Experimental demonstration of higher-order Laguerre-Gauss mode interferometry

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The compatibility of higher-order Laguerre-Gauss (LG) modes with interferometric technologies commonly used in gravitational wave detectors is investigated. In this paper, we present the first experimental results concerning the performance of the LG_{33} mode in optical resonators. We show that the Pound-Drever-Hall error signal for a LG_{33} mode in a linear optical resonator is identical to that of the more commonly used LG_{00} mode, and demonstrate the feedback control of the resonator with a LG_{33} mode. We succeeded to increase the mode purity of a LG_{33} mode generated using a spatial-light modulator from 51% to 99% upon transmission through a linear optical resonator. We further report the experimental verification that a triangular optical resonator does not transmit helical LG modes.

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I. INTRODUCTION

The sensitivity of second generation gravitational wave detectors such as Advanced LIGO [1] and Advanced Virgo [2] will be limited by the thermal noises of the test masses [3], therefore the gravitational wave community is involved in research into methods for reducing the effects of this noise. One proposed method for thermal noise reduction is to use so-called ‘‘flat beams’’ in the main interferometer, in place of the currently standard fundamental LG_{00} beam [4]. This method is currently investigated for possible upgrades of second generation detectors [5], as well as for third generation detectors [6]. Flat beams provide wider intensity distributions for the same optical losses and can therefore average better over the mirror surface distortions caused by the thermal motions. A number of different flat beam shapes have been proposed, such as mesa beams [7], conical beams [8], and higher-order Laguerre-Gauss (LG) beams [9]. One advantage of higher-order LG beams over the other flat beams is their potential compatibility with the currently used spherical mirror surfaces. For this reason, we find it worthwhile to further investigate the compatibility of higher-order LG beams with gravitational wave interferometer technology. Some of us recently showed that the potential detection rate of binary neutron star inspiral systems with the Advanced Virgo detector could be increased by a factor of 2.1 if the LG_{33} beam was used in place of the LG_{00} beam [10]. In addition to the thermal noise benefits, the wider intensity distributions of higher-order LG beams have been shown to reduce the magnitude of thermal aberrations of optics within the interferometers [11]. This would reduce the extent to which thermal compensation systems are relied upon in future detectors to reach design sensitivity. The improved thermal noise characteristics of higher-order LG beams over the fundamental beam also makes the technology a strong candidate for improving the sensitivity of optical clock experiments, which currently also approach the mirror thermal noise limit [12].

An investigation using numerical simulations, into the sensing and control signals for length and alignment with a LG_{33} beam in advanced detectors yielded positive results, indicating that the LG_{33} beam performed as well if not better than the LG_{00} beam in all of the examined criteria [10]. We present the results of an experimental follow-on study into the interferometric performance of higher-order LG beams, in order to assess how much of the potential for sensitivity improvement is realizable in practice. The first crucial test for the interferometric performance of LG_{33} modes is their compatibility with mode cleaner technology. We show that the mode cleaner effect works equivalently for the LG_{33} mode as for the LG_{00} mode in a linear optical resonator as depicted in Fig. 2. We also demonstrate the incompatibility of helical LG modes with three mirror mode cleaners.

II. PERFORMANCE OF HIGHER-ORDER LAGUERRE-GAUSS BEAMS IN MODE CLEANER CAVITIES

Laguerre-Gauss modes represent a complete set of solutions to the paraxial wave equation, and as such are well suited to modeling the eigenmodes of spherical optical resonators [13]. There is some lack of consensus in current literature about the exact naming of LG mode functions. Much of the literature relating to higher-order LG modes refers to modes with spiral phase fronts, which carry orbital angular momenta $l\hbar$ per photon, where l is the azimuthal mode index [14–16]. Equation (1) shows in cylindrical polar coordinates the normalized form of the complex amplitude of this mode set, which in the following will be referred to as *helical* LG modes.

