

***c*-functions in the Born-Infeld extended new massive gravity**İbrahim Güllü,^{*} Tahsin Çağrı Şişman,[†] and Bayram Tekin[‡]*Department of Physics, Middle East Technical University, 06531, Ankara, Turkey*

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We derive and study the equations of motion of the Born-Infeld extension of new massive gravity for globally and asymptotically (anti-)de Sitter spaces, and show that the assumptions of the null-energy condition and holography (that bounds the *c*-function) lead to two simple *c*-functions, one of which is equivalent to the *c*-function of Einstein's gravity. We also show that, at the fixed point, the *c*-function gives the central charge of the Virasoro algebra and the coefficient of the Weyl anomaly up to a constant.

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I. INTRODUCTION

New massive gravity (NMG) that provides a nonlinear extension to the Pauli-Fierz massive gravity was introduced in [1] with the action

$$I_{\text{NMG}} = \frac{1}{\kappa^2} \int d^3x \sqrt{-\det g} \left[\sigma R + \frac{1}{m^2} \left(R_{\mu\nu}^2 - \frac{3}{8} R^2 \right) - 2\lambda m^2 \right]. \quad (1)$$

At the linearized level, this theory is unitary for both flat and (a)dS spaces [1–5] for $\sigma = -1$. The question arises as to what possible additional higher-derivative terms beyond the quadratic ones can be added to this action keeping the unitarity intact. In flat spaces, at the linearized level, such terms will not change the unitarity property. But, for other backgrounds one has to find a guiding principle, since higher-derivative terms could enter in many different forms and combinations. In [6], AdS/CFT correspondence along with the existence of a holographic *c*-theorem was used to extend NMG to $O(R^3)$ and $O(R^4)$. At each order, a new coupling constant is introduced with this procedure. In [7], another proposal of extending NMG to all orders was made which is based on a Born-Infeld (BI) type action, and reads as

$$I_{\text{BI-NMG}} = -\frac{4m^2}{\kappa^2} \int d^3x \left[\sqrt{-\det \left(g + \frac{\sigma}{m^2} G \right)} - \left(1 - \frac{\lambda}{2} \right) \sqrt{-\det g} \right], \quad (2)$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ and $\sigma = \pm 1$. What is quite interesting about (2) is that at the first order it gives the cosmological Einstein-Hilbert action, at the second order it gives the NMG action, and at the third and fourth orders it gives the action introduced in [6] upon fixing the arbitrary coupling constants in the latter. See Appendix A for $O(R^4)$ matching of (2) with that of [6], $O(R^3)$ matching was

already shown in [7]. We expect that this matching will work at any order.

BI type actions, with their singularity-free solutions, and due to their nice properties such as ghost freedom and causal propagation, have appeared in physics ranging from Maxwell theory to gravity and string theory. The fact that with such a simple action (2) one can describe nonlinearly massive gravity, albeit in three dimensions, is remarkable. As we shall see below, what is also interesting with the BI-NMG is that, unlike the NMG, for a given viable λ there is a unique vacuum solution with an effective cosmological constant Λ . In the case of NMG and other finite order deformations of it with higher curvatures, one has to deal with degenerate vacua [two for NMG with cosmological constants $\Lambda_{\pm} = -2m^2(\sigma \pm \sqrt{1 + \lambda})$]. In quantum field theory, degenerate vacua would normally cause no problem, but for gravity it is not clear at all which vacuum should be chosen: as far as energy is concerned, one cannot compare spaces which asymptote to different constant curvature manifolds. (Note that in NMG, at $\lambda = -1$, degeneracy is lifted and one has a unique vacuum. See the solutions of this theory [1,8]. But, clearly $\lambda = -1$ is a very special point.)

In this work, we find the field equations of BI-NMG, and study the constant curvature and asymptotically constant curvature spaces. More specifically, with the assumption of the null-energy condition [9], we show that the BI-NMG action has two simple *c*-functions. One of these *c*-functions is just like the *c*-function of Einstein's gravity and at infinity coincides with the central charge of the Virasoro algebra and the Weyl anomaly coefficient up to a constant.

The layout of the paper is as follows. In Sec. II, we give the equations of motion for the exact BI-NMG theory and find constant curvature solutions. In Sec. III, we investigate the *c*-function of the BI-NMG theory, and compute the central charge and the Weyl anomaly coefficient. Some of the computations are expounded upon in the appendices.

