Estimates for $X(4350)$ decays from the effective Lagrangian approach

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The strong and electromagnetic decays of $X(4350)$ with quantum numbers $J^P = 0^{++}$ and 2^{++} have been studied by using the effective Lagrangian approach. The coupling constant between $X(4350)$ and $D_s^* D_{s0}^*$ is determined with the help of the compositeness condition which means that $X(4350)$ is a bound state of $D_s^* D_{s0}^*$. Other coupling constants applied in the calculation are determined phenomenologically. Our numerical results show that, using the present data, the possibility that $X(4350)$ is a $D_s^* D_{s0}^*$ molecule cannot be ruled out within the present model.

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I. INTRODUCTION

Recently, a hidden charm resonance named $X(4350)$ was observed by the Belle Collaboration in the analysis of the $\gamma\gamma \to \phi J/\psi$ process [1]. The mass and natural width of this resonance are measured to be $(4350.6^{+4.6}_{-5.1}(\mathrm{stat}) \pm 0.7(\mathrm{syst}))\ \mathrm{MeV}/c^2$ and $(13.3^{+17.9}_{-9.1}(\mathrm{stat}) \pm 4.1(\mathrm{syst}))$ MeV, respectively. The product of its two-photon decay width and branching fraction to $\phi J/\psi$ is $(6.7^{+3.2}_{-2.4}(\mathrm{stat}) \pm 1.1(\mathrm{syst}))$ eV for $J^{PC} = 0^{++}$, or $(1.5^{+0.7}_{-0.6}(\mathrm{stat}) \pm 0.3(\mathrm{syst}))$ eV for $J^{PC} = 2^{++}$. In the literature, the structure of $X(4350)$ has been proposed to be a $c\bar{c}s\bar{s}$ teraquark state with $J^{PC} = 2^{++}$ [2], a $D_s^* D_{s0}^*$ molecular state [3], a P-wave charmonium state χ''_{c2} [4], and a scalar $\bar{c}c$ and $D_s^* \bar{D}_s^*$ mixing state [5]. And concerning the quantum numbers of the final states $J/\psi\phi$, $X(4350)$ can also have quantum numbers $J^{PC} = 1^{-+}$. In Ref. [6], it was shown that $X(4350)$ cannot be a 1^{-+} exotic $D_s^* D_{s0}^*$ molecular state from the QCD sum rule calculation although a dynamical calculation from the potential model indicates that the S-wave $D_s^* D_{s0}^*$ bound state with positive charge parity may exist [7]. In this paper, we will accept it as a bound state of $D_s^* D_{s0}^*$ to study its strong and electromagnetic decays in the effective Lagrangian approach in the case of $J^{PC} = 0^{++}$ and 2^{++}.

Since the mass of $X(4350)$ is about 80 MeV below the threshold of $D_s^* D_{s0}^*$ ($m_{D_{s0}^*} = 2317.8 \pm 0.6$ MeV and $m_{D_s^*} = 2112.3 \pm 0.5$ MeV [8]), it is reasonable to regard it as a bound state of $D_{s0}^* D_s^*$. And because the quantum numbers of D_{s0}^* and D_s^* are $J^P = 0^+$ and $J^P = 1^-$, respectively, to form a bound state with quantum numbers $J^{PC} = 0^{++}$ or 2^{++}, the coupling between $X(4350)$ and its constituents should be a P wave. To determine the effective coupling constant between $X(4350)$ and it constituents $D_s^* D_{s0}^*$, as in our previous work (for example, Ref. [9]), we resort to the compositeness condition $Z_X = 0$ (Z_X as the wave function renormalization constant of $X(4350)$) which was early used in the study of a deuteron as a bound state of a proton and a neutron [10,11] and is being widely used by particle physicists (see the references in [9]). Recently, this method has been applied to study the properties of some "exotic" hadrons [9,12–20] and some conclusions were yielded comparing with data. For other interactions, we write down the general effective Lagrangian and determine the coupling constants with the help of data, theoretical calculations, the $SU(4)$ relation, or the vector meson dominance (VMD).

As in our previous work [9,12–20], we introduce a correlation function including a scale parameter Λ_X to illustrate the distribution of the constituents in the bound state $X(4350)$. The parameter Λ_X is varied to find the physical region where the data can be understood. In the physical region of Λ_X, the partial widths for strong and electromagnetic decays are yielded.

This paper is organized as follows: In Sec. II we will provide the theoretical framework used in this paper. We will present the analytic forms for the radiative and strong decay matrix elements and partial widths of $X(4350)$ in Sec. III. And, the last section is our numerical results and discussions.

II. THEORETICAL FRAMEWORK

In this section, we will propose the theoretical framework for the calculation of the strong and electromagnetic decays of $X(4350)$.

A. The molecular structure of $X(4350)$

As was mentioned above, we regard $X(4350)$ as a $D_s^* D_{s0}^*$ bound state. And concerning the experimental status, we accept the quantum numbers of $X(4350)$ as $J^P = 0^{++}$ and 2^{++}. For the scalar case, one can write the free Lagrangian of $X(4350)$ as

$$\mathcal{L}^S_{\mathrm{free}} = \tfrac{1}{2}\partial_\mu X \partial_\mu X - \tfrac{1}{2} m_X^2 X^2, \qquad (1)$$

with m_X as the mass of $X(4350)$. The propagator of $X(4350)$ can be easily written as

$$G_F(x) = \int \frac{d^4 p}{(2\pi)^4} \frac{i}{p^2 - m_X^2 - i\epsilon} e^{-ip\cdot x}, \qquad (2)$$