$$u^{\mathrm{hel}}_{p,l}(r,\phi,z)=\frac{1}{w(z)}\sqrt{\frac{2p!}{\pi(|l|+p)!}}e^{\mathrm{i}(2p+|l|+1)\Psi(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|}\times L_p^{(|l|)}\left(\frac{2r^2}{w(z)^2}\right)e^{-\mathrm{i}k((r^2)/(2q(z)))+\mathrm{i}l\phi}\qquad(1)$$

where $p \geq 0$ is the radial mode index, l is the azimuthal mode index, Ψ is the Gouy phase, w is the beam radius, k is the wave number, and q is the complex Gaussian beam parameter [17]. $L_p^{(|l|)}$ are the associated Laguerre polynomials. An alternative form of LG modes with a sinusoidal amplitude dependence in azimuthal angle can be used equally well. The normalized form of the complex amplitude of this mode set is shown in Eq. (2); we will refer to this mode set as *sinusoidal* LG modes. The symbols are as defined in Eq. (1), and δ is the Kronecker delta [17].

$$u_{p,l}^{\sin}(r, \phi, z) = \frac{2}{w(z)}\sqrt{\frac{2p!}{1 + \delta_{0l}\pi(|l| + p)!}} \times e^{\mathrm{i}(2p+|l|+1)\Psi(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|} L_p^{(|l|)}\left(\frac{2r^2}{w(z)^2}\right) \times e^{-\mathrm{i}k((r^2)/(2q(z)))}\left\{\begin{matrix}\sin(l\phi)\\ \cos(l\phi)\end{matrix}\right\}. \quad (2)$$

A complete set of sinusoidal solutions consists of the functions as given in Eq. (2) using $\cos(l\phi)$ for $l \geq 0$ and $\sin(l\phi)$ if $l < 0$. Higher-order LG modes of both sets offer improvements in thermal noise for gravitational wave interferometers compared to the LG_{00} mode, however the advantage is greater for the helical modes than for the sinusoidal modes [11].

A number of different methods for generating higher-order LG modes have been demonstrated [16]. However, so far the optimization of higher-order LG beam sources has largely been in a different direction to that which is required by the gravitational wave detector community. For example, the use of LG beams in the cold atoms and optics fields often requires high-speed manipulation of the beam parameters and positions, whereas the use of LG modes in high-precision interferometry depends on mode purity and stability. One of the leading candidate methods for the latter is the use of diffractive optics, or *phase plates* for conversion from a LG_{00} mode to a higher-order LG mode, due to their stability, as well as potentially high conversion efficiency and output mode purity. Other conversion methods include using computer generated holograms [18], spatial-light modulators [19], and astigmatic mode converters [15]. However, none of these mode conversion methods are perfect, and some light inevitably remains in unwanted modes. An effectively pure and stable higher-order LG mode light source for gravitational wave interferometers can possibly be achieved with the implementation of *mode cleaner cavities*.

In practice, mode cleaners take the form of medium- to high-finesse optical resonators which are feedback controlled to remain on resonance for a chosen laser mode [20]. Mode cleaners are used in several locations in gravitational wave interferometers [21]. So-called *premode cleaners* are used in the initial frequency stabilization chain of the laser. These typically employ small, monolithic spacers in air. The beam then passes the *input mode cleaners*, suspended optical cavities in vacuum whose main function is to filter beam geometry fluctuations (also called beam-jitter noise). Modern laser interferometers also include optical cavities in the main interferometer, which act as additional mode cleaning cavities. Often a small in-vacuum output mode cleaner is then used to filter the light leaving the interferometer before it reaches the photo detectors. Mode cleaners can in principle be used to increase the spatial purity of any Gaussian mode. Experimental verification of the compatibility of higher-order LG beams with mode cleaner technology is of paramount importance for determining the future prospects for LG beams in gravitational wave interferometers.

Currently, a triangular arrangement is favored for the mode cleaners in gravitational wave detectors as it allows one to spatially separate the injected beam from the reflected beam, enabling a length control error signal to be measured in reflection without the need for polarizing optics. However, triangular cavities are not ideal for use with higher-order LG modes.

