PHYSICAL REVIEW D **82,** 016007 (2010)

Location and direction dependent effects in collider physics from noncommutativity

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(Received 16 June 2010; published 28 July 2010)

We examine the leading order noncommutative corrections to the differential and total cross sections for $e^+e^- \rightarrow q\bar{q}$. After averaging over the Earth's rotation, the results depend on the latitude for the collider, as well as the direction of the incoming beam. They also depend on the scale and direction of the noncommutativity. Using data from LEP, we exclude regions in the parameter space spanned by the noncommutative scale and angle relative to the Earth's axis. We also investigate possible implications for phenomenology at the future International Linear Collider.

DOI: 10.1103/PhysRevD.82.016007 PACS numbers: 11.10.Nx, 13.66.−a

I. INTRODUCTION

Motivated in part by quantum gravity [1], it has been of recent interest to examine field theories, and, in particular, the standard model of particle physics, on noncommutative space-time backgrounds. Noncommutative versions of the standard model have been proposed, which have the potential of explaining its gauge algebra and fermion representations [2]. Two popular noncommutative generalizations of the standard model were given by Chaichian *et al.* [3] and Calmet *et al.* [4]. In both cases the geometry is generated by Heisenberg algebras, i.e.,

$$[\mathbf{x}^\mu, \mathbf{x}^\nu] = i\Theta^{\mu\nu}, \qquad (1.1)$$

where $\mathbf{x}^\mu$, $\mu, \nu, \ldots = 0, 1, 2, 3$, are operator analogues of space-time coordinates and $\Theta^{\mu\nu} = -\Theta^{\nu\mu}$ are central elements which are independent of $\mathbf{x}^\mu$. Also, both approaches rely on the Moyal-Weyl star product realization of the algebra. One often distinguishes two cases: space-space noncommutativity associated with Θ^{ij}, $i, j, k, \ldots = 1, 2, 3$, and time-space noncommutativity associated with Θ^{0i}. Then two noncommutative energy scales $\Lambda_{\rm SS}$, $\Lambda_{\rm TS}$, and two unit vectors v_i, w_i can be defined using

$$\Theta^{ij} = \frac{1}{2\Lambda_{\rm SS}^2}\epsilon_{ijk}v_k \qquad \Theta^{0i} = \frac{1}{\Lambda_{\rm TS}^2}w_i. \qquad (1.2)$$

Bounds on $\Lambda_{\rm SS}$ and $\Lambda_{\rm TS}$ have been obtained from atomic physics, collider physics, and astrophysics. (See, for example, [5].) v_i and w_i correspond to fixed directions in space. If the noncommutative scales become accessible in collider physics, then information about these directions may be obtainable, as we shall illustrate in this article.

We first make some brief remarks about the two approaches to the noncommutative standard model mentioned above. In the approach of [3], one enlarges the standard model gauge group to the noncommutative analogue of $U(3) \times U(2) \times U(1)$, thus introducing gauge bosons in addition to those of the standard model. New symmetry breakings and Higgs scalars are then also required. The model was shown to be one-loop renormalizable [6]. The approach of [4] does not involve introducing any additional gauge bosons or symmetry breakings. It instead relies upon a map, known as the Seiberg-Witten map [7], between commutative and noncommutative gauge theories, from which one can obtain corrections to the standard model interactions [8]. These corrections have been computed up to second order in $\Theta^{\mu\nu}$ [9]. In contrast with the former model, one-loop corrections are not well understood in this approach. Although the model is anomaly free up to one-loop order [10], only the pure gauge sector of this theory has been shown to be renormalizable at this order [11].

