

Preventing eternity in phantom inflationChao-Jun Feng,^{1,*} Xin-Zhou Li,^{1,†} and Emmanuel N. Saridakis^{2,‡}¹*Shanghai United Center for Astrophysics (SUCA), Shanghai Normal University, 100 Guilin Road, Shanghai 200234, People's Republic of China*²*College of Mathematics and Physics, Chongqing University of Posts and Telecommunications, Chongqing, 400065, People's Republic of China*

(Received 23 April 2010; published 27 July 2010)

We have investigated the necessary conditions that prevent phantom inflation from being eternal. Allowing additionally for a nonminimal coupling between the phantom field and gravity, we present the slow-climb requirements, perform an analysis of the fluctuations, and finally we extract the overall conditions that are necessary in order to prevent eternity. Furthermore, we verify our results by solving explicitly the cosmological equations in a simple example of an exponential potential, formulating the classical motion plus the stochastic effect of the fluctuations through Langevin equations. Our analysis shows that phantom inflation can be finite without the need of additional exotic mechanisms.

DOI: [10.1103/PhysRevD.82.023526](https://doi.org/10.1103/PhysRevD.82.023526)

PACS numbers: 98.80.Cq, 98.80.-k

I. INTRODUCTION

After almost three decades of extensive research, inflation is now considered to be a crucial part of the cosmological history of the Universe [1], having affected indelibly its observational features. Introducing a scalar field, the inflaton, and a suitable potential, one can make various scenarios of inflation realization in conventional, as well as in higher-dimensional frameworks [2–4]. Additionally, one could generalize the aforementioned paradigm, allowing for a nonminimal interaction of the scalar field with gravity [5], since nonminimal inflation could improve the obtained perturbation spectrum [6,7].

One important subject that has to be addressed in this paradigm is that of the exit from the inflationary epoch, that is to examine whether inflation can be eternal or not. In particular, in the new inflation scenario it was shown that the procedure could be eternal since the “false” vacuum (in which the field lies during inflation) is never dominated by the “true” one (the approach of which causes the end of inflation) [8,9]. Additionally, even in advanced scenarios, such as the chaotic inflation, where there is no false vacuum state, slow-roll eternity is also possible [10] due to a different mechanism. In particular, in this model-subclass the inflaton is classically rolling down its potential slope, however the quantum fluctuations can conditionally drive it upwards and thus inflation will never end [11–13]. Thus, one must in general examine the conditions for the realization of eternity [14].

An interesting class of inflation scenarios [15–22] is achieved through the use of phantom fields [23], inspired by the wide use of such fields to explain the late-time universe acceleration [24]. The simplest realization of phantom fields is the use of a negative kinetic term in the

Lagrangian, but this could lead their quantum theory to be problematic, due to the causality and stability problems and the possible spontaneous breakdown of the vacuum into phantoms and conventional particles [25,26]. However, one could consider that the phantom fields arise through an effective description of a nonphantom fundamental (probably higher-dimensional) underlying theory, consistently with the basic requirements of quantum field theory [27]. Indeed actions with phantomlike behavior may arise in supergravity [28], scalar tensor gravity [29], higher derivative gravity [30], braneworld [31], k-field [32], stringy [33] and others scenarios [34,35].

The peculiar nature of phantom fields requires the inflation paradigm to be suitably redesigned. In particular, since phantoms behave inversely in potential slopes, climbing up along them, in order to avoid an early-time Big Rip singularity [36], one must use potentials with maxima instead of minima, and the slow-roll parameters are replaced by the “slow-climb” ones [37]. However, even with potentials bounded from above, the problem of eternal inflation still exists, and one should examine in detail the possible exits from the inflationary epoch [18–22].

In the present work we are interested in investigating the necessary conditions that prevent phantom inflation from being eternal, going beyond the basic requirements of bounded-from-above potentials. In particular, we examine whether the quantum fluctuations could affect the classical motion towards the potential maximum, preventing inflation to the end. Furthermore, in order to be general we allow for a nonminimal coupling of the phantom field with gravity, since this interaction could also affect the eternity conditions, similarly to the canonical case [38].

The plan of this work is the following: In Sec. II we present the phantom-inflation scenario under the conditions of slow-climb. In Sec. III we perform a fluctuation analysis and we extract the conditions for preventing eter-

*fengcj@shnu.edu.cn

†kychz@shnu.edu.cn

‡msaridak@phys.uoa.gr

nality, while in Sec. IV we verify our results by solving explicitly the Langevin equations for the cosmological evolution in a simple example. Finally, Sec. V is devoted to the summary of the obtained results.

II. PHANTOM SLOW-CLIMB INFLATION

Let us present briefly the cosmological scenario of non-minimal phantom inflation [20], focusing on the conditions required for its longtime, efficient duration. The action of a universe constituted by a phantom scalar field φ is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) - \frac{1}{2} f(R) \varphi^2 \right], \quad (1)$$

with $V(\varphi)$ the corresponding potential, and where for simplicity we have set $8\pi G = M_p^{-2} = 1$. As usual R is the Ricci scalar, and $f(R)$ is the function describing the coupling of the phantom field to gravity. Throughout this work we consider a flat Friedmann-Robertson-Walker geometry with the unperturbed metric

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \quad (2)$$

with $a(t)$ the scale factor and t the comoving time. Thus, defining the Hubble parameter as $H \equiv \dot{a}/a$ the scalar curvature reads:

$$R = 6(\dot{H} + 2H^2), \quad (3)$$

where the dot denotes the derivative with respect to t .