II. EQUATIONS OF MOTION AND CONSTANT CURVATURE SOLUTIONS

To find the equations of motion it is somewhat better (if less compact) to expand the determinant with the help of

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$$\det A = \frac{1}{6}[(\text{Tr}A)^3 - 3 \text{Tr}A \text{Tr}(A^2) + 2 \text{Tr}(A^3)], \quad (3)$$

which is valid for a generic 3×3 matrix. Then, one computes

$$\begin{aligned} \text{Tr}\left(1 + \frac{\sigma}{m^2} g^{-1} G\right) &= 3 - \frac{\sigma}{2m^2} R, \\ \text{Tr}(g^{-1} G)^2 &= R_{\mu\nu}^2 - \frac{1}{4} R^2, \\ \text{Tr}(g^{-1} G)^3 &= R^{\mu\nu} R_{\nu}{}^{\alpha} R_{\alpha\mu} - \frac{3}{2} R R_{\mu\nu}^2 + \frac{3}{8} R^3, \end{aligned} \quad (4)$$

which reduces the action to [10]

$$I_{\text{BI-NMG}} = -\frac{4m^2}{\kappa^2} \int d^3x \sqrt{-\det g} F(R, K, S), \quad (5)$$

where

$$F(R, K, S) \equiv \sqrt{1 - \frac{\sigma}{2m^2} \left(R + \frac{\sigma}{m^2} K - \frac{1}{12m^4} S\right)} - \left(1 - \frac{\lambda}{2}\right), \quad (6)$$

$$K \equiv R_{\mu\nu}^2 - \frac{1}{2} R^2, \quad S \equiv 8R^{\mu\nu} R_{\mu\alpha} R^{\alpha}{}_{\nu} - 6R R_{\mu\nu}^2 + R^3. \quad (7)$$

After a quite lengthy calculation which we partly give in Appendix B, one arrives at

$$\begin{aligned} -\frac{\kappa^2}{8m^2} T_{\mu\nu} &= -\frac{1}{2} F g_{\mu\nu} + (g_{\mu\nu} \square - \nabla_{\mu} \nabla_{\nu}) F_R + F_R R_{\mu\nu} - \frac{\sigma}{m^2} \{2\nabla_{\alpha} \nabla_{\mu} (F_R R^{\alpha}{}_{\nu}) - g_{\mu\nu} \nabla_{\beta} \nabla_{\alpha} (F_R R^{\alpha\beta}) - \square(F_R R_{\mu\nu}) \\ &\quad - 2F_R R_{\nu}{}^{\alpha} R_{\mu\alpha} + g_{\mu\nu} \square(F_R R) - \nabla_{\mu} \nabla_{\nu} (F_R R) + F_R R R_{\mu\nu}\} - \frac{1}{2m^4} \{4F_R R^{\rho}{}_{\mu} R_{\rho\alpha} R^{\alpha}{}_{\nu} \\ &\quad + 2g_{\mu\nu} \nabla_{\alpha} \nabla_{\beta} (F_R R^{\beta\rho} R^{\alpha}{}_{\rho}) + 2\square(F_R R_{\nu}{}^{\rho} R_{\mu\rho}) - 4\nabla_{\alpha} \nabla_{\mu} (F_R R_{\nu}{}^{\rho} R^{\alpha}{}_{\rho}) + 2\nabla_{\alpha} \nabla_{\mu} (F_R R R^{\alpha}{}_{\nu}) \\ &\quad - g_{\mu\nu} \nabla_{\alpha} \nabla_{\beta} (F_R R R^{\alpha\beta}) - \square(F_R R R_{\mu\nu}) - 2F_R R R_{\nu}{}^{\rho} R_{\mu\rho} - g_{\mu\nu} \square(F_R R^2_{\alpha\beta}) + \nabla_{\nu} \nabla_{\mu} (F_R R^2_{\alpha\beta}) - F_R R^2_{\alpha\beta} R_{\mu\nu} \\ &\quad + \frac{1}{2} g_{\mu\nu} \square(F_R R^2) - \frac{1}{2} \nabla_{\mu} \nabla_{\nu} (F_R R^2) + \frac{1}{2} F_R R^2 R_{\mu\nu}\}, \end{aligned} \quad (8)$$

where

$$F_R = \frac{\partial F}{\partial R} = -\frac{\sigma}{4m^2 [F + (1 - \frac{\lambda}{2})]}. \quad (9)$$

Clearly, flat spacetime is a solution for these equations when $\lambda = 0$, since the Riemann tensor is zero which yields $F = 0$, and $F_R = -\frac{\sigma}{4m^2}$. In order to find the constant curvature solutions, let

$$\begin{aligned} R_{\mu\rho\nu\sigma} &= \Lambda(g_{\mu\nu} g_{\rho\sigma} - g_{\mu\sigma} g_{\nu\rho}), \\ R_{\rho\sigma} &= 2\Lambda g_{\rho\sigma}, \quad R = 6\Lambda. \end{aligned} \quad (10)$$