Here we shall follow the approach of [4], and obtain all leading noncommutative corrections to the differential and total cross sections for the example of $e^+e^- \rightarrow q\bar{q}$ at tree level. These corrections are second order in Θ^{0i}. Assuming that w_i in (1.2) corresponds to a fixed direction relative to some frame external to the earth, and not the lab frame, we must average over the Earth's rotation. (Presumably, other effects due to the Earth's motion relative to w_i are much smaller.) Results for the cross sections then depend on the latitude for the collider, as well as the direction of the incoming beam. We can obtain an analytic expression for the averaged leading order noncommutative correction to the total cross section in terms of these quantities, along with the noncommutative scale $\Lambda_{\rm TS}$, and the projection w_Z of w_i along the Earth's axis. Using data from LEP, we are then able to exclude regions in the parameter space spanned by $\Lambda_{\rm TS}$ and w_Z for different detectors. Here we also investigate possible implications of the noncommutative corrections for phenomenology at the International Linear Collider (ILC), a future high energy e^+e^- linear collider with $\sqrt{s} = 500$ GeV–1 TeV. Since the corrections depend on both the location of the ILC and its beam direction, there can be an optimal site and beam direction for observing noncommutative effects.

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Before proceeding, we comment on several works where noncommutative corrections for the annihilation of e^+e^- to fermion-antifermion pairs, and where earth rotational effects in collider physics were already considered. $e^+e^- \to e^+e^-$ was examined in [12] in a version of noncommutative QED that did not rely on the Seiberg-Witten map. The calculations =included Z-boson exchange, although the Z-boson vertex was not obtained from a noncommutative standard model Lagrangian. Electron-positron scattering was reconsidered in [13] within the framework of the noncommutative standard model [4], which gives a specific form for the Z-boson vertex. Electron scattering was examined in another approach to noncommutative QED in [14]. Earth rotation effects were not taken into account in these works. Such effects were considered for collider physics in [15], using the example of Higgs pair production. Earth rotation effects were also illustrated in [16] for $e^-e^+ \to \gamma\gamma$ and in [17] for $e^+e^- \to \mu^+\mu^-$. The investigations in [15–17] were conducted within the context of noncommutative QED, and Z-boson exchange was not included. Scattering amplitudes were only expanded up to first order in $\Theta^{\mu\nu}$ in the above mentioned works, although the leading order corrections to the cross section for e^+e^- to fermion-antifermion pairs are quadratic. So additional terms can contribute to the cross section at this order. In our work, upon applying the noncommutative standard model of [4], and not just noncommutative QED, we shall include all second order contributions to the scattering amplitude, in addition to taking into account Earth rotational effects.

The outline for the rest of this article is as follows: In Sec. II we compute the noncommutative corrections for photon and Z-boson exchange. We average over the Earth's rotation in Sec. III and apply the results to LEP and ILC in Sec. IV.

II. APPLICATION OF THE NONCOMMUTATIVE STANDARD MODEL

We first give the noncommutative Feynman rules for the relevant vertices γff and Zff up to second order in $\Theta^{\mu\nu}$. There are no noncommutative corrections to the propagators when one uses the Moyal-Weyl star product.

The Feynman rule for vertex γff is given by

$$V_\mu^{\gamma ff} = ieQ_f\Gamma_\mu, \tag{2.1}$$

where Q_f denotes the fermion charge and one can expand Γ_μ in a power series in the noncommutativity tensor $\Theta^{\mu\nu}$,

$$\Gamma_\mu = \gamma_\mu + \Gamma_\mu^{(1)} + \Gamma_\mu^{(2)} + \ldots. \tag{2.2}$$

Here and in what follows, the dots denoting terms that are more than second order in $\Theta^{\mu\nu}$. The first and second order terms were found in [8,9], respectively, to be

$$\begin{aligned}\Gamma_\mu^{(1)} &= \frac{i}{2}[(k\Theta)_\mu \not{p}_{\rm in}(1-4c_\psi^{(1)}) + 2(k\Theta)_\mu \not{k}(c_A^{(1)} - c_\psi^{(1)}) \\ &\quad - (p_{\rm in}\Theta)_\mu \not{k} - (k\Theta p_{\rm in})\gamma_\mu], \\ \Gamma_\mu^{(2)} &= \frac{1}{8}(k\Theta p_{\rm in})[(k\Theta)_\mu \not{p}_{\rm in}(1-16c_\psi^{(2)}) \\ &\quad + 4(k\Theta)_\mu \not{k}(c_A^{(1)} - 2c_\psi^{(2)}) - (p_{\rm in}\Theta)_\mu \not{k} - (k\Theta p_{\rm in})\gamma_\mu],\end{aligned} \tag{2.3}$$