Variation of the action (1) leads to the two Friedmann equations

$$3H^2 = -\frac{1}{2} \dot{\varphi}^2 + V + \frac{1}{2} f \varphi^2 + 3H^2 \left[\frac{d}{dt} \left(\frac{f' \varphi^2}{H} \right) - f' \varphi^2 \right] \quad (4)$$

$$-2\dot{H} = -\dot{\varphi}^2 + H^3 \frac{d}{dt} \left(\frac{f' \varphi^2}{H^2} \right) - \frac{d^2}{dt^2} (f' \varphi^2), \quad (5)$$

where the prime denotes differentiation with respect to the corresponding argument, that is $f' = df/dR$ and $V' = dV/d\varphi$. Additionally, the evolution equation for the scalar field writes

$$\ddot{\varphi} + 3H\dot{\varphi} - V' - f\varphi = 0. \quad (6)$$

As we discussed in the Introduction, the phantom fields require potentials bounded from above, since they climb upwards the potential slopes. Therefore, in order to acquire a longtime inflation in phantom cosmology we impose the following slow-climb conditions [37],

$$|\dot{H}| \ll H^2, \quad |\dot{\varphi}| \ll 3H|\dot{\varphi}|, \quad (7)$$

which corresponds to the slow-roll conditions of canonical inflation [2]. After some algebra, and assuming potential domination ($|-\frac{\dot{\varphi}^2}{2}| \ll V$) to simplify the calculations, the

slow-climb conditions write as

$$\begin{aligned} |f' \varphi^2 \dot{H}| &\ll V, \\ \left| V'' + f - \frac{3Hf'\dot{R}}{f} \frac{f\varphi}{V' + f\varphi} \right| &\ll 9H^2. \end{aligned} \quad (8)$$

Furthermore, in the usual case where f is a monomial of R , for instance $f \sim R^n$, we obtain $3H(\log f)' \dot{R} \approx 6n\dot{H}$. Therefore, if $|f\varphi| \ll |V'|$ or $|f\varphi| \gg |V'|$, the third term on the left-hand side of the second equation in (8) can be neglected. Thus, the aforementioned expressions are simplified to

$$|f' \varphi^2 \dot{H}| \ll V, \quad |V'' + f| \ll 9H^2. \quad (9)$$

In summary, under these slow-climb conditions, the first Friedmann equation (4) becomes

$$3H^2 = \frac{1}{2} (f - 6f'H^2) \varphi^2 + V, \quad (10)$$

while the phantom field equation of motion, for the two examined limiting cases, is simplified as

$$3H\dot{\varphi} = V', \quad |f\varphi| \ll |V'| \text{ (Case I)}, \quad (11)$$

$$3H\dot{\varphi} = f\varphi, \quad |f\varphi| \gg |V'| \text{ (Case II)}. \quad (12)$$

At this stage we introduce the standard dimensionless slow-climb parameters as [21,22]

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = M_p^2 \frac{V''}{V}, \quad (13)$$

and following [38] we define a new dimensionless slow-climb parameter

$$\Delta \equiv M_p^2 \frac{f}{V} \quad (14)$$

to account for the nonminimal coupling, where we have recovered the Planck mass to indicate that these parameters are indeed dimensionless. Using these parameters, the slow-climb conditions (9) become

$$\epsilon \Delta \varphi^2 \ll 1, \quad \eta + \Delta \ll 1, \quad (15)$$

having also used for simplicity $f' \sim f/R$, although this is not necessary. Therefore, if $\epsilon, \eta, \Delta \ll 1$, the slow-climb conditions (9) are indeed satisfied.

In order to continue, we consider explicitly the usual ansatz for $f(R)$ of the literature [5,7], namely $f = \xi R$, with ξ the coupling parameter. Thus, the Friedmann equation (10) becomes

$$3H^2 = \frac{V}{1 - \xi \varphi^2}, \quad (16)$$

and the slow-climb parameter Δ reads

$$\Delta = \frac{2\xi(2 - \epsilon)}{(1 - \xi \varphi^2)}. \quad (17)$$

As we see, $\Delta \ll 1$ requires $\xi \ll 1$ in the model at hand, a condition which is usually satisfied in all nonminimal scenarios. Finally, differentiating the Friedmann equation (10), we deduce that in the case $|f\varphi| \ll |V'|$ (Case I), that is $\Delta\varphi^2 \ll 1$, the slow-climb parameter ϵ becomes

$$\epsilon = -\frac{V'^2}{2V^2} \left[1 - \left(1 - \frac{2V}{V'\varphi} \right) \xi \varphi^2 \right], \quad (18)$$

while in the case of $|f\varphi| \gg |V'|$ (Case II), i.e. $\Delta\varphi^2 \gg 1$, it becomes

$$\epsilon = -\frac{f\varphi V'}{2V^2} \left[1 - \left(1 - \frac{2V}{V'\varphi} \right) \xi \varphi^2 \right]. \quad (19)$$

Note that in the latter case the condition $\Delta\varphi^2 \gg 1$ requires the field values to be large and therefore, without loss of generality, in the following we consider large-field inflation.

III. FLUCTUATIONS AND CONDITIONS FOR PREVENTING ETERNALITY

In the previous section we extracted the basic conditions for an efficient longtime, but noneternal phantom inflation. However, as we discussed in the Introduction, even if one manages to stop inflation at the classical level using suitable potentials, the backreaction of the metric plus the inflaton's quantum fluctuations on the background space-time could make the inflaton field follow a Brownian motion in which half of the time the inflaton field in a given domain will jump downwards, instead of drifting up to the potential. Thus, the necessary conditions for preventing eternity in phantom inflation will arise through examination of the overall effects of the classical behavior plus the fluctuations.

In order to calculate the quantum fluctuation of the inflaton, we expand the action (1) to second order, since the action approach guarantees the correct normalization for the quantization of fluctuations. It is convenient to work in the Arnowitt-Deser-Misner (ADM) formalism and write the metric as

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (20)$$

where N is the lapse function and N^i is the shift vector. Note that such perturbations have been studied in a different framework, for the minimal case, in [21,22].

