Constraints on cold dark matter accelerating cosmologies and cluster formation

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We discuss the properties of homogeneous and isotropic flat cosmologies in which the present accelerating stage is powered only by the gravitationally induced creation of cold dark matter (CCDM) particles ($\Omega_m = 1$). For some matter creation rates proposed in the literature, we show that the main cosmological functions such as the scale factor of the universe, the Hubble expansion rate, the growth factor, and the cluster formation rate are analytically defined. The best CCDM scenario has only one free parameter and our joint analysis involving baryonic acoustic oscillations + cosmic microwave background (CMB) + SNe Ia data yields $\tilde{\Omega}_m = 0.28 \pm 0.01 \, (1\sigma)$, where $\tilde{\Omega}_m$ is the observed matter density parameter. In particular, this implies that the model has no dark energy but the part of the matter that is effectively clustering is in good agreement with the latest determinations from the large-scale structure. The growth of perturbation and the formation of galaxy clusters in such scenarios are also investigated. Despite the fact that both scenarios may share the same Hubble expansion, we find that matter creation cosmologies predict stronger small scale dynamics which implies a faster growth rate of perturbations with respect to the usual ΛCDM cosmology. Such results point to the possibility of a crucial observational test confronting CCDM with ΛCDM scenarios through a more detailed analysis involving CMB, weak lensing, as well as the large-scale structure.

I. INTRODUCTION

The analyses of high quality cosmological data (e.g. supernovae type Ia, cosmic microwave background (CMB), galaxy clustering, etc.) have converged towards a cosmic expansion history that involves a spatially flat geometry and some sort of dark energy in order to explain the recent accelerating expansion of the universe [1–7]. The simplest dark energy candidate corresponds to a cosmological constant, Λ (see [8–10] for reviews). In the standard concordance ΛCDM model, the overall cosmic fluid contains baryons, cold dark matter, plus a vacuum energy that fits accurately the current observational data and thus it provides an excellent scenario to describe the observed universe.

On the other hand, the concordance model suffers from, among others [11], two fundamental problems: (a) *The fine-tuning problem*, i.e., the fact that the observed value of the vacuum energy density ($\rho_\Lambda = \Lambda c^2/8\pi G \simeq 10^{-47} \, \mathrm{GeV}^4$) is more than 120 orders of magnitude below the natural value estimated using quantum field theory [8], and (b) *the coincidence problem* [12], i.e., the fact that the matter energy density and the vacuum energy density are of the same order just prior to the present epoch. Such problems have inspired many authors to propose alternative candidates to dark energy such as $\Lambda(t)$ cosmologies, quintessence, k essence, vector fields, phantom, tachyons, Chaplygin gas, and the list goes on (see [13–28] and references therein).

Nowadays, the nature of the dark energy is considered one of the most fundamental and difficult problems in the interface uniting astronomy, particle physics, and cosmology. However, there are other possibilities. For instance, one may consider nonstandard gravity theories where the present accelerating stage of the universe is driven only by cold dark matter, that is, with no appealing to the existence of dark energy. Such a reduction of the so-called dark sector is naturally obtained in the so-called $f(R)$ gravity theories [29] (see, however, [30]). Even in the framework of the standard general relativity theory, it is also possible to reduce the dark sector by considering the presence of inhomogeneities [31], quartessence models [32], as well as the gravitationally induced particle creation mechanism [33–48]. In what follows we focus our attention to the last approach by considering the gravitational creation of cold dark matter in the expanding Universe.

The basic microscopic description for gravitational particle production in an expanding universe has been investigated in the literature by many authors starting with Schrödinger [49]. In the late 1960s, independently of Schrödinger's work, this issue was investigated by Parker and others [33,34] by considering the Bogoliubov mode-mixing technique in the context of quantum field theory in curved spacetime. The basic physical effect is that a classical nonstationary background influences bosonic or fermionic quantum fields in such a way that their masses become time dependent (see, e.g., [50] for more recent works). For a real scalar field in a flat Friedmann-Lemaître-Robertson-Walker (FLRW) geometry described in conformal coordinates, for example, the key result is that the effective mass reads [51] $m_{\mathrm{eff}}^2(\eta) \equiv m^2 a^2 - a''/a$, where m is the ''Minkowskian'' constant mass of the scalar field,

$a(\eta)$ is the scale factor, and the derivatives are computed with respect to the conformal time. This time-dependent mass accounts for the interaction between the scalar field and the geometry of the Universe. When the field is quantized, this leads to particle creation, with the energy for newly created particles being supplied by the classical, time-varying gravitational background.

Macroscopically, the first self-consistent formulation for matter creation was put forward by Prigogine and co-workers [35] and somewhat clarified by Calvão, Lima, and Waga [36] (see also [37]). It was shown that matter creation, at the expense of the gravitational field, is also macroscopically described by a negative pressure, and, potentially, can accelerate the Universe. Within this framework, various interesting features of cosmologies with particle creation have been discussed in [38–44] (see also [45] for recent studies on this subject). More recently, the corresponding effects on the global dynamics of the particle creation regime have been investigated extensively by a number of authors (see [46–48] and references therein). In particular, it was also found that a subclass of such models depends only on one free parameter and that the evolution of the scale factor is exactly the same of ΛCDM models [48]. Naturally, in order to know if creation of cold dark matter (CCDM) models provides a realistic description of the observed Universe, its viability needs to be tested by discussing all the constraints imposed from current observations both for background and perturbative levels (structure formation).

In this context, the basic aim of the present work is twofold. First, we place constraints on the main parameters of CCDM cosmologies by performing a joint likelihood analysis involving the shift parameter of the cosmic microwave background (CMB) [3] and the observed baryonic acoustic oscillations (BAO) [5], and the latest SNIa data (Constitution) [7]. Second, for a wide class of matter creation cosmologies, we also develop the linear approach for the density perturbations. Analytical solutions for the differential equation governing the evolution of the growth factor and some properties of the large-scale structure (cluster formation) are also discussed and compared to the predictions of the ΛCDM model.

The paper is planned as follows. The basic theoretical elements of the problem are presented in Sec. II, where we introduce the basic FLRW cosmological equations for CCDM cosmologies. In Secs. III and IV we present the specific CCDM scenarios and derive the constraints on their parameters based on a statistical joint analysis involving the shift parameter of the cosmic microwave background (CMB) [3], the baryonic acoustic oscillations (BAO) [5], and the latest SNIa data (Constitution) [7]. In Sec. V we study the evolution of linear perturbations, while in Sec. VI we present the corresponding theoretical predictions regarding the formation of the galaxy clusters with basis on the Press-Schechter formalism. Finally, in

Sec. VII we draw our conclusions. Throughout the paper we consider $H_0 = 70.5$ km/sec/Mpc as given by the WMAP five-years data [3].

