrary holographic screen to describe inflation, we instead use a global screen as in [5] to describe acceleration of the universe. However, when it comes to inflation we need to account for the local fluctuations of the scalar fields, thus a local screen appears to be necessary. It remains to be seen whether one can use a global screen to describe background inflation and local screens to describe local fluctuations.

Another way to circumvent the above problem of negative temperature is to modify the derivation of the Einstein equations. Since a negative temperature is caused by a negative Tolman-Komar mass, we may try to replace the Tolman-Komar mass by another stress tensor

$$a(T_{ab} - bg_{ab}T), \tag{7}$$

so as long as $(1-b)\rho + 3bp \geq 0$, we will not have the negative temperature problem. For instance, for the extremal case $p = -\rho$, we need $4b \leq 1$. One choice is $b = \frac{1}{4}$, and this choice requires the curvature part in (4) to be replaced by $R_{ab} - \frac{1}{4}g_{ab}R$. Interestingly, we will not obtain the Einstein equations, but rather new equations containing the traceless part of the Einstein equations.

No matter how we modify the stress tensor, we shall start with the equipartition theorem and use a modified definition of temperature as well as the proposition that the number of bits proportional to the area of the screen. Our aim is to derive an integral over the volume Σ enclosed by screen S, and the integrand must contain a term proportional to the scalar curvature (as required by the modification of the stress tensor). In the following, we shall prove that the scalar curvature is unattainable, thus we have a proof of a no-go theorem.

To be more general, we use a rank-two tensor dS^{ab} to denote the area element on the surface S, and the temperature vector in (1) is replaced by a tensor t_{ab}. We consider a generalization of equipartition relation on the holographic screen as

$$M = \frac{1}{2}\int_S TdN = c\int_S t^{ab}dS_{ab}, \tag{8}$$

where t^{ab} is antisymmetrized with respect to ab and its contraction with dS_{ab} represents the temperature on the screen times a modification of the number of bits and an area element. Components t^{ab} are functions of only the local Killing vector ξ and its higher-order covariant derivatives up to order n. Actually, the formula in (2) can be recast into the form (8), if one notices that

$$\begin{aligned}\int_S e^{\phi}\nabla\phi\cdot dS &= \int_S \frac{N^a}{\sqrt{-\xi^c\xi_c}}\xi^b\nabla_a\xi_b dS\\ &= -\int_S \frac{N^a}{\sqrt{-\xi^c\xi_c}}\xi^b\nabla_b\xi_a dS,\\ &= -\int_S N^{[a}\hat{\xi}^{b]}\nabla_b\xi_a dS,\end{aligned} \tag{9}$$

where $\hat{\xi}^b \equiv \frac{\xi^b}{\sqrt{-\xi^c\xi_c}}$ is a timelike vector with unit norm and $[a, b]$ denotes the antisymmetrization between a, b, due to the fact that $\nabla_b\xi_a = \nabla_{[b}\xi_{a]}$. With the same assumption as in [1] that $\hat{\xi}^b$ is also normal to S, then $N^{[a}\hat{\xi}^{b]}$ constitutes the binormal of the space like screen S. Combined with the surface area ds, $N^{[a}\hat{\xi}^{b]}dS$ is just dual to dS_{ab} used in (8) and t_{ab} is dual to $\nabla_a\xi_b$.

In the following, we will prove that when the volume of a region surrounded by S goes to zero, the integral (8) cannot be expressed as a volume integral

$$\int_\Sigma d\Sigma_a(c_1R^a{}_b + c_2\delta^a_bR)\xi^b \tag{10}$$

except when t_{ab} contains a term proportional to $\nabla_a\xi_b$, and $c_2 = 0$ in this case.

To begin with, we elaborate more on the property of local Killing vector ξ. The local Killing vector at point p is defined by

$$(\nabla_a\xi_b + \nabla_b\xi_a)|_{\mathrm{p}} = 0. \tag{11}$$

It should be noted that this equation does not fix the local Killing vector; in our proof we require further that the (n + 1)th and lower-order covariant derivatives of the local Killing vector possess the same property as a Killing vector. Namely, the (n + 1)th and lower-order covariant derivatives of the local Killing vector can be expressed by a linear combination of ξ and $\nabla\xi$ through $\nabla_a\nabla_b\xi_c = R_{cbad}\xi^d$. Without this requirement, the high-order covariant derivatives of the local Killing vector can take arbitrary value, and our proof becomes easier. This requirement can be fulfilled because one can always choose a local initial frame at point p such that $g_{ab}|_{\rm p} = \eta_{ab}$ and $\Gamma^a_{bc}|_{\rm p} = 0$. Then at a nearby point p′, the Killing equation can be written as

$$\partial_a\xi_b + \partial_b\xi_a - \frac{2}{3}(R^c{}_{abd} + R^c{}_{bad})|_{\rm p}\delta x^d\xi_c + \cdots = 0, \tag{12}$$

where we have used the covariant expansion of the connection in the neighborhood of p and $\delta x^d = x^d|_{\rm p'} - x^d|_{\rm p}$. The solution of the above equation in general exists. If we truncate the infinite expansion up to the power $(\delta x)^{\rm n}$, then the corresponding solution of the truncated equation is the local Killing vector we are after.

