PHYSICAL REVIEW D **82,** 025010 (2010)

Massless chiral supermultiplets of higher spins and the θ-twistor

M. Chaichian,[1] A. Tureanu,[1] and A. A. Zheltukhin[2,3,4]

[1]*Department of Physics, University of Helsinki and Helsinki Institute of Physics, P. O. Box 64, FIN-00014 Helsinki, Finland*
[2]*Kharkov Institute of Physics Technology, 61108 Kharkov, Ukraine*
[3]*Department of Physics, Stockholm University, 10 691 Stockholm, Sweden*
[4]*NORDITA, Roslagstullsbacken 23, 106 91, Stockholm, Sweden*
(Received 25 April 2010; published 15 July 2010)

Recently N. Berkovits, motivated by the supertwistor description of $\mathcal{N} = 4$ $D = 4$ super Yang-Mills, considered the generalization of the $\mathcal{N} = 1$ $D = 4$ θ-twistor construction to $D = 10$ and applied it for a compact covariant description of $\mathcal{N} = 1$ $D = 10$ super Yang-Mills. This supports the relevance of the θ-twistor as a supersymmetric twistor alternative to the well-known supertwistor. The minimal breaking of superconformal symmetry is an inherent property of the θ-twistor received from its fermionic components, described by a Grassmannian vector instead of a Grassmannian scalar in the supertwistor. The θ-twistor description of the $\mathcal{N} = 1$ $D = 4$ massless chiral supermultiplets $(S, S + 1/2)$ with spins $S = 0, 1/2, 1, 3/2, 2, \ldots$ is considered here. The description permits to restore the auxiliary F fields of the chiral supermultiplets absent in the supertwistor approach. The proposed formalism is naturally generalized to $\mathcal{N} = 4$ $D = 4$ and can be used for an off-shell description of the corresponding super Yang-Mills theory.

DOI: 10.1103/PhysRevD.82.025010 PACS numbers: 12.60.Jv, 11.30.Pb, 11.25.Tq

I. INTRODUCTION

The supertwistor is a supersymmetric generalization of the Penrose twistor [1] constructed by Ferber at the end of the 1970s [2,3]. The recent discovery of the supertwistor role in computing $\mathcal{N} = 4$ $D = 4$ multigluon amplitudes in super Yang-Mills theory [4–8] and their connection with the twistor strings strongly extended the range of the previous applications of the Penrose twistor program (see, e.g., [9–16]). Concerning the role of the supertwistor in superstring and super p-brane theory we recall that their use made possible the covariant quantization of tensionless superstrings and supermembranes, as well as the proof of the absence of the critical dimension in their quantum description. After this progress, the supersymmetric twistor variables had been used to construct the twistor-like Lagrangian and Hamiltonian for the $D = 10$ Green-Schwarz superstring (and $D = 11$ supermembrane), to solve the problem of the Lorentz covariant splitting of its first- and second-class constraints and to find the irreducible covariant realization of the κ-symmetry generators [17]. However, the covariant classical Becchi-Rouet-Stora-Tyutin (BRST) charge of the superstring, derived in the approach, has turned out to be such a complicated function of the canonical variables that blocked the transition to a quantum BRST operator. The twistor transform of the superstring action [17] considered in [18] presented it in the supertwistor form with the $D = 10$ supertwistor realized as the fundamental representation of the $OSp(32|1)$ supergroup [19]. This supergroup realizes the superconformal transformations like the superconformal group $SU(2, 2|1)$ in the $D = 4$ Minkowski space extended by the Grassmannian spinor coordinates θ. Some promising tools towards the solution of this problem are connected with the approach [20] using the $D = 10$ pure spinors, previously considered in [21–23], in the role of the discussed twistor-like variables (see also [24]). The approach made possible computing the superstring amplitudes [25], however its relation with the Green-Schwarz superstring is still open. These and other known results show the important role of the concept of twistor in the superstring and Yang-Mills theories, and stimulate its further development accompanied by unification with supersymmetry and higher dimensions.

In the recent paper [26], Berkovits proposed to generalize the well-known supertwistor description of the $\mathcal{N} = 4$ $D = 4$ super Yang-Mills theory to the case $\mathcal{N} = 1$ $D = 10$ super Yang-Mills theory to get its compact covariant description. The proposal was partially stimulated by the papers [27–29], showing many similar features between Yang-Mills and superstring theories in $D = 10$. In [26] were discussed new $D = 10$ supertwistor variables $Z = (\lambda^\alpha, \mu_\alpha, \Gamma^m)$, where λ^α and μ_α are constrained spinors and Γ^m is the Grassmannian 10-vector $\Gamma^m = (\lambda\gamma^m\theta)$ substituted for the Grassmannian scalar $\eta = (\lambda\theta)$ used in the supertwistor. It results in a relation between the scalar superfield $\Phi(Z)$ and the $D = 10$ superfield Yang-Mills vertex operator $\lambda^\alpha A_\alpha(x, \theta)$ of the pure spinor superstring formalism. In addition, the cubic super Yang-Mills amplitude was found to be proportional to the integral of $\Phi^3(Z)$ over the Z-space. The observations [26] shed a new light on the connection between the GS and Ramond-Neveu-Schwarz strings with the $D = 10$ super Yang-Mills theories.

The supertwistor variables $Z = (\lambda^\alpha, \mu_\alpha, \Gamma^m)$ discussed in [26] are the generalization to the case $D = 10$ of the $D = 4$ $\mathcal{N} = 1$ supersymmetric θ-twistor $\boldsymbol{\Xi}$ studied in

[30]. The Ξ components are presented by the triple $\Xi_{\mathcal{A}} = (-il_\alpha, \bar{\nu}^{\dot{\alpha}}, 2\sqrt{2}\bar{\eta}_m)$ including the known Penrose chiral spinor $\bar{\nu}^{\dot{\alpha}}$, accompanied by the new $D = 4$ Weyl spinor l_α and the complex Grassmannian 4-vector $\bar{\eta}_m$. The θ-twistor components l_α and $\bar{\eta}_m$ are defined by the general solution of the real null constraint $\Xi_{\mathcal{A}}\bar{\Xi}^{\mathcal{A}} = 0$, where $\bar{\Xi}^{\mathcal{A}} \equiv (\Xi_{\mathcal{A}})^* = (\nu^\alpha, i\bar{l}_{\dot{\alpha}}, 2\sqrt{2}\eta^m)$ is the complex conjugate of $\Xi_{\mathcal{A}}$. The solution of the constraint fixes l_α and $\bar{\eta}_m$ in the form

$$l_\alpha = y_{\alpha\dot{\alpha}}\bar{\nu}^{\dot{\alpha}}, \qquad \bar{\eta}_m = -\frac{1}{2}(\theta\sigma_m\bar{\nu}),$$
$$y_{\alpha\dot{\alpha}} = x_{\alpha\dot{\alpha}} - 2i\theta_\alpha\bar{\theta}_{\dot{\alpha}},$$

where $y_{\alpha\dot{\alpha}} \equiv y_m\sigma^m_{\alpha\dot{\alpha}}$ and θ_α are known coordinates of the chiral superspace (y_m, θ_α) [31]. The vector $\bar{\eta}_m$ and its complex conjugate $\eta_m = (\bar{\eta}_m)^* = -\frac{1}{2}(\nu\sigma_m\bar{\theta})$ form the real $D = 4$ Grassmannian vector $\psi_m = \eta_m + \bar{\eta}_m = -\frac{1}{2}(\bar{\nu}\gamma_m\theta)$ first introduced in [32] and later used for $D = 10$ in [28] and denoted by Γ_m in [26]. The name θ-twistor used in [30] for the triple $\Xi_{\mathcal{A}}$ was motivated by its difference from the supertwistor [2]. The difference is that the θ-twistor is covariant only under transformations of the maximal subgroup of the $D = 4$ superconformal group $SU(2, 2|1)$ including the super-Poincaré group, dilatation together with the phase and the axial γ_5 transformations, as it was shown in [30]. The broken symmetries of the θ-twistor superspace turn out to be the superconformal boosts. Taking into account that both the $D = 10$ super Yang-Mills theory and the $D = 10$ θ-twistor are not superconformally covariant, it seems instructive to study the structure of the $\mathcal{N} = 1$ $D = 4$ massless superfields $F(\Xi)$ in the superspace created by the θ-twistor Ξ. This is the main aim of the present paper.