II. CREATION COLD DARK MATTER (CCDM) COSMOLOGIES: BASIC EQUATIONS

The nontrivial cosmological equations for the mixture of radiation, baryons and cold dark matter (with creation of dark matter particles), and the energy conservation laws for each component have been investigated thoroughly by [37–39,46–48]. In this framework, for a spatially flat FLRW geometry the basic equations which govern the global dynamics of the universe in the matter dominated era ($\rho_{rad} \equiv 0$) are given by [46–48]

$$H^2 = \frac{8\pi G}{3}(\rho_{bar} + \rho_{dm}) = \frac{8\pi G}{3}\rho_m, \qquad (1)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_m + 3p_c), \qquad (2)$$

$$\dot{\rho}_{bar} + 3H\rho_{bar} = 0, \qquad \dot{\rho}_{dm} + 3H\rho_{dm} = \rho_{dm}\Gamma, \quad (3)$$

which implies that

$$\dot{\rho}_m + 3H\rho_m = \rho_{dm}\Gamma, \qquad \text{where } \rho_m = \rho_{dm} + \rho_{bar}. \tag{4}$$

In the above set of differential equations, an overdot denotes derivative with respect to time, ρ_{bar} and ρ_{dm} are the baryonic and dark matter energy densities, $H = \dot{a}/a$ is the Hubble parameter, whereas p_c corresponds to the creation pressure. The quantity Γ is the so-called creation rate of the cold dark matter and it has units of $(time)^{-1}$. The creation pressure is defined in terms of the creation rate and other physical quantities. For adiabatic creation of cold dark matter, it has been found [36,37] that the creation pressure is given by (see also Prigogine *et al.* [35])

$$p_c = -\frac{\rho_{dm}\Gamma}{3H}. \tag{5}$$

Using Eqs. (1) and (4) we can obtain the following useful formula:

$$\dot{H} + \frac{3}{2}H^2 = \frac{4\pi G\rho_{dm}}{3}\frac{\Gamma}{H}. \tag{6}$$

Expressions (4)–(6) show how the matter creation rate, Γ, modifies the basic quantities of Einstein-de Sitter cosmology. Note that if the creation rate is negligible, $\Gamma \ll H$, then, the creation pressure is negligible, $\rho_m \propto a^{-3}$, and the solution of the above equation reduces to that of the Einstein-de Sitter model, namely, $H(t) = 2/3t$.

III. TWO SPECIFIC CCDM MODELS

Although the precise functional form of $\Gamma(t)$ is still missing, a number of different phenomenological parame-

trizations have been proposed in the literature treating the time-dependent $\Gamma(t)$ function ([46–48] and references therein). In what follows, we consider two different flat matter creation models, namely, Lima, Silva, and Santos (hereafter LSS [46]) and Lima, Jesus, and Oliveira (hereafter LJO [48]), respectively. With the aid of the current observational data, we will also put stringent constraints on their free parameters.

A. The LSS model

In the LSS scenario, the functional form of Γ is phenomenologically parametrized by the following linear expansion in the Hubble parameter (see LSS for more details):

$$\Gamma = 3\gamma H_0 + 3\beta H. \tag{7}$$

It is worth noticing that in its original formulation by LSS, both the baryonic and radiative contributions were neglected ($\rho_{\text{bar}} = \rho_{\text{rad}} = 0$). Such incompleteness was cured in a subsequent paper by Steigman *et al.* [47]. The advantage of the LSS model is that it is analytically described, and due to this feature we will consider it as an interesting and preparatory toy model for more physical CCDM scenarios.

The mass density in LSS reduces to the dark matter density, $\rho_m = \rho_{\text{dm}}$ [see Eq. (4)]. Without going into the details of that model, let us present its main features. Based on the latter considerations the basic Friedmann equation becomes [see Eq. (6)]

$$\dot{H} + \frac{3}{2}H^2\left(1 - \beta - \frac{\gamma H_0}{H}\right) = 0. \tag{8}$$

Performing the integration of Eq. (8) we derive the following Hubble function:

$$H(t) = H_0\left(\frac{\gamma}{1-\beta}\right)\frac{e^{(3\gamma H_0/2)t}}{(e^{(3\gamma H_0/2)t} - 1)}, \tag{9}$$

and by integrating the latter we obtain the evolution of the scale factor

$$a(t) = \frac{1}{(1+z)} = \left[\left(\frac{1-\gamma-\beta}{\gamma}\right)\left(e^{3\gamma H_0 t/2} - 1\right)\right]^{2/3(1-\beta)}. \tag{10}$$

Evaluating Eq. (10) at the present time ($a \equiv 1$ or $z = 0$), we can define the present age of the Universe:

$$t_0 = \frac{2}{3\gamma H_0}\ln\left(\frac{1-\beta}{1-\gamma-\beta}\right). \tag{11}$$

Now utilizing Eqs. (9) and (10), we find after some algebra that the normalized Hubble flow takes the following form [47]:

$$E(a) = \frac{H(a)}{H_0} = \frac{\gamma}{1-\beta} + \frac{1-\gamma-\beta}{1-\beta}a^{-3(1-\beta)/2}. \tag{12}$$

We now study the conditions under which an inflection

point exists in our past, implying an acceleration phase of the scale factor. This crucial period in the cosmic history corresponds to $\ddot{a}(t_I) = 0$ and $t_I < t_0$. Differentiating twice Eq. (10), we simply have

$$a_I = \frac{1}{1+z_I} = \left[\frac{2\gamma}{(1-3\beta)(1-\gamma-\beta)}\right]^{-2/3(1-\beta)}. \tag{13}$$

Note that if $\gamma = 0$, there is no transition from an early decelerating to a late time accelerating Universe [47]. In this case, one can also prove that the expansion of the Universe always decelerates for $0 \leq \beta < 1/3$ and always accelerates for $1/3 < \beta \leq 1$. The latter condition implies that the parameter γ has to obey the following restriction $0 < \gamma \leq 1$. When $\gamma = 0$, negative values of β mean destruction of particles, and, for completeness, such a possibility will also be considered in the discussion of some background tests (see Sec. IV).