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which satisfies

$$(\partial^2 + m^2)G_F(x) = -i\delta^{(4)}(x), \tag{3}$$

while for tensor resonance we have the free Lagrangian as [21]

$$\mathcal{L}^T_{\rm free} = -\tfrac{1}{2}X_{\mu\nu}D^{\mu\nu;\lambda\sigma}X_{\lambda\sigma}, \tag{4}$$

where the symmetric tensor $X_{\mu\nu} = X_{\nu\mu}$ denotes the $J^{PC} = 2^{++}$ field for $X(4350)$ and

$$\begin{aligned} D^{\mu\nu;\lambda\sigma} = {}& (\Box + m_X^2)\{\tfrac{1}{2}(g^{\mu\lambda}g^{\nu\sigma} + g^{\nu\lambda}g^{\mu\sigma}) - g^{\mu\nu}g^{\lambda\sigma}\} \\ & + g^{\lambda\sigma}\partial^\mu\partial^\nu + g^{\mu\nu}\partial^\lambda\partial^\sigma - \tfrac{1}{2}(g^{\nu\sigma}\partial^\mu\partial^\lambda \\ & + g^{\nu\lambda}\partial^\mu\partial^\sigma + g^{\mu\sigma}\partial^\nu\partial^\lambda + g^{\mu\lambda}\partial^\nu\partial^\sigma). \end{aligned} \tag{5}$$

The propagator for $X_{\mu\nu}(4350)$ is obtained as

$$\begin{aligned} G_{\mu\nu;\lambda\sigma}(x) &= \int \frac{d^4p}{(2\pi)^4}\frac{i}{p^2 - m_X^2 - i\epsilon}P_{\mu\nu;\lambda\sigma}e^{-ip\cdot x}, \\ P_{\mu\nu;\lambda\sigma} &= \frac{1}{2}(P_{\mu\lambda}P_{\nu\sigma} + P_{\mu\sigma}P_{\nu\lambda}) - \frac{1}{3}P_{\mu\nu}P_{\lambda\sigma}, \\ P_{\mu\nu} &= -g_{\mu\nu} + \frac{p_\mu p_\nu}{m_X^2}, \\ D^{\mu\nu;\lambda\sigma}G_{\lambda\sigma}{}^{\alpha\beta} &= -i\frac{1}{2}(g^{\mu\alpha}g^{\nu\beta} + g^{\nu\alpha}g^{\mu\beta})\delta^{(4)}(x). \end{aligned} \tag{6}$$

With respect to the discussions given in the first section, one can write the effective Lagrangian describing the interaction between $X(4350)$ and $D_s^*D_{s0}^*$ as

$$\begin{aligned} \mathcal{L}^S_{\rm int} = {}& \frac{i}{\sqrt{2}}g_SX(x)\int dx_1dx_2C_{\mu\mu}(x_1,x_2)\Phi_X((x_1-x_2)^2) \\ & \times\delta(x-\omega_vx_1-\omega_sx_2), \\ \mathcal{L}^T_{\rm int} = {}& \frac{i}{\sqrt{2}}g_TX^{\mu\nu}(x)\int dx_1dx_2\Big[C_{\mu\nu}(x_1,x_2) \\ & + C_{\nu\mu}(x_1,x_2) - \frac{1}{4}g_{\mu\nu}C_{\alpha\alpha}(x_1,x_2)\Big]\Phi_X((x_1-x_2)^2) \\ & \times\delta(x-\omega_vx_1-\omega_sx_2), \end{aligned} \tag{7}$$

where $\mathcal{L}^S_{\rm int}$ is for the scalar resonance case while $\mathcal{L}^T_{\rm int}$ is for the tensor resonance case. g_S and g_T are the effective coupling constants for the interaction between $X(4350)$ and $D_s^*D_{s0}^*$ in the scalar and tensor resonance cases, respectively. ω_v and ω_s are mass ratios which are defined as

$$\omega_v = \frac{m_{D_s^*}}{m_{D_s^*} + m_{D_{s0}^*}}, \qquad \omega_s = \frac{m_{D_{s0}^*}}{m_{D_s^*} + m_{D_{s0}^*}}. \tag{8}$$

$\Phi_X((x_1-x_2)^2)$ is a correlation function which illustrates the distribution of the constituents in the bound state. The Fourier transform of the correlation function reads

$$\Phi_X(y^2) = \int\frac{d^4p}{(2\pi)^4}\tilde{\Phi}_X(p^2)e^{-ip\cdot y}. \tag{9}$$

To write down Lagrangian (7), for simplicity, we have defined the tensor $C_{\mu\nu}$ as a function of the constituents with the explicit form

$$C_{\mu\nu}(x_1,x_2) = D^{*+}_{s;\mu}(x_1)\partial_\nu D^{*-}_{s0}(x_2) + D^{*-}_{s;\nu}(x_1)\partial_\mu D^{*+}_{s0}(x_2). \tag{10}$$

The coupling constants g_S and g_T can be determined with the help of the compositeness condition $Z_X = 0$ with Z_X as the wave function renormalization constant of $X(4350)$ which is defined as the residual of the $X(4350)$ propagator, i.e.,

$$Z_X = 1 - g_X^2\frac{d}{dp^2}\Sigma_X(p^2)|_{p^2=m_X^2}, \tag{11}$$

where $g_X = g_S$ for the scalar case while $g_X = g_T$ for the tensor case. For scalar resonance $X(4350)$, $g_S^2\Sigma_S(p^2) = \Pi_S(p^2)$ is its mass operator. But for the tensor resonance $X(4350)$, $g_T^2\Sigma_T(p^2)$ relates to its mass operator via the relation

$$\Pi_T^{\mu\nu;\alpha\beta}(p^2) = \tfrac{1}{2}(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha})g_T^2\Sigma_T(p^2) + \cdots, \tag{12}$$

where $\cdots$ denotes terms do not contribute to the mass renormalization of $X(4350)$. The mass operator of $X(4350)$ is illustrated by Fig. 1.

Concerning the Feynman diagram depicted in Fig. 1 one can calculate the mass operator explicitly. To get the numerical result of the coupling constant g_X, an explicit form of $\tilde{\Phi}_X(p^2)$ is necessary. Throughout this paper, we take the Gaussian form

$$\tilde{\Phi}_X(p^2) = \exp(p^2/\Lambda_X^2), \tag{13}$$

where the size parameter Λ_X parametrizes the distribution of the constituents inside the molecule. In the following calculation, we will find the physical value of Λ_X by comparing our calculation of the product of $X(4350)$ to two-photon partial width and the branching fraction to $J/\psi\phi$ with data. It should be noted that choice (13) is not unique. In principle any choice of $\tilde{\Phi}_X(p^2)$, as long as it renders the integral convergent sufficiently fast in the ultraviolet region, is reasonable. In this sense, $\tilde{\Phi}_X(p^2)$ can be regarded as a regulator which makes the ultraviolet divergent integral well defined.