With these, F becomes

$$F(R, K, S) = \sqrt{\left(1 - \frac{\sigma\Lambda}{m^2}\right)^3 - \left(1 - \frac{\lambda}{2}\right)}, \quad (11)$$

where the cosmological constant is restricted as $\frac{\sigma}{m^2} \Lambda \leq 1$. Putting these in the equation of motion, one obtains

$$\begin{aligned} 0 &= \left(-\frac{1}{2} F - \frac{\sigma\Lambda}{2m^2 [F + (1 - \frac{\lambda}{2})]} + \frac{\Lambda^2}{m^4 [F + (1 - \frac{\lambda}{2})]} \right. \\ &\quad \left. - \frac{\sigma\Lambda^3}{2m^6 [F + (1 - \frac{\lambda}{2})]}\right) g_{\mu\nu}. \end{aligned} \quad (12)$$

which puts another constraint $\sigma \frac{\Lambda}{m^2} \neq 1$. Let us define $x \equiv -\sigma \frac{\Lambda}{m^2}$, then the above equation can be put in the following

form

$$(1+x)^2 = \left(1 - \frac{\lambda}{2}\right) \sqrt{(1+x)^3}. \quad (13)$$

which restricts λ as $\lambda < 2$. Here, note that $(1 - \frac{\lambda}{2})$ cannot be equal to zero, since it implies $\sigma \frac{\Lambda}{m^2} = 1$ which then leads to vanishing F . It is important to note that BI-NMG theory does not restrict λ for generic, that is nonmaximally symmetric, spaces. Solving (13) gives

$$\Lambda = \sigma m^2 \lambda \left(1 - \frac{\lambda}{4}\right), \quad \lambda < 2. \quad (14)$$

Unlike the NMG case, as we mentioned in the introduction, for a viable λ , there is a *unique* vacuum solution. For $\sigma = +1$, Λ and λ have the same signs; for $\sigma = -1$, they have the opposite signs. Minimum of Λ occurs at $\lambda = 2$, but this point is not allowed. (Note that the discussion above works exactly for static BTZ black holes: namely, they are solutions of this theory.)

III. c-FUNCTIONS OF THE BI-NMG THEORY

A. Central charges

In order to discuss the degrees of freedom for the two-dimensional CFT theory that is living on the boundary of AdS₃, let us determine the central charges. In pure Einstein gravity in AdS, the global SO(2, 2) symmetry is enlarged to

two copies of an infinite dimensional symmetry (Virasoro algebra) which has the following central charge [11]

$$c = \frac{3\ell}{2G_3}, \quad (15)$$

where ℓ is the AdS length defined as $\Lambda \equiv -\frac{1}{\ell^2}$, and G_3 is the three-dimensional Newton's constant which is related to κ as $G_3 = \frac{\kappa^2}{16\pi}$. In understanding (15), one has to be careful about the role played by the symmetry and the role played by the dynamics, that is the field equations. To get (15), certain falloff conditions on how the space asymptotes AdS₃ are specified. Therefore, as the theory and the asymptotic falloff conditions change, c changes. For higher-derivative gravity theories, central charge is given as [12–15]

$$c = \frac{\ell}{2G_3} g_{\mu\nu} \frac{\partial \mathcal{L}_3}{\partial R_{\mu\nu}}, \quad (16)$$

where \mathcal{L}_3 in the BI-NMG gravity is $\mathcal{L}_3 = -4m^2 F(R, K, S)$. Then,

$$\begin{aligned} \frac{\partial \mathcal{L}_3}{\partial R_{\mu\nu}} = & \frac{\sigma}{[F + (1 - \frac{\lambda}{2})]} \left\{ \left[1 - \frac{\sigma}{m^2} R + \frac{1}{2m^4} \left(R_{\alpha\beta}^2 - \frac{1}{2} R^2 \right) \right] \right. \\ & \left. \times g^{\mu\nu} + \frac{2}{m^2} \left(\sigma + \frac{1}{2m^2} R \right) R^{\mu\nu} - \frac{2}{m^4} R_{\mu\alpha} R^{\alpha\nu} \right\}, \end{aligned}$$

and using (10) with $\Lambda \equiv -\frac{1}{\ell^2}$, the central charge becomes

$$c = \frac{3\sigma\ell}{2G_3} \sqrt{1 + \frac{\sigma}{\ell^2 m^2}} = \frac{3\sigma}{4G_3\ell} (2 - \lambda). \quad (17)$$

where $\lambda < 2$. For c to be positive, σ should be positive: This seems to be in apparent conflict with the unitarity of NMG which required $\sigma = -1$. But, one should still study the bulk unitarity of $\sigma = +1$ theory in BI-NMG, which we have not done yet.