B. The LJO model

Now, let us consider an alternative CCDM scenario that includes the baryonic and radiative components, and was also proposed with the intention to solve the cluster problem in such a framework [48]. In this context, the particle creation rate reads

$$\Gamma = 3\tilde{\Omega}_\Lambda\left(\frac{\rho_{\text{co}}}{\rho_{\text{dm}}}\right)H, \tag{14}$$

where $\tilde{\Omega}_\Lambda$ (called α in the [48]) is a constant, $\rho_{\text{co}} = 3H_0^2/8\pi G$ is the present-day value of the critical density, and the factor 3 has been maintained for mathematical convenience. We stress that such CCDM scenario mimics the global dynamics of the traditional Λ cosmology [48]. Indeed, the combination of Eqs. (1), (6), and (14) leads to the following formula:

$$\dot{H} + \tfrac{3}{2}H^2 = \tfrac{3}{2}\tilde{\Omega}_\Lambda H_0^2, \tag{15}$$

a solution of which is given by

$$H(t) = \sqrt{\tilde{\Omega}_\Lambda}H_0\coth\left(\frac{3H_0\sqrt{\tilde{\Omega}_\Lambda}}{2}t\right). \tag{16}$$

Using now the definition of the Hubble parameter $H \equiv \dot{a}/a$, the scale factor of the universe $a(t)$, normalized to unity at the present epoch, evolves with time as

$$a(t) = \left(\frac{\tilde{\Omega}_m}{\tilde{\Omega}_\Lambda}\right)^{1/3}\sinh^{2/3}\left(\frac{3H_0\sqrt{\tilde{\Omega}_\Lambda}}{2}t\right), \tag{17}$$

where $\tilde{\Omega}_m = 1 - \tilde{\Omega}_\Lambda \equiv 1 - \alpha$ can be viewed as the effective matter density parameter at the present time. For an arbitrary redshift, $\tilde{\Omega}_m(z)$ measures the mass density that is effectively clustering [48]. The cosmic time is related with the scale factor as

$$t(a) = \frac{2}{3\sqrt{\tilde{\Omega}_\Lambda}H_0}\sinh^{-1}\left(\sqrt{\frac{\tilde{\Omega}_\Lambda}{\tilde{\Omega}_m}}a^{3/2}\right). \qquad (18)$$

Combining the above equations we can define the normalized Hubble expansion as a function of the scale factor:

$$E(a) = \frac{H(a)}{H_0} = (\tilde{\Omega}_m a^{-3} + \tilde{\Omega}_\Lambda)^{1/2}. \qquad (19)$$

Obviously, the LJO model contains only one free parameter. The inflection point [namely, the point where the Hubble expansion changes from the decelerating to the accelerating regime, $\ddot{a}(t_I) = 0$] takes place at

$$t_I = \frac{2}{3\sqrt{\tilde{\Omega}_\Lambda}H_0}\sinh^{-1}\left(\sqrt{\frac{1}{2}}\right), \qquad a_I = \left[\frac{\tilde{\Omega}_m}{2\tilde{\Omega}_\Lambda}\right]^{1/3}. \quad (20)$$

In what follows we constrain the free parameters of such models coming from the background tests. For the sake of clarity, the study of the perturbed models and the associated constraints coming from galaxy cluster formation will be postponed to Secs. V and VI.

IV. BACKGROUND TESTS: BAO, CMB SHIFT PARAMETERS, AND SNE IA DATA

In this section we briefly present the statistical analysis used in order to constrain the matter creation models presented in the previous section. First of all, we consider the baryonic acoustic oscillations (BAO). This kind of estimator is produced by pressure (acoustic) waves in the photon-baryon plasma in the early universe, generated by dark matter overdensities. Evidence of this excess was recently found in the clustering properties of the luminous SDSS red-galaxies [5] and it provides a "standard ruler" with which we can constrain dark energy models. In particular, we use the following estimator: $A(\mathbf{p}) = \{\sqrt{\Omega_m}/[z_s^2 E(a_s)]^{1/3}\}\{\int_{a_s}^1 (da)/[a^2 E(a)]\}^{2/3}$, measured from the SDSS data to be $A = 0.469 \pm 0.017$, where $z_s = 0.35$ [or $a_s = (1 + z_s)^{-1} \simeq 0.75$]. Therefore, the corresponding χ^2_{BAO} function is simply written

$$\chi^2_{\text{BAO}}(\mathbf{p}) = \frac{[A(\mathbf{p}) - 0.469]^2}{0.017^2}, \qquad (21)$$

where $\mathbf{p}$ is a vector containing the cosmological parameters that we want to fit.

On the other hand, a very accurate and deep geometrical probe of dark energy is the angular scale of the sound horizon at the last scattering surface as encoded in the location l_1^{TT} of the first peak of the cosmic microwave background (CMB) temperature perturbation spectrum. This probe is described by the CMB shift parameter [52,53] and is defined as $R = \sqrt{\Omega_m}\int_{a_{ls}}^1\{(da)/[a^2 E(a)]\}$. The shift parameter measured from the WMAP five-years data [3] is $R = 1.71 \pm 0.019$ at $z_{ls} = 1090$ [or $a_{ls} = (1 +$

$z_{ls})^{-1} \simeq 9.17 \times 10^{-4}$]. In this case, the χ^2_{cmb} function is given:

$$\chi^2_{\text{cmb}}(\mathbf{p}) = \frac{[R(\mathbf{p}) - 1.71]^2}{0.019^2}. \qquad (22)$$

Note that the measured CMB shift parameter is somewhat model dependent but mostly to models which are not included in our analysis; for example, in the case where massive neutrinos are included or when there is a strongly varying equation of state parameter. The robustness of the shift parameter was tested and discussed in [54].

Finally, we use the "Constitution set" of 397 type Ia supernovae of Hicken *et al.* [7]. In order to avoid possible problems related with the local bulk flow, we use a subsample of the overall sample in which we select those SNIa with $z > 0.023$. This subsample contains 351 entries. The corresponding χ^2_{SNIa} function becomes

$$\chi^2_{\text{SNIa}}(\mathbf{p}) = \sum_{i=1}^{351}\left[\frac{\mu^{\text{th}}(a_i, \mathbf{p}) - \mu^{\text{obs}}(a_i)}{\sigma_i}\right]^2, \qquad (23)$$

where $a_i = (1 + z_i)^{-1}$ is the observed scale factor of the Universe, z_i is the observed redshift, μ is the distance modulus $\mu = m - M = 5\log d_L + 25$, and $d_L(a, \mathbf{p})$ is the luminosity distance $d_L(a, \mathbf{p}) = \frac{c}{a}\int_a^1 \frac{dy}{y^2 H(y)}$, where c is the speed of light. We can combine the above probes by using a joint likelihood analysis: $\mathcal{L}_{\text{tot}}(\mathbf{p}) = \mathcal{L}_{\text{BAO}} \times \mathcal{L}_{\text{cmb}} \times \mathcal{L}_{\text{SNIa}}$ or $\chi^2_{\text{tot}}(\mathbf{p}) = \chi^2_{\text{BAO}} + \chi^2_{\text{cmb}} + \chi^2_{\text{SNIa}}$ in order to put even further constraints on the parameter space used.[1]