First let us make some geometric comments, explaining the origin of the θ-twistor and how it differs from the supertwistor. The supertwistor is the projective triple including two commuting spinors and the additional Grassmannian Lorentz scalar $\eta = \nu^\alpha\theta_\alpha$ representing the Grassmannian component of the supe rtwistor contributed by the spinor coordinate θ_α of the $D = 4$ $\mathcal{N} = 1$ superspace. Using the projection η instead of θ_α reduces the spin structure represented by the supertwistor and, as a result, the massless chiral supermultiplets lose their auxiliary F-field, yielding the on-shell supersymmetry transformations discussed in [2]. The restriction may be overcome by the transition to the θ-twistor.[1] Contrary to the supertwistor, the Grassmannian components of the θ-twistor are represented by the composite Grassmannian (or Ramond) vector $(\theta\sigma_m\bar{\nu})$ [32] composed of the spinors θ_α, $\bar{\nu}_{\dot{\alpha}}$.

[1]For simplicity, we discuss the $N = 1$ supersymmetry, but the θ-twistor generalization for the $SU(N)$ symmetry group is automatically achieved by the substitution of θ^i_α for θ_α, where i is the fundamental representation index.

There is a very simple algebraic reason for the existence of the projective θ-twistor. For a given Weyl spinor θ_α with a fixed chirality one can match it either with the same chirality Penrose spinor ν^α, or with the opposite chirality spinor $\bar{\nu}_{\dot{\alpha}}$. In the first case we obtain the Lorentz scalar $(\theta\nu)$, and, respectively, the Lorentz vector $(\theta\sigma_m\bar{\nu})$ in the second case. The first possibility results in the supertwistor, while the second leads to the θ-twistor. The supertwistor components additional to the Penrose spinor ν_α are produced by the *left projection* of the chiral superspace coordinates $(y_{\alpha\dot{\alpha}}, \theta_\alpha)$ on ν^α. The two projections form the *double* $(\frac{1}{2}, 0)$ whose unification with ν^α yields the projective *triple* linearly realizing the supersymmetry transformations and called supertwistor [2]. Because of the *non-Hermiticity* of the matrix $y_{\alpha\dot{\alpha}}$, formed by the chiral coordinates, we observe the alternative possibility to extend the chiral superspace by the *right multiplication* of its coordinates on the c.c. spinor $\bar{\nu}^{\dot{\alpha}}$. This way yields a new projective *triple*, formed by the *double* $(\frac{1}{2}, 1)$ and $\bar{\nu}^{\dot{\alpha}}$, called the θ-twistor [30] and forming a new linear representation of the supersymmetry. The vector component of the new *double* $(\frac{1}{2}, 1)$ is $(\theta\sigma_m\bar{\nu})$ and it represents the Grassmannian component of the θ-twistor. Thus, the two different *doubles* $(\frac{1}{2}, 0)$ and $(\frac{1}{2}, 1)$ suggest two independent supersymmetric generalizations of the bosonic Penrose twistor called the supertwistor and the θ-twistor, respectively. The θ-twistor and the supertwistor turn out to be general solutions of different supersymmetric constraints [30], generalizing the standard chirality constraint to superspaces extended either by ν_α or $\bar{\nu}_{\dot{\alpha}}$. The Grassmannian vector $\bar{\eta}_m = -\frac{1}{2}(\theta\sigma_m\bar{\nu})$ plays the role of the Grassmannian scalar $\eta = \nu^\alpha\theta_\alpha$ in the superfields $F(\Xi)$ on the θ-twistor space. The $\bar{\eta}_m$ expansion of $F(\Xi)$ generates a complete set of component fields contrary to the supertwistor set, produced by the η expansion. It is because $\eta^2 = 0$, but $\bar{\eta}_m^2$ is proportional to θ^2. Thus, the superfields in the θ-twistor formalism preserve their auxiliary fields and give an off-shell description of the chiral supermultiplets.

Here we investigate the component structure of the scalar superfield $F(\Xi)$ generated by its expansion with respect to both $\bar{\nu}_{\dot{\alpha}}$ and $\bar{\eta}_m$, and find the structure to be associated with an infinite chain of massless chiral supermultiplets $(S, S + \frac{1}{2})$ with spin $S = 0, 1/2, 1, 3/2, 2, \ldots$. The chain includes the well-known massless scalar $(0, \frac{1}{2})$, vector $(\frac{1}{2}, 1)$ and other higher spin massless supermultiplets previously studied in [33–40] and many other papers. We prove that the $\bar{\eta}_m$ expansion of generalized chiral superfields in the θ-twistor superspace turns out to be equivalent to the power series expansion in θ_α.

II. THE PENROSE TWISTOR

To present the θ-twistor construction [30] in a clearer form we start here from the complexified Minkowski space with its Lorentz group locally isomorphic to $SL(2C) \times SL(2C)$. The positive chirality Weyl spinor ν_α and its complex conjugate $\bar{\nu}_{\dot{\alpha}}$ with negative chirality form the fundamental representation $(\frac{1}{2}, 0)$ and its c.c. $(0, \frac{1}{2})$ of the group. Such spinors have been used by Penrose to construct twistors [1]. The complex coordinates z_m of a point in the complexified Minkowski space are represented by the non-Hermitian 2×2 matrix $z_{\alpha\dot{\alpha}} \equiv z_m(\sigma^m)_{\alpha\dot{\alpha}}$, where $\sigma^m = (1, \vec{\sigma})$, with $\vec{\sigma}$ being the 2×2 Pauli spin matrices [31]. For our objective it is convenient to introduce the Penrose twistors starting from a subspace of holomorphic functions $f(z_{\alpha\dot{\alpha}}, \bar{\nu}_{\dot{\alpha}})$ which satisfy the constraint [30]

$$\bar{\nu}_{\dot{\alpha}} \frac{\partial}{\partial z_{\alpha\dot{\alpha}}} f(z, \bar{\nu}) = 0. \tag{1}$$

The general solution of (1) is given by arbitrary functions $f(l_\alpha, \bar{\nu}_{\dot{\alpha}})$ depending on an effective spinor variable l_α defined by the Penrose incidence relation [1]

$$l_\alpha - z_{\alpha\dot{\alpha}} \bar{\nu}^{\dot{\alpha}} = 0. \tag{2}$$

The relation (2) is invariant under the shifts of $z_{\alpha\dot{\alpha}}$ by the complex null vector $\lambda_\alpha \bar{\nu}_{\dot{\alpha}}$, where λ_α is an arbitrary spinor. When $\bar{\nu}_{\dot{\alpha}}$ is fixed and λ_α varies, a complex totally null plane in the Minkowski space is swept. Penrose called such a null plane the α-plane. One could think about the α-plane as the world volume swept by a null three-brane [41]. The pair of spinors $\bar{\nu}_{\dot{\alpha}}, l_\alpha$ composes the four-dimensional complex object called the twistor Ξ_A or its c.c. $\bar{\Xi}^A$

$$\Xi_A = (-il_\alpha, \bar{\nu}^{\dot{\alpha}}), \qquad \bar{\Xi}^A \equiv (\Xi_A)^* = (\nu^\alpha, i\bar{l}_{\dot{\alpha}}). \tag{3}$$

Penrose indicated that the complex points z are incident with the twistor. It means that if ν and l are fixed and Eq. (2) is considered as equation for z then its general solution is given by z belonging to a two-dimensional complex α-plane in four-dimensional complex Minkowski space. The twistor Ξ_A and its c.c. $\bar{\Xi}^A$ (3) yield the quadratic Hermitian form

$$\begin{aligned}\Xi_A \bar{\Xi}^A &= i[-l_\alpha \nu^\alpha + \bar{\nu}^{\dot{\alpha}} \bar{l}_{\dot{\alpha}}] = -i\nu^\alpha [z_{\alpha\dot{\beta}} - \bar{z}_{\dot{\beta}\alpha}] \bar{\nu}^{\dot{\beta}} \\ &= -i\nu(z - z^\dagger)\bar{\nu}\end{aligned} \tag{4}$$

that vanishes for the Hermitian z-matrices: $z_{\alpha\dot{\beta}} = \bar{z}_{\beta\dot{\alpha}}$. The case corresponds to the real Minkowski space and, respectively, converts the twistor to the null twistor. The real space-time point $x_{\alpha\dot{\alpha}}$ defined by the Hermitian matrix z is associated with the real light vector $\nu_\alpha \bar{\nu}_{\dot{\alpha}}$ going through it. The light vector is represented by a point on the Riemann sphere interpreted as a projective line CP^1 belonging to a subspace of the null twistors embedded into the complex projective 3-space CP^3. Thus, any point in the real space-time is represented by a Riemann sphere in the projective null twistor space. The construction described above admits a straightforward supersymmetric generalization resulting in the θ-twistor.

