Planck-scale modifications to electrodynamics characterized by a spacelike symmetry-breaking vector

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In the study of Planck-scale ("quantum-gravity-induced") violations of Lorentz symmetry, an important role was played by the deformed-electrodynamics model introduced by Myers and Pospelov. Its reliance on conventional effective quantum field theory, and its description of symmetry-violation effects simply in terms of a four-vector with a nonzero component only in the time direction, rendered it an ideal target for experimentalists and a natural concept-testing ground for many theorists. At this point however the experimental limits on the single Myers-Pospelov parameter, after improving steadily over these past few years, are "super-Planckian"; i.e. they take the model out of actual interest from a conventional quantum-gravity perspective. In light of this we here argue that it may be appropriate to move on to the next level of complexity, still with vectorial symmetry violation but adopting a generic four-vector. We also offer a preliminary characterization of the phenomenology of this more general framework, sufficient to expose a rather significant increase in complexity with respect to the original Myers-Pospelov setup. Most of these novel features are linked to the presence of spatial anisotropy, which is particularly pronounced when the symmetry-breaking vector is spacelike, and they are such that they reduce the bound-setting power of certain types of observations in astrophysics.

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I. INTRODUCTION

A large effort has been devoted over the last decade (see, e.g., Refs. [1–14], and references therein) toward establishing that it is possible to actually study experimentally some minute effects introduced at the ultrahigh "Planck scale" $M_P(\simeq 1.2\times 10^{28}$ eV), the scale expected to characterize quantum-gravity effects. At this point the scopes of this "quantum-gravity phenomenology" [15] extend over a rather large ensemble of candidate quantum-gravity effects, inspired by (and/or formalized within) several models that are believed to be relevant for the understanding of the quantum-gravity problem. We here focus on one of these research programs which has been driven by a model first introduced by Myers and Pospelov [16], as a candidate description of the Lorentz-symmetry-violation effects that are expected in some approaches to the quantum-gravity problem [1,2,7,12]. This model adopts effective field theory for the description of Lorentz-symmetry-violation effects that are suppressed by a single power of the Planck scale (linear in $1/M_P$) and its proposal was primarily grounded on the observation [16] that there is a unique such correction term which could be added to Maxwell theory,

$$\Delta\mathcal{L}_{QG}=\frac{1}{2M_P}n^{\alpha}F_{\alpha\delta}n^{\sigma}\partial_{\sigma}(n_{\beta}\varepsilon^{\beta\delta\gamma\lambda}F_{\gamma\lambda}),\tag{1}$$

if one enforces some relatively weak assumptions, including gauge invariance and the characterization of the symmetry-breaking structure in terms of an external four-vector n^{α}.

Myers and Pospelov provided an even more definite and manageable framework by restricting their attention [16] to the case in which the four-vector n_{α} only has a time component, $n_{\alpha}=(n_0,0,0,0)$. Then, upon introducing the convenient notation $\xi\equiv(n_0)^3$, one arrives at the following modified Maxwell Lagrangian density:

$$\mathcal{L}_{\rm MP}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{\xi}{2M_P}\varepsilon^{jkl}F_{0j}\partial_0F_{kl},\tag{2}$$

and, in particular, it is then possible to exploit the simplifications provided by spatial isotropy. This Myers-Pospelov effective-field-theory model of Planck-scale-modified electromagnetism has attracted much attention over the last few years. For phenomenologists it provided an ideal target (see, e.g., Refs. [15,17–20], and references therein), because of the presence of a single parameter and because (unlike most other fashionable proposals for the study of the quantum-gravity problem [15]) its reliance on standard effective field theory poses no challenges at the level of "physical interpretation" of the formalism.

This vigorous effort of investigation of the Myers-Pospelov model has produced a quick pace of improvement of experimental bounds, and, while the rough estimate invited by a quantum-gravity intuition [15,17,18] would be $\xi\sim 1$, the Myers-Pospelov parameter ξ is now constrained to be much smaller than 1, with some analyses [19,20] even suggesting a bound at the level $\xi<10^{-15}$. We here observe that however these bounds are not applicable to the general correction term $\Delta\mathcal{L}_{QG}$ of Eq. (1), since they exploit significantly the spatial isotropy regained by the *ad hoc* choice $n_{\alpha}=(n_0,0,0,0)$. And actually this *ad hoc* choice is only available for a restricted class of frames of reference: even imposing "by brute force" $n_\alpha = (n_0, 0, 0, 0)$ in some desired frame of reference, then the four-vector n_α will of course still acquire a spatial component in other (boosted) frames. Since the main strategy for constraining the Myers-Pospelov parameter has relied on various astrophysics observations, conducted in different "laboratory frames," these are concerns that necessarily must be investigated, at least in order to establish to which extent those limits are vulnerable to the presence of a (perhaps small, but necessarily nonzero) spatial component in frames other than the "preferred frame."

In the next section we therefore propose a phenomenology centered on the more general form of the $\Delta\mathcal{L}_{QG}$ of Eq. (1), involving an arbitrary (four-parameter) four-vector n_α, and we describe the resulting equations of motion for the electromagnetic field. Since the types of data that are most useful and are likely to still be most useful to set bounds on this framework concern regimes that involve classical electromagnetic waves, we shall here be satisfied with an analysis confined at the level of some modified Maxwell equation for classical electromagnetic waves. In this respect we adopt the same perspective of the original analysis by Myers and Pospelov [16], but for our purposes it is valuable to provide, as we shall, a more detailed description of the Planck-scale modifications of classical electromagnetic waves, whereas Ref. [16] focused exclusively on the form of the dispersion ("on-shell") relation.