Performing now our statistical analysis, we attempt to put constraints on the free parameters. In particular, we sample $\beta \in [-0.2, 0.2]$ and $\gamma \in [0.1, 1]$ in steps of 0.001. Thus, the overall likelihood function peaks at $\beta = -0.15 \pm 0.007$ and $\gamma = 0.86 \pm 0.01$ with $\chi^2_{\text{tot}}(\beta, \gamma) = 432.5$ (dof = 351). Using the latter cosmological parameters the corresponding current age of the universe is found to be $t_0 \sim 14.8$ Gyr while the inflection point is located at $a_I \simeq 0.44$. This corresponds to a redshift $z_I \simeq 1.26$, which is substantially higher than in the case of the concordance model. If we now impose $\beta = 0$, the SNIa likelihood analysis provides a best fit value of $\gamma = 0.66 \pm 0.02$, in agreement with [47]. Note that [47] used only the SNIa data. We find that, although our SNIa likelihood analysis provides a similar γ value to that found in [47], our joint likelihood function peaks at $\gamma = 0.41 \pm 0.01$ but with a poor quality fit: the reduced χ^2_{tot} is $\simeq 3$.

This simply means that the functional form $E(a) = \gamma + (1 - \gamma)a^{-3/2}$ is unable to fit the observational data simultaneously at low and high redshifts. We confirm this point

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[1]Likelihoods are normalized to their maximum values. We will report 1σ uncertainties of the fitted parameters. The overall number of data points used is $N_{\text{tot}} = 353$ and the degrees of freedom dof $= N_{\text{tot}} - n_{\text{fit}}$, with n_{fit} the number of fitted parameters, which varies for the different models.

by using the CMB shift parameter only, finding that the corresponding likelihood function peaks at $\gamma \simeq 0.20$. This value is ~ 3.4 times larger than that provided by the SNIa + BAO solution $\gamma \simeq 0.68$ (SNIa only points $\gamma \simeq 0.66$). This fact alone suggests that for the class of models with a constant creation rate $\Gamma = 3\gamma H_0$, we are unable to provide a quality fit of the basic cosmological data in all the relevant redshift ranges.

Now, let us constrain the LJO model with the observational data from background tests. From our joint analysis involving BAO + CMB shift parameter + SNe Ia data, we find that the best fit value (within 1σ uncertainty) is $\tilde{\Omega}_m = 0.28 \pm 0.01$ with $\chi^2_{\text{tot}}(\Omega_m) \simeq 431.5$ (dof = 352). This result is in good agreement with recent studies [1–3,6,7] of the concordance Λ cosmology. Therefore, the current age of the universe is $t_0 \simeq 13.9$ Gyr while the inflection point is located at $a_I \simeq 0.58$ (hence at redshift $z_I \simeq 0.72$).

Summing up, although solving the acceleration and age problems, the LSS type scenarios is endowed with severe difficulties even for the set of background tests analyzed here. On the other hand, the LJO scenario provides good fits for the SNe + BAO + CMB shift parameter. A more detailed analysis on this issue (background tests) will be published in a forthcoming paper.

V. MATTER DENSITY PERTURBATIONS IN CCDM COSMOLOGIES

Let us now derive the basic equation that governs the evolution of the mass density contrast, modeled here as an ideal fluid in a "quasi"-Newtonian or neo-Newtonian framework [55]. In this approach, background equations are formulated in a way that the isotropic pressure becomes dynamically relevant even for FLRW cosmologies where all the physical properties may depend only on time. This allows us to describe an accelerated expansion of the Universe powered by an effective negative pressure in a Newtonian framework. Naturally, even considering that the neo-Newtonian approach reproduces the general relativity (GR) background dynamics exactly, small differences may occur at the perturbative level. However, as discussed by Reis [56], the GR first-order perturbation dynamics and its neo-Newtonian counterpart coincide exactly for a vanishing sound speed. Indeed, for constant equations of state it has been demonstrated that the correct large-scale behavior in the synchronous gauge are also reproduced. This should also be expected since the observational data correspond to modes that are well inside the Hubble radius, for which the use of a neo-Newtonian approach seems to be adequate (for recent papers in this subject see [57] and references therein).

A. Basic formalism

In what follows we will adopt a nonrelativistic description with basis in an extended continuity equation together with the Euler and Poisson equations. In virtue of the particle creation process they take the form

$$\left(\frac{\partial \rho}{\partial t}\right)_r + \nabla_r \cdot (\rho \mathbf{u}) = \rho \Gamma, \tag{24}$$

$$\left(\frac{\partial \mathbf{u}}{\partial t}\right)_r + (\mathbf{u} \cdot \nabla_r)\mathbf{u} = -\nabla \Phi, \tag{25}$$

and

$$\nabla_r^2 \Phi = 4\pi G \rho - \Lambda_{\text{eff}}, \tag{26}$$

where $(\mathbf{r}, t)$ are the proper coordinates, $\mathbf{u}$ is the velocity of a fluid element of volume, ρ is the mass density, and Φ is the gravitational potential. Since we are working within the context of cosmological models with matter creation we have modified the continuity equation (24) by including the "standard" source term. Utilizing Eqs. (2) and (5), the quantity Λ_{eff} is defined as

$$\Lambda_{\text{eff}} = \Lambda - 12\pi G p_c = \Lambda + 4\pi G \rho_{\text{dm}} \frac{\Gamma}{H}, \tag{27}$$

where for completeness and future comparison to the present cosmic concordance model (ΛCDM), we have also included the cosmological Λ term.

Now, by changing from proper $(\mathbf{r}, t)$ to comoving $(\mathbf{x}, t)$ variables, the fluid velocity becomes [58]

$$\mathbf{u} = \dot{a}\mathbf{x} + a\dot{\mathbf{x}} = \dot{a}\mathbf{x} + \mathbf{v}(\mathbf{x}, t), \tag{28}$$

while the corresponding differential operators take the following forms:

$$\nabla_x \equiv \nabla = a\nabla_r, \tag{29}$$

and

$$\left(\frac{\partial}{\partial t}\right)_x \equiv \frac{\partial}{\partial t} = \left(\frac{\partial}{\partial t}\right)_r + H\mathbf{x} \cdot \nabla, \tag{30}$$

where $\mathbf{x} = \mathbf{r}/a$ and $\mathbf{v}(\mathbf{x}, t)$ is the peculiar velocity with respect to the general expansion. Note that the mass density is written as

$$\rho = \rho_m(t)[1 + \delta(\mathbf{x}, t)]. \tag{31}$$

In this context, using Eqs. (4), (29), and (30) and neglecting second order terms ($\delta \ll 1$ and $v \ll u$), it is routine to rewrite Eqs. (24)–(26)

$$\ddot{a}\mathbf{x} + \frac{\partial \mathbf{v}}{\partial t} + H\mathbf{v} = -\frac{\nabla \Phi}{a}, \tag{32}$$

$$\nabla \cdot \mathbf{v} = -a\left(\frac{\partial \delta}{\partial t} + \frac{\psi \delta}{\rho_m}\right), \tag{33}$$

$$\frac{1}{a^2}\nabla^2 \Phi = 4\pi G \rho_m(1 + \delta) - \Lambda_{\text{eff}}. \tag{34}$$

