Lemaitre-Tolman-Bondi dust spacetimes: Symmetry properties and some extensions to the dissipative case

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We consider extensions of Lemaitre-Tolman-Bondi (LTB) spacetimes to the dissipative case. We have previously carried out a systematic study on LTB. This study is based on two different aspects of LTB. On the one hand, a symmetry property of LTB will be presented. On the other hand, the description of LTB in terms of some fundamental scalar functions (structure scalars) appearing in the orthogonal splitting of Riemann tensor will be provided. We shall consider as natural generalizations of LTB (hereafter referred to as GLTB) either those metrics admitting some similar kind of symmetry as LTB, or those sharing structure scalars with similar dependence on the metric.

I. INTRODUCTION

Lemaitre-Tolman-Bondi (LTB) dust models [1–3] are among the oldest and most interesting solutions to Einstein equations. They describe spherically symmetric distribution of inhomogeneous nondissipative dust (see [4,5] for a detailed description of these spacetimes).

They have been used as cosmological models (see [6–9] and references therein), in the study of gravitational collapse and the problem of the cosmic censorship [10–16], and in quantum gravity [17,18].

A renewed interest in LTB has appeared, in relation with recent observations of type Ia supernovae, indicating that the expansion of the universe is accelerating. Indeed, even if it is true that there is a general consensus to invoke dark energy as a source of antigravity for understanding the cosmic acceleration, it is also true that a growing number of researchers consider that inhomogeneities can account for the observed cosmic acceleration, without invoking dark energy (see [19–25] and references therein).

Now, in spite of all their interest, LTB spacetimes present an important limitation, namely: they do not admit dissipative fluxes. This is a serious shortcoming since it is already an established fact that gravitational collapse is a highly dissipative process (see [26–28] and references therein). This dissipation is required to account for the very large (negative) binding energy of the resulting compact object (of the order of -10^{53} erg).

Indeed, it appears that the only plausible mechanism to carry away the bulk of the binding energy of the collapsing star, leading to a neutron star or black hole is neutrino emission [29].

Dissipation processes are usually treated invoking two possible (opposite) approximations: diffusion and streaming out.

In the diffusion approximation, it is assumed that the energy flux of radiation (as that of thermal conduction) is proportional to the gradient of temperature. In this regime, an equation of transport should be assumed in order to obtain the temperature distribution for each model.

The diffusion approximation is in general very sensible, since it applies whenever the mean free path of particles responsible for the propagation of energy is very small as compared with the typical length of the object, a circumstance found very often in astrophysical scenarios.

In fact, for a main sequence star such as the sun, the mean free path of photons at the center, is of the order of 2 cm. Also, the mean free path of trapped neutrinos in compact cores of densities about 10^{12} g.cm^{-3}. becomes smaller than the size of the stellar core [30,31].

Furthermore, the observational data collected from supernovae 1987A indicate that the regime of radiation transport prevailing during the emission process is closer to the diffusion approximation than to the streaming out limit [32].

However, in many other circumstances, the mean free path of particles transporting energy may be large enough to justify the free streaming approximation. Therefore, we shall include simultaneously both limiting cases of radiative transport (diffusion and streaming out), allowing us to describe a wide range of situations.

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On the other hand, at cosmological scales, even if it is true that cold dark matter is noncollisional and strongly dominated by rest mass, so that pressure and heat flux terms (of kinetic nature) are negligible, it could be interesting to test the stability of conclusions and results based on the assumptions above, with respect to small deviations from those assumptions. In this sense, GLTB's with arbitrarily small (but nonvanishing) dissipative fluxes could be very helpful.

From all of the above, the motivations to generalize LTB spacetimes to admit dissipative fluxes are clearly justified.

Therefore, it is our goal in this manuscript to consider possible generalizations of LTB spacetimes to the dissipative case. As it should be obvious, such generalizations are not unique. As a "qualitative" guide in our endeavor we shall look for GLTB's "as similar" as possible to LTB's. More precise definitions of what we mean by qualitative guide and "as similar as possible," will be provided later.

Thus, for example, exact solutions to Einstein equations describing dissipative geodesic fluids have been found in [33–37], however all of them are shear-free and we know that a distinct property of LTB is its shear, accordingly we shall search for shearing GLTB. Shearing dissipative geodesic fluids may be found in [38,39], though they do not become LTB in the nondissipative case.

Our search will be based on two different types of arguments. On the one hand, symmetry arguments. We shall find a symmetry property of LTB spacetimes and we shall assume that the corresponding generalizations (GLTB) describing dissipative dust, share the same kind of symmetry. On the other hand, we shall describe LTB in terms of some scalar functions which emerge from the orthogonal splitting of the Riemann tensor. We shall assume that two of these scalar functions share the same form (with respect to metric functions) in both LTB and GLTB.

In the specific case of localized configurations we have to assume that our fluid distribution is bounded by a spherical surface. In order to avoid thin shells on such a boundary surface, Darmois [40] conditions should be imposed.

We would like to emphasize that, even though some specific examples are exhibited, our main goal in this work consists of providing different techniques to obtain exact solutions representing geodesic radiating fluids whose properties are in some respect similar to LTB.

II. FLUID DISTRIBUTION, KINEMATICAL VARIABLES, AND BASIC EQUATIONS

We consider a spherically symmetric distribution of geodesic fluid, which may be bounded by a spherical surface Σ, or not. The fluid is assumed to be pure dust undergoing dissipation in the form of heat flow (diffusion approximation) and outgoing null fluid (streaming out limit).

Choosing comoving coordinates, the general metric can be written (in the case of bounded configurations such a line element applies to the fluid inside Σ) as

$$ds^2 = -dt^2 + B^2 dr^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where B and R are functions of t and r and are assumed positive. We number the coordinates $x^0 = t$, $x^1 = r$, $x^2 = \theta$, and $x^3 = \phi$. Observe that B is dimensionless, whereas R has the same dimension as r. Also observe that t is the proper time.

