PHYSICAL REVIEW D **82,** 024002 (2010)

Static solutions with nontrivial boundaries for the Einstein-Gauss-Bonnet theory in vacuum

Gustavo Dotti,[1,5,*] Julio Oliva,[2,†] and Ricardo Troncoso[3,4,‡]

[1]*FaMAF, Universidad Nacional de Córdoba, Ciudad Universitaria, (5000) Córdoba, Argentina*
[2]*Instituto de Física, Facultad de Ciencias Físicas, Universidad Austral de Chile, Valdivia, Chile*
[3]*Centro de Estudios Científicos (CECS), Casilla 1469, Valdivia, Chile*
[4]*Centro de Ingeniería de la Innovación del CECS (CIN), Valdivia, Chile*
[5]*Instituto de Física Enrique Gaviola, CONICET, Córdoba, Argentina*
(Received 29 April 2010; published 1 July 2010)

The classification of a certain class of static solutions for the Einstein-Gauss-Bonnet theory in vacuum is performed in $d \geq 5$ dimensions. The class of metrics under consideration is such that the spacelike section is a warped product of the real line and an arbitrary base manifold. It is shown that for a generic value of the Gauss-Bonnet coupling, the base manifold must be necessarily Einstein, with an additional restriction on its Weyl tensor for $d > 5$. The boundary admits a wider class of geometries only in the special case when the Gauss-Bonnet coupling is such that the theory admits a unique maximally symmetric solution. The additional freedom in the boundary metric enlarges the class of allowed geometries in the bulk, which are classified within three main branches, containing new black holes and wormholes in vacuum.

DOI: 10.1103/PhysRevD.82.024002 PACS numbers: 04.50.−h

I. INTRODUCTION

The asymptotic properties of spacetime play a crucial role for a suitable definition of energy in gravitation, which has been a subtle issue since the early days of general relativity (see, e.g. [1]). Nowadays, understanding the asymptotic structure of spacetime becomes a fundamental problem by itself. In the case of a negative cosmological constant, the asymptotic behavior of gravity is particularly interesting, and a renewed interest has been raised in view of the AdS/CFT correspondence, which is a conjectured duality between gravity on asymptotically AdS spacetimes and conformal field theory (for a review see e.g., [2]). In this context, it is natural wondering about the possible freedom in the choice of the metric at the boundary, where the dual theory is defined. As a simple example, one can consider the following class of d-dimensional static metrics in bulk

$$ds^2 = -f^2(r)dt^2 + \frac{dr^2}{g^2(r)} + r^2 d\Sigma^2_{(d-2)}, \tag{1.1}$$

where

$$d\Sigma^2_{(d-2)} = \tilde{g}_{ij}(x)dx^i dx^j \tag{1.2}$$

is the line element[1] of the "base" manifold $\Sigma_{(d-2)}$ of $d-2$ dimensions.

The Einstein equations with cosmological constant Λ in vacuum are then solved for

$$f^2 = g^2 = \gamma - \frac{\mu}{r^{d-3}} - \frac{2\Lambda}{(d-1)(d-2)} r^2, \tag{1.3}$$

provided the geometry of $\Sigma_{(d-2)}$ is restricted to be that of an Einstein manifold, fulfilling

$$\tilde{R}^i{}_j = (d-3)\gamma\delta^i_j, \tag{1.4}$$

where the constant γ can be normalized to ± 1 or zero [3–5]. Thus, if the cosmological constant is non-negative, solutions of the form (1.1), with (1.3) and (1.4), describe black holes only for $\gamma = 1$ and $\mu > 0$; otherwise they possess naked singularities. Remarkably, for the asymptotically AdS case, the solution describes black holes for any value of γ provided μ is bounded from below [6–8], widening the possibilities in order to define a dual theory at the boundary, whose metric is of the form $R \times \Sigma_{(d-2)}$.

In dimensions greater than four, general relativity (GR) is not the only option to describe gravity. Indeed, a natural and conservative generalization of GR, being the most general theory of gravity leading to second order field equations for the metric is described by the Lovelock action, which possesses nonlinear terms in the curvature in a precise combination [9]. The simplest case corresponds to the so-called Einstein-Gauss-Bonnet (EGB) theory, whose action is quadratic in the curvature, and it is given by

$$I = \int \sqrt{-g}d^dx\Big[c_1 R - 2c_0 + \frac{c_2}{2}(R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu} - 4R^{\mu\nu}R_{\mu\nu} + R^2)\Big], \tag{1.5}$$

so that apart from the Newton and cosmological constants, the theory possesses an additional coupling c_2 associated with the quadratic terms. The field equations read

*gdotti@famaf.unc.edu.ar
†julio.oliva@docentes.uach.cl
‡troncoso@cecs.cl

[1]Here x^i correspond to local "angular" coordinates, and hereafter a tilde is used on geometrical objects intrinsically defined on $\Sigma_{(d-2)}$.

$$c_2 H_{\mu\nu} + c_1 G_{\mu\nu} + c_0 g_{\mu\nu} = 0, \tag{1.6}$$

where $G_{\mu\nu}$ is the Einstein tensor, and

$$H_{\mu\nu} := R R_{\mu\nu} - 2R_{\mu\rho}R^{\rho}{}_{\nu} - 2R^{\delta}{}_{\rho}R^{\rho}{}_{\mu\delta\nu} + R_{\mu\rho\delta\gamma}R_{\nu}{}^{\rho\delta\gamma} + \frac{1}{d-4} H g_{\mu\nu}, \tag{1.7}$$

with

$$H := H^{\mu}{}_{\mu} = \frac{(4-d)}{4}(R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu} - 4R^{\mu\nu}R_{\mu\nu} + R^2), \tag{1.8}$$

identically vanishes in $d < 5$ dimensions.

In terms of the vielbein $e^a = e^a_\mu dx^\mu$ and the curvature 2-form $R^{ab} = \frac{1}{2}R^{ab}{}_{\mu\nu}dx^\mu dx^\nu$, the field equations read

$$\mathcal{E}_a := \epsilon_{ab_1 \ldots b_{d-1}}[a_2 R^{b_1 b_2} R^{b_3 b_4} + 2a_1 R^{b_1 b_2} e^{b_3} e^{b_4} + a_0 e^{b_1} e^{b_2} e^{b_3} e^{b_4}] e^{b_5} \ldots e^{b_{d-1}} = 0, \tag{1.9}$$

where the wedge product between forms is understood. The relation between the constants α_j in (1.9) and c_j in (1.6) is

$$c_0 = \frac{a_0}{2}(d-1)!, \qquad c_1 = -2(d-3)! a_1, \qquad c_2 = -2(d-5)! a_2. \tag{1.10}$$

Generically, the field equations of the EGB theory admit two different maximally symmetric solutions—(A)dS or Minkowski—fulfilling[2]

$$R^{\alpha\beta}{}_{\gamma\delta} = \lambda \delta^{\alpha\beta}_{\gamma\delta}, \tag{1.11}$$

with two different radii, determined by

$$\lambda_{\pm} = \frac{a_1}{a_2}\left(-1 \pm \sqrt{1 - \frac{a_2 a_0}{a_1^2}}\right). \tag{1.12}$$

In the limit of vanishing Gauss-Bonnet coupling, $a_2 \to 0$, the branch with negative sign in (1.12) diverges, whereas the other gives the expected GR limit, i.e., $\lambda_+ = -\frac{a_0}{2a_1}$.