Following the notations of [58], we write the gravitational potential as

$$\Phi = \phi(\mathbf{x}, t) + \tfrac{2}{3}\pi G\rho_m a^2 x^2 - \tfrac{1}{6}\Lambda_{\text{eff}} a^2 x^2. \quad (35)$$

Thus utilizing Eqs. (1), (5), and (27) we derive after some algebra that

$$\frac{\partial \mathbf{v}}{\partial t} + H\mathbf{v} = -\frac{\nabla \phi}{a}, \quad (36)$$

and

$$\nabla^2 \phi = 4\pi G a^2 \rho_m \delta. \quad (37)$$

Finally, by taking the divergence of Eq. (36) and using Eqs. (33) and (37), we obtain the time evolution equation for the growth factor $D(t) \equiv \delta/A(\mathbf{x})$ [where $A(\mathbf{x})$ is an arbitrary function]:

$$\ddot{D} + (2H + Q)\dot{D} - (4\pi G\rho_m - 2HQ - \dot{Q})D = 0, \quad (38)$$

where

$$Q(t) = \frac{\rho_{\text{dm}}\Gamma}{\rho_m}. \quad (39)$$

Now, it is clear how the matter creation term affects the growth factor via the function $Q(t)$.

It should be noticed that the background creation pressure affects both the global dynamics as well as the fluctuations via $Q(t)$. In the above results, it was also implicitly assumed that the produced particles have negligible velocities as measured by the observers in the comoving framework which implies that any additional internal pressure term is negligible. In fact, this is a reasonable assumption since we are working with a nonrelativistic phase of universe expansion (matter era). The second assumption is that the matter production is strictly homogeneous which implies that possible contributions of $Q(t)$ at the perturbative level are practically zero. Thus, within the context of the latter assumptions, the advantage of employing a neo-Newtonian approach is a gain in simplicity and transparency of all computations.

In order to solve the above differential equation we need to know the functional form of the quantity $Q(t)$ (or Γ). In the case of a negligible matter creation rate $\Gamma \ll H$ [$Q(t) \sim 0$], the above equation reduces to the usual time evolution equation for the mass density contrast [58] for which the growth factor is $D_{EdS}(a) = a$. As one may check, by solving Eq. (38) for the Λ cosmology [$Q(t) = 0$], we also derive the well-known perturbation growth factor (see [58]):

$$D_\Lambda(a) = \frac{5\Omega_m E(a)}{2} \int_0^a \frac{dx}{x^3 E^3(x)}, \quad (40)$$

where $\Omega_m = 1 - \Omega_\Lambda$ is the matter density parameter and $E(a) = H(a)/H_0$ is the normalized Hubble function

$$E(a) = (\Omega_\Lambda + \Omega_m a^{-3})^{1/2}. \quad (41)$$

It is also interesting to mention here that one can explicitly derive Eq. (38) in the framework of cosmological models with a time-varying vacuum energy density [59]. Note that for simplicity throughout the rest of the paper we will use geometrical units ($c = 8\pi G \equiv 1$). In the analytical treatment below, unless explicitly stated, we consider $\Lambda = 0$.

B. Growth of fluctuations in CCDM models: Analytical solutions

Let us now discuss thoroughly the time evolution equation of the mass density contrast in the linear regime in order to test the implications of CCDM models on the structure formation process. In order to illustrate our results with a basic application, we also consider later the formation of galaxy clusters through the so-called Press-Schetcher formalism.

1. Fluctuations in the LSS Model

Using the time evolution equation for the mass density contrast as given by (38), we now derive the growth factor of fluctuations for LSS cosmology. To begin with, we change variables from t to a new one defined by the following transformation:

$$y = e^{nt/2} - 1 \quad \text{with } n = 3\gamma H_0. \quad (42)$$

In this context, using Eqs. (1), (4), (7), (9), (10), (39), and (42) we obtain

$$H(y) = \frac{n(y + 1)}{\mu y}, \qquad \rho_m(y) = \frac{3n^2(y + 1)^2}{\mu^2 y^2},$$

$$Q(y) = \frac{n[3y - (\mu - 3)]}{\mu y}, \qquad \dot{Q}(y) = \frac{n^2(\mu - 3)(y + 1)}{2\mu y^2}, \quad (43)$$

where $\mu = 3(1 - \beta)$. We can now rewrite Eq. (38) as

$$\mu^2(y + 1)y^2 D'' + \mu y f(y)D' + 2g(y)D = 0, \quad (44)$$

where prime denotes derivatives with respect to y and

$$f(y) = (10 + \mu)(y + 1) - 3\mu$$
$$g(y) = 9(y + 1) - 7\mu + \mu^2. \quad (45)$$

Factors of n drop at the end of the calculation. Notice that this variable is related with the scale factor as

$$y = \frac{3\gamma}{\mu - 3\gamma} a^{\mu/2}. \quad (46)$$

We find that Eq. (44) has a growing solution of the form

$$D(y) = \mathcal{C}y^{(-10+3\mu+\sqrt{\Delta})/2\mu} F\left(\nu_1, \nu_1 + \frac{2\sqrt{7}}{\mu}, \nu_2, -y\right), \quad (47)$$

where

$$\nu_1 = \frac{3}{2} - \frac{\sqrt{7}}{\mu} + \frac{\sqrt{\Delta}}{2\mu} \qquad \nu_2 = 1 + \frac{\sqrt{\Delta}}{\mu}, \qquad (48)$$

$$\Delta = 28 - 4\mu + \mu^2.$$

Notice that the quantity F is the hypergeometric function and $\mathcal{C}$ is a constant. Inserting Eq. (46) into Eq. (47), we obtain the growth factor $D(a)$ as a function of the scale factor:

$$D(a) = C_1 a^{(-10+3\mu+\sqrt{\Delta})/4} F\left(\nu_1, \nu_1 + \frac{2\sqrt{7}}{\mu}, \nu_2, -\frac{3\gamma a^{\mu/2}}{\mu - 3\gamma}\right), \qquad (49)$$

where

$$C_1 = \mathcal{C}\left(\frac{3\gamma}{\mu - 3\gamma}\right)^{(-10+3\mu+\sqrt{\Delta})/2\mu}. \qquad (50)$$