The energy-momentum $T_{\alpha\beta}$ (inside Σ if the system is bounded) is assumed to have the form

$$T_{\alpha\beta} = \mu V_\alpha V_\beta + q_\alpha V_\beta + V_\alpha q_\beta + \epsilon l_\alpha l_\beta, \quad (2)$$

where μ is the energy density, q^α the heat flux, ϵ the radiation density, V^α the four-velocity of the fluid, and l^α a null four-vector. These quantities satisfy

$$V^\alpha V_\alpha = -1, \quad V^\alpha q_\alpha = 0, \quad l^\alpha V_\alpha = -1, \quad l^\alpha l_\alpha = 0. \quad (3)$$

Since we have chosen a comoving coordinate system, we have

$$V^\alpha = A^{-1}\delta_0^\alpha, \quad q^\alpha = qB^{-1}\delta_1^\alpha,$$
$$l^\alpha = A^{-1}\delta_0^\alpha + B^{-1}\delta_1^\alpha, \quad (4)$$

where q is a function of t and r, $q^\alpha = q\chi^\alpha$ and χ^α a unit four-vector along the radial direction, satisfying

$$\chi^\alpha \chi_\alpha = 1, \quad \chi^\alpha V_\alpha = 0, \quad \chi^\alpha = B^{-1}\delta_1^\alpha. \quad (5)$$

It may be more convenient to write (2) in the form

$$T_{\alpha\beta} = \tilde{\mu}V_\alpha V_\beta + \tilde{q}(V_\alpha \chi_\beta + \chi_\alpha V_\beta) + \epsilon\chi_\alpha \chi_\beta, \quad (6)$$

with

$$\tilde{\mu} = \mu + \epsilon, \quad \tilde{q} = q + \epsilon.$$

A. Einstein equations

For (1) and (6), Einstein equations

$$G_{\alpha\beta} = 8\pi T_{\alpha\beta}, \quad (7)$$

read:

$$8\pi T_{00} = 8\pi\tilde{\mu}$$
$$= \left(2\frac{\dot{B}}{B} + \frac{\dot{R}}{R}\right)\frac{\dot{R}}{R} - \left(\frac{1}{B}\right)^2\left[2\frac{R''}{R} + \left(\frac{R'}{R}\right)^2\right.$$
$$\left. - 2\frac{B'}{B}\frac{R'}{R} - \left(\frac{B}{R}\right)^2\right], \quad (8)$$

$$8\pi T_{01} = -8\pi\tilde{q}B = -2\left(\frac{\dot{R}'}{R} - \frac{\dot{B}}{B}\frac{R'}{R}\right), \quad (9)$$

$$8\pi T_{11} = 8\pi\epsilon B^2 = -B^2\left[2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2\right] + \left(\frac{R'}{R}\right)^2 - \left(\frac{B}{R}\right)^2, \tag{10}$$

$$8\pi T_{22} = 0 = -R^2\left(\frac{\ddot{B}}{B} + \frac{\ddot{R}}{R} + \frac{\dot{B}}{B}\frac{\dot{R}}{R}\right) + \left(\frac{R}{B}\right)^2\left(\frac{R''}{R} - \frac{B'}{B}\frac{R'}{R}\right), \tag{11}$$

where dots and primes denote derivatives with respect to t and r, respectively. Observe that if $\epsilon \neq 0$, the dissipative dust behaves as an anisotropic fluid with vanishing tangential stresses [41].

B. Kinematical variables and the mass function

The expansion Θ is given by

$$\Theta = V^\alpha_{;\alpha} = \left(\frac{\dot{B}}{B} + 2\frac{\dot{R}}{R}\right), \tag{12}$$

and for the shear we have (remember that the four-acceleration vanishes)

$$\sigma_{\alpha\beta} = V_{(\alpha;\beta)} - \frac{1}{3}\Theta h_{\alpha\beta}, \tag{13}$$

where $h_{\alpha\beta} = g_{\alpha\beta} + V_\alpha V_\beta$.

Using (4) we obtain the nonvanishing components of (13)

$$\sigma_{11} = \frac{2}{3}B^2\sigma, \qquad \sigma_{22} = \frac{\sigma_{33}}{\sin^2\theta} = -\frac{1}{3}R^2\sigma, \tag{14}$$

with

$$\sigma^{\alpha\beta}\sigma_{\alpha\beta} = \frac{2}{3}\sigma^2, \tag{15}$$

being

$$\sigma = \left(\frac{\dot{B}}{B} - \frac{\dot{R}}{R}\right). \tag{16}$$

$\sigma_{\alpha\beta}$ may be also written as

$$\sigma_{\alpha\beta} = \sigma\left(\chi_\alpha\chi_\beta - \frac{1}{3}h_{\alpha\beta}\right). \tag{17}$$

Next, the mass function $m(t, r)$ introduced by Misner and Sharp [42] (see also [43]) is given by

$$m = \frac{(R)^3}{2}R_{23}{}^{23} = \frac{R}{2}\left[\dot{R}^2 - \left(\frac{R'}{B}\right)^2 + 1\right]. \tag{18}$$

We can define the velocity U of the collapsing fluid as the variation of the areal radius (R) with respect to proper time, i.e.

$$U = \dot{R}. \tag{19}$$

Then (18) can be rewritten as

$$E \equiv \frac{R'}{B} = \left[1 + U^2 - \frac{2m(t, r)}{R}\right]^{1/2}. \tag{20}$$

With the above we can express (9) as

$$4\pi\tilde{q} = E\left[\frac{1}{3R'}(\Theta - \sigma)' - \frac{\sigma}{R}\right]. \tag{21}$$

From (18)

$$\dot{m} = -4\pi(\epsilon U + \tilde{q}E)R^2, \tag{22}$$

and

$$m' = 4\pi\left(\tilde{\mu} + \tilde{q}\frac{U}{E}\right)R'R^2. \tag{23}$$

Equation (23) may be integrated to obtain

$$m = \int_0^r 4\pi R^2\left(\tilde{\mu} + \tilde{q}\frac{U}{E}\right)R'dr \tag{24}$$

(assuming a regular center to the distribution, so $m(0) = 0$). We may partially integrate (24) to obtain

$$\frac{3m}{R^3} = 4\pi\tilde{\mu} - \frac{4\pi}{R^3}\int_0^r R^3\left(\tilde{\mu}' - 3\tilde{q}\frac{R'U}{RE}\right)dr. \tag{25}$$

C. The exterior spacetime and junction conditions

In the case of bounded configurations, we assume that outside Σ we have the Vaidya spacetime (i.e. we assume all outgoing radiation is massless), or in the dissipationless case the Schwarzschild spacetime, described by

$$ds^2 = -\left[1 - \frac{2M(v)}{\rho}\right]dv^2 - 2d\rho dv + \rho^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{26}$$

where $M(v)$ denotes the total mass (which is constant in the Schwarzschild case) and v is the retarded time.

The matching of the full nonadiabatic sphere (including viscosity) to the Vaidya spacetime, on the surface $r = r_\Sigma = $ const, was discussed in [44] (for the discussion of the shear-free case see [45,46]). However, observe that we are now including a null fluid within the star configuration.