where we ignore the fermion mass and take all momenta, k, $p_{\rm in}$, and $p_{\rm out}$, to be incoming. $p_{\rm in}$ and $p_{\rm out}$ are associated with incoming and outgoing fermions, respectively. We have adopted the notation $(k\Theta)_\mu = k^\nu\Theta_{\nu\mu}$ and $(k\Theta p) = k^\nu\Theta_{\nu\mu}p^\mu$. $c_\psi^{(1)}$, $c_A^{(1)}$, and $c_\psi^{(2)}$ are arbitrary constants which originate from ambiguities in the Seiberg-Witten map. They do not appear in the vertex when the incoming and outgoing fermions are evaluated on-shell (at energies well above the fermion mass). In that case, the vertex reduces to

$$V_\mu^{\gamma ff}|_{\text{on-shell}} = ieQ_f\gamma_\mu \mathcal{I}(p_{\rm out}, p_{\rm in}), \tag{2.4}$$

where

$$\mathcal{I}(p_{\rm out}, p_{\rm in}) = 1 + \frac{i}{2}(p_{\rm out}\Theta p_{\rm in}) + \frac{1}{8}(p_{\rm out}\Theta p_{\rm in})^2 + \ldots. \tag{2.5}$$

The sign in front of the second term changes upon switching the ingoing momenta k to outgoing.

The Feynman rule for vertex Zff is obtained by replacing $Q_f\gamma_\mu$ in (2.4) by $(c_{V,f} - c_{A,f}\gamma_5)\gamma_\mu/\sin 2\theta_W$, where

$$\begin{aligned}c_{V,f} &= \tfrac{1}{2}(c_{L,f} + c_{R,f}) = T_{3,f} - 2Q_f\sin^2\theta_W, \\ c_{A,f} &= \tfrac{1}{2}(c_{L,f} - c_{R,f}) = T_{3,f},\end{aligned} \tag{2.6}$$

θ_W is the Weinberg angle and $T_{3,f}$ denotes the fermion weak isospin. The on-shell vertex is then

$$V_\mu^{Zff}|_{\text{on-shell}} = \frac{ie}{\sin 2\theta_W}(c_{V,f} - c_{A,f}\gamma_5)\gamma_\mu \mathcal{I}(p_{\rm out}, p_{\rm in}). \tag{2.7}$$

Up to second order, both noncommutative on-shell vertices (2.4) and (2.7) are related to the commutative on-shell vertices by the same factor $\mathcal{I}(p_{\rm out}, p_{\rm in})$. It follows that noncommutative scattering amplitudes for $e^-e^+ \to q\bar{q}$ associated with γ and Z exchanges are related to their commutative counterparts by a common factor. Then the total noncommutative scattering amplitude at tree level $\mathcal{M}^{\mathcal{NC}}$ is related to total commutative scattering amplitude $\mathcal{M}$ by

$$\mathcal{M}^{\mathcal{NC}} = \mathcal{I}(p_{\rm out}, p_{\rm in})^* \mathcal{I}(p'_{\rm out}, p'_{\rm in})\mathcal{M}, \tag{2.8}$$

where the primed momenta are associated with the created fermions, and $\mathcal{M}$ is the corresponding standard model amplitude. The leading noncommutative corrections to the squared-amplitude are second order in $\Theta^{\mu\nu}$,

$$|\mathcal{M}^{\mathcal{NC}}|^2 = \left[1 + \frac{1}{2}(p_{\text{out}}\Theta p_{\text{in}})^2 + \frac{1}{2}(p'_{\text{out}}\Theta p'_{\text{in}})^2 + \cdots\right]|\mathcal{M}|^2. \tag{2.9}$$

Only the space-time components of $\Theta_{\mu\nu}$ contribute in the center-of-mass frame for beam on beam scattering, where

$$p_{\text{in}} = \left(\frac{\sqrt{s}}{2}, \vec{p}\right), \qquad p_{\text{out}} = \left(\frac{\sqrt{s}}{2}, -\vec{p}\right), \tag{2.10}$$

and similarly,

$$p'_{\text{in}} = \left(\frac{\sqrt{s}}{2}, \vec{p}'\right), \qquad p'_{\text{out}} = \left(\frac{\sqrt{s}}{2}, -\vec{p}'\right). \tag{2.11}$$