B. c-functions

Let us recall that Zamolodchikov [16] proved, under certain symmetry assumptions, that any nonconformal two-dimensional theory has a so called c -function which monotonically decreases under the renormalization group flow towards lower energies and matches that of the central charge of the corresponding conformal field theory that appears at the *fixed* points of the flow. Beyond two dimensions, it is hard to prove the existence of a c -theorem in general. But, one can explicitly construct c -functions in certain theories [17] that satisfy the null energy condition, $T_{\mu\nu} \zeta^\mu \zeta^\nu \geq 0$, where ζ is an arbitrary null vector. Following [17], consider the domain wall ansatz:

$$ds^2 = e^{2A(r)} (-dt^2 + dx^2) + dr^2, \quad (18)$$

and plug it to the field equations of pure Einstein gravity to get

$$\frac{2A''}{\kappa^2} = T'_t - T'_r \leq 0. \quad (19)$$

The c -function can simply be defined as

$$c(r) = \frac{24\pi}{\kappa^2 A'(r)} = \frac{3}{2G_3 A'(r)}. \quad (20)$$

With this definition, $c(r)$ satisfies a monotonic increase towards higher energy, for increasing r . Its prefactor is arbitrary, but one can fix it in such a way that one gets the central charge of the Virasoro algebra that appears in the boundary of AdS₃.

Plugging (18) to (8), one arrives

$$\begin{aligned} & \left(\frac{2m}{\kappa^2} \right) \frac{[A'' + (A')^2 + \sigma m^2] A''}{\sqrt{[m^2 + \sigma(A')^2][A'' + (A')^2 + \sigma m^2]^2}} \\ & = T'_t - T'_r \leq 0, \end{aligned} \quad (21)$$

where m is taken to be positive. In order to understand the constraints, let us look at

$$F_R = - \frac{\sigma m}{4\sqrt{[m^2 + \sigma(A')^2][A'' + (A')^2 + \sigma m^2]^2}}, \quad (22)$$

which appears in the equation of motion. Finiteness and reality of F_R set two constraints

$$[A'' + (A')^2 + \sigma m^2] \neq 0, \quad m^2 + \sigma(A')^2 > 0. \quad (23)$$

First of all, observe that the second constraint is automatically satisfied for the $\sigma = +1$ case, but implies a bound for A' for the $\sigma = -1$ case. Let us discuss what the first constraint implies: $[A'' + (A')^2 + \sigma m^2]$ cannot change sign for $r \in (-\infty, \infty)$. The sign of A'' and the sign of $[A'' + (A')^2 + \sigma m^2]$ are correlated because of (21): namely, for both values of σ , they should have the opposite signs. Therefore, the sign of A'' , also, cannot change. Hence, A' is either monotonically increasing or decreasing. Let us discuss the $\sigma = +1$ and $\sigma = -1$ cases separately:

- (i) $\sigma = +1$ case: If one chooses $A'' \geq 0$, then $[A'' + (A')^2 + m^2]$ is positive which conflicts with (21). So, this choice is forbidden. The other option: $A'' \leq 0$ leads to $A'' + (A')^2 + m^2 > 0$ which only puts an upper bound on $|A''|$. Therefore, A' is a monotonically decreasing function. To see the implications of the second constraint, we should emphasize that the AdS/CFT correspondence requires that the spacetime is asymptotically AdS such that as $r \rightarrow \infty$ (the UV region), $A(r)$ takes the form $A(r) = r\sqrt{|\Lambda|}$. This gives a lower bound of $\sqrt{|\Lambda|}$ at $r \rightarrow \infty$ for A' . Therefore, A' is positive at every r . Then, if one assumes that the spacetime at $r \rightarrow -\infty$ (the IR region) is also an AdS spacetime, A' is bounded from above. To sum up, for $\sigma = +1$, $A'' \leq 0$ is proven and A' is a monotonically decreasing function (a possible form is depicted in the first plot of Fig. 1). It is important to understand the role played by the

AdS/CFT correspondence here: without it we would still have $A'' \leq 0$, but A' need not be bounded.

- (ii) $\sigma = -1$ case: The second constraint can be written as $m > |A'| > -m$. Monotonic and bounded behavior of A' also requires that $A'' = 0$ as $r \rightarrow \pm\infty$. If one chooses $A'' \leq 0$, then $[A'' + (A')^2 - m^2]$ should be positive; however, this is not possible with $m^2 - (A')^2 > 0$. So, $A'' \leq 0$ is ruled out. Consider the other case: $A'' \geq 0$, which implies $m^2 - (A')^2 > A''$. Therefore, A' is a monotonically increasing function. Here, the fact that $A'(+\infty)$ and $A'(-\infty)$ are allowed to take positive and negative values, respectively, leads to the interesting case of having two AdS boundaries (this means $A(r) \rightarrow \infty$). Overall, for $\sigma = -1$, we proved that $A'' \geq 0$ and so A' is a monotonically increasing function (a possible form is shown in the second plot of Fig. 1).