Now, from the continuity of the first differential form it follows (see [44] for details),

$$dt \overset{\Sigma}{=} dv\left(1 - \frac{2M(v)}{\rho}\right), \tag{27}$$

$$R \overset{\Sigma}{=} \rho(v), \tag{28}$$

and

$$\left(\frac{dv}{dt}\right)^{-2} \overset{\Sigma}{=} \left(1 - \frac{2M(v)}{\rho} + 2\frac{d\rho}{dv}\right). \tag{29}$$

Whereas the continuity of the second differential form produces

$$m(t, r) \overset{\Sigma}{=} M(v), \tag{30}$$

and

$$2\left(\frac{\dot{R}'}{R} - \frac{\dot{B}}{B}\frac{R'}{R}\right) \overset{\Sigma}{=} -B\left[2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2\right] + \frac{1}{B}\left[\left(\frac{R'}{R}\right)^2 - \left(\frac{B}{R}\right)^2\right],$$
(31)

where $\overset{\Sigma}{=}$ means that both sides of the equation are evaluated on Σ (observe a misprint in eq. (40) in [44] and a slight difference in notation).

Comparing (31) with (9) and (10) one obtains

$$q \overset{\Sigma}{=} 0.$$
(32)

Thus the matching of (1) and (26) on Σ implies (30) and (32).

Also, we have

$$\epsilon \overset{\Sigma}{=} \frac{L}{4\pi\rho^2},$$
(33)

where (32) has been used and L_Σ denotes the total luminosity of the sphere as measured on its surface, which is given by

$$L \overset{\Sigma}{=} L_\infty\left(1 - \frac{2m}{\rho} + 2\frac{d\rho}{d\upsilon}\right)^{-1},$$
(34)

and where

$$L_\infty = \frac{dM}{d\upsilon} \overset{\Sigma}{=} -\left[\frac{dm}{dt}\left(\frac{d\upsilon}{dt}\right)^{-1}\right],$$
(35)

is the total luminosity measured by an observer at rest at infinity.

The boundary redshift z_Σ is given by

$$\frac{d\upsilon}{dt} \overset{\Sigma}{=} 1 + z,$$
(36)

with

$$\frac{d\upsilon}{dt} \overset{\Sigma}{=} \left(\frac{R'}{B} + \dot{R}\right)^{-1}.$$
(37)

Therefore, the time of formation of the black hole is given by

$$\left(\frac{R'}{B} + \dot{R}\right) \overset{\Sigma}{=} E + U \overset{\Sigma}{=} 0.$$
(38)

Also observe that from (29), (34), and (37) it follows

$$L \overset{\Sigma}{=} \frac{L_\infty}{(E + U)^2},$$
(39)

and from (19), (20), (29), and (37)

$$\frac{d\rho}{d\upsilon} \overset{\Sigma}{=} U(U + E).$$
(40)

Finally, it is worth noticing that in the diffussion approximation ($\epsilon = 0$, $q \neq 0$) it follows at once from (33) that the total luminosity (L_Σ) vanishes, even though there are nonvanishing dissipative fluxes (q) within the sphere.

This result is an obvious consequence of the dust condition (vanishing hydrodynamic pressure).

D. Weyl tensor

The Weyl tensor is defined through the Riemann tensor $R^\rho_{\alpha\beta\mu}$, the Ricci tensor $R_{\alpha\beta}$ and the curvature scalar $\mathcal{R}$, as:

$$C^\rho_{\alpha\beta\mu} = R^\rho_{\alpha\beta\mu} - \tfrac{1}{2}R^\rho_\beta g_{\alpha\mu} + \tfrac{1}{2}R_{\alpha\beta}\delta^\rho_\mu - \tfrac{1}{2}R_{\alpha\mu}\delta^\rho_\beta$$
$$+ \tfrac{1}{2}R^\rho_\mu g_{\alpha\beta} + \tfrac{1}{6}\mathcal{R}(\delta^\rho_\beta g_{\alpha\mu} - g_{\alpha\beta}\delta^\rho_\mu).$$
(41)

The electric part of Weyl tensor is defined by

$$E_{\alpha\beta} = C_{\alpha\mu\beta\nu}V^\mu V^\nu,$$
(42)

with the following nonvanishing components

$$E_{11} = \tfrac{2}{3}B^2\mathcal{E}, \qquad E_{22} = -\tfrac{1}{3}R^2\mathcal{E}, \qquad E_{33} = E_{22}\sin^2\theta,$$
(43)

where

$$\mathcal{E} = \frac{1}{2}\left[\frac{\ddot{R}}{R} - \frac{\ddot{B}}{B} - \left(\frac{\dot{R}}{R} - \frac{\dot{B}}{B}\right)\frac{\dot{R}}{R}\right]$$
$$+ \frac{1}{2B^2}\left[-\frac{R''}{R} + \left(\frac{B'}{B} + \frac{R'}{R}\right)\frac{R'}{R}\right] - \frac{1}{2R^2}.$$
(44)

Observe that we may also write $E_{\alpha\beta}$ as

$$E_{\alpha\beta} = \mathcal{E}(\chi_\alpha\chi_\beta - \tfrac{1}{3}h_{\alpha\beta}).$$
(45)

Finally, using (8), (10), and (11) with (18) and (44) we obtain

$$\frac{3m}{R^3} = 4\pi(\tilde{\mu} - \epsilon) - \mathcal{E}.$$
(46)

E. Evolution equations for the expansion and the shear, Bianchi identities, and a constraint equation for the Weyl tensor

For the system under consideration two equations describing the evolution of the expansion (Raychaudhury) and the shear (Ellis [47,48], respectively, can be easily derived (see [49] for details), they read:

$$\dot{\Theta} + \tfrac{1}{3}\Theta^2 + \tfrac{2}{3}\sigma^2 = -4\pi(\tilde{\mu} + \epsilon),$$
(47)

$$\dot{\sigma} + \tfrac{1}{3}\sigma^2 + \tfrac{2}{3}\Theta\sigma = 4\pi\epsilon - \mathcal{E}.$$
(48)

The two independent components of Bianchi identities for the system under consideration read (see [50] for details):

$$\dot{\tilde{\mu}} + (\tilde{\mu} + \epsilon)\frac{\dot{B}}{B} + 2\frac{\tilde{\mu}\dot{R}}{R} + \frac{\tilde{q}'}{B} + 2\frac{\tilde{q}R'}{BR} = 0,$$
(49)

$$\dot{\tilde{q}} + \frac{\epsilon'}{B} + 2\tilde{q}\left(\frac{\dot{B}}{B} + \frac{\dot{R}}{R}\right) + 2\frac{R'\epsilon}{RB} = 0.$$
(50)

Finally, the following constraint equation for the Weyl tensor may be derived from the Bianchi identities (e.g. see [27] for details)

$$[\mathcal{E} - 4\pi(\tilde{\mu} - \epsilon)]' + (\mathcal{E} + 4\pi\epsilon)\frac{3R'}{R} = -12\pi\tilde{q}B\frac{\dot{R}}{R}. \quad (51)$$

III. STRUCTURE SCALARS

As we mentioned in the Introduction, part of our analysis will be based on a set of scalar functions which appear in a natural way in the orthogonal splitting of the Riemann tensor (see [51] for details).