In Sec. III we investigate the features that are likely to be most relevant from the phenomenology perspective, which concern dispersion, birefringence, and a possible longitudinal component. In Sec. IV we provide a rough quantitative characterization of the effects introduced by the spatial components of n_α, focusing mainly on cases with a spacelike symmetry-breaking vector and stressing that the magnitude of the effects is not exclusively governed by the magnitude of the spatial components of n_α: there are direction-dependent (anisotropic) effects, and even small values of the spatial components of n_α produce large effects within a certain range of directions. Section V offers some closing remarks.

II. MODIFIED MAXWELL EQUATIONS AND ANALOGY WITH ANISOTROPIC MEDIA

By adding the Planck-scale correction term (1) to Maxwell's Lagrangian we arrive at a modified Lagrangian density for electrodynamics of the form

$$\mathcal{L}_{QG} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2M_P}n^\alpha F_{\alpha\delta}n^\sigma\partial_\sigma(n_\beta\varepsilon^{\beta\delta\gamma\lambda}F_{\gamma\lambda}), \tag{3}$$

from which one easily derives the associated modified Maxwell equations:

$$0 = \vec{\nabla}\times\vec{B} - \frac{\partial\vec{E}}{\partial t} - \frac{2}{M_P}\left(n_0\frac{\partial}{\partial t} - \vec{n}\cdot\vec{\nabla}\right)^2(\vec{n}\times\vec{E} + n_0\vec{B}), \tag{4}$$

$$0 = \vec{\nabla}\cdot\vec{E} + \frac{2}{M_P}\left(n_0\frac{\partial}{\partial t} - \vec{n}\cdot\vec{\nabla}\right)^2\vec{n}\cdot\vec{B}, \tag{5}$$

$$0 = \vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t}, \tag{6}$$

$$0 = \vec{\nabla}\cdot\vec{B}. \tag{7}$$

For the case of plane waves, in which we are primarily interested, these modified Maxwell equations take the form

$$\begin{aligned}
&\vec{k}\times\vec{B} = -\omega\vec{E} - i\frac{2}{M_P}(n_0\omega + \vec{n}\cdot\vec{k})^2(\vec{n}\times\vec{E} + n_0\vec{B}),\\
&\vec{k}\times\vec{E} = \omega\vec{B},\\
&\vec{k}\cdot\vec{E} = -i\frac{2}{M_P}(n_0\omega + \vec{n}\cdot\vec{k})^2\vec{n}\cdot\vec{B},\\
&\vec{k}\cdot\vec{B} = 0.
\end{aligned} \tag{8}$$

Interestingly these equations are rather similar to the ones that govern ordinary propagation of electromagnetic radiation in certain anisotropic media [21]. In particular, one could view (8) as equations of propagation in a material with polarization vector

$$\vec{P} = \frac{2}{M_P}\omega\left|\left(\vec{n} + n_0\frac{\vec{k}}{\omega}\right)\right|\left(n_0 + \vec{n}\cdot\frac{\vec{k}}{\omega}\right)^2(-i\hat{v})\times\vec{E}, \tag{9}$$

where we introduced the notation

$$\hat{v} = \frac{\vec{n} + n_0\frac{\vec{k}}{\omega}}{|\vec{n} + n_0\frac{\vec{k}}{\omega}|}$$

and essentially we noticed that $\vec{P}$ can be written in terms of a susceptivity tensor χ that can be expressed in terms of $\hat{v}$ as follows:

$$\begin{aligned}
\chi &\equiv \chi(\vec{n}, n_0, \vec{k}, \omega)\\
&= \frac{2}{M_P}\omega\left|\left(\vec{n} + n_0\frac{\vec{k}}{\omega}\right)\right|\left(n_0 + \vec{n}\cdot\frac{\vec{k}}{\omega}\right)^2\\
&\quad\times\begin{bmatrix} 0 & i\hat{v}_3 & -i\hat{v}_2\\ -i\hat{v}_3 & 0 & i\hat{v}_1\\ i\hat{v}_2 & -i\hat{v}_1 & 0\end{bmatrix}.
\end{aligned} \tag{10}$$

Clearly the availability of a strict analogy between our model and propagation in anisotropic media is confined to the ideal case of propagation of plane waves, since the susceptivity tensor χ which we formally introduced depends on the wave vector $\vec{k}$ (so the propagation of generic waves, spread over different wave vectors, could not be characterized in terms of a susceptivity tensor). And even restricting one's attention on plane waves there are some peculiarities that characterize our Planck-scale-deformed propagation of electromagnetic waves, as a result of the fact that the relation between polarization and the electric field depends on $\vec{k}$ and ω.

