Vacuum phenomenology of the chiral partner of the nucleon in a linear sigma model with vector mesons

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We investigate a linear sigma model with global chiral $U(2)_R \times U(2)_L$ symmetry. The mesonic degrees of freedom are the standard scalar and pseudoscalar mesons and the vector and axial-vector mesons. The baryonic degrees of freedom are the nucleon, N, and its chiral partner, N^*, which is usually identified with $N(1535)$. The chiral partner is incorporated in the so-called mirror assignment, where the nucleon mass is not solely generated by the chiral condensate but also by a chirally invariant mass term, m_0. The presence of (axial-) vector fields modifies the expressions for the axial-coupling constants of the nucleon, g_A^N, and its partner, $g_A^{N^*}$. Using experimental data for the decays $N^* \to N\pi$ and $a_1 \to \pi\gamma$, as well as lattice results for $g_A^{N^*}$ we infer that in our model $m_0 \sim 500$ MeV, i.e., an appreciable amount of the nucleon mass originates from sources other than the chiral condensate. We test our model by evaluating the decay $N^* \to N\eta$ and the s-wave nucleon-pion scattering lengths $a_0^{(\pm)}$.

I. INTRODUCTION

The theory of the strong interaction, quantum chromodynamics (QCD), has a global chiral $U(N_f)_R \times U(N_f)_L$ symmetry, for N_f flavors of massless quarks. This symmetry is spontaneously broken in the vacuum, which has important consequences for hadron phenomenology. Because of confinement of color charges, all low-energy hadronic properties, such as masses, decay widths, scattering lengths, etc. cannot be inferred from perturbative QCD calculations. Therefore, effective chiral models are widely used in order to study the vacuum properties of hadrons. Viable candidates should obey a well-defined set of low-energy theorems [1–3], but they may still differ in some interesting aspects such as the generation of the nucleon mass and the behavior at nonzero temperature T and chemical potential μ.

A nucleon mass term $\sim m_N \bar{\Psi}\Psi$ explicitly breaks the chiral $U(N_f)_R \times U(N_f)_L$ symmetry and thus should not occur in a chiral linear sigma model. Therefore, in the standard linear sigma model of Refs. [2,4], the nucleon mass is (mostly) generated by the chiral condensate, $\langle \bar{q}q \rangle$. (A small contribution also arises from the explicit breaking of chiral symmetry due to the nonzero current quark masses.) Similarly, in the framework of QCD sum rules Ioffe [5] formulated a connection between the quark condensate and the nucleon mass, now called Ioffe formula: $m_N \sim -4\pi^2 \Lambda_B^{-2}\langle \bar{q}q \rangle$, where $\Lambda_B \simeq 1$ GeV.

However, also other condensates exist, e.g. a gluon condensate, and it is not yet known to what extent they contribute to the nucleon mass [6]. This problem can be studied in a chiral model via the so-called mirror assignment for the chiral partner of the nucleon, which was first discussed in Ref. [4] and extensively analyzed in Refs. [7,8]. In this assignment, there exists a chirally invariant mass term $\sim m_0$ which does *not* originate from the quark condensate. The mirror assignment has been subsequently used in Ref. [9] to study the properties of cold and dense nuclear matter.

In this work we consider a linear sigma model with global chiral $U(2)_R \times U(2)_L$ symmetry which includes scalar and pseudoscalar mesons as well as vector and axial-vector mesons [10]. We extend this model by including the nucleon and its chiral partner in the mirror assignment. The most natural candidate for the chiral partner of the nucleon is the resonance $N(1535)$ which is the lightest state with the correct quantum numbers ($J^P = \frac{1}{2}^-$) listed in the PDG [11]. We also investigate two other possibilities: the well-identified resonance $N(1650)$ and a speculative, very broad, and not yet discovered resonance with mass about 1.2 GeV, which has been proposed in Ref. [9].

We first study their axial charges which have been the focus of interest in recent studies of hadron phenomenology [see Ref. [12] and refs. therein]. We show that, in the present model, including (axial-) vector mesons drastically changes the relations of the original model [7]. Without (axial-) vector mesons, N and N^* have opposite axial charge, $g_A^N = -g_A^{N^*} \leq 1$. (We remind the reader that, in the so-called "naive assignment," where the nucleon partner transforms just as the nucleon, one has $g_A^N = g_A^{N^*} = 1$

[8]). With (axial-) vector mesons, this is no longer true and we are free to adjust the two axial charges independently, employing experimental knowledge about g_A^N and recent lattice QCD data for $g_A^{N^*}$ [13].

Using the decays $N^* \to N\pi$ and $a_1 \to \pi\gamma$ to determine the other parameters of the model, the mass parameter turns out to be $m_0 \sim 500$ MeV. This value is between the one derived in Ref. [7] and the one from Ref. [9].

We then test our model studying the decay $N^* \to N\eta$ and pion-nucleon scattering. For $N(1535)$ as chiral partner of the nucleon, the decay width $N^* \to N\eta$ comes out too small, while for $N(1650)$, it agrees well with experimental data. Pion-nucleon scattering has been studied in a large variety of approaches [see Refs. [14–17] and refs. therein]. Here, we evaluate the scattering lengths in the framework of the mirror assignment. We find that the isospin-odd s-wave scattering length $a_0^{(-)}$ is in good agreement with experimental data, while the isospin-even scattering length $a_0^{(+)}$ strongly depends on the value for the mass of the sigma meson.

Finally, we discuss two possible extensions of our work. The first is an enlarged mixing scenario. A second pair of chiral partners is added, e.g. $N(1440)$ and $N(1650)$, which also mix with $N(939)$ and $N(1535)$. The second is the generalization of the chirally invariant mass term $\sim m_0$ to a dilatation-invariant mass term. In this case, we argue that m_0 is a sum of two contributions, arising from the tetraquark and the gluon condensates, respectively. The dilatation-invariant mass term also couples a tetraquark state to the nucleon. We discuss possible implications for nuclear physics and the behavior of the nucleon mass at nonzero temperature.

This paper is organized as follows. In Sec. II we present the Lagrangian of our model and the expressions for the axial charges, the decay widths $N^* \to N\pi$ and $N^* \to N\eta$, and the s-wave scattering lengths. Section III contains our results. In Sec. IV, we present a short summary of our work and discuss the two possible extensions mentioned above, i.e., the enlarged mixing scenario and the dilatation-invariant mass term. Details of our calculations are relegated to the appendices.

Our units are $\hbar = c = 1$, the metric tensor is $g^{\mu\nu} = \text{diag}(+, -, -, -)$.

II. THE MODEL AND ITS IMPLICATIONS

A. The Lagrangian

In this section we present the chirally symmetric linear sigma model considered in this work. It contains scalar,

pseudoscalar, vector, and axial-vector fields, as well as nucleons and their chiral partners including all globally symmetric terms up to fourth order, see Refs. [10,18]. While higher-order terms are in principle possible, we do not consider them here. In fact, one can argue that they should be absent in dilatation-invariant theories, cf. the discussion in Sec. IV.

The scalar and pseudoscalar fields are included in the matrix

$$\Phi = \sum_{a=0}^{3} \phi_a t_a = (\sigma + i\eta_N)t^0 + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{t}, \quad (1)$$

where $\vec{t} = \vec{\tau}/2$, with the vector of Pauli matrices $\vec{\tau}$, and $t^0 = \mathbf{1}_2/2$. Under the global $U(2)_R \times U(2)_L$ chiral symmetry, Φ transforms as $\Phi \to U_L \Phi U_R^\dagger$. The vector and axial-vector fields are represented by the matrices

$$V^\mu = \sum_{a=0}^{3} V_a^\mu t_a = \omega^\mu t^0 + \vec{\rho}^\mu \cdot \vec{t}, \quad (2a)$$

$$A^\mu = \sum_{a=0}^{3} A_a^\mu t_a = f_1^\mu t^0 + \vec{a}_1^\mu \cdot \vec{t}. \quad (2b)$$

From these fields, we define right- and left-handed vector fields $R^\mu \equiv V^\mu - A^\mu$, $L^\mu \equiv V^\mu + A^\mu$. Under global $U(2)_R \times U(2)_L$ transformations, these fields behave as $R^\mu \to U_R R^\mu U_R^\dagger$, $L^\mu \to U_L L^\mu U_L^\dagger$.

The identification of mesons with particles listed in Ref. [11] is straightforward in the pseudoscalar and (axial-) vector sectors, as already indicated in Eqs. (1) and (2): the fields $\vec{\pi}$ and η_N correspond to the pion and the $SU(2)$ counterpart of the η meson, $\eta_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$, with a mass of about 700 MeV. This value can be obtained by "unmixing" the physical η and η' mesons, which also contain $\bar{s}s$ contributions. The fields ω^μ and $\vec{\rho}^\mu$ represent the $\omega(782)$ and $\rho(770)$ vector mesons, respectively, and the fields f_1^μ and $\vec{a}_1^\mu$ represent the $f_1(1285)$ and $a_1(1260)$ axial-vector mesons, respectively. (In principle, the physical ω and f_1 states also contain $\bar{s}s$ contributions, however their admixture is negligible small.)