Then using (1.2),

$$(p_{\text{out}}\Theta p_{\text{in}}) = \frac{\sqrt{s}}{\Lambda_{\text{TS}}^2}\vec{p}\cdot\vec{w}, \qquad (p'_{\text{out}}\Theta p'_{\text{in}}) = \frac{\sqrt{s}}{\Lambda_{\text{TS}}^2}\vec{p}'\cdot\vec{w}, \tag{2.12}$$

where $\vec{w} = \{w_i\}$. In terms of unit vectors $\hat{p} = \vec{p}/|p|$ and $\hat{p}' = \vec{p}'/|p'|$, one can then write the noncommutative differential cross section $d\sigma^{\mathcal{NC}}/d\Omega$ for $e^+e^- \to q\bar{q}$ according to

$$\frac{d\sigma^{\mathcal{NC}}}{d\Omega} = \left[1 + \frac{1}{8}\left(\frac{s}{\Lambda_{\text{TS}}^2}\right)^2\{(\hat{p}\cdot\vec{w})^2 + (\hat{p}'\cdot\vec{w})^2\} + \cdots\right] \times \frac{d\sigma}{d\Omega}, \tag{2.13}$$

where $d\sigma/d\Omega$ is the standard model differential cross section. This expression is valid at lowest order in perturbation theory provided that $\Lambda_{\text{TS}} \gtrsim \sqrt{s}$. The standard model differential cross section and total cross section σ_{total} are well known [18]

$$\frac{d\sigma}{d\Omega} = \frac{N_c\alpha^2 s}{16}\{F(s)(1+\cos\beta)^2 + G(s)(1-\cos\beta)^2\}, \tag{2.14}$$

$$\sigma_{\text{total}} = \frac{N_c\alpha^2\pi s}{3}(F(s) + G(s)), \tag{2.15}$$

where β denotes the scattering angle, $N_c = 3$ is the number of colors,

$$F(s) = |A_{LL}(s)|^2 + |A_{RR}(s)|^2, \quad G(s) = |A_{LR}(s)|^2 + |A_{RL}(s)|^2, \tag{2.16}$$

$$A_{ij}(s) = \frac{Q_e Q_f}{s} + \frac{c_{i,e}c_{j,f}}{\sin^2\theta_W\cos^2\theta_W}\frac{1}{s - M_Z^2 - iM_Z\Gamma_Z}, \quad i, j = L, R, \tag{2.17}$$

and we have included the correction due to the decay width Γ_Z for Z.

III. EARTH ROTATIONAL EFFECTS

Now we take into account Earth rotational effects. This is necessary since $\hat{p}$ and $\hat{p}'$ are defined in the lab frame, while $\vec{w}$ is a fixed direction in space, and so the Earth's rotation implies that the scalar products appearing in (2.13) are not constant. As was reported in [15], this can lead to a day-night asymmetry in the cross section. Since such time-dependent experimental data are not readily available, we shall average $(\hat{p}\cdot\vec{w})^2$ and $(\hat{p}'\cdot\vec{w})^2$ in (2.13) over a full day.

Following [19], denote by $(\hat{X}, \hat{Y}, \hat{Z})$ a nonrotating basis, with $\hat{Z}$ parallel to the Earth's axis along the north direction. To a good approximation, this basis spans an inertial frame. The transformation to a basis $(\hat{x}, \hat{y}, \hat{z})$ attached to a point on the Earth's surface at any time t was given by

$$\begin{pmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{pmatrix} = \begin{pmatrix}\cos\chi\cos\Omega t & \cos\chi\sin\Omega t & -\sin\chi\\ -\sin\Omega t & \cos\Omega t & 0\\ \sin\chi\cos\Omega t & \sin\chi\sin\Omega t & \cos\chi\end{pmatrix}\begin{pmatrix}\hat{X}\\ \hat{Y}\\ \hat{Z}\end{pmatrix}, \tag{3.1}$$