2. Fluctuations in LJO models

Let us now proceed further to solve analytically the differential Eq. (38) in order to investigate the matter fluctuation field of the LJO model in the linear regime. To do so, we change variables from t to y according to the transformation

$$y = \coth\left(\frac{3H_0\sqrt{\tilde{\Omega}_\Lambda}}{2}t\right). \qquad (51)$$

Using (51) we find, after some calculations, that Eq. (38) takes on the form

$$3y^2(y^2 - 1)^2 D'' + 2yf(y)D' - 2g(y)D = 0, \qquad (52)$$

where

$$f(y) = (y^2 - 1)(y^2 - 3), \qquad g(y) = (y^4 - 7y^2 + 3). \qquad (53)$$

Note that factors of H_0 drop at the end of the calculation.

In deriving Eq. (52) we have substituted the various terms in (38) as a function of the new variable [see Eq. (51)]. For instance, from Eqs. (1), (16), and (39), we have

$$H(y) = \sqrt{\tilde{\Omega}_\Lambda}H_0 y, \qquad \rho_m(y) = 3\tilde{\Omega}_\Lambda H_0^2 y^2,$$

$$Q(y) = \frac{3\sqrt{\tilde{\Omega}_\Lambda}H_0}{y}, \qquad \dot{Q}(y) = \frac{9\sqrt{\tilde{\Omega}_\Lambda}H_0(y^2 - 1)}{2y^2}. \qquad (54)$$

In this framework, the differential equation (52) can be brought into the following expression:

$$6x^2(x + 1)^2 \frac{d^2D}{dx^2} + xq(x)\frac{dD}{dx} - b(x)D = 0, \qquad (55)$$

$$q(x) = (x + 1)(5x - 4)b(x) = (x^2 - 5x - 3), \qquad (56)$$

where

$$x = y^2 - 1 = \left(\frac{\tilde{\Omega}_m}{\tilde{\Omega}_\Lambda}\right)a^{-3}, \qquad x + 1 = y^2 = \frac{E^2(a)}{\tilde{\Omega}_\Lambda}, \qquad (57)$$

in which we have used Eqs. (16), (17), and (19).

The solution of the main differential equation (55) is

$$D(x) = \mathcal{C}\frac{x^{(5-\sqrt{7})/6}}{x + 1} F\left[\kappa, \kappa + \frac{5}{6}, 2\left(\kappa + \frac{7}{6}\right), -x\right], \qquad (58)$$

where $\kappa = -(4 + \sqrt{7})/6$ and $\mathcal{C}$ is a constant. Inserting Eq. (57) into Eq. (58), we finally obtain the growth factor $D(a)$ as a function of the scale factor:

$$D(a) = C_1 \frac{a^{-(5-\sqrt{7})/2}}{E^2(a)} F\left[\kappa, \kappa + \frac{5}{6}, 2\left(\kappa + \frac{7}{6}\right), -\frac{\tilde{\Omega}_m}{\tilde{\Omega}_\Lambda}a^{-3}\right], \qquad (59)$$

where

$$C_1 = \mathcal{C}\tilde{\Omega}_\Lambda\left(\frac{\tilde{\Omega}_m}{\tilde{\Omega}_\Lambda}\right)^{(5-\sqrt{7})/6}. \qquad (60)$$

We would like to stress here that the above solution of the growth factor is valid when $\tilde{\Omega}_\Lambda \neq 0$ (or $\tilde{\Omega}_m \neq 1$).

C. Linear growth: Analysis and summary

Let us now discuss the evolution of the growth factor as a function of the redshift [$D(z)$, $z = a^{-1} - 1$] as predicted by the two CCDM cosmologies [see Eqs. (49) and (59)]. For comparison, we also present the growth factor provided by the standard ΛCDM cosmology [see Eq. (40)].

In Fig. 1 we display the basic results. First, we notice that the growth factors are normalized to unity at the present time ($z = 0$). It is also evident that the LSS growth factor (dashed line) is much greater with respect to the other two models, LJO (dotted line) and Λ (solid line). In this framework, we find that for $z \leq 0.8$ the LSS growth factor (dashed line hereafter LSS$_1$) starts to decay, implying that cosmic structures cannot be formed via gravitational instability. The same general behavior seems to hold also for the LSS model with $\beta = 0$ and $\gamma = 0.66$ (dot-dashed line hereafter LSS$_2$).

Let us now compare the predictions of LJO and ΛCDM growth factors. First, we stress a remarkable result: despite the fact that the two models share the same global dynamics [compare Eqs. (19) and (41)], they trace differently the evolution of the matter fluctuation field. In particular, close to the present epoch ($z < 0.4$) the LJO growth factor (dot-dashed line) reaches almost a plateau, which means that the matter fluctuations are effectively frozen. Between $0.4 \leq z < 1.6$, the growth factor in the LJO model is greater than that of the concordance ΛCDM cosmology. In particular,
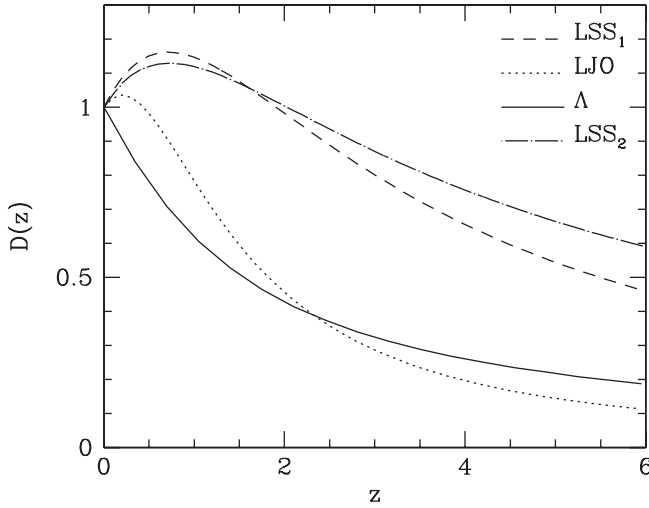
FIG. 1. The evolution of the growth factor. The lines correspond to LSS_1 [dashed line, $(\beta, \gamma) = (-0.15, 0.86)$], LSS_2 [dot-dashed line, $(\beta, \gamma) = (0, 0.66)$], LJO (dotted line), and Λ cosmology (solid line). Note that the growth factors of all models were scaled to unity at the present time. Despite the fact that LJO and ΛCDM models share the same global dynamics, we see that they trace differently the evolution of the matter fluctuation field (compare the dotted and solid lines).

the corresponding deviation $(1 - D/D_\Lambda)\%$ lies in the interval $[-35, -19]\%$. As an example, prior to the inflection point ($z_I \sim 0.72$) we find $\sim -28\%$.