III. SUPERSYMMETRY AND THE θ-TWISTOR

Supersymmetry implies the extension of the real Minkowski space coordinates $x_{\alpha\dot{\alpha}}$ by the Grassmannian spinor coordinates θ_α and $\bar{\theta}_{\dot{\alpha}}$. The corresponding superspace has the coordinates $(x_{\alpha\dot{\alpha}}, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$ and is invariant under the super-Poincaré symmetry [31]. The superspace may also be extended by the addition of the Penrose spinors ν_α and $\bar{\nu}_{\dot{\alpha}}$. Using the conventions [16] we define the supersymmetry transformations in the $D = 4$ $\mathcal{N} = 1$ superspace as follows:

$$\delta\theta_\alpha = \varepsilon_\alpha, \qquad \delta x_{\alpha\dot{\alpha}} = 2i(\varepsilon_\alpha \bar{\theta}_{\dot{\alpha}} - \theta_\alpha \bar{\varepsilon}_{\dot{\alpha}}), \qquad \delta\nu_\alpha = 0, \tag{5}$$

where ν_α, $\bar{\nu}_{\dot{\alpha}}$ are not transformed. The odd D^α, $\bar{D}^{\dot{\alpha}}$ and even $\partial^{\dot{\alpha}\alpha} \equiv \frac{\partial}{\partial x_{\alpha\dot{\alpha}}}$ derivatives

$$\begin{aligned}D^\alpha &= \frac{\partial}{\partial\theta_\alpha} - 2i\bar{\theta}_{\dot{\alpha}} \partial^{\dot{\alpha}\alpha}, \\ \bar{D}^{\dot{\alpha}} &\equiv -(D^\alpha)^* = \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} - 2i\theta_\alpha \partial^{\dot{\alpha}\alpha}, \\ \{D^\alpha, \bar{D}^{\dot{\alpha}}\} &= -4i\partial^{\dot{\alpha}\alpha}\end{aligned} \tag{6}$$

in the superspace are also invariant under the transformations (5). A function $F(x, \theta, \bar{\theta})$ in the superspace is known as a superfield. The superfields satisfying the chiral constraint

$$\begin{aligned}\bar{D}^{\dot{\alpha}} F(x, \theta, \bar{\theta}) = 0 \rightarrow F = F(y, \theta), \\ y_{\alpha\dot{\alpha}} = x_{\alpha\dot{\alpha}} - 2i\theta_\alpha \bar{\theta}_{\dot{\alpha}}\end{aligned} \tag{7}$$

are important for the supersymmetric Yang-Mills theory, supergravity and superstring. The general solution of (7) is given by a chiral superfield $F = F(y, \theta)$ depending on the complex coordinates $y_{\alpha\dot{\alpha}}$ whose imaginary part is the nilpotent monomial $(-2\theta_\alpha \bar{\theta}_{\dot{\alpha}})$. The subspace $(y_{\alpha\dot{\alpha}}, \theta_\alpha)$ called the *chiral* superspace [31] is closed under the supersymmetry transformations

$$\delta\theta_\alpha = \varepsilon_\alpha, \qquad \delta y_{\alpha\dot{\alpha}} = -4i\theta_\alpha \bar{\varepsilon}_{\dot{\alpha}} \tag{8}$$

and preserves the Cartan-Volkov differential one-form $\omega_{\alpha\dot{\alpha}}$

$$\omega_{\alpha\dot{\alpha}} = dy_{\alpha\dot{\alpha}} + 4id\theta_\alpha \bar{\theta}_{\dot{\alpha}}, \qquad \delta\omega_{\alpha\dot{\alpha}} = 0. \tag{9}$$

After transition to the chiral space the θ-twistor is introduced by a natural generalization of the above considered twistor construction. It implies the extension of any chiral superfield $F = F(y, \theta)$ to a generalized chiral superfield $F(y, \theta, \bar{\nu})$ depending on the new argument $\bar{\nu}$. The extension allows to generalize the constraint (1) to the supersymmetric one

$$\bar{\nu}_{\dot{\alpha}} \frac{\partial}{\partial y_{\alpha\dot{\alpha}}} F(y, \bar{\nu}, \theta) = 0 \rightarrow F = F(l_\alpha, \bar{\nu}_{\dot{\alpha}}, \theta_\alpha), \tag{10}$$

consistent with the chiral constraint (7). The constraint (10) is satisfied by any chiral superfield $F(l_\alpha, \bar\nu_{\dot\alpha}, \theta_\alpha)$ depending on the new spinor l_α defined by the generalized *incidence relation*

$$l_\alpha - y_{\alpha\dot\alpha}\bar\nu^{\dot\alpha} = 0. \tag{11}$$

The transformations of the spinor l_α (11) under the supersymmetry (8) are nonlinear

$$\delta l_\alpha = -4i\theta_\alpha(\bar\nu^{\dot\beta}\bar\varepsilon_{\dot\beta}), \qquad \delta\theta_\alpha = \varepsilon_\alpha, \qquad \delta\bar\nu_{\dot\alpha} = 0 \tag{12}$$

and reveal the spinor triple l_α, $\bar\nu_{\dot\alpha}$, θ_α as a new representation of the supersymmetry. The transformations (12) are presented as linear by the transition to the new superpartner $(\theta_\alpha\bar\nu_{\dot\beta})$ of l_α

$$\delta l_\alpha = 4i(\theta_\alpha\bar\nu_{\dot\beta})\bar\varepsilon^{\dot\beta}, \qquad \delta(\theta_\alpha\bar\nu_{\dot\beta}) = \varepsilon_\alpha\bar\nu_{\dot\beta}, \quad \delta\bar\nu_{\dot\alpha} = 0 \tag{13}$$

which is the Lorentz vector. The equivalent linear form of the transformations (13) is

$$\delta l_\alpha = -4i(\sigma_m\bar\varepsilon)_\alpha\bar\eta^m, \qquad \delta\bar\eta_m = -\frac{1}{2}(\varepsilon\sigma_m\bar\nu), \qquad \delta\bar\nu_{\dot\alpha} = 0. \tag{14}$$

The Grassmannian vector $\bar\eta_m$ in (14) and its c.c. η_m are the *composite* Ramond vectors

$$\eta_m \equiv -\frac{1}{2}(\nu\sigma_m\bar\theta), \qquad \bar\eta_m = (\eta_m)^* = -\frac{1}{2}(\theta\sigma_m\bar\nu),$$
$$\nu_\beta\bar\theta_{\dot\alpha} \equiv \eta_{\beta\dot\alpha} = (\sigma^m)_{\beta\dot\alpha}\eta_m, \qquad \eta_m\eta_n + \eta_n\eta_m = 0 \tag{15}$$

introduced in [32] (see details in [42]) to prove the equivalence between superparticles and spinning particles based on the observation that $\bar\eta_m$ (15) solves the Dirac constraint

$$\bar\eta_m(\bar\nu\tilde\sigma^m\lambda) = 0, \tag{16}$$

where $(\bar\nu\tilde\sigma^m\lambda)$ is the tangent vector of the Penrose α-plane.

