Derivation of the gauge link in light cone gauge PHYSICAL REVIEW D 82, 014018 (2010)
Derivation of the gauge link in light cone gauge

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In light cone gauge, a gauge link at light cone infinity is necessary for transverse momentum-dependent parton distribution to restore the gauge invariance in some specific boundary conditions. We derive such transverse gauge link in a more regular and general method. We find the gauge link at light cone infinity naturally arises from the contribution of the pinched poles: one is from the quark propagator and the other is hidden in the gauge vector field in light cone gauge. Actually, in the amplitude level, we have obtained a more general gauge link over the hypersurface at light cone infinity which is beyond the transverse direction. The difference of such gauge link between semi-inclusive deep inelastic scattering and Drell-Yan processes can also be obtained directly and clearly in our derivation.

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I. INTRODUCTION

Nucleon structure functions are physical observables and can be measured in deep inelastic scattering (DIS). In the naive parton model [\[1](#page-8-0)], the structure functions are expressed in terms of the probability of finding quarks and gluons in the parent nucleon. In collinear QCD factorization formulas, such structure functions can be given by compact operator matrix elements of the target [\[2\]](#page-8-1)

$$
q(x) = \frac{1}{2} \int \frac{dy^-}{2\pi} e^{-ixp^+y^-} \langle P|\bar{\psi}(y^-,\vec{0}_\perp)\hat{\psi}\mathcal{L}[y^-,\vec{0}_\perp;0,\vec{0}_\perp]
$$

$$
\times \psi(0,\vec{0}_\perp)|P\rangle,
$$
 (1)

where

$$
\mathcal{L}[y^-, \vec{0}_\perp; 0, \vec{0}_\perp] \equiv P \exp\left(-ig \int_0^{y^-} d\xi^- A^+(\xi^-, \vec{0}_\perp)\right)
$$
\n(2)

is the gauge link between the quark fields, which arises from final state interactions between the struck quark and the target spectators. In Eqs. ([1](#page-0-1)) and [\(2](#page-0-2)), all fields are evaluated at equal $y^+ = 0$. Since structure functions, as physical observables, should not be dependent on the gauge that we choose, it is necessary to introduce such gauge link to ensure the gauge invariance of matrix element. In the light cone gauge $A^+=0$, where the pathordered exponential in Eq. [\(2](#page-0-2)) reduces to unity, we can identify the quark distribution in Eq. ([1\)](#page-0-1) as a probability distribution as we made in naive parton model. Actually, in collinear structure function such as Eq. ([1\)](#page-0-1), we can always select a clever gauge to vanish the gauge link. But when we consider the transverse-momentum dependent quark distribution, such naive manipulation will result in inconsistency. In the nonsingular gauge, in which the gauge potential vanishes at the space-time infinity, the

transverse-momentum parton distribution is defined in the literature as [\[2](#page-8-1)–[4\]](#page-8-2)

$$
q(x, \vec{k}_{\perp}) = \frac{1}{2} \int \frac{dy^{-}}{2\pi} \frac{d^{2} \vec{y}_{\perp}}{(2\pi)^{2}} e^{-ixp^{+}y^{-} + i\vec{k}_{\perp} \cdot \vec{y}_{\perp}} \langle P | \bar{\psi}(y^{-}, \vec{y}_{\perp}) \hat{\psi} \rangle
$$

$$
\times \mathcal{L}^{\dagger}[\infty, \vec{y}_{\perp}; y^{-}, \vec{y}_{\perp}] \mathcal{L}[\infty, \vec{0}_{\perp}; 0, \vec{0}_{\perp}]
$$

$$
\times \psi(0, \vec{0}_{\perp}) | P \rangle,
$$
 (3)

where

$$
\mathcal{L}\left[\infty, \vec{y}_{\perp}; y^{-}, \vec{y}_{\perp}\right] \equiv P \exp\left(-ig \int_{y^{-}}^{\infty} d\xi^{-} A^{+}(\xi^{-}, \vec{y}_{\perp})\right),\tag{4}
$$

and all fields are evaluated at equal $y^+ = 0$. From Lorentz invariance, parity invariance and time reversal invariance, the transverse-momentum parton distribution can be decomposed into the following expressions:

$$
q(x, \vec{k}_{\perp}) = f(x, k_{\perp}) + \vec{S} \cdot (\hat{\vec{p}} \times \vec{k}_{\perp}) f_{1T}^{\perp}(x, k_{\perp}) / M, \quad (5)
$$

where \vec{S} is the spin of the target nucleon and $\hat{\vec{p}}$ is a unit vector along the direction of the target momentum in infinite momentum frame. The function $f_{1T}^{\perp}(x, k_{\perp})$ is just the Sivers function and can contribute to single spin asymmetries. It is verified in Ref. [\[5\]](#page-8-3) that the Sivers function vanishes unless there is the gauge link in Eq. [\(4\)](#page-0-3), which is yielded by the final state interactions [[6\]](#page-8-4). In the light cone gauge, however, it seems as if the gauge link in Eq. [\(4\)](#page-0-3) would become unity and the final interaction vanish accordingly, too. Hence, there will be inconsistent results from different gauges, which is impossible, since physical observables should not depend on the gauge by choice. Ji and Yuan in [\[7\]](#page-8-5) have shown that the final state interaction effects in single spin asymmetry can be recovered properly in the light cone gauge by taking into account a transverse gauge link at $y^- = +\infty$. Further in [[8\]](#page-8-6), Belitsky, Ji, and Yuan demonstrate the existence of extra leading twist [*g](#page-0-4)aojh79@ustc.edu.cn contributions from transverse components of the gauge

potential at the light cone infinity. It turns out that these contributions just form a transverse gauge link in light cone gauge. In this paper, we will give another more regular and systematic method to obtain such transverse gauge link in light cone gauge. We find the gauge link at light cone infinity will arise naturally from the pinched poles, one of which is provided by the quark propagator and the other is hidden in the gauge vector field in light cone gauge. Actually, it turns out that we obtain a more general gauge link over hypersurface $y^- = \infty$, instead of only transverse gauge link. The difference of such gauge link between semi-inclusive deep inelastic scattering (SIDIS) and Drell-Yan (DY) processes can also be shown directly and clearly in our derivation. The paper is organized as follows: in the next section, we will introduce some kinetics definitions and notations which will be involved all through our paper. In Sec. III, we would like to give a brief review on the singularity in light cone gauge and different prescriptions for different light cone pole structures. Then in Sec. IV, we will devote to deriving the gauge link in light cone gauge in SIDIS process. In Sec. V, we will deal with the DY process and compare it with the SIDIS process. A very short summary is given in the end. Other relevant work on the transverse gauge link can be found in the literature [\[9](#page-8-7)[,10\]](#page-8-8).