With these discussions, we can calculate the effective coupling constant g_X numerically. In the typical nonperturbative region $\Lambda_X = 1.0 \sim 2.0$ GeV, using the central

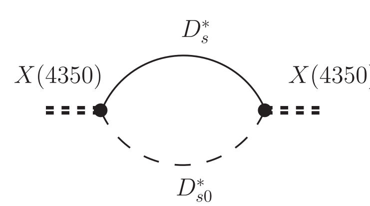

FIG. 1. The mass operator for $X(4350)$.

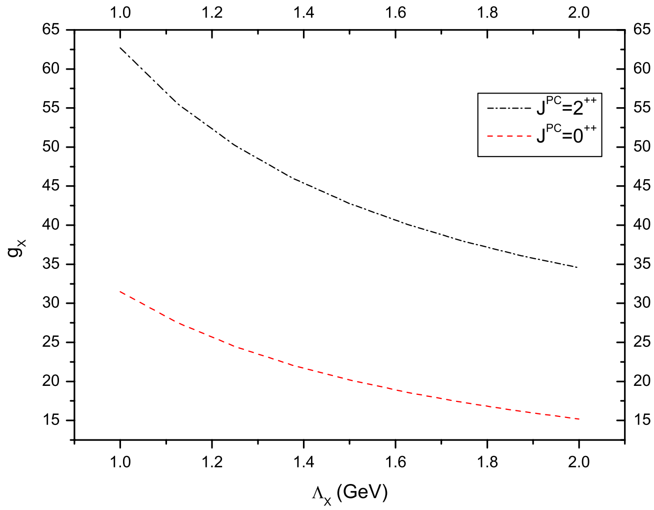

FIG. 2 (color online). Λ_X dependence of the coupling constant g_X.

value of $X(4350)$ mass, our numerical result is found to be

$$g_S = 31.49\text{–}15.19, \qquad g_T = 62.70\text{–}34.54. \tag{14}$$

In Fig. 2 we plot the Λ_X dependence of the coupling constants. One can see that both coupling constants decrease against Λ_X. This can be understood from the momentum integral of the mass operator. For scalar $X(4350)$, the loop integral is quadratically divergent so the derivative of the mass operator which is proportional to the inverse of g_S^2 increases against Λ_X which means the coupling constant g_S decreases against Λ_X. A similar argument can be given for g_T.

B. Effective Lagrangian for strong and electromagnetic decays of $X(4350)$

The effective Lagrangian for the study of strong and electromagnetic decays of $X(4350)$ consists of two parts: the electromagnetic part $\mathcal{L}_{\rm em}$ and the strong part $\mathcal{L}_{\rm str}$.

The electromagnetic interaction Lagrangian $\mathcal{L}_{\rm em}$ includes five parts: $\mathcal{L}_{\rm em}^{\rm NL}$ from the gauge of the nonlocal and derivative coupling of Eq. (7), $\mathcal{L}_{\rm em}^{\rm gauge}$ from the gauge of the kinetic terms of the charged constituents D_{s0}^* and D_s^*, the electromagnetic interaction Lagrangian $\mathcal{L}_{\rm em}^{SV}$ including D_{s0}^* and D_s^*, $\mathcal{L}_{\rm em}^{AV}$ for electromagnetic interaction including D_{s1} and D_s^*, and $\mathcal{L}_{\rm em}^{AS}$ for electromagnetic interaction including D_{s1} and D_{s0}^*.

One can write $\mathcal{L}_{\rm em}^{\rm NL}$ by substituting $C_{\mu\nu}$ in Eq. (7) with $C_{\mu\nu}^{\rm gauge}$

$$\begin{aligned} C_{\mu\nu}^{\rm gauge}(x_1, x_2) = {}& e^{-ieI(x_1,x_2;P)} D_{s;\mu}^{*+}(x_1) \\ & \times (\partial_\nu + ieA_\nu(x_2)) D_{s0}^{*-}(x_2) \\ & + e^{ieI(x_1,x_2;P)} D_{s;\nu}^{*-}(x_1)(\partial_\mu - ieA_\mu(x_2)) \\ & \times D_{s0}^{*+}(x_2), \end{aligned} \tag{15}$$

where the Wilson's line $I(x, y, P)$ is defined as

$$I(x, y; P) = \int_y^x dz_\mu A^\mu(z). \tag{16}$$

In our following calculation, the nonlocal vertex with one photon is necessary. The nonlocal vertex with one photon comes from two sources: One is from the covariant derivative and the other one is from the expansion of the Wilson's line. One can easily derive the Feynman rule for the nonlocal vertex with one photon which comes from the covariant derivative. But to derive the Feynman rule for a photon from Wilson's line, one may use the path-independent prescription suggested in [22,23].

The electromagnetic vertex $\mathcal{L}_{\rm em}^{\rm gauge}$ from the gauge of the kinetic terms of the charged constituents can be easily written as

$$\begin{aligned} \mathcal{L}_{\rm em}^{\rm gauge} = {}& ieA_\mu (D_{s0}^{*-} \overleftrightarrow{\partial}_\mu D_{s0}^{*+}) + ieA_\mu [-D_{s;\alpha}^{*-} \overleftrightarrow{\partial}_\mu D_{s;\alpha}^{*+} \\ & + \tfrac{1}{2} D_{s;\alpha}^{*-} \overleftrightarrow{\partial}_\alpha D_{s;\mu}^{*+} + \tfrac{1}{2} D_{s;\mu}^{*-} \overleftrightarrow{\partial}_\alpha D_{s;\alpha}^{*+}]. \end{aligned} \tag{17}$$