We see that the supersymmetrization of the complex Minkowski space, matching $y_{\alpha\dot\alpha}$ with θ_α, yields the supersymmetrization of the Penrose twistor by the matching of l_α with $\bar\eta_m$.

As a result, the incidence relation (2) gets its Grassmannian counterpart

$$l_\alpha - y_{\alpha\dot\alpha}\bar\nu^{\dot\alpha} = 0, \qquad \bar\eta_m + \frac{1}{2}(\theta\sigma_m\bar\nu) = 0. \tag{17}$$

Thus, the Penrose twistor space CP^3 is extended up to the superspace $CP^{3|2_{\rm spin}}$ by the addition of the fermionic sector presented by two independent complex components of the composite vector $\bar\eta_m$ associated with θ_α.[2] It is a consequence of the composite structure of $\bar\eta_m$ (15). To see it one can use the spinor basis associated with the supersymmetric triple (14). The basis is formed by a Newman-Penrose dyad ν^α and υ^α [1] whose components satisfy the condition $\nu^\alpha\upsilon_\alpha = 1$. Then θ_α may be decomposed in the dyad basis

$$\theta_\alpha = \eta\upsilon_\alpha - \chi\nu_\alpha, \qquad \eta \equiv (\nu^\alpha\theta_\alpha), \qquad \chi \equiv (\upsilon^\alpha\theta_\alpha), \tag{18}$$

where the complex numbers η and χ are the θ coordinates. So, the spinor θ_α is equivalently represented by two complex numbers, η and χ. The substitution of the θ-decomposition (18) in the definition of $\bar\eta_m$ (15) yields its decomposition

$$\bar\eta_m = -\frac{1}{2}[\eta(\upsilon\sigma_m\bar\nu) - \chi(\nu\sigma_m\bar\nu)] \tag{19}$$

in only two basis vectors $(\upsilon\sigma_m\bar\nu)$ and $(\nu\sigma_m\bar\nu)$ out of the complete Newman-Penrose *vector* basis constructed from the Newman-Penrose dyad ν_α, υ_α and their complex conjugate (see details in [43,44]). In this vector basis originating from the spinor $\bar\nu^{\dot\alpha}$, belonging to the supermultiplet (14), the composite Ramond vector $\bar\eta_m$ (15) turns out to be represented by the same pair of the complex numbers ν and χ as the spinor θ_α (18). These two complex numbers form the fermionic sector of $CP^{3|2_{\rm spin}}$. The composite structure of $\bar\eta_m$ puts severe restrictions on its monomials. The monomials of degree greater than two vanish because they are proportional monomials formed by the Weyl spinor θ_α (see also Eq. (41)). Such a vanishing never occurs for an arbitrary Grassmannian vector ψ_m, because its maximal monomial $\psi_0\psi_1\psi_2\psi_3$ has the degree equal four.

Thus we obtain the generalization of the twistor (3) that includes the Ramond vector $\bar\eta_m$. The corresponding supersymmetric triple has the following form:

$$\Xi_{\mathcal{A}} \equiv (-il_\alpha, \bar\nu^{\dot\alpha}, 2\sqrt{2}\bar\eta_m), \qquad \bar\Xi^{\mathcal{A}} \equiv (\Xi_{\mathcal{A}})^* = (\nu^\alpha, i\bar l_{\dot\alpha}, 2\sqrt{2}\eta^m). \tag{20}$$

The triple, called the θ-twistor, yields a new supersymmetric generalization of isotropic Penrose twistor. One can check that the quadratic Hermitian form in the θ-twistor superspace

[2] We use the notation $2_{\rm spin}$ to show the *spinor* structure of the fermionic sector of $CP^{3|2_{\rm spin}}$. The standard notation $CP^{3|\mathcal{N}}$ of the twistor superspace shows that its fermionic sector is represented by the Lorentz *scalars* belonging to the fundamental representation of the *internal* $SU(\mathcal{N})$ symmetry of the Yang-Mills theory [4].

$$\Xi_{\mathcal{A}}\bar{\Xi}'^{\mathcal{A}} \equiv i[-l_\alpha \nu'^\alpha + \bar{\nu}^{\dot{\alpha}}\bar{l}'_{\dot{\alpha}} - 8i\bar{\eta}_m\eta'^m],$$
$$\bar{\Xi}'^{\mathcal{A}} \equiv (\Xi'_{\mathcal{A}})^* = (\nu'^\alpha, i\bar{l}'_{\dot{\alpha}}, 2\sqrt{2}\eta'^m), \tag{21}$$
$$\bar{l}'_{\dot{\alpha}} = \bar{y}_{\dot{\alpha}\alpha}\nu'^\alpha, \qquad \eta'_m = -\frac{1}{2}(\nu'\sigma_m\bar{\theta}),$$

built of coordinates of points $\Xi_{\mathcal{A}}$ and $\bar{\Xi}'^{\mathcal{A}}$, vanishes (like the form (4)) because of the supersymmetric incidence relations (17). Next we compare the θ-twistor with the supertwistor [2].

IV. THE SUPERTWISTOR

The presented derivation of the θ-twistor may be applied for the supertwistor derivation. A minor change is to start from an equivalent definition of the twistor using the constraint

$$\nu_\alpha \frac{\partial}{\partial z_{\alpha\dot{\alpha}}} f(z, \nu) = 0, \tag{22}$$

which differs from (1) by the substitution of ν_α for $\bar{\nu}_{\dot{\alpha}}$. The general solution of (22) is given by the functions $f(\bar{q}_{\dot{\alpha}}, \nu_\alpha)$ depending on the new spinor $\bar{q}_{\dot{\alpha}}$ defined by the incidence relation

$$\bar{q}_{\dot{\alpha}} - \nu^\alpha z_{\alpha\dot{\alpha}} = 0. \tag{23}$$

The comparison of the complex conjugate of Eq. (23) with (2) shows their independence

$$q_\alpha - (\bar{\nu}\bar{z})_\alpha = 0, \qquad l_\alpha - (z\bar{\nu})_\alpha = 0 \tag{24}$$

in general and their coincidence for the case of Hermitian matrices z: $\bar{z} = z^T$.

Because the chiral superspace has the non-Hermitian coordinate matrix $y_{\alpha\dot{\alpha}}$, the supersymmetric generalizations of the two coincidence relations (24) will not be equivalent and in fact they will generate the supertwistor and θ-twistor, respectively. In fact, the incidence condition (23) defines another twistor Z_A

$$Z_A = (-iq_\alpha, \bar{\nu}^{\dot{\alpha}}), \qquad \bar{Z}^A \equiv (Z_A)^* = (\nu^\alpha, i\bar{q}_{\dot{\alpha}}) \tag{25}$$

which includes the spinor q_α instead of l_α (3).

To construct the supersymmetric generalization of the twistor (25) we go back to the chiral superfields $F = F(y, \bar{\nu}, \theta)$, but replace their argument $\bar{\nu}_{\dot{\alpha}}$ by ν_α and, respectively, the constraint (10) by the constraint

$$\nu_\alpha \frac{\partial}{\partial y_{\alpha\dot{\alpha}}} F(y, \nu, \theta) = 0 \to F = F(\bar{q}_{\dot{\alpha}}, \nu_\alpha, \theta_\alpha), \tag{26}$$

which is also supersymmetric and consistent with the chiral constraint (7). The general solution of the constraint (26) is given by the chiral superfields $F(\bar{q}_{\dot{\alpha}}, \nu_\alpha, \theta_\alpha)$ depending on the new spinor $\bar{q}_{\dot{\alpha}}$ defined by the incidence relation (23) with $y_{\alpha\dot{\alpha}}$ substituted for $z_{\alpha\dot{\alpha}}$

$$\bar{q}_{\dot{\alpha}} - \nu^\alpha y_{\alpha\dot{\alpha}} = 0. \tag{27}$$

The transformations of the spinor $\bar{q}_{\dot{\alpha}}$ (27) under the supersymmetry (8) are nonlinear

$$\delta\bar{q}_{\dot{\alpha}} = -4i(\nu^\beta\theta_\beta)\bar{\varepsilon}_{\dot{\alpha}}, \qquad \delta\theta_\alpha = \varepsilon_\alpha, \qquad \delta\nu_\alpha = 0. \tag{28}$$

However, the transformations (28) may be easily brought to the linear form

$$\delta\bar{q}_{\dot{\alpha}} = -4i\eta\bar{\varepsilon}_{\dot{\alpha}}, \qquad \delta\eta = \nu^\alpha\varepsilon_\alpha, \qquad \delta\nu_\alpha = 0, \tag{29}$$

after the contraction of the second equation in (28) with ν^α and transition to the *scalar* variable η (18), representing only half of the spinor θ_α components. The second part of the θ-components described by the complex number χ is lost under the transition.