Thus, let us introduce the tensors $Y_{\alpha\beta}$ and $X_{\alpha\beta}$ (which are elements of that splitting [52,53]), defined by:

$$Y_{\alpha\beta} = R_{\alpha\gamma\beta\delta}V^\gamma V^\delta, \quad (52)$$

and

$$X_{\alpha\beta} = {}^*R^*_{\alpha\gamma\beta\delta}V^\gamma V^\delta = \tfrac{1}{2}\eta_{\alpha\gamma}{}^{\epsilon\rho}R^*_{\epsilon\rho\beta\delta}V^\gamma V^\delta, \quad (53)$$

where $R^*_{\alpha\beta\gamma\delta} = \tfrac{1}{2}\eta_{\epsilon\rho\gamma\delta}R_{\alpha\beta}{}^{\epsilon\rho}$ and $\eta_{\epsilon\rho\gamma\delta}$ denotes the Levi-Civita tensor.

Tensors $Y_{\alpha\beta}$ and $X_{\alpha\beta}$ may also be expressed through their traces and their trace-free parts, as

$$Y_{\alpha\beta} = \tfrac{1}{3}Y_T h_{\alpha\beta} + Y_{TF}(\chi_\alpha\chi_\beta - \tfrac{1}{3}h_{\alpha\beta}), \quad (54)$$

$$X_{\alpha\beta} = \tfrac{1}{3}X_T h_{\alpha\beta} + X_{TF}(\chi_\alpha\chi_\beta - \tfrac{1}{3}h_{\alpha\beta}). \quad (55)$$

Then from (52)–(55) and using (6), (7), (41), and (45) we obtain

$$Y_T = 4\pi(\tilde{\mu} + \epsilon), \qquad Y_{TF} = \mathcal{E} - 4\pi\epsilon, \quad (56)$$

$$X_T = 8\pi\tilde{\mu}, \qquad X_{TF} = -\mathcal{E} - 4\pi\epsilon. \quad (57)$$

Also, combining (25) and (46) with (56) we may write

$$Y_{TF} = -8\pi\epsilon + \frac{4\pi}{R^3}\int_0^r R^3\left(\tilde{\mu}' - 3\tilde{q}\frac{UR'}{RE}\right)dr. \quad (58)$$

Thus the scalar Y_{TF} may be expressed through the Weyl tensor and the outgoing null fluid or in terms of the null radiation, the density inhomogeneity and the dissipative variables. It is worth recalling that a link between Y_{TF} and the Tolman mass has been established in [51]. Also, it has been shown that in the geodesic case, Y_{TF} controls the stability of the shear-free condition [49].

Scalars (56) and (57) for the line element (1) are

$$Y_T = -2\frac{\ddot{R}}{R} - \frac{\ddot{B}}{B}, \qquad Y_{TF} = \frac{\ddot{R}}{R} - \frac{\ddot{B}}{B}, \quad (59)$$

$$X_T = \left(2\frac{\dot{B}}{B} + \frac{\dot{R}}{R}\right)\frac{\dot{R}}{R} - \frac{1}{B^2}\left[2\frac{R''}{R} + \left(\frac{R'}{R}\right)^2 - 2\frac{B'}{B}\frac{R'}{R}\right] + \frac{1}{R^2}, \quad (60)$$

$$X_{TF} = \left(\frac{\dot{R}}{R} - \frac{\dot{B}}{B}\right)\frac{\dot{R}}{R} + \frac{1}{B^2}\left[\frac{R''}{R} - \left(\frac{B'}{B} + \frac{R'}{R}\right)\frac{R'}{R}\right] + \frac{1}{R^2}. \quad (61)$$

In terms of these scalar functions, Eqs. (47), (48), and (51), become

$$\dot{\Theta} + \tfrac{1}{3}\Theta^2 + \tfrac{2}{3}\sigma^2 = -Y_T, \quad (62)$$

$$\dot{\sigma} + \tfrac{1}{3}\sigma^2 + \tfrac{2}{3}\Theta\sigma = -Y_{TF}, \quad (63)$$

$$(4\pi\tilde{\mu} + X_{TF})' = -\frac{3R'}{R}X_{TF} + 4\pi\tilde{q}(\Theta - \sigma)B. \quad (64)$$

Now, from (64) in the nondissipative case we have

$$(4\pi\mu + X_{TF})' = -\frac{3R'}{R}X_{TF}, \quad (65)$$

implying that if $X_{TF} = 0$ then $\mu' = 0$. On the other hand if $\mu' = 0$ then

$$(X_{TF})' = -\frac{3R'}{R}X_{TF}, \quad (66)$$

producing

$$X_{TF} = \frac{f(t)}{R^3}. \quad (67)$$

Since X_{TF} must be regular everywhere within the sphere, we must put $f(t) = 0$, therefore we have $\mu' = 0 \Longleftrightarrow X_{TF} = 0$. In other words, in absence of dissipation, energy density inhomogeneity is controlled by the scalar X_{TF}. This last result is also valid for an anisotropic fluid [51].

In the case of nondissipative dust, it can be shown that $\sigma = 0$ implies $X_{TF} = 0$. Indeed, in this particular case it follows from (63) that $Y_{TF} = 0$, which because of (56) and (57) produces $X_{TF} = 0$.

We shall next analyze in some detail some properties of LTB spacetimes.

IV. LEMAITRE-TOLMAN-BONDI METRIC

In order to obtain the general form of LTB, we assume a geodesic, dissipationless fluid ($q = \epsilon = 0$). Then we find after integration of (9),

$$B(t, r) = \frac{R'}{(1 + \kappa(r))^{1/2}}, \quad (68)$$

where κ is an arbitrary function of r.

Feeding back (68) in (1) we obtain the Lemaitre-Tolman-Bondi metric (LTB):

$$ds^2 = -dt^2 + \frac{(R')^2}{1 + \kappa(r)}dr^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (69)$$

The metric (69) is usually associated with an inhomogeneous dust source (and so we shall do here), however it is

worth mentioning that the most general source compatible with LTB spacetimes is an anisotropic fluid [4,54].