Unfortunately, the identification of the σ and $\vec{a}_0$ fields is controversial, the possibilities being the pairs $\{f_0(600), a_0(980)\}$ and $\{f_0(1370), a_0(1450)\}$. In Sec. IV B a more detailed discussion of this problem is presented. In the present work, the scalar assignment affects only the isospin-even πN scattering length and we study its dependence on the sigma mass. The Lagrangian describing the meson fields reads

$$\mathcal{L}_{\text{mes}} = \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda_2 (\Phi^\dagger \Phi)^2] - \lambda_1 (\text{Tr}[\Phi^\dagger \Phi])^2 + c(\det\Phi^\dagger + \det\Phi) + h_0 \text{Tr}[(\Phi^\dagger + \Phi)]$$

$$- \frac{1}{4} \text{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \frac{m_1^2}{2} \text{Tr}[(L^\mu)^2 + (R^\mu)^2] + \frac{h_1}{2} \text{Tr}[\Phi^\dagger \Phi] \text{Tr}[(L^\mu)^2 + (R^\mu)^2]$$

$$+ h_2 \text{Tr}[\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger] + 2h_3 \text{Tr}[\Phi R_\mu \Phi^\dagger L^\mu] + \mathcal{L}_3 + \mathcal{L}_4, \quad (3)$$

where $D^\mu \Phi = \partial^\mu \Phi + ig_1(\Phi R^\mu - L^\mu \Phi)$, and $R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu$, $L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu$ are the field-strength tensors of the vector fields. The terms $\mathcal{L}_3$ and $\mathcal{L}_4$ describe three- and four-particle interactions of the (axial-) vector fields [10], which are not important for this work. We list them in Appendix A. For $c = h_0 = 0$, the Lagrangian $\mathcal{L}_{\mathrm{mes}}$ is invariant under global $U(2)_R \times U(2)_L$ transformations. For $c \neq 0$, the $U(1)_A$ symmetry, where $A = L - R$, is explicitly broken, thus parametrizing the $U(1)_A$ anomaly of QCD. For $h_0 \neq 0$, the $U(2)_R \times U(2)_L$ symmetry is explicitly broken to the vectorial subgroup $U(2)_V$, where $V = L + R$.

The chiral condensate $\varphi = \langle 0|\sigma|0\rangle = Zf_\pi$ emerges upon spontaneous chiral symmetry breaking in the mesonic sector. The parameter $f_\pi = 92.4$ MeV is the pion decay constant and Z is the wave function renormalization constant of the pseudoscalar fields [10,19], also related to π-a_1 mixing, see Appendix B for more details.

We now turn to the baryon sector which involves the baryon doublets Ψ_1 and Ψ_2, where Ψ_1 has positive parity and Ψ_2 negative parity. In the mirror assignment they transform as follows:

$$\Psi_{1R} \to U_R \Psi_{1R}, \qquad \Psi_{1L} \to U_L \Psi_{1L},$$
$$\Psi_{2R} \to U_L \Psi_{2R}, \qquad \Psi_{2L} \to U_R \Psi_{2L}, \qquad (4)$$

i.e., Ψ_2 transforms in a ''mirror way'' under chiral transformations [4,7]. These field transformations allow to write down a baryonic Lagrangian with a chirally invariant mass term for the fermions, parametrized by m_0:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{bar}} = &\ \bar{\Psi}_{1L} i\gamma_\mu D^\mu_{1L} \Psi_{1L} + \bar{\Psi}_{1R} i\gamma_\mu D^\mu_{1R} \Psi_{1R} \\
&+ \bar{\Psi}_{2L} i\gamma_\mu D^\mu_{2R} \Psi_{2L} + \bar{\Psi}_{2R} i\gamma_\mu D^\mu_{2L} \Psi_{2R} \\
&- \hat{g}_1 (\bar{\Psi}_{1L} \Phi \Psi_{1R} + \bar{\Psi}_{1R} \Phi^\dagger \Psi_{1L}) \\
&- \hat{g}_2 (\bar{\Psi}_{2L} \Phi^\dagger \Psi_{2R} + \bar{\Psi}_{2R} \Phi \Psi_{2L}) \\
&- m_0 (\bar{\Psi}_{1L} \Psi_{2R} - \bar{\Psi}_{1R} \Psi_{2L} - \bar{\Psi}_{2L} \Psi_{1R} + \bar{\Psi}_{2R} \Psi_{1L}),
\end{aligned}
$$
$$(5)$$

where $D^\mu_{1R} = \partial^\mu - ic_1 R^\mu$, $D^\mu_{1L} = \partial^\mu - ic_1 L^\mu$, and $D^\mu_{2R} = \partial^\mu - ic_2 R^\mu$, $D^\mu_{2L} = \partial^\mu - ic_2 L^\mu$ are the covariant derivatives for the nucleonic fields, with the coupling constants c_1 and c_2. (Note that in the case of local chiral symmetry one has $c_1 = c_2 = g_1$). The interaction of the baryonic fields with the scalar and pseudoscalar mesons is parametrized by $\hat{g}_1$ and $\hat{g}_2$.

The term proportional to m_0 generates a mixing between the fields Ψ_1 and Ψ_2. The physical fields N and N^*, referring to the nucleon and its chiral partner, arise by diagonalizing the corresponding mass matrix in the Lagrangian (5):

$$\begin{pmatrix} N \\ N^* \end{pmatrix} = \frac{1}{\sqrt{2\cosh\delta}} \begin{pmatrix} e^{\delta/2} & \gamma_5 e^{-\delta/2} \\ \gamma_5 e^{-\delta/2} & -e^{\delta/2} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}. \quad (6)$$

The masses of the nucleon and its partner are obtained as

$$m_{N,N^*} = \sqrt{m_0^2 + [\tfrac{1}{4}(\hat{g}_1 + \hat{g}_2)\varphi]^2} \pm \tfrac{1}{4}(\hat{g}_1 - \hat{g}_2)\varphi. \quad (7)$$

The coupling constants $\hat{g}_{1,2}$ are uniquely determined by the values of m_N, m_{N^*}, and the parameter m_0,

$$\hat{g}_{1,2} = \frac{1}{\varphi} [\pm (m_N - m_{N^*}) + \sqrt{(m_N + m_{N^*})^2 - 4m_0^2}]. \quad (8)$$

From Eq. (7) one observes that, in the chirally restored phase where $\varphi \to 0$, the masses of the nucleon and its partner become degenerate, $m_N = m_{N^*} = m_0$. The mass splitting is generated by breaking chiral symmetry, $\varphi \neq 0$.

Note that the nucleon mass *cannot* be expressed as $m_N = m_0 + \lambda\varphi$, thus m_0 should not be interpreted as a linear contribution to the nucleon mass. Such a linearization is only possible in the case when m_0 dominates or the chiral condensate dominates. As we shall see, this does not happen and both quantities are sizable.

The parameter δ in Eq. (6) is related to the masses and the parameter m_0 by the expression

$$\cosh\delta = \frac{m_N + m_{N^*}}{2m_0}. \quad (9)$$

When $\delta \to \infty$, corresponding to $m_0 \to 0$, there is no mixing and $N = \Psi_1$, $N^* = -\Psi_2$. In this case, $m_N = \hat{g}_1 \varphi / 2$ and $m_{N^*} = \hat{g}_2 \varphi / 2$, thus the nucleon mass is solely generated by the chiral condensate as in the standard linear sigma model of Refs. [2,4] with the naive assignment for the baryons.

B. Axial coupling constants

The expressions for the axial coupling constants of the nucleon and the partner are derived in Appendix C. The result is

$$g_A^N = \frac{1}{2\cosh\delta} (g_A^{(1)} e^\delta + g_A^{(2)} e^{-\delta}), \qquad (10)$$
$$g_A^{N^*} = \frac{1}{2\cosh\delta} (g_A^{(1)} e^{-\delta} + g_A^{(2)} e^\delta),$$

where

$$g_A^{(1)} = 1 - \frac{c_1}{g_1}\left(1 - \frac{1}{Z^2}\right), \qquad g_A^{(2)} = -1 + \frac{c_2}{g_1}\left(1 - \frac{1}{Z^2}\right) \quad (11)$$

are the axial coupling constants of the bare fields Ψ_1 and Ψ_2. At this point, it should be emphasized that the interaction with the (axial-) vector mesons generates additional contributions to g_A^N and $g_A^{N^*}$, proportional to c_1 and c_2. We now discuss several limiting cases, using the fact that Z is required to be larger than 1, cf. Eq. (B7):

(i) *Local chiral symmetry:* In this case, the coupling constants $c_1 = c_2 = g_1$. This implies $g_A^N = -g_A^{N^*} = Z^{-2} \tanh\delta < 1$, which is at odds with the experimental value $g_A^N = 1.267 \pm 0.004$ [11].

(ii) *Decoupling of vector mesons:* Here, $Z = 1$ and $c_1 = c_2 = 0$, and we obtain the results of Ref. [7]: $g_A^N = -g_A^{N^*} = \tanh\delta$. In the limit $\delta \to \infty$, this reduces to $g_A^N = 1$ and $g_A^{N^*} = -1$. Also in this case the experimental value for g_A^N cannot be obtained for any choice of the parameters. Moreover, a positive value of $g_A^{N^*}$, as found in the lattice simulation of Ref. [13], is also impossible.

(iii) *Decoupling of the chiral partner:* This is achieved in the limit $\delta \to \infty$, where $N = \Psi_1$ and $N^* = -\Psi_2$. One has $g_A^N = g_A^{(1)}$ and $g_A^{N^*} = g_A^{(2)}$. Since $Z > 1$, it is evident that the ratio c_1/g_1 must be negative in order to obtain the experimental value $g_A^N = 1.267 \pm 0.004$ [11].

Note that, in the case of local chiral symmetry, the axial charge of the nucleon can be also correctly reproduced when introducing dimension-6 terms in the Lagrangian $\mathcal{L}_{\text{bar}}$, cf. Refs. [1,2,17,20,21], because the coefficients of these so-called Weinberg-Tomozawa (WT) terms [22,23] can be adjusted accordingly. However, such WT terms naturally arise when integrating out the axial-vector mesons from our Lagrangian, just as in chiral perturbation theory [15]. In this sense, it would be double-counting to simultaneously consider axial-vector mesons and WT terms. Our generalization to a *global* chiral symmetry allows a description of the axial charge without explicitly introducing WT terms.