For a first level of characterization of these peculiarities we can formally think of our $\hat{v}$ as an effective direction of anisotropy, in which case one obtains a close analogy between our theory and the established description of propagation of ordinary electromagnetic waves in gyrotropic optically active media [22]. Indeed for gyrotropic media with both natural and induced optical activity, the polarization vector can be written as [23]

$$\vec{P} = -if(\hat{k} \times \vec{E}) - ig(\hat{g} \times \vec{E}), \tag{11}$$

where $\hat{g}$ identifies the direction of the external field which induces optical activity (gyrotropic axis), while f and g are two coefficients for the magnitude of the effect. The case of propagation in an inactive dielectric is obtained for $f = g = 0$, while for $f \neq 0$, $g = 0$ one has natural optical activity, and the case $f = 0$, $g \neq 0$ gives pure induced gyrotropy. As shown above, in our model $\vec{P}$ can be written in the same form, with $\hat{g} \to \hat{n}$, and

$$f \to \frac{2}{M_P}|\vec{k}|\left(n_0 + \frac{\vec{n}\cdot\vec{k}}{\omega}\right)^2 n_0, \tag{12}$$

$$g \to \frac{2}{M_P}\omega\left(n_0 + \frac{\vec{n}\cdot\vec{k}}{\omega}\right)^2 |\vec{n}|. \tag{13}$$

So the peculiarity of our model resides in the dependence of both f and g on the frequency and wave vector of the wave, and different regimes of our model end up producing effects that resemble the ones found in different types of anisotropic materials. For plane waves propagating with k_μ orthogonal to n_α [i.e. $(n_0 + \frac{\vec{n}\cdot\vec{k}}{\omega}) = 0$], both f and g vanish and the system behaves classically (inactive dielectric). If $|\vec{n}| = 0$ (which, as mentioned, is the case of the original Myers-Pospelov model [16]), then $g = 0$, and the system behaves like a naturally optically active medium. In the opposite limit, $n_0 = 0$, one has $f = 0$, i.e. a medium with pure induced gyrotropy.

III. DISPERSION, BIREFRINGENCE, AND LONGITUDINAL COMPONENT

Already on the basis of established features for the original Myers-Pospelov model (our case $|\vec{n}| = 0$) we must expect that the speed of propagation of our Planck-scale-deformed electromagnetic waves should depend on their wavelength and on polarization. For the more general case $|\vec{n}| \neq 0$ we shall also characterize a dependence of these effects on the angle formed by the wave vector and the vector $\vec{n}$. Moreover, while the field still has only two degrees of freedom, a longitudinal component will in general be present: the presence of a longitudinal component is prevented when both gauge invariance and Lorentz symmetry hold, but our framework (while being gauge invariant) clearly breaks Lorentz symmetry. There was no longitudinal component for solutions of the original Myers-Pospelov model, but only in some sense accidentally, as an indirect result of the adopted simplification of spatial isotropy ($|\vec{n}| = 0$).

We shall characterize these features, at leading order in M_P^{-1}, by examining the equation of motion for the electric field in momentum space that is obtained from our modified Maxwell equations (8):

$$-\vec{k}(\vec{k}\cdot\vec{E}) + \vec{k}^2\vec{E} = \omega^2\vec{E} + \frac{2i}{M_P}\omega(n_0\omega + \vec{n}\cdot\vec{k})^2 \times \left|\vec{n} + n_0\frac{\vec{k}}{\omega}\right|(\hat{v}\times\vec{E}). \tag{14}$$

In particular, from this one easily infers that at leading order the two on-shell conditions (we have indeed birefringence) depend on $\vec{k}$ and n_α as follows:

$$\omega \simeq |\vec{k}| \pm \frac{1}{M_P}|\vec{k}|^2\left(n_0 + \frac{\vec{n}\cdot\vec{k}}{|\vec{k}|}\right)^3, \tag{15}$$

where the sign choice $\pm$ codifies the difference between the two on-shell conditions.

A. Restricting to the spatially isotropic case

For the case $|\vec{n}| = 0$ [i.e. $n_\alpha = (n_0, 0, 0, 0)$] Eq. (15) of course reproduces the dispersion relation originally obtained by Myers and Pospelov:

$$\omega \simeq |\vec{k}| \pm \frac{1}{M_P}|\vec{k}|^2(n_0)^3. \tag{16}$$

And from Eq. (14) one then easily infers that the "normalized field eigenstates" (waves "on shell" with intensity 1) are circularly polarized:

$$\vec{E}_\pm = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \pm i \\ 1 \end{pmatrix}, \tag{17}$$

where we are using three-dimensional Jones-vector notation[1] and we are assuming that the field propagates along the $\hat{x}$ direction. As mentioned, these characteristics of on-shell waves in our framework establish an analogy with the case of ordinary electromagnetic plane waves propagating in a naturally optically active material.

[1] In the notation of Jones three-dimensional vectors the field $\vec{E}(x,t) = \mathrm{Re}[(E_x\hat{x} + E_y\hat{y} + E_z\hat{z})e^{i(\vec{k}\cdot x - \omega t)}]$, with E_x, E_y, and E_z complex numbers, is represented as

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}.$$

B. Spatial anisotropy in the case with no time component for the symmetry-breaking vector

It is valuable to first compare the Myers-Pospelov/spatially isotropic case to the opposite regime $n_0 = 0$, $n_\alpha = (0, n_x, n_y, n_z)$. In this case, $\vec{n}$, the spatial part of n_α, plays a role that is closely analogous to the role of the gyrotropic axis for ordinary propagation in crystals. The dispersion relation takes the form

$$\omega \simeq |\vec{k}| \pm \frac{1}{M_P}|\vec{k}|^2\left(\frac{\vec{n}\cdot\vec{k}}{|\vec{k}|}\right)^3, \tag{18}$$

so that evidently there is a strong dependence of dispersion on the angle between the wave vector and the spatial part of symmetry-breaking vector. For waves propagating in a direction orthogonal to $\vec{n}$ the dispersion is completely absent (no difference from the undeformed theory), while of course the dispersion reaches its maximum magnitude for fields propagating along the $\hat{n}$ direction. These two limiting cases, $\vec{k}\cdot\vec{n} = 0$ and $\vec{k}\times\vec{n} = 0$, are also peculiar in that for them the field does not acquire a longitudinal component, but a (ultrasmall, "Planck-scale suppressed," but nonzero) longitudinal component is present in all other cases.