If the Gauss-Bonnet coupling is such that the square root in (1.12) vanishes, i.e.,

$$a_2 = \frac{a_1^2}{a_0}, \tag{1.13}$$

the EGB theory admits a unique maximally symmetric vacuum. This case is naturally singled out as "special," since the theory admits solutions with a relaxed asymptotic behavior as compared with the standard one of GR [10].

Concerning the possible freedom in the choice of boundary metrics for the class of static spacetimes of the form (1.1), it can be seen that the presence of quadratic terms in the action generically leads to strong restrictions on geometry of the boundary, determined by Σ_{d-2}, since it has to be Einstein with supplementary conditions involving its Weyl tensor [11–13]. Nevertheless, in the special case (1.13), the EGB theory admits a wider class of boundary metrics, such that Σ_{d-2} is not necessarily Einstein. The additional freedom in the boundary metric enlarges the class of allowed geometries in the bulk, which are classified within three main branches, containing new black holes and wormholes in vacuum.

The class of static metrics of the form (1.1) with (1.2), solves the field equations of the EGB theory in d dimensions according to the following scheme.

A. $d = 5$ dimensions

(i) *Generic class:* For an arbitrary value of the Gauss-Bonnet coupling a_2, the metric (1.1) solves the EGB field equations provided the base manifold Σ_3 is necessarily of constant curvature γ (normalized to ± 1, 0), i.e.,

$$\tilde{R}^{ij}{}_{kl} = \gamma \delta^{ij}_{kl}, \tag{1.14}$$

and

$$f^2 = g^2(r) = \gamma + \frac{a_1}{a_2} r^2 \left[1 \pm \sqrt{\left(1 - \frac{a_2 a_0}{a_1^2}\right) + \frac{\mu}{r^4}}\right], \tag{1.15}$$

where μ is an integration constant.

(ii) *Special class:* In the special case where the Gauss-Bonnet coupling is given by (1.13), The bulk geometries split into three main branches according to the geometry of Σ_3:

(ii.a) *Black holes:* For an arbitrary base manifold, i.e.,

$$\Sigma_3\text{: arbitrary}, \tag{1.16}$$

the metric (1.1) solves the field equations provided

$$f^2 = g^2 = \sigma r^2 - \mu, \qquad \sigma := \frac{a_0}{a_1}, \tag{1.17}$$

where μ is an integration constant.

(ii.b.1) *Wormholes:* For base manifolds Σ_3 of constant nonvanishing Ricci scalar,

$$\tilde{R} = 6\gamma, \tag{1.18}$$

the metric (1.1) with

$$f^2(r) = (\sqrt{\sigma} r + a\sqrt{\sigma r^2 + \gamma})^2, \tag{1.19}$$

$$g^2(r) = \sigma r^2 + \gamma, \tag{1.20}$$

is a solution of the field equations, where a is an integration constant.

[2] Here $\delta^{\alpha\beta}_{\gamma\delta} := \delta^\alpha_\gamma \delta^\beta_\delta - \delta^\alpha_\delta \delta^\beta_\gamma$.

(ii.b.2) *Spacetime horns:* If the base manifold Σ_3 has *vanishing* Ricci scalar, i.e.,

$$\tilde{R} = 0, \tag{1.21}$$

the solution is given by

$$f^2(r) = \left(a\sqrt{\sigma}r + \frac{1}{\sqrt{\sigma}r}\right)^2, \tag{1.22}$$

$$g^2(r) = \sigma r^2, \tag{1.23}$$

with a an integration constant.

(iii) *Degeneracy:* If Σ_3 is of constant curvature, i.e.,

$$\tilde{R}^{ij}{}_{kl} = \gamma\delta^{ij}_{kl}, \tag{1.24}$$

then

$$g^2(r) = \sigma r^2 + \gamma, \tag{1.25}$$

$$f^2(r)\text{: an arbitrary function.} \tag{1.26}$$

B. $d = 6$ dimensions

(i) *Generic class:* For arbitrary values of the Gauss-Bonnet coupling the metric (1.1) solves the EGB field equations provided the base manifold Σ_4 is Einstein, i.e.,

$$\tilde{R}^{i}{}_{j} = 3\gamma\delta^{i}_{j} \tag{1.27}$$

(with γ normalized to ±1, 0) with the following (scalar) condition:

$$\tilde{R}^{ij}{}_{kl}\tilde{R}^{kl}{}_{ij} - 4\tilde{R}_{ij}\tilde{R}^{ij} + \tilde{R}^2 - 24\xi = 0, \tag{1.28}$$

and

$$\begin{aligned} f^2(r) &= g^2(r) \\ &= \gamma + \frac{a_1}{a_2}r^2 \\ &\quad\times\left[1 \pm \sqrt{\left(1 - \frac{a_2 a_0}{a_1^2}\right) + \frac{\mu}{r^5} + \frac{a_2^2}{a_1^2}\frac{(\gamma^2 - \xi)}{r^4}}\right], \end{aligned} \tag{1.29}$$

where ξ and μ are integration constants.

(ii) *Special class:* In the special case in which the Gauss-Bonnet coupling is given by (1.13), the solution splits into three main branches according to the geometry of Σ_4:

(ii.a.1) *Black holes:* The base manifold Σ_4 has the same restrictions as in the generic case, i.e.,

$$\tilde{R}^{i}{}_{j} = 3\gamma\delta^{i}_{j}, \tag{1.30}$$

and

$$\tilde{R}^{ij}{}_{kl}\tilde{R}^{kl}{}_{ij} - 4\tilde{R}_{ij}\tilde{R}^{ij} + \tilde{R}^2 - 24\xi = 0, \tag{1.31}$$

with f^2 and g^2 given by

$$f^2(r) = g^2(r) = \gamma + \frac{a_1}{a_2}r^2\left[1 \pm \sqrt{\frac{\mu}{r^5} + \frac{a_2^2}{a_1^2}\frac{(\gamma^2 - \xi)}{r^4}}\right], \tag{1.32}$$

possessing a slower falloff at infinity as compared with (1.29).

For the remaining branches, the base manifold Σ_4 *is no longer restricted to be Einstein*, but instead fulfills the following scalar condition:

$$\tilde{R}^{ij}{}_{kl}\tilde{R}^{kl}{}_{ij} - 4\tilde{R}_{ij}\tilde{R}^{ij} + \tilde{R}^2 - 4\gamma\tilde{R} + 24\gamma^2 = 0, \tag{1.33}$$

and g^2 is given by

$$g^2(r) = \sigma r^2 + \gamma, \qquad \sigma := \frac{a_0}{a_1}. \tag{1.34}$$

The form of the remaining function $f^2(r)$ is then precisely determined according to the following cases:

(ii.a.2) *Special black holes:* The base manifold is such that

$$\Sigma_4\text{: no additional restriction besides (I.33)},$$

and

$$f^2(r) = g^2(r) = \sigma r^2 + \gamma. \tag{1.35}$$

(ii.b.1) *Wormholes:* The base manifold Σ_4, besides (1.33), has a nonvanishing constant Ricci scalar,

$$\tilde{R} = 12\gamma,$$

where γ is rescaled to ±1, and the metric is given by

$$g^2(r) = \sigma r^2 + \gamma,$$

and

$$f^2(r) = \begin{cases} (a\sqrt{\sigma r^2 - 1} + 1 - \sqrt{\sigma r^2 - 1}\tan^{-1}(\frac{1}{\sqrt{\sigma r^2 - 1}}))^2\colon \gamma = -1 \\ (a\sqrt{\sigma r^2 + 1} + 1 - \sqrt{\sigma r^2 + 1}\tanh^{-1}(\frac{1}{\sqrt{\sigma r^2 + 1}}))^2\colon \gamma = 1 \end{cases}, \tag{1.36}$$

with a an integration constant.