As a result, the Grassmannian counterpart of the incidence relation (27) takes the form

$$\eta - (\nu^\alpha\theta_\alpha) = 0 \tag{30}$$

and we obtain the supersymmetrical *incidence relations* associated with the spinor $\bar{q}_{\dot{\alpha}}$

$$\bar{q}_{\dot{\alpha}} - \nu^\alpha y_{\alpha\dot{\alpha}} = 0, \qquad \eta - (\nu^\alpha\theta_\alpha) = 0, \tag{31}$$

previously obtained in [2]. These incidence relations generate the supersymmetric triples $Z_{\mathcal{A}}$ and $\bar{Z}^{\mathcal{A}}$ unifying ν_α, $\bar{\nu}_{\dot{\alpha}}$ with q_α, $\bar{q}_{\dot{\alpha}}$, η, $\bar{\eta}$

$$Z_{\mathcal{A}} \equiv (-iq_\alpha, \bar{\nu}^{\dot{\alpha}}, 2\bar{\eta}), \qquad \bar{Z}^{\mathcal{A}} \equiv (\nu^\alpha, i\bar{q}_{\dot{\alpha}}, 2\eta). \tag{32}$$

The triples $Z_{\mathcal{A}}$ and $\bar{Z}^{\mathcal{A}}$ coincide with the $D = 4$ $\mathcal{N} = 1$ *supertwistor* [2] and its c.c., realizing the well-known supersymmetric generalization of the projective Penrose twistor.

The supersymmetric Hermitian quadratic form [2] in the supertwistor space

$$Z_{\mathcal{A}}\bar{Z}'^{\mathcal{A}} = i[-q_\alpha\nu'^\alpha + \bar{\nu}^{\dot{\alpha}}\bar{q}'_{\dot{\alpha}} - 4i\bar{\eta}\eta'],$$
$$\bar{Z}'^{\mathcal{A}} \equiv (Z'_{\mathcal{A}})^* = (\nu'^\alpha, i\bar{q}'_{\dot{\alpha}}, 2\eta'), \tag{33}$$
$$\bar{q}'_{\dot{\alpha}} = \nu'^\alpha y_{\alpha\dot{\alpha}}, \qquad \eta' = \nu'^\alpha\theta_\alpha,$$

where $Z_{\mathcal{A}}$ and $\bar{Z}'^{\mathcal{A}}$ describes different points of the twistor space, vanishes after using the incidence relations (31). Because of (31) and (17), the nonlinear relations

$$l_\alpha = q_\alpha - 4i\theta_\alpha\bar{\eta},$$
$$4i\bar{\eta}\eta' = -4i(\nu'_\alpha\bar{\nu}_{\dot{\alpha}})\theta^\alpha\bar{\theta}^{\dot{\alpha}} = 2i(\bar{\nu}\tilde{\sigma}_m\theta)(\nu'\sigma^m\bar{\theta}) \tag{34}$$
$$= -8i\bar{\eta}_m\eta'_m$$

are fulfilled and result in the equality of the quadratic forms (21) and (33)

$$\Xi_{\mathcal{A}}\bar{\Xi}'^{\mathcal{A}}|_{(17)} = Z_{\mathcal{A}}\bar{Z}'^{\mathcal{A}}|_{(31)} \tag{35}$$

on the hypersurfaces of the corresponding incidence relations.

Thus, we established that the extension of the complex chiral superspace $(y_{\alpha\dot{\alpha}}, \theta_\alpha)$ by the Penrose spinor ν_α, having *the same* chirality as θ_α, generates the supertwistor [2] and the projective superspace $CP^{3|1}$. On the contrary, the extension of the same chiral space by the Penrose spinor $\bar\nu_{\dot\alpha}$ having *the opposite* chirality to θ_α yields the θ-twistor [30] and the projective superspace $CP^{3|2_{\rm spin}}$. The principal new property of the θ-twistor is that its fermionic sector forms a *vector* representation of the Lorentz group in contrast to the supertwistor, whose fermionic sector is represented by a Lorentz *scalar*. In view of this difference the quadratic form (21) is invariant only under the maximal subgroup of the superconformal group formed by the supersymmetry (14), the scaling and phase symmetries

$$l'_\beta = e^{\varphi} l_\beta, \qquad \bar l'_{\dot\beta} = e^{\varphi^*}\bar l_{\dot\beta}, \qquad \nu'_\beta = e^{-\varphi}\nu_\beta,$$
$$\bar\nu'_{\dot\beta} = e^{-\varphi^*}\bar\nu_{\dot\beta}, \qquad \theta'_\beta = e^{\varphi}\theta_\beta, \qquad \bar\theta'_{\dot\beta} = e^{\varphi^*}\bar\theta_{\dot\beta}, \quad (36)$$
$$\bar\eta'_m = e^{2i\varphi_I}\bar\eta_m, \qquad \eta'_m = e^{-2i\varphi_I}\eta_m$$

described by the complex parameter $\varphi = \varphi_R + i\varphi_I$, as well as the γ_5 rotations

$$\theta'_\beta = e^{i\lambda}\theta_\beta, \qquad \bar\theta'_{\dot\beta} = e^{-i\lambda}\bar\theta_{\dot\beta}. \quad (37)$$

The Hermitian form (21) is not invariant under the superconformal boosts S^α and $\bar S^{\dot\alpha}$ as it follows from the superconformal boost transformations [30] of the coordinates $y_{\alpha\dot\alpha}$, θ_α

$$\delta y_{\alpha\dot\alpha} = 4i\theta_\alpha(\xi^\beta y_{\beta\dot\alpha}),$$
$$\delta\theta_\alpha = -y_{\alpha\dot\beta}\bar\xi^{\dot\beta} + 4i\theta_\alpha(\xi^\beta\theta_\beta) \quad (38)$$

forming the base chiral superspace. In fact, the chiral index $\dot\beta$ of $y_{\alpha\dot\beta}$ in $\delta\theta_\alpha$ (38) is contracted with the index of the transformation parameter $\bar\xi^{\dot\beta}$. Thus, it is not possible to transform $y_{\alpha\dot\beta}$ into l_α, belonging to the $\Xi_{\mathcal A}$-triple (20), by the contraction of $y_{\alpha\dot\beta}$ with $\bar\nu^{\dot\beta}$. On the contrary, $y_{\alpha\dot\beta}$ can be contracted with ν^α to be transformed into $\bar q_{\dot\beta}$ belonging to the $\bar Z^{\mathcal A}$ triple (32).

We see that the difference in the *chiralities* of the $\bar\nu$ and θ spinors forming the triple $\Xi_{\mathcal A}$ obstructs the superconformal boost realization by the θ-twistor.