We find convenient to describe the field eigenstates in an orthonormal basis that takes into account the relative orientation of the vectors $\vec{k}$ and $\vec{n}$:

$$\left\{\frac{\vec{k}}{|\vec{k}|}, \frac{\hat{n}\times\vec{k}}{\sqrt{k^2-(\hat{n}\cdot\vec{k})^2}}, \frac{-\vec{k}(\vec{k}\cdot\hat{n})+\hat{n}|\vec{k}|^2}{|\vec{k}|\sqrt{k^2-(\hat{n}\cdot\vec{k})^2}}\right\}. \tag{19}$$

By adopting this basis we have that the first component of the field is longitudinal, while the other two components lie in the plane orthogonal to the propagation direction. And from Eq. (14) one then easily finds that, for a generic wave vector $\vec{k}$, in this basis

$$\vec{E}_\pm = \begin{pmatrix} \mp\frac{1}{M_P}\frac{|\vec{k}-(\vec{k}\cdot\hat{n})\hat{n}|(\vec{k}\cdot\vec{n})^2|\vec{n}|}{|\vec{k}|^2} \\ \frac{\imath}{2\sqrt{2}|\vec{k}|^2}\left(\pm 2|\vec{k}|^2 + \frac{2}{M_P}(\vec{k}\cdot\vec{n})|\vec{n}|^2|\vec{k}-(\vec{k}\cdot\hat{n})\hat{n}|^2\right) \\ \frac{1}{2\sqrt{2}|\vec{k}|^2}\left(2|\vec{k}|^2 \mp \frac{2}{M_P}|\vec{k}-(\vec{k}\cdot\hat{n})\hat{n}|^2(\vec{k}\cdot\vec{n})|\vec{n}|^2\right)\end{pmatrix}. \tag{20}$$

C. General case

If both the time and spatial parts of the symmetry-breaking four-vector are nonzero, then we are in the most general scenario for our framework, and of course the dispersion relation is the one of (15),

$$\omega \simeq |\vec{k}| \pm \frac{1}{M_P}|\vec{k}|^2\left(n_0 + \frac{\vec{n}\cdot\vec{k}}{|\vec{k}|}\right)^3.$$

Notice that there is no dispersion and no birefringence when $\vec{n}\cdot\vec{k} = -n_0|\vec{k}|$, and from this we infer that if n_α is spacelike (or lightlike) there must necessarily be a "blind direction" (where the dispersion relation has classical form). In the next section we shall attempt to characterize the range of directions in the neighborhood of the blind direction where a significant suppression of the nonclassical effects occurs.

It is also interesting to examine the special case of waves propagating in a direction orthogonal to $\vec{n}$. In this case the dispersion relation takes Myers-Pospelov form, $\omega \simeq |\vec{k}| \pm M_P^{-1}|\vec{k}|^2(n_0)^3$, but the field eigenstates are still different (if $|\vec{n}| \neq 0$) from the ones found in the Myers-Pospelov model:

$$\vec{E}_\pm = \begin{pmatrix} \mp\frac{\sqrt{2}}{M_P}|\vec{k}||\vec{n}|n_0^2 \\ \pm\frac{i}{\sqrt{2}} + \frac{i}{\sqrt{2}M_P}|\vec{k}||\vec{n}|^2 n_0 \\ \frac{1}{\sqrt{2}} \mp \frac{1}{\sqrt{2}M_P}|\vec{k}||\vec{n}|^2 n_0 \end{pmatrix}, \tag{21}$$

which is an elliptically polarized field, rotating in a plane not perpendicular to the propagation direction.

The solutions of the original Myers-Pospelov proposal, which we find convenient to still write in the notation of Jones three-vectors[2]

$$\vec{E}_\pm = \begin{pmatrix} 0 \\ \pm i \\ 1 \end{pmatrix}, \tag{22}$$

emerge in our more general framework when $\vec{k}$ is parallel to $\vec{n}$. But the corresponding dispersion relation still carries a dependence on $|\vec{n}|$:

$$\omega \simeq |\vec{k}| \pm \frac{1}{M_P}|\vec{k}|^2(n_0 + \epsilon_{\vec{k},\vec{n}}|\vec{n}|)^3, \tag{23}$$

where $\epsilon_{\vec{k},\vec{n}} = 1$ if $\vec{k}$ is parallel to $\vec{n}$ while $\epsilon_{\vec{k},\vec{n}} = -1$ if $\vec{k}$ is antiparallel to $\vec{n}$. These cases in which the propagation direction is parallel (or antiparallel) to $\vec{n}$ are the only ones where one finds field eigenstates with circular polarization (of course in the plane orthogonal to the propagation direction, which is also the plane orthogonal to $\vec{n}$), if n_α is timelike and $|\vec{n}| \neq 0$, $n_0 \neq 0$. For spacelike (or timelike) n_α there is also another case with a vanishing longitudinal component, the case of propagation directions such that $\vec{n}\cdot\vec{k} = -n_0|\vec{k}|$ (for which, as already stressed above, all anomalous effects disappear).