C. Decay widths

We now turn to the decays $N^* \to N\pi$ and $N^* \to N\eta$. The calculation of the tree-level decay width for $N^* \to N\pi$ from the Lagrangian (5) is straightforward. However, the decay $N^* \to N\eta$ cannot be directly evaluated because of the absence of the s quark. In order to proceed, we have to take into account that

$$\eta = \eta_N \cos\phi_P + \eta_S \sin\phi_P, \tag{12}$$

where $\eta_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$, $\eta_S \equiv \bar{s}s$ and ϕ_P lies between $-32°$ and $-45°$ [24]. Then, the decay amplitude $\mathcal{A}_{N^* \to N\eta}$ can be expressed as

$$\mathcal{A}_{N^* \to N\eta} = \mathcal{A}_{N^* \to N\eta_N} \cos\phi_P + \mathcal{A}_{N^* \to N\eta_S} \sin\phi_P. \tag{13}$$

In the following, we assume that the OZI-suppressed amplitude $\mathcal{A}_{N^* \to N\eta_S}$ is small, so that to good approximation the decay width $\Gamma_{N^* \to N\eta} \simeq \cos^2\phi_P \Gamma_{N^* \to N\eta_N}$. Note that the *physical* η meson mass, $m_\eta = 547$ MeV, enters $\Gamma_{N^* \to N\eta}$. Therefore, also the decay width $\Gamma_{N^* \to N\eta_N}$ has to be evaluated for the physical mass m_η, not for m_{η_N}.

The expression for the decay width $N^* \to NP$, where $P = \pi, \eta$, is (for details, see Appendix C)

$$\Gamma_{N^* \to NP} = \lambda_P \frac{k_P}{2\pi} \frac{m_N}{m_{N^*}} \frac{Z^2}{32\cosh^2\delta} \left\{ w^2(c_1 + c_2)^2 \left[(m_{N^*}^2 - m_N^2 - m_P^2)\frac{E_P}{m_N} + m_P^2\left(1 - \frac{E_N}{m_N}\right) \right] + (\hat{g}_1 - \hat{g}_2)^2\left(\frac{E_N}{m_N} + 1\right) \right.$$
$$\left. + 2w(\hat{g}_1 - \hat{g}_2)(c_1 + c_2)\left(\frac{m_{N^*}^2 - m_N^2 - m_P^2}{2m_N} + E_P\right) \right\}. \tag{14}$$

The coefficients $\lambda_\pi = 3$, $\lambda_\eta = \cos^2\phi_P$, $w \equiv g_1\varphi/m_{a_1}^2$, and the momentum of the pseudoscalar particle is given by

$$k_P = \frac{1}{2m_{N^*}}\sqrt{(m_{N^*}^2 - m_N^2 - m_P^2)^2 - 4m_N^2 m_P^2}. \tag{15}$$

The energies are $E_P = \sqrt{k_P^2 + m_P^2}$ and $E_N = \sqrt{k_P^2 + m_N^2}$, because the momenta of the nucleon and the pseudoscalar particles are equal in the rest frame of N^*.

It is important to stress that, in the mirror assignment, the only way to obtain a nonzero $N^*N\pi$ coupling is a nonzero value of the parameter m_0. In fact, the coupling is proportional to $\cosh^{-1}\delta \propto m_0$, i.e., when m_0 increases, also the decay width increases.

In the naive assignment, in which the field Ψ_2 transforms just like the field Ψ_1, a term proportional to m_0 is not possible, because it would break chiral symmetry. In this case, a mixing term of the form $\propto \bar{\Psi}_2\gamma^5\Phi\Psi_1 + \text{h.c.}$ is allowed. This leads to a term $\propto \bar{\Psi}_2\gamma^5(\sigma + i\gamma^5\vec{\pi}\cdot\vec{t})\Psi_1$, where the pion is coupled to Ψ_1 and Ψ_2 in a chirally symmetric way. However, the very same term also generates a mixing of Ψ_2 and Ψ_1 due to the nonzero vacuum expectation value of the field $\sigma = \sigma_0$. When performing the diagonalization one obtains two physical fields N and N^*, to be identified with the nucleon and a negative-parity state such as $N^*(1535)$. In terms of the physical fields N^* and N the coupling $\bar{N}^*i\vec{\pi}\cdot\vec{t}N$ vanishes; for the explicit calculation see Ref. [8]. Thus, in the naive assignment and in the minimal framework with only one multiplet of scalar and pseudoscalar fields the decay $N^* \to N\pi$ vanishes. One could go beyond this minimal setup: a possibility is to include the (axial-) vector mesons into the Lagrangian of the naive assignment. In this way a nonzero derivative coupling $\propto \bar{N}^*\gamma^\mu\partial_\mu\vec{\pi}\cdot\vec{t}N$ survives. A complete study of this scenario, involving also the scattering lengths, is in preparation.

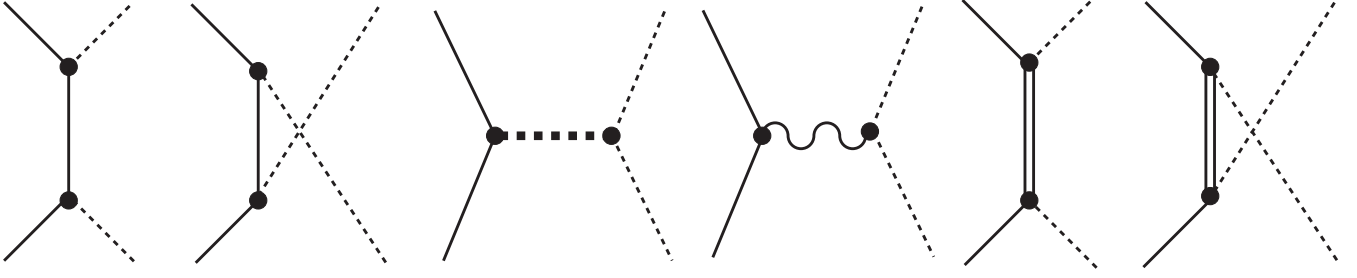Alternatively, the inclusion of a second (or more) multiplet(s) of (pseudo-)scalar mesons, see Refs. [25,26],

FIG. 1. Tree-level diagrams contributing to πN scattering. Dashed lines represent the pion, the bold dashed line the σ meson, the wavy line the ρ meson, full lines the nucleon, and double full lines the N^*, respectively.

coupled to the baryon fields also leads to a nonvanishing coupling between N^*, the nucleon, and the pion.

D. πN scattering lengths

The general form of the πN scattering amplitude is [16]

$$T_{ab} = [A^{(+)} + \tfrac{1}{2}(q_1^\mu + q_2^\mu)\gamma_\mu B^{(+)}]\delta_{ab}$$
$$+ [A^{(-)} + \tfrac{1}{2}(q_1^\mu + q_2^\mu)\gamma_\mu B^{(-)}]i\epsilon_{bac}\tau_c, \quad (16)$$

where the subscripts a and b refer to the isospin of the initial and final states and the superscripts (+) and (−) denote the isospin-even and isospin-odd amplitudes, respectively. The πN scattering amplitudes, $A^{(\pm)}$ and $B^{(\pm)}$, evaluated from the Lagrangian (5) at tree level, involve

exchange of σ and ρ mesons in the t-channel and intermediate N and N^* states in the s- and u-channels, cf. Fig. 1.

The s-wave scattering lengths, $a_0^{(\pm)}$, are given by

$$a_0^{(\pm)} = \frac{1}{4\pi(1 + m_\pi/m_N)}(A_0^{(\pm)} + m_\pi B_0^{(\pm)}), \quad (17)$$

where the subscript 0 at the amplitudes $A^{(\pm)}$, $B^{(\pm)}$ indicates that they are taken at threshold, i.e., for the following values of the Mandelstam variables s, t, u: $s = (m_N + m_\pi)^2$, $t = 0$, $u = (m_N - m_\pi)^2$.

The explicit expression for the isospin-even scattering length can be obtained from Eq. (17) by applying the Feynman rules resulting from the Lagrangians (3) and (5) to the diagrams shown in Fig. 1. The result is

$$a_0^{(+)} = \frac{1}{4\pi(1 + \frac{m_\pi}{m_N})}\left(\frac{Z}{2\cosh\delta}\right)^2\left(-\frac{1}{2}\left[\hat{g}_1 - \hat{g}_2 + \frac{Zf_\pi}{2}w(c_1 + c_2)(\hat{g}_2 - \hat{g}_1)\right]^2 \frac{(m_N + m_{N^*})(m_N^2 + m_\pi^2 - m_{N^*}^2)}{(m_N^2 + m_\pi^2 - m_{N^*}^2)^2 - 4m_N^2 m_\pi^2}\right.$$

$$- w(c_1 + c_2)(\hat{g}_1 - \hat{g}_2) + \frac{Zf_\pi}{4}(\hat{g}_1 - \hat{g}_2)w^2(c_1 + c_2)^2 - w(c_1e^\delta - c_2e^{-\delta})(\hat{g}_1e^\delta + \hat{g}_2e^{-\delta})$$

$$+ w^2 m_N(c_1e^\delta - c_2e^{-\delta})^2 + (\hat{g}_1e^\delta - \hat{g}_2e^{-\delta})\frac{\cosh\delta}{Zf_\pi}\left\{1 + \frac{m_\pi^2}{m_\sigma^2}\frac{1}{Z^4}\left[Z^2 - 2 + 2(Z^2 - 1)\left(1 - \frac{Z^2 m_1^2}{m_{a_1}^2}\right)\right]\right\}$$

$$+ m_\pi\left\{\left[\hat{g}_1 - \hat{g}_2 + \frac{Zf_\pi}{2}w(c_1 + c_2)(\hat{g}_2 - \hat{g}_1)\right]^2 \frac{m_N m_\pi}{(m_N^2 + m_\pi^2 - m_{N^*}^2)^2 - 4m_N^2 m_\pi^2}\right.$$

$$\left.\left. + [\hat{g}_1e^\delta + \hat{g}_2e^{-\delta} - 2m_N w(c_1e^\delta - c_2e^{-\delta})]^2 \frac{m_N}{m_\pi}\frac{1}{m_\pi^2 - 4m_N^2}\right\}\right). \quad (18)$$

Similarly, the expression for the isospin-odd scattering length is given by

$$a_0^{(-)} = \frac{1}{4\pi(1 + \frac{m_\pi}{m_N})}\left(\frac{Z}{2\cosh\delta}\right)^2\left(\left[\hat{g}_1 - \hat{g}_2 + \frac{Zf_\pi}{2}w(c_1 + c_2)(\hat{g}_2 - \hat{g}_1)\right]^2 \frac{(m_N + m_{N^*})m_N m_\pi}{(m_N^2 + m_\pi^2 - m_{N^*}^2)^2 - 4m_N^2 m_\pi^2}\right.$$

$$+ \frac{m_\pi}{2}\left\{\left[\hat{g}_1 - \hat{g}_2 + \frac{Zf_\pi}{2}w(c_1 + c_2)(\hat{g}_2 - \hat{g}_1)\right]^2 \frac{m_N^2 + m_\pi^2 - m_{N^*}^2}{(m_N^2 + m_\pi^2 - m_{N^*}^2)^2 - 4m_N^2 m_\pi^2}\right.$$

$$- [\hat{g}_1e^\delta + \hat{g}_2e^{-\delta} - 2m_N w(c_1e^\delta - c_2e^{-\delta})]^2 \frac{1}{m_\pi^2 - 4m_N^2} - w^2[(c_1 + c_2)^2 - (c_1e^\delta - c_2e^{-\delta})^2]$$

$$\left.\left. + \frac{g_1}{m_\rho^2}\frac{4\cosh\delta}{Z^2}(c_1e^\delta - c_2e^{-\delta})\right\}\right). \quad (19)$$

Although it is not obvious from these expressions, one can show that the s-wave scattering lengths $a_0^{(\pm)}$ vanish in the chiral limit, as required by low-energy theorems for theories with spontaneously broken chiral symmetry.