Next, the "Euclidean" condition at the center requires that the perimeter of an infinitesimally small circle around the center be given by $2\pi l$ where l is the proper radius of the circle, given in turn by

$$l = \int_0^r B dr. \tag{70}$$

On the other hand, the perimeter of any circle, as it follows from (1), is given by $2\pi R$ implying that in the neighborhood of the center we must impose the condition:

$$R' \overset{r=0}{=} B, \tag{71}$$

or, using (68)

$$\kappa' \overset{r=0}{=} 0. \tag{72}$$

Now from (18) and (68) we obtain

$$\dot{R}^2 = \frac{2m}{R} + \kappa(r), \tag{73}$$

and from (22) and (23)

$$m = m(r), \tag{74}$$

$$m' = 4\pi\mu R^2 R'. \tag{75}$$

Also, the Bianchi identity (49) in this case reads

$$\dot{\mu} + \mu\left(\frac{\dot{B}}{B} + 2\frac{\dot{R}}{R}\right) = 0, \tag{76}$$

producing

$$\mu = \frac{h(r)}{BR^2}, \tag{77}$$

or, using (68)

$$\mu = \frac{3h(r)(1 + \kappa(r))^{1/2}}{(R^3)'}, \tag{78}$$

where $h(r)$ is a function of integration.

Depending on κ there are three possible solutions of (73):

(1) $\kappa = 0$ (Parabolic case)

$$R^{3/2} = (2m)^{1/2}\eta^3, \qquad \tfrac{2}{3}\eta^3 = t - t_{\rm bb}(r). \tag{79}$$

(2) $\kappa > 0$ (Hyperbolic case)

$$R = \frac{m}{\kappa}(\cosh\eta - 1),$$

$$\frac{m}{\kappa^{3/2}}(\sinh\eta - \eta) = t - t_{\rm bb}(r). \tag{80}$$

(3) $\kappa < 0$ (Elliptic case)

$$R = \frac{m}{|\kappa|}(1 - \cos\eta),$$

$$\frac{m}{|\kappa|^{3/2}}(\eta - \sin\eta) = t - t_{\rm bb}(r). \tag{81}$$

Where $t_{\rm bb}(r)$ is an integration function of r giving the value of time for which $R(t, r) = 0$, not to be confused with the center of symmetry $R(t, 0)$.

In order to prescribe an explicit model, we have to provide the three functions $\kappa(r)$, $m(r)$, and $t_{\rm bb}(r)$. However, since (69) is invariant under transformations of the form $r = r(\tilde{r})$, we only need two functions of r.

Scalars Y_T, $Y_{\rm TF}$, $X_{\rm TF}$, and X_T for (69) are

$$Y_{\rm TF} = \frac{\ddot{R}}{R} - \frac{\ddot{R}'}{R'}, \qquad Y_T = -2\frac{\ddot{R}}{R} - \frac{\ddot{R}'}{R'}, \tag{82}$$

and

$$X_{\rm TF} = \frac{\dot{R}^2}{R^2} - \frac{\dot{R}}{R}\frac{\dot{R}'}{R'} - \frac{\kappa}{R^2} + \frac{\kappa'}{2RR'}, \tag{83}$$

$$X_T = \frac{2\dot{R}\dot{R}'}{RR'} + \frac{\dot{R}^2}{R^2} - \frac{\kappa}{R^2} - \frac{\kappa'}{RR'}. \tag{84}$$

A. Symmetry properties of LTB spacetimes

It is a simple matter to check that besides the Killing vectors associated with spherical symmetry, LTB spacetimes admit no further Killing vectors. Also, it is not difficult to verify that they do not admit conformal Killing vectors either. It should be clear that such nonadmittance refers to the general form of LTB, not to any specific solution belonging to LTB spacetime.

However, as we shall see below, a specific kind of symmetry may be ascribed to LTB spacetimes.

Proper matter collineation

It can be shown [55] that in the pure dust case, there exists a vector field ξ such that for any LTB spacetime

$$\mathcal{L}_\xi T_{\alpha\beta} = 0, \tag{85}$$

where $\mathcal{L}_\xi$ denotes the Lie derivative with respect to the vector field ξ, which can be shown to be of the form

$$\xi^\rho = \xi^0\delta_t^\rho + \xi^1\delta_r^\rho. \tag{86}$$

whose components are given by

$$\xi^0 = F(t), \tag{87}$$

$$\xi^1 = -\left(2\dot{F}(t) + F(t)\frac{\dot{\mu}}{\mu}\right)\left(\frac{\mu}{\mu'}\right), \tag{88}$$

where $F(t)$ is an arbitrary function. Thus for any LTB spacetime, we can always find a vector field ξ satisfying (85).

It should be stressed that ξ is *not* a Killing vector field, i.e. it defines a proper matter collineation.

V. GENERALIZING LTB TO THE DISSIPATIVE CASE

From the previous section it should be clear that LTB spacetimes explicitly exclude dissipative fluxes (either in the diffusion approximation or in the streaming out limit).

We shall now approach the problem of extending LTB so as to include dissipative fluxes (such spacetimes will be referred to as GLTB). As a necessary condition we shall require that all GLTB's become LTB in the limit when dissipative fluxes vanish.

Since we are searching for spacetimes as "close" as possible to LTB, we shall consider geodesic, shearing dust with $\tilde{q} \neq 0$. It is worth recalling that the pure dust (without dissipation) condition implies that the fluid is geodesic, however this is no longer true for the dissipative case. Therefore the geodesic condition here is nonredundant.

Then integration of (9), produces now:

$$B(t, r) = \frac{R'}{(1 + K(t, r))^{1/2}}, \tag{89}$$

with

$$1 + K(t, r) = \left[\int 4\pi \tilde{q} R dt + C(r) \right]^2. \tag{90}$$

Since in the nondissipative case (89) should become (68), it follows

$$C(r) = (1 + \kappa(r))^{1/2}, \tag{91}$$

and the Euclidean condition at the center, implies

$$C(r) \overset{r=0}{=} 1. \tag{92}$$

Feeding back (89) into (1) and using (90), we obtain for the line element

$$ds^2 = -dt^2 + \frac{(R')^2}{[\int 4\pi \tilde{q} R dt + C(r)]^2} dr^2$$
$$+ R^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{93}$$

or, using (91)

$$ds^2 = -dt^2 + \frac{(R')^2}{[\int 4\pi \tilde{q} R dt + (1 + \kappa(r))^{1/2}]^2} dr^2$$
$$+ R^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{94}$$

Scalars Y_T, Y_{TF}, X_T, and X_{TF} for (93) are

$$Y_{\mathrm{TF}} = \frac{\ddot{R}}{R} - \frac{\ddot{R}'}{R'} + \frac{\dot{K}}{1 + K}\left(\frac{\dot{R}'}{R'} - \frac{3}{4}\frac{\dot{K}}{1 + K}\right) + \frac{\ddot{K}}{2(1 + K)}, \tag{95}$$

$$Y_T = -2\frac{\ddot{R}}{R} - \frac{\ddot{R}'}{R'} + \frac{\dot{K}}{1 + K}\left(\frac{\dot{R}'}{R'} - \frac{3}{4}\frac{\dot{K}}{1 + K}\right) + \frac{\ddot{K}}{2(1 + K)}, \tag{96}$$

$$X_T = \frac{2\dot{R}\dot{R}'}{RR'} + \frac{\dot{R}^2}{R^2} - \frac{K}{R^2} - \frac{K'}{RR'} - \frac{\dot{R}\dot{K}}{R(1 + K)}, \tag{97}$$

$$X_{\mathrm{TF}} = \frac{\dot{R}^2}{R^2} - \frac{\dot{R}}{R}\frac{\dot{R}'}{R'} - \frac{K}{R^2} + \frac{K'}{2RR'} + \frac{\dot{R}\dot{K}}{2R(1 + K)}. \tag{98}$$

In order to proceed further and to obtain specific families of solutions, we need to impose additional conditions. As mentioned before, the criteria to select such conditions will be dictated by the requirement that the obtained solution represents the "closest" possible situation to LTB spacetime, including dissipative fluxes.