Finally let us note down the general result for the field eigenstates, for generic propagation directions such that $\vec{k}$ is not along the $\vec{n}$ direction ($|\vec{k}\cdot\vec{n}| < |\vec{k}||\vec{n}|$), which in our basis (19) is

[2] In the limit in which $\vec{k}$ has the same direction of $\vec{n}$ the expression (19) is not well-defined, since the transverse components collapse. This simply means that any pair of orthonormal vectors in the transverse plane can be used to complete the basis and the form of, e.g., (22) is independent on this choice.

$$\vec{E}_{\pm} = \begin{pmatrix} \mp \frac{2}{M_P} \frac{|\vec{n}|(\vec{k}\cdot\vec{n}+|\vec{k}|n_0)^2|\vec{k}-(\vec{k}\cdot\hat{n})\hat{n}|}{\sqrt{2}|\vec{k}|^2} \\ \pm \frac{i}{\sqrt{2}} + \frac{i}{\sqrt{2}M_P} \frac{|n|^2|\vec{k}-(\vec{k}\cdot\hat{n})\hat{n}|^2(\vec{k}\cdot\vec{n}+|\vec{k}|n_0)}{|\vec{k}|^2} \\ \frac{1}{\sqrt{2}} \mp \frac{1}{\sqrt{2}M_P} \frac{|\vec{n}|^2|\vec{k}-(\vec{k}\cdot\hat{n})\hat{n}|^2(\vec{k}\cdot\vec{n}+|\vec{k}|n_0)}{k^2} \end{pmatrix}. \quad (24)$$

This general form of the eigenstates, as well as the corresponding general form (15) of the dispersion relation, can be naturally described in terms of the analogy discussed in Sec. II with propagation of ordinary waves in gyrotropic media [23–25], but of course this analogy is here of mere academic interest.

IV. OPPORTUNITIES AND CHALLENGES FOR PHENOMENOLOGY

The characterization of Planck-scale-deformed electromagnetic waves given in the previous section is sufficient for the most used and efficacious phenomenological analyses. For example, one can rely on the fact that a wave of this sort emitted with a definite linear polarization after long propagation times ends up losing any trace of the original linear polarization, because of the combined effect of dispersion and birefringence [17,26,27] (also see Refs. [28,29]). Indeed some of the stringent bounds on the single parameter of the original Myers-Pospelov proposal have been established [17,26,27] by exploiting this polarization-erasing effect, using observations of polarized light from distant radio galaxies.

By placing the Myers-Pospelov proposal within the broader framework of a generic symmetry-breaking four-vector n_α we have characterized the possibility of effects that are in many ways similar to the ones of the original Myers-Pospelov proposal, but with the addition of spatial anisotropy. And the example of observations of polarized light from distant radio galaxies can easily illustrate how the spatial anisotropy may reduce the strength of the implications of some observations. In particular, the observation of polarized light from a single distant radio galaxy already produces definite bounds on a spatially isotropic polarization-erasing effect, but in our more general framework, while one clearly still finds polarization-erasing effects similar to the ones of the original Myers-Pospelov proposal, these effects depend on the direction of propagation. As stressed in the previous section, in the cases with spacelike (or lightlike) symmetry-breaking vector one even finds blind directions, i.e. propagation directions where no polarization-erasing effect is produced. In principle within our more general framework a single observation of polarized light from a distant radio galaxy can at best provide information on the relative strength of different components of n_α but without setting any absolute bound on the overall magnitude of the deformation. More insightful bounds can be obtained by combining different observations, associated with different directions of propagation, but still producing results whose significance is partly weakened by the lack of spatial isotropy, and only at the price of handling carefully the fact that of course the components of n_α change in going from one laboratory frame to another (and therefore different data sets must be first rendered comparable by mapping them all to a single reference frame).

It seems that the best strategies for constraining our more general framework should rely either on data on a large sample of directions of propagation, as in the collection of sources considered in Refs. [30,31], or on data that characterize at once a sizable range of directions of propagation, as is the case for certain types of studies done in cosmology, such as the study reported in Ref. [32]. Note however that the scopes of the type of studies we are here advocating are significantly different from both the ones of Ref. [32] and the ones of Refs. [30,31]. The cosmology studies we advocate would look at roughly the same characteristics of cosmic microwave background radiation polarization, but for our more general Myers-Pospelov-inspired framework, whereas Ref. [32] exploited the simplification afforded by the spatial isotropy of the original Myers-Pospelov setup. And concerning studies combining data from several astrophysics sources, the specific wavelength dependence of our framework, which was not considered[3] in Refs. [30,31], will have to play an important role, by introducing a strong premium for UV sources.

Since it appears that from the viewpoint of phenomenology the most challenging aspect of our framework is indeed the spatial anisotropy and particularly the mentioned blind directions, in Figs. 1 and 2 we provide a more quantitative characterization of these features. For definiteness in these figures we took $n_0 > 0$. And we only considered cases with n^α spacelike ($|\vec{n}| > n_0$), since these clearly are the cases for which the previous literature [focused on the Myers-Pospelov case $n_\alpha = (n_0, 0, 0, 0)$] informs less reliably our intuition.