where Ω is the Earth's sidereal frequency and $0 \le \chi \le \pi$. To identify the directions $(\hat{x}, \hat{y}, \hat{z})$, consider the cases $\chi = 0$ and $\chi = \pi/2$, which we identify with the North Pole and Equator, respectively. $\hat{z} \parallel \hat{Z}$ when $\chi = 0$, and therefore, $\hat{z}$ points normal to the Earth's surface. $\hat{x}$ is antiparallel to $\hat{Z}$ when $\chi = \pi/2$, and thus $\hat{x}$ and $\hat{y}$ point south and east, respectively.

Assuming no vertical component to the particle momentum $\vec{p}$ in the lab frame, we have

$$\hat{p} = \cos\phi\hat{x} + \sin\phi\hat{y}. \tag{3.2}$$

Taking $\vec{w} = w_X\hat{X} + w_Y\hat{Y} + w_Z\hat{Z}$, the time average of $(\vec{w}\cdot\hat{p})^2$ is

$$\langle(\hat{p}\cdot\vec{w})^2\rangle = \tfrac{1}{2}(\cos^2\phi\cos^2\chi + \sin^2\phi)(1 - w_Z^2) + \cos^2\phi\sin^2\chi w_Z^2, \tag{3.3}$$

and so the average leading order correction to the standard model differential cross section (2.14) is

$$\left\langle\delta\frac{d\sigma}{d\Omega}\right\rangle = \left(\frac{s}{4\Lambda_{\text{TS}}^2}\right)^2\{(\cos^2\phi\cos^2\chi + \sin^2\phi + \cos^2(\phi+\beta)\cos^2\chi + \sin^2(\phi+\beta))(1-w_Z^2) + 2(\cos^2\phi + \cos^2(\phi+\beta))\sin^2\chi w_Z^2\}\frac{d\sigma}{d\Omega}. \tag{3.4}$$

Note that this correction is always positive. Upon integrating this plus (2.14) over the scattering angle, we obtain the following analytic formula for the average leading order correction to the total cross section (2.15):

$$\langle \delta\sigma_{\rm total}\rangle = \frac{s^2}{5120\Lambda_{\rm TS}^4}\{15\pi r(s)[(3w_Z^2-1)\cos 2\chi - (w_Z^2+1)]\sin 2\phi + 32[2(w_Z^2+1)\cos 2\phi - (5(\cos 2\chi + 1) + 3\cos 2(\phi+\chi))w_Z^2 + (1-3w_Z^2)\cos 2(\phi-\chi) - 5\cos 2\chi + \cos 2(\phi+\chi) + 15]\}\sigma_{\rm total}, \tag{3.5}$$

where

$$r(s) = \frac{F(s)-G(s)}{F(s)+G(s)}. \tag{3.6}$$

IV. NUMERICAL RESULTS FOR COLLIDER PHYSICS

We now apply the results from LEP. Our calculations for the corrections to total cross section of $e^-e^+ \rightarrow q\bar{q}$ can be compared to measurements made at the four detectors ALEPH, DELPHI, OPAL, and L3, which were spaced at 90° degree intervals around the ring. Since (3.5) is unchanged for $\phi \rightarrow \phi + \pi$, only two distinct answers are obtained for the four of the detectors. Below we will apply the results of ALEPH and OPAL.

For $\sqrt{s} \simeq 189$ GeV, we get a contribution of ~10.8 pb to the total standard model cross section $\sigma_{\rm total}$ from $q = u$, c, and ~10.1 pb from $q = d, s, b$, where we have cut the region $|\cos\beta| > 0.95$ from the integration corresponding to data analysis of LEP experiments. The latitude for CERN is 46.234°, corresponding to $\chi = 0.764$ radians.