Triangular cavities behave differently from linear cavities in several ways. One important difference is that after one full round trip in a triangular cavity any beam is mirrored about the vertical axis, which means that only light fields with symmetry about this axis can constructively interfere and be fully resonant. The intensity patterns of LG_{33} modes are symmetric regarding a mirroring around the vertical axis. However, the phase cross sections in general are not, as is shown in Fig. 1. Both types of sinusoidal modes show the required symmetry about the vertical axis, but helical modes do not. The antisymmetric sinusoidal mode will be resonant in a cavity with an optical path length difference of $\lambda/2$ from one resonant for the symmetric mode. In other words, a cavity tuned to be resonant for one type of sinusoidal mode will be antiresonant for the respective other. Furthermore, any helical LG mode can be understood to be a sum of two sinusoidal LG modes, by considering Eqs. (1) and (2) and the identity $\exp(\mathrm{i}x) = \cos(x) + \mathrm{i}\sin(x)$. We thus expect that in the case of a helical LG input beam, the mode cleaner cavity can be tuned to a length at which one of the constituent sinusoidal

FIG. 1. Transverse phase distributions of the helical (left panel), vertically symmetric sinusoidal (center panel), and vertically antisymmetric sinusoidal (right panel) LG_{33} modes. The color represents the phase, in a range from 0 (white) to 2π (black).

LG modes will be resonant and thus transmitted while the other constituent sinusoidal LG mode will be exactly antiresonant and thus reflected, i.e., the helical LG beam would be decomposed into the two constituent sinusoidal modes. This effect can be generalized to optical resonators with any number of mirrors. In all resonators with an *even* number of mirrors, the effect of mirroring the beam about the vertical axis is canceled out in one full round trip, so we expect these to transmit helical LG beams. In all resonators with an *odd* number of mirrors, the effect does not cancel out, so we do not expect them to transmit helical LG beams.

Another important difference between linear and triangular mode cleaner cavities is that the latter feature a spherically curved mirror which is probed by the circulating beam under an angle (not normal incidence). This results in a breaking of the symmetry about the azimuthal angle for the mode cleaner eigenmodes. This is not usually a problem for fundamental mode operation, since an astigmatic LG_{00} mode is still an eigenmode of the cavity. Higher-order LG modes on the other hand are not eigenmodes of astigmatic cavities [22]. The mode shape of even sinusoidal LG beams degenerates upon transmission through a triangular mode cleaner as a result of the astigmatism. There are two possible solutions to this problem; to use linear cavities exclusively, or to design nonastigmatic mode cleaner cavities with four or more mirrors. Some work has already been done to design nonastigmatic mode cleaner cavities for fundamental mode operation [23], which should be investigated for use with higher-order LG modes. One possibility may be to implement aspherical mirrors to build a nonastigmatic mode cleaner for higher-order LG modes. It should be noted that using only linear cavities as mode cleaners incurs the additional complication of using polarizing optics to extract the control signals in reflection.

As a result of these considerations, the main experimental setup described here makes use of a linear mode cleaner cavity instead of a triangular cavity. We have however also experimentally verified the nontransmission of helical modes through a triangular cavity (see Sec. III B). The finesse of the linear cavity was chosen to be low in comparison with some gravitational wave detector input mode cleaners, as shown in Table I. While higher finesse cavities can give a stronger suppression of misalignment modes, it is interesting to see the large improvement that can already be gained through the use of a low-finesse mode cleaner.

III. LABORATORY DEMONSTRATION

In order to investigate the interferometric performance of the LG_{33} beam in a laboratory, it was necessary to produce a reasonably pure example of such a beam. We used a computer-controlled liquid-crystal-on-silicon spatial-light modulator (SLM) for LG beam preparation, as demonstrated in [19], because of the availability and adaptability of such devices. We expect the SLM to be replaced by a passive, etched phase plate in eventual implementations of LG_{33} modes in gravitational wave detectors.

The report on the laboratory investigation is comprised of two parts; the first concerning the performance of the sinusoidal and helical LG_{33} beams in a linear mode cleaner, and the second concerning specifically the performance of the helical LG_{33} beam in a triangular mode cleaner.

A. LG mode performance in a linear mode cleaner

The experimental setup for the investigation into the performance of the LG_{33} mode in a linear mode cleaner is shown in Fig. 2. The 1064 nm laser light is passed through quarter and half wave plates to set the polarization vector to the optimum orientation for use with the SLM. An electro-optic modulator is used to imprint a 15 MHz modulation on the light to enable length control of the mode cleaner with the Pound-Drever-Hall (PDH) method [27]. The light is then reflected from the modulating surface of the SLM, where the phase characteristics of the desired LG mode are imprinted on the beam. The resulting beam is then passed through a telescope to match the beam to the mode cleaner eigenmode.