II. SOME DEFINITIONS AND NOTATIONS

In studying SIDIS or DY process, it is convenient to choose the light cone coordinate system in which we introduce two lightlike vectors n^{μ} and \bar{n}^{μ} ,

$$
n^{\mu} = (0, 1, \vec{0}_{\perp}), \qquad \bar{n}^{\mu} = (1, 0, \vec{0}_{\perp}), \qquad n \cdot \bar{n} = 1. (6)
$$

With these basis vectors, we may write any vector k^{μ} as $(k^+, k^-, \vec{k}_\perp)$, where $k^+ = k \cdot n$, $k^- = k \cdot \bar{n}$. For example, in SIDIS process, we choose the proton infinite momentum frame, in which the proton's momentum and the virtual photon's momentum are given by, respectively,

$$
p^{\mu} = p^{+} \bar{n}^{\mu}, \qquad q^{\mu} = -x_{B} p^{\mu} + \frac{Q^{2}}{2x_{B} p^{+}} n^{\mu}. \tag{7}
$$

where $x_B = Q^2/2p \cdot q$ and $Q^2 = -q^2$.

In order to make the derivation more compact and elegant in the following sections, let us introduce some notations. For any momentum vector k^{μ} and the gauge potential vector A^{μ} , we will manipulate the following decomposition:

$$
k^{\mu} = \tilde{k}^{\mu} + x p^{\mu}, \qquad A^{\mu} = \tilde{A}^{\mu} + A^{+} \bar{n}^{\mu},
$$
 (8)

where $\tilde{k}^{\mu} = (0, k^{-}, \vec{k}_{\perp}), \quad x = k^{+}/p^{+}, \text{ and } \tilde{A}^{\mu} =$ $(0, A^{-}, \vec{A}_{\perp})$. For any coordinate vector y^{μ} , we will make the dual decomposition,

$$
y^{\mu} = \dot{y}^{\mu} + y^{-} n^{\mu}, \tag{9}
$$

where $\dot{y}^{\mu} = (y^+, 0, \vec{y}_\perp)$. When there is no confusion, we

will rewrite y^{μ} as (y^{-}, \dot{y}) . With such notations, we have $k \cdot$ $y = \tilde{k} \cdot \dot{y} + xp^{+}y^{-}$, and in light cone gauge where $A^{+} =$ 0, we also have $A^{\mu} = \tilde{A}^{\mu}$. It should be noted that in light cone coordinate, the covariant vector and contravariant vector are related by $A^+ = A_-, A^- = A_+$ and $A^\perp = -A_+.$

III. SPURIOUS SINGULARITY IN LIGHT CONE GAUGE VIEW

The light cone gauge $n \cdot A = 0$ is widely used in perturbative QCD calculations [[11](#page-8-9),[12](#page-8-10)], and under such a physical gauge condition, the probability interpretation is expected to hold. The Yang-Mills theories, quantized in light cone gauge, have been studied by several authors [\[13](#page-8-11)[,14\]](#page-8-12). However, when we calculate with the gauge propagator in such gauge in perturbation theory, we have to introduce some spurious pole to regularize associated light cone singularity. There have been a variety of pre-scriptions suggested to handle the singularities [[15](#page-8-13)–[19](#page-8-14)], in which most attempts were pragmatic. The literature [\[8,](#page-8-6)[20\]](#page-8-15) states that in general, in light cone gauge, the gauge potential can not be arbitrarily set to vanish at the infinity, the spurious singularities, characteristic of all the axial gauges, are physically related to the boundary conditions that one can impose on the potentials at the infinity. In our paper, we will consider three different boundary conditions as in [\[8\]](#page-8-6), i.e.

Advanced:
$$
\tilde{A}(\infty, \dot{y}) = 0
$$
,
Retarded: $\tilde{A}(-\infty, \dot{y}) = 0$, (10)

Antisymmetric: $\tilde{A}(-\infty, \dot{y}) + \tilde{A}(\infty, \dot{y}) = 0.$

The typical integration we will meet with in our derivation is the Fourier transformation of the gauge potential, such as

$$
\tilde{A}_{\rho}(k^{+}, \dot{y}) \equiv \int_{-\infty}^{\infty} dy^{-} e^{ik^{+}y^{-}} \tilde{A}_{\rho}(y^{-}, \dot{y}). \tag{11}
$$

Manipulating this integration by parts, we obtain

$$
\int_{-\infty}^{\infty} dy^- e^{ik^+ y^-} \tilde{A}_{\rho}(y^-, \dot{y})
$$

=
$$
\left[\frac{i}{k^+} \right] \int_{-\infty}^{\infty} dy^- e^{ik^+ y^-} \partial^+ \tilde{A}_{\rho}(y^-, \dot{y}),
$$
 (12)

where $\partial^+ = \partial^- = \partial/\partial y^-$. Since the boundary condition is set, the term $\left[\frac{i}{k^+}\right]$ can be regularized by definite prescription,

Advanced:
$$
\left[\frac{i}{k^+}\right] = \frac{i}{k^+ - i\epsilon}
$$
,
\nRetarded: $\left[\frac{i}{k^+}\right] = \frac{i}{k^+ + i\epsilon}$, (13)
\nAntisymmetric: $\left[\frac{i}{k^+}\right] = \frac{1}{2}\left(\frac{i}{k^+ + i\epsilon} + \frac{i}{k^+ - i\epsilon}\right)$,

where the last propose is just the conventional principal

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value regulation when the antisymmetry boundary condition is assigned. Hence, we notice that there is a secret pole structure in gauge potential in momentum space. We will show that it is just this pole that will contribute to the final gauge link at the light cone infinity.

The easiest way to illustrate the validity of such regularization is just to set

$$
\tilde{A}_{\rho}(y^{-}) = \begin{cases}\n\text{Advanced:} & \theta(-y^{-}), \\
\text{Retarted:} & \theta(y^{-}), \\
\text{Antysymmetry:} & \frac{1}{2}[\theta(y^{-}) - \theta(-y^{-})],\n\end{cases}
$$
\n(14)

where the function $\theta(y^-)$ is the usual step function. It is a trivial exercise to show that they can result in the proper pole structure as we present in Eq. [\(10](#page-1-0)).

As we mentioned above, in the light cone gauge, we can not impose on the gauge potential the boundary condition both $\tilde{A}_{\rho}(\alpha, \dot{y}) = 0$ and $\tilde{A}_{\rho}(\alpha, \dot{y}) = 0$. We can only choose either of them as the boundary condition to remove the residual gauge freedom and the other one will be subjected to satisfy the field equation or the request that the total gauge energy momentum is finite. However, as a matter of fact, we can still impose a weaker condition, that the gauge potential must be a pure gauge. In the Abelian case,

$$
\tilde{A}_{\rho}(\pm\infty,\dot{y}) = \tilde{\partial}_{\rho}\phi(\pm\infty,\dot{y})
$$
\n(15)

or in the non-Abelian case

$$
\tilde{A}_{\rho}(\pm\infty,\dot{y}) = \omega^{-1}(\pm\infty,\dot{y})\tilde{\delta}_{\rho}\omega(\pm\infty,\dot{y}),\qquad(16)
$$

where $\omega = \exp(i\phi)$. In the non-Abelian case, $\tilde{A}_{\rho} = \tilde{A}_{\rho}^{a} t^{a}$ and $\phi \equiv \phi^a t^a$ where t^a are the generators of non-Abelian group in the fundamental representation. Keeping the leading term in the Tailor expansion of ω around ϕ , we recover the same expression as Eq. ([15](#page-2-0)) in the Abelian case. It follows that

$$
\phi(+\infty, \dot{y}) = -\int_{\dot{y}}^{\dot{\infty}} d\dot{\xi} \cdot \tilde{A}(+\infty, \dot{\xi}), \qquad (17)
$$

where the integral runs over any path on the hypersurface $y = \infty$. Notice that this equation always holds for Abelian gauge potential, and holds for the non-Abelian case only when the ϕ is small. It will be interesting thing to investigate what the nonleading terms contribute to in the non-Abelian case, which is beyond the scope of this paper. We will show that the linear term, such as in Eq. (17) , will lead to the gauge link at the light cone infinity.