III. RESULTS AND DISCUSSION

In this section we present our results. We first discuss the case where the resonance $N(1535)$ is interpreted as the chiral partner of the nucleon. This is the most natural assignment because this resonance is the lightest with the correct quantum numbers. We then consider some important limiting cases. Finally, we also discuss two different assignments: the resonance $N(1650)$, which is the next heavier state with the correct quantum numbers listed in Ref. [11], and a *speculative* candidate $N(1200)$ with a mass $M_{N^*} \sim 1200$ MeV and a very large width $\Gamma_{N^* \to N\pi} \gtrsim 800$ MeV, such as to have avoided experimental detection up to now [9].

A. $N(1535)$ as partner

The resonance $N(1535)$ has a mass $m_{N^*} = (1535 \pm 10)$ MeV [11]. The theoretical expressions for g_A^N, $g_A^{N^*}$, $\Gamma_{N^* \to N\pi}$, $\Gamma_{a_1 \to \pi\gamma}$ depend on the four parameters c_1, c_2, Z, and m_0. Here, Z is the only parameter entering from the meson sector, see Appendix B.

We determine the parameters c_1, c_2, Z, and m_0 by using the experimental results [11] for the decay width $\Gamma_{N^* \to N\pi}^{\exp} = (67.5 \pm 23.6)$ MeV, the radiative decay of the $a_1(1260)$ meson, $\Gamma_{a_1 \to \pi\gamma}^{\exp} = (0.640 \pm 0.246)$ MeV, and the axial coupling constant $g_A^{N,\exp} = 1.267 \pm 0.004$, as well as the lattice result $g_A^{N^*,\text{latt}} = 0.2 \pm 0.3$ [13]. With the help of a standard χ^2 procedure it is also possible to determine the errors for the obtained parameters:

$$c_1 = -3.0 \pm 0.6, \qquad c_2 = 11.6 \pm 3.6,$$
$$Z = 1.67 \pm 0.2, \qquad (20)$$

and

$$m_0 = (460 \pm 136) \text{ MeV}. \qquad (21)$$

The coupling constants $\hat{g}_1$ and $\hat{g}_2$ can be deduced from Eq. (8),

$$\hat{g}_1 = 11.0 \pm 1.5, \qquad \hat{g}_2 = 18.8 \pm 2.4. \qquad (22)$$

The value obtained for m_0 is larger than the one originally found in Ref. [7] and points to a sizable contribution of other condensates to the nucleon mass. However, because of the nonlinear relation (7) between the nucleon mass, m_0, and the chiral condensate, when switching off m_0 the nucleon mass is not simply by an amount m_0 smaller than the physical value, rather $m_N = \hat{g}_1 \varphi / 2 \simeq 850$ MeV, and thus only slightly smaller than 939 MeV. The Ioffe formula is thus still approximately justified also in this context. On the other hand, when varying φ from 0 to the physical value Zf_π, the nucleon mass goes from $m_0 = 460$ MeV to 939 MeV. Interestingly, the coupling constant c_2 which parametrizes the interaction of the nucleon's partner with the (axial-) vector mesons is larger than the constant c_1 which parametrizes the interaction of the nucleon with the (axial-) vector mesons. Nevertheless, when compared with the coupling $g_1 \sim 6$ (similar in all models with vector mesons and pions) the constants c_1 and c_2 are $|c_1| \sim g_1/2$, $c_2 \sim 2g_1$ i.e., they are related to g_1 by some numerical factor of order one. The direct comparison of c_1 and c_2 leads to $|c_1| \sim c_2/4$.

We now test the validity of our model by considering the πN scattering lengths (some preliminary results were already presented in Ref. [27]). The quantity $a_0^{(-)}$ depends on c_1, c_2, Z, and m_0, and in addition on m_ρ and g_1. The latter is a function of Z and m_{a_1}, cf. Eq. (B7). The values of m_ρ and m_{a_1} are known to reasonably good precision [11], and thus our uncertainty in determining $a_0^{(-)}$ is small. (This will be different for $a_0^{(+)}$ which also depends on the poorly known value of the σ meson mass, m_σ.) We obtain

$$a_0^{(-)} = (6.04 \pm 0.63) \cdot 10^{-4} \text{ MeV}^{-1}, \qquad (23)$$

in agreement with the experimental value measured by the ETH Zürich-Neuchatel-PSI collaboration in pionic hydrogen and deuterium X-ray experiments [28]:

$$a_{0,\exp}^{(-)} = (6.4 \pm 0.1) \cdot 10^{-4} \text{ MeV}^{-1}. \qquad (24)$$

An even better agreement is expected when including the Δ resonance [17].

The scattering length $a_0^{(+)}$ depends also on c_1, c_2, Z, and m_0, but in addition on m_1 and m_σ. The former parametrizes the contribution to the ρ mass which does not originate from the chiral condensate: $m_\rho^2 = m_1^2 + \frac{\phi^2}{2}(h_1 + h_2 + h_3)$. Notice that in the present theoretical framework with global chiral symmetry the KSFR relation [29] is obtained for $m_1 = 0$, $h_1 + h_2 + h_3 = g_1^2/Z^2$. A physically reasonable range of values for m_1 is between 0 and m_ρ. For the lower boundary, the mass of the ρ meson is exclusively generated by chiral symmetry breaking, thus it becomes massless when $\varphi \to 0$. This is similar to Georgi's vector limit [30] or Brown-Rho scaling [31]. In principle, the mass of the σ meson varies over a wide range of values; we could choose $m_\sigma \sim 0.4$ GeV or 1.37 GeV, according to the assignment $f_0(600)$ and $f_0(1370)$.

Since the allowed range of values for m_1 and m_σ is large, we choose to plot the scattering length $a_0^{(+)}$ as function of m_1 for different choices of m_σ; the result is shown in Fig. 2. The experimental result [28]

$$a_{0,\exp}^{(+)} = (-8.8 \pm 7.2) \cdot 10^{-6} \text{ MeV}^{-1} \qquad (25)$$

is shown as grey (yellow) band. One observes that for small values of m_σ one requires a large value for m_1 in order to reproduce experimental data. For increasing m_σ, the re-
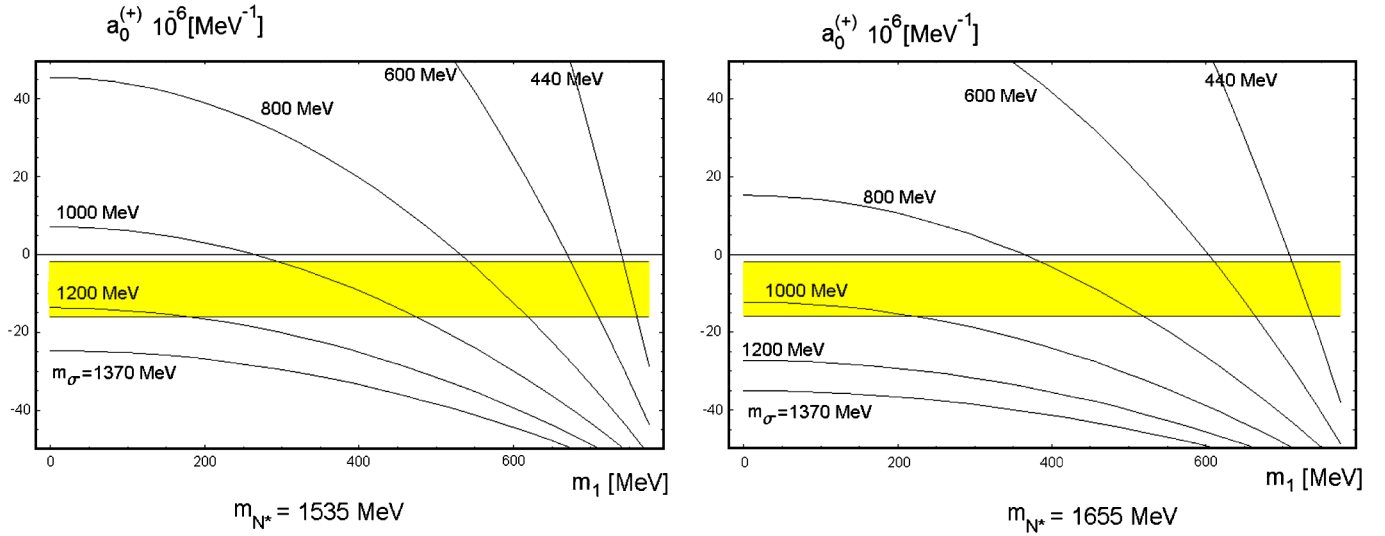
FIG. 2 (color online).   The isospin-even scattering length $a_0^{(+)}$ as a function of m_1 for fixed values of m_σ, for the assignment $N^* = N(1535)$ (left panel) and $N^* = N(1650)$ (right panel). The experimental range is shown by the grey (yellow) band.

quired values for m_1 decrease. For $m_\sigma \gtrsim 1.37$ GeV, $a_{0,\text{exp}}^{(+)}$ cannot be reproduced for any value of m_1. This, however, does not exclude a heavy σ meson, rather, it indicates that an additional light scalar-isoscalar resonance needs to be included as discussed in Sec. IV B.