The above analysis shows that the assumption of the null-energy condition leads to a simple c -function in the BI-NMG theory. As shown above, for $\sigma = +1$, $A'' \leq 0$ and for $\sigma = -1$, $A'' \geq 0$. Then, the c -function for the BI-NMG theory can be defined as in the Einstein's gravity:

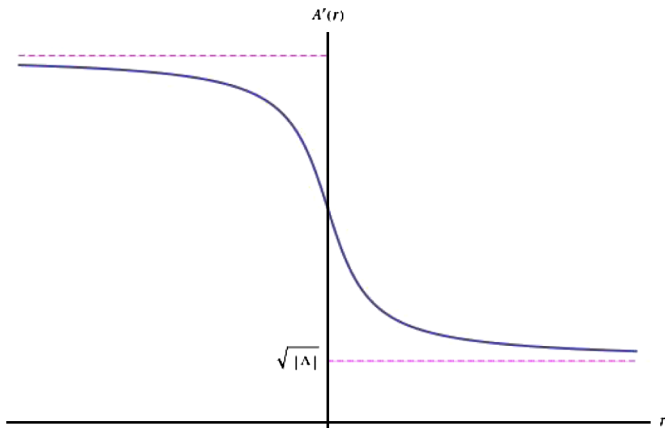
$$c(r) = \frac{3\sigma}{2G_3 A'(r)} \Rightarrow \frac{dc}{dr} \geq 0, \quad (24)$$

which at $r \rightarrow \infty$ gives the central charge of the Virasoro algebra (17) (and the Weyl anomaly to be discussed below) upto a factor. However, this is not the only possibility. Another c -function can be determined by considering the inequality

$$\frac{\sigma A''}{\sqrt{[m^2 + \sigma(A')^2]}} \leq 0, \quad (25)$$

which is also implied by (21). Then, the c -function is found by simply integrating the above form as

$$c(r) = -\sigma \arctan\left(\frac{A'(r)}{\sqrt{[m^2 + \sigma(A')^2]}}\right), \quad (26)$$



which is also a monotonically increasing function. Note that a minus sign is introduced to get the increasing behavior. The meaning of this c -function (26) in its exact form is not immediately clear, but an expansion of (25) around large m^2 gives at the desired order the c -functions introduced by [6] (Note that at each order $(A')^n$ the c -functions of [6] have arbitrary parameters, while these are fixed in our case.).

C. Weyl anomaly

The computation of the coefficient of Weyl anomaly can be done as follows [18,19]. The Euclidean metric

$$ds^2 = \frac{dr^2}{1 - \Lambda r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (27)$$

solves the equations of motion with $\Lambda = \sigma m^2 \lambda (1 - \frac{\lambda}{4})$, $\lambda < 2$. Here, note that AdS space is obtained for $\sigma = +1$ with the bound $\lambda < 0$, and for $\sigma = -1$ with the bound $2 > \lambda > 0$. With this metric our action reads

$$I_{\text{BI}} = \left(-\frac{4m^2}{\kappa^2}\right) \left[\left(1 - \frac{\sigma\Lambda}{m^2}\right)^{3/2} - \left(1 - \frac{\lambda}{2}\right) \right] \int d^3x \sqrt{\text{det}g}. \quad (28)$$

Defining $\Lambda \equiv -\frac{1}{\ell^2}$, and putting a cutoff (L), one arrives at

$$I_{\text{BI}} = \left(\frac{16\pi m^2}{\kappa^2 \ell}\right) \lambda \left(1 - \frac{\lambda}{4}\right) \left(1 - \frac{\lambda}{2}\right) \frac{L^2}{2} \left[\sqrt{\left(\frac{\ell}{L}\right)^2 + 1} - \left(\frac{\ell}{L}\right)^2 \text{arcsinh}\left(\frac{L}{\ell}\right) \right], \quad (29)$$

which is valid for both $\sigma = +1$ and $\sigma = -1$ [Note that $\text{sign}(\lambda) = -\sigma$]. As $L \rightarrow \infty$, after dropping a quadratic divergence, the action becomes

$$I_{\text{BI}} \approx -\left(\frac{8\pi m^2 \ell}{\kappa^2}\right) \lambda \left(1 - \frac{\lambda}{2}\right) \left(1 - \frac{\lambda}{4}\right) \ln\left(\frac{2L}{\ell}\right) \equiv \frac{c}{3} \ln\left(\frac{2L}{\ell}\right), \quad (30)$$