The action (1) becomes

$$\begin{aligned} S = \frac{1}{2} \int dt dx^3 \sqrt{h} [& NR^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2) \\ & - N^{-1}(\dot{\varphi} - N^i \partial_i \varphi)^2 + N h^{ij} \partial_i \varphi \partial_j \varphi \\ & - N(2V + f\varphi^2)], \end{aligned} \quad (21)$$

where $h = \det h_{ij}$ and the symmetric tensor E_{ij} is defined as

$$E_{ij} = \frac{1}{2}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i), \quad E = E^i{}_i. \quad (22)$$

In (21) $R^{(3)}$ is the three-dimensional Ricci curvature, which is computed from the metric h_{ij} , and $K_{ij} = E_{ij}/N$ is the extrinsic curvature. In the following we work in the spatially-flat gauge and we neglect the tensor perturbations. Thus, we write

$$\varphi(t, x) = \bar{\varphi}(t) + \delta\varphi(t, x), \quad h_{ij} = a^2 \delta_{ij}, \quad (23)$$

where $\bar{\varphi}(t)$ is the background value of the scalar field and $\delta\varphi$ is a small fluctuation around the background value.

In the ADM formalism one can consider N and N^i as Lagrange multipliers, and in order to obtain the action for ξ one needs to solve the constraint equations for N and N^i and substitute the result back in the action. The equations of motion for N^i and N are the momentum and Hamiltonian constraints

$$\nabla_i [(1 - f'\varphi^2)N^{-1}(E^i{}_j - \delta^i{}_j E)] + N^{-1}(\dot{\varphi} - N^i \partial_i \varphi) \partial_j \varphi = 0 \quad (24)$$

and

$$\begin{aligned} R^{(3)} - (1 - 2f'\varphi^2)N^{-2}(E_{ij}E^{ij} - E^2) + N^{-2}(\dot{\varphi} - N^i \partial_i \varphi)^2 \\ - 2V - f\varphi^2 + h^{ij} \partial_i \varphi \partial_j \varphi = 0. \end{aligned} \quad (25)$$

We now decompose N^i into

$$N^i = \partial^i \psi + N_T^i \quad (26)$$

with $\partial_i N_T^i = 0$, and we define

$$N_1 \equiv N - 1, \quad (27)$$

where N_1 , N_T^i , $\psi \sim \mathcal{O}(\delta\varphi)$. Thus, inserting these expansions into (24) and (25), we can obtain the solutions up to the first order in ξ . In particular, in the usual case $f = \xi R$, we can derive the first-order solutions similarly to the Appendix of [38]. Simplifying the notation using φ to denote the background value $\bar{\varphi}$, we finally acquire:

$$N_1 = -\frac{\delta\varphi}{1 - \xi\varphi^2} \left(\frac{\dot{\varphi}}{2H} + 2\xi\varphi \right), \quad N_T^i = 0, \quad (28)$$

and

$$\begin{aligned} (1 - \xi\varphi^2)\partial^2 \psi = -N_1 \frac{\dot{\varphi}^2}{2H} + \frac{\dot{\varphi}}{2H} \delta\dot{\varphi} \\ + \left(\frac{3}{2} \frac{\dot{\varphi}}{H} + 6\xi\varphi - \frac{V'}{2H^2} \right) H \delta\varphi, \end{aligned} \quad (29)$$

with suitable boundary conditions. Furthermore, we obtain the exact background dynamical equation

$$3H^2(1 - \xi\varphi^2) = -\frac{1}{2}\dot{\varphi}^2 + V, \quad (30)$$

which coincides with expression (16) in the slow-climb limit.

Now, in order to find the quadratic action for $\delta\varphi$, we need to insert relations (28) and (29) in the action (21) and expand it up to second order. However, as we can see these expressions for N and N^i are subleading in the slow-climb limit ($\dot{\varphi}^2 \ll H^2$) and large-field inflation ($\varphi^2 \gg 1$), comparing to $\delta\varphi$ (on the other hand, if the momentum of the inflaton was comparable with its energy density, namely $|\dot{\varphi}| \sim H$, the quantum fluctuation of the background would become significant and could cause instabilities on the background). Therefore, it is adequate to consider just the action (1) for $\delta\phi$ in the de Sitter background, resulting in the second-order action [38]

$$S_2 = \frac{1}{2} \int d^4x a^3 [-\delta\dot{\varphi}^2 + (\nabla\delta\varphi)^2 - V''\delta\varphi^2 - 12\xi H^2\delta\varphi^2]. \quad (31)$$

Moreover, introducing the Fourier transform of $\delta\varphi$ through $\delta\varphi_k$, the perturbation equation writes

$$\delta\ddot{\varphi}_k + 3H\delta\dot{\varphi}_k + \frac{k^2}{a^2}\delta\varphi_k = 0, \quad (32)$$

where we have used $\eta \ll 1$ and $\Delta \ll 1$. Therefore, as we observe, the quantum fluctuations in a Hubble time have the same value as in the canonical case [13,38]

$$\delta_q\varphi \approx \frac{H}{2\pi}. \quad (33)$$

Expression (33) provides the quantum fluctuations of the inflaton in one Hubble time. On the other hand, it is known that usually the classical motion of the inflaton during one Hubble time is given by [2,4]

$$|\delta_c\varphi| \approx |\dot{\varphi}H^{-1}| \sim \frac{|V'|}{3H^2}(1 + \Delta\varphi^2). \quad (34)$$

Thus, we deduce that if the quantum fluctuations are larger than the classical ones, namely $\delta_q\varphi > |\delta_c\varphi|$, then inflation will be eternal. Therefore, the necessary conditions for exiting phantom inflation are to use the suitably defined and bounded-from-above potentials of the phantom-inflation literature [15–22], plus the condition $\delta_q\varphi < |\delta_c\varphi|$. Thus, since slow-climb always requires $\Delta\varphi^2 \ll 1$ and the validity of (16), the condition that prevents eternality reads

$$\left| \frac{dV(\varphi)}{d\varphi} \right| \geq |V(\varphi)|^{3/2} \left(1 + \frac{3\xi}{2}\varphi^2 \right). \quad (35)$$

This condition restricts the potential-forms that can give rise to a finite inflation, or inversely, for a given potential it determines the bounds inside which the field can move, in order to avoid eternality. Finally, in the limit $\xi \rightarrow 0$ the above relation provides the corresponding condition for the minimal phantom inflation.