The light transmitted through the mode cleaner is passed through a beam splitter, and analyzed at the two ports with a CCD camera and a photodiode, respectively. The signal

TABLE I. Input mode cleaner parameters for some gravitational wave detectors, as well as those used in this work. TEM_{01} suppression factors and throughput percentages are given in terms of light power. The finesse and TEM_{01} suppression factors of the mode cleaners used in this work were chosen to be lower than those of the large-scale mode cleaners.

Mode cleaner	Finesse	FSR	TEM_{01} suppression	Throughput
GEO MC1 [24]	2700	37.48 MHz	1325	80%
GEO MC2 [24]	1900	37.12 MHz	937	72%
Virgo IMC [25]	1181	1.044 MHz	···	86.6%
AdLIGO IMC [26]	500	17.96 MHz	···	···
Linear MC	172	714 MHz	50.1	63%
Triangular MC	300	714 MHz	87.6	99%

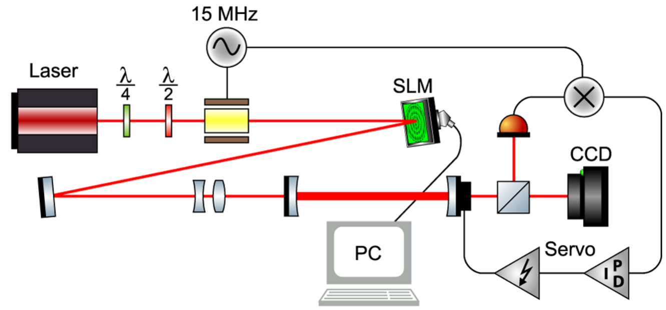

FIG. 2 (color online). The experimental setup for mode cleaning a SLM generated higher-order LG beam. The LG_{00} input beam is converted to a higher-order LG beam by the SLM. The resulting beam is passed through a mode-matching telescope into the linear cavity. The transmitted light is used to generate an error signal which is fed back to the Piezo-electric transducer attached to the curved end mirror to control the length of the cavity. The transmitted beam is simultaneously imaged on the CCD camera.

from the photodiode is mixed down with the 15 MHz signal to generate the PDH error signal, which is then fed back to a Piezo-electric transducer attached to the mode cleaner end mirror, via a servo and high-voltage amplifier. In this way, the length of the mode cleaner cavity can be controlled to maintain the resonance condition for a given mode order. In typical implementations of mode cleaners in gravitational wave interferometers, the error signal is taken in reflection. For this work, however, the mode cleaner cavity was of a low enough finesse to allow the error signal to be taken in transmission.

The PDH error signal for a sinusoidal LG_{33} input beam is shown in Fig. 3. This signal was recorded from the output of the mixer while scanning over the LG_{33} resonance of the linear mode cleaner. The error signal is equivalent to that generated when the input beam is a LG_{00} beam, confirming the result in [10], and thus allowed a robust feedback control of the cavity length. This is a significant result, as the PDH control loop method is a fundamental technique in the operation of gravitational wave interferometers.

The CCD camera was used to record intensity images of the transmitted beams while the mode cleaner was controlled to be resonant for the LG_{33} mode. Figure 4 shows the input and output beam intensity patterns for both helical and sinusoidal LG_{33} beams. The images indicate that the output modes are more symmetrical, and have a higher intensity in the innermost bright radial fringe relative to the others; a feature that is characteristic of LG_{33} modes. The typical method for measuring the output mode purity would be to pass the output beam through another cavity and observe the magnitudes of different mode order resonances [28]. This method in its original form is not ideal for the work described in this paper however, since in this case the performance of the mode in a cavity is itself being investigated. Instead, we have been able to estimate the mode content based on the intensity pattern alone using numerical simulations. The light transmitted through the cavity can be described well using eigenmodes of said