One can generally write the effective Lagrangian $\mathcal{L}_{\rm em}^{SV}$ for electromagnetic interaction including D_{s0}^* and D_s^* as

$$\mathcal{L}_{\rm em}^{SV} = eg_{D_{s0}^* D_s^* \gamma} [\tilde{V}_{\mu\nu}^- D_{s0}^{*+} - \tilde{V}_{\mu\nu}^+ D_{s0}^{*-}] F_{\mu\nu}, \tag{18}$$

where $\tilde{V}_{\mu\nu}^\pm$ is the gauged field strength tensor for $D_s^{*\pm}$ with definition $\tilde{V}_{\mu\nu}^\pm = (\partial_\mu \mp ieA_\mu) D_{s;\nu}^{*\pm} - (\partial_\nu \mp ieA_\nu) D_{s;\mu}^{*\pm}$. And similarly, the general effective Lagrangian $\mathcal{L}_{\rm em}^{AV}$ and $\mathcal{L}_{\rm em}^{AS}$ can be written as

$$\begin{aligned} \mathcal{L}_{\rm em}^{AV} &= eg_{D_{s1} D_s^* \gamma} \epsilon_{\mu\nu\alpha\beta} [D_{s1;\mu}^- D_{s;\nu}^{*+} - D_{s1;\mu}^+ D_{s;\nu}^{*-}] F_{\alpha\beta}, \\ \mathcal{L}_{\rm em}^{AS} &= -ieg_{D_{s1} D_{s0}^* \gamma} \epsilon_{\mu\nu\alpha\beta} [D_{s0}^{*+} \tilde{D}_{s1;\mu\nu}^- - D_{s0}^{*-} \tilde{D}_{s1;\mu\nu}^+] F_{\alpha\beta}. \end{aligned} \tag{19}$$

Similar to the definition of $\tilde{V}_{\mu\nu}^\pm$, we have defined the gauged field strength tensor for $D_{s1}^\pm$ with definition $\tilde{D}_{s1;\mu\nu}^\pm = (\partial_\mu \mp ieA_\mu) D_{s1;\nu}^{*\pm} - (\partial_\nu \mp ieA_\nu) D_{s1;\mu}^{*\pm}$.

The relevant coupling constants can be determined phenomenologically. Confined by the experimental status, one cannot fix $g_{D_{s0}^* D_s^* \gamma}$ from the data, so we turn to the theoretical calculations (for example, Ref. [9] and references therein). From the literature, one can see that the minimal result of the theoretical calculation of $D_{s0}^* \to D_s^* \gamma$ decay width is 0.2 KeV. From this decay width, we get $g_{D_{s0}^* D_s^* \gamma} \geq 3.02 \times 10^{-2}\ \text{GeV}^{-1}$.

The coupling constants $g_{D_{s1} D_s^* \gamma}$ and $g_{D_{s1} D_{s0}^* \gamma}$ can be determined by using the heavy quark effective theory (HQET) and branching ratio for relevant processes. First, let us consider the decay of $D_{s1} \to D_s \gamma$. The effective Lagrangian for this process can be written as

$$\mathcal{L}_{\rm em}^{D_{s1} D_s \gamma} = ieg_{D_{s1} D_s \gamma} [D_s^+ D_{s1;\mu\nu}^- - D_s^- D_{s1;\mu\nu}^+] F_{\mu\nu}, \tag{20}$$

where $D_{s1;\mu\nu} = \partial_\mu D_{s1;\nu} - \partial_\nu D_{s1;\mu}$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. From this Lagrangian, one can express the decay width as

$$\Gamma(D_{s1} \to D_s\gamma) = \frac{\alpha_{\rm em} g^2_{D_{s1}D_s\gamma}}{6m^3_{D_{s1}}}(m^2_{D_{s1}} - m^2_{D_s})^3. \quad (21)$$

The numerical result of the decay width has been evaluated by several groups. From the references given in Ref. [12], we see all the results are larger than 0.6 KeV, so that we have $g_{D_{s1}D_s\gamma} \geq 2.67 \times 10^{-2}\ \mathrm{GeV}^{-1}$. The coupling constant $g_{D_{s1}D_s^*\gamma}$ relates to $g_{D_{s1}D_s\gamma}$ via HQET as

$$\frac{g_{D_{s1}D_s\gamma}}{g_{D_{s1}D_s^*\gamma}} = \frac{1}{m_{D_{s1}}}\frac{\sqrt{m_{D_s}}}{\sqrt{m_{D_s^*}}} = 3.92 \times 10^{-1}\ \mathrm{GeV}^{-1}, \quad (22)$$

so that we have $g_{D_{s1}D_s^*\gamma} = 6.81 \times 10^{-2}$. The coupling constant $g_{D_{s1}D_{s0}^*\gamma}$ can be determined by using the relevant branching ratio given by the PDG [8]. From (19) we have

$$\Gamma(D_{s1} \to D_{s0}^*\gamma) = \frac{2\alpha_{\rm em} g^2_{D_{s1}D_{s0}^*\gamma}}{3m^3_{D_{s1}}}(m^2_{D_{s1}} - m^2_{D_{s0}^*})^3. \quad (23)$$

Using the central value of the branching ratio we have $\Gamma(D_{s1} \to D_{s0}^*\gamma)/\Gamma(D_{s1} \to D_s\gamma) \simeq 0.21$ which leads to $g_{D_{s1}D_{s0}^*\gamma} = 3.53 \times 10^{-2}\ \mathrm{GeV}^{-1}$.