For the decay $N^* \rightarrow N\eta$, we obtain with Eq. (14) the result

$$\Gamma_{N^*\rightarrow N\eta} = (10.9 \pm 3.8) \text{ MeV}, \qquad (26)$$

where the error also takes into account the uncertainty in the pseudoscalar mixing angle $\phi_P = -38.7° \pm 6°$. We observe that $\Gamma_{N^*\rightarrow N\eta}$ is about a factor 7 smaller than $\Gamma_{N^*\rightarrow N\pi}$, which is in reasonable agreement with the naive expectation based on the relation $\lambda_\eta/\lambda_\pi = \cos^2\phi_P/3 \simeq 0.097$. However, it is clearly smaller than the experimental value $\Gamma_{N^*\rightarrow N\eta}^{\text{exp}} = (78.7 \pm 24.3)$ MeV [11]. The agreement could be improved if one generalizes our discussion to the $SU(3)$ case and includes a large OZI-violating contribution, or if one considers an enlarged mixing scenario as discussed in Sec. IV A.

B. Limiting cases

We now consider three important limiting cases. In all of these $N(1535)$ is taken as chiral partner of the nucleon.

(i) *Local chiral symmetry:* This case is obtained by setting $g_1 = c_1 = c_2$ and $h_1 = h_2 = h_3 = 0$. As a consequence, $m_\rho = m_1$, $m_{a_1}^2 = m_\rho^2 + (g_1\varphi)^2$, $Z = m_{a_1}/m_\rho$. Using the experimental values for $\Gamma_{N^*\rightarrow N\pi}$ and $\Gamma_{a_1\rightarrow\gamma\pi}$ one obtains

$$m_0 = (730 \pm 229) \text{ MeV}. \qquad (27)$$

As a consequence, $g_A^N = -g_A^{N^*} \equiv Z^{-2}\tanh\delta = 0.33 \pm 0.02$, both at odds with experimental and

lattice data. The scattering length $a_0^{(-)}$ is in the range of the experimental data, $a_0^{(-)} = (4.9 \pm 1.7) \times 10^{-4}$ MeV^{-1}. Since $m_1 = m_\rho$ is fixed, the isospin-even scattering length only depends on m_σ. Thus, for a given value of m_σ, we obtain a single value with theoretical errors: $a_0^{(+)} = (7.06 \pm 3.12) \times 10^{-6}$ MeV^{-1} for $m_\sigma = 1.37$ GeV and $a_0^{(+)} = (4.46 \pm 0.11) \times 10^{-5}$ MeV^{-1} for $m_\sigma = 0.44$ GeV, which is outside the range of the experimental error band. As already argued in Refs. [10,21] we conclude that the case of local chiral symmetry (in the present model without higher-order terms) is not capable of properly reproducing low-energy phenomenology.

(ii) *Decoupling of vector mesons:* This corresponds to $g_1 = c_1 = c_2 = h_1 = h_2 = h_3 = 0$, and thus $Z = 1$ and $w = 0$. Using the decay width $\Gamma_{N^*\rightarrow N\pi} = (67.5 \pm 23.6)$ MeV one obtains

$$m_0 = (262 \pm 46) \text{ MeV}, \qquad (28)$$

in agreement with Ref. [7]. As a result $g_A^N = -g_A^{N^*} = 0.97 \pm 0.01$, in disagreement with both experimental and lattice data. The description of the scattering lengths also becomes worse; the isospin-odd scattering length $a_0^{(-)} = (5.7 \pm 0.47) \times 10^{-4}$ MeV^{-1}, which is just outside the experimental error band. Also in this case, the isospin-even scattering length assumes a single value (with theoretical errors) for given m_σ: $a_0^{(+)} = (1.08 \pm 0.05) \times 10^{-4}$ MeV^{-1} for $m_\sigma = 1.37$ GeV and $a_0^{(+)} = (-7.55 \pm 0.19) \times 10^{-4}$ MeV^{-1} for $m_\sigma = 0.44$ GeV, i.e., 2 orders of magnitude away from the experimental value. We thus conclude that vec-

tor mesons cannot be omitted for a correct description of pion-nucleon scattering lengths.

(iii) *Decoupling of the chiral partner:* This is obtained by sending $m_0 \to 0$ or $\delta \to \infty$. The partner decouples and we are left with a linear σ model with vector and axial-vector mesons. The decay $\Gamma_{N^* \to N\pi}$ vanishes in this case and is obviously at odds with the experiment. Using the experimental values for g_A^N and $\Gamma_{a_1 \to \gamma\pi}$ to fix the parameters c_1 and Z (c_2 and $\hat{g}_2$ play no role here because of the decoupling of the partner), we obtain

$$c_1 = -2.59 \pm 0.51, \qquad Z = 1.66 \pm 0.2. \quad (29)$$

The scattering lengths are

$$a_0^{(-)} = (5.99 \pm 0.66) \times 10^{-4} \text{ MeV}^{-1}, \quad (30)$$

and $a_0^{(+)}$ shows a similar behavior as shown in Fig. 2. We conclude that the role of the partner is marginal in improving the scattering lengths. It could be omitted, unless one wants to consider, in the framework of the mirror assignment, its decay into nucleon and pseudoscalar particles.

C. Other candidates

In this subsection, we discuss two more exotic possibilities for the chiral partner of the nucleon.

(i) *N(1650) as partner.* The resonance $N(1650)$ has a mass $m_{N^*} = (1655 \pm 15)$ MeV and a decay width $\Gamma_{N^* \to N\pi}^{\text{exp}} = (128 \pm 44)$ MeV [11]. The axial coupling constant measured in the lattice simulation of Ref. [13] reads $g_A^{N^*, \text{latt}} = 0.55 \pm 0.2$. By following the previous steps we obtain

$$c_1 = -3.3 \pm 0.7, \qquad c_2 = 14.8 \pm 3.4,$$
$$Z = 1.67 \pm 0.2, \quad (31)$$

and

$$m_0 = (709 \pm 157) \text{ MeV}. \quad (32)$$

This leads to the coupling constants

$$\hat{g}_1 = 9.45 \pm 1.81, \qquad \hat{g}_2 = 18.68 \pm 2.68. \quad (33)$$

In this case m_0 is even larger than before. However, as in the case of $N(1535)$, the quantity $\hat{g}_1 \varphi/2 \simeq 730$ MeV is still sizable and similar to m_0. The result for the isospin-odd scattering length

$$a_0^{(-)} = (5.90 \pm 0.46) \times 10^{-4} \text{ MeV}^{-1} \quad (34)$$

is similar to the case of $N(1535)$. The isospin-even scattering lengths behave similarly as before, cf. Fig. 2, however, slightly smaller values for m_σ are required in order to reproduce the experimental data.

The decay width into $N\eta$ is $\Gamma_{N^* \to N\eta} = (18.3 \pm 8.5)$ MeV, which should be compared to $\Gamma_{N^* \to N\eta}^{\text{exp}} = (10.7 \pm 6.7)$ MeV. Thus, in this case the decay width is in agreement with the experimental value. However, we then face the problem of how to describe the $N(1535)$ resonance, cf. the discussion in Sec. IV A.

(ii) *Speculative candidate N(1200) as partner.* We consider a *speculative* candidate $N(1200)$ with a mass $m_{N^*} \sim 1200$ MeV and a very large width $\Gamma_{N^* \to N\pi} \gtrsim 800$ MeV, such as to have avoided experimental detection up to now. The reason for its introduction was motivated by properties of nuclear matter [9] and further on investigated in Ref. [32] in the context of asymmetric nuclear matter present in a neutron star. Regardless of the precise value of the axial coupling constant of the partner (which is unknown for this hypothetical resonance) one obtains $m_0 > 1$ GeV. This, in turn, implies a large interaction of N and N^*. As a consequence, both scattering lengths turn out to be off by two order of magnitudes: $a_0^{(-)} \sim 10^{-2}$ MeV^{-1} and $a_0^{(+)} \sim 10^{-4}$ MeV^{-1}. Thus, we are led to *discard* the possibility that a hypothetical, not yet discovered $N(1200)$ exists.

IV. SUMMARY AND OUTLOOK

In this paper, we investigated a linear sigma model with global chiral $U(2)_R \times U(2)_L$ symmetry, where the mesonic degrees of freedom are the standard scalar and pseudoscalar mesons and the vector and axial-vector mesons. In addition to the mesons, we included baryonic degrees of freedom, namely, the nucleon and its chiral partner, which is incorporated in the model in the so-called mirror assignment.

We used this model to study the origin of the mass of the nucleon, the assignment and decay properties of its chiral partner and the pion-nucleon scattering lengths. The mass of the nucleon results as an interplay of the chiral condensate and a chirally invariant baryonic mass term, proportional to the parameter m_0. When the chiral partner of the nucleon is identified with the resonance $N^* \equiv N(1535)$, the parameter $m_0 \simeq 500$ MeV is obtained as a result of a fitting procedure which involves the three experimentally measured quantities $N^* \to N\pi$, $a_1 \to \pi\gamma$, g_A^N, and the quantity $g_A^{N^*}$ evaluated on the lattice. The isospin-odd scattering length $a_0^{(-)}$ is then fixed and found to be in good agreement with experimental data. The isospin-even scattering length depends, in addition, strongly on the mass of the σ meson, see Fig. 2 and the discussion in Sec. IV B. The decay width $N^* \to N\eta$ turns out to be a factor of 8 smaller than the experimental value.

The obtained value $m_0 \simeq 500$ MeV implies that a sizable amount of the nucleon mass does not originate from the chiral condensate. As this result is subject to the

assumptions and the validity of the employed chiral model, most notably due to the identification of the chiral partner with $N(1535)$ and to the mathematical properties of the mirror assignment, future studies of other scenarios, incorporating new results both from the experiment and the lattice, are necessary to further clarify this important issue of hadron physics.