IV. LANGEVIN ANALYSIS FOR THE NONMINIMAL SLOW-CLIMB PHANTOM SCENARIO

In the previous section we extracted the general condition that prevents eternality in phantom inflation, estimating separately the effects of the classical motion and of the quantum fluctuations. In this section we will try to verify the aforementioned results, solving explicitly the cosmological equations, formulating the classical motion plus the stochastic effect of the quantum fluctuations through a Langevin analysis [39]. In order to be able to provide analytical results we will use the toy example of the exponential potential $V(\varphi) = V_0 e^{\lambda\varphi}$, with $\lambda > 0$, which satisfies the basic requirements for phantom inflation.

The overall evolution of the phantom field, including quantum fluctuations, is modeled through a random walk, and therefore it can be described by the following Langevin equation [39],

$$3H\dot{\varphi} - V'(\varphi) - 12\xi H^2\varphi = \frac{3}{2\pi} H^{5/2} n(t), \quad (36)$$

where $n(t)$ is a Gaussian white noise normalized as

$$\langle n(t) \rangle = 0, \quad \langle n(t)n(t') \rangle = \delta(t - t'). \quad (37)$$

As can be seen, $n(t)$ has dimensions of mass to the power of one half. Using the exponential potential, and taking the approximation $3H^2 \approx V_0$ during inflation, which means we do not consider the backreaction from the space-time to the classical evolution of the inflaton and focus on the quantum fluctuation of the inflaton itself which is modeled by the stochastic process, then we get

$$\dot{\varphi} - (\lambda e^{\lambda\varphi} + 4\xi\varphi) \sqrt{\frac{V_0}{3}} = qn(t), \quad (38)$$

where we have defined $q \equiv \frac{H^{3/2}}{2\pi}$.

In Eq. (38), if the term on the right-hand side is absent we recover the usual slow-climb equation of motion and the inflaton will follow a classical trajectory $\varphi_c(t)$. Therefore, we expand the field $\varphi(t)$ around its classical value $\varphi_c(t)$ up to order $\mathcal{O}(q^2)$, namely

$$\varphi(t) = \varphi_c(t) + q\varphi_1(t) + q^2\varphi_2(t) + \mathcal{O}(q^3). \quad (39)$$

Substituting this expansion into (38) and setting the coefficients of the q -powers to zero, we acquire the equations

$$\dot{\varphi}_c = \sqrt{V_0/3}(\lambda e^{\lambda\varphi_c} + 4\xi\varphi_c) \quad (40)$$

$$\dot{\varphi}_1 = \sqrt{V_0/3}(\lambda^2 e^{\lambda\varphi_c} + 4\xi)\varphi_1 + n(t) \quad (41)$$

$$\dot{\varphi}_2 = \sqrt{V_0/3} \left[\frac{1}{2} \lambda^3 e^{\lambda\varphi_c} \varphi_1^2 + (\lambda^2 e^{\lambda\varphi_c} + 4\xi)\varphi_2 \right]. \quad (42)$$

These three equations can be solved analytically in Case I and Case II of (11) and (12), namely, for $|f\varphi| \ll |V'|$ and

$|f\varphi| \gg |V'|$ respectively. The explicit solutions are presented in the Appendix.

For case I, the condition for the Hubble parameter not to be changed significantly by the quantum noise (see Appendix A 1) reads

$$\varphi_0 \lesssim \lambda^{-1} \ln V_0^{-1}, \quad (43)$$

while for Case II the corresponding condition (see Appendix A 2) reads

$$\varphi_0 \lesssim \lambda^{-1} \ln V_0^{-1} + \lambda^{-1} \ln(\sqrt{\xi}/\lambda). \quad (44)$$

In other words, if these conditions are satisfied, that is if the inflaton remains smaller than these critical values, then inflation will not be eternal.

Let us now compare these expressions with the condition (35) derived in the previous section. Applying (35) in the case of the exponential potential of the present section, and keeping up to zeroth order in terms of ξ (since otherwise we obtain transcendental equations), we acquire

$$\varphi_0 \lesssim \lambda^{-1} \ln V_0^{-1} + 2\lambda^{-1} \ln \lambda. \quad (45)$$

Clearly, this expression is consistent with both (43) and (44), and the slight differences arise from the performed assumptions that were necessary in order to solve the Langevin equation. Additionally, going to first order in ξ in (35), one can numerically show the agreement too. Therefore, we conclude that the results of the previous sections are indeed reliable.

V. CONCLUSIONS

In this work we investigated the necessary conditions that prevent phantom inflation from being eternal, going beyond the basic conditions of slow-climb behavior. In particular, even using potentials bounded from above and with suitable slopes, which give rise to slow climbing, quantum fluctuations could still lead inflation to be eternal. Thus, after presenting the slow-climb conditions, we performed an analysis of the fluctuations, extracting the overall conditions that are necessary for preventing eternity. Finally, in order to be general, we moreover allowed for a nonminimal coupling of the phantom field with gravity.