Of course, this last requirement is still vague enough and allows a great deal of possibilities. In this work we shall focus on two possible extensions. On the one hand, we shall consider extensions based on the characterization of LTB spacetimes in terms of the structure scalars. On the other hand, we shall propose extensions of LTB to the dissipative case, based on the symmetry property discussed above. However, before considering that, let us describe the treatment of the transport equation in the pure diffusive case.

VI. THE TRANSPORT EQUATION

In the diffusion approximation ($\epsilon = 0$, $\tilde{q} = q$), we shall use a transport equation derived from the Müller-Israel-Stewart second order phenomenological theory for dissipative fluids [56–59].

Indeed, it is well known that the Maxwell-Fourier law for radiation flux leads to a parabolic equation (diffusion equation) which predicts propagation of perturbations with infinite speed (see [60–63] and references therein). This simple fact is at the origin of the pathologies [64] found in the approaches of Eckart [65] and Landau [66] for relativistic dissipative processes. To overcome such difficulties, various relativistic theories with nonvanishing relaxation times have been proposed in the past [56–59,67,68]. The important point is that all these theories provide a heat transport equation which is not of Maxwell-Fourier type but of Cattaneo type [69], leading thereby to a hyperbolic equation for the propagation of thermal perturbations.

The corresponding transport equation for the heat flux reads

$$\beta h^{\alpha\rho} V^\gamma q_{\rho;\gamma} + q^\alpha = -Kh^{\alpha\rho}(\mathcal{T}_{,\rho} + \mathcal{T}a_\rho)$$
$$- \frac{1}{2}\kappa\mathcal{T}^2\left(\frac{\beta V^\rho}{K}\mathcal{T}^2\right)_{;\rho} q^\alpha, \tag{99}$$

where K denotes the thermal conductivity, and $\mathcal{T}$ and β denote temperature and relaxation time, respectively. In the case $\beta = 0$ we recover the Eckart-Landau equation.

Observe that, due to the symmetry of the problem, Eq. (99) only has one independent component, which may be written as

$$\beta \dot{q} + q = -\frac{K}{B} \mathcal{T}' - \frac{1}{2} \beta q \Theta + \beta \frac{\dot{\mathcal{T}}}{\mathcal{T}} q. \qquad (100)$$

In the truncated version of the theory, the last term in (99) is absent (see for example [70]), and (100) becomes

$$\beta \dot{q} + q = -\frac{K}{B} \mathcal{T}'. \qquad (101)$$

Finally, observe that if $\epsilon = 0$ we obtain from (50)

$$q = \frac{g(r)}{B^2 R^2}, \qquad (102)$$

where $g(r)$ is an arbitrary function. Then using (102) in (100) or (101) we obtain (up to the function $g(r)$) the temperature distribution of the fluid in terms of metric functions.

VII. EXTENSIONS OF LTB BASED ON STRUCTURE SCALARS

From an inspection of (62), (63), (82)–(84), and (95)–(98), the following remarks are in order:

(i) For any geodesic fluid (dissipative or not), the evolution of Θ and σ is fully controlled by Y_{TF} and Y_T.

(ii) In LTB, scalars Y_T and Y_{TF} do not contain κ.

(iii) In GLTB, scalars Y_T and Y_{TF} differ from their expressions in LTB, by the same term.

Based on the comments above and on the requirement of "maximal" similarity between a LTB and its corresponding GLTB, let us assume that scalars Y_T and Y_{TF} share the same expression (in terms of R) in both cases.

Then it follows at once from (82) and (95) [or (96)],

$$\frac{\dot{K}}{1+K}\left[\frac{\dot{R}'}{R'} - \frac{3}{4}\frac{\dot{K}}{(1+K)}\right] + \frac{\ddot{K}}{2(1+K)} = 0. \qquad (103)$$

Integrating (103) we obtain

$$\frac{R'\dot{K}^{1/2}}{(1+K)^{3/4}} = C_1(r), \qquad (104)$$

or using (90) and (91)

$$C_1(r) = \frac{(8\pi\tilde{q}R)^{1/2}R'}{(1+\kappa(r))^{1/2} + \int 4\pi\tilde{q}Rdt}, \qquad (105)$$

where $C_1(r)$ is an arbitrary integration function.

Next, integrating (104) we obtain

$$K + 1 = \frac{4}{[C_1(r)^2 \int \frac{dt}{(R')^2} + C_2(r)]^2}, \qquad (106)$$

where $C_2(r)$ is another integration function, which may be related to $C(r)$ as follows. Taking the t derivative of (106) we have

$$\dot{K} = -\frac{8C_1^2(r)}{(R')^2[C_1^2(r) \int \frac{dt}{(R')^2} + C_2(r)]^3}, \qquad (107)$$

then, combining (90) with (107) it follows

$$2\pi\tilde{q} = \frac{C_1^2(r)}{R(R')^2[C_1^2(r) \int \frac{dt}{(R')^2} + C_2(r)]^2}, \qquad (108)$$

finally, comparing (105) and (108) we also find

$$C_2(r) = \frac{2(1 - 4\pi\tilde{q}RR'^2)}{C(r) + \int 4\pi\tilde{q}Rdt}. \qquad (109)$$

Thus, the assumption above on Y_T and Y_{TF} allows us to obtain a GLTB from any given LTB. Indeed, from (108), taking the time and radial dependence of R from any given LTB, we obtain the dissipative flux of the corresponding GLTB, up to the two functions of r, C_1 and C_2. The former must satisfy the regularity condition $C_1(0) = 0$, and its vanishing brings back to the starting "seed" LTB solution. The remaining physical variables follow from the field equations.

Let us now illustrate the method above, with an example inspired in the parabolic subclass of LTB.