(ii.b.2)*Spacetime horns:* If the base manifold Σ_4 has *vanishing* Ricci scalar,

$$\tilde{R} = 0, \tag{1.37}$$

the solution is given by

$$f^2(r) = \left(a\sqrt{\sigma}r + \frac{1}{\sqrt{\sigma}r^2}\right)^2, \tag{1.38}$$

$$g^2(r) = \sigma r^2, \tag{1.39}$$

where a is an integration constant.

(iii) *Degeneracy:* The base manifold Σ_4 is of constant curvature,

$$\tilde{R}^{ij}{}_{kl} = \gamma\delta^{ij}_{kl}, \tag{1.40}$$

and

$$g^2(r) = \sigma r^2 + \gamma, \tag{1.41}$$

$$f^2(r)\text{: an arbitrary function.} \tag{1.42}$$

The purpose of this paper is extending this classification to higher dimensions. The class of static metrics in Eq. (1.1) with a base manifold Σ_{d-2} solves the EGB field equations $d > 6$ dimensions according to the following.

C. $d \geq 7$ dimensions

(i) *Generic class:* For a generic value of the Gauss-Bonnet coupling a_2, the most general solution of the EGB field equations (1.6) within the class of metrics under consideration, given by (1.1), is such that the following is the case.
The base manifold Σ_{d-2} *must be* Einstein,

$$\tilde{R}^i{}_j = (d-3)\gamma\delta^i_j \tag{1.43}$$

(with γ normalized to ± 1, 0), and simultaneously fulfills the following (tensorial) condition on its Weyl tensor,

$$\tilde{C}^{ik}{}_{lm}\tilde{C}^{lm}{}_{jk} = \frac{(d-3)!}{(d-6)!}(\xi - \gamma^2)\delta^i_j, \tag{1.44}$$

with

$$f^2 = g^2 = \gamma + \frac{a_1}{a_2}r^2 \times \left[1 \pm \sqrt{1 - \frac{a_2 a_0}{a_1^2} + \frac{\mu}{r^{d-1}} + \frac{a_2^2}{a_1^2}\frac{(\gamma^2 - \xi)}{r^4}}\right], \tag{1.45}$$

where ξ and μ are integration constants.

(ii) *Special class:* If the Gauss-Bonnet coupling is given by (1.13), there are three main branches of solutions in the bulk according to the geometry of Σ_{d-2}:

(ii.a.1) *Black holes:* The base manifold Σ_{d-2} has the same restrictions as in the generic case, i.e.,

$$\tilde{R}^i{}_j = (d-3)\gamma\delta^i_j, \tag{1.46}$$

and also fulfills

$$\tilde{C}^{ik}{}_{lm}\tilde{C}^{lm}{}_{jk} = \frac{(d-3)!}{(d-6)!}(\xi - \gamma^2)\delta^i_j, \tag{1.47}$$

with f^2 and g^2 given by

$$f^2 = g^2 = \gamma + \frac{a_1}{a_2}r^2\left[1 \pm \sqrt{\frac{\mu}{r^{d-1}} + \frac{a_2^2}{a_1^2}\frac{(\gamma^2 - \xi)}{r^4}}\right], \tag{1.48}$$

where ξ and μ are integration constants. Note that the asymptotic behavior of (1.48) is slower than that of the generic case in (1.45).
For the remaining branches, the base manifold Σ_{d-2} *is no longer restricted to be Einstein*, but instead fulfills a scalar condition:

$$\tilde{H} + \frac{\gamma}{2}\frac{(d-4)!}{(d-7)!}\left[\tilde{R} - \frac{\gamma}{2}(d-2)(d-3)\right] = 0, \tag{1.49}$$

where $\tilde{H}$ is proportional to the Gauss-Bonnet invariant of Σ_{d-2}, as defined in Eq. (1.8), i.e.,

$$\tilde{H} := \tilde{H}^i{}_i = \frac{(6-d)}{4}(\tilde{R}^{ijkl}\tilde{R}_{ijkl} - 4\tilde{R}^{ij}\tilde{R}_{ij} + \tilde{R}^2),$$

and g^2 is given by

$$g^2(r) = \sigma r^2 + \gamma, \qquad \sigma := \frac{a_0}{a_1}, \tag{1.50}$$

where γ is a constant normalized to ± 1, 0.
The function $f^2(r)$ becomes determined according to the following cases.

(ii.a.2) *Special black holes:* The base manifold Σ_{d-2}, satisfies the Euclidean EGB equation for the special case (1.13) in $d-2$ dimensions, i.e.,

$$\tilde{H}^i{}_j - \gamma(d-5)(d-6)\tilde{G}^i{}_j - \frac{\gamma^2(d-3)!}{4(d-7)!}\delta^i_j = 0, \tag{1.51}$$

admitting a unique maximally symmetric solution of curvature γ, whose trace reduces to (1.49), and

$$f^2(r) = g^2(r) = \sigma r^2 + \gamma.$$

(ii.b.1) *Wormholes:* The base manifold Σ_{d-2} has constant nonvanishing Ricci scalar

$$\tilde{R} = (d-2)(d-3)\gamma, \tag{1.52}$$

and also satisfies the generic Euclidean EGB equation

$$\tilde{H}^i{}_j - (d-5)(d-6)\left(\gamma - \frac{J}{2}\right)\tilde{G}^i{}_j + \frac{\gamma(J-\gamma)}{4}\frac{(d-3)!}{(d-7)!}\delta^i_j = 0, \quad (1.53)$$

where J is an integration constant. Note that, by virtue of (1.52) the trace of (1.53) reduces to (1.51) giving no additional constraints on the geometry of Σ_{d-2}. The bulk geometry is then determined by

$$g^2(r) = \sigma r^2 + \gamma,$$

and $f^2(r)$ fulfills the generalized Legendre equation, given by

$$r(\sigma r^2 + \gamma)f'' + [(d-4)\sigma r^2 + (d-5)\gamma]f' - \left((d-4)\sigma r - \frac{(d-5)(d-6)J}{4r}\right)f(r) = 0. \quad (1.54)$$

The general solution then reads

$$f^2(r) = r^{6-d}[aP_\nu{}^\mu(\sqrt{1+\gamma\sigma r^2}) + bQ_\nu{}^\mu(\sqrt{1+\gamma\sigma r^2})]^2, \quad (1.55)$$

where $P_\nu{}^\mu(x)$ and $Q_\nu{}^\mu(x)$ are the generalized Legendre functions of first and second kind, respectively, with

$$\mu := \frac{1}{2}\sqrt{(d-6)^2 - \frac{J}{\gamma}(d-5)(d-6)}, \quad (1.56)$$

$$\nu := \frac{d}{2} - 2, \quad (1.57)$$

and a, b are integration constants.