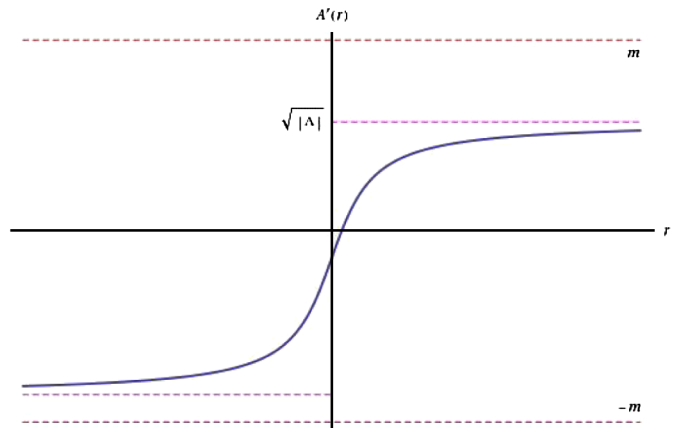


FIG. 1 (color online). The first graph gives a possible behavior for $A'(r)$ in the $\sigma = +1$ case, while the second one shows a possible behavior for $A'(r)$ in the $\sigma = -1$ case.

where $\frac{\xi}{3}$ is the coefficient of Weyl anomaly. Therefore, the central charge reads

$$c = \left(\frac{3\sigma}{4G_3\ell}\right)(2 - \lambda), \quad (31)$$

which is exactly equal to the one obtained from Wald's formula (17).

IV. CONCLUSION

We derived the equations of motion of the BI-NMG theory and discussed the global AdS and asymptotically AdS solutions. We have shown that assuming the null-energy condition, the theory admits simple c -functions in the spirit introduced by Zamolodchikov. For asymptotically AdS spaces, the value of the c -function at the boundary matches up to a constant the central charge of the asymptotic symmetry algebra and the coefficient of the Weyl anomaly. We have also shown that the BI-NMG theory is free from the vacuum degeneracy problem of the NMG theory. We still have not yet proven that BI-NMG theory has both a unitary bulk and a unitary boundary theory, which was the problem in NMG [1] and the $O(R^3)$ extended NMG [6]. After the completion of this manuscript, two related works [20,21] have appeared. In [20], a detailed discussion on the AdS black hole solutions

of the $O(R^3)$ extension of [6] and the BI-NMG theory as well as the computation of the central charge are given. Some of our results overlap with [20]. In [21], an infinite order extension of NMG is discussed by requiring the existence of a c -function. That c -function can be reproduced by expanding (25).

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APPENDIX A: $O(R^4)$ EXPANSION OF THE BI ACTION

First, one should choose $\sigma = -1$. Using the expansion,

$$\begin{aligned} [\det(1 + A)]^{1/2} &= 1 + \frac{1}{2}\text{Tr}A - \frac{1}{4}\text{Tr}(A^2) + \frac{1}{8}(\text{Tr}A)^2 \\ &+ \frac{1}{6}\text{Tr}(A^3) - \frac{1}{8}\text{Tr}A \text{Tr}(A^2) + \frac{1}{48}(\text{Tr}A)^3 \\ &- \frac{1}{8}\text{Tr}(A^4) + \frac{1}{32}[\text{Tr}(A^2)]^2 \\ &+ \frac{1}{12}\text{Tr}A \text{Tr}(A^3) - \frac{1}{32}(\text{Tr}A)^2\text{Tr}(A^2) \\ &+ \frac{1}{384}(\text{Tr}A)^4 + O(A^5). \end{aligned}$$

Then, $O(R^4)$ contributions to the Born-Infeld action becomes

$$\begin{aligned} O(R^4): \frac{1}{m^8} &\left[-\frac{1}{8}\text{Tr}(g^{-1}Gg^{-1}Gg^{-1}Gg^{-1}G) + \frac{1}{32}[\text{Tr}(g^{-1}Gg^{-1}G)]^2 + \frac{1}{12}\text{Tr}(g^{-1}Gg^{-1}Gg^{-1}G)\text{Tr}(g^{-1}G) \right. \\ &- \frac{1}{32}[\text{Tr}(g^{-1}G)]^2\text{Tr}(g^{-1}Gg^{-1}G) + \frac{1}{384}[\text{Tr}(g^{-1}G)]^4 \left. \right] = -\frac{1}{8m^8} \left[G^{\mu\nu}G_{\nu\alpha}G^{\alpha\beta}G_{\beta\mu} - \frac{1}{4}(G^2_{\mu\nu})^2 \right. \\ &- \frac{2}{3}(G^{\mu\nu}G_{\nu\alpha}G^{\alpha\mu})G^\beta_\beta + \frac{1}{4}(G^\beta_\beta)^2G^2_{\mu\nu} - \frac{1}{48}(G^\beta_\beta)^4 \left. \right]. \end{aligned}$$