The figures highlight the key role played by the angle θ between $\vec{n}$, the spatial part of the symmetry-breaking vector, and the direction of propagation [$\cos\theta \equiv \vec{n}\cdot\vec{k}/(|\vec{n}||\vec{k}|)$], and they mainly intend to characterize the behavior of the correction to the dispersion relation (and associated birefringence), but also describe the behavior of the longitudinal component of the field. From the figures one easily recognizes several characteristic features, some of which we had already pointed out in our discussion of some relevant formulas:

[3]References [30,31] focused on terms of dimension 4 that could be consistently included in the "standard model extension" program [33,34]. The original formulation of the standard model extension [33] was devised so that the Lagrangian density would involve only terms of dimension 4 or lower, and therefore did not include the $\Delta\mathcal{L}_{QG}$ of Eq. (1). In recent years a generalization of the standard model extension has been adopted (see, e.g., Ref. [34]), allowing also for the presence of terms of dimension 5, like $\Delta\mathcal{L}_{QG}$, and 6.

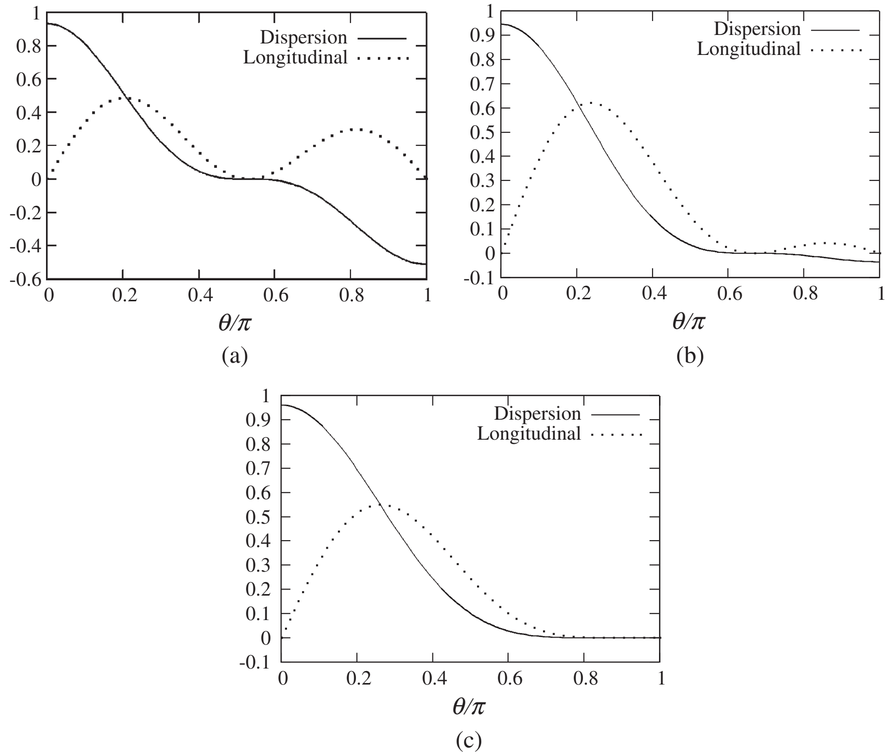

FIG. 1. Behavior of the longitudinal component of the field (dotted line), and of the nonclassical part of the dispersion law (continuous line) as functions of $\frac{\theta}{\pi}$. For the behavior of the dispersive effects we simply show (up to an irrelevant overall factor introduced for visibility) the function $(n_0 + |\vec{n}|\cos(\theta))^3$, which indeed gives the dependence of these effects on the propagation direction, and similarly for the longitudinal component we show (up to another irrelevant factor introduced for visibility) $(n_0 + |\vec{n}|\cos(\theta))^2\sin(\theta)$, which indeed gives the dependence of the longitudinal component on the propagation direction. For panel (a) we took $|\vec{n}| = 1$, $n_0 = 0.1$; for panel (b) we took $|\vec{n}| = 1$, $n_0 = 0.5$; and for panel (c) we took $|\vec{n}| = 1$, $n_0 = 0.9$.