(ii.b.2) *Spacetime horns:* The base manifold Σ_{d-2} has vanishing Ricci scalar

$$\tilde{R} = 0, \quad (1.58)$$

and also satisfies the Euclidean EGB equation devoid of the volume term

$$\tilde{H}^i{}_j + \frac{J(d-5)(d-6)}{2}\tilde{G}^i{}_j = 0, \quad (1.59)$$

with J an integration constant. As in the previous case, the vanishing of the Ricci scalar of Σ_{d-2} makes the trace of (1.59) reduce to (1.51) (with $\gamma = 0$), without additional conditions on Σ_{d-2}. The bulk geometry is given by

$$g^2(r) = \sigma r^2 + \gamma,$$

and $f^2(r)$ fulfills Eq. (1.54) with $\gamma = 0$, i.e.,

$$\sigma r^3 f'' + (d-4)\sigma r^2 f' - \left((d-4)\sigma r - \frac{(d-5)(d-6)J}{4r}\right)f(r) = 0, \quad (1.60)$$

whose general solution is

$$f^2(r) := r^{5-d}\left[aJ_\alpha\left(\frac{1}{r}\sqrt{\frac{(d-5)(d-6)}{4\sigma}}\right) + bY_\alpha\left(\frac{1}{r}\sqrt{\frac{(d-5)(d-6)}{4\sigma}}\right)\right]^2. \quad (1.61)$$

Here $J_\alpha(x)$ and $Y_\alpha(x)$ are the Bessel functions of the first and second kind, respectively, with

$$\alpha := -\frac{d-3}{2}, \quad (1.62)$$

and a, b are integration constants.

(iii) *Degeneracy:* The base manifold Σ_4 is of constant curvature,

$$\tilde{R}^{ij}{}_{kl} = \gamma\delta^{ij}_{kl}, \quad (1.63)$$

and

$$g^2(r) = \sigma r^2 + \gamma, \quad (1.64)$$

$$f^2(r)\text{: an arbitrary function.} \quad (1.65)$$

This concludes the classification.

II. DERIVATION OF THE CLASSIFICATION SCHEME

In order to prove the previous classification, it is convenient to work with differential forms. The field equations for the EGB theory (1.5) are given by (1.9), and in the case $a_2 = 0$ Eq. (1.9) reduces to the Einstein equations with cosmological constant.

For the metric given in (1.1) the vielbein can be chosen as

$$e^0 = f(r)dt, \qquad e^1 = \frac{dr}{g(r)}, \qquad e^m = r\tilde{e}^m,$$

where $\tilde{e}^m$ is the vielbein of the base manifold Σ_{d-2}, so that $m = 2, 3, \ldots, d-1$, and the curvature 2-form is then given by

$$R^{01} = -\left(gg'\frac{f'}{f} + g^2\frac{f''}{f}\right)e^0e^1, \quad (2.1)$$

$$R^{0m} = -\left(g^2\frac{f'}{fr}\right)e^0e^m, \quad (2.2)$$

$$R^{1m} = -\frac{1}{2}\frac{(g^2)'}{r}e^1e^m, \quad (2.3)$$

$$R^{mn} = \tilde{R}^{mn} - \frac{g^2}{r^2}e^me^n, \quad (2.4)$$

where $\tilde{R}^{mn}$ stands for the curvature of Σ_{d-2}.

To proceed with the classification, we first solve the constraint $\mathcal{E}_0 = 0$. One then finds that the analysis naturally splits into two cases, one involving generic theories and the other restricted to the special class of theories defined by (1.13). Solving the remaining field equations in each branch completes the classification.

A. Solving the constraint

The equation $\mathcal{E}_0 = 0$ reads

$$\epsilon_{m_1\ldots m_n}[a_2(d-5)\tilde{R}^{m_1m_2}\tilde{R}^{m_3m_4} + B_0\tilde{R}^{m_1m_2}\tilde{e}^{m_3}\tilde{e}^{m_4} + A_0\tilde{e}^{m_1}\tilde{e}^{m_2}\tilde{e}^{m_3}\tilde{e}^{m_4}]\tilde{e}^{m_5}\ldots\tilde{e}^{m_n} = 0, \quad (2.5)$$

where $n = d-2$ is the dimension of the base manifold, and $A_0(r)$, $B_0(r)$ are functions constructed out from $g^2(r)$ and its derivative (see Appendix A). Taking a derivative of this equation with respect to r, one obtains the following consistency condition:

$$\epsilon_{m_1\ldots m_n}[B_0'\tilde{R}^{m_1m_2} + A_0'\tilde{e}^{m_1}\tilde{e}^{m_2}]\tilde{e}^{m_3}\ldots\tilde{e}^{m_n} = 0, \quad (2.6)$$

and since $\tilde{R}^{mn}$ and $\tilde{e}^m$ depend only on the coordinates of Σ_n, one obtains that

$$A_0' = -\gamma B_0', \quad (2.7)$$

where γ is a constant. Equation (2.7) implies that

$$A_0 = -\gamma B_0 - (d-5)a_2\xi, \quad (2.8)$$

where ξ is a new integration constant that has been conveniently rescaled.

Inserting (2.7) in (2.6) then gives the following condition:

$$B_0'\epsilon_{m_1\ldots m_n}(\tilde{R}^{m_1m_2} - \gamma\tilde{e}^{m_1}\tilde{e}^{m_2})\tilde{e}^{m_4}\ldots\tilde{e}^{m_n} = 0, \quad (2.9)$$

which means that the analysis splits into two cases: $B_0' \neq 0$ and $B_0' = 0$.

1. *The constraint $\mathcal{E}_0 = 0$ in the generic case ($B_0' \neq 0$)*

If B_0' is nonvanishing, the condition (2.9) reduces to

$$\epsilon_{m_1\ldots m_n}[\tilde{R}^{m_1m_2} - \gamma\tilde{e}^{m_1}\tilde{e}^{m_2}]\tilde{e}^{m_3}\ldots\tilde{e}^{m_n} = 0, \quad (2.10)$$

which means that the Ricci scalar of the base manifold $\tilde{R}$ is a constant, i.e.,

$$\tilde{R} = n(n-1)\gamma. \quad (2.11)$$

Inserting (2.8) and (2.10) in the constraint (2.5) gives an additional condition being quadratic in the curvature of the base manifold:

$$(d-5)a_2\epsilon_{m_1\ldots m_n}(\tilde{R}^{m_1m_2}\tilde{R}^{m_3m_4} - \xi\tilde{e}^{m_1}\tilde{e}^{m_2}\tilde{e}^{m_3}\tilde{e}^{m_4})\tilde{e}^{m_5}\ldots\tilde{e}^{m_n} = 0. \quad (2.12)$$

Equations (2.11) and (2.12) restrict the geometry of Σ_n, whereas (2.8) is a first order equation for $g^2(r)$ whose solution is

$$g^2(r) = \gamma + \frac{a_1}{a_2}r^2\left[1 \pm \sqrt{1 - \frac{a_2a_0}{a_1^2} + \frac{\mu}{r^{d-1}} + \frac{a_2^2}{a_1^2}\frac{(\gamma^2-\xi)}{r^4}}\right], \quad (2.13)$$

with μ an integration constant.

Note that we have *not* assumed any relation between the coupling constants of the theory, and this is why these conditions apply in the *generic* case.