Using (4) and

$$\begin{aligned} G^{\mu\nu}G_{\nu\alpha}G^{\alpha\beta}G_{\beta\mu} &= R^{\mu\nu}R_{\nu\alpha}R^{\alpha\beta}R_{\beta\mu} - 2R^{\mu\nu}R_{\nu\alpha}R^\alpha_\mu R \\ &+ \frac{3}{2}R^2_{\mu\nu}R^2 - \frac{5}{16}R^4, \end{aligned}$$

one can obtain

$$\begin{aligned} O(R^4): &= -\frac{1}{8m^8} \left[R^{\mu\nu}R_{\nu\alpha}R^{\alpha\beta}R_{\beta\mu} - \frac{5}{3}R^{\mu\nu}R_{\nu\alpha}R^\alpha_\mu R \right. \\ &+ \frac{19}{16}R^2_{\mu\nu}R^2 - \frac{1}{4}(R^2_{\mu\nu})^2 - \frac{169}{768}R^4 \left. \right]. \end{aligned}$$

The coefficients of the above form solve the equations of consistency for $O(R^4)$ extension in [6].

APPENDIX B: DERIVING THE EQUATIONS OF MOTION

In order to obtain equations of motion, let us consider the following ($\sigma = -1$) action

$$I = I_{\text{BI-NMG}} + I_{\text{matter}}.$$

Then,

$$\begin{aligned} \frac{\delta I_{\text{BI-NMG}}}{\delta g^{\mu\nu}} &= -\frac{4m^2}{\kappa^2} \int d^3x \sqrt{-\det g} \left[-\frac{1}{2}Fg_{\mu\nu} \right. \\ &+ F_R \left(\frac{\delta R}{\delta g^{\mu\nu}} - \frac{1}{m^2} \frac{\delta K}{\delta g^{\mu\nu}} - \frac{1}{6m^4} \frac{\delta S}{\delta g^{\mu\nu}} \right) \left. \right], \end{aligned}$$

where $\frac{\delta F}{\delta R}$ is defined as F_R and given (9). δK reads as

$$\begin{aligned} \delta K &= \delta R^2_{\mu\nu} - R\delta R \\ &= (2R^\alpha_\nu R_{\mu\alpha} - RR_{\mu\nu})\delta g^{\mu\nu} - (2R^\alpha_\nu \nabla_\mu \nabla_\alpha \\ &- g_{\mu\nu}R^{\alpha\beta} \nabla_\beta \nabla_\alpha - R_{\mu\nu} \square)\delta g^{\mu\nu} - R(g_{\mu\nu} \square \\ &- \nabla_\mu \nabla_\nu)\delta g^{\mu\nu}. \end{aligned}$$

The δS term is

$$\begin{aligned}
\delta S &= 8\delta(R^{\mu\nu}R_{\mu\alpha}R^{\alpha\nu}) - 6\delta(RR^2_{\mu\nu}) + \delta(R^3) \\
&= 3[8R^{\beta}_{\mu}R_{\beta\alpha}R^{\alpha\nu} - 4RR^{\alpha}_{\nu}R_{\mu\alpha} + R_{\mu\nu}(R^2 - 2R^2_{\alpha\beta})]\delta g^{\mu\nu} + 12(g_{\mu\nu}R^{\alpha\rho}R^{\beta}_{\rho}\nabla_{\beta}\nabla_{\alpha} + R_{\mu}{}^{\rho}R_{\nu\rho}\square \\
&\quad - 2R^{\alpha\rho}R_{\nu\rho}\nabla_{\mu}\nabla_{\alpha})\delta g^{\mu\nu} - 6R(g_{\mu\nu}R^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha} + R_{\mu\nu}\square - 2R^{\alpha}_{\nu}\nabla_{\mu}\nabla_{\alpha})\delta g^{\mu\nu} + 3(R^2 - 2R^2_{\alpha\beta})(g_{\mu\nu}\square \\
&\quad - \nabla_{\mu}\nabla_{\nu})\delta g^{\mu\nu}.
\end{aligned}$$