FIG. 1. The tree diagram in DIS process.

IV. GAUGE LINK IN LIGHT CONE GAUGE IN SIDIS IN SIDIS

In DIS process, the hadronic tensor is defined by

$$
W^{\mu\nu} = \frac{1}{4\pi} \sum_{X} \int \frac{d^3 p_J}{(2\pi)^3} (2\pi)^4 \delta^{(4)}(P_X + p_J - p - q)
$$

× $\langle P|j^{\mu}(0)|p_J, X\rangle\langle p_J, X|j^{\nu}(0)|P\rangle.$ (18)

The tree scattering amplitude corresponding to Fig. [1](#page-2-2) reads

$$
M_0^{\mu} = \langle p_J, X | j^{\mu}(0) | P \rangle_{(0)} = \bar{u}(k+q) \gamma^{\mu} \langle X | \psi(0) | P \rangle, \tag{19}
$$

where k denotes the momentum of intial quark scattered by the photon with momentum q .

The one-gluon amplitude in light cone gauge corresponding to Fig. [2](#page-2-3) reads

$$
M_1^{\mu} = \int \frac{d^4 k_1}{(2\pi)^4} \int d^4 y_1 e^{i(k-k_1)\cdot y_1} \bar{u}(k+q) \gamma^{\rho_1} \times \frac{k_1 + \phi}{(k_1 + q)^2 + i\epsilon} \langle X | \tilde{A}_{\rho_1}(y_1) \gamma^{\mu} \psi(0) | P \rangle.
$$
 (20)

The quark propagator can be decomposed into two parts,

$$
\frac{\cancel{k}_1 + \cancel{q}}{(\cancel{k}_1 + \cancel{q})^2 + i\epsilon} = \frac{1}{2p \cdot (\hat{k}_1 + \cancel{q})} \bigg[\frac{\cancel{\hat{k}}_1 + \cancel{q}}{(x_1 - \hat{x}_1 + i\epsilon)} + \cancel{p} \bigg],\tag{21}
$$

where $\hat{k}_1 = (\hat{x}_1 p^+, k_1^-, k_{1\perp})$ with $\hat{x}_1 = \hat{k}^+/p^+$ $x_B + k_\perp^2/2p \cdot (k_1 + q)$ is determined by the on-shell condition $(\vec{k}_1 + q)^2 = 0$. Actually, to obtain the Eq. ([21](#page-2-4)), we have neglected the contribution

FIG. 2. The one-gluon exchange diagram in DIS process.

$$
\frac{\Theta(-k_1^- - q^-)}{2p \cdot (\hat{k}_1 + q)} \bigg[\frac{\hat{k}_1 + \phi}{(x_1 - \hat{x}_1 - i\epsilon)} + \phi \bigg], \qquad (22)
$$

which will contribute at higher twist level since they vanish in the limit $q^- \rightarrow +\infty$. The last term in Eq. [\(21\)](#page-2-4) is the socalled ''contact'' term of a normal propagator which does not propagate along the light cone coordinate [\[21\]](#page-8-16). Such a contact term will always result in higher twist contribution and does not contribute to gauge link at all. Hence, when we are considering the leading twist contribution in our following derivation, we can just drop such contact terms and only keep the pole terms, i.e. the first term in Eq. [\(21\)](#page-2-4):

$$
\hat{M}_1^{\mu} = \int \frac{d^3 \tilde{k}_1}{(2\pi)^4} \int d^3 \dot{y}_1 \int \frac{p^+ dx_1}{2\pi} \int dy_1^- \times e^{i(\tilde{k} - \tilde{k}_1) \cdot \dot{y}_1 + i(x - x_1)p^+ y^-} \frac{1}{2p \cdot (\hat{k}_1 + q)} \bar{u}(\hat{k} + q) \gamma^{\rho_1} \times \frac{\hat{k}_1 + \phi}{(x_1 - \hat{x}_1 + i\epsilon)} \langle X | \tilde{A}_{\rho_1}(y_1) \gamma^{\mu} \psi(0) | P \rangle, \tag{23}
$$

where another notation \hat{M}^{μ}_1 with extra an^is introduced to remind us that the only pole term is kept, and we have also separate the integral over x_1 and y_1^- from the others which means we will finish integrating them out first in the following. Before proceeding further, we should first choose a specific boundary condition for the gauge potential \tilde{A}_{ρ} at infinity. Let us start with the retarded boundary condition $\tilde{A}(\infty, \dot{y}) = 0$. Using the Eq. [\(12\)](#page-1-1) accordingly which corresponds to retarded boundary condition, we have

$$
\hat{M}_1^{\mu} = \int \frac{d^3 \tilde{k}_1}{(2\pi)^4} \int d^3 y_1 \int \frac{dx_1}{2\pi} \int dy_1^-
$$
\n
$$
\times e^{i(\tilde{k} - \tilde{k}_1) \cdot y_1 + i(x - x_1)p^+ y^-} \bar{u}(k + q) \gamma^{\rho_1}
$$
\n
$$
\times \frac{\hat{k}_1 + \phi}{2p \cdot (\hat{k}_1 + q)} \frac{1}{(x_1 - \hat{x}_1 + i\epsilon)} \frac{i}{(x - x_1 + i\epsilon)}
$$
\n
$$
\times \langle X | \partial^+ \tilde{A}_{\rho_1}(y_1) \gamma^\mu \psi(0) | P \rangle.
$$
\n(24)

Now, we can finish integrating over x_1 and y_1^- first,

$$
\int \frac{dx_1}{2\pi} \int dy_1^- e^{i(x-x_1)p^+y^-} \frac{1}{(x_1 - \hat{x}_1 + i\epsilon)} \frac{i}{(x - x_1 + i\epsilon)}
$$

\n
$$
\times \partial^+ \tilde{A}_{\rho_1}(y_1)
$$

\n
$$
= \frac{1}{x - \hat{x}_1} \int dy_1^- (\theta(y^-) e^{i(x-\hat{x}_1)p^+y^-} + \theta(-y^-)) \partial^+ \tilde{A}_{\rho_1}(y_1)
$$

\n
$$
= \frac{1}{x - \hat{x}_1} \int dy_1^- (\theta(y^-) + \theta(-y^-)) \partial^+ \tilde{A}_{\rho_1}(y_1)
$$