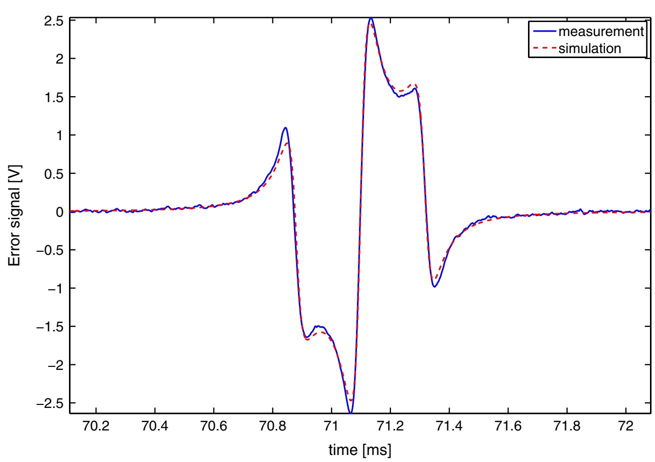

FIG. 3 (color online). The blue trace shows the PDH error signal from the linear cavity, set up as shown in Fig. 2, with a sinusoidal LG_{33} input beam. The red dashed trace shows the PDH error signal for the same optical setup as simulated in the frequency domain simulation software FINESSE [29]. While there are small discrepancies between the two traces, the primary features are identical, as predicted in [10].

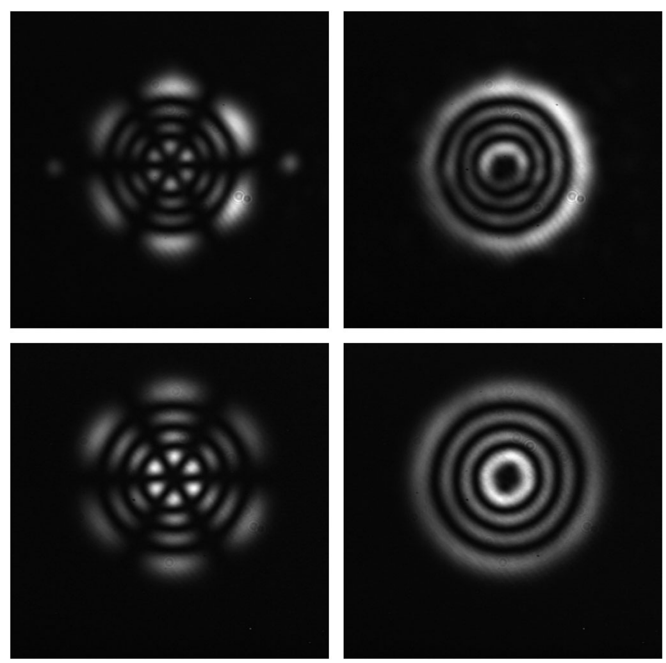

FIG. 4. The measured intensity patterns of the sinusoidal (left column) and helical (right column) LG_{33} beams before (upper row) and after (lower row) transmission through the linear mode cleaner. The increase in mode purity upon transmission is already evident in the increased symmetry. The remaining asymmetry apparently is a result of the inaccuracy in the manual alignment of the input beam to the mode cleaner. This effect is the same for both images but more visually apparent in the case of the helical mode.