Our main result is expression (35), which is the condition restricting the potential-forms that can give rise to a finite inflation, or inversely the condition determining the bounds inside which the field can move in a given, slow-climb potential, in order to avoid eternity. Note that in our analysis we did not need any additional mechanism in order to exit eternal phantom inflation, such as the use of an extra scalar [21], the imposition of strong backreaction [22], the consideration of multiuniverses [18], or the use of specially-designed braneworld models with brane/flux annihilation [19].

Furthermore, in order to verify the obtained results, we solved explicitly the cosmological system in a simple

example of an exponential potential, formulating the classical motion plus the stochastic effect of the quantum fluctuations through Langevin equations. Requiring finite parameters in the inflation we resulted to similar conditions with those obtained by the above fluctuation-analysis procedure.

Let us make a comment here, on the limits of applicability of our analysis. First of all, as we have mentioned, the phantom field must be smaller than the Planck scale, thus its backreaction will be small and not capable of bringing inflation to eternity (in Langevin-equation terms, this means that the expansion around the classical trajectory (39) is valid). However, in an inflating universe, even if the examined region satisfies these conditions, its neighboring regions can have very high densities, and thus one could ask whether this behavior could bring about strong quantum effects in the examined region too. Therefore, we have to make an additional assumption, namely, that the initially low-density, slow-roll-inflating region has been already causally disconnected from its possible high-density neighboring regions, and the possible interactions lie outside the horizon. In such a case, the inflation of the observable universe will not be led to eternity.

Phantom fields could have interesting implications either in inflation or in describing the late-time acceleration of the Universe. Although their quantum behavior could be problematic at first, one can consider the phantoms to arise through an effective description of a nonphantom, fundamental, higher-dimensional, underlying theory, consistently with the basic requirements of quantum field theory. Therefore, the examination of their cosmological implications is valuable and can improve our understanding of nature. In these lines, the fact that phantom inflation can be noneternal makes the scenario at hand a candidate for the description of the early universe.

ACKNOWLEDGMENTS

This work is supported by National Education Foundation of China Grant No. 2009312711004 and Shanghai Natural Science Foundation, China Grant No. 10ZR1422000.

APPENDIX A: SOLUTION OF THE LANGEVIN EQUATIONS

Since we are dealing with stochastic variables, we perform the average of any physical quantity by defining the statistical measure. In particular, we use the Fokker-Planck approach and define the measure to be the physical volume of the Hubble patch, and thus the average is defined as

$$\langle H(t) \rangle_p = \frac{\langle H(t) e^{3N(t)} \rangle}{\langle e^{3N(t)} \rangle}, \quad N(t) = \int_0^t H(t') dt'. \quad (A1)$$

Since the Hubble patch that is eternally inflating will have an exponentially larger physical volume, taking the largest

weight in the average at late times, the physical volume can be a good measure to characterize eternal inflation. Therefore, the average $\langle H(t) \rangle_p$ could be significantly changed by quantum fluctuations if eternal inflation is realized. Furthermore, we shall use the functional technique developed in [39] and define a generating functional

$$W_t[\mu] = \ln \langle e^{M_t[\mu]} \rangle, \quad M_t[\mu] = \int_0^t \mu(t') H(t') dt'. \quad (\text{A2})$$

Thus, $\langle H(t) \rangle_p$ can be evaluated by functionally differentiating $W_t[\mu]$ with respect to μ and setting $\mu = 3$, resulting in the following equations up to $\mathcal{O}(q^2)$:

$$\langle H(t) \rangle_p = \left. \frac{\delta W_t[\mu]}{\delta \mu(t)} \right|_{\mu(t)=3} = \langle H(t) \rangle + 3 \int_0^t \langle \langle H(t) H(t') \rangle \rangle dt', \quad (\text{A3})$$

$$\langle \langle H(t) H(t') \rangle \rangle = \langle H(t) H(t') \rangle - \langle H(t) \rangle_p \langle H(t') \rangle_p. \quad (\text{A4})$$

After these definitions we can proceed to the solution of the Langevin equations.

1. Case I: $|f\varphi| \ll |V'|$

In this case, the phantom field can be regarded as minimally coupled to gravity and the solution to (40) writes

$$\varphi(t) = \varphi_c(t) + q e^{\lambda \varphi_c(t)} \Xi(t) + q^2 e^{\lambda \varphi_c(t)} \Pi(t) \quad (\text{A5})$$

with

$$\varphi_c(t) = -\lambda^{-1} \ln[e^{-\lambda \varphi_0} - \lambda^2 t \sqrt{V_0/3}], \quad (\text{A6})$$

where the subscript 0 denotes the initial value of the field (at $t = 0$). In (A5) we have defined the quantities

$$\begin{aligned} \Xi(t) &= \int_0^t n(t') e^{-\lambda \varphi_c(t')} dt' \\ &= \int_0^t n(t') [e^{-\lambda \varphi_0} - \lambda^2 t' \sqrt{V_0/3}] dt' \end{aligned} \quad (\text{A7})$$

and

$$\Pi(t) = \frac{\lambda^3}{2} \sqrt{\frac{V_0}{3}} \int_0^t e^{2\lambda \varphi_c(t')} \Xi^2(t') dt', \quad (\text{A8})$$

where $\Xi(t)$ is a new stochastic variable normalized as

$$\langle \Xi(t) \rangle = 0, \quad (\text{A9})$$

$$\langle \Xi(t) \Xi(t') \rangle = \frac{e^{-3\lambda \varphi_0}}{\lambda^2 \sqrt{3V_0}} \{1 - [1 - \lambda^2 e^{\lambda \varphi_0} \sqrt{V_0/3} \min(t, t')]^3\}. \quad (\text{A10})$$