In addition to the Lagrangian (7), the strong part $\mathcal{L}_{\rm str}$ involves the VVV-type Lagrangian describing the interaction of three vector mesons, the SVV-type Lagrangian describing the interaction of one scalar meson with two vector mesons, the SSV-type Lagrangian describing the interaction of two scalar mesons with one vector meson, the AVV-type Lagrangian for the interaction of one axial vector with two vector mesons, and the ASV-type Lagrangian for axial-vector–scalar-vector meson interaction, i.e.,

$$\begin{aligned}\mathcal{L}^{VVV}_{\rm str} = {}& ig_{\psi D_s^* D_s^*}[D^{*-}_{s;\mu}(D^{*+}_{s;\nu}\overleftrightarrow{\partial}_\mu\psi_\nu) + D^{*+}_{s;\mu}(\psi_\nu\overleftrightarrow{\partial}_\mu D^{*-}_{s;\nu}) \\ & + \psi_\mu(D^{*-}_{s;\nu}\overleftrightarrow{\partial}_\mu D^{*+}_{s;\nu})] + ig_{\phi D_s^* D_s^*}[D^{*-}_{s;\mu}(D^{*+}_{s;\nu}\overleftrightarrow{\partial}_\mu\phi_\nu) \\ & + D^{*+}_{s;\mu}(\phi_\nu\overleftrightarrow{\partial}_\mu D^{*-}_{s;\nu}) + \phi_\mu(D^{*-}_{s;\nu}\overleftrightarrow{\partial}_\mu D^{*+}_{s;\nu})], \quad (24)\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{SVV}_{\rm str} = {}& g_{\psi D_{s0}^* D_s^*}[D^{*-}_{s0}D^{*+}_{s;\mu\nu} - D^{*+}_{s0}D^{*-}_{s;\mu\nu}]\psi_{\mu\nu} \\ & + g_{\phi D_{s0}^* D_s^*}[D^{*-}_{s0}D^{*+}_{s;\mu\nu} - D^{*+}_{s0}D^{*-}_{s;\mu\nu}]\phi_{\mu\nu}, \quad (25)\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{SSV}_{\rm str} = {}& -ig_{\psi D_{s0}^* D_{s0}^*}\psi_\mu(D^{*-}_{s0}\overleftrightarrow{\partial}_\mu D^{*+}_{s0}) \\ & - ig_{\phi D_{s0}^* D_{s0}^*}\phi_\mu(D^{*-}_{s0}\overleftrightarrow{\partial}_\mu D^{*+}_{s0}), \quad (26)\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{AVV}_{\rm str} = {}& -g_{\psi D_s^* D_{s1}}\epsilon_{\mu\nu\alpha\beta}[D^-_{s1;\mu}D^{*+}_{s;\nu} - D^+_{s1;\mu}D^{*-}_{s;\nu}]\psi_{\alpha\beta} \\ & - g_{\phi D_s^* D_{s1}}\epsilon_{\mu\nu\alpha\beta}[D^-_{s1;\mu}D^{*+}_{s;\nu} - D^+_{s1;\mu}D^{*-}_{s;\nu}]\phi_{\alpha\beta}, \quad (27)\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{ASV}_{\rm str} = {}& -ig_{\psi D_{s0}^* D_{s1}}\epsilon_{\mu\nu\lambda\sigma}[D^{*-}_{s0}\psi_{\mu\nu}D^+_{s1;\lambda\sigma} \\ & - D^{*+}_{s0}\psi_{\mu\nu}D^-_{s1;\lambda\sigma}] \\ & - ig_{\phi D_{s0}^* D_{s1}}\epsilon_{\mu\nu\lambda\sigma}[D^{*-}_{s0}\phi_{\mu\nu}D^+_{s1;\lambda\sigma} \\ & - D^{*+}_{s0}\phi_{\mu\nu}D^-_{s1;\lambda\sigma}]. \quad (28)\end{aligned}$$

Because of our less knowledge, we cannot determine these coupling constants from data. Here, we resort to the VMD model [24]. In the VMD model, the virtual photon in the process $e^-D^{*+}_{s0} \to e^-D^{*+}_{s0}$ is coupled to vector mesons ϕ and J/ψ, which are then coupled to D^{*+}_{s0}. For zero momentum transfer, one has relation

$$\sum_{V=\phi,\psi}\frac{\gamma_V g_{VD_{s0}^*D_{s0}^*}}{m_V^2} = e, \quad (29)$$

where γ_V is the photon-vector-meson mixing amplitude

$$\mathcal{L}_{V\text{-}\gamma\ \rm mixing} = \gamma_V V_\mu A_\mu, \quad (30)$$

which can be determined from $V \to e^+e^-$ decay width, i.e.,

$$\Gamma_{Vee} = \frac{\alpha_{\rm em}\gamma_V^2}{3m_V^3}, \quad (31)$$

where we did not include the electron mass since it is much smaller than the vector meson mass. For the ϕ meson, using $\Gamma(\phi \to e^+e^-) = 2.97 \times 10^{-4} \times 4.26$ MeV [8] we have $\gamma_\phi = 23\,472.3\ \mathrm{MeV}^2$, while $\gamma_\psi = 259\,965.8\ \mathrm{MeV}^2$ when $\Gamma(\psi \to e^+e^-) = 5.94\% \times 93.2$ KeV [8] is applied. Concerning the fact that the virtual photon sees the charge of the charm quark in the D^{*+}_{s0} meson through $\psi D^*_{s0}D^*_{s0}$ coupling and the charge of the antistrange quark in D^{*+}_{s0} meson through $\phi D^*_{s0}D^*_{s0}$ coupling, we have relations

$$\frac{\gamma_\psi g_{\psi D_{s0}^*D_{s0}^*}}{m_\psi^2} = \frac{2}{3}e, \qquad \frac{\gamma_\phi g_{\phi D_{s0}^*D_{s0}^*}}{m_\phi^2} = \frac{1}{3}e. \quad (32)$$

From these relations we have $g_{\psi D_{s0}^*D_{s0}^*} = 7.45$ and $g_{\phi D_{s0}^*D_{s0}^*} = 4.47$. To determine coupling constants $g_{VD_{s0}^*D_s^*}$, we make an extension to the VMD model used above, i.e., substituting Eq. (29) with

$$\sum_{V=\phi,\psi}\frac{\gamma_V g_{VD_iD_j}}{m_V^2} = eg_{D_iD_j\gamma}, \quad (33)$$

where D_i and D_j denote the relevant charmed-strange mesons. Similarly, Eqs. (32) should also be extended to

$$\frac{\gamma_\psi g_{\psi D_iD_j}}{m_\psi^2} = \frac{2}{3}eg_{D_iD_j\gamma}, \qquad \frac{\gamma_\phi g_{\phi D_iD_j}}{m_\phi^2} = \frac{1}{3}eg_{D_iD_j\gamma}, \quad (34)$$

from which we yield the relevant coupling constants as

$$g_{\psi D_i D_j} = 7.45 \times g_{D_i D_j \gamma}, \qquad g_{\phi D_i D_j} = 4.47 \times g_{D_i D_j \gamma}. \tag{35}$$