V. MASSLESS CHIRAL SUPERMULTIPLETS OF HIGHER SPIN FIELDS

The superfields $F(\bar Z^{\mathcal A})$ and $F(\Xi_{\mathcal A})$ describe massless supermultiplets because they satisfy the Klein-Gordon equations

$$\partial_m\partial^m F(\bar Z) = 0, \qquad \partial_m\partial^m F(\Xi) = 0, \quad (39)$$

where $\partial_m \equiv (\sigma_m)_{\dot\alpha\alpha}\partial^{\dot\alpha\alpha} \equiv (\sigma_m)_{\alpha\dot\alpha}\frac{\partial}{\partial x_{\alpha\dot\alpha}}$, $\partial^{\dot\alpha\alpha} = -\frac{1}{2}\tilde\sigma_m^{\dot\alpha\alpha}\partial^m$. It follows from the identities $\sigma_{m\alpha\dot\alpha}\tilde\sigma^{m\beta\dot\beta} = -2\delta_\alpha^\beta\delta_{\dot\alpha}^{\dot\beta}$ and $\bar\nu^{\dot\alpha}\bar\nu_{\dot\alpha} = 0$. The holomorphic superfield $F(\Xi_{\mathcal A})$ depending on the $\Xi_{\mathcal A}$-triple is expanded in the finite power series in the Grassmannian vector $\bar\eta_m$

$$F(\Xi_{\mathcal A}) \equiv F(-il_\alpha, \bar\nu^{\dot\alpha}, 2\sqrt{2}\bar\eta_m)$$
$$= f_0(-iy_{\beta\dot\beta}\bar\nu^{\dot\beta}, \bar\nu^{\dot\beta}) + \bar\eta_m f^m(-iy_{\beta\dot\beta}\bar\nu^{\dot\beta}, \bar\nu^{\dot\beta})$$
$$+ \bar\eta_m\bar\eta_n f_2^{nm}(-iy_{\beta\dot\beta}\bar\nu^{\dot\beta}, \bar\nu^{\dot\beta}). \quad (40)$$

The monomial $\bar\eta_m\bar\eta_n\bar\eta_l$ of the third degree and higher monomials vanish in view of the composite structure of $\bar\eta_m$, defined by the incidence relation (17) and discussed in the previous section. As a result, the monomials constructed of $\bar\eta_m$ have to be proportional to the monomials constructed of the spinor θ_α. The direct calculation of the $\bar\eta_m$-monomials using the definition (17) and the spinor algebra [31] yields the desired relations

$$\bar\eta_m = -\frac{1}{2}(\theta\sigma_m\bar\nu),$$
$$\bar\eta_m\bar\eta_n \equiv \frac{1}{2}[\bar\eta_m\bar\eta_n - \bar\eta_n\bar\eta_m] = \frac{1}{4}(\bar\nu\tilde\sigma_{mn}\bar\nu)\theta^2, \quad (41)$$
$$\bar\eta_m\bar\eta_n\bar\eta_l = \theta_\alpha\theta_\beta\theta_\gamma = 0,$$

where $\theta^2 \equiv \theta^\alpha\theta_\alpha$. Because of the correspondence (41) one can rename the functions in (40)

$$\bar\eta_m f^m = -\frac{1}{2}(\theta\sigma_m\bar\nu)f^m \equiv -2\theta_\lambda f^\lambda,$$
$$\bar\eta_m\bar\eta_n f_2^{nm} = \frac{1}{4}\theta^2(\bar\nu\tilde\sigma_{mn}\bar\nu)f_2^{nm} \equiv \theta^2 f_2 \quad (42)$$

and present (40) in the equivalent form of the power series expansion in θ_α

$$F(\Xi_{\mathcal A}) \equiv F(-il_\alpha, \bar\nu^{\dot\alpha}, \bar\eta_m)$$
$$= f_0(-iy_{\beta\dot\beta}\bar\nu^{\dot\beta}, \bar\nu^{\dot\beta}) - 2\theta_\lambda f^\lambda(-iy_{\beta\dot\beta}\bar\nu^{\dot\beta}, \bar\nu^{\dot\beta})$$
$$+ \theta^2 f_2(-iy_{\beta\dot\beta}\bar\nu^{\dot\beta}, \bar\nu^{\dot\beta}). \quad (43)$$

The expansion (43) has the form of a chiral superfield expansion including the *auxiliary* field f_2 that *identically vanishes* in the supertwistor approach [2]. It implies the existence of an equivalent representation $\tilde F(\Xi_A)$ of the superfield $F(\Xi_{\mathcal A})$

$$F(\Xi_{\mathcal A}) = \tilde F(\Xi_A), \quad (44)$$

which depends on the supersymmetric entirely spinor triple Ξ_A

$$\Xi_{\mathcal A} \equiv (-il_\alpha, \bar\nu^{\dot\alpha}, \theta^\alpha), \qquad \bar\Xi^{\mathcal A} \equiv (\Xi_{\mathcal A})^* = (\nu^\alpha, i\bar l_{\dot\alpha}, \bar\theta^{\dot\alpha}) \quad (45)$$

associated with the $\Xi_{\mathcal A}$-triple. This correspondence between the $\Xi_{\mathcal A}$ and Ξ_A triples is the reason to use the same name, θ-twistor, for the spinor triple Ξ_A (45). Consequently, in the following the symbol tilde over the superfields depending on Ξ_A will be dropped.

We proved that the θ-twistor introduces a natural extension of the base chiral superspace $(y_{\alpha\dot\alpha}, \theta_\alpha)$ and its use preserves all components in the chiral superfields depending on $\bar{\nu}^{\dot\alpha}$. The superfield $\tilde{F}(\Xi_A)$ (44) depends on the commuting spinor $\bar{\nu}^{\dot\alpha}$ and the coefficients of power expansion of $\tilde{F}(\Xi_A)$ in the spinor degrees are chiral superfields $\Phi^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y,\theta)$ with arbitrary number of dotted spinor indices. To find these fields one can use Penrose's idea of the contour integration applied to $\tilde{F}(\Xi_A)$ that permits to integrate out the $\bar{\nu}$ dependence. The idea was previously extended to the supertwistor in [2] and the extension may be applied for the superfields depending on the θ-twistor. The corresponding contour integral defining the superfield $\Phi^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y,\theta)$ is given by the expression

$$\Phi^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y,\theta)=\oint(d\bar{\nu}^{\dot\gamma}\bar{\nu}_{\dot\gamma})\bar{\nu}^{\dot\alpha_1}\ldots\bar{\nu}^{\dot\alpha_{2S}}F(\bar{\nu}^{\dot\beta},-i\bar{\nu}^{\dot\gamma}y_{\beta\dot\gamma},\theta_\beta), \tag{46}$$

where we omit the tilde over $F(\Xi)$ (44) and assume its homogeneity degree equal to $-2(S+1)$. The $\bar{\nu}$-contour encloses the singularities of F in (46) for each fixed point (y,θ). The substitution of (43) into (46) results in the expansion

$$\Phi^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y,\theta)=f_0^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y)-2\theta_\lambda f^{\lambda\dot\alpha_1\ldots\dot\alpha_{2S}}(y)+\theta^2 f_2^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y), \tag{47}$$

where the component functions are defined by the integrals

$$\begin{aligned}f_0^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y)&=\oint(d\bar{\nu}^{\dot\gamma}\bar{\nu}_{\dot\gamma})\bar{\nu}^{\dot\alpha_1}\ldots\bar{\nu}^{\dot\alpha_{2S}}f_0(-iy_{\beta\dot\beta}\bar{\nu}^{\dot\beta},\bar{\nu}^{\dot\beta}),\\ f^{\lambda\dot\alpha_1\ldots\dot\alpha_{2S}}(y)&=\oint(d\bar{\nu}^{\dot\gamma}\bar{\nu}_{\dot\gamma})\bar{\nu}^{\dot\alpha_1}\ldots\bar{\nu}^{\dot\alpha_{2S}}f^\lambda(-iy_{\beta\dot\beta}\bar{\nu}^{\dot\beta},\bar{\nu}^{\dot\beta}),\\ f_2^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y)&=\oint(d\bar{\nu}^{\dot\gamma}\bar{\nu}_{\dot\gamma})\bar{\nu}^{\dot\alpha_1}\ldots\bar{\nu}^{\dot\alpha_{2S}}f_2(-iy_{\beta\dot\beta}\bar{\nu}^{\dot\beta},\bar{\nu}^{\dot\beta})\end{aligned} \tag{48}$$

with $f^{\lambda\dot\alpha_1\ldots\dot\alpha_{2S}}(y)$ and $f_{0,2}^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y)$ satisfying the chiral Dirac equations

$$\partial_{\alpha\dot\alpha_k}f^{\lambda\dot\alpha_1..\dot\alpha_k..\dot\alpha_{2S}}(x)=\partial_{\alpha\dot\alpha_k}f_{0,2}^{\dot\alpha_1..\dot\alpha_k..\dot\alpha_{2S}}(x)=0,\quad (k=1,2,\ldots,2s). \tag{49}$$