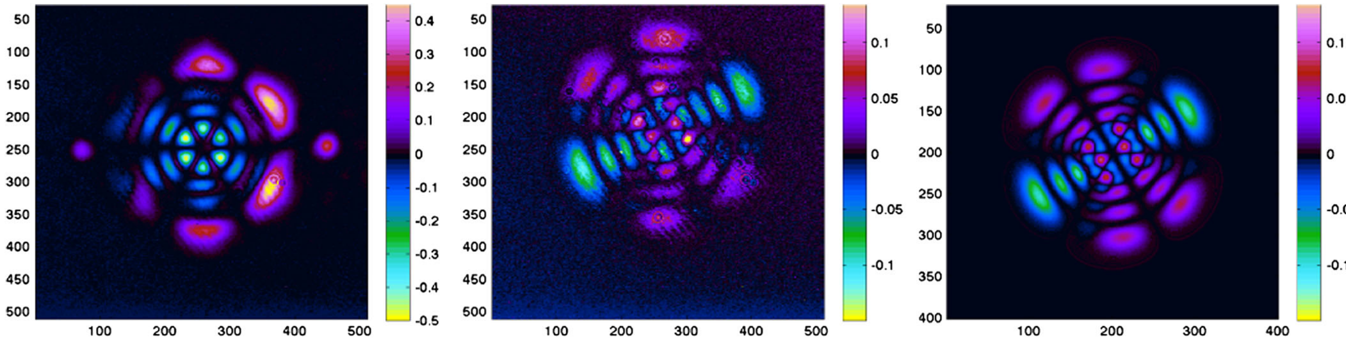

FIG. 5 (color online). Residuals from best fits between intensity patterns and a theoretically ideal sinusoidal LG_{33} intensity pattern. From left to right: the residual for the measured input LG_{33} beam, the residual for the measured output LG_{33} beam, the residual for an output LG_{33} pattern generated with a numerical model including a misalignment of the input beam to the cavity.

cavity. We were able to construct a numerical model representing the measured intensity pattern as a sum of LG eigenmodes using the following steps. We initially performed nonlinear fits of the LG_{33} eigenmode to the measured intensity pattern. The model function was a pure LG_{33} beam and the beam center as well as the beam size have been adjusted by the fitting algorithm. This step effectively produces a calibration of the position and pixel size of the CCD sensor. Once the beam position and size in the coordinate system of the sensor are known, the LG_{33} model can be subtracted from the measured data. Figure 5 shows the residuals of this step: The left panel shows the residual between the input sinusoidal LG_{33} beam intensity pattern shown in Fig. 4 and a theoretical LG_{33} mode. The central panel shows the equivalent residual for the output sinusoidal LG_{33} beam intensity pattern. It can be seen that the scale of the residuals is less for the output LG_{33} beam than for the input beam. The residual of the transmitted beam also indicates that the mode deformation is dominated by a misalignment of the injected beam into the mode cleaner. Using the interferometer simulation FINESSE [29] a model of the setup was used to search the alignment parameter space. It was possible to create a beam pattern like the measured pattern when the model included an input beam misaligned by $\alpha_x = -100\ \mu$rad in the horizontal plane, and $\alpha_y = 60\ \mu$rad in the vertical plane. The residual pattern between the intensity pattern calculated with the FINESSE simulation, and an ideal LG_{33} mode is shown in the right-hand panel of Fig. 5. Based on the FINESSE result we were able to produce a very good numerical model of the transmitted field amplitude and estimate the mode content by separately evaluating the overlap integrals between the complex field amplitude of the model and the field amplitudes of all LG eigenmodes. The results for the sinusoidal beam are shown in Table II. Our model predicts that 99% of the light power is in the LG_{33} mode and most of the remaining light power is distributed in other modes of the order 9. A similar analysis for the helical mode gave effectively the same results for the mode purity.

TABLE II. Mode decomposition of the numerical model of the sinusoidal LG_{33} beam transmitted through the linear mode cleaner, under an input beam misalignment of $-100\ \mu$rad in the horizontal axis, and 60 μrad in the vertical axis. The majority of the beam power is in the desired sinusoidal LG_{33} mode, with the rest almost entirely concentrated in other modes of order 9.

$u_{lp}^{\sin}$ mode	3, 3	4, −1	2, −5	4, 1	2, 5	other
Power	99%	0.4%	0.3%	0.1%	0.1%	<10 ppm

Since the transmitted beam was to 99% in a single mode we were able to make an accurate estimate of the input mode purity by comparing the throughput of the LG_{33} modes to that of the LG_{00} mode. Once the intrinsic optical losses of the mode cleaner cavity were taken into account, we estimated the input mode purity to be 51% for the sinusoidal LG_{33} beam, and 66% for the helical LG_{33} beam. It should be noted that examples of higher-order LG modes with mode purities likely to be above 70% have been created previously directly with SLMs using a more thoroughly optimized conversion procedure, for example, in [19], although in this case the authors refrain from quoting an experimentally measured purity. However, this is the first time a purity improvement of a Laguerre-Gauss mode using an optical resonator to an estimated 99% has been reported in the scientific literature. The demonstrated mode purity is limited in first order by the manual alignment of the input beam and can very likely be improved using a standard automatic alignment system.