\n
$$
+ \text{ higher twist}
$$

\n
$$
= \frac{1}{x - \hat{x}_1} \tilde{A}_{\rho_1}(+\infty, \hat{y}_1) + \text{ higher twist},
$$
 (25)

where only the leading term in the Tailor expansion of the

phase factor $e^{i(x-\hat{x}_1)p^+y^-}$ is kept, because the other terms are proportional to $(x - \hat{x}_1)^n = [k_{\perp}^2/2p \cdot (k + q)$ $k_{1\perp}^2/2p \cdot (k_1 + q)^{n} \quad (n \ge 1)$, which will contribute at higher twist level. Only keep leading twist contribution and inserting Eq. ([25](#page-3-0)) into Eq. ([24](#page-3-1)), we have

$$
\hat{M}_1 = \int \frac{d^3 \tilde{k}_1}{(2\pi)^4} \int d^3 \dot{y}_1 e^{i(\tilde{k} - \tilde{k}_1) \cdot \dot{y}_1} \bar{u}(k + q) \gamma^{\rho_1} \times \frac{\hat{k}_1 + \phi}{2p \cdot (\hat{k}_1 + q)} \frac{1}{x - \hat{x}_1} \langle X | \tilde{A}_{\rho_1}(+\infty, \dot{y}_1) \psi(0) | P \rangle.
$$
\n(26)

Using Eq. ([15](#page-2-0)) and performing the integration by parts over \dot{y}_1 where $\tilde{\delta}_{\rho} \rightarrow -i(\tilde{k} - \tilde{k}_1)_{\rho}$, we obtain

$$
\hat{M}_1 = \int \frac{d^3 \tilde{k}_1}{(2\pi)^4} \int d^3 \dot{y}_1 e^{i(\tilde{k} - \tilde{k}_1) \cdot \dot{y}_1} \bar{u}(k + q) (\tilde{k} - \tilde{k}_1)
$$
\n
$$
\times \frac{\hat{k}_1 + \phi}{2p \cdot (\hat{k}_1 + q)} \frac{-i}{x - \hat{x}_1} \langle X | \phi(+\infty, \dot{y}_1) \psi(0) | P \rangle. \tag{27}
$$

To carry out the matrix algebra further, we note that

$$
\tilde{\mathbf{k}} - \tilde{\mathbf{k}}_1 = (\mathbf{k} + \mathbf{q}) - (\hat{\mathbf{k}}_1 + \mathbf{q}) - (x - \hat{x}_1)\mathbf{p},
$$
 (28)

together with the on-shell conditions

$$
\bar{u}(k+q)(k+q) = 0
$$
, and $(\hat{k}_1 + q)^2 = 0$. (29)

Using these equations, we reduce the \hat{M}_1 into

$$
\hat{M}_1 = \int \frac{d^3 \tilde{k}_1}{(2\pi)^4} \int d^3 \dot{y}_1 e^{i(\tilde{k} - \tilde{k}_1) \cdot \dot{y}_1} \bar{u}(k + q) \cancel{p}(\hat{k}_1 + q)
$$
\n
$$
\times \frac{-i}{2p \cdot (\hat{k}_1 + q)} \langle X | \phi(+\infty, \dot{y}_1) \psi(0) | P \rangle
$$
\n
$$
= \bar{u}(k + q) \langle X | i \phi(+\infty, 0) \psi(0) | P \rangle
$$
\n
$$
+ \int \frac{d^3 \tilde{k}_1}{(2\pi)^4} \int d^3 \dot{y}_1 e^{i(\tilde{k}_2 - \tilde{k}_1) \cdot \dot{y}_1} \bar{u}(k + q) (\tilde{k} - \tilde{k}_1) \cancel{p}
$$
\n
$$
\times \frac{-i}{2p \cdot (\hat{k}_1 + q)} \langle X | \phi(+\infty, \dot{y}_1) \psi(0) | P \rangle. \tag{30}
$$

Since the last term in Eq. ([30](#page-3-2)) only contributes to higher twist, keeping only the leading twist contribution, we finally obtain

$$
\hat{M}_1 = \bar{u}(k+q)\langle X|i\phi(+\infty,0)\psi(0)|P\rangle. \tag{31}
$$

So far, the previous derivations have been restricted to the retarded boundary condition where $A(-\infty, \dot{y}) = 0$, now let us turn to the other two boundary conditions. When we assign the advanced boundary condition, $\tilde{A}(+\infty, \dot{y}) = 0$, which means that we should choose the advanced one in Eqs. [\(12\)](#page-1-1) and [\(13\)](#page-1-2). Such a sign change in the pole structure will lead to replacing the integration in Eq. (25) (25) by

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$$
\int \frac{dx_1}{2\pi} \int dy_1^- e^{i(x-x_1)p^+y^-} \frac{1}{(x_1 - \hat{x}_1 + i\epsilon)} \frac{i}{(x - x_1 - i\epsilon)}
$$

\n
$$
\times \partial^+ \tilde{A}_{\rho_1}(y_1)
$$

\n
$$
= \frac{1}{x - \hat{x}_1} \int dy_1^- (\theta(y^-) e^{i(x-\hat{x}_1)p^+y^-} - \theta(y^-)) \partial^+ \tilde{A}_{\rho_1}(y_1)
$$

\n
$$
= \text{higher twist.}
$$
\n(32)

We note that, different from retarded case, the leading contributions from two poles have canceled each other completely, and there will be no gauge link at all. As shown by [\[8](#page-8-6)], all final state interactions have been included into the initial state light cone wave functions.

If we choose the antisymmetry boundary condition, which corresponds to the principal value regularization, we have

$$
\int \frac{dx_1}{2\pi} \int dy_1^- e^{i(x-x_1)p^+y^-} \frac{1}{(x_1 - \hat{x}_1 + i\epsilon)} \text{PV} \frac{i}{(x - x_1)}
$$

\n
$$
\times \partial^+ \tilde{A}_{\rho_1}(y_1)
$$

\n
$$
= \frac{1}{x - \hat{x}_1} \int dy_1^- \frac{1}{2} (2\theta(y^-) e^{i(x-\hat{x}_1)p^+y^-} - \theta(y^-)
$$

\n
$$
+ \theta(-y^-)) \partial^+ \tilde{A}_{\rho_1}(y_1)
$$

\n
$$
= \frac{1}{x - \hat{x}_1} \int dy_1^- \frac{1}{2} (\theta(y^-) + \theta(-y^-)) \partial^+ \tilde{A}_{\rho_1}(y_1)
$$

\n
$$
+ \text{ higher twist}
$$

\n
$$
= \frac{1}{x - \hat{x}_1} \tilde{A}_{\rho_1}(+\infty, \hat{y}_1) + \text{ higher twist},
$$
 (33)

where PV denotes principal value. The above result appear the same as the one in the retarded boundary condition. The difference between retarded and principal value regularization is that final state scattering effects appear only through the gauge link in principal regularization, while

FIG. 3. The two-gluon exchange diagram in DIS process.

they appear through both the gauge link and initial light cone wave functions in retarded regularization. Such detailed discussion and illustration can be found in Ref. [[8\]](#page-8-6). In the above derivation, we notice that the pinched poles are needed to pick up the gauge potential at the light cone infinity, which will be shown to result in the gauge link that we expect. In the following, we will only concentrate on the retarded boundary condition in the following derivation.