The Hubble parameter reads

$$\begin{aligned} H(t) &= H_c(t) + q \sqrt{\frac{V_0}{3}} \frac{\lambda}{2} e^{3\lambda \varphi_c(t)/2} \Xi(t) \\ &\quad + q^2 \sqrt{\frac{V_0}{3}} \frac{\lambda}{8} e^{3\lambda \varphi_c(t)/2} [\lambda e^{\lambda \varphi_c(t)} \Xi^2(t) + 4\Pi(t)], \end{aligned} \quad (\text{A11})$$

where $H_c(t) = \sqrt{V(\varphi_c(t))/3} = e^{\lambda \varphi_c/2} \sqrt{V_0/3}$. Using (A9)–(A11), we can further obtain

$$\langle H(t) \rangle = \langle H_c(t) \rangle + q^2 \frac{e^{-2\lambda \varphi_0}}{8} e^{3\lambda \varphi_c(t)/2} (e^{\lambda[\varphi_c(t) - \varphi_0]} - 1) \quad (\text{A12})$$

and

$$\begin{aligned} &3 \int_0^t \langle \langle H(t) H(t') \rangle \rangle \\ &= \frac{q^2 e^{-\lambda \varphi_c(t)}}{10\lambda^2} \{5e^{3\lambda[\varphi_c(t) - \varphi_0]} + 1 - 6e^{5\lambda[\varphi_c(t) - \varphi_0]/2}\}. \end{aligned} \quad (\text{A13})$$

Now, in the limit $t \ll t_0 \equiv \lambda^{-2} e^{-\lambda \varphi_0} \sqrt{3/V_0}$ we can acquire the leading-order behavior of $\langle H(t) \rangle_p$ in terms of t as

$$\langle H(t) \rangle_p = \langle H(t=0) \rangle_p + \frac{e^{\lambda \varphi_0/2}}{2} \sqrt{\frac{V_0}{3}} \left(\frac{t}{t_0}\right) + \frac{q^2 e^{-\lambda \varphi_0/2}}{8} \left(\frac{t}{t_0}\right)^2, \quad (\text{A14})$$

where the second term arises from expanding the classical motion $H_c(t)$, while the last term comes from the quantum correction (A12). Note that the contribution from (A13) is of the order of $(t/t_0)^2$. Requiring the Hubble parameter not to be changed significantly by the quantum noise, we need to impose

$$\frac{e^{\lambda \varphi_0/2}}{2} \sqrt{\frac{V_0}{3}} \lesssim \frac{q^2 e^{-\lambda \varphi_0/2}}{8}, \quad (\text{A15})$$

which provides the bound when $(t/t_0) \ll 1$ as

$$\varphi_0 \lesssim \lambda^{-1} \ln V_0^{-1}. \quad (\text{A16})$$

2. Case II: $|f\varphi| \gg |V'|$

In this case, the solution to (40) reads

$$\varphi(t) = \varphi_c(t) + q \varphi_c(t) \Xi(t) + q^2 \varphi_c(t) \frac{\varphi_{20}}{\varphi_0} \quad (\text{A17})$$

with

$$\varphi_c(t) = \varphi_0 \exp(4\xi t \sqrt{V_0/3}), \quad (\text{A18})$$

where $\varphi_{20} \equiv \varphi_2(t=0)$, and similarly to the previous subsection we have defined

$$\begin{aligned}\Xi(t) &= \int_0^t n(t') \varphi_c^{-1}(t') dt' \\ &= \varphi_0^{-1} \int_0^t n(t') \exp(-4\xi t' \sqrt{V_0/3}) dt',\end{aligned}\quad (\text{A19})$$

normalized as

$$\langle \Xi(t) \rangle = 0, \quad (\text{A20})$$

$$\langle \Xi(t) \Xi(t') \rangle = \frac{\varphi_0^{-2}}{8\xi\sqrt{V_0/3}} \{1 - \exp[-8\xi\sqrt{V_0/3} \min(t, t')]\}. \quad (\text{A21})$$

The Hubble parameter is

$$\begin{aligned}H(t) &= H_c(t) + q\sqrt{\frac{V_0}{3}} \frac{\lambda}{2} e^{\lambda\varphi_c(t)/2} \varphi_c(t) \Xi(t) \\ &\quad + q^2 \sqrt{\frac{V_0}{3}} \frac{\lambda}{8} e^{3\lambda\varphi_c(t)/2} \left[\lambda\varphi_c^2(t) \Xi^2(t) + 4\varphi_c(t) \frac{\varphi_{20}}{\varphi_0} \right],\end{aligned}\quad (\text{A22})$$

where $H_c(t) = e^{\lambda\varphi_c/2} \sqrt{V_0/3}$. Moreover we obtain

$$\begin{aligned}\langle H(t) \rangle &= \langle H_c(t) \rangle + q^2 \sqrt{\frac{V_0}{3}} \frac{\lambda}{8} e^{3\lambda\varphi_c(t)/2} \\ &\quad \times \left[4\varphi_c(t) \frac{\varphi_{20}}{\varphi_0} + \frac{\lambda}{8\xi\sqrt{V_0/3}} \left[\left(\frac{\varphi_c(t)}{\varphi_0} \right)^2 - 1 \right] \right]\end{aligned}\quad (\text{A23})$$

and

$$\begin{aligned}3 \int_0^t \langle \langle H(t) H(t') \rangle \rangle &= \frac{3q^2 \lambda^2 e^{\lambda\varphi_c/2}}{128\xi^2} \left(\frac{\varphi_c}{\varphi_0} \right) \\ &\quad \times \left\{ \frac{2}{\lambda\varphi_0} (e^{\lambda\varphi_c/2} - e^{\lambda\varphi_0/2}) + \left(\frac{\varphi_0}{\varphi_c} e^{\lambda\varphi_c/2} - e^{\lambda\varphi_0/2} \right) \right. \\ &\quad \left. + \frac{\lambda\varphi_0}{2} \left[\text{Ei}\left(-\frac{\lambda\varphi_c}{2}\right) - \text{Ei}\left(-\frac{\lambda\varphi_0}{2}\right) \right] \right\},\end{aligned}\quad (\text{A24})$$

where Ei is the exponential integral function.