To fix the magnitude of coupling constants $g_{VD_s^* D_s^*}$, we resort to the $SU(4)$ relation as was used in Ref. [25] from which we have relations

$$g_{\psi D_s^* D_s^*} = \frac{2}{\sqrt{3}} g_{\phi D_s^* D_s^*} = g_{\psi D^* D^*} = 7.64. \tag{36}$$

To fix the relative signs for the relevant effective Lagrangian, one can use heavy hadron chiral perturbation theory (HHChPT) including the D_{s0}^* and D_{s1} mesons [26]. But even with this consideration, the relative signs of $\mathcal{L}_{\rm em}^{AS}$ and $\mathcal{L}_{\rm str}^{ASV}$ to the other terms cannot be determined. We leave this as an ambiguity and discuss different cases in the following calculation. In summary, our framework of the interaction Lagrangian is

$$\mathcal{L}_{\rm int} = \mathcal{L}_{\rm em}^{\rm NL} + \mathcal{L}_{\rm em}^{\rm gauge} + \mathcal{L}_{\rm em}^{SV} + \mathcal{L}_{\rm em}^{AV} + \mathcal{L}_{\rm str}^{VVV} + \mathcal{L}_{\rm str}^{SVV} + \mathcal{L}_{\rm str}^{SSV} + \mathcal{L}_{\rm str}^{AVV} + \mathcal{L}_{\rm int}^{ASV}, \tag{37}$$

$$\mathcal{L}_{\rm int}^{ASV} = \pm[\mathcal{L}_{\rm em}^{AS} + \mathcal{L}_{\rm str}^{ASV}]. \tag{38}$$

Up to now, we have fixed all the coupling constants that are necessary for our following calculation of the electromagnetic and strong decays of $X(4350)$.

III. ELECTROMAGNETIC AND STRONG DECAYS OF $X(4350)$

In this section, we will present the general forms of the matrix elements and partial widths for the electromagnetic and strong decays of $X(4350)$ and the Feynman diagrams included in our calculation.

A. Electromagnetic decay of $X(4350)$

The four kinds of diagrams depicted in Fig. 3 and their corresponding crossing ones should be taken into account to study $X(4350) \to 2\gamma$ decay. Figures 3(a) and 3(b) are from the final state interaction due to the exchange of D_s^*, D_{s1}, and D_{s0}^*; Fig. 3(c) arises from the gauge of the nonlocal and derivative coupling between $X(4350)$ and its constituents $D_s^* D_{s0}^*$, but Fig. 3(d) is from the Lagrangian (18).

For $X(4350)$ with quantum numbers $J^{PC} = 0^{++}$, concerning the $U(1)_{\rm em}$ gauge invariance and the transverseness of the photon polarization vector, one can write down the matrix element for the decay of $X \to 2\gamma$ as

$$iM_S^{\rm em} = ie^2 F_{X_s\to 2\gamma}\Big[g_{\alpha\beta} - \frac{q_{2\alpha}q_{1\beta}}{q_1\cdot q_2}\Big]\epsilon_\alpha(q_1)\epsilon_\beta(q_2), \tag{39}$$

while for the tensor meson $X(4350)$ with quantum numbers $J^{PC} = 2^{++}$, its polarization vector satisfies $\epsilon^{\mu\nu} = \epsilon^{\nu\mu}$ and $\epsilon_\mu^\mu = 0$, so that the matrix element for electromagnetic decay can be written as [27,28]

$$\begin{aligned} iM_T^{\rm em} = ie^2\Big\{ & F_{T\to 2\gamma}^{(0)}\Big[g_{\alpha\beta} - \frac{q_{2\alpha}q_{1\beta}}{q_1\cdot q_2}\Big]\frac{q_\mu q_\nu}{q^2} \\ & + F_{T\to 2\gamma}^{(2)}\Big[\Big(g_{\mu\alpha} - \frac{q_\mu q_\alpha}{q^2}\Big)\Big(g_{\nu\beta} - \frac{q_\nu q_\beta}{q^2}\Big) \\ & + \Big(g_{\mu\beta} - \frac{q_\mu q_\beta}{q^2}\Big)\Big(g_{\nu\alpha} - \frac{q_\nu q_\alpha}{q^2}\Big)\Big]\Big\} \\ & \times \epsilon_{\mu\nu}(p)\epsilon_\alpha(q_1)\epsilon_\beta(q_2), \end{aligned} \tag{40}$$

where $q = q_1 - q_2$. From Eqs. (39) and (40) we express the decay widths for $X(4350)$ as

$$\begin{aligned} \Gamma_S(X\to 2\gamma) &= \frac{2\pi}{m_X}\alpha_{\rm em}^2 F_{X_S\to 2\gamma}^2, \\ \Gamma_T(X\to 2\gamma) &= \frac{\pi}{15 m_X}\alpha_{\rm em}^2 (5F_{X_T\to 2\gamma}^{(0)2} - 4F_{X_T\to 2\gamma}^{(0)}F_{X_T\to 2\gamma}^{(2)} \\ &\quad + 32F_{X_T\to 2\gamma}^{(2)2}), \end{aligned} \tag{41}$$

where the subindices S and T denote the scalar and tensor resonance $X(4350)$, respectively. To get the last equation, we have applied the sum of the polarization vector for the tensor meson [29]

$$\sum_{\rm polar}\epsilon_{\mu_1\nu_1}(p)\epsilon^*_{\mu_2\nu_2}(p) = \frac{1}{2}(\theta_{\mu_1\mu_2}\theta_{\nu_1\nu_2} + \theta_{\mu_1\nu_2}\theta_{\nu_1\mu_2}) - \frac{1}{3}\theta_{\mu_1\nu_1}\theta_{\mu_2\nu_2}, \tag{42}$$

where $\theta_{\mu\nu} = -g_{\mu\nu} + (p_\mu p_\nu/m_X^2)$.