Now let us consider further the two-gluon exchange scattering amplitude in Fig. [3,](#page-4-0)

$$
M_{2}^{\mu} = \int \frac{d^{4}k_{2}}{(2\pi)^{4}} \frac{d^{4}k_{1}}{(2\pi)^{4}} \int d^{4}y_{2}d^{4}y_{1}
$$

\n
$$
\times e^{i(k-k_{2}) \cdot y_{2} + i(k_{2} - k_{1}) \cdot y_{1}} \bar{u}(k+q)\gamma^{\rho_{2}}
$$

\n
$$
\times \frac{k_{2} + \phi}{(k_{2} + q)^{2} + i\epsilon} \gamma^{\rho_{1}} \frac{k_{1} + \phi}{(k_{1} + q)^{2} + i\epsilon}
$$

\n
$$
\times \langle X|\tilde{A}_{\rho_{2}}(y_{2})\tilde{A}_{\rho_{1}}(y_{1})\gamma^{\mu}\psi(0)|P\rangle.
$$
 (34)

Just following what we did in the M_1^{μ} , we drop the contact terms which do not contribute in leading twist level and label the residual terms as \hat{M}_2 , which is given by

$$
\hat{M}_{2}^{\mu} = \int \frac{d^{3}\tilde{k}_{2}}{(2\pi)^{3}} \frac{d^{3}\tilde{k}_{1}}{(2\pi)^{3}} \int d^{3}\dot{y}_{2} d^{3}\dot{y}_{1} \int \frac{p^{+}dx_{2}}{2\pi} \frac{p^{+}dx_{1}}{2\pi} \int dy_{2}^{-}dy_{1}^{-}e^{i(\tilde{k}-\tilde{k}_{2})\cdot\dot{y}_{2}+i(\tilde{k}_{2}-\tilde{k}_{1})\cdot\dot{y}_{1}+i(x-x_{2})p^{+}y_{2}^{-}+i(x_{2}-x_{1})p^{+}y_{1}^{-}} \bar{u}(k+q)\gamma^{\rho_{2}} \times \frac{\hat{k}_{2} + \phi}{2p \cdot (\hat{k}_{2} + q)} \gamma^{\rho_{1}} \frac{\hat{k}_{1} + \phi}{2p \cdot (\hat{k}_{1} + q)} \frac{1}{(x_{2} - \hat{x}_{2} + i\epsilon)} \frac{1}{(x_{1} - \hat{x}_{1} + i\epsilon)} \langle X|\tilde{A}_{\rho_{2}}(y_{2})\tilde{A}_{\rho_{1}}(y_{1})\gamma^{\mu}\psi(0)|P\rangle.
$$
\n(35)

Still with the help of the regularization in Eqs. [\(12\)](#page-1-1) and ([13](#page-1-2)), let us do integrating over x_1 and y_1^- first,

$$
\hat{M}_{2}^{\mu} = \int \frac{d^{3}\tilde{k}_{2}}{(2\pi)^{3}} \frac{d^{3}\tilde{k}_{1}}{(2\pi)^{3}} \int d^{3}\dot{y}_{2} d^{3}\dot{y}_{1} \int \frac{dx_{2}}{2\pi} \frac{p^{+}dx_{1}}{2\pi} \int dy_{2}^{-} dy_{1}^{-} e^{i(\tilde{k}-\tilde{k}_{2}) \cdot \dot{y}_{2} + i(\tilde{k}_{2}-\tilde{k}_{1}) \cdot \dot{y}_{1} + i(x-x_{2})p^{+}y_{2}^{-} + i(x_{2}-x_{1})p^{+}y_{1}^{-} \tilde{u}(k+q)\gamma^{\rho_{2}} \times \frac{\hat{k}_{2} + \phi}{2p \cdot (\hat{k}_{2} + q)} \gamma^{\rho_{1}} \frac{\hat{k}_{1} + \phi}{2p \cdot (\hat{k}_{1} + q)} \frac{1}{(x_{2} - \hat{x}_{2} + i\epsilon)} \frac{i}{(\hat{x} - x_{2} + i\epsilon)} \frac{1}{(x_{1} - \hat{x}_{1} + i\epsilon)} \langle X|\partial^{+}\tilde{A}_{\rho_{2}}(y_{2})\tilde{A}_{\rho_{1}}(y_{1})\gamma^{\mu} \psi(0)|P\rangle \n= \int \frac{d^{3}\tilde{k}_{2}}{(2\pi)^{3}} \frac{d^{3}\tilde{k}_{1}}{(2\pi)^{3}} \int d^{3}\dot{y}_{2} d^{3}\dot{y}_{1} \int \frac{p^{+}dx_{1}}{2\pi} \int dy_{1}^{-} e^{i(\tilde{k}-\tilde{k}_{2}) \cdot \dot{y}_{2} + i(\tilde{k}_{2}-\tilde{k}_{1}) \cdot \dot{y}_{1} + i(x-x_{1})p^{+}y_{1}^{-} \times \tilde{u}(k+q)\gamma^{\rho_{2}} \frac{\hat{k}_{2} + \phi}{2p \cdot (\hat{k}_{2} + q)} \gamma^{\rho_{1}} \times \frac{\hat{k}_{1} + \phi}{2p \cdot (\hat{k}_{1} + q)} \frac{1}{(x - \hat{x}_{2} + i\epsilon)} \frac{1}{(x_{1} - \hat{x}_{1} + i\epsilon)} \langle X|\tilde{A}_{\rho_{2}}(+\infty, \dot{y}_{2})\tilde{A}_{\rho_{1}}(y_{1})\gamma^{\mu} \psi(0)|P\rangle.
$$
\n(36)

Further integrating over x_2 and y_2^- , which is totally the same as what we did with x_1 and y_1^- . The results read

$$
\hat{M}_{2}^{\mu} = \int \frac{d^{3}\tilde{k}_{2}}{(2\pi)^{3}} \frac{d^{3}\tilde{k}_{1}}{(2\pi)^{3}} \int d^{3}\dot{y}_{2} d^{3}\dot{y}_{1} e^{i(\tilde{k}-\tilde{k}_{2})\cdot\dot{y}_{2}+i(\tilde{k}_{2}-\tilde{k}_{1})\cdot\dot{y}_{1}} \bar{u}(k+q)\gamma^{\rho_{2}} \frac{\hat{k}_{2}+q}{2p\cdot(\hat{k}_{2}+q)} \gamma^{\rho_{1}} \frac{\hat{k}_{1}+q}{2p\cdot(\hat{k}_{1}+q)} \frac{1}{(x-\hat{x}_{2}+i\epsilon)} \times \frac{1}{(x-\hat{x}_{1}+i\epsilon)} \langle X|\tilde{A}_{\rho_{2}}(+\infty,\dot{y}_{2})\tilde{A}_{\rho_{1}}(+\infty,\dot{y}_{1})\gamma^{\mu}\psi(0)|P\rangle \n= \int \frac{d^{3}\tilde{k}_{2}}{(2\pi)^{3}} \frac{d^{3}\tilde{k}_{1}}{(2\pi)^{3}} \int d^{3}\dot{y}_{2} d^{3}\dot{y}_{1} e^{i(\tilde{k}-\tilde{k}_{2})\cdot\dot{y}_{2}+i(\tilde{k}_{2}-\tilde{k}_{1})\cdot\dot{y}_{1}} \bar{u}(k+q)\gamma^{\rho_{2}} \frac{\hat{k}_{2}+q}{2p\cdot(\hat{k}_{2}+q)} \gamma^{\rho_{1}} \frac{\hat{k}_{1}+q}{2p\cdot(\hat{k}_{1}+q)} \frac{1}{(x-\hat{x}_{2}+i\epsilon)} \times \frac{1}{(x-\hat{x}_{1}+i\epsilon)} \langle X|\tilde{\partial}_{\rho_{2}}\phi(+\infty,\dot{y}_{2})\tilde{\partial}_{\rho_{1}}\phi(+\infty,\dot{y}_{1})\gamma^{\mu}\psi(0)|P\rangle.
$$
\n(37)