In the limit $t \ll t_0 \equiv \sqrt{3/V_0}/(4\xi)$ we can obtain the leading-order behavior of $\langle H(t) \rangle_p$ in terms of t as

$$\begin{aligned}\langle H(t) \rangle_p &= \langle H(t=0) \rangle_p + \frac{\lambda\varphi_0 e^{\lambda\varphi_0/2}}{2} \sqrt{\frac{V_0}{3}} \left(\frac{t}{t_0} \right) \\ &\quad + q^2 \sqrt{\frac{V_0}{3}} \frac{\lambda}{8} e^{3\lambda\varphi_0/2} \\ &\quad \times \left[\frac{\lambda}{4\xi\sqrt{V_0/3}} + 4\varphi_{20} + 6\lambda\varphi_0\varphi_{20} \right] \left(\frac{t}{t_0} \right),\end{aligned}$$

where the second term arises from expanding the classical motion $H_c(t)$ and the last term comes from the quantum correction (A23). Note that the contribution from (A24) is of the order of $(t/t_0)^2$. Requiring the Hubble parameter not to be changed significantly by the quantum noise, we impose

$$4\varphi_0 \lesssim \frac{q^2 \lambda e^{\lambda\varphi_0}}{4\xi\sqrt{V_0/3}}, \quad (\text{A25})$$

where we have used that $\xi \ll 1$ and $\varphi_0 \gg \Delta^{-1/2} \sim \xi^{-1/2}$. Thus, we conclude that at $(t/t_0) \ll 1$:

$$\varphi_0 \lesssim \lambda^{-1} \ln V_0^{-1} + \lambda^{-1} \ln(\sqrt{\xi}/\lambda). \quad (\text{A26})$$

-
- [1] A. H. Guth, *Phys. Rev. D* **23**, 347 (1981); A. D. Linde, *Phys. Lett. B* **108**, 389 (1982); A. J. Albrecht and P. J. Steinhardt, *Phys. Rev. Lett.* **48**, 1220 (1982).
- [2] A. D. Linde, *Particle Physics and Inflationary Cosmology* (CRC Press, Boca Raton, USA, 1990).
- [3] D. Baumann and L. McAllister, *Annu. Rev. Nucl. Part. Sci.* **59**, 67 (2009).
- [4] J. E. Lidsey *et al.*, *Rev. Mod. Phys.* **69**, 373 (1997).
- [5] V. Sahni and S. Habib, *Phys. Rev. Lett.* **81**, 1766 (1998); J. P. Uzan, *Phys. Rev. D* **59**, 123510 (1999); F. Perrotta, C. Baccigalupi, and S. Matarrese, *Phys. Rev. D* **61**, 023507 (1999); V. Faraoni, *Phys. Rev. D* **68**, 063508 (2003); E. Elizalde, S. Nojiri, and S. Odintsov, *Phys. Rev. D* **70**, 043539 (2004); S. Nojiri, S. D. Odintsov, and M. Sami, *Phys. Rev. D* **74**, 046004 (2006); M. R. Setare and E. N. Saridakis, *J. Cosmol. Astropart. Phys.* **03** (2009) 002; E. N. Saridakis and S. V. Sushkov, *Phys. Rev. D* **81**, 083510 (2010).
- [6] V. Faraoni, *Phys. Rev. D* **62**, 023504 (2000); M. Li, *J. Cosmol. Astropart. Phys.* **10** (2006) 003; B. Chen, M. Li, T. Wang, and Y. Wang, *Mod. Phys. Lett. A* **22**, 1987 (2007).
- [7] Y. S. Piao, Q. G. Huang, X. m. Zhang, and Y. Z. Zhang, *Phys. Lett. B* **570**, 1 (2003); X. Z. Li and X. H. Zhai, *Phys. Rev. D* **67**, 067501 (2003); D. J. Liu and X. Z. Li, *Phys. Rev. D* **70**, 123504 (2004); K. Nozari and B. Fazlpour, *J. Cosmol. Astropart. Phys.* **11** (2007) 006; S. C. Park and S. Yamaguchi, *J. Cosmol. Astropart. Phys.* **08**, (2008) 009; F. Bauer and D. A. Demir, *Phys. Lett. B* **665**, 222 (2008); G. Gupta, E. N. Saridakis, and A. A. Sen, *Phys. Rev. D* **79**,