The further expansion of $\Phi^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y,\theta)$ (47) at the real point x_m is given by

$$\begin{aligned}\Phi^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y,\theta)=&f_0^{\dot\alpha_1\ldots\dot\alpha_{2S}}(x)-2\theta_\lambda f^{\lambda\dot\alpha_1\ldots\dot\alpha_{2S}}(x)\\&-2i\theta_\gamma\bar\theta_{\dot\gamma}\partial^{\dot\gamma\gamma}f_0^{\dot\alpha_1\ldots\dot\alpha_{2S}}(x)\\&-2i\theta^2\bar\theta_{\dot\gamma}\partial^{\dot\gamma\lambda}f_\lambda^{\dot\alpha_1\ldots\dot\alpha_{2S}}(x)+\theta^2f_2^{\dot\alpha_1\ldots\dot\alpha_{2S}}(x),\end{aligned} \tag{50}$$

where the term $\frac{1}{2}\theta^2\bar\theta^2\partial^{\dot\gamma\gamma}\partial_{\gamma\dot\gamma}f_0^{\dot\alpha_1\ldots\dot\alpha_{2S}}(x)$ was dropped because of the zero mass constraint (39)

$$\Box\Phi^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y,\theta)=0\rightarrow\Box f_0^{\dot\alpha_1\ldots\dot\alpha_{2S}}(x)=0. \tag{51}$$

For sewing together these results with the well-known case of the scalar supermultiplet, corresponding to $S=0$, we rename the component f-fields by the generally accepted notations [31]

$$\begin{aligned}f_0^{\dot\alpha_1\ldots\dot\alpha_{2S}}&=\sqrt{2}A^{\dot\alpha_1\ldots\dot\alpha_{2S}}\equiv\sqrt{2}A^{\cdots},\\ f_2^{\dot\alpha_1\ldots\dot\alpha_{2S}}&=\sqrt{2}F^{\dot\alpha_1\ldots\dot\alpha_{2S}}\equiv\sqrt{2}F^{\cdots},\\ f_\lambda^{\dot\alpha_1\ldots\dot\alpha_{2S}}&=\psi_\lambda^{\dot\alpha_1\ldots\dot\alpha_{2S}}\equiv\psi_{\lambda}^{\cdots},\end{aligned} \tag{52}$$

where $(\ldots)\equiv(\dot\alpha_1\ldots\dot\alpha_{2S})$. Then we find the superfield $\frac{1}{\sqrt{2}}\Phi^{\cdots}(y,\theta)$ to describe the massless chiral multiplet [31] for the case $S=0$. For $S\neq0$, the superfield (50) represents the chiral supermultiplets of massless higher spin fields with the particle spin content

$$\left(\frac{1}{2},1\right),\left(1,\frac{3}{2}\right),\left(\frac{3}{2},2\right),\ldots,\left(S,S+\frac{1}{2}\right)$$

accompanied by the corresponding auxiliary fields for any integer or half-integer spin $S=\frac{1}{2},1,\frac{3}{2},2,\ldots$. In various approaches the supermultiplets were discussed in the papers [33–40] and many others. The supersymmetry transformations for the higher spin multiplet (50) presented in the notations (52) take the form

$$\begin{aligned}&\delta A^{\cdots}=\sqrt{2}\varepsilon^\lambda\psi_\lambda^{\cdots},\qquad\delta F^{\cdots}=i\sqrt{2}(\bar\varepsilon\tilde\sigma_m\partial^m\psi^{\cdots})\\&\delta\psi_\lambda^{\cdots}=i\sqrt{2}(\sigma_m\bar\varepsilon)_\lambda\partial^mA^{\cdots}+\sqrt{2}\varepsilon_\lambda F^{\cdots}\end{aligned} \tag{53}$$

and coincide with the transformation rules for the $S=0$ chiral multiplet of weight $n=\frac{1}{2}$ [31] if we put $A^{\cdots}=A$, $F^{\cdots}=F$ and $\psi_\lambda^{\cdots}=\psi_\lambda$ in (53).

It was above noted that the θ-twistor superspace is invariant under the axial rotations (37). These phase transformations generate the R-symmetry transformations for $\tilde{F}(\Xi)$ (43)

$$\tilde{F}'(-il_\alpha,\bar\nu^{\dot\alpha},e^{i\varphi}\theta^\alpha)=e^{2in\varphi}\tilde{F}(-il_\alpha,\bar\nu^{\dot\alpha},\theta^\alpha), \tag{54}$$

where n is the corresponding R number. Then taking into account the representation (46) we get the R-symmetry transformation of the generalized chiral superfield $\Phi^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y,\theta)$

$$\Phi'^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y,\theta)=e^{2in\varphi}\Phi^{\dot\alpha_1\ldots\dot\alpha_{2S}}(y,e^{-i\varphi}\theta). \tag{55}$$

We emphasize that the θ-twistor restores the off-shell representation for the superfields of massless higher spin chiral supermultiplets, which is lost in the supertwistor description. The off-shell spinor superfield $\Phi^{\dot\alpha}(y,\theta)$ describes the $\mathcal{N}=1$ $D=4$ gluon and gluino with fixed helicities. The complex conjugate superfield describes the gluon and gluino with opposite helicities. Thus, the states with opposite helicities are associated with the holomorphic $\Xi_{\mathcal{A}}$ and the antiholomorphic parts $\bar\Xi^{\mathcal{A}}$ of the complete θ-twistor space. The construction permits a direct generalization to the case of $\mathcal{N}=4$ $D=4$ super Yang-Mills.

VI. CONCLUSION

The new concept of the supersymmetric θ-twistor, alternative to the well-known supertwistor, and its physical applications have been discussed. The fermionic sector of the θ-twistor is represented by the composite Grassmannian Lorentz vector $(\bar{\nu}\gamma_m\theta)$, providing the extension of the Penrose projective twistor space to a new projective superspace different from the supertwistor space. The new property of the proposed supersymmetrization of the Penrose twistor consists of the presence of the Lorentz *vector* in the fermionic sector of the θ-twistor. The supertwistor fermionic sector is represented by a Lorentz *scalar*. The $\mathcal{N} = 1$ θ-twistor considered here is naturally generalized to include the internal symmetry, similar to the extension for the supertwistor case. This is achieved by the substitution of θ^i_α for θ_α in the θ-twistor components, where the index i belongs to the fundamental representation of the group $SU(N)$. The substitution yields a new composite Grassmannian vector $(\bar{\nu}\gamma_m\theta^i)$ carrying both the space-time and internal symmetry indices. The corresponding extension of the projective CP^3 space yields a new extended projective space, whose fermionic sector *mixes* the space-time and internal symmetries. This property deserves further study along the line developed in [45]. The θ-twistor discussed here is not covariant under the superconformal boosts contrary to the supertwistor, but it is covariant under the maximal subgroup of the superconformal group. The superconformal boosts appear to be a broken symmetry of the θ-twistor space. This breaking correlates with the Gross-Wess effect of the *conformal* symmetry breaking in the *scale* invariant amplitudes of the scalar-*spinor* and scalar-*vector* particles scattering [46]. An attractive property of the breaking is that it is accompanied by restoration of the auxiliary fields of the chiral supermultiplets, absent in the supertwistor description. Having observed this property, we applied the θ-twistor for the construction of the physical supermultiplets describing the $\mathcal{N} = 1$ $D = 4$ physical fields. As a result, we find an infinite chain of massless higher spin $\mathcal{N} = 1$ $D = 4$ chiral supermultiplets $(S, S + 1/2)$ *containing* their auxiliary F fields. The above-mentioned extension of the θ-twistor superspace to describe the $SU(N)$ group degrees of freedom shows a way of an off-shell description of the $\mathcal{N} = 4$ $D = 4$ super Yang-Mills theory. This extension has to be accompanied by the auxiliary F field restoration, resulting in the superconformal symmetry breaking. The breaking is present in the $\mathcal{N} = 1$ $D = 10$ super Yang-Mills theory, but the recent proposal of Berkovits [26] to built its θ-twistor description stimulates the θ-twistor description of $\mathcal{N} = 4$ $D = 4$ super Yang-Mills. Another application [42] of the θ-twistor superspace is connected with a possible way to solve the problem of Lorentz symmetry breaking in supersymmetric non(anti) commutative geometry, alternative to using the twisted Hopf algebra construction recently proposed in [47,48]. In general, it might be that the exact superconformal symmetry is not realized in nature. At least we know that the (super)conformal gravity is plagued with ghosts when its perturbation quantization is considered [49,50]. Then the θ-twistor superspace, minimally breaking superconformal symmetry, but preserving the super-Poincaré and other important symmetries, might be used as a natural base space-time for new supersymmetric model building.