Utilizing the Stokes theorem, the right-hand side of Eq. (8) is transformed into

$$\int_S t^{ab}dS_{ab} = 2\int_\Sigma \nabla_b t^{ab}d\Sigma_a, \tag{13}$$

where Σ is a three dimensional region containing point p and enclosed by the screen. The integrand can be expressed as

$$\nabla_b t^{ab} = \frac{\partial t^{ab}}{\partial\xi_c}\nabla_b\xi_c + \frac{\partial t^{ab}}{\partial\nabla_d\xi_c}\nabla_b\nabla_d\xi_c + \cdots + \frac{\partial t^{ab}}{\partial(\nabla_{d_{\rm n}}\cdots\nabla_{d_1}\xi_c)}\nabla_b\nabla_{d_{\rm n}}\cdots\nabla_{d_1}\xi_c. \tag{14}$$

As the volume of Σ goes to zero, the integrand approaches its value at point p. According to our setup, the high-order covariant derivatives of ξ at point p can be replaced by a linear combination of $\xi|_{\rm p}$ and $\nabla\xi|_{\rm p}$. We notice that

$$\nabla_b\nabla_{d_{\rm n}}\cdots\nabla_{d_1}\xi_c|_{\rm p} = (\nabla_b\nabla_{d_{\rm n}}\cdots\nabla_{d_3}R_{cd_1d_2a})|_{\rm p}\xi^a + \cdots, \tag{15}$$

where there is a term containing the (n + 1)th-order metric derivative which cannot be canceled by other terms on the right-hand side of Eq. (14), since these terms are composed by $g, \partial g\cdots\partial^n g$. For n > 1, this term is linearly independent of the Ricci tensor and the Ricci scalar containing only the second-order metric derivative. Therefore, to generate terms containing no $\partial^n g$, $n > 2$, the generalized temperature t^{ab} can only depend on ξ and $\nabla\xi$. Now the remaining terms are

$$\nabla_b t^{ab}|_{\rm p} = \frac{\partial t^{ab}}{\partial\xi_c}\nabla_b\xi_c + \frac{\partial t^{ab}}{\partial\nabla_d\xi_c}R_{cdbe}\xi^e|_{\rm p}, \tag{16}$$

where we have used the relation $\nabla_a\nabla_b\xi_c = R_{cbad}\xi^d$. To understand the above expression, we choose a frame where the local Killing vector can be written as $\xi^a\partial_a = \partial_t$ with t identified as the time coordinate in this new frame. Then $\nabla_d\xi_c$ are functions of ∂g. We note that the first term on the right-hand side of Eq. (16) contains only ∂g. For Eq. (16) to be identified with $(c_1R^a_b + c_2\delta^a_bR)\xi^b$, this term should vanish; in other words, $\partial t^{ab}/\partial\xi = 0$, since the relevant terms should be proportional to the Riemann tensor. The last term is proportional to the Riemann tensor with its coefficients being functions of g and ∂g. By expanding (10) to a polynomial composed of g, ∂g and $\partial^2 g$, it is clear that each term in this expansion contains only the second metric derivative. Thus $\partial t^{ab}/\partial(\nabla_d\xi_c)$ should be independent of ∂g or $\nabla\xi$, since in (16) $\partial t^{ab}/\partial(\nabla_d\xi_c)$ is multiplied by the Riemann tensor already containing the second derivatives of the metric. We find that $\partial t^{ab}/\partial(\nabla_d\xi_c)$ can only be a metric function. Inheriting $\nabla_d\xi_c$'s antisymmetric nature about indices d, c, $\partial t^{ab}/\partial(\nabla_d\xi_c)$ should be antisymmetrical with respect to a, b and d, c. Taking into account all of these, it is a unique possibility that $\partial t^{ab}/\partial(\nabla_d\xi_c) \propto g^{ad}g^{bc} - g^{ac}g^{bd}$ leads to the conclusion that $t_{ab} \propto \nabla_a\xi_b$. This implies that only the Ricci tensor appears in (13), and c_2 always vanishes in (10); this concludes our proof of the no-go theorem.

In conclusion, we have shown that in the original derivation of the Einstein equations by Verlinde, a negative temperature must be introduced for an accelerated expanding region, thus introducing vexing physical problems. One may try to modify Verlinde's derivation by modifying the definition of temperature, and we have shown that no modification is appropriate to generate the correct Einstein equations.

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Note added.—This no-go theorem is valid for inflation models utilizing a fluid with negative Tolman-Komar mass; it is not valid for $f(R)$ inflation. In the latter case, inflation is driven by higher-order derivative terms; these terms may be introduced by using a modified temperature. We thank Qing Guo Huang for a discussion on this point.

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