Thus, we choose

$$R(t, r) = f(r)(T(r) - t)^{2/3}. \qquad (110)$$

where $f(r)$ and $T(r)$ correspond to any specific parabolic LTB solution. However, care must be exercised in not identifying $f(r)$ with the mass function as it happens in LTB.

Using (110) we may write the integral $\int \frac{dt}{(R')^2}$ as

$$I = \int \frac{dt}{(R')^2} = \int \frac{(T(r) - t)^{2/3} dt}{[f'(r)(T(r) - t) + \frac{2}{3}f(r)T'(r)]^2}, \qquad (111)$$

introducing the variable $T(r) - t = \tau^3$ in (111) and integrating we obtain

$$I = -3 \int \frac{\tau^4 d\tau}{(a\tau^3 + b)^2}$$

$$= \frac{\tau^2}{(a^2\tau^3 + ab)} + \frac{1}{3\sqrt[3]{ba^5}}\left[2\sqrt{3}\arctan\left(\frac{1 - 2\sqrt[3]{\frac{a}{b}}\tau}{\sqrt{3}}\right)\right.$$

$$\left. - \ln\left(\frac{a\tau^3 + b}{(\sqrt[3]{a}\tau + \sqrt[3]{b})^3}\right)\right], \qquad (112)$$

where $a = f'(r)$ and $b = \frac{2}{3}f(r)T'(r)$.

Feeding back (112) into (108), using (110), we find

$$2\pi\tilde{q} = \frac{C_1^2(r)f(r)^{-1}}{\{C_1^2(r)[\frac{\tau^2}{a} + \frac{a\tau^3+b}{3\sqrt[3]{ba^5}}[2\sqrt{3}\arctan(\frac{1-2\sqrt[3]{\frac{a}{b}}\tau}{\sqrt{3}}) - \ln(\frac{a\tau^3+b}{(\sqrt[3]{a\tau}+\sqrt[3]{b})^3})]] + C_2(r)(a\tau^3 + b)\}^2}. \tag{113}$$

Thus, the function C_1 controls the magnitude of the dissipation and can be chosen as small as desired in the case when perturbations of LTB are required. From the above it is evident that these GLTB become LTB when dissipative fluxes vanish. Also, it should be clear that if the physical properties of the seed LTB are reasonable, so will be the properties of the corresponding GLTB, at least for a sufficiently small value of the function $C_1(r)$. Finally, in the particular case ($\epsilon = 0$) the temperature profile of the model can be obtained from (100) [or (101) and (113)].

VIII. EXTENSIONS OF LTB BASED ON SYMMETRY PROPERTIES

In this section we shall propose another approach to obtain GLTB spacetimes. It consists in assuming that any GLTB spacetime shares the same symmetry property described in Sec. IV for the corresponding LTB.

Thus, let us assume now that our GLTB admits a proper matter collineation.

Then, from the energy-momentum tensor for dust with dissipation (6), we obtain, using (85), the three equations:

$$\xi^0\dot{\tilde{\mu}} + \xi^1\tilde{\mu}' + 2\tilde{\mu}\xi^0_{,0} - 2\tilde{q}B\xi^1_{,0} = 0, \tag{114}$$

$$-\xi^0\dot{\tilde{q}} - \xi^1\tilde{q}' + \tilde{\mu}\frac{\xi^0_{,1}}{B} - \tilde{q}\left(\xi^0_{,0} + \xi^0\frac{\dot{B}}{B} + \xi^1\frac{B'}{B} + \xi^1_{,1}\right)$$
$$+ \epsilon B\xi^1_{,0} = 0, \tag{115}$$

$$\xi^0\dot{\epsilon} + \xi^1\epsilon' - 2\tilde{q}\frac{\xi^0_{,1}}{B} + 2\epsilon\left(\xi^0\frac{\dot{B}}{B} + \xi^1\frac{B'}{B} + \xi^1_{,1}\right) = 0. \tag{116}$$

We shall consider two separated subcases, namely:
(i) Dissipation in purely diffusion approximation ($q \neq 0$, $\epsilon = 0$, $\tilde{q} = q$).
(ii) Dissipation in the streaming out limit ($\epsilon \neq 0$, $q = 0$, $\tilde{q} = \epsilon$).

A. Diffusion approximation

From (87) and (89), we can write

$$8\pi qB = \frac{\dot{K}}{1 + K}\frac{R'}{R}. \tag{117}$$

Combining (89), (102), and (117) produces

$$8\pi g(r) = RR'^2\frac{\dot{K}}{(1 + K)^{3/2}}. \tag{118}$$

Next, from (116) we obtain $\xi^0 = F(t)$, as in LTB, and (114) and (115) take the form

$$F(t)\dot{\mu} + \xi^1\mu' + 2\mu\dot{F}(t) - 2qB\xi^1_{,0} = 0, \tag{119}$$

$$F(t)\dot{q} + \xi^1q' + q\left(\dot{F}(t) + F(t)\frac{\dot{B}}{B} + \xi^1\frac{B'}{B} + \xi^1_{,1}\right) = 0. \tag{120}$$

This last equation can be rewritten in the form

$$\xi^1[\ln(qB\xi^1)]' + F(t)[\ln(qBF(t))]\dot{} = 0. \tag{121}$$

Multiplying (121) by qB, it becomes:

$$(qB\xi^1)' + (qBF)\dot{} = 0, \tag{122}$$

a partial solution of which may be written as

$$qB\xi^1 = -\dot{\psi}(t, r), \tag{123}$$

$$qBF(t) = \psi'(t, r). \tag{124}$$

From (117) and (124) we have

$$\psi'(t, r) = \frac{F(t)}{8\pi}\frac{\dot{K}}{1 + K}\frac{R'}{R}. \tag{125}$$

Then, using (102) in (124) with (125), and taking (89) into account, we get

$$8\pi g(r) = RR'^2\frac{\dot{K}}{(1 + K)^{3/2}}, \tag{126}$$

which is exactly (118).

More generally, it can be shown that integrability conditions for (119) and (122) can always be satisfied. In the particular case $F(t) = 1$, we obtain

$$\xi^1 = -\frac{(\ln qB)\dot{} + \frac{1}{2}(\frac{\mu'}{qB})\dot{}}{(\ln qB)' + \frac{1}{2}(\frac{\mu'}{qB})'}.$$

In other words the existence of a proper matter collineation is consistent with the general form of GLTB in the diffusion limit.

We can integrate (118) to obtain

$$\frac{1}{(1 + K)^{1/2}} = -4\pi g(r)\int\frac{dt}{RR'^2} + c(r), \tag{127}$$

where $c(r)$ is an arbitrary function.