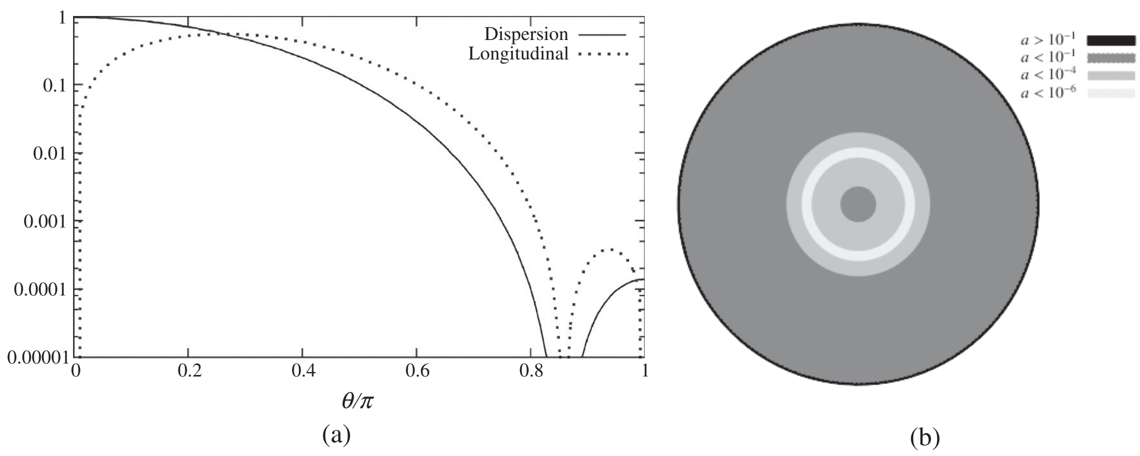

FIG. 2. In (a) we show the same case already shown in Fig. 1(c) ($|\vec{n}| = 1$, $n_0 = 0.9$), but in a logarithmic plot which allows one to better appreciate the sizable partial blindness present between $\theta \simeq 0.8\pi$ to $\theta \simeq \pi$. [Notice that, this being a logarithmic plot, we characterize the dispersive effects by the absolute value of $(n_0 + |\vec{n}|\cos(\theta))^3$, whereas Fig. 1(c) showed also the behavior of the sign of $|(n_0 + |\vec{n}|\cos(\theta))^3|$.] We give in (b) an intuitive quantitative characterization (polar plot of one-half of the sky) of the portions of the sky that would typically have a certain blindness level, still for the case $|\vec{n}| = 1$, $n_0 = 0.9$. The directions of "total blindness" form a circle in the sky, and for any given "level of blindness" one has an associated circular crown of corresponding thickness. Interestingly nearly the whole of the half of the sky shown in (b) has blindness amounting to a least suppression by a factor of 10, and the portion of the sky with blindness of 10^{-4} is evidently non-negligible. Note however that (b) was drawn assuming one is looking in the direction of $\vec{n}$, so essentially concerns $\pi/2 \leq \theta \leq \pi$, which for $n_0 > 0$ is the portion of the sky where most "blindness" is found.

(i) the longitudinal component vanishes both for $\theta = 0$ and for $\theta = \pi$;
(ii) the magnitude of the dispersive effects is greatest for $\theta = 0$ (would have been greatest for $\theta = \pi$ if we had chosen $n_0 < 0$);
(iii) in all figures one clearly notices the "blind" value of θ, $\theta = \theta_0$, with θ_0 such that $n_0 + |\vec{n}| \cos\theta_0 = 0$, where both the longitudinal component and the dispersive effects vanish;
(iv) the smallness of the dispersive effects persists for a sizable range of values of θ in some neighborhood of $\theta = \theta_0$.

This last point was particularly surprising for us: at the qualitative level we expected of course that the dispersive effects would be small in some neighborhood of the blind direction, but somehow we envisaged this neighborhood would be very small. Instead one typically finds sizable regions of "partial (but significant) blindness." In order to render this feature more visible in Fig. 2(a) we show in logarithmic scale the same case already shown in Fig. 1(c), with $n_0/|\vec{n}| = 0.9$ and blind direction $\theta_0 \simeq 0.86\pi$. Comparable "blindness features" are found for all cases with spacelike n_α. It is noteworthy that in Fig. 1(c) one sees a rather persistent suppression of the dispersive effects by more than 4 orders of magnitude, all the way from $\theta \simeq 0.8\pi$ to $\theta \simeq \pi$. In Fig. 2(b) we give an intuitive quantitative characterization of the portions of the sky that would typically have a certain "blindness level," still for the case of Figs. 1(c) and 2(a). It is noteworthy that in Fig. 2(b) one finds that a suppression as large as 10^{-4} is found for a rather large fraction of the sky, of about 10%. And suppression of 10^{-1}, which is less impressive but still of course very significant, is found in a large fraction of the sky.

V. CLOSING REMARKS

The fast pace of improvement of the phenomenology of the Myers-Pospelov proposal [16] exploited the spatial isotropy regained by the *ad hoc* choice $n_\alpha = (n_0, 0, 0, 0)$. It should be noticed that the analysis we reported here is in principle relevant even for the case of timelike n_α, where $n_\alpha = (n_0, 0, 0, 0)$ is possible in one class of frames: for frames boosted with respect to a frame with $n_\alpha = (n_0, 0, 0, 0)$ there would of course be a spatial component for n_α. There is recent literature [35–37] on frameworks that could implement observer-independent departures from Lorentz symmetry, but this is clearly not the case of the Myers-Pospelov setup. Phenomenologists who have analyzed the Myers-Pospelov proposal are well aware of this frame dependence, and they have neglected it only in light of the fact that the different laboratory frames where the data were being collected are connected by relatively small boosts. For the case of timelike n_α our analysis is therefore at least valuable in as much as it allows one to actually estimate the size of corrections that are being neglected by assuming that $n_\alpha = (n_0, 0, 0, 0)$ in all of these laboratory frames.