2. *The constraint $\mathcal{E}_0 = 0$ in the special case ($B_0' = 0$)*

If B_0' vanishes, Eq. (2.9) is trivially solved. On the other hand, Eq. (2.7) implies

$$A_0' = B_0' = 0, \quad (2.14)$$

and it is easy to see, from the expressions for A_0 and B_0 in the Appendix A, that this equation can be fulfilled only if the Gauss-Bonnet coupling is fixed as

$$a_2 = \frac{a_1^2}{a_0}, \quad (2.15)$$

which corresponds to the special class of theories (1.13). In this case $g^2(r)$ is given by

$$g^2(r) = \sigma r^2 + \gamma, \quad (2.16)$$

where we have defined

$$\sigma := \frac{a_0}{a_1}. \quad (2.17)$$

Therefore, since the functions A_0 and B_0 reduce to

$$A_0 = (d-5)a_2\gamma^2, \quad (2.18)$$

$$B_0 = -2(d-5)a_2\gamma. \quad (2.19)$$

Equation (2.5) gives the following scalar restriction on the base manifold:

$$(d-5)a_2\epsilon_{m_1\ldots m_n}[\tilde{R}^{m_1m_2} - \gamma\tilde{e}^{m_1}\tilde{e}^{m_2}] \times [\tilde{R}^{m_3m_4} - \gamma\tilde{e}^{m_3}\tilde{e}^{m_4}]\tilde{e}^{m_5}\ldots\tilde{e}^{m_n} = 0. \quad (2.20)$$

Note that this last condition on Σ_n is weaker than the ones obtained in the generic case (2.11) and (2.12). One should keep in mind that Eq. (2.20) applies only for the *special theories* fulfilling (2.15).

B. Solving the remaining equations

The equation $\mathcal{E}_1 = 0$ reduces to

$$\epsilon_{m_1\ldots m_n}[(d-5)a_2\tilde{R}^{m_1m_2}\tilde{R}^{m_3m_4} + B_1\tilde{R}^{m_1m_2}\tilde{e}^{m_3}\tilde{e}^{m_4} + A_1\tilde{e}^{m_1}\tilde{e}^{m_2}\tilde{e}^{m_3}\tilde{e}^{m_4}]\tilde{e}^{m_5}\ldots\tilde{e}^{m_n} = 0, \quad (2.21)$$

where A_1 and B_1 are functions of r, f, g, and their derivatives (see Appendix A). Subtracting (2.21) from (2.5), the quadratic terms cancel out, and we obtain

$$\epsilon_{m_1\ldots m_n}[(B_0 - B_1)\tilde{R}^{m_1m_2} + (A_0 - A_1)\tilde{e}^{m_1}\tilde{e}^{m_2}]\tilde{e}^{m_3}\ldots\tilde{e}^{m_n} = 0. \tag{2.22}$$

The projection of the EGB field equations (1.9) on Σ_n, $\mathcal{E}_m = 0$, reads

$$\epsilon_{mm_2\ldots m_n}[(d-5)(d-6)a_2\tilde{R}^{m_2m_3}\tilde{R}^{m_4m_5} + C\tilde{R}^{m_2m_3}\tilde{e}^{m_4}\tilde{e}^{m_5} + D\tilde{e}^{m_2}\tilde{e}^{m_3}\tilde{e}^{m_4}\tilde{e}^{m_5}]\tilde{e}^{m_6}\ldots\tilde{e}^{m_n} = 0, \tag{2.23}$$

where again C and D are functions of r, f, g, and their derivatives, given in Appendix A.

We will solve (2.22) and (2.23) for the generic and special cases separately.

1. Radial and angular equations: Generic case

Introducing (2.10) in (2.22) we obtain

$$(B_0 - B_1)\gamma + (A_0 - A_1) = 0, \tag{2.24}$$

which reduces to

$$\frac{d}{dr}\left[\ln\frac{g(r)}{f(r)}\right][g^2 - (\sigma r^2 + \gamma)] = 0, \tag{2.25}$$

Note that since in the generic case the function g^2 is given by (2.13), the second factor in (2.25) does not vanish in general. This implies that $f^2(r)$ is proportional to $g^2(r)$, and the constant of proportionality can be reabsorbed by a time rescaling, so that

$$f^2(r) = g^2(r), \tag{2.26}$$

where $g^2(r)$ is given in (2.13).

Let us now solve the remaining equations $\mathcal{E}_m = 0$. By virtue of (2.26), the functions C and D fulfill the following relation:

$$D = -\gamma C - (d-5)(d-6)a_2\xi. \tag{2.27}$$

Taking a derivative of (2.23) with respect to r we obtain

$$C'\epsilon_{mm_2\ldots m_n}[\tilde{R}^{m_2m_3} - \gamma\tilde{e}^{m_2}\tilde{e}^{m_3}]\tilde{e}^{m_4}\ldots\tilde{e}^{m_n} = 0, \tag{2.28}$$

and since it is straightforward to check that $C' \neq 0$ for the generic case, this equation is solved provided

$$\epsilon_{mm_2\ldots m_n}[\tilde{R}^{m_2m_3} - \gamma\tilde{e}^{m_2}\tilde{e}^{m_3}]\tilde{e}^{m_4}\ldots\tilde{e}^{m_n} = 0, \tag{2.29}$$

which means that the base manifold must be Einstein.

Furthermore, if we use the latter equation and (2.27), then Eq. (2.23) reads

$$(d-5)(d-6)a_2\epsilon_{mm_2\ldots m_n}[\tilde{R}^{m_2m_3}\tilde{R}^{m_4m_5} - \xi\tilde{e}^{m_2}\tilde{e}^{m_3}\tilde{e}^{m_4}\tilde{e}^{m_5}]\tilde{e}^{m_6}\ldots\tilde{e}^{m_n} = 0. \tag{2.30}$$

It is simple to verify (see Appendix B) that for an Einstein manifold (2.29), this last equation reduces to (1.47).

This concludes the proof of the classification in the generic case (i), which includes the case (ii.a.1) when the condition (1.13) is further fulfilled.

2. Radial and angular equations: Special case

Using (2.16) in (2.22) gives

$$a_2\frac{d}{dr}\left[\ln\frac{g(r)}{f(r)}\right]\epsilon_{m_1\ldots m_n}[\tilde{R}^{m_1m_2} - \gamma\tilde{e}^{m_1}\tilde{e}^{m_2}]\tilde{e}^{m_3}\ldots\tilde{e}^{m_n} = 0, \tag{2.31}$$

which means that the analysis splits in the following two cases.

(ii.a.2): This is the case where the first factor in (2.31) vanishes. Hence, after a rescaling of time, one obtains

$$f^2(r) = g^2(r) = \sigma r^2 + \gamma, \tag{2.32}$$

with σ given by (2.17). Replacing (2.32) in $\mathcal{E}_m = 0$ implies that the metric of the base manifold fulfills the following equation:

$$(d-5)(d-6)a_2\epsilon_{mm_2\ldots m_n}[\tilde{R}^{m_2m_3} - \gamma\tilde{e}^{m_2}\tilde{e}^{m_3}] \times [\tilde{R}^{m_4m_5} - \gamma\tilde{e}^{m_4}\tilde{e}^{m_5}]\tilde{e}^{m_6}\ldots\tilde{e}^{m_n} = 0. \tag{2.33}$$

It is worth pointing out that Eq. (2.33) is the same (Euclidean) EGB equation for the special case (2.15), but in $n = d-2$ dimensions. Once expressed in terms of tensors in $d > 6$ dimensions, Eq. (2.33) reads

$$\delta^{ik_1k_2k_3k_4}_{jl_1l_2l_3l_4}(\tilde{R}^{l_1l_2}{}_{k_1k_2} - \gamma\delta^{l_1l_2}_{k_1k_2})(\tilde{R}^{l_3l_4}{}_{k_3k_4} - \gamma\delta^{l_3l_4}_{k_3k_4}) = 0, \tag{2.34}$$

which reduces to (1.51). This corresponds to the case (ii.a.2) of the classification.