Now, we are in a position to perform integrating over \tilde{k}_2 and \dot{y}_2 . Thanks to the integration by parts and the algebras given in Eqs. (28) (28) (28) and (29) , we obtain

$$
\hat{M}_{2}^{\mu} = \int \frac{d^{3}\tilde{k}_{1}}{(2\pi)^{3}} \int d^{3}y_{1} e^{i(\tilde{k}-\tilde{k}_{1})\cdot\dot{y}_{1}} \bar{u}(k+q)\gamma^{\rho_{1}} \frac{\hat{k}_{1} + \phi}{2p \cdot (\hat{k}_{1} + q)} \frac{1}{(x - \hat{x}_{1} + i\epsilon)} \langle X | i\phi(+\infty, \dot{y}_{1})\tilde{\theta}_{\rho_{1}}\phi(+\infty, \dot{y}_{1})\gamma^{\mu}\psi(0) | P \rangle
$$

\n
$$
= \int \frac{d^{3}\tilde{k}_{1}}{(2\pi)^{3}} \int d^{3}y_{1} e^{i(\tilde{k}_{2} - \tilde{k}_{1})\cdot\dot{y}_{1}} \bar{u}(k+q)\gamma^{\rho_{1}} \frac{\hat{k}_{1} + \phi}{2p \cdot (\hat{k}_{1} + q)} \frac{1}{(x - \hat{x}_{1} + i\epsilon)} \langle X | \frac{i}{2} \tilde{\partial}_{\rho_{1}}\phi^{2}(+\infty, \dot{y}_{1})\gamma^{\mu}\psi(0) | P \rangle.
$$
 (38)

Repeat what we did with \tilde{k}_2 and \dot{y}_2 above, and we can finish integrating over \tilde{k}_1 and \dot{y}_1 and finally arrive at

$$
\hat{M}_{2}^{\mu} = \bar{u}(k+q)\langle X|\frac{i^{2}}{2!}\phi^{2}(+\infty,0)\gamma^{\mu}\psi(0)|P\rangle.
$$
\n(39)

All through calculating \hat{M}_2 , as we did with \hat{M}_1 , we have neglected the higher twist contributions and only keep the leading twist terms. From M_1 to M_2 , it is obvious that our procedure can be easily extended to *n*-gluon exchange amplitude M_n in Fig. [4,](#page-6-0) which is given by

$$
\hat{M}_{n} = \int \prod_{j=1}^{n} \frac{d^{3} \tilde{k}_{j}}{(2\pi)^{3}} d^{3} \dot{y}_{j} e^{i(\tilde{k}_{n} - \tilde{k}_{n-1}) \cdot \dot{y}_{n} + i(\tilde{k}_{n-1} - \tilde{k}_{n-2}) \cdot \dot{y}_{n-1} + \dots + i(\tilde{k}_{2} - \tilde{k}_{1}) \cdot \dot{y}_{2}} \prod_{j=1}^{n} \frac{p^{+} dx_{j}}{2\pi} dy_{j}^{-} e^{i(x_{n+1} - x_{n})p^{+} y_{n}^{-} + i(x_{n} - x_{n-1})p^{+} y_{n-1}^{-} + \dots + i(x_{2} - x_{1})p^{+} y_{1}^{-}} \times \bar{u}(k+q) \gamma^{\rho_{n}} \frac{\hat{k}_{n} + q}{2p \cdot (\hat{k}_{n} + q)} \cdots \gamma^{\rho_{1}} \frac{\hat{k}_{1} + q}{2p \cdot (\hat{k}_{1} + q)} \frac{1}{(x_{n} - \hat{x}_{n} + i\epsilon)} \cdots \frac{1}{(x_{1} - \hat{x}_{1} + i\epsilon)}
$$
\n
$$
\times \langle X | \tilde{A}_{\rho_{n}}(y_{n}) \tilde{A}_{\rho_{n-1}}(y_{n-1}) \cdots \tilde{A}_{\rho_{1}}(y_{1}) \psi(0) | P \rangle.
$$
\n(40)

We first finish integrating from x_n , y_n^- to x_1 , y_1^- one by one. Keeping the leading twist contribution, we have

$$
\hat{M}_{n} = \int \prod_{j=1}^{n} \frac{d^{3} \tilde{k}_{j}}{(2\pi)^{3}} d^{3} y_{j} e^{i(\tilde{k}_{n+1} - \tilde{k}_{n}) \cdot \hat{y}_{n} + i(\tilde{k}_{n} - \tilde{k}_{n-1}) \cdot \hat{y}_{n-1} + \dots + i(\tilde{k}_{2} - \tilde{k}_{1}) \cdot \hat{y}_{2}} \bar{u}(k+q) \gamma^{\rho_{n}} \frac{\hat{k}_{n} + \phi}{2p \cdot (\hat{k}_{n} + q)} \cdots \gamma^{\rho_{1}} \frac{\hat{k}_{1} + \phi}{2p \cdot (\hat{k}_{1} + q)} \times \frac{1}{(\hat{x}_{n+1} - \hat{x}_{n})} \cdots \frac{1}{(\hat{x}_{2} - \hat{x}_{1})} \langle X | \tilde{A}_{\rho_{n}}(+\infty, \dot{y}_{n}) \tilde{A}_{\rho_{n-1}}(+\infty, \dot{y}_{n-1}) \cdots \tilde{A}_{\rho_{1}}(+\infty, \dot{y}_{1}) \psi(0) | P \rangle, \tag{41}
$$

or using

$$
\tilde{A}_{\rho_n}(+\infty, \dot{y}_n) = \tilde{\partial}_{\rho_n} \phi(+\infty, \dot{y}_n),\tag{42}
$$

we rewrite it as

$$
\hat{M}_{n} = \int \prod_{j=1}^{n} \frac{d^{3} \tilde{k}_{j}}{(2\pi)^{3}} d^{3} \dot{y}_{j} e^{i(\tilde{k}_{n+1} - \tilde{k}_{n}) \cdot \dot{y}_{n} + i(\tilde{k}_{n} - \tilde{k}_{n-1}) \cdot \dot{y}_{n-1} + \dots + i(\tilde{k}_{2} - \tilde{k}_{1}) \cdot \dot{y}_{2}} \bar{u}(k+q) \gamma^{\rho_{n}} \frac{\hat{k}_{n} + \phi}{2p \cdot (\hat{k}_{n} + q)} \cdots \gamma^{\rho_{1}} \frac{\hat{k}_{1} + \phi}{2p \cdot (\hat{k}_{1} + q)} \times \frac{1}{(\hat{x}_{n+1} - \hat{x}_{n})} \cdots \frac{1}{(\hat{x}_{2} - \hat{x}_{1})} \langle X | \tilde{\partial}_{\rho_{n}} \phi(+\infty, \dot{y}_{n}) \tilde{\partial}_{\rho_{n-1}} \phi(+\infty, \dot{y}_{n-1}) \cdots \tilde{\partial}_{\rho_{1}} \phi(+\infty, \dot{y}_{1}) \psi(0) | P \rangle.
$$
\n(43)

FIG. 4. The n-gluon exchange diagram in DIS process.