ACKNOWLEDGMENTS

We thank I. Bengtsson, P. Di Vecchia, F. Hassan, P. Howe, S. Theisen, K. Tod, P. Townsend, D. Uvarov and J. W. van Holten for useful discussions. This work was partially supported by the Grants of the Academy of Finland, under the Projects No. 121720 and 127626, and Nordita.

[1] R. Penrose, J. Math. Phys. (N.Y.) **8**, 345 (1967); R. Penrose and W. Rindler, *Spinor and Twistor Methods in Space-time Geometry* (Cambridge University Press, Cambridge, United Kingdom, 1986).
[2] A. Ferber, Nucl. Phys. **B132**, 55 (1978).
[3] E. Witten, Phys. Lett. B **77**, 394 (1978).
[4] E. Witten, Commun. Math. Phys. **252**, 189 (2004).
[5] V. P. Nair, arXiv:hep-th/0506120.
[6] N. Berkovits, Phys. Rev. Lett. **93**, 011601 (2004).
[7] R. Roiban, M. Spradlin, and A. Volovich, J. High Energy Phys. 04 (2004) 012.
[8] G. Georgiou and V. V. Khoze, J. High Energy Phys. 05 (2004) 070.
[9] M. F. Atiyah, V. G. Drinfel'd, N. J. Hitchin, and Yu. I. Manin, Phys. Lett. A **65**, 185 (1978).
[10] E. Corrigan, D. Fairlie, P. Goddard, and S. Templeton, Nucl. Phys. **B140**, 31 (1978).
[11] T. Shirafuji, Prog. Theor. Phys. **70**, 18 (1983).
[12] I. Bengtsson and M. Cederwal, Nucl. Phys. **B302**, 81 (1988).
[13] W. Siegel, arXiv:hep-th/0404255.
[14] W. Siegel, Phys. Rev. D **52**, 1042 (1995).
[15] I. Bengtsson and A. A. Zheltukhin, Phys. Lett. B **570**, 222 (2003).
[16] D. V. Uvarov and A. A. Zheltukhin, J. High Energy Phys. 08 (2002) 008; 03 (2004) 063; Phys. Lett. B **545**, 183 (2002); **565**, 229 (2003).
[17] I. A. Bandos and A. A. Zheltukhin, JETP Lett. **54**, 421 (1991); Phys. At. Nucl. **56**, 113 (1993); Phys. Part. Nucl. **25**, 453 (1994); Classical Quantum Gravity **12**, 609

(1995).
[18] D. V. Uvarov, J. Phys. A **42**, 115204 (2009).
[19] J. van Holten and A. van Proyen, J. Phys. A **15**, 3763 (1982).
[20] N. Berkovits, J. High Energy Phys. 04 (2000) 018; 01 (2008) 065.
[21] B. E. W. Nilsson, Classical Quantum Gravity **3**, L41 (1986).
[22] P. S. Howe, Phys. Lett. B **258**, 141 (1991); **273**, 90 (1991).
[23] M. Tonin, Phys. Lett. B **266**, 312 (1991).
[24] N. Berkovits and S. Cherkis, J. High Energy Phys. 12 (2004) 049.
[25] N. Berkovits and B. Carlini Vallio, J. High Energy Phys. 07 (2000) 015; N. Berkovits, J. High Energy Phys. 08 (2001) 026; 09 (2004) 047; 01 (2006) 005; N. Berkovits and C. R. Mafra, Phys. Rev. Lett. **96**, 011602 (2006).
[26] N. Berkovits, J. High Energy Phys. 04 (2010) 67.
[27] E. Witten, Nucl. Phys. **B266**, 245 (1986).
[28] D. P. Sorokin, V. I. Tkach, D. V. Volkov, and A. A. Zheltukhin, Phys. Lett. B **216**, 302 (1989).
[29] N. Berkovits, Nucl. Phys. **B350**, 193 (1991).
[30] A. A. Zheltukhin, Phys. Lett. B **658**, 82 (2007).
[31] J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton University Press, Princeton, New Jersey, 1992).
[32] D. V. Volkov and A. A. Zheltukhin, JETP Lett. **48**, 63 (1988); Lett. Math. Phys. **17**, 141 (1989); Nucl. Phys. **B335**, 723 (1990).
[33] V. I. Ogievetskii and E. Sokachev, J. Phys. A **10**, 2021 (1977); V. I. Ogievetskii and L. Mezinchesky, Sov. Phys. Usp. **18**, 960 (1975).
[34] C. Fronsdal, Phys. Rev. D **18**, 3624 (1978); J. Fang and C. Fronsdal, Phys. Rev. D **18**, 3630 (1978).
[35] E. S. Fradkin and M. A. Vasiliev, Nuovo Cimento Lett. **25**, 79 (1979); M. A. Vasiliev, Ann. Phys. (N.Y.) **190**, 59 (1989).
[36] F. A. Berends, J. W. van Holten, P. van Niewenhuizen, and B. de Wit, Phys. Lett. B **83**, 188 (1979).
[37] S. J. Gates and W. Siegel, Nucl. Phys. **B164**, 484 (1980).
[38] B. de Wit and J. W. van Holten, Nucl. Phys. **B155**, 530 (1979).
[39] S. M. Kuzenko, A. G. Sibiryakov, and V. V. Postnikov, JETP Lett. **57**, 534 (1993); S. M. Kuzenko and A. G. Sibiryakov, JETP Lett. **57**, 53 (1993).
[40] S. J. Gates, S. M. Kuzenko, and A. G. Sibiryakov, Phys. Lett. B **394**, 343 (1997).
[41] A. A. Zheltukhin, Sov. J. Nucl. Phys. **48**, 375 (1988); JETP Lett. **46**, 262 (1987).
[42] A. A. Zheltukhin, Mod. Phys. Lett. A **21**, 2117 (2006).
[43] O. E. Gusev and A. A. Zheltukhin, JETP Lett. **64**, 487 (1996).
[44] U. Lindström and A. A. Zheltukhin, J. High Energy Phys. 01 (2002) 034.
[45] S. Giddings and P. Nelson, Phys. Rev. Lett. **59**, 2619 (1987).
[46] D. Gross and J. Wess, Phys. Rev. D **2**, 753 (1970).
[47] M. Chaichian, P. P. Kulish, K. Nishijima, and A. Tureanu, Phys. Lett. B **604**, 98 (2004).
[48] M. Chaichian, P. Presnajder, and A. Tureanu, Phys. Rev. Lett. **94**, 151602 (2005).
[49] M. Kaku, P. K. Townsend, and P. van Nieuwenhuizen, Phys. Lett. B **69**, 304 (1977); Phys. Rev. D **17**, 3179 (1978).
[50] M. Kaku, Phys. Rev. D **27**, 2809 (1983).