It should also be noted that the results presented in this work are based on a tree-level calculation. The inclusion of loops represents a task for the future. Nevertheless, we expect that the results will not change qualitatively: On the one hand, while the dimensionless couplings of the model g_1, c_1, c_2, $\hat{g}_1$, $\hat{g}_2$ are large, the contribution of loops is suppressed according to large–N_c arguments [33]. On the other hand, in our model we have included from the very beginning the relevant resonances which contribute as virtual states to processes, thus reducing the effects of loops in the model. To clarify the latter point, consider the ρ meson exchange in πN scattering. In an approach in which the ρ meson is not directly included, its contribution could only be obtained after a corresponding loop resummation, while in our approach it is taken directly into account by a tree-level exchange diagram.

We studied three important limiting cases: (i) In the framework of local chiral symmetry it is not possible to correctly reproduce low-energy phenomenology. (ii) It is not admissible to neglect (axial-) vector mesons. They are crucial in order to obtain a correct description of the axial coupling constants and πN scattering lengths. (iii) The role of the partner N^* has only a marginal influence on the scattering lengths.

We have also tested other assignments for the partner of the nucleon: a broad, not-yet discovered partner with a mass of about 1.2 GeV must be excluded on the basis of scattering data. The well-established resonance $N(1650)$ provides qualitatively similar results as $N(1535)$ and, in this case, the theoretical value of the decay width $N(1650) \to N\eta$ is in agreement with the experimental one. However, in this scenario it is not clear how $N(1535)$ fits into the baryonic resonance spectrum. This issue is discussed in Sec. IV A presented below. In Sec. IV B we discuss the origin of m_0 in terms of tetraquark and gluon condensates and the implications for future studies.

A. Outlook 1: Enlarged mixing scenario

In this section we briefly describe open problems of the previous results and present a possible outlook to improve the theoretical description.

A simultaneous description of both resonances $N(1525)$ and $N(1650)$ requires an extension of the model. In the framework of the mirror assignment, instead of only two bare nucleon fields Ψ_1 and Ψ_2 one should include two additional bare fields Ψ_3 and Ψ_4 with positive and negative parity, respectively. The latter two are assumed to trans-

form like Ψ_1 and Ψ_2 in Eq. (4). The interesting part of the enlarged Lagrangian are the bilinear chirally invariant mass terms:

$$\begin{aligned} \mathcal{L}_{\text{mass}} = {} & m_0^{(1,2)}(\bar{\Psi}_2 \gamma^5 \Psi_1 - \bar{\Psi}_1 \gamma^5 \Psi_2) \\ & + m_0^{(3,4)}(\bar{\Psi}_4 \gamma^5 \Psi_3 - \bar{\Psi}_3 \gamma^5 \Psi_4) \\ & + m_0^{(1,4)}(\bar{\Psi}_4 \gamma^5 \Psi_1 - \bar{\Psi}_1 \gamma^5 \Psi_4) \\ & + m_0^{(2,3)}(\bar{\Psi}_2 \gamma^5 \Psi_3 - \bar{\Psi}_3 \gamma^5 \Psi_2). \end{aligned} \quad (35)$$

In the limit $m_0^{(1,4)} = m_0^{(2,3)} = 0$ the bare fields Ψ_1 and Ψ_2 do not mix with the fields Ψ_3 and Ψ_4. The fields Ψ_1 and Ψ_2 generate the states $N(939)$ and $N(1535)$, just as described in this paper with $m_0^{(1,2)} = m_0$, while the fields Ψ_3 and Ψ_4 generate the states $N(1440)$ and $N(1650)$, which are regarded as chiral partners. The term proportional to $m_0^{(3,4)}$ induces a decay of the form $N(1650) \to N(1440)\pi$ (or η), but still $N(1650)$ and $N(1440)$ do not decay into $N\pi(\eta)$.

When in addition the coefficients $m_0^{(1,4)}$ and $m_0^{(2,3)}$ are nonzero, a more complicated mixing scenario involving four bare fields arises. As a consequence, it is possible to account for the decay of both resonances $N(1550)$ and $N(1650)$ into $N\pi(\eta)$. Moreover, it is well conceivable that the anomalously small value of the decay width $N(1550) \to N\eta$ arises because of destructive interference. Interestingly, a mixing of bare configurations generating $N(1535)$ and $N(1650)$ is necessary also at the level of the quark model [34]. Note that in the framework of the generalized mixing scenario, the fields $N(1535)$ and $N(1650)$ are chiral partners of $N(939)$ and $N(1440)$. Because of mixing phenomena, it is not possible to isolate the chiral partner of the nucleon, which is present in both resonances $N(1535)$ and $N(1650)$. However, also in this case the nonzero decay widths of both fields $N(1535)$ and $N(1650)$ are obtained as a result of nonvanishing m_0-like parameters.

The mixing scenario outlined above may look at first sight not very useful, because it involves too many new parameters. However, a quick counting shows that this is not the case. In addition to the four mass parameters $m_0^{(i,j)}$, we have the already discussed parameters c_1, c_2, $\hat{g}_1$, and $\hat{g}_2$, plus similar parameters c_3, c_4, $\hat{g}_3$, and $\hat{g}_4$ which describe the interactions of $\Psi_{3,4}$ with mesons. These 12 parameters can be used to describe the following 14 quantities: the masses of the states $N \equiv N(939)$, $N(1535)$, $N(1440)$, $N(1650)$, the decay widths $N(1535) \to N\pi$, $N(1535) \to N\eta$, $N(1650) \to N\pi$, $N(1650) \to N\eta$, $N(1440) \to N\pi$, $N(1440) \to N\eta$ (the latter by taking into account the nonzero width of the $N(1440)$ resonance), and the four axial coupling constants g_A^N, $g_A^{N(1535)}$, $g_A^{N(1440)}$, and $g_A^{N(1650)}$. A detailed study of this enlarged scenario, in which the four lightest $J^P = \frac{1}{2}^\pm$ baryonic resonances are simultaneously included, will be performed in the future.

B. Outlook 2: Origin of m_0

The scattering length $a_0^{(+)}$ shows a strong dependence on the mass of the σ meson. A similar situation occurs for $\pi\pi$ scattering at low energies [10]. While a light σ is favored by the scattering data, many other studies show that the σ meson—as the chiral partner of the pion in the linear sigma model—should be placed above 1 GeV and identified with the resonance $f_0(1370)$ rather than the light $f_0(600)$ [see Refs. [35,36] and refs. therein]. Indeed, also in the framework of the linear sigma model used in this paper, the decay width $f_0(600) \rightarrow \pi\pi$ turns out to be too small when the latter is identified with the chiral partner of the pion [10].

When identifying σ with $f_0(1370)$, two possibilities are left for $f_0(600)$: (i) It is a dynamically generated state arising from the pion-pion interaction. The remaining scalar states below 1 GeV, $f_0(980)$, $a_0(980)$, and $K_0^*(800)$ can be interpreted similarly. (ii) The state $f_0(600)$ is predominantly composed of a diquark $[u, d]$ (in the flavor and color antitriplet representation) and an antidiquark $[\bar{u}, \bar{d}]$, i.e., $f_0(600) \simeq [\bar{u}, \bar{d}][u, d]$. In this case the light scalar states $f_0(600)$, $f_0(980)$, $a_0(980)$, and $K_0^*(800)$ form an additional tetraquark nonet [37–41]. Note that in both cases the resonance $f_0(600)$—which is needed to explain $\pi\pi$ and πN scattering experiments and also to understand the nucleon-nucleon interaction potential—is *not* the chiral partner of the pion. In the following we concentrate on the implications of scenario (ii) at a qualitative level, leaving a more detailed study for the future. First, a short digression on the dilaton field is necessary.

Dilatation invariance of the QCD Lagrangian in the chiral limit is broken by quantum effects. This situation can be taken into account in the framework of a chiral model by introducing the dilaton field G [42]. The corresponding dilaton potential reflects the trace anomaly of QCD as underlying theory and has the form $V(G) \propto G^4(\log\frac{G}{\Lambda_G} - \frac{1}{4})$, where $\Lambda_G \sim \Lambda_{QCD}$ is the only dimensional quantity which appears in the full effective Lagrangian in the chiral limit. Because of the nonzero expectation value of G, a shift $G \rightarrow G_0 + G$ is necessary: the fluctuations around the minimum correspond to the scalar glueball, whose mass is placed at $M_G \sim 1.7$ GeV by lattice QCD calculations [43] and by various phenomenological studies [44]. [Beyond the chiral limit, also the parameter h_0 in Eq. (3), which describes explicit symmetry breaking due to the nonzero valence quark masses, appears as an additional dimensionful quantity.]

We assume that, in the chiral limit, the full interaction potential $V(\Phi, L_\mu, R_\mu, \Psi_1, \Psi_2, G, \chi)$ is dilatation invariant up to the term $\propto \log\frac{G}{\Lambda_G}$ and that it is finite for any finite value of the fields, i.e., only terms of the kind $G^2 \text{Tr}[\Phi^\dagger\Phi]$, $\text{Tr}[\Phi^\dagger\Phi]^2, \ldots$ are retained. By performing the shift $G \rightarrow G_0 + G$, the term $G^2 \text{Tr}[\Phi^\dagger\Phi]$ becomes $G_0^2 \text{Tr}[\Phi^\dagger\Phi] + \ldots$, where the dots refer to glueball-meson interactions.

Identifying $\mu^2 \sim G_0^2$, a term $G_0^2 \text{Tr}[\Phi^\dagger\Phi]$ is already present in our Lagrangian (3), but the glueball-hadron interactions are neglected. Note that a term of the kind $G^{-4} \text{Tr}[\partial_\mu\Phi^\dagger\partial^\mu\Phi]^2$ is not allowed because of our assumption that the potential is finite. Following this line of arguments, our Lagrangian (3) cannot contain operators of order higher than four [36], because such operators must be generated from terms with inverse powers of G. E.g., upon shifting G, the above mentioned term would generate an order-eight operator of the kind $G_0^{-4} \text{Tr}[\partial_\mu\Phi^\dagger\partial^\mu\Phi]^2$.