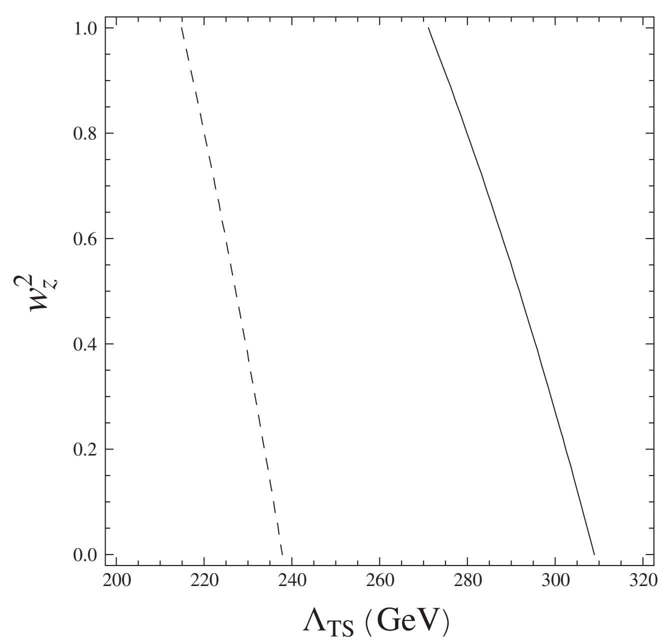

FIG. 1. Constraints on parameters $\Lambda_{\rm TS}$ and w_Z^2 from ALEPH (dashed line) and OPAL (solid line) data. The left-hand side of each of the contours is excluded.