B. Strong decay of $X(4350)$

We should take into account the Feynman diagrams illustrated in Fig. 4 in the study of the strong decay of

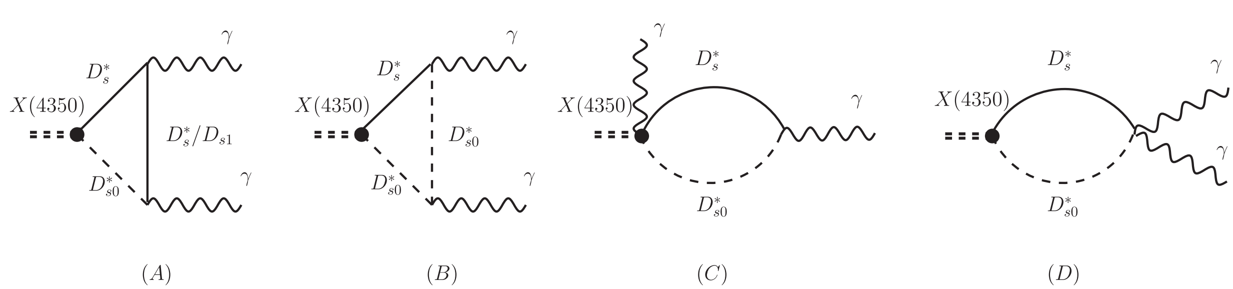

FIG. 3. Feynman diagrams for decay $X(4350) \to \gamma\gamma$ (cross diagrams should be included).

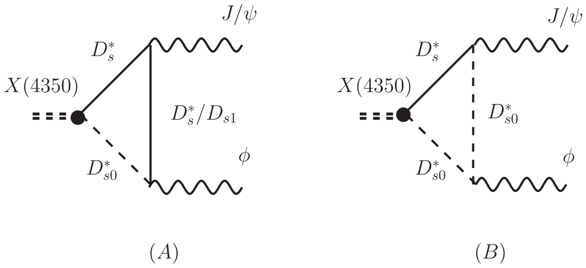

FIG. 4. Feynman diagrams for decay $X(4350) \to J/\psi\phi$ (cross diagrams should be included).

$X(4350) \to J/\psi\phi$. Furthermore, in addition to these diagrams, their crossing ones should also be included.

Compared to the electromagnetic case, the expression for the matrix element of strong decay is more complicated because the constraint from the gauge invariance is released. When $X(4350)$ is regarded as a scalar resonance, the matrix element for the strong decay of $X \to V_\alpha(q_1)V_\beta(q_2)$ can be written as

$$iM_S^{\rm str} = i\left[G_{X_s\to V_1V_2}g_{\alpha\beta} + F_{X_s\to V_1V_2}\frac{q_{2\alpha}q_{1\beta}}{q_1\cdot q_2}\right]\epsilon_\alpha(q_1)\epsilon_\beta(q_2). \tag{43}$$

One can show that, when the gauge invariance is imposed, $G_{X_s\to V_1V_2} = -F_{X_s\to V_1V_2}$ so expression (43) becomes (39). Similarly, without the constraint from the gauge invariance, in the tensor case, one can write the matrix element for the strong decay of $X_{\mu\nu} \to V_\alpha(q_1)V_\beta(q_2)$ as

$$\begin{aligned} iM_T^{\rm str} = i[&F^{(1)}_{X_T\to V_1V_2}g_{\alpha\beta}q_\mu q_\nu \\ &+ F^{(2)}_{X_T\to V_1V_2}(g_{\mu\alpha}g_{\nu\beta} + g_{\nu\alpha}g_{\mu\beta}) \\ &+ F^{(3)}_{X_T\to V_1V_2}(g_{\mu\alpha}q_\nu q_{1\beta} + g_{\nu\alpha}q_\mu q_{1\beta}) \\ &+ F^{(4)}_{X_T\to V_1V_2}(g_{\mu\beta}q_\nu q_{2\alpha} + g_{\nu\beta}q_\mu q_{2\alpha}) \\ &+ F^{(5)}_{X_T\to V_1V_2}q_\mu q_\nu q_{2\alpha}q_{1\beta}]\epsilon_{\mu\nu}(p)\epsilon_\alpha(q_1)\epsilon_\beta(q_2). \end{aligned} \tag{44}$$

One can prove that when the final vector mesons are both massless particles and the gauge invariance is imposed the following relations can be reduced:

$$\begin{aligned} F^{(3)}_{X_T\to V_1V_2} &= -F^{(4)}_{X_T\to V_1V_2} = \frac{1}{2q_1\cdot q_2}F^{(2)}_{X_T\to V_1V_2}, \\ F^{(5)}_{X_T\to V_1V_2} &= -\frac{1}{q_1\cdot q_2}F^{(3)}_{X_T\to V_1V_2} - \frac{1}{2(q_1\cdot q_2)^2}F^{(2)}_{X_T\to V_1V_2}. \end{aligned} \tag{45}$$

So expression (40) for the electromagnetic decay matrix element can be yielded.

With the help of (42) one can get the analytic forms for the strong decay as

$$\begin{aligned} \Gamma_S(X\to J/\psi\phi) &= \frac{1}{16\pi m_X^3}\lambda^{1/2}(m_X^2, m_\psi^2, m_\phi^2)\left\{G^2_{X_s\to V_1V_2}[2+\omega^2] - 2G_{X_s\to V_1V_2}F_{X_s\to V_1V_2}[1-\omega^2] + F^2_{X_s\to V_1V_2}\left[\omega - \frac{1}{\omega}\right]^2\right\}, \\ \Gamma_T(X\to J/\psi\phi) &= \frac{1}{80\pi m_X^3}\lambda^{1/2}(m_X^2, m_\psi^2, m_\phi^2)\sum_{i\geq j=1}^{5}\{C_{ij}F^{(i)}_{X_T\to V_1V_2}F^{(j)}_{X_T\to V_1V_2}\}, \end{aligned} \tag{46}$$

where $\omega = q_1\cdot q_2/(m_\psi m_\phi) = (m_X^2 - m_\psi^2 - m_\phi^2)/(2m_\psi m_\phi)$. λ is the Källen function and C_{ij} are functions of the relevant masses of initial and final states which will be given in the Appendix.