(ii.b) and (iii): In the case when the first factor of (2.31) does not vanish, i.e., when $f(r)$ is *not* proportional to $g(r)$, Eq. (2.31) reduces to

$$\epsilon_{m_1\ldots m_n}[\tilde{R}^{m_1m_2} - \gamma\tilde{e}^{m_1}\tilde{e}^{m_2}]\tilde{e}^{m_3}\ldots\tilde{e}^{m_n} = 0, \tag{2.35}$$

which means that the Ricci scalar of Σ_n is a constant,

$$\tilde{R} = n(n-1)\gamma. \tag{2.36}$$

The angular Eq. (2.23) in this case reads

$$\begin{aligned}\mathcal{E}_m := {}& \epsilon_{mm_2\ldots m_n}[\tilde{R}^{m_2m_3} - \gamma\tilde{e}^{m_2}\tilde{e}^{m_3}] \times [\tilde{R}^{m_4m_5} - \gamma\tilde{e}^{m_4}\tilde{e}^{m_5}]\tilde{e}^{m_6}\ldots\tilde{e}^{m_n} \\ &+ \frac{\mathcal{D}[f(r)]}{f(r)}\epsilon_{mm_2\ldots m_n}[\tilde{R}^{m_2m_3} - \gamma\tilde{e}^{m_2}\tilde{e}^{m_3}]\tilde{e}^{m_4}\ldots\tilde{e}^{m_n} \\ ={}& 0,\end{aligned} \tag{2.37}$$

where $\mathcal{D}$ is the linear differential operator defined by

$$\mathcal{D}[f(r)] := \frac{4}{(d-5)(d-6)}[-r^2(\sigma r^2+\gamma)f'' - r((d-4)\sigma r^2+(d-5)\gamma)f' + \sigma r^2(d-4)f(r)]. \tag{2.38}$$

Taking a derivative of Eq. (2.37) with respect to r leads us to consider the following two subcases:

(ii.b): If $\mathcal{D}[f(r)] = Jf(r)$, where J is a constant, then (2.37) reduces to

$$\epsilon_{mm_2\ldots m_n}[\tilde{R}^{m_2m_3} - \gamma\tilde{e}^{m_2}\tilde{e}^{m_3}][\tilde{R}^{m_4m_5} - \gamma\tilde{e}^{m_4}\tilde{e}^{m_5}]\tilde{e}^{m_6}\ldots\tilde{e}^{m_{d-3}} + J\epsilon_{mm_2\ldots m_n}[\tilde{R}^{m_2m_3} - \gamma\tilde{e}^{m_2}\tilde{e}^{m_3}]\tilde{e}^{m_4}\ldots\tilde{e}^{m_{d-3}} = 0. \tag{2.39}$$

This means that the base manifold also fulfills a Euclidean EGB equation for a generic choice of the Gauss-Bonnet coupling in $n = d-2$ dimensions, where the constant J measures the departure of (2.39) from the special case. The function $f(r)$ solves the following equation:

$$r^2(\sigma r^2+\gamma)f'' + r((d-4)\sigma r^2 + (d-5)\gamma)f' + \left(\frac{(d-5)(d-6)J}{4} - \sigma(d-4)r^2\right)f = 0, \tag{2.40}$$

whose integration depends on the value of γ.

(i) *(ii.b.1):* For $\gamma \neq 0$ the solution of (2.40) is given by

$$f(r) = r^{3-(d/2)}[aP_\nu{}^\mu(\sqrt{\gamma\sigma r^2+1}) + bQ_\nu{}^\mu(\sqrt{\gamma\sigma r^2+1})],$$

where $P_\nu{}^\mu(x)$ and $Q_\nu{}^\mu(x)$ are the generalized Legendre functions of the first and second kind, respectively, with

$$\mu := \frac{1}{2}\sqrt{(d-6)^2 - \frac{J}{\gamma}(d-5)(d-6)}, \tag{2.41}$$

$$\nu := \frac{d}{2} - 2, \tag{2.42}$$

and a, b are integration constants.

(ii) *(ii.b.2):* For $\gamma = 0$, Eq. (2.40) integrates as

$$f(r) := r^{-(d-5)/2}\left[aJ_\alpha\left(\frac{1}{r}\sqrt{\frac{(d-5)(d-6)}{4\sigma}}\right) + bY_\alpha\left(\frac{1}{r}\sqrt{\frac{(d-5)(d-6)}{4\sigma}}\right)\right], \tag{2.43}$$

where $J_\alpha(x)$ and $Y_\alpha(x)$ are the Bessel functions of the first and second kind, respectively, with

$$\alpha := -\frac{d-3}{2}. \tag{2.44}$$

This concludes the proof corresponding to the cases (ii.b.1) and (ii.b.2).

(iii): If $\mathcal{D}[f(r)]/f(r)$ is not a constant, then Eq. (2.37) is solved provided the base manifold simultaneously fulfills the Einstein and the EGB equations in the special case with the same cosmological constant, i.e.,

$$\epsilon_{mm_2\ldots m_n}[\tilde{R}^{m_2m_3} - \gamma\tilde{e}^{m_2}\tilde{e}^{m_3}]\tilde{e}^{m_4}\ldots\tilde{e}^{m_n} = 0, \tag{2.45}$$

$$\epsilon_{mm_2\ldots m_n}[\tilde{R}^{m_2m_3} - \gamma\tilde{e}^{m_2}\tilde{e}^{m_3}][\tilde{R}^{m_4m_5} - \gamma\tilde{e}^{m_4}\tilde{e}^{m_5}] \times \tilde{e}^{m_6}\ldots\tilde{e}^{m_n} = 0; \tag{2.46}$$

and $f(r)$ becomes an arbitrary function.

For a Euclidean Einstein manifold fulfilling (2.45) and (2.46) reduces to

$$\tilde{C}^{ij}{}_{lm}\tilde{C}^{lm}{}_{jk} = 0, \tag{2.47}$$

which implies that Σ_n must be of constant curvature γ (see Appendix B), i.e.,

$$\tilde{R}^{ij}{}_{kl} = \gamma\delta^{ij}_{kl}. \tag{2.48}$$

This ends the proof of the classification.

III. DISCUSSION

In this paper, the class of static metrics given by (1.1) that solves the EGB field equations in vacuum for $d \geq 5$ dimensions has been classified. It was shown that for a generic value of the Gauss-Bonnet coupling, the base manifold must be necessarily Einstein, with an additional restriction on its Weyl tensor if $d > 5$. The boundary admits a wider class of geometries only in the special case when the Gauss-Bonnet coupling is given by (1.13), such that the theory admits a unique maximally symmetric solution. The additional freedom in the boundary metric enlarges the class of allowed geometries in the bulk, which are classified within three main branches, containing new black holes and wormholes in vacuum.