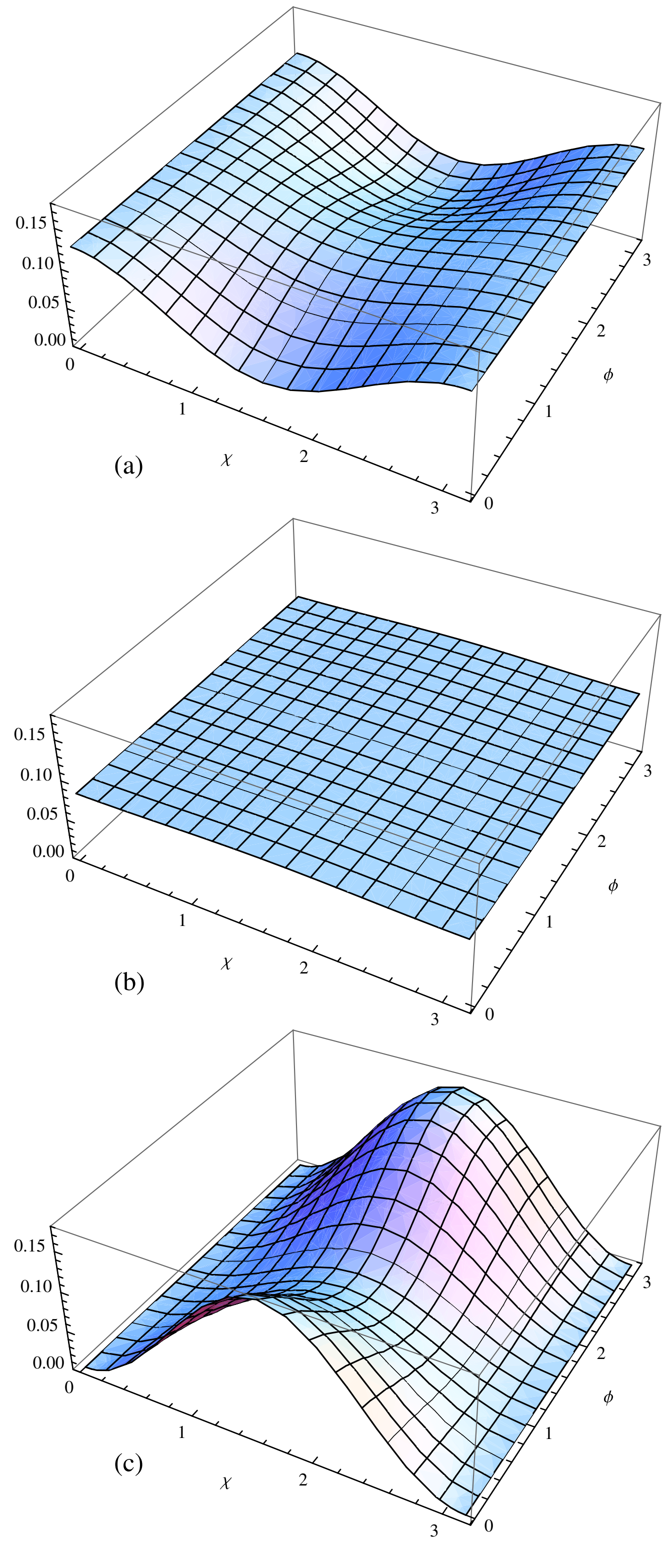

FIG. 2 (color online). The deviations of the total cross section from the standard model one as a function of χ and ϕ for (a) $w_Z^2 = 0$ (top), (b) $w_Z^2 = 0.35$ (middle) and (c) $w_Z^2 = 1.0$ (bottom).

Upon taking $\phi \simeq -\pi/3$ for the ALEPH detector, and summing over all five quark final states, we find the deviation of the total cross section from the standard model one as

$$\frac{\sum_{\text{quarks}} \langle \delta\sigma_{\text{total}} \rangle|_{\text{ALEPH}}}{\sum_{\text{quarks}} \sigma_{\text{total}}} \approx \left(\frac{104.6\ \text{GeV}}{\Lambda_{\text{TS}}}\right)^4 - \left(\frac{79.5\ \text{GeV}}{\Lambda_{\text{TS}}}\right)^4 w_Z^2. \tag{4.1}$$

Similarly, if we take $\phi = -5\pi/6$ for the OPAL detector, we find

$$\frac{\sum_{\text{quarks}} \langle \delta\sigma_{\text{total}} \rangle|_{\text{OPAL}}}{\sum_{\text{quarks}} \sigma_{\text{total}}} \approx \left(\frac{105.3\ \text{GeV}}{\Lambda_{\text{TS}}}\right)^4 - \left(\frac{84.1\ \text{GeV}}{\Lambda_{\text{TS}}}\right)^4 w_Z^2. \tag{4.2}$$

We set (4.1) equal to the error found in the ALEPH results of 3.74% [20] and (4.2) equal to the error found in the OPAL results of 1.35% [21].[1] Fig. 1 shows contour plots for ALEPH (dashed line) and OPAL (solid line) results, respectively. The left-hand side of each of the contours is excluded.

We finally investigate the implications for phenomenology at the ILC with $\sqrt{s} = 500$ GeV–1 TeV. The ILC, with its high precision, can allow us to search noncommutative effects for Λ_{TS} in the range of 1–10 TeV. Since the total cross section depends on both the location of the ILC and its beam direction, there may be an optimal site and beam direction for the ILC for observing noncommutative effects. As an example, we take $\Lambda_{\text{TS}} = 500$ GeV and we calculate the cross section of the process $e^+e^- \to q\bar{q}$ at the ILC, with $\sqrt{s} = 500$ GeV. In Figs. 2(a)–2(c), we show the resulting deviations of the total cross section due to noncommutative effects as a function of χ and ϕ for three different values of w_Z. (Again, we have cut the region $|\cos\beta| > 0.95$.) Figure 2(a), where $w_Z^2 = 0$, shows that the deviation is maximized for an ILC located at the poles ($\chi = 0, \pi$). On the other hand, Fig. 2(c), where $w_Z^2 = 1$, shows the deviation of the cross section is maximized for an ILC located on the Equator ($\chi = \pi/2$) and along the direction to the north ($\phi = 0$). For $w_Z^2 = 1$, the deviation tends to zero for an ILC located at the poles, which is evident from the analytic results (3.4) and (3.5). Aside from other inconveniences, the poles may therefore not be optimal collider sites for seeing noncommutative effects.

[1] Here, we refer to the ALEPH results for $\sqrt{s'/s} > 0.85$ and the OPAL results for $\sqrt{s'/s} > 0.7225$, where s' is the effective center-of-mass energy of e^+e^- collisions. Although the initial state radiations are not taken into account in our analysis, they are not significant for such high cuts on $\sqrt{s'/s}$.

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