Let us now turn to the mass term $\sim m_0$ in Eq. (5),

$$m_0(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{1L}). \quad (36)$$

The parameter m_0 has the dimension of mass and is the only term in the baryon sector, which is not dilatation invariant. In order to render it dilatation invariant while simultaneously preserving chiral symmetry, we can couple it to the chirally invariant dilaton field G. Moreover, in the framework of $U(2)_R \times U(2)_L$ chiral symmetry also the above mentioned tetraquark field, denoted as $\chi \equiv \frac{1}{2} \times [\bar{u}, \bar{d}][u, d]$, is invariant under chiral transformations. We then write the following dilatation-invariant interaction term:

$$(a\chi + bG)(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{1L}), \quad (37)$$

where a and b are dimensionless coupling constants.

When shifting both fields around their vacuum expectation values $\chi \rightarrow \chi_0 + \chi$ and $G \rightarrow G_0 + G$ we recover the term (36) of our Lagrangian by identifying

$$m_0 = a\chi_0 + bG_0, \quad (38)$$

where χ_0 and G_0 are the tetraquark and gluon condensates, respectively.

Note that the present discussion holds true also in the highly excited part of the baryon sector: as described in Refs. [12,25], the heavier the baryons, the less important becomes the quark condensate φ: For two heavy chiral partners B and B^*, one expects a mass degeneracy of the form $m_B \simeq m_{B^*} \simeq m_0$. We expect the gluon condensate G_0 to be the dominant term in this sector, $m_0 \simeq bG_0$. In fact, the tetraquark condensate is also related to the chiral condensate in the vacuum [40,45] and—while potentially important for low-lying states like the nucleon and its partner—its role should also diminish when considering very heavy baryons.

We now return to the nucleon and its partner and concentrate on their interaction with the tetraquark field χ. From the point of low-energy phenomenology, the tetraquark field χ is very interesting because the corresponding excitation is expected to be lighter than the gluonium and the scalar quarkonium states, for instance $m_\chi \sim M_{f_0(600)} \sim$ 0.6 GeV. A nucleon-tetraquark interaction of the kind $a\chi(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{1L})$ arising

from Eq. (37) would then contribute to pion-pion and nucleon-pion scattering and possibly improve the agreement with experimental data.

Moreover, there is also another interesting consequence: in virtue of Eq. (37) the state χ appears as intermediate state in nucleon-nucleon interactions and, due to its small mass, is likely to play an important role in the one-meson exchange picture for the nucleon-nucleon potential. This raises the interesting question whether a *tetraquark* is the scalar state which mediates the middle-range attraction among nucleons, in contrast to the standard picture where this task is performed by a quark-antiquark state. Let us further elucidate this picture by a simple and intuitive example. Let us consider the nucleon as a quark-diquark bound state. The standard picture of one-boson exchange in the nucleon-nucleon interaction consists of exchanging the two quarks between the nucleons. However, one could well imagine that instead of the quarks one exchanges the two diquarks between the nucleons. Note that these diquarks are in the correct color and flavor antitriplet representations in order to form a tetraquark of the type suggested by Jaffe [37], such as the meson χ discussed here. A full analysis must include a detailed study of mixing between all scalar states.

As a last subject we discuss how the nucleon mass might evolve at nonzero temperature and density. In particular, in the high-density region of the so-called "quarkyonic phase" [46] hadrons are confined but chiral symmetry is (almost) restored, i.e., the chiral condensate (approximately) vanishes. What are the properties of the nucleon in this phase? In the framework of the Lagrangian (5), when $\varphi \rightarrow 0$, the masses of both the nucleon and its partner approach a constant value m_0. Then, the first naive answer is that we expect a nucleon mass of about 500 MeV in this phase. The situation is, however, more complicated than this. In fact, as discussed in this section the term m_0 is not simply a constant but is related to other condensates. The behavior of these condensates at nonzero T and μ is then crucial for the determination of the nucleon mass. Interestingly, in Ref. [45] it is shown that the tetraquark condensate does *not* vanish but rather increases for increasing T. A future study at nonzero T and μ must include both the tetraquark and the gluon condensate in the same framework.

APPENDIX A: VECTOR-MESON SELF-INTERACTIONS

In this appendix, we present the terms $\mathcal{L}_3$ and $\mathcal{L}_4$ of Eq. (3):

$$\mathcal{L}_3 = -2ig_2(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\})$$
$$- 2g_3(\mathrm{Tr}[(\partial_\mu L_\nu + \partial_\nu L_\mu)\{L^\mu, L^\nu\}]$$
$$+ \mathrm{Tr}[(\partial_\mu R_\nu + \partial_\nu R_\mu)\{R^\mu, R^\nu\}]), \tag{A1}$$

and

$$\mathcal{L}_4 = g_4\{\mathrm{Tr}[L^\mu L^\nu L_\mu L_\nu] + \mathrm{Tr}[R^\mu R^\nu R_\mu R_\nu]\}$$
$$+ g_5\{\mathrm{Tr}[L^\mu L_\mu L^\nu L_\nu] + \mathrm{Tr}[R^\mu R_\mu R^\nu R_\nu]\}$$
$$+ g_6\mathrm{Tr}[R^\mu R_\mu]\mathrm{Tr}[L^\nu L_\nu] + g_7\{\mathrm{Tr}[L^\mu L_\mu]\mathrm{Tr}[L^\nu L_\nu]$$
$$+ \mathrm{Tr}[R^\mu R_\mu]\mathrm{Tr}[R^\nu R_\nu]\}. \tag{A2}$$

The coupling constants g_k with $k = 2, \ldots, 7$ are not relevant for the present work.

APPENDIX B: MESON SECTOR

In the mesonic Lagrangian (3), there are ten parameters: λ_1, λ_2, c, h_0, h_1, h_2, h_3, μ^2, g_1, and m_1. In the following, we describe how to relate them to the physical meson masses and the pion decay constant.

If chiral symmetry is spontaneously broken, the scalar-isoscalar field σ develops a nonvanishing vacuum expectation value (v.e.v.), $\langle\sigma\rangle \equiv \varphi$, the so-called chiral condensate. In order to proceed, we have to shift σ by its v.e.v., $\sigma \rightarrow \varphi + \sigma$. The chiral condensate is identified with the minimum of the potential energy density $V(\varphi)$, cf. Eq. (3):

$$V(\varphi) = \frac{1}{2}(\mu^2 - c)\varphi^2 + \frac{1}{4}\left(\lambda_1 + \frac{\lambda_2}{2}\right)\varphi^4 - h_0\varphi, \tag{B1}$$

$$0 = \frac{dV}{d\varphi} = \left[\mu^2 - c + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\varphi^2\right]\varphi - h_0. \tag{B2}$$

After the shift $\sigma \rightarrow \varphi + \sigma$ a mixing term between axial-vector and pseudoscalar mesons arises; for instance between a_1-meson and pion it is of the form $-g_1\vec{a}_1^\mu \cdot \partial_\mu\vec{\pi}$. The standard way to treat this term is to eliminate it by a shift of the axial-vector fields. Then, in order to recover the canonical normalization of the pseudoscalar fields, one has to introduce a corresponding wave function renormalization factor. For a_1-meson and pion this operation has the form

$$\vec{a}_1^\mu \rightarrow \vec{a}_1^\mu + Zw\partial^\mu\vec{\pi}, \qquad \vec{\pi} \rightarrow Z\vec{\pi}, \quad \text{where } w = \frac{g_1\varphi}{m_{a_1}^2},$$

$$Z^2 = \frac{m_{a_1}^2}{m_{a_1}^2 - (g_1\varphi)^2}. \tag{B3}$$

The meson masses are then given by

$$m_\sigma^2 = \mu^2 - c + 3\left(\lambda_1 + \frac{\lambda_2}{2}\right)\varphi^2,$$

$$m_{a_0}^2 = \mu^2 + c + \left(\lambda_1 + 3\frac{\lambda_2}{2}\right)\varphi^2, \tag{B4}$$

$$m_{\eta_N}^2 = Z^2\left[\mu^2 + c + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\varphi^2\right],$$

$$m_\pi^2 = Z^2\left[\mu^2 - c + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\varphi^2\right] = \frac{Z^2 h_0}{\varphi}, \tag{B5}$$

$$m_\omega^2 = m_\rho^2 = m_1^2 + \frac{\varphi^2}{2}(h_1 + h_2 + h_3),$$

$$m_{f_1}^2 = m_{a_1}^2 = m_1^2 + (g_1\varphi)^2 + \frac{\varphi^2}{2}(h_1 + h_2 - h_3). \tag{B6}$$

Note that only the linear combination $h_1 + h_2$ enters these equations, so only nine out of the original ten parameters are determined by the meson masses. However, in the following considerations, only the sum $h_1 + h_2$ will enter, so we do not need to determine h_1 and h_2 independently. We therefore have six physical meson masses in order to determine nine parameters. A seventh physical quantity is the pion decay constant, f_π, which we determine from the axial current, $J_{A\mu}^a = \frac{\varphi}{Z}\partial_\mu\pi^a + \ldots \equiv f_\pi\partial_\mu\pi^a + \ldots$, i.e., $\varphi = Zf_\pi$.

This leaves us with two independent parameters, which turn out to be g_1 and m_1. The latter only enters the isospin-even pion-nucleon scattering length. We shall leave it as a free parameter to study the dependence of $a_0^{(+)}$ on m_1.

For the sake of convenience, we shall replace the coupling constant g_1 by the pseudoscalar wave function renormalization factor Z. This is achieved with the help of the relation (B3),

$$g_1(Z) = \frac{m_{a_1}}{Zf_\pi}\sqrt{1 - \frac{1}{Z^2}}. \tag{B7}$$

In this work we fix $m_{a_1} = 1.23$ GeV which is the central value quoted in Ref. [11]. In fact, it is technically easier to use Z than g_1 as independent parameter: while g_1 is a unique function of Z, the function $Z(g_1)$ would be multivalued.