In the five-dimensional case, the classification was performed in [12], including a thorough analysis of the geometrically well-behaved solutions including black holes, wormholes, and spacetime horns. It was also shown that these solutions have finite Euclidean action (regularized through the boundary terms proposed in [14,15]), which reduces to the free energy in the case of black holes and vanishes in the remaining cases. The mass was also obtained from the corresponding conserved charge written as a surface integral. For a generic choice of the Gauss-Bonnet coupling, the solution was obtained in [16] assuming the base manifold to be of constant curvature, and in the spherically symmetric case Eq. (1.15) reduces to the well-known solution of Boulware and Deser [17]. In the special case, in which the Gauss-Bonnet coupling is given by (1.13), the Lagrangian can be written as a Chern-Simons form [18] and its locally supersymmetric extension is known [19,20]. For the special case, when the cosmological constant is negative ($\sigma > 0$) this solution, corresponding to the branch (ii.a), describes a black hole [21,22], which for spherical symmetry, reduces to the one found in [17,23]. It can also be seen that the black hole metric still solves the field equations even in the presence of a nontrivial fully antisymmetric torsion [24]. For the branch (ii.b.1) with $\gamma = -1$, the solution with $|a| < 1$ corresponds to the wormhole in vacuum found in [25]. It has also been shown that if the base manifold is given by the hyperbolic space in three dimensions, i.e., $\Sigma_3 = H_3$ with no identifications, this metric describes a smooth gravitational soliton [26]. If $|a| = 1$, the solution reduces to a different kind of wormholes possessing inequivalent asymptotic regions. For the branch (ii.b.2), if $a \geq 0$ the solution describes a "spacetime horn" [12].

In the six-dimensional case, the classification was carried out in [13]. For a generic choice of the Gauss-Bonnet coupling, besides the mass parameter μ, an independent integration constant ξ appears. The base manifold Σ_4 has to be Einstein with an additional scalar condition on its geometry, given by (1.28), which means that the Euler density of Σ_4 must be constant. Therefore, if one assumes that Σ_4 is compact and without boundary, integration of Eq. (1.28) on Σ_4 gives a topological restriction on the base manifold, constraining the new parameter to be $\xi = \frac{4}{3}\pi^2 \frac{\chi(\Sigma_4)}{\mathcal{V}_4}$, where $\chi(\Sigma_4)$ is the Euler characteristic of the base manifold and $\mathcal{V}_4$ stands for its volume. Note that the term proportional to r^{-4} inside the square root in the metric (1.29) vanishes if and only if the base manifold is of constant curvature. It is worth pointing out that this term severely modifies the asymptotic behavior of the metric. Depending on the value of the parameters, this spacetime can describe black holes being asymptotically locally (A) dS or flat. The asymptotic behavior of the metric is further relaxed in the special case (1.13) [see (ii.a.1)], which for a constant curvature base manifold Σ_4 reduces to the solution found in [22].

When (1.13) is fulfilled, it was shown that the restriction that Σ_4 be Einstein can be circumvented [case (ii)]. For the case (ii.a.2), the geometry of the base manifold is as relaxed as possible, since it has to fulfill just a single scalar equation, given by (1.33). Remarkably, if the Ricci scalar is further required to be a nonvanishing constant $\gamma = \pm 1$, for negative cosmological constant, wormholes in vacuum also exist in six dimensions, provided $a^2 < \frac{\pi^2}{4}$ [case (ii.b.1) with $\gamma = -1$], and the volume of the base manifold turns out to be fixed in terms of the Euler characteristic, according to $\chi(\Sigma_4) = \frac{3}{4\pi^2}\mathcal{V}_4$. In the case of $\gamma = 0$, i.e., if the base manifold Σ_4 has vanishing Ricci scalar, one obtains that $\chi(\Sigma_4) = 0$, and for $a \geq 0$ the metric looks like a "spacetime horn." In the six-dimensional case this classification has been further explored in [27] for the case in which the functions f^2 and g^2 are also time dependent.

The classification of these solutions presents special features in $d = 5$ and 6 dimensions, and as explained above, a common pattern arises in higher dimensions. In the case of $d \geq 7$, for a generic choice of the Gauss-Bonnet coupling [case (i)] it was found that the base manifold has to be Einstein, fulfilling the additional condition (1.44), in agreement with [11]. Apart from the mass parameter μ, an additional integration constant ξ appears. For spherical symmetry, one recovers the result found by Boulware and Deser [17]. The gravitational stability in the spherically symmetric case was analyzed in [28], where it was found that, contrary to what happens for higher-dimensional spherical black holes in GR, in the asymptotically flat five- and six-dimensional cases, there is a critical mass below which the black holes become unstable [29,30]. If the base manifold Σ_{d-2} is of constant curvature, then the condition (1.44) implies that $\gamma^2 - \xi = 0$, and one recovers the results found by Cai [16]. The difference $\gamma^2 - \xi$ parametrizes the deviation of the base manifold from being of constant curvature, and it is worth pointing out that if $\xi \neq \gamma^2$, the metric given by (1.45) acquires an additional term of order r^{-4} within the square root in (1.45) regardless of the spacetime dimension, so that the metric possesses a slower falloff at infinity as compared with ones with base manifolds of constant curvature. In the asymptotic region, the behavior of the metric is further relaxed in the special case (1.13) [10] [see (ii.a.2)], and for base manifolds Σ_{d-2} of constant curvature, the solution reduces to the one found in [22].

It was shown that for the special choice (1.13), the restriction that Σ_{d-2} be Einstein can be surmounted [case (ii)]. In the case (ii.a.2), the geometry of the base manifold turns out to be as relaxed as possible, since it has to fulfill just Eq. (1.51), corresponding to the Euclidean EGB equation for the special case (1.13) in $d - 2$ dimensions, admitting a unique maximally symmetric solution of curvature γ.

For the choice (1.13), if the base manifold Σ_{d-2} has constant Ricci scalar and fulfills the Euclidean EGB equation in $d - 2$ dimensions in (1.53), which depends on an integration constant J, one recovers cases (ii.b) for which the metric is expressed in terms of generalized Legendre functions for $\gamma = \pm 1$ [Eq. (1.55) of case (ii.b.1)], and Bessel functions for $\gamma = 0$ [Eq. (1.61) of case (ii.b.1)]. In the case of $J = 0$ Eq. (1.53) reduces to (1.51), and the metric explicitly acquires the following form:

$$f(r) = ar + \frac{b}{r^{d-4}}, \tag{3.1}$$

for $\gamma = 0$, and

$$f(r) = a\sqrt{\sigma r^2 + \gamma} + b\frac{h(r)}{r^{d-6}}, \tag{3.2}$$

with

$$h(r) = \begin{cases} 2\sigma r^2 + \gamma: & d = 7 \\ 3\sigma r^2 + \gamma - 3\sigma r^2\sqrt{\gamma\sigma r^2 + 1}\tanh^{-1}((\gamma\sigma r^2 + 1)^{-1/2}): & d = 8 \\ 8\sigma^2 r^4 + 4\sigma\gamma r^2 - 1: & d = 9 \\ 2 - 5\sigma r^2\gamma - 15\sigma^2 r^4 + 15\sigma r^4\sqrt{\gamma\sigma r^2 + 1}\tanh^{-1}((\gamma\sigma r^2 + 1)^{-1/2}): & d = 10 \\ 16\sigma^3 r^6 + 8\sigma^2\gamma r^4 - 2\sigma\gamma^2 r^2 - \gamma^3: & d = 11 \end{cases}, \tag{3.3}$$

for $\gamma = \pm 1$.