As a specific example, let us choose for $R(t, r)$ the form for the parabolic subclass of LTB given by (110), then we obtain

$$\frac{1}{(1 + K)^{1/2}} = -\frac{4\pi g(r)}{f(r)f'(r)[f'(r)(T(r) - t) + \frac{2}{3}f(r)T'(r)]} + c(r), \tag{128}$$

and introducing this last expression into (102) we have

$$q = \frac{g(r)}{f^2(r)(T(r) - t)^{2/3}}\left\{-\frac{4\pi g(r)}{f(r)f'(r)} + c(r)\right.$$

$$\times \left[f'(r)(T(r) - t) + \frac{2}{3}f(r)T'(r)\right]\right\}^{-2}. \quad (129)$$

where we have used (89).

So far the GLTB thus obtained, is defined up to four functions of r, which by the invariance with respect to the transformation of the radial coordinate reduces to three. Two of them can be taken exactly as the corresponding in the LTB seed model, and the remaining one is obtained via Eqs. (122)–(124), from the assumption that ξ^1 has the same radial dependence as in the LTB case. Observe that due to the junction condition (32) we have to impose $g(r) \overset{\Sigma}{=} 0$. Finally, the temperature profile can be easily obtained from (100) [or (101) and (129)].

B. Streaming out approximation

In the case $q = 0$ and $\epsilon \neq 0$, we obtain from Bianchi identities (49) and (50)

$$\dot{\mu} + \mu\left(\frac{\dot{B}}{B} + 2\frac{\dot{R}}{R}\right) = 0, \quad (130)$$

producing

$$\mu = \frac{j(r)}{BR^2}, \quad (131)$$

exactly as in the LTB case. However, using (89)–(91) we get in this case

$$\mu = \frac{3j(r)[\int 4\pi\epsilon R dt + (1 + \kappa(r))^{1/2}]}{(R^3)'}, \quad (132)$$

where $j(r)$ is a function of integration.

Next, in this approximation (114)–(116) take the form

$$\xi^0(\mu + \epsilon)^{\cdot} + \xi^1(\mu + \epsilon)' + 2(\mu + \epsilon)\xi^0_{,0} - 2\epsilon B\xi^1_{,0} = 0, \quad (133)$$

$$\xi^0\dot{\epsilon} + \xi^1\epsilon' - (\mu + \epsilon)\frac{\xi^0_{,1}}{B}$$
$$+ \epsilon\left(\xi^0_{,0} + \xi^0\frac{\dot{B}}{B} + \xi^1\frac{B'}{B} + \xi^1_{,1}\right) - \epsilon B\xi^1_{,0} = 0, \quad (134)$$

$$\xi^0\dot{\epsilon} + \xi^1\epsilon' - 2\epsilon\frac{\xi^0_{,1}}{B} + 2\epsilon\left(\xi^0\frac{\dot{B}}{B} + \xi^1\frac{B'}{B} + \xi^1_{,1}\right) = 0. \quad (135)$$

From (133)–(135) we find

$$\xi^0\dot{\mu} + \xi^1\mu' + 2\mu\xi^0_{,0} + 2\mu\frac{\xi^0_{,1}}{B} = 0, \quad (136)$$

producing

$$\xi^1 = -\frac{\mu}{\mu'}\left(2\dot{F}(r, t) + F(r, t)\frac{\dot{\mu}}{\mu} + \frac{2F'(r, t)}{B}\right), \quad (137)$$

with $\xi^0 = F(r, t)$.

Following the requirement of maximal similarity between a LTB and its corresponding GLTB, let us assume $\xi^0 = F(t)$, in which case (137), becomes (88).

Under that condition, we have from (133)–(135)

$$F(t)(\mu + \epsilon)^{\cdot} + \xi^1(\mu + \epsilon)' + 2(\mu + \epsilon)\dot{F}(t) - 2\epsilon B\xi^1_{,0} = 0, \quad (138)$$

$$F(t)\dot{\epsilon} + \xi^1\epsilon' + \epsilon\left(\dot{F}(t) + F(t)\frac{\dot{B}}{B} + \xi^1\frac{B'}{B} + \xi^1_{,1}\right)$$
$$- \epsilon B\xi^1_{,0} = 0, \quad (139)$$

$$F(t)\dot{\epsilon} + \xi^1\epsilon' + 2\epsilon\left(F(t)\frac{\dot{B}}{B} + \xi^1\frac{B'}{B} + \xi^1_{,1}\right) = 0. \quad (140)$$

This last equation can be written as

$$F(t)[\ln(\epsilon B^2)]^{\cdot} + \xi^1[\ln(\epsilon(B\xi^1)^2)]' = 0, \quad (141)$$

or in the form

$$F(t)(\epsilon B^2)^{\cdot} + \frac{1}{\xi^1}(\epsilon(B\xi^1)^2)' = 0. \quad (142)$$

Also, in this case, we can write (50) in the form

$$[\ln(\epsilon(BR)^2)]^{\cdot} + \frac{1}{B}[\ln(\epsilon R^2)]' = 0, \quad (143)$$

or,

$$[\epsilon(BR)^2]^{\cdot} + B(\epsilon R^2)' = 0. \quad (144)$$

The method to obtain GLTB from any given LTB (in the streaming out approximation) may now be sketched as follows:

(i) Take any specific seed LTB with a given vector field ξ defining a proper matter collineation (since $F(t)$ is arbitrary we put it equal to one).

(ii) Replace the term $(\epsilon B^2)^{\cdot}$ in (142) by its expression obtained from (144).

(iii) Assume that the radial dependence of ξ^1 is the same as the corresponding to the LTB seed solution.

(iv) From the form of R corresponding to the seed LTB, integrate (142) with respect to r to obtain ϵ. Any remaining arbitrary function of t may be found from junction conditions.

IX. CONCLUSIONS

We have presented different alternatives for obtaining spherically symmetric, geodesic, dissipative dust solutions to Einstein equations (GLTB). All these alternatives are oriented to produce solutions with specific similarities to the nondissipative case (LTB). For doing so we have first carried out a study on LTB spacetimes based, on the one

hand on some symmetry properties of LTB, and on the other on their characterization in terms of the structure scalars.

The generalizations based on structure scalars produce a dissipative model for each seed LTB, such that dissipative variables are arbitrary only up to a single function of r.

However, the generalizations based on symmetry properties have more degrees of freedom, which may be specified depending on the specific problem under consideration.

For the purely diffusion case the temperature profile may be obtained from the transport equation considered in Sec. IV.

At the present time we foresee two interesting applications of GLTB's: One, the testing of any result obtained from any specific LTB against the presence of small but nonvanishing dissipative fluxes, such dissipative perturba-tions being represented by exact solutions to the Einstein equations; and two, the study of relaxational effects on different aspects of collapse via the transport equation.

Finally, as already stressed in the Introduction, let us recall that our intention here is not to produce specific solutions with a distinct physical interpretation, but rather to pave the way for finding such solutions.

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