It remains to determine Z. For this purpose we use the decay width $a_1 \to \pi\gamma$. The experimental value quoted by the PDG is $\Gamma_{a_1\to\pi\gamma}^{\exp} = (640 \pm 246)$ keV [11]. The theoretical expression is obtained by minimal coupling of the photon in the meson sector and only depends on Z:

$$\Gamma_{a_1\to\pi\gamma} = \frac{\alpha}{24}m_{a_1}(Z^2 - 1)\left(1 - \frac{m_\pi^2}{m_{a_1}^2}\right)^3, \tag{B8}$$

where $\alpha = 1/137$. Using the experimental value quoted above we derive $Z = 1.67 \pm 0.19$. The quantities m_1 and Z are the only independent parameters from the mesonic sector, which enter the determination of the axial coupling

constants of the nucleon and its chiral partner, the decay widths for $N^* \to N\pi$, and the $N\pi$ scattering lengths. Because of the large uncertainty, we shall employ Eq. (B8) together with the constraints from the baryon sector, cf. Sec. III, to perform a simultaneous fit of all relevant parameters in the baryonic sector, i.e., c_1, c_2, Z, and m_0.

It should be noted that the inclusion of the axial-vector degrees of freedom is the ultimate reason which allows for a correct determination of the axial-coupling constants g_A^N and g_A^{N*}. We can easily convince ourselves of this fact by assuming the contrary, i.e., studying the case where the axial-vector mesons are absent. This can be achieved either by setting g_1 to zero, or by sending the a_1 mass to infinity. In both cases, $Z = [1 - (g_1\varphi/m_{a_1})^2]^{-1/2} \to 1 + O[(g_1\varphi/m_{a_1})^2]$. Then, from Eq. (11), we obtain $g_A^{(1)} = -g_A^{(2)} = 1$, and the physical axial coupling constants are $g_A^{(N)} = -g_A^{(N*)} = \tanh\delta \leq 1$, in contradiction to the experimental values.

The next question is, whether the experimental value of the a_1 mass is not too large compared to the "natural scale" of the problem, so that the correct description of the axial-coupling constants is impossible. The natural scale is given by the scale of chiral symmetry breaking, i.e., by the value of φ, possibly multiplied by a constant of order one. If we take the natural scale to be $g_1\varphi \sim g_1 f_\pi \simeq 600$ MeV, then indeed $g_1\varphi/m_{a_1} \sim 1$, i.e., the a_1 mass is not too large compared to the natural scale of the problem. This can also be seen from the fact that the $a_1 \to \pi\gamma$ decay requires $Z \simeq 1.67 > 1$, i.e., $g_1\varphi$ must be of order m_{a_1}. If m_{a_1} were large, a fit of $g_A^{(N*)}$ to the lattice data would lead to an unnaturally large c_2. But this problem does not emerge because m_{a_1} is not large when compared to the natural scale of the model.

APPENDIX C: DETAILS OF THE CALCULATIONS

1. Axial coupling constants

From the baryonic Lagrangian (5) we select the terms which are relevant for the derivation of the baryonic axial coupling constants:

$$\mathcal{L}^{ax} = i\bar\Psi_1\gamma^\mu\partial_\mu\Psi_1 + i\bar\Psi_2\gamma^\mu\partial_\mu\Psi_2 - c_1\bar\Psi_1\gamma^\mu\gamma^5\vec{t}$$
$$\cdot\vec{a}_{1\mu}\Psi_1 + c_2\bar\Psi_2\gamma^\mu\gamma^5\vec{t}\cdot\vec{a}_{1\mu}\Psi_2,$$

in which the interactions of Ψ_1 and Ψ_2 with the a_1-meson are retained. After performing the shift of the axial field $\vec{a}_1^\mu \to \vec{a}_1^\mu + Zw\partial^\mu\vec{\pi}$ we obtain:

$$\mathcal{L}^{ax} = i\bar\Psi_1\gamma^\mu\partial_\mu\Psi_1 + i\bar\Psi_2\gamma^\mu\partial_\mu\Psi_2 - Zwc_1\bar\Psi_1\gamma^\mu\gamma^5\vec{t}$$
$$\cdot\partial_\mu\vec{\pi}\Psi_1 + Zwc_2\bar\Psi_2\gamma^\mu\gamma^5\vec{t}\cdot\partial_\mu\vec{\pi}\Psi_2 + \ldots. \tag{C1}$$

The axial current is calculated as

$$\mathcal{J}^i_{A\mu} = \frac{\partial \mathcal{L}}{\partial(\partial^\mu \Psi_1)}(\delta\Psi_1)^i + \frac{\partial \mathcal{L}}{\partial(\partial^\mu \Psi_2)}(\delta\Psi_2)^i$$
$$+ \frac{\partial \mathcal{L}}{\partial(\partial^\mu \pi^j)}(\delta\pi^j)^i, \tag{C2}$$

where $(\delta\Psi_1)^i = i\gamma^5 t^i \Psi_1$, $(\delta\Psi_2)^i = -i\gamma^5 t^i \Psi_2$, $(\delta\pi^j)^i = \delta^{ij}(\sigma + \varphi)/Z$.

We obtain:

$$\mathcal{J}^i_{A\mu} = g^{(1)}_A \bar\Psi_1 \gamma_\mu \gamma^5 t^i \Psi_1 + g^{(2)}_A \bar\Psi_2 \gamma_\mu \gamma^5 t^i \Psi_2 + \dots, \tag{C3}$$

where, taking into account that $w = (1 - Z^{-2})/(g_1\varphi)$:

$$g^{(1)}_A = 1 - \varphi w c_1 = 1 - \frac{c_1}{g_1}\left(1 - \frac{1}{Z^2}\right),$$
$$g^{(2)}_A = -1 + \frac{c_2}{g_1}\left(1 - \frac{1}{Z^2}\right), \tag{C4}$$

which are Eqs. (11).

In order to obtain the axial coupling constants of the physical fields, we make use of Eq. (6):

$$\mathcal{J}^i_{A\mu} = g^N_A \bar{N}\gamma_\mu \gamma^5 t^i N e^\delta + g^{N^*}_A \bar{N}^* \gamma_\mu \gamma^5 t^i N^* e^{-\delta} + \dots \tag{C5}$$

where

$$g^N_A = \frac{1}{2\cosh\delta}(e^\delta g^{(1)}_A + e^{-\delta} g^{(2)}_A),$$
$$g^{N^*}_A = \frac{1}{2\cosh\delta}(e^{-\delta} g^{(1)}_A + e^\delta g^{(2)}_A), \tag{C6}$$

which are Eqs. (10).

2. Decay widths

After the field transformation $\sigma \to \varphi + \sigma$ and (B3) discussed in Appendix B have been performed, we isolate the terms relevant for the decay $N^* \to NP$. In the following, we only discuss $P = \pi^0$, η_N, the other isospin components can be obtained similarly:

$$\mathcal{L}_{NN^*P} = iA\bar{N}^*NP + B\bar{N}^*\gamma^\mu N\partial_\mu P - iA\bar{N}N^*P$$
$$+ B\bar{N}\gamma^\mu N^* \partial_\mu P. \tag{C7}$$

where

$$A = -\frac{Z(\hat{g}_1 - \hat{g}_2)}{4\cosh\delta}, \qquad B = -\frac{Zw(c_1 + c_2)}{4\cosh\delta}. \tag{C8}$$

The decay amplitude for the process $N^* \to NP$ reads

$$-iM_{\alpha\beta} = i\bar{u}^N_\beta(\vec{k}_1)Cu^{N^*}_\alpha(\vec{k} = 0), \tag{C9}$$

where $C = -iA + iB\gamma_\rho k^\rho_2$. Averaging over initial states and summing over final states, we obtain the following squared amplitude:

$$\overline{|-iM_{N^* \to NP}|^2} = \frac{1}{2}\sum_{\alpha,\beta}|-iM_{\alpha\beta}|^2 = \frac{1}{2}\sum_{\alpha,\beta}[\bar{u}^N_\beta(\vec{k}_1)Cu^{N^*}_\alpha(\vec{k} = 0)][\bar{u}^{N^*}_\alpha(\vec{k} = 0)C'u^N_\beta(\vec{k}_1)], \tag{C10}$$

with $C' = iA - iB\gamma_\rho k^\rho_2 = -C$. Using the well-known properties of the traces of γ matrices leads to the result

$$\overline{|-iM|^2} = \frac{1}{2}\sum_{\alpha,\beta}|-iM_{\alpha\beta}|^2 = \frac{1}{2}\,\text{Tr}\left[C\frac{\gamma^\mu k_\mu + m_{N^*}}{2m_{N^*}}C'\frac{\gamma^\nu k_{1,\nu} + m_N}{2m_N}\right]$$
$$= \frac{A^2}{2}\,\text{Tr}\left[\frac{\gamma^\mu k_\mu + m_{N^*}}{2m_{N^*}}\frac{\gamma^\mu k_{1,\mu} + m_N}{2m_N}\right] + \frac{B^2}{2}\,\text{Tr}\left[\gamma_\rho k^\rho_2 \frac{\gamma^\mu k_\mu + m_{N^*}}{2m_{N^*}}\gamma_\rho k^\rho_2 \frac{\gamma^\mu k_{1,\mu} + m_N}{2m_N}\right]$$
$$- AB\,\text{Tr}\left[\frac{\gamma^\mu k_\mu + m_{N^*}}{2m_{N^*}}\gamma_\rho k^\rho_2 \frac{\gamma^\mu k_{1,\mu} + m_N}{2m_N}\right]$$
$$= \frac{A^2}{2}\left(\frac{E_N}{m_N} + 1\right) + \frac{B^2}{2}\left[(m^2_{N^*} - m^2_N - m^2_P)\frac{E_P}{m_N} + m^2_P\left(1 - \frac{E_N}{m_N}\right)\right] - AB\left(\frac{m^2_{N^*} - m^2_N - m^2_P}{2m_N} + E_P\right). \tag{C11}$$

The full decay width is obtained including all isospin states for the pion and by replacing the unphysical state η_N with the physical η meson. The result is

$$\Gamma_{N^* \to NP} = \lambda_P \frac{k_P}{2\pi}\frac{m_N}{m_{N^*}}\overline{|-iM_{N^* \to NP}|^2}, \tag{C12}$$

which leads to Eq. (14).

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