B. Helical LG mode performance in a triangular mode cleaner

In order to test the effect described in Sec. II, whereby we expected helical LG beams to be decomposed into their constituent sinusoidal LG modes upon interaction with a triangular mode cleaner, we placed a cavity of the standard triangular pre-mode cleaner design [30] after the linear mode cleaner as shown in Fig. 6. The triangular mode cleaner was scanned with the sinusoidal LG_{33} beam input

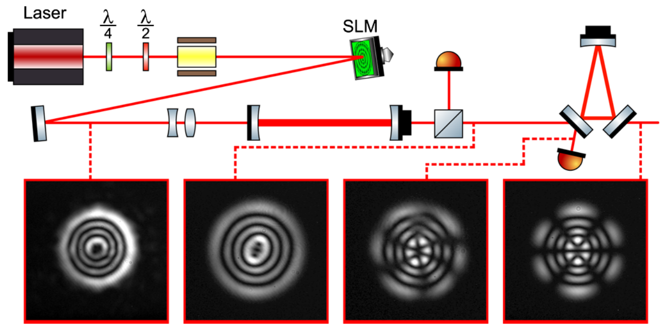

FIG. 6 (color online). The experimental setup for transmitting a helical LG_{33} mode through a triangular mode cleaner, showing the intensity pattern of the beams at various locations in the setup (note that the images shown here are contrast enhanced to show the pattern more clearly). From left to right: helical LG_{33} after the SLM, helical LG_{33} after transmission through the linear mode cleaner, beam reflected from the triangular cavity, and beam transmitted through the triangular cavity.

and the helical LG_{33} beam input successively. As expected, extra resonances at half a free-spectral range were observed when the input was changed from sinusoidal LG_{33} to helical LG_{33}. The triangular mode cleaner was then feedback controlled in similar fashion to the linear mode cleaner, with the PDH error signal being this time obtained from the light reflected from the cavity input mirror. Figure 6 shows images of the input, transmitted and reflected beams at one of these resonances (the beam after the linear cavity was of slightly lower quality than that shown in Fig. 4 as less time was spent on the alignment optimization for this experiment). We observed that the beam transmitted through the triangular cavity was nearly a vertically symmetric sinusoidal LG_{33} mode. The stronger vertical central section compared to the one shown in Fig. 4 is caused by astigmatism due to the curved end mirror [22]. The reflected beam was a superposition of all the modes rejected by the mode cleaner, and was therefore of lower mode purity than the transmitted mode. However, the vertically antisymmetric LG_{33} mode can be seen to be the dominant mode present in the reflected light. The measurement was repeated for the alternative resonance point, where as expected the dominant type of the transmitted and reflected mode was reversed. The helical input beam is decomposed into the constituent sinusoidal modes upon interaction with the triangular mode cleaner, as predicted. We therefore conclude that in order for helical LG_{33} modes to be compatible with gravitational wave interferometers, the mode cleaners used must be linear, or at least be comprised of an even number of mirrors. If helical Laguerre-Gauss beams are to be implemented in second generation detector upgrades or third generation detectors, this result shows that at the very least a redesign of the mode cleaners from the first and second generation detectors will be necessary, since these are currently triangular cavities. As mode cleaners are present in gravitational wave interferometers in several places, this constitutes a significant consideration for the overall optical design of the detectors for which LG mode technology is considered.

IV. CONCLUSION

Research into the interferometric performance of LG modes is important for the gravitational wave community, as LG modes offer a thermal noise advantage over the fundamental LG_{00} mode that can improve the sensitivity achievable by future detectors [9]. Simulations have already shown promising results for the interferometric performance of LG modes [10]. This article provides experimental verification for some of these predictions: We have demonstrated the generation of a PDH error signal from a linear mode cleaner injected with both helical and sinusoidal LG_{33} modes equivalent to the error signal obtained with a LG_{00} mode. We used this error signal to successfully demonstrate longitudinal control of the linear mode cleaner cavity at resonance for the LG_{33} mode; a vital technique for the operation of gravitational wave interferometers with LG_{33} modes. We also showed an increase in the purity of a sinusoidal LG_{33} mode from 51% to 99% upon transmission through a linear mode cleaner, demonstrating that very high-purity LG_{33} mode light sources can be produced in this way. Furthermore, we have demonstrated the decomposition of a helical LG_{33} mode into the constituent sinusoidal LG_{33} modes with a triangular mode cleaner; a result which has a strong impact on the choice of the optical design of future detectors.