As explained in [12,13], in five and six dimensions, respectively, and extended here to any dimension $d \geq 7$, for the EGB theory with special choice of the Gauss-Bonnet coupling (1.13), metrics of the form

$$ds^2 = -f^2(r)dt^2 + \frac{dr^2}{\sigma r^2 + \gamma} + r^2 d\Sigma^2_{(d-2)}, \tag{3.4}$$

with $\sigma = \frac{a_0}{a_1}$, and $\gamma = \pm 1, 0$, may acquire degeneracy [case (iii)]. Degeneracy occurs for the metric (3.4) when the base manifold Σ_{d-2} is of constant curvature γ, since the EGB equations turn out to be solved *for an arbitrary function* $f^2(r)$. Thus, in particular, the Lifshitz spacetimes in [31–33] fall within this class.

This kind of degeneracy is a known feature of a wide class of theories [34]. A similar degeneracy has been found in the context of Birkhoff's theorem for the EGB theory in vacuum [35,36], and also for theories containing dilaton and an axion fields coupled with a Gauss-Bonnet term [37].

From the point of view of the AdS/CFT correspondence [2], the dual CFT is expected to have a behavior that strongly depends on the choice of the base manifold Σ_{d-2}. Note that the existence of wormholes with AdS asymptotics, as the ones reported here, raises some puzzles within this context [38–40]. Nevertheless, in five dimensions, some interesting results have been found in [41]. The EGB theory also admits wormhole solutions in the presence of matter that fulfill the standard energy conditions [42–45]. From the gravity side of the correspondence, the addition of a Gauss-Bonnet term in the action has recently attracted a lot of attention concerning the hydrodynamic limit of the dual CFT [46–57].

The EGB theory also possesses rotating solutions with a nontrivial geometry at the boundary [58]. Currently, a wide spectrum of solutions in vacuum is known, including black strings and black p-branes [59–63], spontaneous compactifications [64–71], metrics with a nontrivial jump in the extrinsic curvature [72–74], and even solutions with nontrivial torsion [24,75,76].

ACKNOWLEDGMENTS

We thank Steve Willison for helpful comments. This research is partially funded by Fondecyt Grants No. 1085322, No. 1095098, No. 11090281, by the Conicyt grant "Southern Theoretical Physics Laboratory" ACT-91, by the CONICET grant PIP 112-200801-02479 from CONICET and by Grants No. 05/B384 and No. 05/B253 from Universidad Nacional de Córdoba. G. D. is supported by CONICET. The Centro de Estudios Científicos (CECS) is funded by the Chilean Government through the Millennium Science Initiative and the Centers of Excellence Base Financing Program of Conicyt. CECS is also supported by a group of private companies which at present includes Antofagasta Minerals, Arauco, Empresas CMPC, Indura, Naviera Ultragas, and Telefónica del Sur. CIN is funded by Conicyt and the Gobierno Regional de Los Ríos.

APPENDIX A: FUNCTIONS APPEARING IN THE FIELD EQUATIONS

Here we present the expressions for the functions appearing in the EGB field equations. For the constraint $\mathcal{E}_0 = 0$ in (2.5), the corresponding functions are defined by

$$A_0 := r^{-d+6}[a_0 r^{d-1} - 2a_1 r^{d-3} g^2 + a_2 r^{d-5} g^4]', \tag{A1}$$

$$B_0 := 2r^{-d+6}[a_1 r^{d-3} - a_2 r^{d-5} g^2]', \tag{A2}$$

and for the radial equation $\mathcal{E}_1 = 0$ (2.21) those are

$$A_1(r) := a_0(d-1)r^4 - 2a_1 g^2 r^2\left((d-3) + 2\frac{f'}{f}r\right) + a_2 g^4\left((d-5) + 4\frac{f'}{f}r\right), \tag{A3}$$

$$B_1(r) := -2a_2 g^2\left((d-5) + 2\frac{f'}{f}r\right) + 2(d-3)a_1 r^2. \tag{A4}$$

For the projection of the EGB field equations along the base manifold Σ_n, $\mathcal{E}_m = 0$ in (2.23), the corresponding functions are given by

$$C := -2a_2 r^2\left[(g^2)'\frac{f'}{f} + 2g^2\frac{f''}{f} + (d-5)\left(2g^2\frac{f'}{rf} + r^{5-d}(g^2 r^{d-6})'\right)\right] + 2(d-3)(d-4)a_1 r^2, \tag{A5}$$

$$D := (d-1)(d-2)a_0 r^4 - 2a_1 r^2\Big[(d-3)r^{5-d}(g^2 r^{d-4})' + \frac{r}{f}(2(d-3)g^2 f' + (g^2)' f' r + 2g^2 f'' r)\Big] + a_2 r\Big[(d-5)r^{6-d}(g^4 r^{d-6})' + 4(d-5)g^4\frac{f'}{f} + 2(g^4)'\frac{f'}{f}r + (g^4)'\frac{f'}{f}r + 4g^4\frac{f''}{f}r\Big]. \tag{A6}$$

APPENDIX B: SOME USEFUL GEOMETRICAL IDENTITIES

An n-dimensional Einstein manifold Σ_n fulfills

$$R^i{}_j = \gamma(n-1)\delta^i_j. \tag{B1}$$

In this case the Einstein tensor reads

$$G^i_j = -\frac{(n-2)(n-1)}{2}\gamma\delta^i_j. \tag{B2}$$

and for $n > 3$ the Weyl tensor defined as the "trace-free part" of the Riemann tensor

$$C^{ij}{}_{kl} := R^{ij}{}_{kl} - \frac{4}{n-2}\delta^{[i}_{[k}R^{j]}_{l]} + \frac{R}{(n-1)(n-2)}\delta^{ij}_{kl} \tag{B3}$$

reduces to

$$C^{ij}{}_{kl} := R^{ij}{}_{kl} - \gamma\delta^{ij}_{kl}. \tag{B4}$$

Here antisymmetrization is normalized as $T_{[ij]} := \frac{1}{2}(T_{ij} - T_{ji})$.

The Gauss-Bonnet tensor (1.7) can then be expressed as

$$H^i{}_j = C^{ik}{}_{lm}C^{lm}{}_{jk} - \tfrac{1}{4}[C^2 + \gamma^2(n-1)(n-2)(n-3)(n-4)]\delta^i_j, \tag{B5}$$

where $C^2 := C^{ij}{}_{kl}C^{kl}{}_{ij}$. Note that for Euclidean signature $C^2 \geq 0$, and it vanishes only if $C^{ij}{}_{kl} = 0$. Thus, by virtue of (B4), Euclidean Einstein manifolds with $C^2 = 0$ are of constant curvature.

The trace of Eq. (B5) implies that the difference of the Gauss-Bonnet combination and the squared Weyl tensor is a constant, i.e.,

$$R^{ij}_{kl}R^{kl}_{ij} - 4R^{ij}R_{ij} + R^2 - C^2 = \frac{n!}{(n-4)!}\gamma^2, \tag{B6}$$

which is actually valid for $n > 3$. Note that since in four dimensions the Gauss-Bonnet tensor identically vanishes, $H^i{}_j \equiv 0$, Eq. (B5) means that Einstein manifolds fulfill the following identity [77]

$$C^{ik}{}_{lm}C^{lm}{}_{jk} = \frac{C^2}{4}\delta^i_j. \tag{B7}$$

Another useful identity allows writing Eq. (1.49) as

$$\mathcal{R}^{ij}{}_{kl}\mathcal{R}^{kl}{}_{ij} - 4\mathcal{R}_{ij}\mathcal{R}^{ij} + \mathcal{R}^2 = 0, \tag{B8}$$

with

$$\mathcal{R}^{ij}{}_{kl} := \tilde{R}^{ij}{}_{kl} - \gamma\delta^{ij}_{kl}. \tag{B9}$$

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