Continue to integrating over from \tilde{k}_n and \dot{y}_n to \tilde{k}_1 and \dot{y}_1 one by one, we can finally have

$$
\hat{M}_n = \bar{u}(k+q)\langle X|\frac{i^n}{n!}\phi^n(+\infty,0)\psi(0)|P\rangle.
$$
 (44)

As a final step, we should resum to all orders and obtain

$$
\sum_{n=0}^{\infty} \hat{M}_n = \bar{u}(k+q)\langle X| \exp(i\phi(+\infty,0))\psi(0)|P\rangle, \quad (45)
$$

or the more conventional form

$$
\sum_{n=0}^{\infty} \hat{M}_n = \bar{u}(k+q)\langle X|P\exp\left(-i\int_{\dot{y}}^{\dot{\infty}} d\dot{\xi} \cdot \tilde{A}(+\infty, \dot{\xi})\right) \times \psi(0)|P\rangle, \tag{46}
$$

where $P \exp(-i \int_{y}^{\infty} d\dot{\xi} \cdot \tilde{A}(+\infty, \dot{\xi}))$ is just the gauge link .
.
. that we tried to derive. It should be noted that the gauge link we obtain in the final result Eq. [\(46\)](#page-6-1) is over the hypersurface at light cone infinity along any path integral, not restricted along the transverse direction, which means that it is more general than what Belitsky, Ji, and Yuan have obtained in Ref. [[8\]](#page-8-6).

V. GAUGE LINK IN LIGHT CONE GAUGE IN DY

Now let us turn to the DY process, which is represented in Fig. [5](#page-6-2), where, for brevity, we have fixed the target to be a nucleon and the projectile to be just an antiquark, q is the virtual photon's momentum and $q - k$ and p is momentum of the projectile and target, respectively. Such simplifying does not lose any generality when we are only considering

 $\overline{\chi}$

FIG. 5. The one-gluon exchange diagram in DY process.

k,

how to derive the gauge link, but it will be more convenient and manifest to compare with the SIDIS process. We still choose the light cone coordinate system, and use the two lightlike vectors n^{μ} and \bar{n}^{μ} to fix "plus" and "minus" directions. All the notations and conventions are the same as in DIS. The one-gluon exchange amplitude reads

$$
M_{1(DY)}^{\mu} = \int \frac{d^4 k_1}{(2\pi)^4} \int d^4 y_1 e^{i(k-k_1)\cdot y_1} \bar{u}(q-k) \gamma^{\rho_1}
$$

$$
\times \frac{\oint q - \not{k_1}}{(q-k_1)^2 + i\epsilon} \langle X|\tilde{A}_{\rho_1}(y_1)\gamma^{\mu}\psi(0)|P\rangle. \tag{47}
$$

Dropping the contact terms and assigning the retarded boundary condition, we rewrite it as

$$
\hat{M}_{1(DY)}^{\mu} = \int \frac{d^3 \tilde{k}_1}{(2\pi)^4} \int d^3 \dot{y}_1 \int \frac{dx_1}{2\pi} \int dy_1^- e^{i(\tilde{k} - \tilde{k}_1) \cdot \dot{y}_1} \times e^{i(x - x_1)p^+ y^-} \bar{u}(q - k) \gamma^{\rho_1} \frac{q - \hat{k}_1}{2p \cdot (\hat{k}_1 - q)} \times \frac{1}{(x_1 - \hat{x}_1 - i\epsilon)} \frac{i}{(x - x_1 + i\epsilon)} \times \langle X | \partial^+ \tilde{A}_{\rho_1}(y_1) \gamma^\mu \psi(0) | P \rangle.
$$
\n(48)

It should be noticed the difference of the pole structure between Eqs. ([48](#page-6-3)) and ([24](#page-3-1)). Just like we did in the SIDIS, we can finish integrating over x_1 and y_1^- first,

$$
\int \frac{dx_1}{2\pi} \int dy_1^- e^{i(x-x_1)p^+y^-} \frac{1}{(x_1 - \hat{x}_1 - i\epsilon)} \frac{i}{(x - x_1 + i\epsilon)} \partial^+ \tilde{A}_{\rho_1}(y_1)
$$

= $-\frac{1}{x - \hat{x}_1} \int dy_1^- (\theta(-y^-) e^{i(x-\hat{x}_1)p^+y^-} - \theta(-y^-)) \partial^+ \tilde{A}_{\rho_1}(y_1)$
= $-\frac{1}{x - \hat{x}_1} \int dy_1^- (\theta(-y^-) - \theta(-y^-)) \partial^+ \tilde{A}_{\rho_1}(y_1) + \text{higher twist} = \text{higher twist.}$ (49)

Opposite to the case in SIDIS, the retarded boundary condition does not lead to the gauge link at the light cone infinity and hence all the final state interaction effects must be shifted into the initial light cone wave functions. For the advanced boundary condition, we have

$$
\hat{M}_{1(DY)}^{\mu} = \int \frac{d^3 \tilde{k}_1}{(2\pi)^4} \int d^3 y_1 \int \frac{dx_1}{2\pi} \int dy_1^- e^{i(\tilde{k} - \tilde{k}_1) \cdot \dot{y}_1} e^{i(x - x_1)p^+ y^-} \bar{u}(q - k) \gamma^{\rho_1} \frac{\phi - \hat{k}_1}{2p \cdot (\hat{k}_1 - q)} \frac{1}{(x_1 - \hat{x}_1 - i\epsilon)} \frac{i}{(x - x_1 - i\epsilon)} \times \langle X | \partial^+ \tilde{A}_{\rho_1}(y_1) \gamma^\mu \psi(0) | P \rangle.
$$
\n(50)

Finish integrating over x_1 and y_1^- :

$$
\int \frac{dx_1}{2\pi} \int dy_1^- e^{i(x-x_1)p^+y^-} \frac{1}{(x_1 - \hat{x}_1 - i\epsilon)} \frac{i}{(x - x_1 - i\epsilon)} \partial^+ \tilde{A}_{\rho_1}(y_1)
$$

= $-\frac{1}{x - \hat{x}_1} \int dy_1^- (\theta(-y^-) e^{i(x-\hat{x}_1)p^+y^-} + \theta(y^-)) \partial^+ \tilde{A}_{\rho_1}(y_1)$
= $-\frac{1}{x - \hat{x}_1} \int dy_1^- (\theta(-y^-) + \theta(y^-)) \partial^+ \tilde{A}_{\rho_1}(y_1) + \text{higher twist} = \frac{1}{x - \hat{x}_1} \tilde{A}_{\rho_1}(-\infty, \dot{y}_1) + \text{higher twist.}$ (51)