It is worth noting that in the interval of $1.6 \leq z \leq 2.8$ the LJO growth factor tends to the Λ solution. Indeed, at the epoch of $z \sim 1.62$ ($a \sim 0.38$), in which the most distant cluster has been found [60], the deviation $(1 - D/D_\Lambda)\%$ of the growth factor $D(z)$ for the LJO scenario with respect to the Λ solution $D_\Lambda(z)$ is -5%. To this end, for $z > 2.8$ we find that $D(z) < D_\Lambda(z)$ and the corresponding deviation $(1 - D/D_\Lambda)\%$ remains close to $\sim 28\%$. Naturally, one may expect that the differences among the growth factors will affect the predictions related with the formation of the cosmic structures (see the next section). In particular, such results point to the possibility of a crucial observational test confronting CCDM with ΛCDM scenarios in a perturbative level.

VI. THE FORMATION AND EVOLUTION OF COLLAPSED STRUCTURES

In this section we study the cluster formation processes in CCDM cosmologies by using the standard Press-Schechter formalism [61]. In such an approach, the behavior of the matter perturbations is described by assuming that the density contrast behaves like a Gaussian random field. As it is widely known, the cluster distribution basically traces scales that have not yet undergone the non-linear phase of gravitationally clustering, thus simplifying their connections to the initial conditions of cosmic structure formation.

The basic aim here is to estimate the fractional rate of cluster formation (see [62,63]). In particular, these studies introduce a methodology which computes the rate at which mass joins virialized halos (such as galaxy clusters) which grow from small initial perturbations in the universe, with matter fluctuations δ greater than a critical value δ_c.

Now, by assuming that the density fluctuation field, smoothed at the scale R (corresponding to a mass scale of $M = 4\pi\bar{\rho}R^3/3$, with $\bar{\rho}$ the mean background mass density of the Universe), is normally distributed with zero mean, then the probability that the field will have a value δ at any given point in space is

$$\mathcal{P}(\delta, z) = \frac{1}{\sqrt{2\pi}\sigma(R, z)} \exp\left[-\frac{\delta^2}{2\sigma^2(R, z)}\right], \qquad (61)$$

where the variance of the Gaussian field $\sigma^2(R, z)$ is given by

$$\sigma^2(R, z) = \frac{1}{2\pi^2} \int_0^\infty k^2 P(k, z) W^2(kR) dk, \qquad (62)$$

with $P(k, z)$ the power spectrum of density fluctuations which evolves according to $P(k, z) = P(k, 0)D^2(z)$, with $D(z)$ the growing mode of the density fluctuations evolution, normalized such that $D(0) = 1$.

Finally, $W(kR)$ is the top-hat smoothing kernel, given in Fourier space by

$$W(kR) = \frac{3}{(kR)^3}[\sin(kR) - kR\cos(kR)]. \qquad (63)$$

We can now estimate what fraction of the Universe, at some reference redshift z and above some mass threshold M, has collapsed to form bound structures. To this end we need to integrate the probability function given by Eq. (61) over all regions that at some prior redshift had overdensities which by the reference redshift have increased to above the critical value $\delta_c(z)$, which in an Einstein-de Sitter universe is $\simeq 1.686$ and varies slightly for different values of Ω_m [64,65]. Therefore, this fraction is given by [28,63,66,67]

$$\mathcal{F}(M, z) = \int_{\delta_{c(z)}}^\infty \mathcal{P}(\delta, z) d\delta, \qquad (64)$$

and performing the above integration, parametrizing the rms mass fluctuation amplitude at $R = 8h^{-1}$ Mpc which can be expressed as a function of redshift as $\sigma(M, z) = \sigma_8(z) = D(z)\sigma_8$, we obtain

$$\mathcal{F}(z) = \frac{1}{2}\left[1 - \text{erf}\left(\frac{\delta_c}{\sqrt{2}\sigma_8(z)}\right)\right]. \qquad (65)$$

It is worth noticing that the above generic form of Eq. (65) is heavily dependent on the choice of the background cosmology and the matter power spectrum normalization, σ_8. For the sake of comparison, in what follows we consider two values σ_8. The first is the WMAP5 result $\sigma_8 \simeq 0.80$ [3], whereas the second is $\sigma_8 \simeq 0.95$.
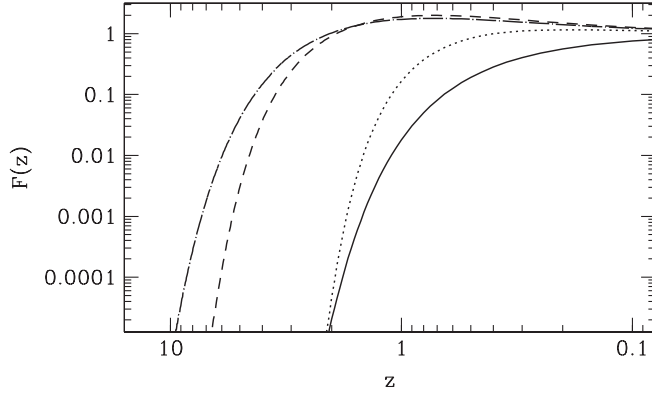
FIG. 2.   The predicted fractional rate of cluster formation as a function of redshift for $\sigma_8 \simeq 0.80$. The lines represent (a) LSS_1 [dashed line, $(\beta, \gamma) = (-0.15, 0.86)$], (b) LSS_2 [dot-dashed line, $(\beta, \gamma) = (0, 0.66)$], (c) LJO (dotted line), and (d) Λ cosmology (solid line). Note that the clusters in LSS-type models form earlier ($z \sim 7 - 10$) in comparison to those produced in the framework of the LJO and ΛCDM models ($z \sim 2$).
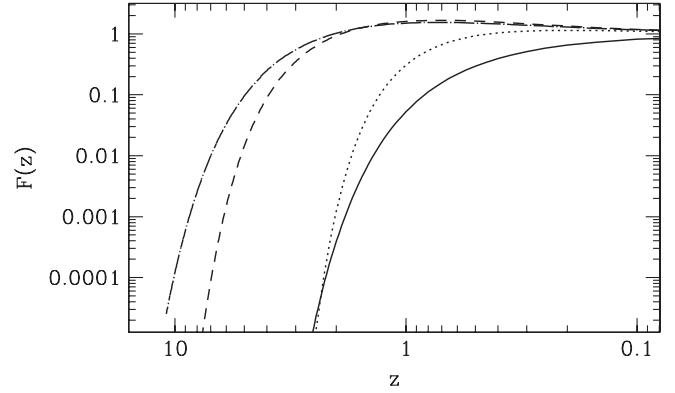

FIG. 3.   The predicted fractional rate of cluster formation as a function of redshift for using a higher value for the normalization of the power spectrum, $\sigma_8 \simeq 0.95$. The meaning of the various lines is the same as Fig. 2. We see that for a larger value of σ_8 the corresponding cluster formation rate is displaced to higher redshifts (compare to Fig. 2).