Then, the equation of motion can be found as

$$\begin{aligned}
-\frac{\kappa^2}{8m^2}T_{\mu\nu} &= -\frac{1}{2}Fg_{\mu\nu} + (g_{\mu\nu}\square - \nabla_{\mu}\nabla_{\nu})F_R + F_R R_{\mu\nu} + \frac{1}{m^2}F_R\left[RR_{\mu\nu} - 2R_{\lambda\nu\alpha\mu}R^{\lambda\alpha} - \square\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\right] \\
&\quad + \frac{1}{m^2}\left[2\nabla_{\alpha}F_R\nabla_{\mu}R^{\alpha\nu} - 2\nabla_{\alpha}F_R\nabla^{\alpha}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) - \nabla_{\nu}F_R\nabla_{\mu}R\right] + \frac{1}{m^2}\left[(2R^{\alpha}_{\nu}\nabla_{\alpha}\nabla_{\mu} - g_{\mu\nu}R^{\alpha\beta}\nabla_{\beta}\nabla_{\alpha} \right. \\
&\quad \left. - R_{\mu\nu}\square) + R(g_{\mu\nu}\square - \nabla_{\mu}\nabla_{\nu})\right]F_R - \frac{1}{m^4}F_R(2R^{\rho}_{\mu}R_{\rho\alpha}R^{\alpha\nu} + [g_{\mu\nu}\nabla_{\alpha}\nabla_{\beta}(R^{\beta\rho}R^{\alpha}_{\rho}) + \square(R_{\nu}{}^{\rho}R_{\mu\rho}) \\
&\quad - 2\nabla_{\alpha}\nabla_{\mu}(R_{\nu}{}^{\rho}R^{\alpha}_{\rho})]) - \frac{1}{2m^4}F_R([2\nabla_{\alpha}\nabla_{\mu}(RR^{\alpha\nu}) - g_{\mu\nu}\nabla_{\alpha}\nabla_{\beta}(RR^{\alpha\beta}) - \square(RR_{\mu\nu})] - 2RR_{\nu}{}^{\rho}R_{\mu\rho}) \\
&\quad + \frac{1}{2m^4}F_R[(g_{\mu\nu}\square - \nabla_{\nu}\nabla_{\mu}) + R_{\mu\nu}]\left(R^2_{\alpha\beta} - \frac{1}{2}R^2\right) - \frac{2}{m^4}\nabla_{\alpha}F_R[\nabla^{\alpha}(R_{\nu}{}^{\rho}R_{\mu\rho}) + g_{\mu\nu}\nabla_{\beta}(R^{\beta\rho}R^{\alpha}_{\rho}) \\
&\quad - \nabla_{\mu}(R_{\nu}{}^{\rho}R^{\alpha}_{\rho})] + \frac{2}{m^4}\nabla_{\alpha}(R_{\nu}{}^{\rho}R^{\alpha}_{\rho})\nabla_{\mu}F_R + \frac{1}{m^4}\nabla_{\alpha}F_R[\nabla^{\alpha}(RR_{\mu\nu}) + g_{\mu\nu}\nabla_{\beta}(RR^{\alpha\beta}) - \nabla_{\mu}(RR^{\alpha\nu})] \\
&\quad - \frac{1}{m^4}\nabla_{\alpha}(RR^{\alpha\nu})\nabla_{\mu}F_R - \frac{1}{2m^4}(\nabla_{\mu}F_R\nabla_{\nu} + \nabla_{\nu}F_R\nabla_{\mu} - 2g_{\mu\nu}\nabla_{\rho}F_R\nabla^{\rho})\left(R^2_{\alpha\beta} - \frac{1}{2}R^2\right) \\
&\quad - \frac{1}{m^4}(g_{\mu\nu}R^{\beta\rho}R^{\alpha}_{\rho}\nabla_{\alpha}\nabla_{\beta} + R_{\nu}{}^{\rho}R_{\mu\rho}\square - 2R_{\nu}{}^{\rho}R^{\alpha}_{\rho}\nabla_{\alpha}\nabla_{\mu})F_R - \frac{1}{2m^4}(2RR^{\alpha}_{\nu}\nabla_{\alpha}\nabla_{\mu} - g_{\mu\nu}RR^{\alpha\beta}\nabla_{\alpha}\nabla_{\beta} \\
&\quad - RR_{\mu\nu}\square)F_R + \frac{1}{2m^4}\left(R^2_{\alpha\beta} - \frac{1}{2}R^2\right)(g_{\mu\nu}\square - \nabla_{\nu}\nabla_{\mu})F_R,
\end{aligned}$$

where

$$\nabla_{\alpha}\nabla_{\mu}R^{\alpha\nu} = \frac{1}{2}\nabla_{\mu}\nabla_{\nu}R + R_{\lambda\mu}R^{\lambda\nu} - R_{\lambda\nu\alpha\mu}R^{\lambda\alpha}$$

is used. This form is more suitable for the linearization and the conserved charge analysis which we shall return in a separate work.

The trace of (8) is also of some use

$$\begin{aligned}
-\frac{\kappa^2}{8m^2}T &= -\frac{3}{2}F + 2\square F_R + F_R R - \frac{1}{m^2}[\nabla_{\beta}\nabla_{\alpha}(F_R R^{\alpha\beta}) + 2F_R R^2_{\mu\nu} - \square(F_R R) - F_R R^2] - \frac{1}{2m^4}\left[4F_R R^{\rho}_{\mu}R_{\rho\alpha}R^{\alpha\mu} \right. \\
&\quad \left. + 2\nabla_{\alpha}\nabla_{\beta}(F_R R^{\beta\rho}R^{\alpha}_{\rho}) - \nabla_{\alpha}\nabla_{\beta}(F_R R R^{\alpha\beta}) - 3F_R R R^2_{\mu\nu} + \frac{1}{2}F_R R^3\right].
\end{aligned}$$

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