If we choose the antisymmetry boundary condition, we have

$$
\int \frac{dx_1}{2\pi} \int dy_1^- e^{i(x-x_1)\rho^+ y^-} \frac{1}{(x_1 - \hat{x}_1 - i\epsilon)} \text{PV} \frac{i}{(x - x_1)} \partial^+ \tilde{A}_{\rho_1}(y_1)
$$
\n
$$
= \frac{1}{x - \hat{x}_1} \int dy_1^- \frac{1}{2} (-2\theta(-y^-) e^{i(x-\hat{x}_1)\rho^+ y^-} - \theta(y^-) + \theta(-y^-)) \partial^+ \tilde{A}_{\rho_1}(y_1)
$$
\n
$$
= -\frac{1}{x - \hat{x}_1} \int dy_1^- \frac{1}{2} (\theta(y^-) + \theta(-y^-)) \partial^+ \tilde{A}_{\rho_1}(y_1) + \text{higher twist} = \frac{1}{x - \hat{x}_1} \tilde{A}_{\rho_1}(-\infty, \dot{y}_1) + \text{ higher twist.} \tag{52}
$$

The difference between advanced and principal value regularization in the DY process is that final state scattering effects appear only through the gauge link in principal value regularization, while they appear through both the gauge link and initial light cone wave functions in advanced regularization. It follows that

$$
\hat{M}_{1(DY)} = \bar{u}(q-k)\langle X| \left(-i \int_{-\infty}^{y} d\dot{\xi} \cdot \tilde{A}(-\infty, \dot{\xi})\right) \psi(0) | P \rangle. \tag{53}
$$

Just as in the SIDIS process, the pinched poles are indispensable to produce a finite contribution in the leading twist level. Following the same line as the one carried out in SIDIS, we show that the gauge link in advanced or antisymmetry boundary conditions is given by

$$
\sum_{n=0}^{\infty} \hat{M}_{n(DY)} = \bar{u}(q-k)\langle X|P\exp\left(-i\int_{-\infty}^{y} d\dot{\xi} \cdot \tilde{A}(-\infty, \dot{\xi})\right) \times \psi(0)|P\rangle.
$$
\n(54)

It should be noted that the light cone infinity $y^- = +\infty$ has been replaced by $y^- = -\infty$, reflecting that the gauge link arises from the initial state interactions rather than from the final state.

To summarize, in light cone gauge, we should choose a specific boundary condition first to fix the residual gauge freedom. Using the proper regularization corresponding to specific boundary condition, we can obtain the residual gauge link at infinity along the light cone coordinate. We find the gauge link at light cone infinity arises naturally from the pinched poles: one is from the quark propagator and the other is hidden in the gauge vector field in light cone gauge. Actually, it turns out that we obtain a more general gauge link over hypersurface $y^- = \infty$, which is beyond the transverse gauge link. The difference of such gauge link between SIDIS and DY processes can also be obtained directly and clearly in our derivation. We expect our regularization method will also be valuable to make it possible to perform higher twist calculations in light cone gauge more unambiguously.

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- [1] R.P. Feynman, *Photon Hadron Interactions* (W. A. Benjamin, New York, 1971).
- [2] J.C. Collins, D.E. Soper, [Nucl. Phys.](http://dx.doi.org/10.1016/0550-3213(82)90021-9) **B194**, 445 [\(1982\)](http://dx.doi.org/10.1016/0550-3213(82)90021-9).
- [3] J. C. Collins, Adv. Ser. Dir. High Energy Phys. 5, 573 (1989).
- [4] J. C. Collins, Acta Phys. Pol. B 34, 3103 (2003).
- [5] J. C. Collins, Nucl. Phys. **B396**[, 161 \(1993\).](http://dx.doi.org/10.1016/0550-3213(93)90262-N)
- [6] S. J. Brodsky, D. S. Hwang, and I. Schmidt, [Phys. Lett. B](http://dx.doi.org/10.1016/S0370-2693(02)01320-5) 530[, 99 \(2002\)](http://dx.doi.org/10.1016/S0370-2693(02)01320-5).
- [7] X. d. Ji and F. Yuan, *[Phys. Lett. B](http://dx.doi.org/10.1016/S0370-2693(02)02384-5)* **543**, 66 (2002).
- [8] A. V. Belitsky, X. Ji, and F. Yuan, [Nucl. Phys.](http://dx.doi.org/10.1016/S0550-3213(03)00121-4) **B656**, 165 [\(2003\)](http://dx.doi.org/10.1016/S0550-3213(03)00121-4).
- [9] D. Boer, P.J. Mulders, and F. Pijlman [Nucl. Phys.](http://dx.doi.org/10.1016/S0550-3213(03)00527-3) **B667**, [201 \(2003\)](http://dx.doi.org/10.1016/S0550-3213(03)00527-3).
- [10] A. Idilbi and A. Majumder [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.80.054022) 80, 054022 [\(2009\)](http://dx.doi.org/10.1103/PhysRevD.80.054022).
- [11] Y. L. Dokshitzer, D. Diakonov, and S. I. Troian, [Phys. Rep.](http://dx.doi.org/10.1016/0370-1573(80)90043-5) 58[, 269 \(1980\)](http://dx.doi.org/10.1016/0370-1573(80)90043-5).
- [12] A. H. Mueller, Phys. Rep. **73**[, 237 \(1981\).](http://dx.doi.org/10.1016/0370-1573(81)90030-2)
- [13] A. Bassetto, M. Dalbosco, I. Lazzizzera, and R. Soldati, Phys. Rev. D 31[, 2012 \(1985\)](http://dx.doi.org/10.1103/PhysRevD.31.2012).
- [14] G. Leibbrandt, Phys. Rev. D **29**[, 1699 \(1984\).](http://dx.doi.org/10.1103/PhysRevD.29.1699)
- [15] A. Bassetto, I. Lazzizzera, and R. Soldati, [Nucl. Phys.](http://dx.doi.org/10.1016/0550-3213(84)90538-8) B236[, 319 \(1984\)](http://dx.doi.org/10.1016/0550-3213(84)90538-8).
- [16] G. P. Lepage and S. J. Brodsky, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.22.2157) 22, 2157 [\(1980\)](http://dx.doi.org/10.1103/PhysRevD.22.2157).
- [17] G. Leibbrandt, [Rev. Mod. Phys.](http://dx.doi.org/10.1103/RevModPhys.59.1067) **59**, 1067 (1987).
- [18] A. A. Slavnov and S. A. Frolov, Teor. Mat. Fiz. 73, 199 (1987) [\[Theor. Math. Phys.](http://dx.doi.org/10.1007/BF01017585) 73, 1158 (1987)].
- [19] Y. V. Kovchegov, Phys. Rev. D **55**[, 5445 \(1997\).](http://dx.doi.org/10.1103/PhysRevD.55.5445)
- [20] A. Bassetto, I. Lazzizzera, and R. Soldati, [Phys. Lett.](http://dx.doi.org/10.1016/0370-2693(81)90830-3) 107B[, 278 \(1981\)](http://dx.doi.org/10.1016/0370-2693(81)90830-3).
- [21] J. W. Qiu, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.42.30) **42**, 30 (1990).