Finally, we normalize the above probability to give the fraction of mass in bound structures which have already collapsed by the epoch z, divided by the corresponding fraction in structures which have collapsed at the present epoch ($z = 0$),

$$\tilde{F}(z) = \mathcal{F}(z)/\mathcal{F}(0). \qquad (66)$$

In Fig. 2, by assuming the matter power spectrum normalization, $\sigma_8 \simeq 0.80$, we present in a logarithmic scale the behavior of normalized structure formation rate as a function of redshift for the present cosmological models.

In the context of the LSS_1 matter creation model (see dashed line), we find that prior to $z \sim 0.8$ (or $z \sim 1.1$ for the LSS_2 model, dot-dashed line) the cluster formation has effectively terminated due to the fact that the matter fluctuation field, $D(z)$, effectively overs (see Sec. V C). Also, the large amplitude of the LSS fluctuation field (see also Fig. 1) implies that in this model galaxy clusters appear to form much earlier ($z \sim 7$) with respect to the LJO (dot-dashed) and Λ (solid line) cosmological models. Indeed, for the latter cosmological models we find that galaxy clusters formed typically at $z \sim 2.2$. From the observational point of view, it is interesting to point out here that the most distant cluster has been found at $z \sim 1.62$ [60]. We would like to stress that the LSS_2 predicts an early rapid cluster formation that takes place at the redshift of reionization ($z \simeq z_{\text{reion}} \sim 10$), at which the universe was reionized from the neutral state to a fully ionized state. Obviously, this result rules out the LSS_2 model.

We also stress that the amount of cluster abundances between the two models remains almost the same for $1.6 \lesssim z \lesssim 2.2$. This is to be expected due to the fact that the two cosmological models have almost the same growth factors (see also Sec. V C). Then for $z < 1.6$ the LJO cluster

formation rate becomes larger than that of the concordance model and finally it terminates for $z \lesssim 0.4$.

In Fig. 3 we also display the normalized structure formation rate as a function of redshift, but now for a higher value of the power spectrum parameter ($\sigma_8 = 0.95$). Comparing Figs. 2 and 3, we see that the basic effect of a larger value of σ_8 is that the corresponding cluster formation rate moves to higher redshifts.

VII. CONCLUSIONS

In this work, we have investigated (analytically and numerically) the overall dynamics of two cosmological models in which the dark matter creation process provides the late time accelerating phase of the cosmic expansion without the need of dark energy. Such scenarios termed here by LSS [46] and LJO [48] are phenomenologically characterized by two distinct creation rates, Γ (see Sec. III). In the first scenario (LSS), the creation rate is $\Gamma = 3\gamma H_0 + 3\beta H$ while in the second one it is proportional to the Hubble parameter; namely, $\Gamma = 3\tilde{\Omega}_\Lambda (\rho_{\text{co}}/\rho_{\text{dm}})H$, where $\rho_{\text{co}} = 3H_0^2/8\pi G$ is the present-day value of the critical density.

It should also be stressed that the phenomenological approach adopted here cannot determine the mass of the particles, as well as whether their nature is fermionic or bosonic. In order to access the mass of the particles, and, therefore, the nature of the CDM particles, it is necessary to determine the creation rate, Γ, from quantum field theory in FLRW space time. In principle, such a treatment must somewhat incorporate a source of entropy since the matter creation process is truly an irreversible process. In particular, in the case of adiabatic matter creation considered here, both the entropy (S) and the number of particles (N) in a comoving volume increase but the specific entropy (S/N) remains constant [36,37,43].

In this context, by using current observational data (BAOs, CMB shift parameter, and SNIa) we have first performed a joint likelihood analysis in order to put tight constraints on the main cosmological parameters. Subsequently, through a linear analysis we have also studied the growth factor of density perturbations for both classes of creation cold dark matter models, and, finally, by using the Press-Schechter formalism we have discussed how the cluster formation rates evolve in such scenarios.

The result of our joint statistical analysis shows that the fit provided by the LSS model with $\gamma = 0.66$ and $\beta = 0$ turns out to be of poor quality because it is unable to adjust simultaneously the observational data at low and high redshift. This result confirms the conclusion by Steigman *et al.* [47] using only SNe Ia and the determination of $z_{\rm eq}$, the redshift of the epoch of matter radiation equality. For the LSS scenario, we also find that the amplitude and the shape of the linear density contrast are significantly different with respect to those predicted by the LJO and Λ models (see Fig. 1). In particular, for $z \leq 0.8$ the matter fluctuation field of the LSS model practically decays thereby implying that the corresponding cosmic structures cannot be formed via gravitational instability (see also Figs. 2 and 3).

On the other hand, the creation cold dark matter scenario (termed here as LJO) provides good quality fits of the cosmological parameters at all redshifts and it resembles the global dynamics of the concordance ΛCDM cosmology by including only one free parameter. Despite the latter, the corresponding growth factor has the following evolution with respect to that of the usual Λ cosmology: (i) prior to the present epoch ($z < 0.4$) the evolution of the LJO growth factor reaches almost a plateau, which means that the matter fluctuations are effectively frozen; (ii) between $0.4 \leq z < 1.6$, the growth factor in the LJO model is greater than that of the concordance Λ cosmology, while for $1.6 \leq z \leq 2.8$ the two growth factors have converged; and (iii) for $z > 2.8$ we find that $D(z) < D_{\Lambda}(z)$ and the corresponding deviation $(1 - D/D_{\Lambda})\%$ remains close to $\sim 28\%$.

Summarizing, in the case of LSS scenario, the large-scale structures (such as galaxy clusters) form earlier ($z \sim 7 - 10$) with respect to those produced in the framework of the LJO and ΛCDM models ($z \sim 2$). Therefore, in view of the recent cluster observational data, the latter are much more favored as compared to the former. Our basic conclusion is that the LJO creation cold dark matter model which has only one free parameter passes all the tests considered here. However, new constraints from complementary observations need to be investigated in order to see whether this kind of scenario provides a realistic description of the observed Universe.

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