- 123013 (2009).
- [8] P.J. Steinhardt, in *The Very Early Universe, Proceedings of the Nuffield Workshop, Cambridge, 21 June–9 July, 1982*, edited by G.W. Gibbons, S.W. Hawking, and S.T.C. Siklos (Cambridge University Press, Cambridge, 1983), pp. 251–66.
- [9] A. Vilenkin, *Phys. Rev. D* **27**, 2848 (1983).
- [10] A.D. Linde, *Phys. Lett. B* **175**, 395 (1986).
- [11] Y.F. Cai and Y. Wang, *J. Cosmol. Astropart. Phys.* **06** (2007) 022; M. Li and Y. Wang, *J. Cosmol. Astropart. Phys.* **08** (2007) 007; Q.G. Huang, M. Li, and Y. Wang, *J. Cosmol. Astropart. Phys.* **09** (2007) 013; Y.F. Cai and Y. Wang, *J. Cosmol. Astropart. Phys.* **01** (2008) 001; S. Winitzki, *Phys. Rev. D* **78**, 043501 (2008); D.I. Podolsky, *Gravitation Cosmol.* **15**, 69 (2009).
- [12] A.D. Linde, *Lect. Notes Phys.* **738**, 1 (2008).
- [13] A.H. Guth, *J. Phys. A* **40**, 6811 (2007).
- [14] A.H. Guth, *Phys. Rep.* **333**, 555 (2000).
- [15] J.E. Lidsey, *Phys. Rev. D* **70**, 041302 (2004).
- [16] S. Nojiri and S.D. Odintsov, *Gen. Relativ. Gravit.* **38**, 1285 (2006).
- [17] S. Capozziello, S. Nojiri, and S.D. Odintsov, *Phys. Lett. B* **632**, 597 (2006).
- [18] P.F. Gonzalez-Diaz and J.A. Jimenez-Madrid, *Phys. Lett. B* **596**, 16 (2004).
- [19] Y.S. Piao, *Phys. Rev. D* **78**, 023518 (2008).
- [20] E. Elizalde, S. Nojiri, S.D. Odintsov, D. Saez-Gomez, and V. Faraoni, *Phys. Rev. D* **77**, 106005 (2008).
- [21] Y.S. Piao and Y.Z. Zhang, *Phys. Rev. D* **70**, 063513 (2004).
- [22] P. Wu and H.W. Yu, *J. Cosmol. Astropart. Phys.* **05** (2006) 008.
- [23] R.R. Caldwell, *Phys. Lett. B* **545**, 23 (2002); R.R. Caldwell, M. Kamionkowski, and N.N. Weinberg, *Phys. Rev. Lett.* **91**, 071301 (2003); M. Baldi, F. Finelli, and S. Matarrese, *Phys. Rev. D* **72**, 083504 (2005).
- [24] J.G. Hao and X.Z. Li, *Phys. Rev. D* **67**, 107303 (2003); **68**, 043501 (2003); X.Z. Li and J.G. Hao, *Phys. Rev. D* **69**, 107303 (2004); P.F. Gonzalez-Diaz and C.L. Siguenza, *Nucl. Phys.* **B697**, 363 (2004); J.G. Hao and X.Z. Li, *Phys. Rev. D* **70**, 043529 (2004); S. Nojiri and S.D. Odintsov, *Phys. Rev. D* **72**, 023003 (2005); H. Garcia-Compean, G. Garcia-Jimenez, O. Obregon, and C. Ramirez, *J. Cosmol. Astropart. Phys.* **07** (2008) 016; E.N. Saridakis, P.F. Gonzalez-Diaz, and C.L. Siguenza, *Classical Quantum Gravity* **26**, 165003 (2009); X.M. Chen, Y.G. Gong, and E.N. Saridakis, *J. Cosmol. Astropart. Phys.* **04** (2009) 001; S. Dutta and R.J. Scherrer, *Phys. Lett. B* **676**, 12 (2009); E.N. Saridakis, *Nucl. Phys.* **B819**, 116 (2009).
- [25] S.M. Carroll, M. Hoffman, and M. Trodden, *Phys. Rev. D* **68**, 023509 (2003).
- [26] J.M. Cline, S. Jeon, and G.D. Moore, *Phys. Rev. D* **70**, 043543 (2004).
- [27] S. Nojiri and S.D. Odintsov, *Phys. Lett. B* **562**, 147 (2003); **571**, 1 (2003).
- [28] H.P. Nilles, *Phys. Rep.* **110**, 1 (1984).
- [29] B. Boisscau, G. Esposito-Farese, d. Polarski, and A.A. Starobinsky, *Phys. Rev. Lett.* **85**, 2236 (2000).
- [30] M.D. Pollock, *Phys. Lett. B* **215**, 635 (1988).
- [31] V. Sahni and Y. Shtanov, *J. Cosmol. Astropart. Phys.* **11** (2003) 014; D.J. Liu and X.Z. Li, *Phys. Rev. D* **68**, 067301 (2003).
- [32] C. Armendariz-Picon, T. Damour, and V.F. Mukhanov, *Phys. Lett. B* **458**, 209 (1999); T. Chiba, T. Okabe, and M. Yamaguchi, *Phys. Rev. D* **62**, 023511 (2000); P.F. Gonzalez-Diaz, *Phys. Lett. B* **586**, 1 (2004).
- [33] P.H. Frampton, *Phys. Lett. B* **555**, 139 (2003).
- [34] L.P. Chimento and R. Lazkoz, *Phys. Rev. Lett.* **91**, 211301 (2003); J.G. Hao and X.Z. Li, *Phys. Lett. B* **606**, 7 (2005).
- [35] H. Stefancic, *Eur. Phys. J. C* **36**, 523 (2004).
- [36] R. Kallosh, J. Kratochvil, A. Linde, E. Linder, and M. Shmakova, *J. Cosmol. Astropart. Phys.* **10** (2003) 015; P.F. Gonzalez-Diaz, *Phys. Rev. D* **68**, 021303 (2003); S. Nojiri, S.D. Odintsov, and S. Tsujikawa, *Phys. Rev. D* **71**, 063004 (2005).
- [37] M. Sami and A. Toporensky, *Mod. Phys. Lett. A* **19**, 1509 (2004); P. Singh, M. Sami, and N. Dadhich, *Phys. Rev. D* **68**, 023522 (2003).
- [38] C.J. Feng and X.Z. Li, arXiv:0911.3994.
- [39] X. Chen, S. Sarangi, S.H. Henry Tye, and J. Xu, *J. Cosmol. Astropart. Phys.* **11** (2006) 015; S. Gratton and N. Turok, *Phys. Rev. D* **72**, 043507 (2005); A.D. Linde, D.A. Linde, and A. Mezhlumian, *Phys. Rev. D* **49**, 1783 (1994).