IV. NUMERICAL RESULTS AND DISCUSSIONS

With these discussions, the numerical calculation can be performed via standard loop derivation. Since the magnitude of Λ_X is unknown, we vary its magnitude from 0.5 to 4.0 GeV to find its physical region where the data can be understood. In our estimate, we use the central value of the total width, i.e., $\Gamma_X = 13.3$ MeV. And, because it is difficult to determine the relative signs between $\mathcal{L}^{AS}_{\rm em}$ and $\mathcal{L}^{ASV}_{\rm str}$ and other terms, we will consider two cases when we do our numerical calculation, i.e., the last two terms of Eq. (38) give positive and negative contributions to the total Lagrangian. Our results are summarized in Tables I and II.

From the numerical results, one can see that the possibility that $X(4350)$ is a molecular state of $D^*_{s0}D^*_s$ cannot be ruled out in our model. In the case that $X(4350)$ has quantum numbers $J^{PC} = 0^{++}$, the physical region of Λ_X is smaller than the tensor resonance case which means the size of scalar $X(4350)$ is bigger than the tensor one.

We would like to point out that, because we used the minimal values of the theoretical calculation of coupling constants $g_{D^*_{s0}D^*_s\gamma}$ and $g_{D_{s1}D_s\gamma}$, our final results of the partial widths can be regarded as lower limits. This is an ambigu-

TABLE I. Our numerical results in the case of the positive sign of Eq. (38).

J^{PC}	Λ_X (GeV)	Branch product (eV)	$\Gamma_{\rm str}$ (KeV)	$\Gamma_{\rm em}$ (KeV)
0^{++}	0.5–0.7	2.19–10.26	100.9–174.5	0.29–0.78
2^{++}	1.1–1.8	1.24–2.28	285.3–973.5	0.03–0.09

TABLE II. Our numerical results in the case of the negative sign of Eq. (38).

J^{PC}	Λ_X (GeV)	Branch product (eV)	$\Gamma_{\rm str}$ (KeV)	$\Gamma_{\rm em}$ (KeV)
0^{++}	0.5–0.6	7.21–12.74	373.6–391.0	0.26–0.43
2^{++}	1.0–1.9	0.66–2.42	166.0–915.1	0.02–0.19

ity of the present calculation. In fact, the best way to determine these coupling constants is from data, but because of the precision of the data, we cannot in this way. When the magnitudes of coupling constants $g_{D_{s0}^* D_s^* \gamma}$ and $g_{D_{s1} D_s \gamma}$ are improved, the theoretical results of the product of the two-photon decay width and branch fraction to $J/\psi\phi$ should be larger than the present conclusion. In this case, compared to the tensor $X(4350)$, the typical region of Λ_X for scalar resonance can be reduced to an unphysically small region so one can first rule out the possibility of a scalar molecule.

Another ambiguity in our calculation of the product of the two-photon decay width and branch fraction to $J/\psi\phi$ is from the total width of $X(4350)$. Here we apply the central value, i.e., $\Gamma_X = 13.3$ MeV. When a larger total width is applied, the physical region of Λ_X can be enlarged. But this does not affect the partial widths for strong and electromagnetic decays we predicted above in the corresponding region of Λ_X.

Finally, we conclude that, with the present data and in the framework our model, $X(4350)$ can be interpreted as a $D_{s0}^* D_s^*$ molecule.

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APPENDIX: EXPLICIT FORMS FOR THE FUNCTIONS C_{ij}

In this Appendix, I will present the coefficients C_{ij} in formula (46):

$$
\begin{aligned}
C_{11} &= \frac{1}{24m^4m_1^2m_2^2}[5\lambda^2(\lambda+12m_1^2m_2^2)],\\
C_{12} &= \frac{-1}{12m^4m_1^2m_2^2}[\lambda(5\lambda(m^2+m_1^2+m_2^2)+24m^2m_1^2m_2^2)],\\
C_{13} &= \frac{1}{12m^4m_1^2m_2^2}[5\lambda^2((m^2-m_2^2)^2-m_1^4)],\\
C_{14} &= \frac{-1}{12m^4m_1^2m_2^2}[5\lambda^2((m^2-m_1^2)^2-m_2^4)],\\
C_{15} &= \frac{1}{24m^4m_1^2m_2^2}[5\lambda^3(m^2-m_1^2-m_2^2)],\\
C_{22} &= \frac{1}{24m^4m_1^2m_2^2}[5\lambda^2+44m^2(m_1^2+m_2^2)\lambda+528m_1^2m_2^2m^4],\\
C_{23} &= \frac{-1}{12m^4m_1^2m_2^2}[\lambda(5\lambda+44m^2m_1^2)(m^2-m_1^2+m_2^2)],\\
C_{24} &= \frac{1}{12m^4m_1^2m_2^2}[\lambda(5\lambda+44m^2m_2^2)(m^2-m_2^2+m_1^2)],\\
C_{25} &= \frac{-1}{24m^4m_1^2m_2^2}[5\lambda^2(m^4-(m_1^2-m_2^2)^2)],\\
C_{33} &= \frac{1}{24m^4m_1^2m_2^2}[\lambda^2(5\lambda+44m^2m_1^2)],\\
C_{34} &= \frac{-1}{12m^4m_1^2m_2^2}[5\lambda^2(m^4-(m_1^2-m_2^2)^2)],\\
C_{35} &= \frac{1}{24m^4m_1^2m_2^2}[5\lambda^3(m^2-m_2^2+m_1^2)],\\
C_{44} &= \frac{1}{24m^4m_1^2m_2^2}[\lambda^2(5\lambda+44m^2m_2^2)],\\
C_{45} &= \frac{-1}{24m^4m_1^2m_2^2}[5\lambda^3(m^2-m_1^2+m_2^2)],\\
C_{55} &= \frac{1}{96m^4m_1^2m_2^2}[5\lambda^4], \qquad \text{(A1)}
\end{aligned}
$$

where $\lambda = \lambda(m^2, m_1^2, m_2^2)$ is the Källen function and $m = m_X$, $m_1 = m_\psi$, and $m_2 = m_\phi$.

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