The prospects for LG modes in gravitational wave detectors remain intact following the investigation described in this paper. In the future, we will expand this work to use LG modes in systems that combine Michelson interferometers and resonant cavities, and also include alignment control systems. The requirements for mirror surfaces for high-finesse systems with LG modes also needs further investigation to move scrutiny of the interferometric performance of LG modes to the next level, towards a possible implementation in future gravitational wave detectors.

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[1] G. M. Harry (The LIGO Scientific Collaboration), Classical Quantum Gravity **27**, 084006 (2010).
[2] The Virgo Collaboration, Tech. Rep. VIR-027A-09, Virgo, 2009.
[3] S. Rowan, J. Hough, and D. Crooks, Phys. Lett. A **347**, 25 (2005).
[4] J.-Y. Vinet, Classical Quantum Gravity **24**, 3897 (2007).
[5] The LIGO Scientific Collaboration, Tech. Rep. LIGO-T0900276-v4, LSC, 2009.
[6] M. Punturo *et al.*, Classical Quantum Gravity **27**, 084007 (2010).
[7] E. D'Ambrosio, Phys. Rev. D **67**, 102004 (2003).
[8] M. Bondarescu, O. Kogan, and Y. Chen, Phys. Rev. D **78**, 082002 (2008).
[9] B. Mours, E. Tournefier, and J.-Y. Vinet, Classical Quantum Gravity **23**, 5777 (2006).
[10] S. Chelkowski, S. Hild, and A. Freise, Phys. Rev. D **79**, 122002 (2009).
[11] J.-Y. Vinet, Living Rev. Relativity **12**, 1 (2009), http://www.livingreviews.org/lrr-2009-5.
[12] S. A. Webster, M. Oxborrow, S. Pugla, J. Millo, and P. Gill, Phys. Rev. A **77**, 033847 (2008).
[13] A. Siegman, *Lasers* (University Science Books, Sausalito, California, 1986).
[14] G. A. Turnbull, D. A. Robertson, G. M. Smith, L. Allen, and M. J. Padgett, Opt. Commun. **127**, 183 (1996).
[15] J. Courtial and M. J. Padgett, Opt. Commun. **159**, 13 (1999).
[16] S. A. Kennedy, M. J. Szabo, H. Teslow, J. Z. Porterfield, and E. R. Abraham, Phys. Rev. A **66**, 043801 (2002).
[17] A. Freise and K. Strain, Living Rev. Relativity **13**, 1 (2010), http://relativity.livingreviews.org/Articles/lrr-2010-1/.
[18] J. Arlt, K. Dholakia, L. Allen, and M. J. Padgett, J. Mod. Opt. **45**, 1231 (1998).
[19] N. Matsumoto *et al.*, J. Opt. Soc. Am. A **25**, 1642 (2008).
[20] A. Rüdiger *et al.*, Opt. Acta **28**, 641 (1981).
[21] B. Abbott *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **517**, 154 (2004).
[22] P. Fulda *et al.*, (unpublished).
[23] T. Skettrup, J. Opt. A **7**, 645 (2005).
[24] S. Goßler *et al.*, Rev. Sci. Instrum. **74**, 3787 (2003).
[25] A. Genin, J. Marque, B. Swinkels, and G. Vajente, Tech. Rep. VIR-0232A-10, Virgo, 2010.
[26] Advanced LIGO Team, Tech. Rep. LIGO-M060056-v1, LSC, 2009.
[27] E. D. Black, Am. J. Phys. **69**, 79 (2001).
[28] P. Kwee, F. Seifert, B. Willke, and K. Danzmann, Rev. Sci. Instrum. **78**, 073103 (2007).
[29] A. Freise, G. Heinzel *et al.*, Classical Quantum Gravity **21**, S1067 (2004); software available at: http://www.gwoptics.org/finesse.
[30] B. Willke *et al.*, Opt. Lett. **23**, 1704 (1998).