Clearly the most intriguing part of our findings concerns the case of spacelike n_α, which had not been considered in previous works inspired by the Myers-Pospelov proposal. For spacelike n_α one could easily imagine that there would be sizable anisotropy, but it might have been hard to imagine that, for example, a reduction of anomalous effects by 4 orders of magnitude ("partial blindness") could persist for ranges of directions of propagation (with respect to the direction of $\vec{n}$) as large as shown in the previous section. At least at the preliminary level of analysis we here offered it appears that these features might reduce significantly the bound-setting power of most types of observations in astrophysics, which essentially probe a narrow range of directions of propagation. More stringent bounds can be obtained by either combining data from several astrophysics sources or exploiting the fact that certain types of studies in cosmology can be used to investigate anomalous laws of propagation in ways that naturally involve [32] a very broad range of directions of propagation. We expect that even using these techniques the bounds on our more general framework will turn out to be very significantly weaker than the corresponding bounds on the original Myers-Pospelov proposal, possibly as much as an order of magnitude weaker. Considering the tightness of the bounds obtained on the spatially isotropic case, several orders of magnitude beyond $\xi \sim 1$, the magnitude of this weakening of bounds is not going to regain much interest in these models from the quantum-gravity side, at least not according to the naive expectation that would favor values of ξ of order 1. But it should be taken into account by future further investigations of modifications to electromagnetism by dimension-5 correction terms.

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[1] G. Amelino-Camelia *et al.*, Nature (London) **393**, 763 (1998).
[2] R. Gambini and J. Pullin, Phys. Rev. D **59**, 124021 (1999).
[3] B. E. Schaefer, Phys. Rev. Lett. **82**, 4964 (1999).

[4] S. D. Biller *et al.*, Phys. Rev. Lett. **83**, 2108 (1999).
[5] T. Kifune, Astrophys. J. Lett. **518**, L21 (1999).
[6] G. Amelino-Camelia, Nature (London) **398**, 216 (1999).
[7] J. Alfaro, H. A. Morales-Tecotl, and L. F. Urrutia, Phys. Rev. Lett. **84**, 2318 (2000).
[8] G. Amelino-Camelia and T. Piran, arXiv:astro-ph/0008107; Phys. Rev. D **64**, 036005 (2001).
[9] T. Jacobson, S. Liberati, and D. Mattingly, Nature (London) **424**, 1019 (2003).
[10] U. Jacob and T. Piran, Nature Phys. **3**, 87 (2007).
[11] J. Ellis, N. E. Mavromatos, and D. V. Nanopoulos, Phys. Lett. B **674**, 83 (2009).
[12] G. Amelino-Camelia and L. Smolin, Phys. Rev. D **80**, 084017 (2009).
[13] A. Abdo *et al.*, Nature (London) **462**, 331 (2009).
[14] G. Amelino-Camelia, C. Laemmerzahl, F. Mercati, and G. M. Tino, Phys. Rev. Lett. **103**, 171302 (2009).
[15] G. Amelino-Camelia, Lect. Notes Phys. **541**, 1 (2000); arXiv:0806.0339.
[16] R. C. Myers and M. Pospelov, Phys. Rev. Lett. **90**, 211601 (2003).
[17] D. Mattingly, Living Rev. Relativity **8**, 5 (2005).
[18] L. Maccione and S. Liberati, J. Cosmol. Astropart. Phys. 08 (2008) 027.
[19] M. Galaverni and G. Sigl, Phys. Rev. Lett. **100**, 021102 (2008).
[20] L. Maccione and S. Liberati, J. Cosmol. Astropart. Phys. 08 (2008) 027.
[21] M. Born and E. Wolf, *Principles of Optics* (Pergamon, New York, 1970).
[22] L. d. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, New York, 1960).
[23] M. P. Silverman and R. B. Sohn, Am. J. Phys. **54**, 69 (1986).
[24] S. C. McClain, L. W. Hillman, and R. A. Chipman, J. Opt. Soc. Am. A **10**, 2383 (1993).
[25] O. S. Eritsyan, Sov. Phys. Usp. **25**, 919 (1982).
[26] R. J. Gleiser and C. N. Kozameh, arXiv:gr-qc/0102093; Phys. Rev. D **64**, 083007 (2001).
[27] R. J. Gleiser, C. N. Kozameh, and F. Parisi, arXiv:gr-qc/0304048; Classical Quantum Gravity **20**, 4375 (2003).
[28] S. M. Carroll, G. B. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990).
[29] V. A. Kostelecky and M. Mewes, arXiv:hep-ph/0111026; Phys. Rev. Lett. **87**, 251304 (2001).
[30] V. A. Kostelecky and M. Mewes, Phys. Rev. Lett. **87**, 251304 (2001).
[31] V. A. Kostelecky and M. Mewes, Phys. Rev. D **66**, 056005 (2002).
[32] G. Gubitosi, L. Pagano, G. Amelino-Camelia, A. Melchiorri, and A. Cooray, J. Cosmol. Astropart. Phys. 08 (2009) 021.
[33] D. Colladay and V. A. Kostelecky, Phys. Rev. D **58**, 116002 (1998).
[34] V. A. Kostelecky and M. Mewes, Phys. Rev. D **80**, 015020 (2009).
[35] G. Amelino-Camelia, arXiv:gr-qc/0012051; Int. J. Mod. Phys. D **11**, 35 (2002); arXiv:hep-th/0012238; Phys. Lett. B **510**, 255 (2001).
[36] J. Magueijo and L. Smolin, arXiv:gr-qc/0207085; Phys. Rev. D **67**, 044017 (2003).
[37] J. Kowalski-Glikman and S. Nowak, arXiv:hep-th/0204245; Int. J. Mod. Phys. D **12**, 299 (2003).